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STATE-DEPENDENT FORMATION OF INFLATION EXPECTATIONS

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The learning-from-experience model of Malmendier and Nagel [(2016) Quarterly Journal of Economics 131, 53–87] successfully reproduces the heterogeneity in inflation expectations across age groups. However, inflation expectations presumably depend on not only age and experience but also inflation regime. Therefore, this paper proposes an extended learning-from-experience model, in which expectation formation depends on inflation regime. Estimating the model with US data, I find that when inflation is higher and more volatile, households place more weight on recent data when making private forecasts. Moreover, in this regime, households rely more heavily on private forecasts than on public information.

Keywords: Inflation Expectations, Learning from Experience, State Dependence

1. INTRODUCTION

How households form inflation expectations has long been an important question in macroeconomics. In the literature, one of the most discussed inflation expectation features is *heterogeneity* across individual households. Malmendier and Nagel (2016) propose a model in which households form inflation expectations via idiosyncratic learning processes based on their private experiences.[1] Their model demonstrates that different age groups hold differing inflation expectations, as depicted in the left panel of Figure 1, since their lifetime experiences of inflation differ.[2] This model, in general, accurately reproduces the cross-sectional dispersion in inflation expectations between age groups, particularly for the 1970s and 1980s.

Another notable feature in expected inflation data is its *state dependence*. As revealed in the right panel of Figure 1, the co-movement of actual and expected inflation is stronger when inflation volatility is high. More specifically, the correlation coefficient between the two variables is 0.60 when the standard deviation of inflation is higher than a certain threshold and 0.34 when it is below this threshold. Given that the average inflation rate is also generally higher in this period,

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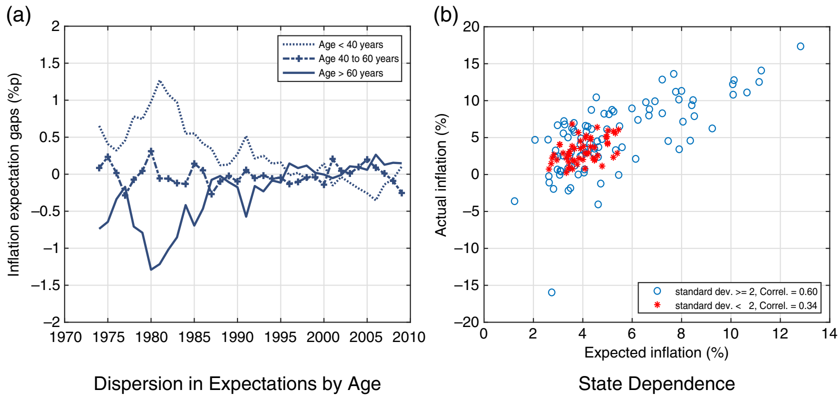

Notes: The left panel demonstrates the deviations of inflation expectations from average expectations across all age groups [Malmendier and Nagel (2016)]. Each point refers to the annual average value. In the right panel, actual inflation refers to the annualized quarterly increase rate of the consumer price index. The standard deviation of inflation is calculated by the eight-quarter rolling window. *Data Sources:* Michigan Consumer Survey, Robert Shiller's website.

FIGURE 1. Dispersion in expectations and state dependence.

it is likely that households form their expectations based on the state of inflation; they seem more affected by current actual inflation in forming expectations when inflation is higher and more volatile.

State dependence in expectations has been examined in previous studies; however, most have been conducted on an aggregate level rather than by examining individual expectations. Empirical studies on cross-sectional dispersion usually focus on periods with significant changes in inflation level and volatility. It is, therefore, crucial to also consider the inflation regime when studying the formation of individual households' expectations. By doing so, the properties of inflation expectations become clearer in terms of cross section and time series.

To investigate state dependence in conjunction with heterogeneity in expectations, this paper modifies the learning-from-experience model, so that its parameters are determined by inflation regimes. In the revised model, households form inflation expectations by combining two information sources: private forecasts and public information. To make private forecasts, like econometricians, they estimate their own simple inflation models, based on the perceived law of motion (PLM). Households consider only the inflation data experienced during their lifetimes to estimate the models; moreover, memories from older experiences tend to fade over time. At the same time, the extent to which they consider these older experiences also depends on the inflation regime. When inflation is high and more volatile, households pay more attention to its recent movement. Meanwhile, they care less about how inflation evolved in the past. To incorporate these properties of private forecasts into the model, households are assumed to have a pair of state-contingent PLMs, the coefficients of which

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are recursively updated with different degrees of learning from each period's new data. Households choose one or a combination of the two PLMs to make private inflation forecasts in relation to the inflation regime.

After households determine their private forecasts, they form inflation expectations by considering public information in addition to the private forecasts. The allocation between private forecasts and public information is determined by households' perceptions of the inflation regime. In this study, households are assumed to be exogenously endowed with information about the probability of which inflation regime prevails during each period.

For model estimation, a two-stage procedure is implemented: to begin, the Markov regime-switching assumption for actual inflation identifies the inflation regimes, a step that, according to the literature, has been adopted extensively by researchers.[3] In the next stage, by employing the state probability information derived in the first stage, the state-dependent model of inflation expectations is estimated using US household survey data. To ensure the robustness of the estimation, the results are analyzed to determine how they change under different identifications of inflation regime and under a different sample period.

The estimation results reveal several interesting findings. First, when inflation is higher and more volatile, households place substantially more weight on recent data than on data from the past when making private forecasts. Second, in the high-inflation regime, households pay more attention to their private forecasts than to other public information when forming inflation expectations. This, in conjunction with cross-sectional properties, suggests that the degree of heterogeneity in inflation expectations expands as households place more weight on their private forecasts. In contrast, dispersion diminishes as each household age group relies more on public information. Third, public information might be more associated with households' common beliefs attained through social learning rather than with exogenous information sources such as professional forecasters' opinions. This finding differs from that of Malmendier and Nagel (2016), who note that both are equivalently related to public information in the model. Finally, the model improvement by adopting state dependence results mainly from a period of stable inflation. This indicates that the parameters that characterize learning from experience and the allocation of private information can be overestimated for such a period when one focuses exclusively on the average values of the parameters. In other words, the role of public information in households' inflation expectations could be underestimated in a stable inflation regime. These findings emphasize the importance of state dependence in the model, because it is not observed in the state-independent framework. Indeed, state dependence is essential for understanding households' inflation expectations at the individual and cohort levels.

This paper builds primarily on Nakov and Nuño (2015), Madeira and Zafar (2015), and Malmendier and Nagel (2016), who study households' heterogeneous expectations and financial decisions under bounded rationality. Dispersion in expectations comprises a key area of research regarding effective monetary policy

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[Mankiw et al. (2004), Andrade et al. (2016)]. Such studies focus specifically on age differences and heterogeneity across experiences as major causes of disagreement in expectations. In terms of state dependence in expectations, this paper follows the theoretical approaches of Marcet and Nicolini (2003), Milani (2014), and Adam et al. (2016), whose studies consider a state-dependent learning gain in agents' PLM during structural breaks in the economy. However, their state-dependent learning gain in an aggregate-level differs from the individual-based and state-contingent PLM employed in this paper. Furthermore, this paper relates to studies by Morris and Shin (2002), Carrillo and Emran (2012), Areosa (2015), and Coibion et al. (2018) examining the role of public information and the allocation of information in setting prices. Following such ideas, the present paper investigates the relationship between degree of heterogeneity and information allocation strategy in the context of state dependence.

The remainder of this paper is organized as follows. Section 2 describes the learning-from-experience model with state dependence and explains how to identify inflation regimes. Section 3 discusses the estimation results, as well as providing sensitivity analyses to the identification of inflation regimes and the different sample period. Moreover, this section demonstrates how state dependence improves the model using a transition matrix. Section 4 concludes.

2. MODEL

This section first describes the state-dependent formation of inflation expectations as an extended version of the learning-from-experience model. Then, it discusses how to identify inflation regimes in which households behave asymmetrically.

2.1. State-dependent Inflation Expectations

This paper assumes that households possess bounded rationality and that they form expectations by following adaptive learning. Households are assumed to estimate the parameters in their PLMs as would econometricians. Based on many other studies, an individual's PLM is assumed to follow the AR(1) process, as expressed by:

$$\pi_{t+1} = \mu^p_{t,r,s} + \rho^p_{t,r,s}\pi_t + \sigma^p_{\eta,s}\eta_{t+1}, \tag{1}$$

where π_t is inflation at t, $\rho^p_{t,r,s} \in (-1, 1)$ is the autocorrelation coefficient, $\mu^p_{t,r,s} \in \mathbb{R}$ is the constant, and $\eta_{t+1} \sim iidN(0, 1)$ is the inflation innovation at $t + 1$.[4] The subscripts t and r denote time and time of birth, respectively. The subscript $s \in \{1, 2\}$ denotes two possible inflation regimes in which households could stand. The coefficients are recursively updated by learning from experience on an individual basis, following

$$b_{t,r,s} = b_{t-1,r,s} + \gamma_{t,r,s}R^{-1}_{t,r,s}x_{t-1}(\pi_t - b'_{t-1,r,s}x_{t-1}), \tag{2}$$

$$R_{t,r,s} = R_{t-1,r,s} + \gamma_{t,r,s}(x_{t-1}x'_{t-1} - R_{t-1,r,s}), \tag{3}$$

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where $b_{t,r,s} \equiv [\mu^p_{t,r,s}, \rho^p_{t,r,s}]'$, $x_t \equiv [1, \pi_t]'$, and $R_{t,r,s}$ denotes the moment matrix for x_t.[5] The parameter $\gamma_{t,r,s}$ denotes a learning gain, which determines the degree of updating that cohort r applies when faced with an unexpected change in inflation.[6]

This model differs from an ordinary adaptive learning model in that its parameters depend on agents' age and inflation regime. Households use different sets of information based on their private experiences to estimate (1). For example, the 30-year-old cohort uses the past 30 years of data to estimate their PLM, while the 60-year-old cohort uses the past 60 years of data. This constitutes learning from experience.

In addition, households can change the type of learning algorithm, since they are affected by economic circumstances. This represents state dependence. State-dependent learning in inflation expectations is often considered in the context of hyperinflation or during structural breaks in inflation [Marcet and Nicolini (2003), Milani (2014)]. Since ordinary least squares (OLS) does not generate useful forecasts, due to its slow adaption to rapidly changing inflation levels, considering the recent movement of inflation is preferable in the high-inflation regime.[7] During stable periods, in contrast, households prefer to use more information from the past, because a larger value of learning gain produces noisier forecasts [Evans and Honkapohja (1993)].[8]

To combine the two aforementioned features, in (2) and (3), the learning gain $\gamma_{t,r,s}$ is assumed to be age-specific and state-dependent, as follows:

$$\gamma_{t,r,s} = \begin{cases} \dfrac{\theta_s}{t-r} & \text{if } t-r \geq \theta_s \\ 1 & \text{if } t-r < \theta_s \end{cases}, \tag{4}$$

where the condition $\gamma_{t,r,s} = 1$ $(t - r < \theta_s)$ indicates that data before an individual's birth year is ignored. The parameter $\theta_s > 0$ has a constant value within a certain inflation regime and determines the shape of the implied function of weight on past inflation data. If $\theta_s = 1$, all observations since birth are treated equally, as in an OLS estimation [Evans and Honkapojha (2001)].[9] However, in this paper, each observation is valued depending on its distance from the current period, because households' earlier memories and experiences tend to fade, and thus, they emphasize the recent movement of inflation. Therefore, parameter θ_s in (4) must be greater than unity, so that the downward slope of the weight function can be incorporated into the PLM estimation. With this assumption, a younger cohort, which has a smaller dataset, is more strongly influenced by recent data. This decreasing gain with $\theta_s > 1$ resembles constant gain learning, which many studies employ [Milani (2007), Adam et al. (2016), Berardi and Galimberti (2017)]: both learning gains generate downward slopes of the weight functions. The primary difference between the two learning gains is that the decreasing gain with $\theta_s > 1$ enables the formulation of weight functions with different slopes for different cohort groups.

