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Robust Combined Adaptive and Variable Structure Adaptive Control of Robot Manipulators

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SUMMARY

The paper addresses the robust adaptive control problem of robot manipulators. The dynamic equations of robot manipulators and their fundamental properties that facilitate analysis and control system design are first reviewed. Then the direct, indirect, and combined direct and indirect adaptive control approaches of robot manipulators are presented. After that, a number of variable structure adaptive control approaches which combines features of the robust design based on variable structure systems with parameter adaptive control, are studied. After that, a new combined adaptive and variable structure adaptive control approach is proposed for the tracking control of robot manipulators under the uncertainty environment. This is a robust, high-performance adaptive control scheme that combines the advantages and overcomes the disadvantages of both types of techniques. Finally, the extensive comparing simulation results are presented to demonstrate the theory study.

KEYWORDS: Adaptive control; Robots; Robust design; Variable structure systems.

1. INTRODUCTION

Robot manipulators are basically multi-degree-of-freedom positioning devices. The robot, as the ‘‘plant to be controlled’’, is a multi-input/multi-output, highly coupled, nonlinear mechatronic system. The main challenges in the robot control problem are the complexity of the dynamics and uncertainties. Robots have to face uncertainty in many dynamic properties, in particular the parameters describing the dynamic properties of grasped payload. Sensitivity to such parameter uncertainty is especially severe in high-speed operations or when controlling direct-drive robots, for which no gear reduction is available to mask effective inertia variations. More generally, if advanced robots are designed to be capable of precisely affecting their environment (e.g. providing accurate force or impedance control), they are likely to exhibit high sensitivity to external forces and load variations. There are basically two distinct philosophies for controlling such uncertain systems: the adaptive control philosophy,[1–8] and the robust control philosophy.[9–13] A robust controller is designed to make the system is not sensitive to all the uncertainties, and the final controller has a fixed-structure. However, robust control is suitable for dealing with only relatively small uncertainty. On the other hand, adaptive control uses on-line identification (identifying the plant parameters in indirect adaptive control using prediction errors and the controller parameters in direct adaptive control using tracking errors) and attempts to ‘learn’ the uncertain parameters of the system. The adaptive approach is applicable to a wider range of parameter variation, but is sensitive to unstructured uncertainty. The question is whether we can combine the different adaptive control approaches and robust control approaches and hence obtain the advantages of both control methods.

Improved system responses in terms of speed and accuracy, as well as robustness in the presence of perturbations, may be possible if direct, indirect and variable structure adaptive controllers were somehow combined. The difficulty arises since the information acquired using the different methods cannot, in general, be combined conveniently to determine the adaptive laws. Combined direct and indirect adaptive control approaches have been proposed by Duarte and Narendra,[7] for linear plants, and by Slotine and Li,[8] for robot manipulators. Further, a combined direct, indirect, and variable structure adaptive method of linear plants is suggested by Narendra and Boskovic.[14] A robust combined direct and variable structure adaptive control scheme of robot manipulators is proposed by Yu.[15]

It is noted that a majority of standard adaptive control theory is only applicable to linear time-invariant systems, and hence is unsuitable for direct application to the robot manipulator control problem. In this paper, two combined control methods for robot manipulators are studied. The first is the combined adaptive control approach which combines direct adaptive control with indirect adaptive control.[8] The second is the robust combined adaptive control approach which combines direct/indirect adaptive control with the robust variable structure adaptive control.[15] The work presented here is an extension of our previous work.[15] The robustness analysis of the proposed combined adaptive control approach, which is not studied before,[15] is presented in this paper. The robust combined adaptive control laws are designed using a special Lyapunov function. The control structure does not rely on the passivity property of manipulators (**Property 2**) and guarantees that the tracking errors decrease to zero asymptotically. The chattering problem is also avoided in this approach. The direct adaptive control law ensures asymptotic stability, while the variable structure adaptive control law improves the transient behaviour and enhances the robustness of the robot system. The combined method improves the transient

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behaviour and robustness to external disturbances and unmodelled dynamics, and also overcomes the chattering problem which is the main drawback of the variable structure method. The fact that the passivity properties of rigid robot manipulators are not used in the control scheme will increase its flexibility. The fast tracking convergence and the robustness to external disturbances and unmodelled dynamics of the robust combined controller are confirmed by extensive computer simulations.

2. MANIPULATOR DYNAMIC EQUATIONS AND USEFUL PROPERTIES

Using the Euler–Lagrangian formulation, the joint-space (robot coordinates) dynamics of an n-link rigid robot manipulator can be written as

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + \tau_d = \tau \qquad (1)$$

where $q \in R^n$ and $\tau \in R^n$ denote generalized displacements and generalized control input forces in robot coordinates, $\tau_d \in R^n$ represents the friction, input torque disturbances, and other unmodelled dynamics, $D(q) \in R^{n\times n}$ is the manipulator inertial matrix, $C(q, \dot{q})\dot{q} \in R^n$ is the vector of centripetal and Coriolis torque and $G(q) \in R^n$ is gravitational torque. It is assumed that only the joint positions and velocities, not accelerations, are available from measurements. The robot manipulator dynamic model, equation (1), possesses a number of important properties that facilitate analysis and control system design. Among these are the following properties:[19, 23]

Property 1. *The inertia matrix D(q): 1) The kinetic energy of a manipulator can be written as* $K(q) = \frac{1}{2}\dot{q}^T D(q)\dot{q}$; 2) *It is symmetric, i.e.* $D^T(q) = D(q)$; 3) *It is a uniformly positive definite matrix*; 4) *It is bounded above and below, i.e.,* $\mu^1(q)I \leq D(q) \leq \mu_2(q)I$, *where* $I \in R^{n\times n}$ *is the identity matrix,* $\mu_1(q) \neq 0$ *and* $\mu_2(q)$ *are scalar constants for a revolute arm, and generally scalar functions of q for an arm containing prismatic joints.*

Property 2. *The Coriolis and Centripetal Terms,* $C(q, \dot{q})\dot{q}$: 1) *It is quadratic in the generalized velocity* $\dot{q}$ *and bounded as follows:* $\|C(q,\dot{q})\dot{q}\| \leq \mu_3(q)\|\dot{q}\|^2$, *where* $\mu_3(q)$ *is a scalar constrant for an all-revolute arm and a scalar function of q for arms containing prismatic joints*; 2) *It may be written in several factorisations, such as* $C(q, \dot{q})\dot{q} = V(q, \dot{q}) = D^1(q)C^1(\dot{q})\dot{q} = D^2(q)[\dot{q}\dot{q}] + D^3(q)[\dot{q}^2]$, where $[\dot{q}\dot{q}] = [\dot{q}_1\dot{q}_2\dot{q}_1\dot{q}_3 \ldots \dot{q}_{n-1}\dot{q}_n]^T \in R^{\frac{n(n-1)}{2}}$, $[\dot{q}^2] = [\dot{q}_1^2\dot{q}_2^2 \ldots \dot{q}_n^2]^T \in R^n$, $D^1(q) \in R^{n\times nn}$, $C^1(\dot{q}) \in R^{nn\times n}$, $D^2(q) \in R^{n\times[\frac{n(n-1)}{2}]}$, *and* $D^3(q) \in R^{n\times n}$; 3). *The two* $n\times n$ *matrices* $D(q)$ *and* $C(q, \dot{q})$ *are not independent; in particular, a suitable definition of* $C(q, \dot{q})$ *makes* $\dot{D}(q) - 2C(q, \dot{q})$ *skew-symmetric. And the following equation is always true, no matter how* $C(q, \dot{q})$ is defined, $X^T\,[\dot{D}(q) - 2C(q, \dot{q})]X = 0$, *where* $X \in R^n$, *is an arbitrary vector. Strongly related to the skew symmetry property is the so-called, Passivity Property*; 4) $C(q, x)y = C(q, y)x$ *for all x and* $y \in R^n$.

Property 3. *The Gravity Term,* $G(q)$: *It is bounded as follows* $\|G(q)\| \leq g_b(q)$ *where* $g_b(q)$ *is a scalar constant for revolute arms and a scalar function of q for arms containing prismatic joints.*

Property 4. *The Friction Term,* $F(\dot{q})$: *Since friction is a local effect, it is reasonable to assume that the friction terms* $F(\dot{q})$ *are uncoupled among the joints so that* $F(\dot{q}) = [f_1(\dot{q}_1), f_2(\dot{q}_2), \ldots, f_n(\dot{q}_n)]^T$, *where* 1) $f_i(\dot{q}_i)$ *is a known scalar function*; 2) $f_i(\dot{q}) = f_{vi} + f_{si} = v_{ci}\dot{q}_i + k_i sgn(\dot{q}_i)$; 3) f_{vi} *represents the viscous (linear) friction part*; 4) f_{si} *represents the static (Coulomb) friction part*; 5) $|f_i(\dot{q}_i)| \leq v_{ci}|\dot{q}_i| + k_i$.

Property 5. *The Disturbance Torque,* τ_d: *The disturbance torque* τ_d *is by definition unknown. It is, however, reasonable to assume that it is bounded by a known function* $\|\tau_d\| \leq d(q, \dot{q})$.

Property 6. *Equation* (1) *is linear with respect to* $\ddot{q}$.

Property 7. *Control inputs in equation* (1), τ_i, *are independent.*

Property 8. *The dynamics equation* (1) *defines a passive mapping from the input* τ *to the generalized velocity* $\dot{q}$.

Property 9. *The structure of the dynamic equation is linear in terms of a suitably selected set of robot and load parameters; that is, if a suitable choice of* $W(q, \dot{q}, \ddot{q})$, $W_0(q, \dot{q}, \ddot{q})$, *and* Θ *is made, the left hand side of equation* (1) *can be represented as follows:* $W(q, \dot{q}, \ddot{q})\Theta + W_0(q, \dot{q}, \ddot{q}) = D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q)$, *where* $W(q, \dot{q}, \ddot{q}) \in R^{n\times p}$ *is a matrix (or regressor matrix) and* $W_0(q, \dot{q}, \ddot{q}) \in R^n$ *is a vector, both composed of known functions, and* $\Theta \in R^p$ *is a vector of unknown constant system parameters.*

Property 10. *The* **Lagrangian** *function L can be written by using the same parameters* Θ *as in* **Property 9** $L = Y(q, \dot{q})\Theta + Y_0(q, \dot{q})$, *where* $W(q, \dot{q}, \ddot{q}) = \frac{d}{dt}(\frac{\partial Y}{\partial \dot{q}}) - \frac{\partial Y}{\partial q}$, $W_0(q, \dot{q}, \ddot{q}) = \frac{d}{dt}(\frac{\partial Y_0}{\partial q}) - \frac{\partial Y_0}{\partial q}$.

Property 11. *The Hamilton function (or total mechanical energy) can be written as follows by using the same parameters* Θ *as in* **Property 9**, $H = W_H(q, \dot{q})\Theta + W_{H0}(q, \dot{q})$, *where* $\dot{W}_H(q, \dot{q}) = \dot{q}^T W(q, \dot{q}, \ddot{q})$ *and* $\dot{W}_{H0} = \dot{q}^T W_0(q, \dot{q}, \ddot{q})$.

Remark 1. *It is noted that the dimension of the parameter space is not unique and the search for the parameterization that minimizes the dimension of the parameter space is an important problem.* **Property 9** *plays a crucial role in the various linear parameterization approaches.*[1–8, 12, 13]

3. COMBINED DIRECT AND INDIRECT ADAPTIVE CONTROL

The direct adaptive controllers use tracking errors of the joint motion to drive the parameter adaptation. The indirect adaptive manipulator controllers use prediction errors on the filtered joint torques to generate parameter estimates to be used in the control law. The combined direct and indirect adaptive controllers use both tracking errors in the joint motions and prediction errors on the filtered torques to drive the parameter adaptation. The direct adaptive control approach and the indirect adaptive control approach are first reviewed. The control structure which can be extended to the combined adaptive control schemes is studied. Then, a combined direct and indirect adaptive control approach is presented. The combined adaptive method uses both the prediction (input) error and the tracking (output) error in the

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adaptation law and improves the performances of the robot.

3.1. Direct adaptive control

Historically, there are many publications on the adaptive control of robot manipulators that avoid the use of the inverse of the estimated inertia matrix and the joint accelerations, using **Property 1 2–6**. All these controllers also use the passivity property (**Property 2** or **8**), except Johansson[4] where the derivative of the inertia matrix is used. However, it is noted that adaptive control without using accelerations and the inverse of the estimated inertia matrix is due to **Property 1**, and not due to the passivity property (**Property 2** or **8**) as is misunderstood in some previous papers. An adaptive control method is presented on Lyapunov stability theory using **Property 1**, the passivity property of robot manipulators (**Property 2** or **8**), and the linearisation property (**Property 9**). This approach is different from the computed torque (inverse dynamics, linearising) approach, since here even with exact knowledge of the parameters, the control law does not linearise the equations of motion of the robot. However, this approach overcomes the main drawbacks of the adaptive computed torque control methods in that it does not require measurement of the manipulator accelerations, nor does it require the inverse of the estimated inertia matrix. Most of the above methods are not strictly derived from Lyapunov stability theory. Johansson[4] gave an adaptive controller which is completely designed from Lyapunov theory.

3.1.1. Structure of the controller. Let the tracking error state be,

$$\tilde{x}=[\dot{\tilde{q}}^T(t) \quad \tilde{q}^T(t)]^T; \quad \tilde{x}\in R^{2n} \tag{2}$$

where $\tilde{q}=q-q_d$. The objective is, given the structure of the manipulator dynamic equations, the desired trajectories $q_d(t)$, $\dot{q}_d(t)$, $\ddot{q}_d(t)$, measurements of the joint position $q(t)$ and velocity $\dot{q}(t)$, and with some or all of the manipulator parameters being unknown, to determine the control input $\tau(t)$ such that all the signals in the control system remain bounded and $\lim_{t\to\infty}\tilde{x}=0$.

