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THE DEPTH OF THE SOLAR CONVECTION ZONE INFERRED FROM HYDRODYNAMICAL MODELS OF THE SURFACE LAYERS

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HYDRODYNAMICAL MODELS OF SOLAR SURFACE CONVECTION

We have obtained detailed 2-dimensional models of the surface layers of solar-type stars from extensive numerical simulations solving the time-dependent, non-linear equations of hydrodynamics for a stratified compressible fluid. The calculations take into account a realistic equation of state (including ionization of H and He as well as formation of H_2-molecules) and use an elaborate scheme to describe multi-dimensional, non-local, frequency-dependent radiative transfer. The hydrodynamical models include the photosphere as well as part of the convective subphotospheric layers, with an open lower boundary located about one pressure scale height ($\approx$ 320 km) below $\tau_{\rm Ross} = 1$, allowing a free flow of gas out of and into the model. A fixed specific entropy, s^*, is (asymptotically) assigned to the gas entering the simulation volume from below, which uniquely determines the effective temperature of the hydrodynamical model. For details about the physical assumptions, numerical method and characteristics of the resulting convective flow at the solar surface see Steffen (1991); Steffen and Freytag (1991).

The results presented in the next section are taken from one of our most recent hydrodynamical model calculations of convection at the solar surface, based on 40 (vertical) by 140 (horizontal) grid points, assuming 2-dimensional Cartesian geometry with periodic lateral boundaries enforcing a repetition of the flow pattern every 5250 km (comfortably larger than the observed maximum size of solar granules, $\approx$ 3000 km). The sequence spans a time interval of 8400 sec, enough to yield statistically stable results.

The purpose of this paper is to demonstrate that under certain assumptions concerning the topology of solar-type convection zones, our hydrodynamical models of stellar *surface convection* provide direct information about the entropy of the deep, adiabatic layers of the convection zone. In combination with standard stellar envelope models based on mixing length theory (MLT), this immediately determines the location of the *base of the convection zone*.

FROM THE SURFACE TO THE BASE OF THE CONVECTION ZONE

Fig. 1a shows the *mean* entropy as a function of depth obtained from the hydrodynamical solar surface model mentioned above by averaging over horizontal planes and over time. Clearly, this model does not extend deep enough to include those layers where the mean stratification of the convection zone is expected to

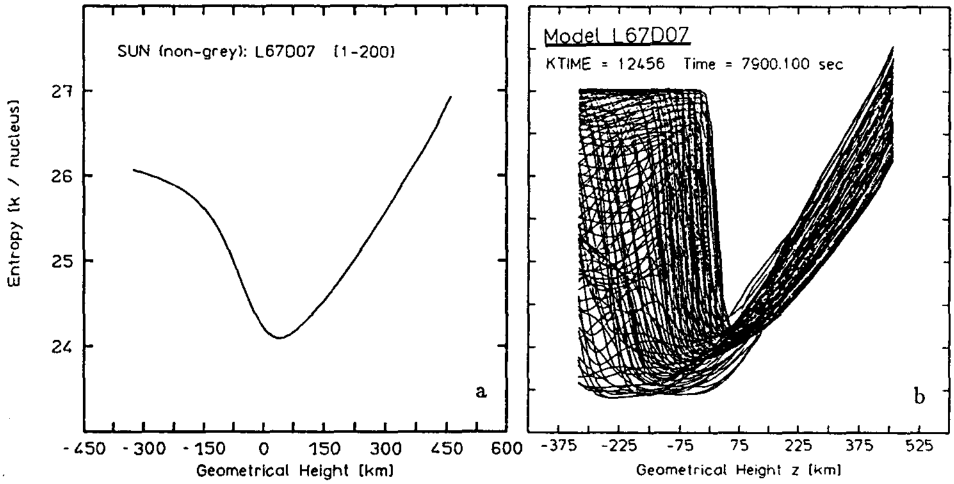

FIGURE I Depth dedendence of entropy in the solar surface layers as obtained from hydrodynamical calculations. The mean entropy (horizontal and temporal average) is shown at left (a), the spatially resolved entropy profiles after a simulated time of 7900 sec are displayed at right (b). Geometrical height zero corresponds to $\tau_{\mathrm{Ross}}=1$.

become adiabatic, more than 2000 km (5 to 6 pressure scale heights) below $\tau_{\mathrm{Ross}}=1$. While the mean entropy stratification of the hydrodynamical model does not permit a determination of the entropy corresponding to the adiabat of the deep convection zone, the *spatially resolved* entropy profiles, displayed in Fig. 1b for an arbitrary instant of the same sequence, contain additional information. We note a remarkable entropy plateau in the subsurface layers, indicating that (in contrast to the narrow downdrafts) the gas in the central regions of broad ascending flows is still thermally isolated from its surroundings in the deepest parts of the simulation volume due to the large optical depths until it reaches the layers around $\tau_{\mathrm{Ross}}=1$ where radiative losses become important.

The height of the entropy plateau is essentially independent of time and corresponds to s^*, the entropy specified with the lower boundary condition to adjust the effective temperature of the hydrodynamical model (see above). We found that the value of s^* required by the simulations to transport the solar surface flux is largely independent of the treatment of radiative transfer (grey / frequency-dependent), the horizontal and vertical extent of the computational domain, and of numerical details of the lower boundary condition. From a variety of different simulations we find for the solar case:
$s^* = 1.730 \pm 0.015 \cdot 10^9$ [erg/g/K] or 27.08 ± 0.25 [Boltzmann units per nucleus], where all results obtained so far lie within the indicated error bars.