For state dependence, the weighting factor θ_s assumes two values, θ_1 and θ_2. The parameter θ_1 in the high regime is larger than θ_2, and the implied weight

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function is steeper, as a result of heightened concerns about the current movement of inflation. More specifically, households are assumed to have two learning equations for the PLM, corresponding to inflation regimes. Each PLM considers different sizes of θ_s and, thus, different weights on past data. Households update the PLMs based on corresponding learning gains for each period, following (2) and (3), and they use the equations to make forecasts for the next period. Lastly, they select one or a combination of the forecasts, given a weight in each period, to form their final private forecasts.[10] Accordingly, it is assumed that households are informed as to which inflation regime they are more likely to occupy.[11] This assumption can be summarized by:

$$E_t^h \pi_{t+1|r} = \xi_t E_t^{h,1} \pi_{t+1|r} + (1 - \xi_t) E_t^{h,2} \pi_{t+1|r}, \tag{5}$$

where $E_t^{h,s} \pi_{t+1|r} = b'_{t,r,s} x_t$, which represents a one-period ahead forecast based on a household's PLM. The parameter ξ_t is a weight on the forecast made by the first PLM, which employs the higher learning gain. The superscript h stands for households' private forecasts.

The cohort effect and state dependence are reflected similarly in learning gains and weights on data for $E_t^h \pi_{t+1|r}$. However, they are distinct in several regards: first, in the learning gain $\frac{\theta_s}{t-r}$, the cohort effect is represented by the denominator $t - r$, and the state dependence appears in the numerator θ_s. As a result, even if the learning gains are identical, they can imply different learning strategies depending on inflation regimes and personal experiences. Second, the cohort effect is deterministic given a specific value θ_s, while state dependence captures changes in θ_s in each period. Third, the cohort effect is defined as the heterogeneity resulting from the difference in personal experiences, which is represented by the length of data. In contrast, state dependence is reflected in the slopes of the weight functions given a certain length of data.[12] Therefore, the state dependence assumption is essential to incorporate the effect in which all age groups become more concerned about recent inflation at the same time given a specific length of data for each age group.

In this paper, the regime-switching concept differs from what is suggested by other studies [see Marcet and Nicolini (2003), Branch and Evans (2007); Milani (2014)], which assume that the learning gain in each period is time varying within one PLM equation. The learning gains in such models are endogenously determined depending on agents' perceptions of the economic situation.[13] The differences between this paper and previous studies evolve from two assumptions: (1) the learning mechanism in this paper is heterogeneous in terms of an agent's age and experiences, and (2) the information about inflation regime is endowed exogenously, such that households adopt a state-contingent learning strategy for each inflation regime. Moreover, the concept of state-dependent expectations in this paper differs from those presented in the works of Maćkowiak and Wiederholt (2009) or Coibion and Gorodnichenko (2015), who relate state dependence to the degree of imperfect information. They explain that information rigidity is

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low in the high-inflation regime, because people allocate more resources to track macro-level shocks.

After households obtain inflation forecasts based on their private experiences, they form inflation expectations by adding the private forecasts and public information, as follows:

$$E_t\pi_{t+1|r} = \beta_s E_t^h \pi_{t+1|r} + \delta' D_t + \sigma_{\varepsilon,s}\varepsilon_{t,r}, \quad \textbf{(6)}$$

where $E_t^h \pi_{t+1|r}$ denotes households' private forecasts, derived by (5), and $\varepsilon_{t,r} \sim iidN(0, 1)$ is the noise in expectations.[14] The time dummies D_t refer to unobserved macro-factors, which affect every agent simultaneously. These are assumed to be public information, such as households' common beliefs, professional forecasters' opinions, or central bank announcements.[15] Hence, the sizes of β_s and δ represent the relative importance of idiosyncratic private and public information in forming expectations, respectively. A higher β_s implies a higher degree of agents' strategic substitutability in forming inflation expectations, such that households intend to emphasize their idiosyncratic forecasts, whose coefficient is β_s. On the other hand, a lower β_s reflects a higher degree of strategic complementarity, because it is optimal for households to value public information when the actions of different agents are strategically complementary [Morris and Shin (2002), Carrillo and Emran (2012), Coibion et al. (2018)].[16]

In line with the parameter θ_s in the learning-from-experience component, the parameters β and σ_ε, which denote the relative weight of private information and the standard deviation of noise in expectations, respectively, are also assumed to perform state-dependent switching between two regimes:

$$\beta_s = \xi_t\beta_1 + (1 - \xi_t)\beta_2, \quad \textbf{(7)}$$

$$\sigma_{\varepsilon,s} = \xi_t\sigma_{\varepsilon,1} + (1 - \xi_t)\sigma_{\varepsilon,2}. \quad \textbf{(8)}$$

Equation (7) implies that households' allocation of information sources to form inflation expectations is determined by inflation regimes. If $\beta_1 > \beta_2$, private information exerts more influence in the high regime than in the low regime, and vice versa. Private and public information are only cumulative under the state-independent framework. However, assuming state dependence and a time-varying β_s can provide insight into the substitution properties between each type of information for forming expectations. For (8), noisier expectations are produced in the high regime with $\sigma_{\varepsilon,1} > \sigma_{\varepsilon,2}$.

2.2. Identification of Inflation Regime

The next stage adds the weight ξ_t, which is related to how households perceive the state of inflation. In studies on adaptive learning with regime switching, most models assume that an endogenous mechanism exists, for a representative household to identify economic regimes. However, an analysis that uses an

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individual-level model cannot incorporate the endogenous mechanism to individual expectations, for practical reasons regarding the computation burden.[17] Therefore, inflation regime information is considered exogenously endowed by public sectors, such as governments and central banks.[18]

To quantify the inflation regime, this paper assumes the weight ξ_t as the relative probability that households are in Regime 1, which is obtained from an actual inflation model. In line with literature that considers the regime-switching inflation model [e.g. Evans and Honkapohja (1993); Amisano and Fagan (2013)], this paper assumes that inflation follows AR(1) in a manner identical to the PLM, with state dependence characterized by two-state Markov regime switching, as follows:

$$\pi_t = \mu_s + \rho_s \pi_{t-1} + \sigma_{u,s} u_t, \tag{9}$$

where μ_s is the constant, ρ_s is the autocorrelation coefficient, and $u_t \sim iidN(0, 1)$ is the inflation innovation. The coefficients and standard deviation switch between two values over time, as the inflation regime changes:

$$\phi_s = \phi_1 S_t + \phi_2 (1 - S_t), \tag{10}$$

where $\phi_s = [\mu_s, \rho_s, \sigma_{u,s}]$ and $S_t = 1$ in Regime 1, and $S_t = 0$ in Regime 2. Thus, when the economy is in Regime 1, μ_s takes μ_1; it takes μ_2 in Regime 2. This is the same for ρ_s and $\sigma_{u,s}$. The inflation regime S_t follows a Markov switching process. In general, S_t depends on S_{t-1}, S_{t-2}, $\cdots$, S_{t-r}, in which case, the process of S_t is named as an r-th order Markov switching process. This paper assumes a two-state inflation regime that follows the first-order Markov switching. A transition probability of inflation regime between $t-1$ and t is assumed to be

$$\Pr[S_t = i | S_{t-1} = i] = p_{ii} = \frac{\exp(\bar{q}_i)}{1 + \exp(\bar{q}_i)}, \tag{11}$$

where $i \in \{1, 2\}$. $\bar{q}_i$ is the parameter to be estimated, and the logistic form represents a constraint for the probability between 0 and 1.[19]

Having derived the state probability under the assumptions (9)–(11), the time-varying weight ξ_t is defined as:

$$\xi_t = \Pr[S_t = 1 | \psi_t]_{L=t}, \tag{12}$$

where $\Pr[S_t = i | \psi_t]_{L=t}$ is a state probability at t, and $\psi_t \in \{\pi_0, \pi_1, \cdots, \pi_t\}$. The probability is a real-time estimate. Thus, only data up to t are available to determine the state probability at period t. The subscript $L = t$ indicates that the probability is estimated based on the data up to t.[20] For this reason, the state probability is recursively estimated with the new data obtained in each period.[21] In this sense, it is necessary to distinguish between the state probability at t based on data up to T, $\Pr[S_t = i | \psi_t]_{L=T}$, and the one based on data up to t, $\Pr[S_t = i | \psi_t]_{L=t}$. The state probability $\Pr[S_t = i | \psi_t]$ represents the latter throughout the rest of the paper.

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3. ESTIMATION

Now, (9) is estimated for the identification of inflation regimes, and (6) is estimated for inflation expectations from Section 2. This section first describes the data used for estimation and then presents and interprets the estimation results for inflation identification and inflation expectations, which form the primary content of this paper. In addition, this section discusses the results of auxiliary estimations to clarify the properties of unobserved macro-factors. Lastly, the section provides comparisons between models, using a transition matrix to determine the extent to which the state-dependent model better matches data than does the state-independent model. The results are also examined to identify the causes of the model's improvement.

3.1. Data

Since 1953, the University of Michigan has conducted the Survey of Consumers report (MSC), a representative survey of inflation expectations. At first, the survey was conducted every 3 years; in 1960, it shifted to quarterly production. Since 1978, however, it has been published every month. In terms of sample design, the survey is based on approximately 500 randomly selected respondents, including a rotating panel. Hence, any single monthly sample consists of two parts: a subsample newly selected in that month and a re-interview subsample, who completed the interview 6 months prior.[22] Survey participants are asked two questions regarding the expected direction and degree of change in prices:

1. During the next 12 months, do you think that prices, in general, will go up, or go down, or stay where they are now?
2. By about what percent do you expect prices to go (up/down) on the average, during the next 12 months?

The analysis in this paper utilizes MSC inflation expectations data, which ranges from 1973 to 2019. In the robustness check, the model is re-estimated with a somewhat shorter sample period up to 2009 to check the sensitivity of the estimates to the data in the recent decade. The 1973–2009 period relies on the dataset used in Malmendier and Nagel (2016), which was released to the public, to allow comparability with their study.[23] For the period 2010–2019, to produce cohort-level data, all observations are used without truncation to better match the cohort-level data of the previous years.[24] The data are adjusted, following Curtin (1996), to overcome deficiencies in the original survey data: first, because respondents in the MSC were not asked specific numbers for expectations when they predicted inflation to decline, the categorical value "down" is assigned to -3% between 1973 and 1979. Second, since the response "same" was, before 1982, often misinterpreted as suggesting that the inflation rate remained the same, a positive value supported by empirical evidence is assumed to adjust the response "same" to the "up" category.[25]

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Respondents are aggregated by age group; sample age groups range from 25 to 74 years old. People born in the same year belong to the same age group, regardless of birth month. To match the frequency of inflation expectations with the consumer price index collected from Robert Shiller's database, monthly inflation expectations are converted to quarterly data by taking the average of them.[26] For the professional forecasts of auxiliary estimations in Section 3.2.3, the Survey of Professional Forecasters (SPF) from the Federal Reserve Bank of Philadelphia is used.

For inflation, an annualized quarter-to-quarter increase rate of the consumer price index is used. The inflation experienced by a 74-year-old person in 1973 is traced to 1899, which is the oldest data required. Thus, the inflation data used for this analysis range from 1899 to 2019.