First consider the case of known parameters. Combining equations (1) and (2), the error dynamics of the robot manipulator as

$$\dot{\tilde{x}}(t)=A_0(t)\tilde{x}(t)+B_0(t)+B_c(t)\tau(t) \tag{3}$$

where

$$A_0(t)=\begin{bmatrix} -D^{-1}(q)C(q,\dot{q}) & 0 \\ I & 0 \end{bmatrix};$$

$$B_0(t)=\begin{bmatrix} -\ddot{q}_d-D^{-1}(q)[G(q)+C(q,\dot{q})\dot{q}_d] \\ 0 \end{bmatrix};$$

$$B_c(t)=\begin{bmatrix} D^{-1}(q) \\ 0 \end{bmatrix}$$

Define the control law as

$$\tau(t)=\tau_n(t)+\tau_l(t) \tag{4}$$

where $\tau_l(t)\in R^n$ is a linear feedback control part which ensures that the time differential of a Lyapunov function to be negative and $\tau_n(t)\in R^n$ is a non-linear feedforward compensation term. Both terms will be decided later from Lyapunov stability theory. Choose the Lyapunov function candidate as

$$V(t)=\frac{1}{2}\tilde{x}^T(t)P_q(q)\tilde{x}(t) \tag{5}$$

where

$P_q(q)=U^TP_D(q)U$;

$$U=\begin{bmatrix} I & P_{12} \\ 0 & I \end{bmatrix};\ P_{12}=P_{cc}^{-1}\Gamma;\ P_D(q)=\begin{bmatrix} D(q) & 0 \\ 0 & P_{cc} \end{bmatrix}$$

with U, $P_D(q)$, $P_q(q)\in R^{2n\times 2n}$, $P_{cc}=P_{cc}^T>0$, $\Gamma=\Gamma^T>0$ are the constant positive $n\times n$ matrices. From **Property 1**, it is known that $V(t)$ is a legitimate Lyapunov function candidate. Differentiating $V(t)$ with respect to time and using equation (3) yield

$$\dot{V}(t)=[U\tilde{x}]^T\begin{bmatrix} D(q)P_{12}\dot{\tilde{q}}+C(q,\dot{q})P_{12}\tilde{q} \\ -D(q)\ddot{q}_d-C(q,\dot{q})\dot{q}_d-G(q)+\tau \\ P_{cc}\dot{\tilde{q}} \end{bmatrix}$$

$$+[U\tilde{x}]^T\begin{bmatrix} \frac{1}{2}\dot{D}(q)-C(q,\dot{q}) & 0 \\ 0 & 0 \end{bmatrix}U\tilde{x}$$

$$=[U\tilde{x}]^T\begin{bmatrix} -D(q)\dot{v}+\frac{1}{2}\dot{D}(q)s-C(q,\dot{q})\dot{q}-G(q)+\tau \\ P_{cc}\dot{\tilde{q}} \end{bmatrix} \tag{6}$$

$$=[U\tilde{x}]^T\begin{bmatrix} -D(q)\dot{v}-C(q,\dot{q})v-G(q)+\tau \\ P_{cc}\dot{\tilde{q}} \end{bmatrix} \tag{7}$$

where $v=\dot{q}_d-P_{12}\tilde{q}$ and $s=\dot{\tilde{q}}+P_{12}\tilde{q}$. Equation (7) uses the passivity property, but equation (6) does not. The non-linear feedforward terms, τ_n, can be defined either without using passivity property,

$$\tau_n=D(q)(\dot{v}-\mu s)-\frac{1}{2}\dot{D}(q)s+C(q,\dot{q})\dot{q}+G(q) \tag{8}$$

or using passivity property **Property 2**,

$$\tau_n=D(q)(\dot{v}-\mu s)+C(q,\dot{q})v+G(q)=W(t)\Theta+W_0(t) \tag{9}$$

where Θ is an unknown parameter vector, definitions of $W(t)$, $W_0(t)$ are obvious, μ is a positive scalar, and the linear feedback control law, τ_l, in both cases, is defined as

$$\tau_l=-(P_{ll}+P_{cc}\Gamma^{-1}P_{cc})s+P_{cc}\tilde{q}=$$
$$-(P_{ll}+P_{cc}\Gamma^{-1}P_{cc})\dot{\tilde{q}}-P_{ll}P_{cc}^{-1}\Gamma\tilde{q} \tag{10}$$

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where $P_{ll} \in R^{n\times n}$ is a symmetric positive definite matrix. Then equation (7) becomes

$$\dot{V} = -2\mu V - \tilde{x}^T Q_\mu \tilde{x} \tag{11}$$

where

$$Q_\mu = \begin{bmatrix} P_{ll} + P_{cc}\Gamma^{-1}P_{cc} & P_{ll}P_{cc}^{-1}\Gamma \\ \Gamma P_{cc}^{-1}P_{ll} & \Gamma P_{cc}^{-1}P_{ll}P_{cc}^{-1}\Gamma - \mu P_{cc} \end{bmatrix},$$

$$Q_0 = \begin{bmatrix} P_{ll} + P_{cc}\Gamma^{-1}P_{cc} & P_{ll}P_{cc}^{-1}\Gamma \\ \Gamma P_{cc}^{-1}P_{ll} & \Gamma P_{cc}^{-1}P_{ll}P_{cc}^{-1}\Gamma \end{bmatrix} \tag{12}$$

Lemma 1. *Since P_{cc}, Γ, $P_{ll} \in R^{n\times n}$ are symmetric positive definite matrices, if a reasonably small μ is chosen, then $Q_\mu = Q_\mu^T > 0$*

Proof. Since P_{ll} and Γ are symmetric positive definite matrices, they can be factored as $P_{ll} = P_l^T P_l$ and $\Gamma^{-1} = \Gamma_1^T \Gamma_1$ with P_l and Γ_1 invertible. When $\mu = 0$, Q_0 can be written as $Q_0 = Q_0^T = Q_c^T Q_c$, with

$$Q_c = \begin{bmatrix} P_l & P_l P_{cc}^{-1}\Gamma \\ -\Gamma_1 P_{cc} & 0 \end{bmatrix}.$$

It can be proven that Q_c is invertible, so that $Q_0 > 0$. Also μ can be chosen small enough to guarantee that Q_μ is still positive de*finite. This completes the proof.*
From **Lemma 1** and equation (11), $\dot{V}(t) \le -2\mu V(t)$. This implies the exponential convergence of V, i.e. $V(t) \le e^{-2\mu t} V(0)$, $\forall t > 0$. Putting it into equation (5) gives $\|\tilde{x}(t)\| \le \sqrt{\frac{2V(0)}{\alpha_p}}\, e^{-\mu t}$, where α_p is the smallest eigenvalue of $P_q(q)$. The above result is summarised in the following theorem:

Theorem 1. *If the conditions in* **Lemma 1** *are satisfied, and the control laws are chosen as equations (4), (9) [or (8)], and (10), then the global exponential stability of the tracking errors, $\tilde{q}$, $\dot{\tilde{q}}$, are guaranteed.*

Remark 2. *If the control law is chosen as equations (4), (9) (or (8)) and (10), the closed loop error dynamics is given by*

$$\begin{cases} D(q)(\dot{s} + \mu s) + C(q, \dot{q})s + (P_{ll} + P_{cc}\Gamma^{-1}P_{cc})s - P_{cc}\tilde{q} = 0 \\ \dot{\tilde{q}} = -P_{12}\tilde{q} + s \end{cases} \tag{13}$$

It is noted that equation (13) is a non-linear equation and the proposed controller, though avoiding the use of $\ddot{q}$ and $\hat{D}^{-1}$, does not linearise the closed loop error dynamics.

3.1.2. Adaptive control design. In the case of unknown parameters, the non-linear control law, equation (9) becomes

$$\tau_n(t) = \tilde{D}(q)(\dot{v} - \mu s) + \hat{C}(q, \dot{q})v + \hat{G}(q) \tag{14}$$

where $\hat{D}$, $\hat{C}$, and $\hat{G}$ have the same structure functions as D, C, and G with estimated parameters $\theta_1, \theta_2, \ldots, \theta_p$, and the error equation becomes

$$D(q)(\dot{s} + \mu s) + C(q, \dot{q})s + (P_{ll} + P_{cc}\Gamma^{-1}P_{cc})s - P_{cc}\tilde{q} = W(t)\tilde{\Theta} \tag{15}$$

where (from **Property 3**),

$$W(t)\hat{\Theta} + W_0(t) = \hat{D}(q)(\dot{v} - \mu s) + \hat{C}(q, \dot{q})v + \hat{G}(q) \tag{16}$$

with $\tilde{\Theta} = \hat{\Theta} - \Theta$. The updating law is chosen as

$$\dot{\hat{\Theta}} = \dot{\tilde{\Theta}} = -K_d W^T(t)s \tag{17}$$

where $K_d \in R^{p\times p}$ is a symmetric positive definite matrix. Then we have the following theorem;

Theorem 2. *The adaptive controller, equations (4), (10), and (14) with the updating law, equation (17), is globally convergent that is $\tilde{q}(t)$ and $\dot{\tilde{q}}(t)$ asymptotically converge to zero and all internal signals are bounded.*

Proof. Consider the Lyapunov function candidate

$$V_a(t) = \frac{1}{2}\tilde{x}^T(t)P_q(q)\tilde{x}(t) + \frac{1}{2}\tilde{\Theta}^T(t)K_d^{-1}\tilde{\Theta}(t) \tag{18}$$

The time derivative of $V_a(t)$ along the error equation (15) with the adaptation law, equation (17) yields

$$\dot{V}_a(t) = -2\mu\tilde{x}^T(t)P_q(q)\tilde{x}(t) - \tilde{x}^T(t)Q_\mu\tilde{x}(t) \tag{19}$$

Thus it can be concluded that $\tilde{x}(t) \in L_2^{2n} \cap {}_\infty^{2n}$ and $\tilde{\Theta} \in L_\infty^p$. From this, it is concluded from equations (4), (10), and (14) that $\tau \in L_\infty^n$. This in turn implies, using equation (1) and **Property 1**, that $\ddot{q} \in L_\infty^n$ and, hence, from equation (15) that $\dot{\tilde{x}} \in L_\infty^{2n}$. Since $\dot{\tilde{x}} \in L_\infty^{2n}$, $\tilde{x}$ is uniformly continuous and the proof is completed using the implication:[21] $\tilde{x}(t)$ is uniformly continuous and $\tilde{x} \in L_2^{2n} \Rightarrow \tilde{x} \rightarrow 0$ *as* $t \rightarrow \infty$.

Remark 3. *It is noted that the above control structure without using the joint accelerations and the inverse of the estimated inertia matrix, is due to* **Property 1**. *The passivity property,* **Property 2**, *is only used to simplify the non-linear feedback control law (comparing equations (9) and (8)).*

3.2. Indirect adaptive control

The indirect adaptive control of robot manipulators, pioneered by Middleton and Goodwin,[5] use prediction errors on the filtered joint torque (as opposed to the joint positions) to generate parameter estimates to be used in the control law. The global tracking convergence of the method has been shown and the adaptive controller is composed of a modified least-square estimator and a modified computed-torque controller. The computation of the adaptive controller again requires the inverse of the estimated inertia matrix. The method was further extended by Li and Slotine,[6] and the requirement of the inverse of the estimated inertia matrix was relaxed by using a projection approach.

3.2.1. Parameter estimation for robot manipulators. Using **Property 9**, equation (1) can be written as $\tau = W(q, \dot{q}, \ddot{q})\Theta + W_0(q, \dot{q}, \ddot{q})$. In indirect adaptive control, the parameters are first estimated from the controlled plant, and then the estimated parameters are used to replace the true but unknown plant parameters in the design of relevant controllers. To achieve this, we operate both sides of the

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above equation by $\frac{\lambda_f}{p+\lambda_f}$, where $p=\frac{d}{dt}$ is the differential operator and λ_f is some positive constant specified by the designer;

$$\tau_f = W_f(q,\dot{q})\Theta + W_{0f}(q,\dot{q}) \quad (20)$$

where $W_f=(\frac{\lambda_f}{p+\lambda_f})W$, $W_{0f}=(\frac{\lambda_f}{p+\lambda_f})W_0$, $\tau_f=(\frac{\lambda_f}{p+\lambda_f})\tau$. It is noted that W_f and W_{0f} are functions of q and $\dot{q}$, but not of $\ddot{q}$. W_f and W_{0f} can be computed using the Lagrangian function L. The Lagrangian Equation can be written in vector form as

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = \tau \quad (21)$$

Multiplying both sides of equation (21) by $\frac{\lambda_f}{p+\lambda_f}$ and using **Property 10**, $L = Y(q,\dot{q})\Theta + Y_0(q,\dot{q})$, gives

$$\tau_f = W_f(q,\dot{q})\Theta + W_{0f}(q,\dot{q}) \quad (22)$$

Thus, if the **Lagrangian** function L is known, it is relatively easy to obtain W_f and W_{0f}. Equation (22) can be used to estimate the unknown parameters as follows. Define the prediction (or input) error as

$$e_f := W_f(q,\dot{q})\hat{\Theta}(t) + W_{0f}(q,\dot{q}) - \tau_f = W_f(q,\dot{q})\hat{\Theta}(t) - W_f(q,\dot{q})\Theta = W_f(q,\dot{q})\tilde{\Theta}(t) \quad (23)$$

The prediction error reflects the error between the currently estimated parameters $\hat{\Theta}(t)$ and the true parameters Θ. There are a number of identification methods[22] to estimate the parameters from equation (23). Here, briefly described are two popular methods, i.e. the gradient estimator and the least square estimator. Both have a common form of parameter update law, namely

$$\dot{\hat{\Theta}}(t) = \dot{\tilde{\Theta}}(t) = -P(t)W_f^T(t)e_f(t) \quad (24)$$

where $P(t)\in R^{p\times p}$ is a constant positive definite matrix in the gradient estimator or a time-varying positive definite gain matrix for the least-square estimator. The gain matrix $P(t)$ determines the estimator performance and as indicated above, may be generated in different forms from different perspectives. However, irrespective of $P(t)$—the prediction error $e_f(t)$ is square integrable (mathematically, $e_f\in L_2^n$), and this is important in proving the global tracking convergence of the various indirect adaptive controllers. The gradient estimator has

$$P(t) = K_d \quad (25)$$

The least-square estimator has $P(t)$ as a time varying matrix:

$$\dot{P}(t) = \mu(t)P(t) - P(t)W_f^T(t)W_f(t)P(t) \quad (26)$$

where $P(0)=P^T(0)>0$, and $\mu(t)>0$ is a scalar quantity. Both estimators have the following the properties.