Now we suggest that s^* may be identified directly with the entropy of the deep, adiabatic convective layers. This idea is based on the qualitative picture of solar-type convection zones proposed by Stein and Nordlund (1989) (see also Spruit et al., 1990; Chan et al., 1991), which is fundamentally different from MLT assumptions. According to this scenario the downdrafts continue all the way from the surface to the bottom of the convection zone, merging to fewer and stronger currents in successively deeper layers. The flow closes only near the base of the convective envelope. Most of the gas elements beginning to ascend from there overturn into neighbouring downflows before reaching the surface. Only a small fraction of gas continues to the surface, reaching the layers corresponding to the location of the lower boundary of our hydrodynamical models without entropy losses, following an adiabat almost up to the visible surface. Hence, s^* obtained from the simulations indicates the entropy of warm, ascending gas throughout the convection zone. This, in turn, is very nearly equal to the *mean* (horizontally averaged) entropy near the base of the convection zone because (i) the downflows are markedly entropy-deficient only near the surface and become continuously diluted by overturning entropy-neutral gas as they reach greater depths, and (ii) the fractional area of the downdrafts decreases with depth.

We have tested this idea by comparison with the results from a 3-dimensional simulation of solar convection by Nordlund and Stein (1991), extending much deeper than our models. Near the bottom of their model (2500 km below $\tau_{\rm Ross}=1$) the *mean* stratification becomes essentially adiabatic, approaching a limiting entropy which indeed closely agrees with the value of s^* derived from our convection models, indicating the validity of the above assumptions.

For comparsion with standard MLT we have calculated solar *envelope* models (not subject to central boundary conditions) including the entire solar convection zone for different values of the mixing length parameter $\alpha = l/H_p$, using the well-known Kippenhahn code in the version described by Schönberner (1983). For the outer layers ($T < 11600$ K) opacities were taken from Cox and Stewart (1970), while in the deeper layers ($T > 16000$ K) we used Los Alamos opacities from Huebner et al. (1977), with a smooth transition between the two regimes. We adopted a chemical composition as similar as possible to that used in the hydrodynamical model: $X=0.7035$, $Y=0.2795$, $Z=0.017$ (by mass). The choice of α uniquely determines the resulting entropy of the deep convective layers, s_0. The smaller α, the larger s_0. The mixing length model approaches the same adiabat as indicated by the hydrodynamical simulations ($s_0=s^*$) if we choose $\alpha = 1.26 \pm 0.05$. The corresponding depth of the convection zone is $D=199000 \pm 8000$ km ($2.05 \cdot 10^6 < T_0 < 2.30 \cdot 10^6$ K). This result is in perfect agreement with helioseismology, which according to Christensen-Dalsgaard et al. (1991) constrains the depth of the solar convection zone to $D=200000 \pm 2000$ km ($2.17 \cdot 10^6 < T_0 < 2.23 \cdot 10^6$ K).

CONCLUDING REMARKS

The results presented above should not be interpreted as a unique calibration of α. We have only demonstrated that a solar mixing length model with $\alpha \approx 1.3$ gives the "right" entropy jump from the surface to the deep convection zone, which is consistent with both the results from hydrodynamical surface convection models and helioseismology. This does not imply that the superadibatic surface layers are also represented correctly by such a mixing length model. A detaild comparison with our hydrodynamical models shows that for no possible choice of α is standard MLT appropriate to quantitatively describe the thermal structure and dynamics of the surface layers of solar-type stars.

Finally, we would like to point out that one has to be very careful when intercomparing α-values of different authors. Unfortunately, the way α enters the formulation of MLT may be different in different codes, so that the numerical value of α required to desrcibe the same physical model may actually differ considerably from one code to another (up to a factor of 2). In addition, the numerical value of α is quite sensitive to the T-τ relation adopted for the atmosphere. Had we used a more realistic atmosphere instead of a grey T-τ relation, we probably would have needed $\alpha \approx 1.5$ to fit the same adiabat.

The resulting depth of the convection zone is sensitive to the choice of opacities. Using Cox and Stewart (1970) opacities through the whole depth range leads to D=188000 ± 8000 km. Moreover, it is clear that our method does not account for possible overshooting at the base of the convection zone.

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REFERENCES

Chan, K.L., Nordlund, Å., Steffen. M., Stein, R.F.: 1991, in *Solar Interior and Atmosphere*, eds. A.N. Cox, W.C. Livingston, M.S. Matthews (University of Arizona Press, Tucson), p. 262-270

Christensen-Dalsgaard, J., Gough, D.O., Thompson, M.J.: 1991, *ApJ* **378**, 413

Cox, A.N., Stewart, J.N.: 1970, *ApJS* **19**, 243

Huebner, W.F., Merts, A.L., Magee, N.H.Jr., Arog, M.F.: 1977, Los Alamos Scientific Laboratory Report No. LA-670-M

Nordlund, Å., Stein, R.F.: 1991, in *Stellar Atmospheres: Beyond Classical Models*, eds. L. Crivellari, I.Hubeny, D.G. Hummer, NATO ASI-Series C, Vol. **341**, p. 263

Schönberner, D.: 1983, *ApJ* **272**, 708

Spruit, H.C., Nordlund, Å., Title, A.M.: 1990, *Ann. Rev. Astron. Astrophys.* **28**, 263-301

Steffen, M.: 1991, in *Stellar Atmospheres: Beyond Classical Models*, eds. L. Crivellari, I. Hubeny, D.G. Hummer, NATO ASI-Series C, Vol. **341**, p. 247

Steffen, M., Freytag, B.: 1991, in *Reviews in Modern Astronomy*, ed. G. Klare, Springer, Heidelberg, Vol. **4**, p. 43

Stein, R.F., Nordlund, Å.: 1989, *ApJ* **342**, L95-L98