3.2. Estimation Results

3.2.1. Identified Inflation Regimes. Table 1 reports the estimation results for inflation regime identification based on the full sample at the final period. Three candidates are considered for the identification model. Model ID_1, in the first column, assumes regime switching in the constant and error terms. ID_2 applies the regime-switching assumption to the error term only. In the last column, ID_3 applies regime switching on all parameters: constant, autocorrelation, and error. In general, only marginal differences exist between models in terms of the magnitude of coefficients. However, in ID_1, the estimated constant μ_2 in Regime 2 is statistically significant and substantially different from μ_1 in Regime 1, given the standard errors of the estimates. Also, information criteria indicate that ID_1 better matches the inflation data than does ID_2. Regarding the autocorrelation coefficient, splitting ρ into ρ_1 and ρ_2 does not improve the model. ID_1 shows a better performance than ID_3 in terms of both information criteria. In addition, the difference between ρ_1 and ρ_2 does not seem significant according to the standard errors of the estimates. Therefore, ID_1 is selected as the baseline identification of the inflation regime, and the model is applied to the recursive estimation.

The estimates for the model ID_1 indicate that p_{11} and p_{22}, which denote the probabilities that each regime is maintained in the next period, are close to unity. This means that both regimes continue for a considerably long period and that the durations of the two regimes are similar.[27] The autocorrelation coefficient ρ is estimated at 0.32, which is low compared to previous studies on the persistence of inflation without regime switching.[28] Considering the estimates in ID_2, identifying the parameter μ separately in each regime could lower the persistence coefficient. In terms of the constant term and the standard deviation of the error term, those in Regime 1 are approximately two times larger than those in Regime 2. Given the estimates and statistical significance, Regime 1 is defined as a state of inflation with a high-inflation level and high-inflation volatility.

Figure 2 depicts the state probability identified by inflation level and volatility, based on available information from each period (i.e. $\{\Pr[S_j = 1|\psi_t]_{L=j}\}_{j=t_0}^{T}$).[29] As

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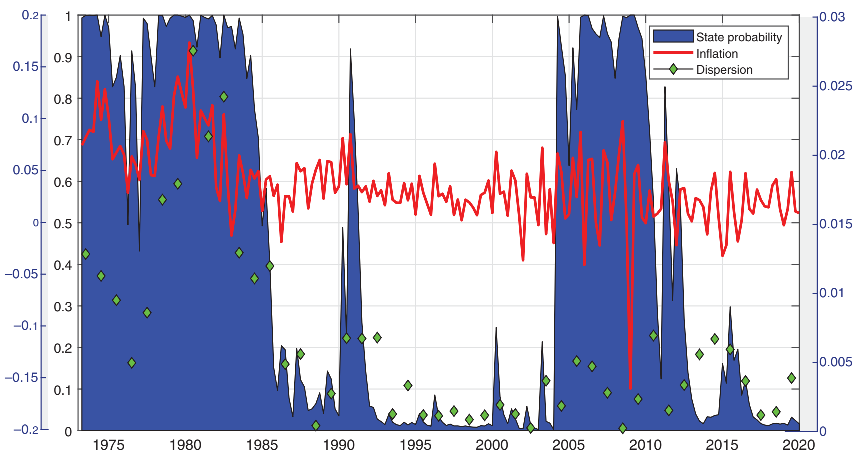
TABLE 1. Estimation results: identification of inflation regimes

	ID_1		ID_2		ID_3	
p_{11}	0.963	(0.023)	0.962	(0.025)	0.963	(0.024)
p_{22}	0.983	(0.010)	0.982	(0.011)	0.983	(0.010)
μ_1	0.038	(0.007)	0.017	(0.002)	0.034	(0.008)
μ_2	0.017	(0.002)			0.018	(0.002)
ρ_1	0.321	(0.061)	0.397	(0.059)	0.406	(0.100)
ρ_2					0.273	(0.078)
σ_{u1}	0.046	(0.004)	0.048	(0.004)	0.046	(0.004)
σ_{u2}	0.020	(0.001)	0.020	(0.001)	0.020	(0.001)
$\ln L$	610.3		604.1		610.8	
AIC	−1206.5		−1196.1		−1205.6	
BIC	−1181.1		−1174.3		−1176.6	

Notes: Models are estimated using maximum likelihood estimation. The estimation results are based on the final sample period, 2019.4Q. AIC = $-2\ln L + 2p$, BIC = $-2\ln L + p\ln n$. $\ln L$, p, and n refer to the log-likelihood, number of parameters, and number of observations, respectively. The numbers in parentheses refer to standard errors.



Notes: The blue area (second left axis) denotes the probability of Regime 1, defined by the period when the inflation level and its volatility are high. The red line (first left axis) represents the annualized quarterly inflation rate. The green dots (right axis) shows the gap in inflation expectations between age<40 years and age>60 years.

FIGURE 2. State Probability of Inflation.

illustrated, the actual inflation level and its volatility are higher in Regime 1.[30] Regime 1 begins during the mid-1970s and lasts until the mid-1980s. It appears again briefly in the early 1990s, reappears in the mid-2000s, and remains until the global financial crisis. Regime 2, characterized by low inflation and low volatility, ranges from the mid-1980s to mid-2000s, consistent with the period

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TABLE 2. Estimation results: inflation expectations

	State-independent	State-dependent			
		SD_1	SD_2	SD_3	SD_4
θ_1	3.319	4.317	3.708	3.824	2.392
	(0.240)	(0.298)	(0.265)	(0.299)	(0.155)
θ_2		1.919	1.911	2.283	
		(0.129)	(0.123)	(0.339)	
β_1	0.616	0.801	0.609	0.634	0.566
	(0.036)	(0.037)	(0.035)	(0.037)	(0.035)
β_2		0.499			
		(0.036)			
$\sigma_{\varepsilon,1}$	0.014	0.018	0.019	0.014	0.019
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$\sigma_{\varepsilon,2}$		0.009	0.009		0.009
		(0.000)	(0.000)		(0.000)
$\ln L$	26737.2	27591.7	27480.6	26741.6	27456.9
AIC	−53092.3	−54795.3	−54575.1	−53099.2	−54529.8
BIC	−51727.0	−53408.5	−53195.5	−51726.7	−53157.3

Notes: Models are estimated using maximum likelihood estimation. The coefficients of time dummies δ' are unreported. The numbers in parentheses refer to standard errors. AIC = $-2\ln L + 2p$, BIC = $-2\ln L + p\ln n$. $\ln L$, p, and n refer to the log-likelihood, number of parameters, and number of observations, respectively. In all cases, $n = 9,400$.

of "great moderation," as many authors describe it.[31] It also appears again after the global financial crisis and lasts until the recent period. Interestingly, the cross-sectional dispersion in inflation expectations between young and old age groups is also high in Regime 1, which is consistent with Mankiw et al. (2004), who note a strong positive relationship between inflation and disagreement in expectations.

3.2.2. Inflation Expectations. This subsection discusses the estimation results for the inflation expectations model; the estimates are reported in Table 2. First, the state-independent model is estimated for the purpose of comparison using the data from Section 3.1. As the first column suggests, θ is estimated at 3.32, which is close to the 3.14 identified by Malmendier and Nagel (2016).[32] The estimate of coefficient β is 0.62, which is marginally lower than the 0.67 produced in their paper. These results reveal two notable findings: first, the estimates are not sensitive to the sample period. This finding is reconfirmed by the sensitivity analysis in Section 3.3. Second, the state-independent model in this paper properly replicates the primary implication of the previous study. The parameter θ is significantly larger than unity and, in turn, strongly supports the properties of learning from experience in forming inflation expectations; households place different weights on recent and past data, depending on their age and lifetime experience. In addition, households form inflation expectations by relying on their idiosyncratic private information, rather than on public information.

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To estimate the state-dependent inflation expectations in (6), four types of models are examined, by changing the composition of certain parameters affected by inflation regimes. The first model, SD_1, applies state dependence to all parameters: the weighting factor θ_s, the coefficient of learning from experience β_s, and the standard deviation $\sigma_{\varepsilon,s}$. SD_2 excludes the possibility of state dependence in coefficient β_s. SD_3 introduces state dependence only for the weighting factor θ_s. The last model assumes state dependence in the variance of noise in expectations $\sigma_{\varepsilon,s}$.

When comparing the models, three interesting features are revealed. First, when comparing SD_4 with the state-independent model, Akaike information criteria (AIC) and Bayesian information criteria (BIC) decline as the state-dependent variance of noise enters the model, which suggests a substantial model improvement. At the same time, θ_s decreases from 3.32 to 2.39, which indicates that a high θ_s in the state-independent model partly absorbs the heterogeneity in the variance of noise that would have been explained by $\sigma_{\varepsilon,s}$. Second, splitting the weighting parameter does not directly improve the model. Despite some lower information criteria of SD_3, compared to the state-independent model, the differences are less significant. However, once the state-dependent variances of noise are given, introducing the state-dependent weighting factor θ_s improves the model. Lastly, the explanatory power of the model becomes stronger as the number of state-dependent coefficients increases. As such, SD_1 is chosen as the baseline model.

For the learning-from-experience component, in the baseline model, the weighting factor θ_1 in Regime 1 is estimated at 4.32, which is approximately two and a half times larger than θ_2 in Regime 2. SD_2 and SD_3 also demonstrate noticeable differences between θ_1 and θ_2; however, this gap widens as additional state-dependent parameters enter the model. The size of parameter θ_s determines the weight matrix for weighted least squares (WLS), as discussed in Section 2.1.[33] Figure 3 illustrates the implied weights on past data determined by parameter θ_s in each regime. The left and right panels depict the slopes of the weight functions for ages 30 and 60 years, respectively. This paper essentially assumes that older people utilize larger datasets to estimate their PLMs. Therefore, the slope of the weight function for 60 years of age is flatter than that of 30 years of age. At the same time, within the same cohort group, the slope in Regime 1 is steeper than that of Regime 2. The difference in slope between Regimes 1 and 2 for 30 years of age is even larger than that between ages 30 and 60 years in Regime 1.[34] This observation indicates that, in forming inflation expectations, the macroeconomic common experience implied in regime-specific learning gain is no less crucial than private experiences.

To focus on the state dependence in the weighting parameter θ_s, a simple exercise is conducted to approximate a constant gain, corresponding to each θ_s on the aggregate level. Although each age group applies a different size of decreasing gain, the average learning gain is constant in terms of a representative household, if the share of each age group in the population is constant over time. The approximate constant gains γ_s are calculated by minimizing the distance between

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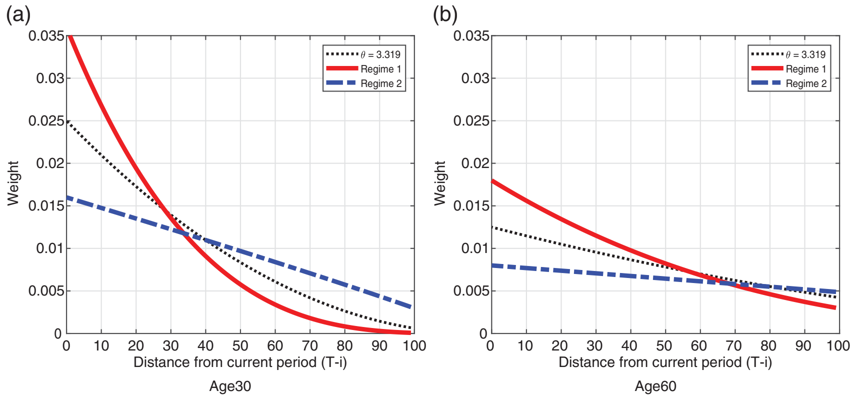

Notes: Each panel shows the implied weights on data, depending on the individual agent's private experiences and inflation regimes. The black dotted line refers to the weights under $\theta = 3.319$, which result from the state-independent model. The y-axis denotes the sizes of weights, and the x-axis denotes distances (quarterly) from the current period. For example, in the case of Regime 1 for the age of 30 years, the weight on data 100 quarters earlier converges to zero.