Lemma 2. *The gradient estimator, equations (24), (25) and the least-square estimator, equations (24), (26), applied to the error system, equation (23), yields the following properties, regardless of the control law: 1) The estimator parameter, $\hat{\Theta}\in L_\infty^p$, and 2) The prediction error, $e_f\in L_2^n$.*

Proof. Choose the Lyapunov function $V(t)=\tilde{\Theta}^T(t)P^{-1}(t)\tilde{\Theta}$

1) For the gradient estimator case: Using equations (23)–(25), the derivative of $V(t)$ is

$$\dot{V}(t) = 2\tilde{\Theta}^T K_d^{-1}\dot{\tilde{\Theta}} = -2\tilde{\Theta}^T W_f^T W_f \tilde{\Theta} = -2e_f^T e_f \quad (27)$$

This leads to $0\le V(t)\le V(0)$, which means that $\tilde{\Theta}\in L_\infty^p \Rightarrow \hat{\Theta}\in L_\infty^p$ (1 of **Lemma 2** above) since the unknown parameter Θ is bounded. Since $V(0)$ is a finite positive constant, integration of V leads to $\int_0^\infty e_f^T(t)e_f(t)dt = V(0)-V(\infty)\le V(0)$, so that $e_f\in L_2^n$ (2 of **Lemma 2**).

2) For least-square estimator case: The relevant derivative of $V(t)$ under the equations (23), (24), and (26) is

$$\dot{V}(t) = 2\tilde{\Theta}^T P^{-1}\dot{\tilde{\Theta}} + \tilde{\Theta}^T \dot{P}^{-1}\tilde{\Theta} = -\mu(t)V(t) - \tilde{\Theta}^T W_f^T W_f \tilde{\Theta} = -\mu(t)V(t) - e_f^T(t)e_f(t) \quad (28)$$

This leads to the desired results.

Remark 4. *The gradient estimator is computationally simple but its convergence is usually slow. The least-square estimator in equation (26) represents a whole class of estimators, including the standard least-square method (corresponding to $\mu(t)=0$), constant-forgetting-factor estimator, constant-trace-forgetting factor, and so on. Comparing equations (27) and (28), it is easy to see that the least-square estimator has a faster convergent rate than the gradient estimator.*

Remark 5. *The convergence of the parameter error needs the persistently exciting condition, namely that there exist positive constants α_1, α_2 and δ such that $\alpha_1 I \le \int_t^{t+\delta} W_f^T(r)W_f(r)dr \le \alpha_2 I$, $\forall t\ge 0$.*

3.2.2. Indirect adaptive control design. The indirect adaptive controller proposed uses a first-order filter to avoid the joint acceleration measurement. The adaptive controller is composed of two parts: a control law and a parameter estimation law. This subsection is focused on the former. The controller part has the following structure,

$$\tau = \tau_m + \tau_\alpha + \tau_b \quad (29)$$

$$\tau_m = \hat{D}(q)[\ddot{q}_d - K_v\dot{\tilde{q}} - K_p\tilde{q}] + \hat{V}(q,\dot{q}) + \hat{G}(q) \quad (30)$$

where τ_a and τ_b are signals dealing with terms arising in the adaptive case due to the commuting of time varying operators and will be decided later. The parameter estimator is given by equation (24), with $P(t)$ generated by equation (25) or equation (26). Substituting equation (29) into equation (1) yields the closed-loop dynamics as

$$\hat{D}(\ddot{\tilde{q}} + K_v\dot{\tilde{q}} + K_p\tilde{q}) = W\tilde{\Theta} + \tau_a + \tau_b \quad (31)$$

Multiplying both sides of equation (23) by $\dfrac{p+\lambda_f}{\lambda_f}$ gives

$$\left(\frac{p+\lambda_f}{\lambda_f}\right)e_f = W\hat{\Theta} + \frac{1}{\lambda_f}W_f\dot{\hat{\Theta}} + W_0 - \tau \quad (32)$$

Combining equations (31) and (32) gives

$$\hat{D}(\ddot{\tilde{q}} + K_v\dot{\tilde{q}} + K_p\tilde{q}) = \frac{p+\lambda_f}{\lambda_f}e_f - \frac{1}{\lambda_f}W_f\dot{\hat{\Theta}} + \tau_a + \tau_b \quad (33)$$

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In order to cancel the term, $\frac{1}{\lambda_f}W_f\dot{\hat{\Theta}}$, in equation (33), let

$$\tau_a = \frac{1}{\lambda_f} W_f \dot{\hat{\Theta}} \tag{34}$$

Further letting

$$\tau_b = \frac{1}{\lambda_f} \hat{D} J^{-1} \frac{d}{dt} [J \hat{D}^{-1}] e_f \tag{35}$$

Equation (33) becomes

$$\ddot{\tilde{q}} + K_v \dot{\tilde{q}} + K_p \tilde{q} = \frac{p+\lambda_f}{\lambda_f} [\hat{D}^{-1} e_f] \tag{36}$$

$\hat{D}^{-1} \in L_\infty + e_f \in L_2 \Rightarrow \frac{1}{\lambda_f} \hat{D}^{-1} e_f \in L_2$, so that $\tilde{q} \in L_\infty$, $\tilde{q}$ and $\dot{\tilde{q}} \in L_2$. Thus using the same mathematical proof of Middleton and Goodwin[5], it can be proved that $e_f \rightarrow 0$, $\tilde{q} \rightarrow 0$, and $\dot{\tilde{q}} \rightarrow 0$. The above results can be summarised as the following theorem.

Theorem 3. *If $\hat{D}^{-1}(q) \in L_\infty$, the desired trajectories q_d, $\dot{q}_d$, and $\ddot{q}_d \in L_\infty$, then the adaptive control law equations (29), (30), (34), and (35), with parameter updating law equations (24) and (25) or (26) guarantees that the prediction (input) error $e_f \rightarrow 0$, the position tracking (output) error $\tilde{q} \rightarrow 0$, and the velocity tracking error $\dot{\tilde{q}} \rightarrow 0$ as time $t \rightarrow \infty$.*

Remark 6. *The additional terms, τ_a and τ_b in equation (29), have their roots in the augmented error adaptive control of linear plants.*[21]

3.3. *Combined adaptive control*

The control law and error equation are same as in direct adaptive control (**Section 3.1**) and are summarized as follows:

$$\begin{aligned} \tau &= W(q, \dot{q}, v, \dot{v})\hat{\Theta} + W_0(q, \dot{q}, v, \dot{v}) + \tau_l \\ &= \hat{D}(q)\dot{v} + \hat{C}(q, \dot{q})v + \hat{G}(q) + \tau_l \end{aligned} \tag{37}$$

$$D(q)\dot{s} + C(q, \dot{q})s + (P_{ll} + P_{cc}\Gamma^{-1}P_{cc})s - P_{cc}\tilde{q} = W(q, \dot{q}, v, \dot{v})\tilde{\Theta} \tag{38}$$

The direct parameter adaptation law:

$$\dot{\hat{\Theta}} = \dot{\hat{\Theta}}_d = -K_d W^T(q, \dot{q}, v, \dot{v})s \tag{39}$$

The indirect parameter adaptation law:

$$\dot{\hat{\Theta}}_I = -P_I(t) W_f^T(q, \dot{q}) e_f(t) \tag{40}$$

where $P_I(t)$ is a constant positive matrix for gradient estimator or time-varying definite gain matrix for least-square estimator.

Since equation (23) is a complicated vector equation and includes centripetal and Coriolis terms, it needs a heavy computation burden. Now we propose a simple method to implement equation (23) using the principle of energy conservation. From **Property 8**,

$$\tau^T \dot{q} = \frac{dH}{dt} \tag{41}$$

where H is a Hamiltonian function representing the total mechanical energy of the robot system. From **Property 11**, we have $\tau^T \dot{q} = \frac{d}{dt}[W_H(q, \dot{q})]\Theta + \frac{d}{dt}[W_{0H}(q, \dot{q})]$. Multiplying both sides of the above equation by $\lambda_f/(p+\lambda_f)$ leads to

$$\begin{aligned} \frac{\lambda_f}{p+\lambda_f}[\tau^T \dot{q}] &= \lambda_f [W_H(q, \dot{q}) - \frac{\lambda_f}{p+\lambda_f}(W_H(q, \dot{q}))]\Theta \\ &+ \lambda_f [W_{0H}(q, \dot{q}) - \frac{\lambda_f}{p+\lambda_f}(W_{0H}(q, \dot{q}))] \end{aligned}$$

This can be written as

$$\tau_{fa} = w_f(q, \dot{q})\Theta + w_{0f}(q, \dot{q}) \tag{42}$$

where

$$\begin{aligned} \tau_{fa} &= \frac{\lambda_f}{p+\lambda_f}[\tau^T \dot{q}];\ w_f(q, \dot{q}) = \lambda_f [W_H(q, \dot{q}) \\ &- \frac{\lambda_f}{p+\lambda_f} W_H(q, \dot{q})];\ w_{0f}(q, \dot{q}) = \lambda_f [W_{0H}(q, \dot{q}) \\ &- \frac{\lambda_f}{p+\lambda_f} W_{0H}(q, \dot{q})] \end{aligned}$$

In this notation, the indirect parameter estimation law, equation (40), becomes

$$\dot{\hat{\Theta}}_I = -P_I(t) w_f^T(q, \dot{q}) e_{fa}(t) \tag{43}$$

where $e_{fa} = \hat{\tau}_{fa} - \tau_{fa} = w_f^T(q, \dot{q})\tilde{\Theta}$.

Remark 7. *Note that the estimation models (24) and (43) are different. Equation (24) is an n dimension vector, while equation (43) is a scalar equation. Using the energy relationship, equation (41), the computation of $w_f \in R^{1\times p}$ and $w_{0f} \in R$ in equation (43) is greatly simplified compared with the computation of $W_f \in R^{n\times p}$ and $W_{0f} \in R^{n\times 1}$ in equation (24), since in equation (43) it is not necessary to compute the complex centripetal and Coriolis terms.*

If the above two adaptation laws are directly added together, the stability analysis is difficult. In order to overcome this difficulty, the following improved indirect adaptation law is introduced:

$$\dot{\hat{\Theta}}_I = -K_d w_f^T(q, \dot{q}) R e_{fa}(t) \tag{44}$$

where R is a positive constant matrix. Now define the combined adaptation law as:

$$\dot{\hat{\Theta}} = -P(t)[W^T(q, \dot{q}, v, \dot{v})s + w_f^T(q, \dot{q})R(t)e_{fa}(t)] \tag{45}$$

The combined adaptive law takes into account information in both the tracking error and the prediction error in estimating the unknown parameters. The structure of the combined adaptive control law and error equation is still the same as that of the direct adaptive controller, i.e. equations (37) – (38). $R=0$ and $P(t)=K_d$ would correspond to the direct adaptive control version (**Section 3.1**). There are a number of methods to generate the combined adaptation gain matrix $P(t)$, e.g. gradient method, least–square method, bounded–gain–forgetting method and cushioned–floor method[6, 22]. The different methods give the different performances under different conditions. For simplicity, here

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we only discuss the simple gradient method, i.e. $P(t)=K_d$ is a constant symmetric positive definite matrix, and choose $R=1$.

Theorem. 4 *For robot dynamics governed by equation (1), if the control law (37), and the combined adaptation law (45) are used, and the desired trasjectories and up to the second order derivatives are bounded, then the following properties hold: (i) The tracking errors $\tilde{q}$ and $\dot{\tilde{q}}$, and the prediction error e_{fa} all globally converge to zero, i.e.* $\lim_{t\to\infty}\tilde{q}=0$, $\lim_{t\to\infty}\dot{\tilde{q}}=0$, $\lim_{t\to\infty} e_{fa}=0$ *with the other signals remaining bounded; (ii) If $q_d^{3}(t)$ is uniformly continuous almost everywhere, then* $\lim_{t\to\infty} W(q,\dot{q}\,v,\dot{v})\tilde{\Theta}=0$*; (iii) If W_d is persistently exciting, then* $\lim_{t\to\infty}\tilde{\Theta}=0$*; (iv) If further $W_f^TW_f>\alpha I_{p\times p}$, then $\tilde{\Theta}$ exponentially converges to zero.*

Proof. Taking the same Lyapunov function candidate as in direct adaptive control, equation (18), the corresponding derivative of the Lyapunov function is

$$\dot{V}_a=\tilde{x}^T(t)P_q(q)\dot{\tilde{x}}(t)+\frac{1}{2}\tilde{x}^T(t)\dot{P}_q(q)\tilde{x}(t)+\tilde{\Theta}^TK_d^{-1}\dot{\tilde{\Theta}}=$$
$$-2\mu\tilde{x}^T(t)P_q(q)\tilde{x}(t)-\tilde{x}^T(t)Q_\mu\tilde{x}(t)-\tilde{\Theta}^Tw_f^T(t)w_f(t)\tilde{\Theta}\le 0 \qquad (46)$$

The boundedness of $\tilde{x}(t)$ and $\tilde{\Theta}(t)$ is found immediately from (18) and (46). Similar to the proof of the **Theorem 2**, it is necessary to prove the boundedness of $\ddot{V}_a(t)$. From the closed-loop error equation (38) and **Property 1**, the boundedness of desired trajectories $\ddot{q}_d$, $\dot{q}_d$, q_d, $\tilde{q}$, $\dot{\tilde{q}}$, and $\tilde{\Theta}$ implies the boundedness of $\ddot{q}$ and $\ddot{\tilde{q}}$. This implies **Lemma 2** guarantees the convergence of $\tilde{q}(t)$, $\dot{\tilde{q}}(t)$, and $e_{fa}(t)$. This proves part (i).