FIGURE 3. Implied weights on past data by age and inflation regime.

implied weight vectors from the baseline model and the constant gain learning model, as follows:

$$\underset{\gamma_s}{\text{argmin}} \sum_{t=1}^{T} \left[\bar{w}_{t,s}(\theta_s) - w_{t,s}^{c}(\gamma_s)\right]^2, \qquad (13)$$

where $\bar{w}_{t,s}(\theta_s)$ refers to the average weight vector across cohort groups, and $w_{t,s}^{c}(\gamma_s)$ refers to the weight vector from constant gain learning, in terms of a representative household. The resulting constant gains from (13), corresponding to θ_1 and θ_2, are 0.0249 and 0.0121, respectively.[35] The learning gain in the high-inflation regime is twice as large as that of the low-inflation regime. The constant gains proposed by other studies closely fall in between the two state-dependent gains of this model.[36]

To justify the assumption that households switch their PLMs depending upon inflation regime, I examine whether households benefit from the state-dependent learning strategy. This can be demonstrated by comparing forecast performances measured with the root mean squared error (RMSE) from the PLM for each case, since households are expected to apply the state-dependent learning strategy to reduce forecast errors. The RMSE of each cohort group r in the state-dependent model is calculated as follows:

$$RMSE_r = \left[\frac{1}{T}\sum_{t=1}^{T}\left(e_{t,r,1}^2 \Pr[S_t = 1|\psi_t] + e_{t,r,2}^2 \Pr[S_t = 2|\psi_t]\right)\right]^{0.5} \qquad (14)$$

where $e_{t,r,s} = \pi_t - E_{t-1}^{h,s}\pi_{t|r}$. Table 3 summarizes the RMSE in each case and indicates the extent of the reduction in the state-dependent PLM. The result reveals

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TABLE 3. Root mean squared error

	All	25–39	40–59	60–75
State-dependent (A)	0.0210	0.0204	0.0210	0.0219
State-independent (B)	0.0216	0.0205	0.0217	0.0228
Improvement ($(A-B)/B \times 100$)	−2.61	−0.46	−3.27	−3.99

Notes: RMSE in the state-independent model is calculated by $\left(\frac{1}{T}\sum_{t=1}^{T} e_{t,r}^2\right)^{0.5}$

that the predicted inflation $E_{t-1}^h \pi_{t|r}$ in the state-dependent model is closer to actual inflation than that in the state-independent model across all age groups. In general, a state-dependent learning strategy achieves a reduction in forecast error of 2.6%. The improvement becomes more significant in the older groups.

Furthermore, the coefficient β_s indicates a considerable difference between regimes. Specifically, SD_1 produces β_1 at 0.80 and β_2 at 0.50, while in the state-independent model, the value is 0.62. Because $\beta_1 > \beta_2$, the private forecasts based on personal experiences are more important when inflation and its volatility are high, as illustrated in the left panel of Figure 4. The contribution rate, which represents the relative share of the private forecasts from dummy variables, amounts to 0.71 in the high regime and 0.53 in the low regime. In contrast, public information, which is captured by time dummies and influences every cohort group simultaneously, accounts for 0.29 and 0.47, respectively. This indicates that households form inflation expectations that depend heavily upon their private forecasts in the high regime. During stable periods, however, households allocate more attention to public information, such as households' common beliefs, professional forecasters' opinions, or central bank announcements, than they do during the high regime.

These results relate to the findings of Madeira and Zafar (2015), who suggest that public information is more relevant for longer horizon forecasts, which are based on long-term movements. In other words, households utilize more public information when they concentrate on the long-term movement of inflation, as in Regime 2 in this paper. Moreover, the higher contribution of public information in Regime 2 could indicate agents' coordination motives arising from a strategic complementarity in forming expectations, as mentioned in Section 2.1 [Morris and Shin (2002), Carrillo and Emran (2012), Coibion et al. (2018)]. Households seem to become more strategically complementary and place more weight on public information in Regime 2, and hence the cross-sectional dispersion of expectations diminishes, accordingly as demonstrated in Figure 2.

This information allocation is more noticeable when compared to that in the state-independent model. The right panel of Figure 4 reveals that, in the state-independent model, the contributions rate changes in the opposite direction from that in the state-dependent model across inflation regimes. Since the contribution rate implies a degree of reliance on an information source in forming expectations,

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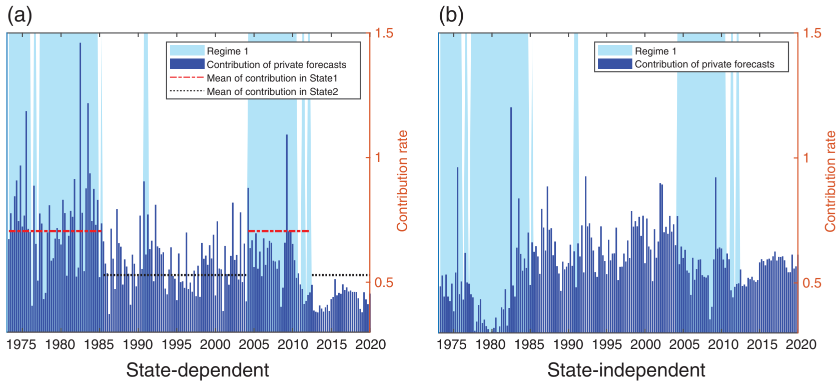

Notes: The contribution rate refers to the relative share of the learning-from-experience component against other macro-factors, in forming inflation expectations. The blue shaded area denotes Regime 1, wherein the state probability $\Pr[S_t = i|\psi_t]$ exceeds 0.5.

FIGURE 4. Contribution rates of private forecasts and common factors.

ignoring state dependence in expectations can be misleading when attempting to understand how households allocate the information source, depending on the state of inflation.

3.2.3. Common Macro-factors. This subsection considers in detail the properties of common factors, represented by $\delta' D_t$. Thus far in this paper, the common factor has been generally defined as public information other than private forecasts. In terms of public information, however, some studies focus on exogenous sources, such as news media or a public sector that affect households' expectations [Caroll (2003), Coibion et al. (2019)]; others highlight the role of households' social learning [Arifovic et al. (2013)]. To clarify the properties of macro-factors in this model, an auxiliary equation is estimated:

$$E_t\pi_{t+1|r} = \beta_s E_t^h \pi_{t+1|r} + (1-\beta_s) f_t + \sigma_{\varepsilon,s}\varepsilon_{t,r}. \quad \textbf{(15)}$$

The common factor f_t is now assumed to be a certain variable, instead of a time dummy. To see the relative share of expectations, the coefficient of f_t is restricted to be $1-\beta_s$. Following Malmendier and Nagel (2016), this paper examines two explanations for the common factors: forecasts of professional forecasters and a common belief through social learning processes. For estimation, SPF data are applied for the first case, and average private forecasts across all cohorts (i.e. $f_t = \bar{E}_t^{\,h}\pi_{t+1} = \frac{1}{50}\sum_r E_t^h \pi_{t+1|r}$) are applied for the second case. With this model, they find that both the forecasts of professional forecasters and social learning are equivalently potential candidates for the common factor, based on similar estimation results between the two cases. This paper checks whether this conclusion still holds in the context of state dependence.

Table 4 reports the estimation results. To begin, the estimates of the state-independent models, in both cases, closely approximate the estimates in Table 2;

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TABLE 4. Estimation results: common macro-factors

	SPF			$\bar{E}_t^h \pi_{t+1}$		
	State-independent	State-dependent		State-independent	State-dependent	
θ_1	3.531	4.631	4.317	3.780	4.591	4.317
	(0.085)	(0.155)	(–)	(0.075)	(0.126)	(–)
θ_2		1.320	1.919		1.148	1.919
		(0.035)	(–)		(0.038)	(–)
β_1	0.785	0.792	0.779	0.634	0.736	0.712
	(0.015)	(0.026)	(0.026)	(0.045)	(0.070)	(0.069)
β_2		0.798	0.814		0.307	0.534
		(0.012)	(0.013)		(0.049)	(0.061)
$\sigma_{\varepsilon,1}$	0.018	0.023	0.023	0.018	0.023	0.023
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$\sigma_{\varepsilon,2}$		0.011	0.011		0.011	0.011
		(0.000)	(0.000)		(0.000)	(0.000)
$\ln L$	24676.3	25798.3	25708.2	24603.7	25715.7	25601.1
AIC	−49346.5	−52290.7	−51408.5	−49201.4	−51419.4	−51194.2
BIC	−49325.1	−52233.5	−51379.9	−49180.0	−51376.5	−51165.6

Notes: The first three columns present the estimation results of (15) with the SPF. The next three columns provide the results with the average private forecasts $\bar{E}_t^h \pi_{t+1}$ in place of time dummies. The last column in each category indicates the result with the fixed θ_s from the estimates in the baseline model. The numbers in parentheses refer to standard errors. AIC = $-2\ln L + 2p$, BIC = $-2\ln L + p\ln n$. $\ln L$, p, and n refer to the log-likelihood, number of parameters, and number of observations, respectively. In all cases, $n = 9,400$.

this indicates that both SPF and common beliefs capture the time dummies fairly well in the state-independent model. However, when assuming state dependence, substantial differences exist between the two cases. In the case of social learning, the results are similar to those of the baseline model: the gap between θ_1 and θ_2 is only slightly wider, and the coefficients β_s are close to those of the baseline model. Moreover, $1-\beta_s$ varies in line with the result that public information draws more attention in Regime 2. In contrast, for SPF, even though state dependence in the weighting factor θ_s is recognized, β_2 is even larger than β_1. This result suggests that households are affected by professional forecasters' opinions to a greater extent in Regime 2 than in Regime 1, unlike the baseline estimation with time dummies.

To ensure the validity of this result, the models are estimated again, with the parameter θ_s fixed at 4.32 and 1.92, the values from the baseline model. The estimation results presented in the third column of each case are generally consistent with the previous unrestricted estimation. First, the parameters β_s with social learning are estimated and proved to be similar to those of the baseline model. Accordingly, $1-\beta_2$ is larger than $1-\beta_1$, which indicates that inflation expectations are more sensitive to common beliefs in the stable inflation regime than in the high regime. In contrast, $1-\beta_1$ in the model with SPF remains larger than

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$1 - \beta_2$; this implies that households care more about public information in the high regime, which is contrary to the primary result presented in Section 3.2.2. The SPF does not confirm what time dummies capture in the model.[37] This finding highlights the discrepancy between households' common beliefs and professional forecasters' opinions about future inflation.[38]

3.3. Sensitivity Analysis

For the robustness of the estimation, sensitivity analyses are conducted with two cases: first, state probability is recalculated with different initial periods for regime identification. Second, the estimation sample excludes the period 2010–2019, which is not analyzed in Malmendier and Nagel (2016). In this way, the sensitivity of the estimates is determined in terms of arbitrary initial periods and data from the previous decade.

3.3.1. Different Initial Periods for Identification of Inflation Regimes. First, this section considers how the primary results change depending on the initial period for identifying inflation regime and state probability. Given that the baseline regime identification in Section 3.2.1 defines the initial period at 1950, inflation regimes are redefined with earlier and later initial periods, ranging from 1940 to 1955. The baseline inflation expectations model, SD_1, is then re-estimated with the state probability from each identification period.

Figure 5 demonstrates the identified inflation regime from each initial period. Identifications (A) and (D) are based on fixed initial periods at 1940 and 1955, respectively. (E) is based on the initial period of 1950, the same as the baseline identification, which is denoted by the bright blue area in each panel. However, (E) uses a rolling fixed window for estimation, while the baseline identification uses a recursively accumulating window. Lastly, the lower-right panel presents the state probability calculated ex post in the full sample period. The identified inflation regimes, as illustrated, are generally similar. In fact, (E) demonstrates that the regimes are almost identical, except for 1985 and 1990, regardless of whether the estimation uses recursive or rolling windows. When identified with the initial period of 1940, the high regime, from 1973 to 1985, is somewhat underestimated, compared to other cases; otherwise, the identified regimes mostly overlap.

To clarify the similarity between identifications, the concordance index is calculated, as suggested by Harding and Pagan (2002), to demonstrate how closely the regimes identified by the models coincide:

$$CI = \frac{1}{T} \sum_{t=1}^{T} \left[S_t^b S_t^i + (1 - S_t^b)(1 - S_t^i) \right], \tag{16}$$

where $S_t^j = 1$ in Regime 1, and $S_t^j = 0$ in Regime 2. S_t^b and S_t^i denote the inflation regimes under the baseline identification and the identification with each initial period, respectively. Regime 1 is defined as the period in which the state probability ξ_t exceeds 0.5. The concordance index ranges between 0 and 1. When

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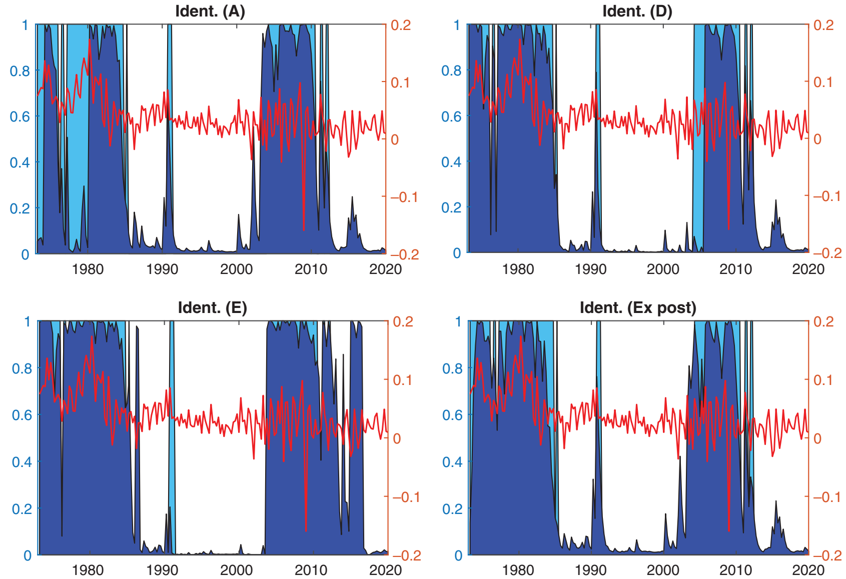

Notes: The bright blue area denotes the period of Regime 1, wherein probability $\Pr[S_t = i|\psi_t]$ exceeds 0.5, which is calculated from the baseline identification model. The dark blue area represents the probability of Regime 1 in each case, and the red line (right axis) indicates actual inflation. The initial periods in (A) and (D) are 1940.1Q and 1955.1Q, respectively. (E) applies a rolling fixed window, starting from 1950.1Q. The last panel depicts the probability calculated ex post in the full sample period.

FIGURE 5. Comparison of identified state probabilities.

S_t^b and S_t^i are perfectly synchronized, $CI = 1$; when unsynchronized, $CI = 0$. The resulting indexes are presented in Table 5. The concordance indexes are relatively low when the identification includes 1940s' data. However, the indexes remain close to unity, which indicates consistency in regime identification.

In terms of the estimation for inflation expectations, the results are presented in Table 5. The results in column (B) and (C) are based on the baseline identification and identification with the initial period of 1940, respectively. The results in column (A) and (B), with earlier initial periods, reveal that the gaps in θ_s and β_s between regimes are larger than in the baseline identification. The remaining estimations provide similar results: θ_1 and θ_2 amount to approximately 4.5 and 1.5, and β_1 and β_2 remain close to 0.8 and 0.4, respectively. In most cases, the gap between the estimates for the two regimes supports the relevance of state dependence in PLM, as well as the allocation of information in expectations. Nevertheless, baseline identification (C) still produces the best performance, in terms of AIC and BIC.

3.3.2. Different Sample Period for Inflation Expectations. To test the sensitivity of the estimates for recent years, (6) is re-estimated with data of the period

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TABLE 5. Estimation results of inflation expectations with different initial periods

	(A)	(B)	(C)	(D)	(E)	Ex post
θ_1	6.558	7.195	4.317	4.492	5.521	4.274
	(0.418)	(0.365)	(0.298)	(0.307)	(0.299)	(0.278)
θ_2	2.546	2.485	1.919	1.924	1.558	1.935
	(0.152)	(0.150)	(0.129)	(0.123)	(0.136)	(0.130)
β_1	1.015	1.069	0.801	0.854	0.920	0.824
	(0.040)	(0.038)	(0.037)	(0.038)	(0.038)	(0.037)
β_2	0.647	0.646	0.499	0.497	0.420	0.540
	(0.036)	(0.035)	(0.036)	(0.037)	(0.040)	(0.036)
$\sigma_{\varepsilon,1}$	0.017	0.017	0.018	0.018	0.015	0.018
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$\sigma_{\varepsilon,2}$	0.010	0.011	0.009	0.009	0.010	0.009
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$\ln L$	27363.3	27324.0	27591.7	27502.2	27142.3	27533.7
AIC	−54260.0	−54260.0	−54795.3	−54616.4	−53896.6	−54679.3
BIC	−52873.2	−52873.2	−53408.5	−53229.6	−52509.8	−53292.5
CI	0.84	0.80	1.00	0.95	0.88	0.91

Notes: The initial period in (A), (B), (C), and (D) is 1940.1Q, 1945.1Q, 1950.1Q, and 1955.1Q, respectively. (E) applies a rolling fixed window, starting from 1950.1Q. The last column uses the state probability calculated in the full sample period. The numbers in parentheses refer to standard errors. AIC = $-2\ln L + 2p$, BIC = $-2\ln L + p\ln n$. $\ln L$, p, and n refer to the log-likelihood, number of parameters, and number of observations, respectively. In all cases, $n = 9,400$. *CI* denotes the concordance index between the identified inflation regimes of the baseline identification (C) and the identification with each initial period.

1973–2009. The identified inflation regimes, in this case, are identical to the baseline model, since the regime identification is carried out in an accumulative way following (12), and hence inflation regimes do not depend on the end of the sample period. The estimation results as of the end of 2009 are presented in Table E1 of Appendix E. As the sample excludes data from the most recent 10-year period, the μ_1 in Regime 1 slightly decreases, and the autocorrelation coefficient ρ_1 increases. Yet, the estimates are, in general, similar to the case with data up to 2019. Moreover, the baseline identification ID_1^s remains preferable in terms of information criteria as revealed in Table E1. As for the inflation regime, the majority of the excluded period belongs to Regime 2, except for 2011, in which inflation sharply increases briefly.

Table 6 presents the estimation results of inflation expectations. In the state-independent model, θ amounts to 3.18, which is smaller than the estimates with data up to 2019, whereas β is marginally larger. In the model SD_1^s, θ_s and β_s are estimated as slightly smaller than in the baseline estimation. The ratios of the parameters between the regimes (i.e. θ_1/θ_2 and β_1/β_2) change from 2.43 and 1.63 to 2.25 and 1.61, respectively, by extending the sample period. Nevertheless, differences in the magnitude of estimates, order of information criteria, and degree

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TABLE 6. Estimation results: inflation expectations (1973–2009)

	State-independent	State-dependent			
		SD_1^s	SD_2^s	SD_3^s	SD_4^s
θ_1	3.176	4.100	3.953	4.012	2.276
	(0.272)	(0.315)	(0.282)	(0.343)	(0.184)
θ_2		1.690	1.667	1.811	
		(0.162)	(0.120)	(0.262)	
β_1	0.620	0.782	0.710	0.674	0.612
	(0.041)	(0.045)	(0.047)	(0.046)	(0.044)
β_2		0.479			
		(0.050)			
$\sigma_{\varepsilon,1}$	0.015	0.018	0.019	0.015	0.019
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$\sigma_{\varepsilon,2}$		0.010	0.010		0.010
		(0.000)	(0.000)		(0.000)
$\ln L$	20424.2	20837.8	20759.2	20431.8	20724.5
AIC	−40546.5	−41367.5	−41212.5	−40559.6	−41144.9
BIC	−39503.2	−40303.5	−40155.4	−39509.4	−40094.7

Notes: Models are estimated using maximum likelihood estimation. The coefficients of time dummies δ' are unreported. The numbers in parentheses refer to standard errors. AIC = $-2\ln L + 2p$, BIC = $-2\ln L + p\ln n$. $\ln L$, p, and n refer to the log-likelihood, number of parameters, and number of observations, respectively. In all cases, $n = 7,400$.

of state dependence are insignificant. Given that the change in sample size for this analysis is significant, the estimates in Section 3.2.2 are robust.[39]

3.4. Comparison of Models with a Transition Matrix

To determine whether assuming state dependence improves the explanatory power of the model, the transition matrices of inflation expectations between the data and models are compared. A transition matrix is known to adequately summarize the properties of longitudinal survey data in terms of persistence and mean reversion [Patton and Timmermann (2010), Vellekoop and Wiederholt (2019)]. To construct the transition matrix for the data, the cohort-level longitudinal survey is produced by identifying individuals who update their expectations between t and t+2, given that the frequency of the rotating panel is 6 months, as discussed in Section 3.1.[40] For the models, inflation expectations are simulated by combining the estimates and actual inflation data, based on (1) to (8).[41] Inflation expectations are divided into six groups, from 1% or less up to 6% or more.

The transition matrix indicates the conditional probability of inflation expectations between the two periods. In Tables 7–9, the first rows represent the relative frequency of answers at $t+2$, given that the answer in the current quarter (t) is 1%. For example, the (1,1) elements of the tables represent the proportion of

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TABLE 7. Transition matrix, data

	1% or less	2%	3%	4%	5%	6% or more
1% or less	13.0	32.5	31.2	16.4	3.1	3.8
2%	8.7	23.8	35.6	21.8	6.6	3.4
3%	3.9	20.1	36.6	24.0	11.1	4.4
4%	3.3	16.3	34.5	28.5	11.6	5.7
5%	4.3	12.9	31.8	25.1	16.1	9.8
6% or more	1.3	12.9	30.7	27.6	17.2	10.3

Notes: The element (i, j) indicates the proportion of cohort groups who answer i% of inflation expectations at time t and j% at time $t+2$.

TABLE 8. Transition matrix, model, state-dependent

	1% or less	2%	3%	4%	5%	6% or more
1% or less	5.3	15.9	28.7	25.1	13.0	12.0
2%	3.8	12.7	28.8	28.5	15.0	11.1
3%	3.6	11.5	26.5	28.5	16.0	13.9
4%	3.3	10.6	25.5	27.7	16.4	16.4
5%	3.0	9.7	23.8	27.4	16.8	19.2
6% or more	2.7	6.3	17.9	24.0	18.1	30.9

Notes: The element (i, j) indicates the proportion of cohort groups who answer i% of inflation expectations at time t and j% at time $t+2$.