In (38), it is only necessary to prove that $D(q)\dot{s}\to 0$ as $t\to\infty$. Utilizing integration by parts.

$$\int_t^{t+c} D(\tau)\dot{s}(\tau)d\tau=D(t+c)s(t+c)-D(t)s(t)$$
$$-\int_t^{t+c}\dot{D}(\tau)s(\tau)d\tau \qquad (47)$$

Notice that $\dot{D}(q)$ can be written as: $\dot{D}=\Sigma_{i=1}^{i=n}\frac{\partial D}{\partial q}\dot{q}_i$. $\dot{q}\in L_\infty^n$ has been proved above, which implies $\dot{D}(q)$ is bounded. Thus, $\lim_{t\to\infty}[\int_t^{t+c}\dot{D}(\tau)s(\tau)d\tau]=0$. From (47), this implies that, $\lim_{t\to\infty}[\int_t^{t+c}D(\tau)\dot{s}(\tau)d\tau]=0$. This implies $\lim_{t\to\infty}[D(q)\dot{s}]=0$. From (38), $\lim_{t\to\infty}[W(q,\dot{q},v,\dot{v})\tilde{\Theta}]\to 0$. This proves part (ii).

The convergence of the estimated parameters to the true parameters can be shown by noting that the adaptation law

$$\dot{\tilde{\Theta}}(t)=-K_d[W^T(t)s+w_f^T(t)w_f(t)\tilde{\Theta}(t)]=-K_dw_f^T(t)w_f(t)\tilde{\Theta}(t)$$
$$-K_dW^T(t)s \qquad (48)$$

represents exponentially stable dyamics[22] with convergent input $-P_dW^T(t)$s.

Using (18), (48), and the condition of part (iv) implies that $\exists\gamma>0$, such that

$$\dot{V}_a(t)+\gamma V_a(t)\le 0 \qquad (49)$$

Similar to **Theorem 1**, exponential stability of the position tracking error $\tilde{q}$, $\dot{\tilde{q}}$, and the parameter error $\tilde{\Theta}$ with a time constant $\frac{1}{\gamma}$ is thus shown. □

4. VARIABLE STRUCTURE ADAPTIVE CONTROL

The variable structure adaptive controllers combine features of the robust design based on variable structure systems with parameter adaptive control. In order to introduce this new design method, the variable structure control design approach is first reviewed.

4.1 Variable structure control design

In variable structure control (VSC), the goal is the same as in adaptive control discussed in **Section 3**. The approach used in accomplishing this goal, is however, different: VSC exploits a variable structure feedback control mechanism where a sliding mode occurs during the transient process. The Lyapunov direct method can be used to design a variable structure controller.[12, 13, 16–18]

The main difference between adaptive control and variable structure control is that parameter estimates are used in the adaptive control, while, instead, switching functions are used in variable structure control. A variable structure control scheme assumes that the unknown manipulator and load parameters, Θ, are known to lie in a bounded set: $\Omega=\{\Theta|\ |\theta_i|\le\bar{\theta}_i,\ i=1,2,\ldots,p\}$. The variable structure controller proposed in Yu[13] is outlined in the following theorem.

Theorem 5. *If the robot manipulator, equation (1), with constant but unknown parameters Θ and with the robust control law:*

$$F(t)=\tilde{x}^T(t)P_1^TW(t)=s(t)^TW(t)=[f_1(t),f_2(t),\ldots,f_p(t)] \qquad (50)$$

$$f_i(t)=\sum_{j=1}^{n}s_j(t)W_{ji}(t)\quad i=1,2,\ldots,p \qquad (51)$$

$$\Theta_v(t)=-F_1(t)\bar{\Theta},\quad \bar{\theta}_i\ge\theta_i\forall i \qquad (52)$$

$$F_1(t)=diag[sgn(f_1),sgn(f_2),\ldots,sgn(f_p)] \qquad (53)$$

$$\tau_n(t)=W(t)\Theta_v(t)+W_0(t) \qquad (54)$$

and (4), (10), where $P_1=[I_{n\times n}P_{12}]\in R^{n\times 2n}$, then for a reasonably small positive constant μ, all the signals in the system are bounded and $\tilde{x}(t)$ tends to zero with at least an exponential rate that is independent of the excitation.

The proof of **Theorem 5** is given in Yu.[13]

Remark 8. *The link between the adaptive control algorithm and the variable structure control algorithm is given by $\nu_1\dot{\Theta}_x=-\nu_2\Theta_x-F(t)$: 1) $\nu_2=0$ and $\nu_1=K_d^{-1}$ leads to the adaptive control scheme proposed in* **Section 3.1**; *2) $\nu_1=0$ and $\nu_2=diag[|f_1|\bar{\theta}_1^{-1},|f_2|\bar{\theta}_2^{-1},\ldots,|f_p|\bar{\theta}_p^{-1}]$ leads to the variable structure control scheme given in* **Theorem 5**.

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4.2. Variable structure adaptive control design

From simulation results, we find that the further $\bar{\Theta}$ is from the true value Θ, the more serious the chattering problem.[13, 19] This observation motivates us to propose the following variable structure adaptive (VSA) control algorithms. The estimated bounds are used to replace the upper bounds in the VS controller, so the controller is called a variable structure adaptive (VSA) controller. The difference between VSA control and VS control is the estimated upper bounds used in the former and the fixed upper bounds used in the latter. All the algorithms in this section have the same control law

$$\tau = W_v(t)\Theta_v + W_{v0}(t) + \tau_i \tag{55}$$

where $W_v(t)$ and $W_{v0}(t)$ satisfy the following equation $W_v(t)\Theta + W_{v0}(t) = D(q)\dot{v} + C(q, \dot{q})v + G(q)$. The VSA law is defined as

$$\Theta_v(t) = F_1(t)\Theta_s(t) \tag{56}$$

where $F = W_v^T s = [f_1, f_2, \ldots, f_p]^T$; $F_1 = diag[sgn(f_1), sgn(f_2), \ldots, sgn(f_p)]$; $f_i = \Sigma_{j=1}^n s_j W_{vji}$, $i = 1, 2, \ldots, p$, $\Theta_s(t) \in R^{p\times 1}$ is an adaptation law of the bounds of the unknown parameters and will be given later. It is easy to see that the ith component of the vector $\Theta_v(t)$ can be expressed as

$$\Theta_{vi}(t) = \Theta_{si}(t) sgn[\sum_{j=1}^{n} s_j(t) W_{vji}(t)], \quad i = 1, 2, \ldots, p \tag{57}$$

The following three choices for the adaptation law $\Theta_s(t)$ are proposed.

Algorithm 1:

$$\dot{\tilde{\Theta}}_s(t) = \dot{\Theta}_s(t) = -P_s F_1(t) F(t) \text{ or } \dot{\tilde{\Theta}}_{si} = \dot{\Theta}_{si} = -P_{si}|\sum_{j=1}^{n} s_j W_{vji}|, \quad i = 1, 2, \ldots, p$$

Algorithm 2:

$$\dot{\Theta}_s(t) = \dot{\Theta}_s(t) = -\Gamma_1 \tilde{\Theta}_s(t) - P_s F_1(t) F(t) \text{ or } \dot{\tilde{\Theta}}_{si} = \dot{\Theta}_{si} = -\Gamma_{1i}\tilde{\Theta}_{si} - P_{si}|\sum_{j=1}^{n} s_j W_{vji}|, \quad i = 1, 2, \ldots, p$$

Algorithm 3:

$$\begin{aligned}\tilde{\Theta}_s(t) = \dot{\Theta}_s(t) &= -\Gamma_1 \Theta_s(t) - P_s F_1(t) F(t) \text{ or } \dot{\tilde{\Theta}}_{si} = \dot{\Theta}_{si} \\ &= -\Gamma_{1i}\Theta_{si} - P_{si}|\sum_{j=1}^{n} s_j W_{vji}|, \quad i = 1, 2, \ldots, p\end{aligned}$$

where P_s and $\Gamma_1 \in R^{p\times p}$ are diagonal positive definite matrices, $\tilde{\Theta}_s(t) = \Theta_s(t) + \Theta$, $\bar{\Theta}_i \geq |\Theta_i|$.

Using control law (55) in the robot dynamics, equation (1), the corresponding error equation is:

$$\begin{cases} D(q)\dot{s} + C(q, \dot{q})s + (P_{ll} + P_{cc}\Gamma^{-1}P_{cc})s - P_{cc}\tilde{q} = W_v(t)\tilde{\Theta} \\ \dot{\tilde{q}} = -P_{12}\tilde{q} + s \end{cases} \tag{58}$$

where $\tilde{\Theta} = \Theta_v(t) - \Theta \in R^p$.

Theorem 6. *For the error equation (58), under the VSA law, equation (56) [or (57)], the following properties hold: i) If the adaptive gain law is chosen as* **Algorithm 1**, *then the tracking error $\tilde{q}(t)$ and $\dot{\tilde{q}}(t)$ converge asymptotically to zero. ii) If* **Algorithm 2** *is chosen, then the conclusion of the* **Algorithm 1** *is true. iii) If* **Algorithm 3** *is chosen, then the system is globally uniformly ultimately bounded, that is, the tracking errors $\tilde{q}(t)$ and $\dot{\tilde{q}}(t)$ are bounded and the boundedness depends on Γ_1.*

Proof: The proof is based on the Lyapunov theory. Define a Lyapunov function candidate as

$$V[\tilde{x}(t), \tilde{\Theta}_s(t)] = \frac{1}{2}\tilde{x}^T(t)P_q(q)\tilde{x}(t) + \frac{1}{2}\tilde{\Theta}_s^T(t)P_s^{-1}\dot{\tilde{\Theta}}_s(t) \tag{59}$$

where $\tilde{x}(t)$ and $P_q(q)$ are defined as before. So $V[\tilde{x}(t), \tilde{\Theta}_s(t)]$ is a legitimate Lyapunov function candidate. Differentiating equation (59) with respect to time along the error equation (58) gives

$$\dot{V} = -\tilde{x}^T(t)Q_0\tilde{x}(t) + \tilde{\Theta}^T(t)F(t) + \tilde{\Theta}_s^T(t)P_s^{-1}\dot{\tilde{\Theta}}_s(t) \tag{60}$$

i) Using **Algorithm 1** in equation (60) leads to

$$\begin{aligned}\dot{V} &= -\tilde{x}^T(t)Q_0\tilde{x}(t) + (\Theta_v - \Theta)^T F(t) - \tilde{\Theta}_s^T(t)F_1(t)F(t) \\ &\leq -\tilde{x}^T(t)Q_0\tilde{x}(t) - \sum_{i=1}^{p}(\bar{\Theta}_i - |\Theta_i|)\sum_{j=1}^{n}s_j(t)W_{vji}(t)| \\ &< -\tilde{x}^T(t)Q_0\tilde{x}(t)\end{aligned}$$

Using similar reasoning as in **Theorem 1** leads to $\lim_{t\to\infty}\tilde{x}(t) = 0 \Rightarrow \lim_{t\to\infty}\tilde{q}(t) = 0$ and $\lim_{t\to\infty}\dot{\tilde{q}}(t) = 0$.

ii) Using **Algorithm 2** in equation (60),

$$\begin{aligned}\dot{V} = &-\tilde{x}^T(t))Q\tilde{x}(t) + \tilde{\Theta}^T(t)F(t) - \tilde{\Theta}_s^T(t)P_s^{-1}\Gamma_1\tilde{\Theta}_s(t) \\ &-\tilde{\Theta}_s^T(t)F_1(t)F(t)) < -\tilde{x}^T(t)Q\tilde{x}(t) \\ &-\tilde{\Theta}_s^T(t)P_s^{-1}\Gamma_1\tilde{\Theta}_s(t)\end{aligned}$$

Since Γ_1 and P_s are positive definite diagonal matrices, $P_s^{-1}\Gamma$ is a positive definite matrix. This implies that $\lim_{t\to\infty}\tilde{x}(t) = 0$ and $\lim_{t\to\infty}\tilde{\Theta}_s(t) = 0$.

iii) Using **Algorithm 3** in equation (60) gives

$$\begin{aligned}\dot{V} &= -\tilde{x}^T(t)Q\tilde{x}(t) - \tilde{\Theta}_s^T(t)P_s^{-1}\Gamma_1\Theta_s(t) - \tilde{\Theta}_s^T(t)F_1(t)F(t) \\ &\leq -\tilde{x}^T(t)Q\tilde{x}(t) - \tilde{\Theta}_s^T(t)P_s^{-1}\Gamma_1\Theta_s(t)\end{aligned}$$

This implies that the system is globally uniformly ultimately bounded. □

Remark 9. Algorithm 1 *is the same as that proposed in the literature.*[16, 17] *It is a special case of the* **Algorithms 2** *and* **3** ($\Gamma_1 = 0$). **Algorithm 2** *is a quite general form. When $\Gamma_1 = 0$, it leads to the* **Algorithm 1**; *when $P_s = 0$ and $\Gamma_1 = \infty$, it leads to the variable structure control scheme.*[12, 13]

Remark 10. Algorithm 1 *guarantees that the tracking errors $\tilde{q}$ and $\dot{\tilde{q}}$ converge asymptotically to zero, and the adaptive error $\tilde{\Theta}_s$ is bounded.* **Algorithm 2** *guarantees that both the tracking errors ($\tilde{q}$ and $\dot{\tilde{q}}$) and the adaptive error ($\tilde{\Theta}_s$) converge asymptotically to zero. They have a same Lyapunov function, but the derivative with respect time of* **Algorithm 2** *is more negative than that of* **Algorithm 1**, *since the extra term $-\tilde{\Theta}_s^T(t)P_s^{-1}\Gamma_1\tilde{\Theta}_s(t)$ appears in* **Algorithm 2**. *This implies that the convergent rate of the* **Algorithm 2** *is faster than that of the* **Algorithm 1**.

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Algorithm 3 *only guarantees that the tracking errors are bounded, that is, the system is only globally uniformly ultimately bounded. If these three* **Algorithms** *are used individually,* **Algorithm 2** *gives better results. However,* **Algorithm 3** *combined with direct adaptive control algorithms proposed in* **Section 3**, *gives a very high performance (see next section).*

Remark 11. Algorithms 1 *and* **3** *use only very general information about the structure of the robot dynamic equation (***Property 9***). The upper bound of the uncertain parameters (Θ) is only used in the stability proof, and is not required in the controller. This avoids the problem of high gains which lead to excessive control signals.* **Algorithm 2** *indirectly uses the upper bound of the uncertainty parameters ($\tilde{\Theta}$), but this is in the adaptive law.*

4.3. Robustness properties

Generally speaking, uncertainties are functions of the system states and may grow beyond any constant bound if the system becomes unstable. For example, the viscous and Coulomb friction forces may be modelled as $F_v\dot{q}+F_c sgn(\dot{q})$. Therefore, from **Properties 4 and 5**, it is reasonable to assume that the uncertainty effects are represented by

$$\|\tau_d\| \le d_0 + d_1\|\dot{\tilde{q}}\| + d_2\|\tilde{q}\| \qquad (61)$$

where $d_0>0$, $d_1>0$, and $d_2>0$ are some constants. To take account of the uncertainties, the controller, equation (55), is modified as

$$\tau_n(t) = W(t)\Theta_v(t) + W_0(t) + \tau_c s + \tau_0 sgn(s) \qquad (62)$$

$$\dot{\tau}_c(t) = -c\|s\|^2; \quad \dot{\tau}_0(t) = -c_0\|s\| \qquad (63)$$

where the switching law $\Theta_v(t)$ is the same as in equation (57), the adaptation law is in the form of **Algorithms 1–3**, and $c>0$ and $c_0>0$ are constants.