TABLE 9. Transition matrix, model, state-independent

	1% or less	2%	3%	4%	5%	6% or more
1% or less	8.8	15.0	22.3	25.4	16.2	12.3
2%	8.0	13.2	20.9	24.0	17.5	16.4
3%	7.3	13.0	20.9	23.1	17.8	17.9
4%	7.3	12.8	20.3	22.0	17.9	19.7
5%	7.2	12.3	19.5	21.1	17.6	22.4
6% or more	6.3	10.3	18.1	20.9	16.5	27.8

Notes: The element (i, j) indicates the proportion of cohort groups who answer i% of inflation expectations at time t and j% at time $t+2$.

cohort groups that answer 1% or less at t and again at $t+2$. Therefore, the diagonal elements represent how many cohorts at t maintain their answers at $t+2$, which implies the persistence of the expectations. The elements in the middle columns indicate the extent to which the cohorts reverts to average expectations, which implies mean-reversion. More specifically, in Table 7, the diagonal elements are generally larger than the off-diagonal elements. In addition, the probability of transition to 2% or 3% at $t+2$ is relatively high, regardless of the answer at t.

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Table 10. Distance between data and models in the transition matrix

	All	Regime 1	Regime 2
State-dependent (A)	36.8	43.3	40.3
State-independent (B)	47.2	43.7	52.6
A/B	0.78	0.99	0.77

Notes: The first column demonstrates the distance between transition matrices from data and models measured by the Euclidean norm across all periods. The second and third columns are the distances of each inflation regime. Regime 1 refers to the period when the state probability $\Pr[S_t = i|\psi_t]$ exceeds 0.5.

A comparison of Table 7 with Tables 8 and 9 reveals that the models generally approximately replicate the properties of micro-level inflation expectations in terms of mean reversion and persistence. However, there are some differences: first, the models suggest lower persistence for the answer 1% or less to 3%. Second, in terms of mean reversion, the probability of households answering 4% at $t+2$ is higher than 2% in the model, in contrast to the actual data. Considering that the transition matrices based on the aggregated survey across all households are generally consistent with those from the models shown in Table F2, these results indicate that the statistical properties of cohort-level data are significantly different between the aggregated sample and the selected sample.[42] To simplify the comparison, the distances between the transition matrices in the data and the models measured by the Euclidean norm are calculated, as displayed in Table 10. The first column presents the distance for all inflation regimes: the models that assume state-dependent better match the data than those that are state-independent. The baseline model SD_1 reduces approximately 22% $(=\frac{47.2-36.8}{47.2})$ of the distance compared to the state-independent model.

To consider the transition matrix in more detail, it is broken down by inflation regime into two parts.[43] As expected, the data indicate higher probabilities of transition to higher expectations under Regime 1, as represented in Table F3 in Appendix F. For example, approximately 67% of households answer 6% or more at $t+2$ under Regime 1; this figure is only 24.4% under Regime 2. Table 10 demonstrates that the transition matrix derived from the state-dependent model is considerably closer to the data than the state-independent model in terms of the distance between the two matrices, especially in Regime 2. However, the model only slightly reduces the distance in Regime 1 by introducing state dependence; this suggests that the improvement in the explanatory power of state dependence derives primarily from Regime 2. In addition, this result indicates that the state-independent model overestimates the size of θ and, thus, by ignoring state dependence, exaggerates the role of learning from experience in expectations for the stable period.

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4. CONCLUSION

This paper proposes a model of inflation expectations that incorporates state-dependent learning from experience and information allocation. More specifically, and as part of the analysis, an inflation model is designed to identify inflation regimes over the analysis period. In doing so, state probability determining whether a certain period belongs to one of the inflation regimes is calculated. The model of inflation expectations is then estimated by combining state probability and US survey data.

In turn, the analysis reveals several interesting findings. First, when inflation and its volatility are high, households adopt learning strategies that more heavily emphasize recent data, because this allows them to better track actual inflation. Second, under the high-inflation regime, households rely upon information from their private experiences, rather than on public information, to form inflation expectations. Furthermore, the public information captured by time dummies seems to be associated with households' common beliefs through social learning during the analysis period. Finally, the state-dependent model is superior to the state-independent model in explaining actual data, in terms of the information criteria and transition matrix. In fact, the latter reveals that the improvement of the model results from a stable inflation regime. This finding indicates that under such a stable regime, the coefficients that characterize learning from experience and allocation to private information can be overestimated. In other words, the role of public information in forming inflation expectations during stable periods can be underestimated, when state dependence is not considered.

NOTES

1. The adaptive learning models are developed to overcome the limitation of rational expectations assumption which presupposes economic agents to have a great deal of knowledge about the economy [Bray and Savin (1986), Marcet and Sargent (1989), Sargent (1993), Evans and Honkapojha (2001)].

2. It differs from Mankiw et al. (2004), who explain the dispersion in expectations with the sticky information theory, or Capistrán and Timmermann (2009), who do so with agents' heterogeneous loss function, which minimizes forecast error.

3. See Evans and Honkapohja (1993), Ayuso et al. (2003), and Amisano and Fagan (2013).

4. Households' PLMs are assumed to be simple. It is reasonable to think that households cannot directly observe exogenous shocks. Therefore, a complex form of ARMA(p,q) would not be suitable. Also, one can consider a higher lag order with $p > 1$, but I leave it for further study.

5. The form of recursive least squares (RLS) is typical. For further detail, see Evans and Honkapojha (2001) or Carceles-Poveda and Giannitsarou (2007).

6. In estimation, I use weighted least squares (WLS) equivalent to RLS. For example, $b_{t,r,s}$ of a 30-year-old person at period t is calculated by least squares using a corresponding weight matrix $\Omega_{t,30,s}$. For more details, see Appendix A. In this way, one does not need to consider how to initialize the RLS algorithm. Even if the RLS is used, the weight on the initial periods for any cohort group converges to zero, and hence, the initialization does not have a critical issue in the model.

7. The OLS learning gives less and less importance to recent events as time passes. Thus, it would take longer for agents to realize that hyperinflation is starting [Marcet and Nicolini (2003)].

8. Pesaran and Timmermann (2007) also suggest that a typical trade-off exists between the bias and variance of forecast errors, regarding the number of observations. In a structural break, parameters

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estimated based on the full sample are biased; those based on the recent sample are unbiased but inefficient due to higher variance.

9. Since the denominator $t - r$ denotes the agent's age, the gain $\gamma_{t,r,s}$ evolves 1, $\frac{1}{2}$, $\frac{1}{3}$, and so on, as time passes under $\theta_s = 1$. Therefore, the weight on each data becomes equal for the agent. This is demonstrated in terms of WLS in Appendix A.

10. Model averaging is often employed to improve forecast performance in situations where the true data generating process is unknown [Timmermann (2006)]. This assumption is similar to those of Berardi (2009) and Busetti et al. (2017), who assume that there are two groups of people who independently form their expectations. In this case, the aggregate expectations in the economy are determined by a weighted average of those two expectations.

11. The issue of regime information is discussed in Section 2.2.

12. For example, the agent aged 30 years with $\theta_2 = 2$ and the agent aged 60 years with $\theta_1 = 4$ apply the same learning gain for the data in current period t. Nevertheless, the agent aged 30 years only uses 30 years of data, while the agent aged 60 years uses 60 years of data.

13. A common example is that households choose a decreasing gain $\gamma_t = 1/t$ if their forecasts are more precise than in the past, and they move to a constant gain $\gamma_t = \bar{\gamma}$ if forecast errors in the previous period are larger than usual.

14. Since inflation data is annualized at a quarterly rate, and the survey asks for inflation expectations over a 1-year (four quarters) horizon, the multi-period forecasts are used to match the forecast horizon.

15. Carrillo and Emran (2012) demonstrate the importance of public information in households' price expectations with microdata. If public information is assumed to be a central bank announcement, the coefficient δ' can be interpreted as the degree of central bank credibility [Dale et al. (2011), Muto (2011), Goy et al. (2020)].

16. Coibion et al. (2018) also find that the degree of agents' strategic complementarity is positively correlated with their preference for public signals over private signals.

17. It is likely that ξ_t is estimated within (6) analogously to Mertens and Nason (2020), instead by using the two-step estimation. However, in this case, it is difficult for the identified regimes to be characterized and for ξ_t to be determined by inflation level and volatility. Alternatively, one might apply a simple endogenous regime identification mechanism from the perspective of a representative household. For example, I can define the high-inflation regime to be when the level or the volatility of inflation in the recent period is higher than a certain threshold as follows:

$$\xi_t = S_t = \begin{cases} 1 & \text{if} \quad m_t^J > \nu_t^m \quad \text{or} \quad d_t^J > \nu_t^d \\ 0 & \text{if} \quad m_t^J \leq \nu_t^m \quad \text{or} \quad d_t^J \leq \nu_t^d, \end{cases}$$

where m_t^J and d_t^J are the mean, $m_t^J = \frac{1}{J}\sum_{j=1}^{J} \pi_{t-j+1}$, and the standard deviation, $d_t^J = \sqrt{\frac{1}{J}\sum_{j=1}^{J}(\pi_{t-j+1} - m_t^J)^2}$, of inflation in the recent period. The thresholds can be assumed to be the mean and the standard deviation of the long-term period, for example, $\nu_t^m = m_t^L$ and $\nu_t^d = d_t^L$, respectively, where $L > J$. The identified regimes and the estimation results for this case are provided in Appendix C.

18. Since it is assumed that households receive public information to form inflation expectations, state information can also be a part of the public information.

19. The form of transition probability could be time varying, or dependent on a certain variable. The estimation of the regime-switching model in this paper is typical. For further details, see Appendix B and Kim and Nelson (1999a).

20. In estimation, this implies that data up to t are used for maximizing the likelihood function.

21. The initial period is fixed at 1950 for the recursive estimation in the baseline analysis. The results under various initial periods are discussed in Section 3.3.1.

22. As of 2019, the former accounts for 64.9%, and the latter accounts for 35.1%.

23. https://voices.uchicago.edu/stefannagel/code-and-data

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24. The truncation of inflation expectations in the range between -5% and +30%, in line with common practice, does not make significant changes in the primary results. In this case, the discarded observations account for 0.4% of all observations.

25. To avoid this bias, starting with the March 1982 survey, an additional question is provided for those who answered "same" in the first place: *"Do you mean that prices will go up at the same rate as now, or that prices, in general, will not go up during the next 12 months?"*

26. http://www.econ.yale.edu/∼shiller/data.htm

27. The expected duration of regime i can be calculated by $E(D) = \frac{1}{1-p_{ii}}$. For further details, see Kim and Nelson (1999a).

28. Fuhrer (2010) and Beechey and Österholm (2012) find that the autocorrelation coefficient is lower than 0.4 only after the mid-2000s. Otherwise, it mostly moves between 0.4 and 0.8.

29. This state probability shows estimates for every end point in recursive estimation. Therefore, the filtered probability of this paper is identical to the probability by the Kalman smoother.

30. As the left panel of Figure C1 reveals, Regime 1 is characterized by a high-inflation level and volatility during the 1970s and 1980s, and by high volatility during the mid-2000s.

31. Kim and Nelson (1999b) identify the starting point of the great moderation as 1984. Other studies define the period similarly: 1984–2007 [Giannone et al. (2010)], 1983–2006 [Malmendier and Nagel (2016)], and 1985–2006 [Milani (2014)].

32. They use the data from 1953 to 2009 in their estimation. For the period 1953–1970, they generate percentage responses from the categorical responses using an econometric methodology.

33. The implied weights are the diagonal elements of the weight matrix. See Appendix A for details.

34. Following (A.7) in Appendix A, the ratio of weights on the current period (t) data between two regimes is calculated by $\frac{\omega_{t,r,1}(t)}{\omega_{t,r,2}(t)} = \frac{\theta_1}{\theta_2}$, and that between ages follows $\frac{\omega_{t,r_1,s}(t)}{\omega_{t,r_2,s}(t)} = \frac{t-r_2}{t-r_1}$. In this particular case, $\frac{\theta_1}{\theta_2} = \frac{4.32}{1.92} > \frac{t-r_2}{t-r_1} = \frac{60}{30}$.

35. When a demographical change, the change in population size of each age group, is considered, the aggregate constant gains on average change to 0.0258 and 0.0118, respectively. For this estimation, the population data provided by US Census Bureau are used. Appendix D illustrates the approximated aggregate constant gains for each period.

36. Milani (2007) and Malmendier and Nagel (2016) estimate the constant gain at 0.0183 and 0.018, respectively. Orphanides and Williams (2005) assume it to be 0.02, and Berardi and Galimberti (2017) calibrate it to match MSC data between 0.01 and 0.02.

37. In the estimation with the data from 1973 to 2009, SPF better matches the macro-factors in the model than the common beliefs through social learning. This result implies that households might be less affected by professional forecasters' opinions and more affected by common beliefs through social learning during the last decade. Further studies are required to clearly determine the properties of the common factors.

38. Coibion et al. (2020) find that professional forecasters and financial markets track macroeconomic developments closely and respond to policy shocks relatively quickly, whereas households are inattentive to inflation dynamics in developed countries.

39. The change in sample size is equivalent to 21% of the original sample size.

40. First, the household samples between the two periods are matched. They are then aggregated by cohort group and quarter. Transition probabilities t to $t+2$ are calculated for each period. Lastly, the final transition matrix is derived by taking the average across the sample period. For transition matrices, the sample period 1980–2019 is applied considering the availability of the data.

41. With the actual inflation data, the state probability, and the estimates of θ_s, private forecasts for each cohort group r, $E_t^h \pi_{t+1|r}$, can be produced by (2)–(3) or the WLS in Appendix A. Each cohort group applies a different learning gain $\gamma_{t,r,s} = \frac{\theta_s}{t-r}$ to produce private forecasts. Next, β_s is applied, and the estimates of δ' and the randomly drawn error terms $\sigma_{\varepsilon,s}\varepsilon_{t,r}$ are added for each period to generate cohort-level inflation expectations. The inflation expectations below 1.5% are assigned to 1%, those in the interval [1.5%, 2.5) to 2%, and so on. This procedure also applies to the state-independent model. Each model is simulated 100 times.

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42. Please note that the transition matrices from the models are based on the estimates from cohort-level data including all households. The transition matrices and the distances between the aggregated data and the models are presented in Tables F1–F2.

43. See Appendix F for the tables related to the inflation regime.

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APPENDIX A: WEIGHT MATRIX FOR WLS

To formulate households' individual forecasts $E_t^h \pi_{t+1|r}$ in (6), WLS is considered in this paper. This is because WLS is more convenient to employ than RLS in (2) and (3), and equivalent to it:

$$b_{t,r,s} = (X'\Omega_{t,r,s}X)^{-1}X'\Omega_{t,r,s}Y, \tag{A.1}$$

where $b_{t,r,s}$ is the estimates of least squares weighted by $\Omega_{t,r,s}$. The weight $\Omega_{t,r,s}$ represents a diagonal matrix consisting of weight components $\omega_{t,r,s}$ imposed on each data. The diagonal element $\omega_{t,r,s}$ can be formulated recursively using the relationship between γ_t and γ_{t+1}. This can be demonstrated by following Malmendier and Nagel (2016).

For simplicity, (2) and (3) can be rewritten by suppressing subscripts as follows:

$$b_t = b_{t-1} + \gamma_t R_t^{-1} x_{t-1}(\pi_t - x'_{t-1} b_{t-1}) \tag{A.2}$$

$$R_t = R_{t-1} + \gamma_t (x_{t-1}x'_{t-1} - R_{t-1}) = (1-\gamma_t)R_{t-1} + \gamma_t x_{t-1}x'_{t-1} \tag{A.3}$$

Multiplying R_t on both side in (A.2) yields

$$\begin{aligned} R_t b_t =& R_t b_{t-1} + \gamma_t x_{t-1}(\pi_t - x'_{t-1}b_{t-1}) \\ =& (R_t - \gamma_t x_{t-1}x'_{t-1})b_{t-1} + \gamma_t x_{t-1}\pi_t \\ =& (1-\gamma_t)R_{t-1}b_{t-1} + \gamma_t x_{t-1}\pi_t \\ =& (1-\gamma_t)[(1-\gamma_{t-1})R_{t-2}b_{t-2} + \gamma_{t-1}x_{t-2}\pi_{t-1}] + \gamma_t x_{t-1}\pi_t \\ =& (1-\gamma_t)(1-\gamma_{t-1})R_{t-2}b_{t-2} + (1-\gamma_t)\gamma_{t-1}x_{t-2}\pi_{t-1} + \gamma_t x_{t-1}\pi_t \\ =& \underbrace{(1-\gamma_t)(1-\gamma_{t-1})\cdots(1-\gamma_r)R_{r-1}b_{r-1}}_{=0} \\ &+ (1-\gamma_t)(1-\gamma_{t-1})\cdots(1-\gamma_{t-(t-r)+1})\gamma_{t-(t-r)}x_{t-(t-r)-1}\pi_{t-(t-r)} + \cdots \\ &+ (1-\gamma_t)\gamma_{t-1}x_{t-2}\pi_{t-1} + \gamma_t x_{t-1}\pi_t. \end{aligned} \tag{A.4}$$

(A.3) can be expressed by:

$$\begin{aligned} R_t =& R_{t-1} + \gamma_t(x_{t-1}x'_{t-1} - R_{t-1}) \\ =& (1-\gamma_t)R_{t-1} + \gamma_t x_{t-1}x'_{t-1} \\ =& (1-\gamma_t)(1-\gamma_{t-1})R_{t-2} + (1-\gamma_t)\gamma_{t-1}x_{t-2}x'_{t-2} + \gamma_t x_{t-1}x'_{t-1} \\ =& \underbrace{(1-\gamma_t)(1-\gamma_{t-1})\cdots(1-\gamma_r)R_{r-1}}_{=0} \\ &+ (1-\gamma_t)(1-\gamma_{t-1})\cdots(1-\gamma_{t-(t-r)+1})\gamma_{t-(t-r)}x_{t-(t-r)-1}x'_{t-(t-r)-1} + \cdots \\ &+ (1-\gamma_t)\gamma_{t-1}x_{t-2}x'_{t-2} + \gamma_t x_{t-1}x'_{t-1}. \end{aligned} \tag{A.5}$$

(A.4) and (A.5) can be rewritten in matrix forms by $R_t b_t = X'\Omega Y$ and $R_t = X'\Omega X$, respectively, where $X = [x_{t-1}, x_{t-2}, \cdots, x_{t-(t-r)-1}]'$ and $Y = [\pi_t, \pi_{t-1}, \cdots, \pi_{t-(t-r)}]'$. As a result,

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the coefficient b_t is calculated by $R_t^{-1}R_tb_t = (X'\Omega X)^{-1}(X'\Omega Y)$, which is equivalent to the WLS estimation with the weight matrix Ω. The weight matrix can thus be expressed by:

$$\Omega = \begin{bmatrix} \omega_t(0) & 0 & \dots & 0 \\ 0 & \omega_t(1) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \omega_t(t-r) \end{bmatrix}$$
$$= \begin{bmatrix} \gamma_t & 0 & \dots & 0 \\ 0 & (1-\gamma_t)\gamma_{t-1} & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & (1-\gamma_t)(1-\gamma_{t-1})\cdots(1-\gamma_{t-(t-r)+1})\gamma_{t-(t-r)} \end{bmatrix}. \quad \textbf{(A.6)}$$

From (A.6), a general relationship between $\omega_t(k)$ and $\omega_t(k-1)$ is derived as follows:

$$\omega_t(k) = \omega_t(k-1)\frac{1-\gamma_{t-k+1}}{\gamma_{t-k+1}}\gamma_{t-k}. \quad \textbf{(A.7)}$$

Given that $\gamma_t = \frac{\theta}{t-r}$, the slope of ω_t between $k-1$ and k, $\frac{1-\gamma_{t-k+1}}{\gamma_{t-k+1}}\gamma_{t-k}$, is simplified by $\frac{t-r-k+1-\theta}{t-r-k}$ and, thus, depends on the value of θ: the weights are identical, $\omega_t(k) = \omega_t(k-1)$, when $\theta = 1$; the weights are decreasing, $\omega_t(k) < \omega_t(k-1)$, when $\theta > 1$.

APPENDIX B: LIKELIHOOD FUNCTION FOR INFLATION REGIME IDENTIFICATION

For the estimation of (9)–(11), the log-likelihood function is evaluated by the following steps.

In the first step, given $\Pr[S_{t-1} = j|\psi_{t-1}]$, the state probability at t $\Pr[S_t = j|\psi_{t-1}]$ based on information up to $t-1$ is predicted by considering transition probabilities as follows:

$$\Pr[S_t = j|\psi_{t-1}] = \sum_{i=0}^{1} \Pr[S_t = j|S_{t-1} = i]\Pr[S_{t-1} = i|\psi_{t-1}]. \quad \textbf{(B.1)}$$

As new inflation data are obtained at period t, the probability density function is evaluated using the predicted state probabilities:

$$\begin{aligned} f(\pi_t|\psi_{t-1}) &= \sum_{S_t=0}^{1} f(\pi_t, S_t|\psi_{t-1}) \\ &= \sum_{S_t=0}^{1} f(\pi_t|S_t, \psi_{t-1})f(S_t|\psi_{t-1}) \\ &= \frac{1}{\sqrt{2\pi\sigma_{u,2}^2}}\exp\left[-\frac{(\pi_t-\mu_2-\rho_2\pi_{t-1})^2}{2\sigma_{u,2}^2}\right]\times\Pr[S_t=0|\psi_{t-1}] \\ &+ \frac{1}{\sqrt{2\pi\sigma_{u,1}^2}}\exp\left[-\frac{(\pi_t-\mu_1-\rho_1\pi_{t-1})^2}{2\sigma_{u,1}^2}\right]\times\Pr[S_t=1|\psi_{t-1}]. \end{aligned} \quad \textbf{(B.2)}$$

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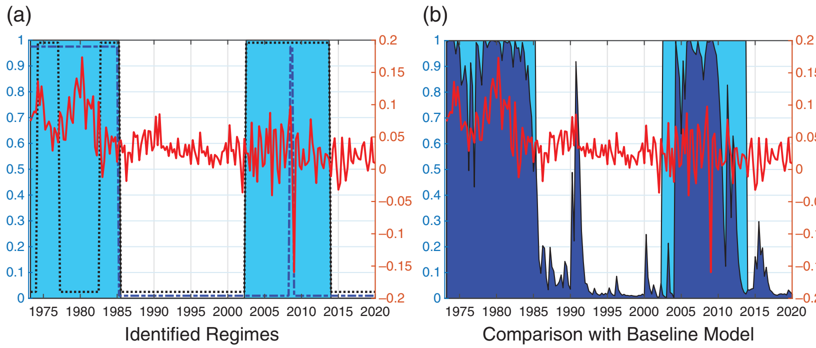

Notes: In the left panel, the black dotted and blue dash-dotted lines denote Regime 1, in terms of the mean and variance of inflation, respectively. The blue shaded area denotes the resulting Regime 1. In the right panel, the dark blue area represents the state probability identified by the baseline identification. In both panels, the red lines (right axis) indicate actual inflation.

FIGURE C1. Alternative identification of inflation regimes

In the next step, the state probabilities are updated by combining the probability density functions and the predicted state probabilities:

$$\begin{aligned}\Pr[S_t=j|\psi_t] &= \sum_{i=0}^{1} \Pr[S_t=j|S_{t-1}=i]\Pr[S_{t-1}=i|\psi_{t-1}] \\ &= \frac{f(\pi_t|S_t=j,\psi_{t-1})\Pr[S_t=j|\psi_{t-1}]}{\sum_{j=0}^{1} f(\pi_t|S_t=j,\psi_{t-1})\Pr[S_t=j|\psi_{t-1}]}. \end{aligned} \quad \textbf{(B.3)}$$

This two-step procedure recursively iterates from $t=1$ to T. The state probability $\Pr[S_0=j|\psi_0]$ to initialize the iteration is usually taken from the steady-state probabilities of S_t: $\Pr[S_0=0|\psi_0]=\frac{1-p_{11}}{1-p_{11}-p_{22}}$ and $\Pr[S_0=1|\psi_0]=\frac{1-p_{22}}{1-p_{11}-p_{22}}$.