The following error equation can be obtained using the control laws equations (55), (56), (62), and (63),

$$\begin{cases} D(q)\dot{s} + C(q,\dot{q})s + (P_{ll} + P_{cc}\Gamma^{-1}P_{cc})s - P_{cc}\tilde{q} = W(t)\tilde{\Theta} + \tau_c s - \tau_d \\ \dot{\tilde{q}} = -P_{12}\tilde{q} + s \end{cases} \qquad (64)$$

Theorem 7. *Considering robot system, equation* (1), *with input uncertainty, equation* (61), *if the control law equations* (55), (56), (62), *and* (63), *and* **Algorithm 1** *(or* **2** *or* **3***) is used, then the conclusions in* **Theorem 6** *are also true.*

Proof: Let the Lyapunov function candidate be

$$V[\tilde{x}(t), \tilde{\Theta}_s(t), \tilde{\tau}_c] = \frac{1}{2}\tilde{x}^T(t)P_q(q)\tilde{x}(t) + \frac{1}{2}\tilde{\Theta}_s^T(t)P_s^{-1}\tilde{\Theta}_s(t)$$

$$+\frac{1}{2}\tilde{\tau}_c^T c^{-1}\tilde{\tau}_c + \frac{1}{2}\tilde{\tau}_0^T c_0^{-1}\tilde{\tau}_0 \qquad (65)$$

where $\tilde{\tau}_c = \bar{\tau}_c - \tau_c$, $\bar{\tau}_c$ is a desired constant value of τ_c, and the other parameters are the same as in equation (59). Differentiating equation (65) with respect to time along the error equation (64) and using **Property 2** gives

$$\dot{V} = -\tilde{x}^T(t)Q\tilde{x}(t) + \tilde{\Theta}^T(t)W^T(t)s(t) + s^T(t)[\tau_c s(t) - \tau_d]$$

$$+\tilde{\Theta}_s^T(t)P_s^{-1}(t)\dot{\tilde{\Theta}}_s(t) + \tilde{\tau}_c c^{-1}\dot{\tilde{\tau}}_c + \tilde{\tau}_0^T c_0^{-1}\dot{\tilde{\tau}}_0 \qquad (66)$$

Since **Algorithm 2** gives a general form of Θ_s, we only prove the theorem for this case. For **Algorithms 1** and **3**, a similar procedure can be followed. From equation (61).

$$-s^T(t)\tau_d(t) \le \|s\|(d_0 + d_1(\|s\| + \lambda_M(P_{12})\|\tilde{q}\|) + d_2\|\tilde{q}\|)$$

$$\le d_0\|s\| + d_1\|s\|^2 + (\lambda_M(P_{12}))d_1 + d_2)\|s\|\|\tilde{q}\| \qquad (67)$$

where λ_M (P_{12}) is the largest eigenvalue of the matrix P_{12}. Putting **Algorithm 2**, equations (56), (63), and (67) into equation (66) gives

$$\dot{V} \le -\tilde{x}^T(t)Q_\mu\tilde{x}(t) - \tilde{\Theta}_s(t)P_s^{-1}\Gamma_1\tilde{\Theta}_s - (\bar{\tau}_c - d_1)\|s\|^2$$
$$-(\bar{\tau}_0 - d_0)\|s\| + (\lambda_M(P_{12})d_1 + d_2)\|s\|\|\tilde{q}\| - \mu\|\tilde{q}\|^2$$

$$\le -\tilde{x}^T(t)Q_\mu\tilde{x}(t) - \tilde{\Theta}_s P_s^{-1}\Gamma_1\tilde{\Theta}_s - [\|s\| \ \ \|\tilde{q}\|]Q_2\begin{bmatrix}\|s\| \\ \|\tilde{q}\|\end{bmatrix} < 0 \qquad (68)$$

where

$$Q_2 = \begin{bmatrix} \bar{\tau}_c - d_1 & -\dfrac{\lambda_M(P_{12})d_1 + d_2}{2} \\ -\dfrac{\lambda_M(P_{12})d_1 + d_2}{2} & \mu \end{bmatrix}$$

and $\mu>0$. Using similar reasoning as in **Lemma 1**, we can choose suitable P_u, P_{cc}, Γ, and μ to guarantee that Q_μ is a positive definite matrix. If we choose $\bar{\tau}_c$ sufficiently large, then the matrix Q_2 is a positive definite. This completes the proof. □

4.4 Continuous approximations of the control laws

The control laws given above are discontinuous and give rise to chattering of the trajectories about the surface $s=0$. The reason resulted in the discontinuous laws is due to the signum function *sgn*(*s*). In general, it is desirable to eliminate this problem and there are two approximate methods. The first one, introduces a thin boundary layer neighbouring the switching surface[11]; that is, the signum function *sgn*(*s*) is replaced by a saturation function *sat*(*s*), which is defined as follows:

$$sat(s) = \begin{cases} 1 \ \ if \ s > \delta \\ \dfrac{s}{\delta} \ if \ |s| < \delta \\ -1 \ \ if \ s < -\delta \end{cases}$$

where δ is the boundary layer thickness. Using similar methods as above, we can prove that both approximate methods give bounded tracking error.

The second one[13] uses $\frac{s}{|s|+\delta}$ to replace the signum function *sgn*(*s*), where δ is a positive constant chosen by the designers. However, both of the above modifications sacrifice the convergence of the closed loop system. A new continuous law[16] uses the function $\frac{s}{|s|+\delta_o e^{-\delta_1 t}}$ to replace the signum function. This approach not only is the problem of existence of solutions due to discontinuities in the control eliminated, but also the tracking errors can be shown to be globally stable.

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5 COMBINED DIRECT AND VARIABLE STRUCTURE ADAPTIVE CONTROL

5.1 Combined adaptive controller design

The combined direct, indirect, and variable structure adaptive control for linear plants with unknown parameters is presented by Narendra[14]. The combined adaptive control can improve the transient behaviour and have strong robustness compared to direct adaptive control; it also avoids the chattering problem and improves the asymptotic behaviour compared with VSA control. This method has been extended to robot manipulator control[15]. The robust analysis of the method is performed in this paper. In the combined adaptive control, we choose the control input $\tau(t)$ and adaptation law as

$$\tau(t) = W_c(t)\hat{\Theta}_c(t) + W_{c0}(t) + \tau_l(t) \tag{69}$$

$$\hat{\Theta}_c(t) = \Theta_d(t) + \Theta_v(t) \tag{70}$$

where $W_c(t)\Theta + W_{c0}(t) = -\frac{1}{2}\dot{D}(q)s(t) + D(q)\dot{v}(t) + C(q, \dot{q})\dot{q} + G(q)$ and linear control law $\tau_l(t)$, reference error s, v, P_{12}, Γ, P_{ll} are defined as before. Both $\Theta_d(t)$ and $\Theta_s(t)$ are adjusted adaptively. $\Theta_d(t)$ can be adjusted as in the direct, indirect adaptive control proposed in **Section 3.1**, or combined adaptive control by the direct and indirect adaptive control discussed in **Section 3.3**. $\Theta_s(t)$ has the form of **Algorithm 3** given in **Section 4**, but the discontinuous signum function is replaced by the continuous function $\frac{s}{|s|+\delta}$. Defining $\Theta_s(t) = \Theta_s(t)$ and $\tilde{\Theta}_d(t) = \Theta_d(t) - \Theta$, we have

$$\dot{\tilde{\Theta}}_d(t) = \dot{\Theta}_d(t) = -K_d W_c^T(t)s(t) \tag{71}$$

$$\Theta_v(t) = F_2(t)\tilde{\Theta}_s(t) = F_2(t)\Theta_s(t) \tag{72}$$

$$\dot{\tilde{\Theta}}_s(t) = \dot{\Theta}_s(t) = -\Gamma_1\bar{\Theta}_s(t) - P_sF_2(t)F(t) = -\Gamma_1\Theta_s(t) - P_s F_2(t)F(t) \tag{73}$$

$$\mathrm{F(t)=W_c^T(t)s(t)=[f_1,\ f_2, \ldots, \ f_p]^T} \tag{74}$$

$$F_2(t) = diag\left[\frac{f_1}{|f_1|+\delta}, \frac{f_2}{|f_2|+\delta}, \ldots, \frac{f_p}{|f_p|+\delta}\right], \ \delta>0 \tag{75}$$

$$F_i = \sum_{j=1}^{n} s_j(t)W_{cji}(t), \ i=1, 2, \ldots, p., \tag{76}$$

where K_d, Γ_1 and P_s are the same as before.

Theorem. 8 *If the robust combined adaptive control laws equations* (69)–(76) *are used to control the robot dynamic equation* (1) *with* $\tau_d = 0$, *then both the tracking error* $\tilde{x}(t)$ *and the gain adjusting parameter,* $\Theta_s(t)$, *converge to zero.*

Proof. Using the control law, equations (69) and (70), in the robot dynamic system, equation (1), leads to the error equation

$$\begin{cases} D(q)P_c\dot{\tilde{x}} + \frac{1}{2}\dot{D}(q)P_c\tilde{x} + (P_u + P_{cc}\Gamma^{-1}P_{cc})P_c\tilde{x} - P_{cc}\tilde{q} = W_c(t)\tilde{\Theta}(t) \\ \dot{\tilde{q}} = -P_{12}P_c\tilde{x} + \tilde{q} \end{cases} \tag{77}$$

where $\tilde{\Theta}(t) = \hat{\Theta}_c(t) - \Theta$, $P_c = [I \ \ P_{12}]$, $\tilde{x} = [\dot{\tilde{q}} \ \tilde{q}]$. The Lyapunov function candidate is chosen as

$$V[\tilde{x}(t), \tilde{\Theta}_d(t), \tilde{\Theta}_s(t)] = \frac{1}{2}\tilde{x}^T(t)P_q(q)\tilde{x}(t) + \frac{1}{2}\tilde{\Theta}_d^T(t)K_d^{-1}\tilde{\Theta}_d(t) + \frac{1}{2}\tilde{\Theta}_s^T(t)P_s^{-1}\tilde{\Theta}_s \tag{78}$$

The time derivative of the Lyapunov function candidate is

$$\dot{V}[\tilde{x}(t), \tilde{\Theta}_d(t), \tilde{\Theta}_s(t)] \le -\tilde{x}^T(t)Q\tilde{x}(t) - \Theta_s^T(t)P_s^{-1}\Theta_s(t) \tag{79}$$

Using the same reasoning as in the proof of **Theorem 2**, we can prove that all the signals in the combined control system are bounded and $\lim_{t\to\infty} \tilde{x}(t)=0$, $\lim_{t\to\infty} \Theta_s(t)=0$.

Remark. 12 $\Theta_s(t)$ *is a transient term, and it only improves transient behaviour of the system. From equation (79), the derivative of Lyapunov function becomes more negative and this will decrease the transient error.*

5.2 Robustness with respect to uncertainty

The robust combined adaptive control scheme proposed in **Section 5.1** can be modified to overcome the uncertainty discussed in **Section 4.3**. The controller is modified as

$$\tau(t) = W_c(t)\,[\Theta_d(t) + \Theta_v(t)] = W_{c0}(t) + \tau_c s + \tau_0 sgn(s) + \tau_l(t) \tag{80}$$

where

$$\dot{\tau}_c = -c\|s\|^2; \quad \dot{\tau} = -C_0\|s\| \tag{81}$$

and $c>0$ and $c_0>0$ are constants, and the other parameters are as in **Section 5.1**.