Lastly, the log-likelihood function is derived by summing up the probability density functions across the sample period as follows:

$$\ln L = \sum_{t=1}^{T} \ln\left[\sum_{S_t=0}^{1} f(\pi_t|S_t,\psi_{t-1})\Pr[S_t|\psi_{t-1}]\right] \quad \textbf{(B.4)}$$

APPENDIX C: ALTERNATIVE REGIME IDENTIFICATION AND ESTIMATION RESULTS

As another example of regime identification, an exercise with a simple endogenous identification mechanism is conducted, by following Footnote 17 in Section 2.2. For this exercise, J and L are set at 20 (5 years) and 80 (20 years), respectively, as suggested by Milani (2014). Figure C1 illustrates the resulting inflation regimes. Regime 1, consisting of high-volatility and high-level periods, lasts until the mid-1980s and reappears in the early 2000s. The first high regime in the 1970s and 1980s is characterized by high volatility as well as

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TABLE C1. Inflation expectations estimation: alternative identification

		State-dependent			
	State-independent	AL_1	AL_2	AL_3	AL_4
θ_1	3.319	4.326	4.092	4.118	3.204
	(0.240)	(0.355)	(0.332)	(0.319)	(0.240)
θ_2		1.946	2.351	2.376	
		(0.295)	(0.243)	(0.256)	
β_1	0.616	0.710	0.642	0.652	0.606
	(0.036)	(0.049)	(0.040)	(0.040)	(0.037)
β_2		0.427			
		(0.079)			
$\sigma_{\varepsilon,1}$	0.014	0.015	0.015	0.014	0.015
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$\sigma_{\varepsilon,2}$		0.013	0.013		0.013
		(0.000)	(0.000)		(0.000)
$\ln L$	26737.2	26773.2	26769.3	26746.0	26760.4
AIC	-53092.3	-53158.5	-53152.5	-53108.0	-53136.8
BIC	-51727.0	-51771.7	-51772.9	-51735.5	-51764.3

Notes: Models are estimated by maximum likelihood estimation. The coefficients of time dummies δ' are not reported herein. The numbers in parentheses refer to standard errors. AIC = $-2\ln L + 2p$, BIC = $-2\ln L + p\ln n$. $\ln L$, p, and n refer to the log-likelihood, number of parameters, and number of observations, respectively. In all cases, $n = 9,400$.

a high level of inflation. In contrast, the second high regime in the 2000s is dominated by high-inflation volatility.

The right panel demonstrates that the regimes identified by the endogenous mechanism are generally consistent with the baseline identification. Despite slight differences in the early 2000s and 2010s, these two regimes mostly overlap. The concordance index appears to be 0.88 in this case.

Table C1 reports the estimation results for (6) with the regimes identified above. Most notably, the smaller information criteria, compared to the baseline model in Table 2, indicate that the regime identification with the Markov switching assumption is more suitable for explaining data. Nevertheless, it is similarly found that the state-dependent models are more accurate than the state-independent model in matching data with this regime identification. In addition, the weighting factor θ_s and coefficient β_s are estimated to be similar to those in the baseline model in terms of the size and gap between regimes in AL_1. However, when compared to AL_2, the state-dependent properties in the allocation of private and public information are not clearly detected with this inflation regime information.

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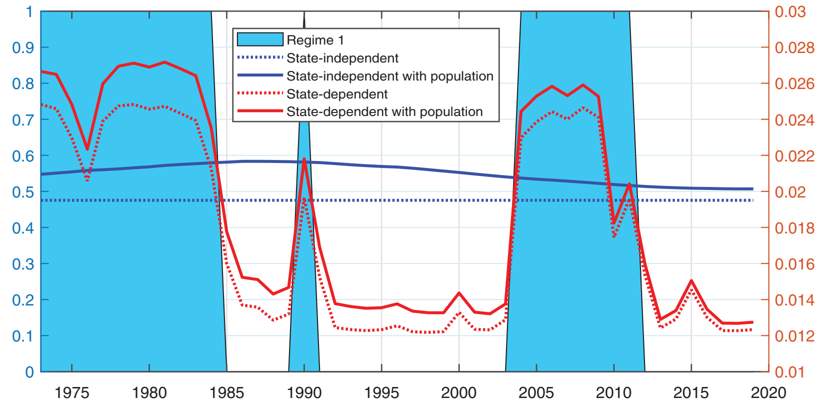
APPENDIX D: AGGREGATE LEARNING GAIN



Notes: The blue area (left axis) denotes the probability of Regime 1. The dotted lines (right axis) represent the aggregate constant gains, approximating the average learning gains in the models. The solid lines indicate the aggregate constant gains with consideration of the population. Changes in the gains are due to the time-varying state probability, as well as demographical changes. To incorporate demographical changes, (13) is applied for each period.

FIGURE D1. Approximated aggregate gain

APPENDIX E: IDENTIFICATION OF INFLATION REGIMES FOR PERIOD 1973–2009

TABLE E1. Estimation results: identification of inflation regimes

	ID_1^s		ID_2^s		ID_3^s	
p_{11}	0.975	(0.020)	0.975	(0.021)	0.975	(0.019)
p_{22}	0.980	(0.012)	0.980	(0.012)	0.981	(0.012)
μ_1	0.035	(0.006)	0.017	(0.002)	0.032	(0.007)
μ_2	0.017	(0.002)			0.018	(0.003)
ρ_1	0.358	(0.063)	0.423	(0.063)	0.410	(0.098)
ρ_2					0.321	(0.082)
$\sigma_{u,1}$	0.046	(0.004)	0.048	(0.004)	0.046	(0.004)
$\sigma_{u,2}$	0.019	(0.001)	0.019	(0.001)	0.019	(0.001)
$\ln L$	519.5		515.0		519.8	
AIC	-1025.0		-1017.9		-1023.5	
BIC	-1000.7		-997.0		-995.7	

Notes: Models are estimated using maximum likelihood estimation. The estimation results are based on the final period, $T = 2009.4Q$. AIC = $-2\ln L + 2p$, BIC = $-2\ln L + p\ln n$. $\ln L$, p, and n refer to the log-likelihood, number of parameters, and number of observations, respectively. The numbers in parentheses refer to standard errors.

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APPENDIX F: TRANSITION MATRICES

TABLE F1. Distance between data and models in the transition matrix (all households)

	All	Regime 1	Regime 2
State-dependent (A)	15.2	24.5	22.9
State-independent (B)	30.8	25.7	38.8
A/B	0.49	0.96	0.59

Notes: The first column demonstrates the distance between transition matrices from data and models measured by the Euclidean norm across all periods. The second and third columns are the distances of each inflation regime. Regime 1 refers to the period when the state probability $\Pr[S_t = i|\psi_t]$ exceeds 0.5.

TABLE F2. Transition matrix, t to $t+2$, data (all households)

Data	1% or less	2%	3%	4%	5%	6% or more
1% or less	5.2	18.2	26.0	24.4	15.1	11.1
2%	3.1	15.4	35.7	26.3	13.9	5.6
3%	3.7	12.1	33.2	28.6	12.9	9.4
4%	2.2	10.1	33.8	28.3	15.0	10.6
5%	2.7	11.3	31.7	27.1	15.0	12.2
6% or more	1.8	12.1	22.9	28.0	17.0	18.2

Regime 1	1% or less	2%	3%	4%	5%	6% or more
1% or less	7.8	15.9	19.7	16.7	18.4	21.4
2%	3.9	10.2	24.4	32.3	22.3	6.9
3%	3.7	8.7	28.2	31.5	15.5	12.4
4%	3.6	6.7	25.4	29.4	18.5	16.4
5%	2.4	5.9	25.7	27.4	18.8	19.8
6% or more	1.9	9.5	19.0	21.4	19.7	28.5

Regime 2	1% or less	2%	3%	4%	5%	6% or more
1% or less	4.1	19.2	28.8	27.9	13.6	6.5
2%	2.8	17.3	39.9	24.1	10.7	5.2
3%	3.7	13.5	35.4	27.4	11.8	8.1
4%	1.6	11.6	37.8	27.8	13.3	7.9
5%	2.8	14.2	35.0	27.0	13.0	8.0
6% or more	1.7	14.0	25.6	32.6	15.1	11.1

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TABLE F3. Transition matrix, t to $t+2$, data

Regime 1	1% or less	2%	3%	4%	5%	6% or more
1% or less	7.1	24.8	32.1	18.4	6.6	11.0
2%	8.1	18.4	29.8	26.1	11.2	6.3
3%	2.2	12.0	29.1	29.2	18.5	9.1
4%	1.4	12.2	27.3	31.6	17.7	9.8
5%	2.1	8.0	25.9	30.6	17.0	16.4
6% or more	1.2	7.7	31.2	27.6	18.3	13.9

Regime 2	1% or less	2%	3%	4%	5%	6% or more
1% or less	15.2	35.3	30.8	15.7	1.8	1.1
2%	8.9	26.0	38.0	20.1	4.7	2.2
3%	4.6	23.5	39.8	21.8	7.9	2.4
4%	4.2	18.1	37.7	27.1	9.0	3.9
5%	5.4	15.4	34.7	22.4	15.6	6.5
6% or more	1.3	15.7	30.4	27.6	16.6	8.3

TABLE F4. Transition matrix, t to $t+2$, model, state-dependent

Regime 1	1% or less	2%	3%	4%	5%	6% or more
1% or less	7.2	11.3	21.5	24.3	18.3	17.5
2%	5.9	10.0	20.0	26.1	19.8	18.1
3%	5.7	9.8	20.7	26.2	19.4	18.2
4%	5.5	9.4	19.9	25.7	19.6	19.9
5%	5.2	9.4	18.7	25.0	19.1	22.7
6% or more	4.8	8.1	17.3	23.5	17.8	28.6

Regime 2	1% or less	2%	3%	4%	5%	6% or more
1% or less	4.5	20.8	38.1	26.4	8.2	2.1
2%	3.3	15.3	34.6	31.4	12.4	3.1
3%	2.9	13.6	32.8	32.0	14.4	4.4
4%	2.8	13.1	32.1	32.3	15.0	4.7
5%	2.5	11.9	31.0	33.3	15.9	5.3
6% or more	1.7	7.8	25.4	35.0	21.8	8.4

TABLE F5. Transition matrix, t to $t+2$, model, state-independent

Regime 1	1% or less	2%	3%	4%	5%	6% or more
1% or less	7.8	11.2	21.6	24.4	18.7	16.2
2%	5.3	10.8	20.5	26.1	20.1	17.2
3%	6.1	12.0	19.2	26.1	19.9	16.8
4%	5.3	11.9	20.0	23.1	21.0	18.7
5%	5.9	10.8	19.8	23.4	18.4	21.5
6% or more	4.5	10.3	17.1	20.6	18.4	29.1

Regime 2	1% or less	2%	3%	4%	5%	6% or more
1% or less	10.1	16.1	23.8	22.3	17.5	10.2
2%	9.2	15.1	22.7	24.4	17.2	11.4
3%	9.0	15.0	22.9	24.0	17.1	12.1
4%	9.0	14.2	23.0	24.9	16.9	12.0
5%	8.7	15.0	21.9	24.8	17.6	12.0
6% or more	7.7	15.5	20.8	24.3	17.9	13.8