Theorem. 9 *If the robust control laws, equations (69)–(75), (80) and (81), are applied to the robot system, equation (1) with uncertainties, equation (61), then the conclusion in* **Theorem 8** *is still true.*

Proof: Following the same line of proof as in **Theorems 7** and **8** leads to the conclusion. □

6 COMPUTER SIMULATION

To compare the effectiveness of the above various adaptive control methods, we present computer simulation results using the two-link manipulator shown in Fig. 1 with the parameters given in Table I. The parameters related to the end-effector, $\theta_1 = m_2$, $\theta_2 = m_2 l_{c2}^2 + I_2$, and $\theta_3 = m_2 l_{c2}$ are assumed to be unknown or changing with time. We suppose that the robot picks up a payload at a time $t=1$ [s], and the

Table I. Dimensions of the fire-truck (metres)

Parameter	Value without payload	Value with payload
1_1	0.432 m	0.432 m
1_{c1}	0.216 m	0.216 m
m_1	15.91 kg	15.91 kg
I_1	0.247 khm^2	0.247 kgm^2
1_2	0.432 m	0.5 m
1_{c2}	0.216 m	0.4 m
m_2	11.36 kg	20 kg
I_2	0.177 kgm^2	0.68 kgm^2

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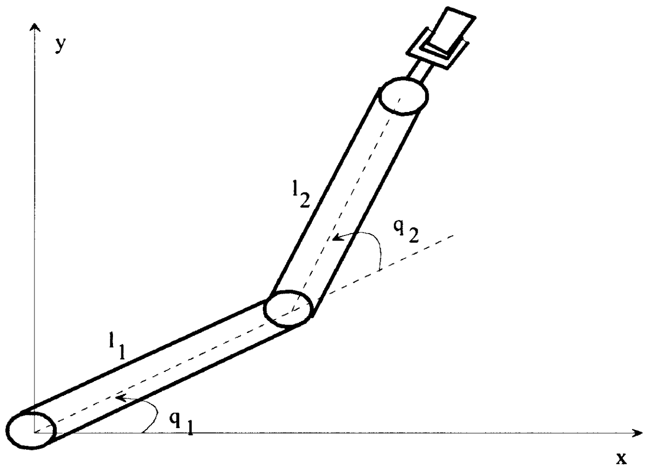

Fig. 1. Two-link Planar Robot Arm

parameters affected by the payload change to $m_2=31.36[kg]$, $I_2=0.88[kgm^2]$, $l_2=0.5[m]$, and $l_{c2}=0.4[m]$ after $t=1[s]$. In all the simulations shown in this paper, the sampling time is 2 ms and the intail errors, $\tilde{q}_1(0)=-1.57$, $\tilde{q}_2(0)=0.57$. All schemes use the same control structure as,

$$\tau(t)=W_c(t)\hat{\Theta}(t) + W_{c0}(t) + \tau_l \tag{82}$$

where $W_c(t)\Theta + W_{c0}(t) = -\frac{1}{2}\dot{D}(q)s(t)+D(q)v(t)+C(q,\dot{q})\dot{q} + G(q)$

In all the simulations, the arm is required to follow the desired positions $q_{d_1}(t)$ and $q_{d_2}(t)$ produced by the reference model $\ddot{q}_{d_1}+2\xi_1\omega_1\dot{q}_{d_1}+\omega_1^2 q_{d_1}=\omega_1^2 r_1$, $\ddot{q}_{d_2}+2\xi_2\omega_2\dot{q}_{d_2}+\omega_2^2 q_{d_2}=\omega_2^2 r_2$, where $\xi_1=\xi_2=0.707$, $\omega_1=\omega_2=4$, and the reference inputs are specified as $r_1(t)=1$, $r_2(t) = 1.5$, $0\leq t<10$; $r_1(t)=-1$, $r_2(t)=-1.5$, $10\leq t<20$. The PD coefficients are chosen as $P_{ll}=10I_{2\times 2}$, $P_{cc}=10\mathrm{I}_{2\times 2}$, $\Gamma=40I_{2\times 2}$. Thus, $P_{12}=\mathrm{P}_{cc}^{-1}\Gamma=4I_{2\times 2}$.

6.2 Ideal case

Here, the ideal case means that the robot system only suffers with initial errors and parameter change but without external disturbances and unmodelled dynamics.

1) Direct adaptive control: In this case, the integral parameter adjusting law and control inputs of the robot manipulator are given as in Figure 2 shows the errors between the outputs of the robot and the model, and control inputs of the robot, with four different integral parameters K_d, during the transient recovery from initial errors, $\tilde{q}_1(0)=-1.57$, $\tilde{q}_2=0.57$. For the convenience of drawing the figures, the output data is selected once every ten sampling time intervals. From Fig. 2, we see that the tracking performance is not improved with increasing the gains of the integral parameters, since the big payload is changed. The simulation results show that $K_d=4I_{3\times 3}$ gives the best tracking performance.

2) VSA control—algorithm 1: In this case, three groups of the simulation results are shown in Fig. 3. The constant δ is chosen as 0.001, 0.05, and 0.2, respectively, and $P_s = 4I_{3\times 3}$. The correspondent adjusting law and control law for the robot are as in. Figure 3 shows that the smaller the value of δ is chosen, the better the tracking performance, but the worse the chattering of the inputs. The tracking errors increase with increasing δ.

3) VSA control—algorithm 2: The adjusting law in this case requires information of the upper bound of the uncertain parameters. The simulation for three groups of the parameters, i) $\bar{\Theta}_{s1} = 32$, $\bar{\Theta}_{s2} = 6$, $\bar{\Theta}_{s3}=13$; ii) $\bar{\Theta}_{s1} = 16$, $\bar{\Theta}_{s2} = 3$, $\bar{\Theta}_{s3} = 6.5$; and iii) $\bar{\Theta}_{s1} = 48$, $\bar{\Theta}_{s2} = 9$, $\bar{\Theta}_{s3} = 19.5$ are given in Figs. 4 i), ii), iii), respectively. The other parameters are chosen as follows: $\Gamma_1 = 2I_{3\times 3}$, $P_s = 4I_{3\times 3}$, and $\delta = 0.001$. Since the first group of parameters are closest to the true parameters during the term that the robot picks up the payload, it shows the best performance. If we increase the values of the parameters, the tracking performance is maintained, but the chattering becomes worse (Fig. 4 iii)). If we decrease the values of the parameters, the chattering is reduced, but the tracking performance becomes worse (Fig. 4 ii)).

4) VSA control—algorithm 3: The parameters are chosen as $P_s = 4I_{3\times 3}$, $\delta = 0.001$, and Γ_1 is chosen as $0.2I_{3\times 3}$, $2I_{3\times 3}$, $8I_{3\times 3}$, respectively, corresponding to Figs. 5 i), ii), and iii), respectively. The larger Γ_1 reduces the chattering of the input, but increases the tracking errors.

5) Combined direct adaptive and VSA control: To test the performarnce of the proposed combined direct adaptive and VSA control (algorithm 3), the following simulation is performed. The constant parameters are chosen as $P_d = 4I_{3\times 3}$, $\delta = 0.001$, $P_s = 4I_{3\times 3}$. The simulation results are shown in Fig. 6 i) combined direct adaptive and VSA—algorithm 3 ($\Gamma_1 = I_{3\times 3}$), ii) combined direct adaptive and VSA—algorithm 3 ($\Gamma_1 = 2I_{3\times 3}$), iii) combined direct adaptive and VSA — algorithm 2 ($\Gamma_1 = 2I_{3\times 3}$, $\Theta_{s1} = 48$, $\Theta_{s_2}=9$,$\Theta_{s3} = 19.5$). When Γ_1 is increased, the control inputs become more smooth with a slight sacrifice in the transition of the tracking errors (comparing Figs 6 (i) and (ii)). When the direct adaptive control algorithm is combined with VSA control algorithm 2, no benefit is obtained (Fig 6 iii)). This can be explained using the Lyapunov function since in this case the tracking errors are not convergent to zero.

Comparing the above simulation results, the combined adaptive control gives the best performance. Increasing the integral parameter gain of the direct adaptive law, K_d, does not improve the tracking performance (Fig. 2). The relationship between K_d and the tracking performance is a complicated problem. Further research is needed to give guidelines for choosing the integral parameter gains. For the other control schemes proposed in this paper, the same problem exists. The positive constant δ has a great influence on the VSA control algorithm 1. Large δ can smooth the control inputs but sacrifice the tracking performance (Fig. 3). In the VSA control algorithm 2, the choice of Θ_s is vital. Large Θ_s will improve the tracking performance but results in bad chattering (Fig. 5). Although increasing Γ_1 or reducing P_s can prevent the chattering, the overall tracking performance will become worse (Fig. 5). Comparing the above five types of control algorithms, the combined direct adaptive and VSA — algorithm 3 control method shows the most promising performance (Fig. 6).

6.2 External disturbances

External disturbances always exist. In the simulation, several disturbances are chosen as follows:

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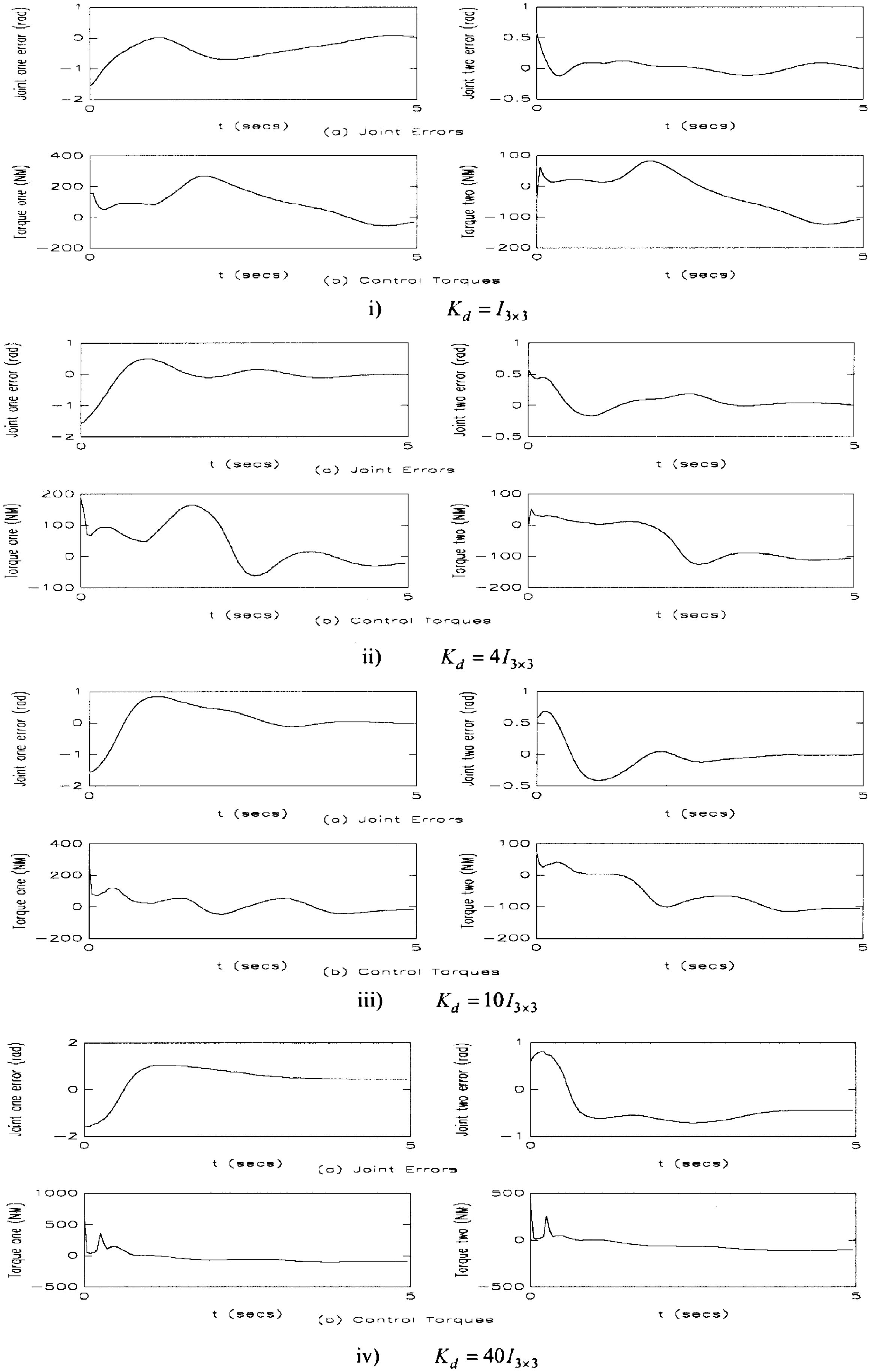

Fig. 2. Direct Adaptive Control

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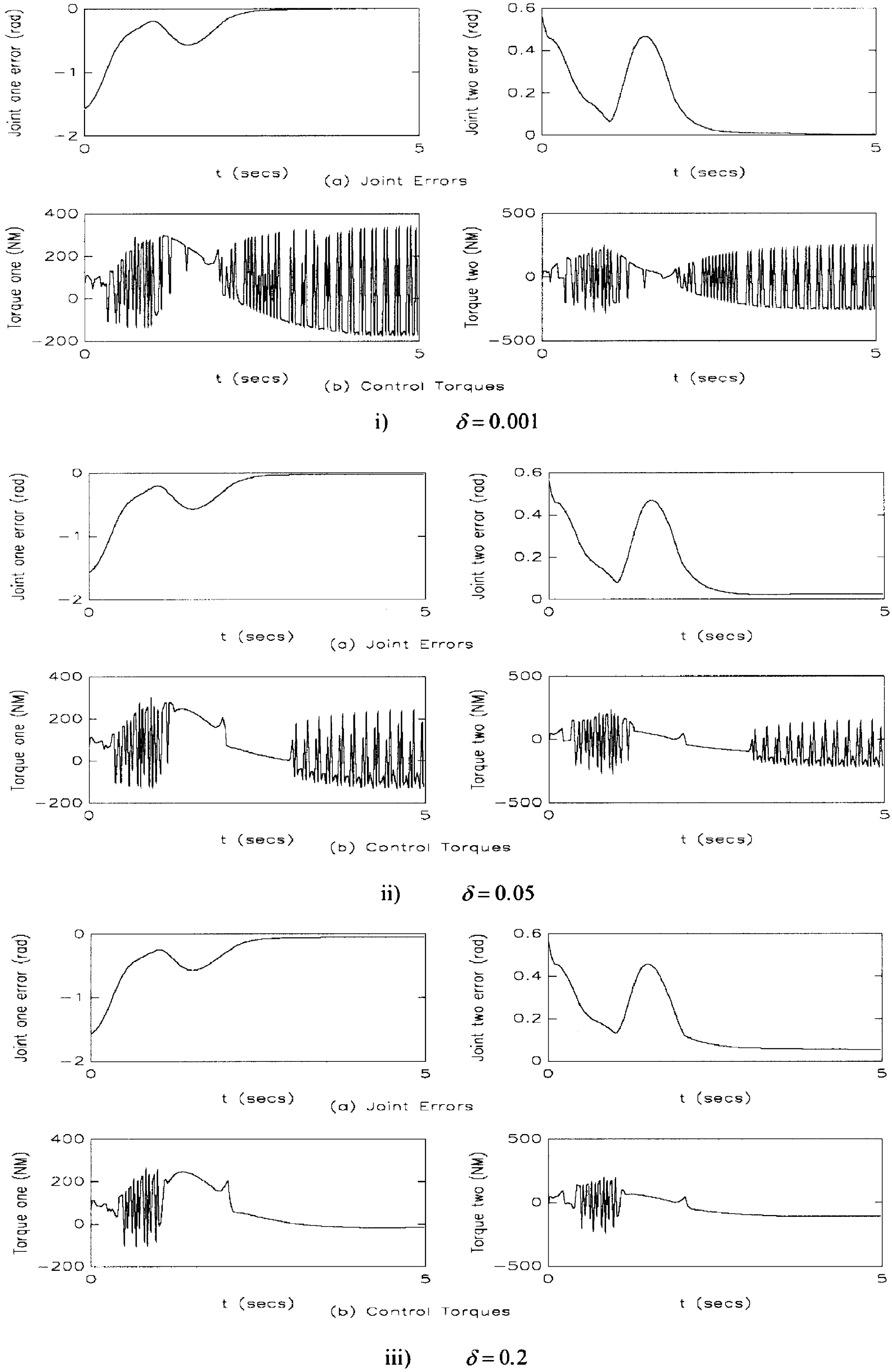

Fig. 3. VSA Control—Algorithm 1 ($P_s = 4I_{3\times3}$)

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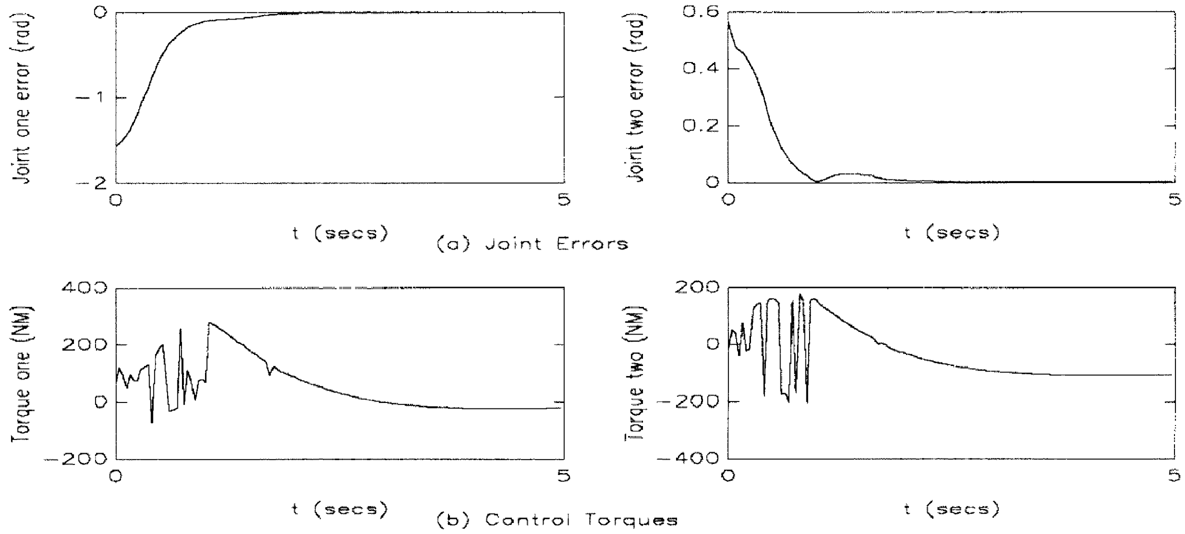
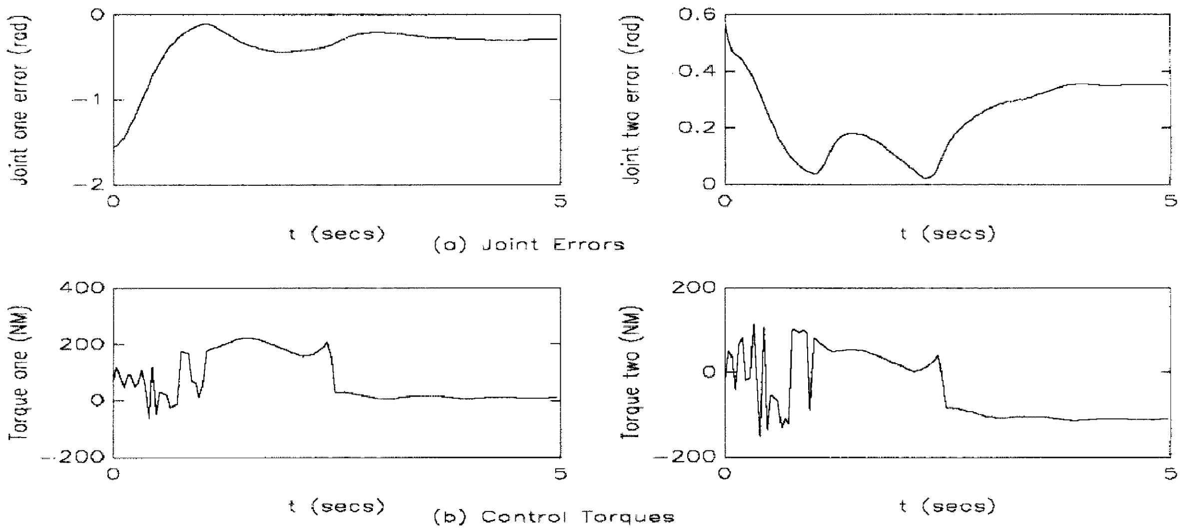
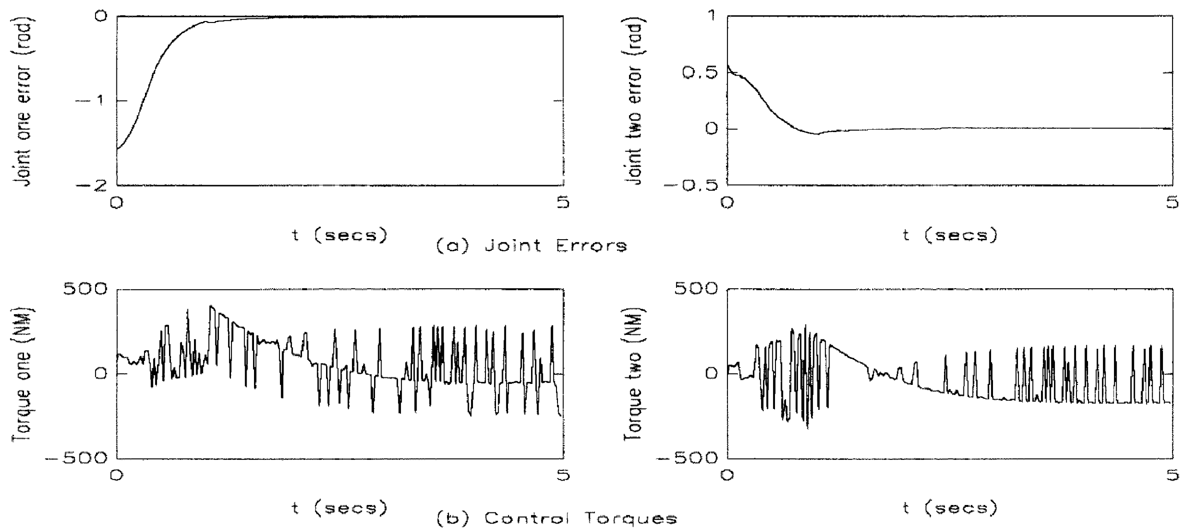


i) $\overline{\Theta}_{s1} = 32,\ \overline{\Theta}_{s2} = 6,\ \overline{\Theta}_{s3} = 13$



ii) $\overline{\Theta}_{s1} = 16,\ \overline{\Theta}_{s2} = 3,\ \overline{\Theta}_{s3} = 6.5$



iii) $\overline{\Theta}_{s1} = 48,\ \overline{\Theta}_{s2} = 9,\ \overline{\Theta}_{s3} = 19.5$

Fig. 4. VSA Control—Algorithm 2 ($\Gamma_1 = 2I_{3\times3}$, $\delta = 0.00$)

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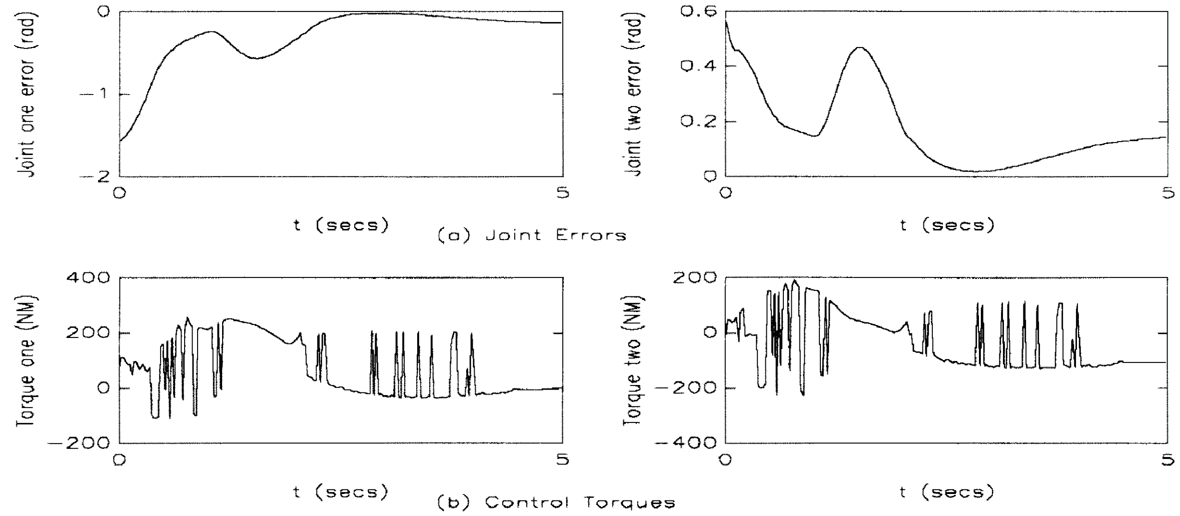
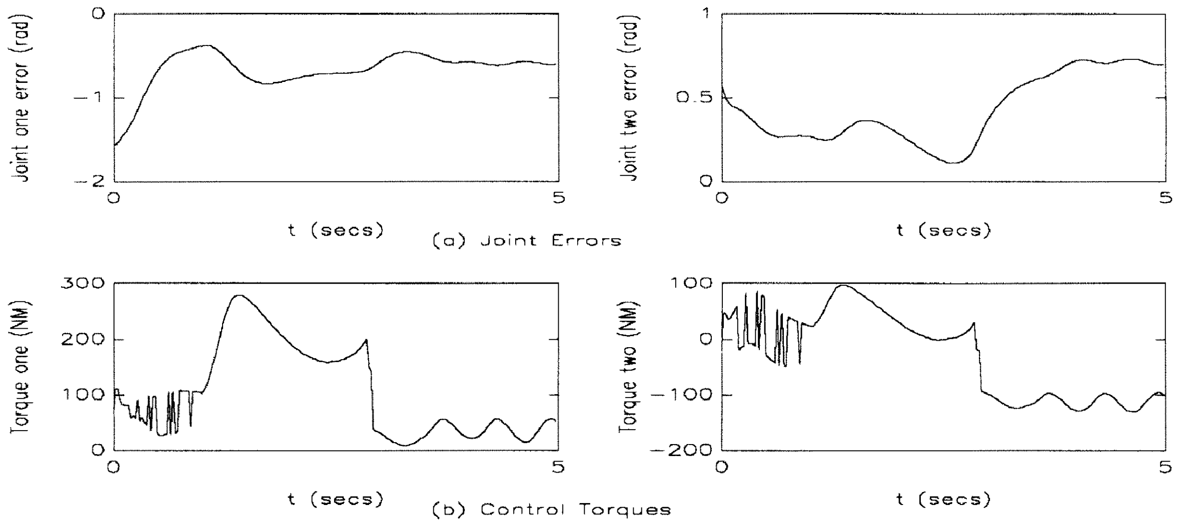
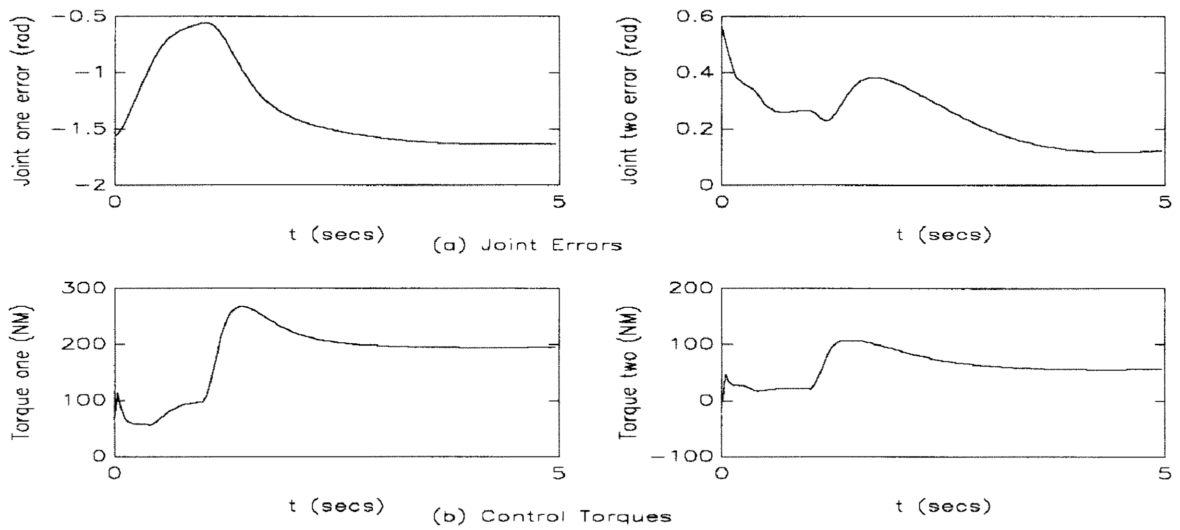


i) $\Gamma = 0.2I_{3\times3}$



ii) $\Gamma = 2I_{3\times3}$



iii) $\Gamma = 8I_{3\times3}$

Fig. 5. VSA Control—Algorithm 3 ($P_s = 4I_{3\times3}$, $\delta = 0.001$)

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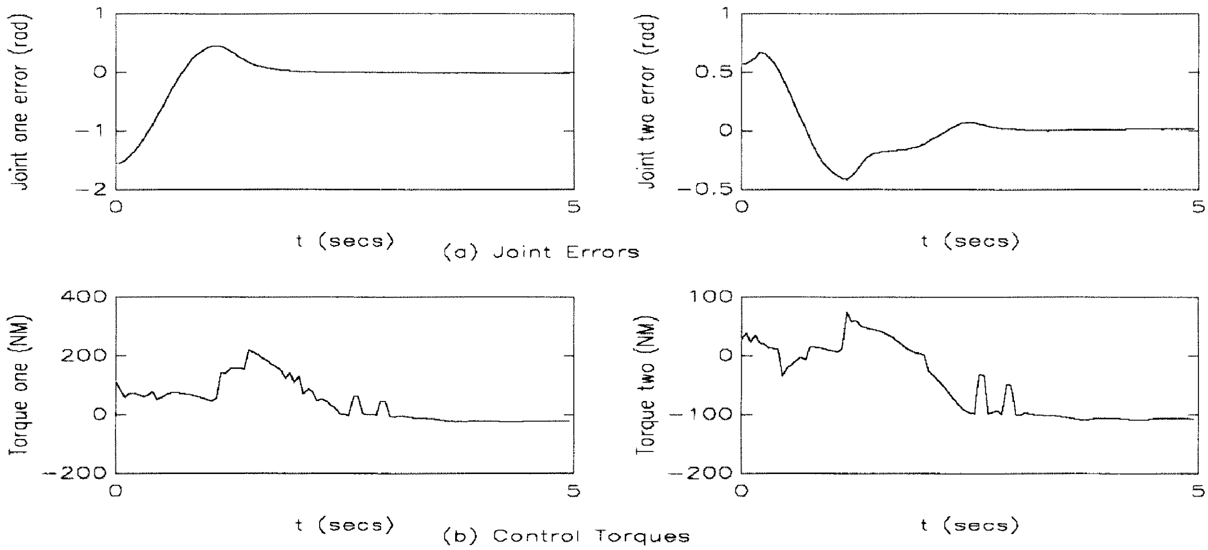
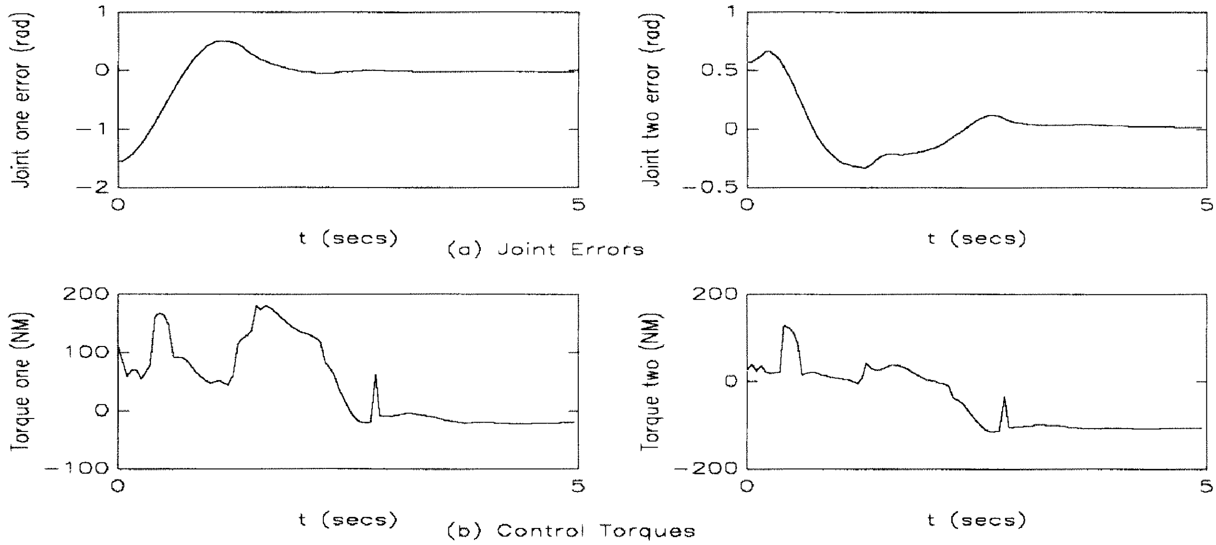
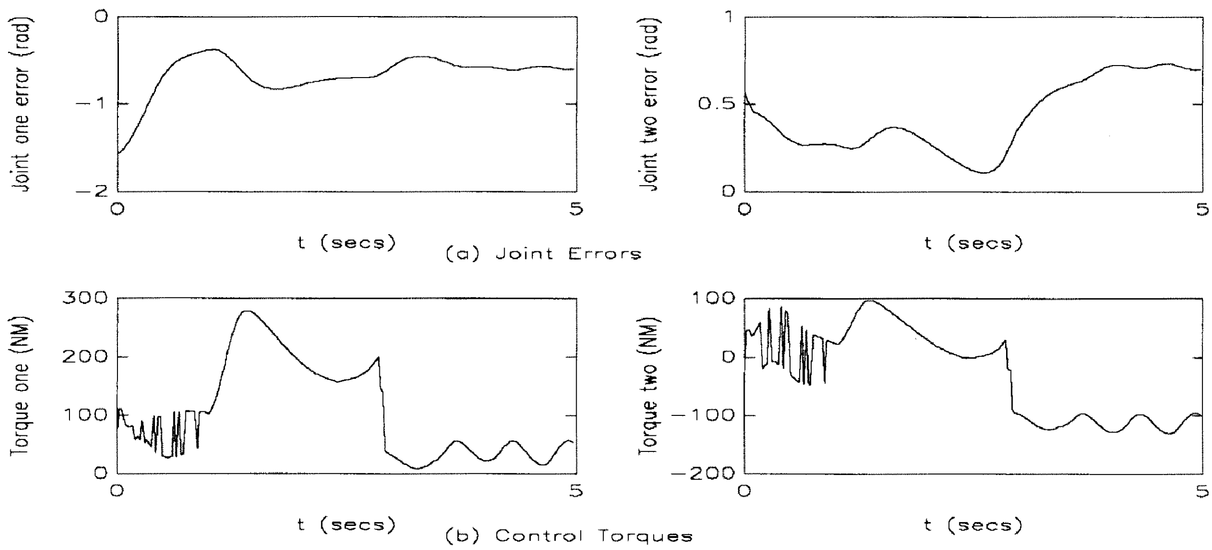


i) Combined Direct and Algorithm 3 ($\Gamma_1 = I_{3\times3}$)



ii) Combined Direct and Algorithm 3 ($\Gamma_1 = 2I_{3\times3}$)



iii) Combined Direct and Algorithm 2 ($\Gamma_1 = 2I_{3\times3}$, $\overline{\Theta}_1 = 16$, $\overline{\Theta}_2 = 3$, $\overline{\Theta}_3 = 6.5$)

Fig. 6. Combined Direct Adaptive and VSA Control ($K_d = 4I_{3\times3}$, $P_s = 4I_{3\times3}$, $\delta = 0.001$)

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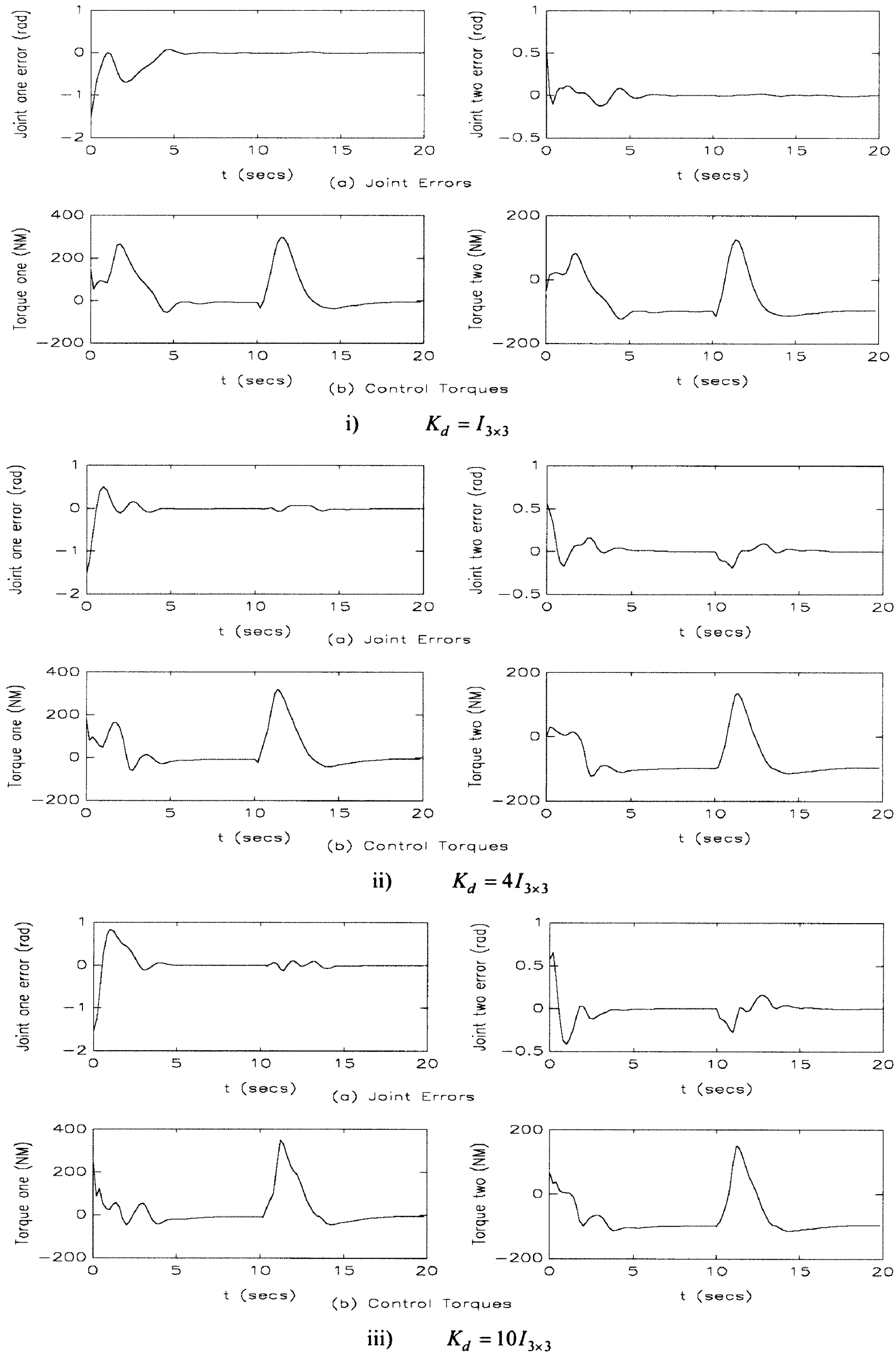

Fig. 7. Direct Adaptive Control with Input Disturbances

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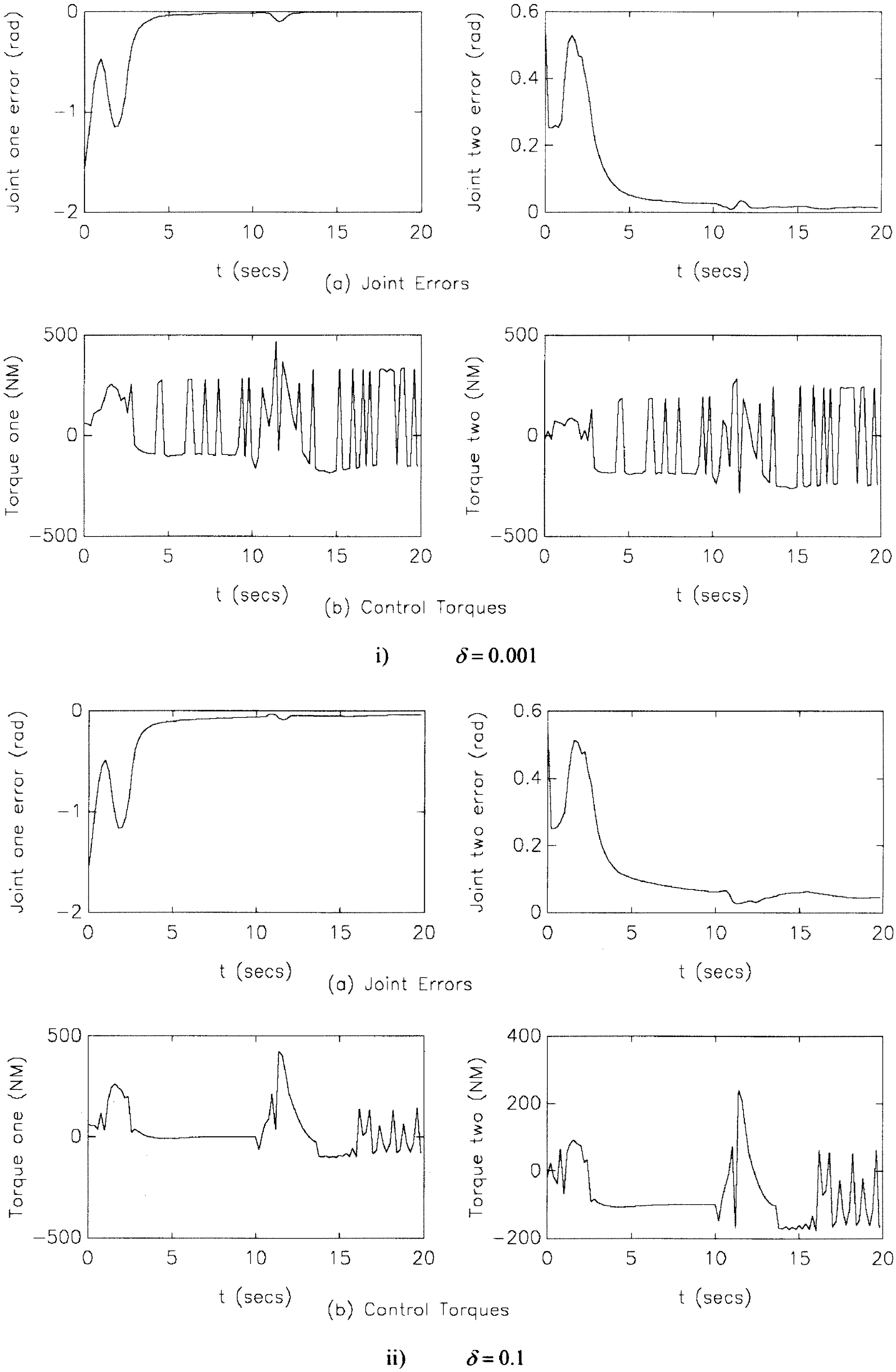

Fig. 8. VSA—Algorithm 1 with Input disturbances ($P_s = I_{3\times3}$)

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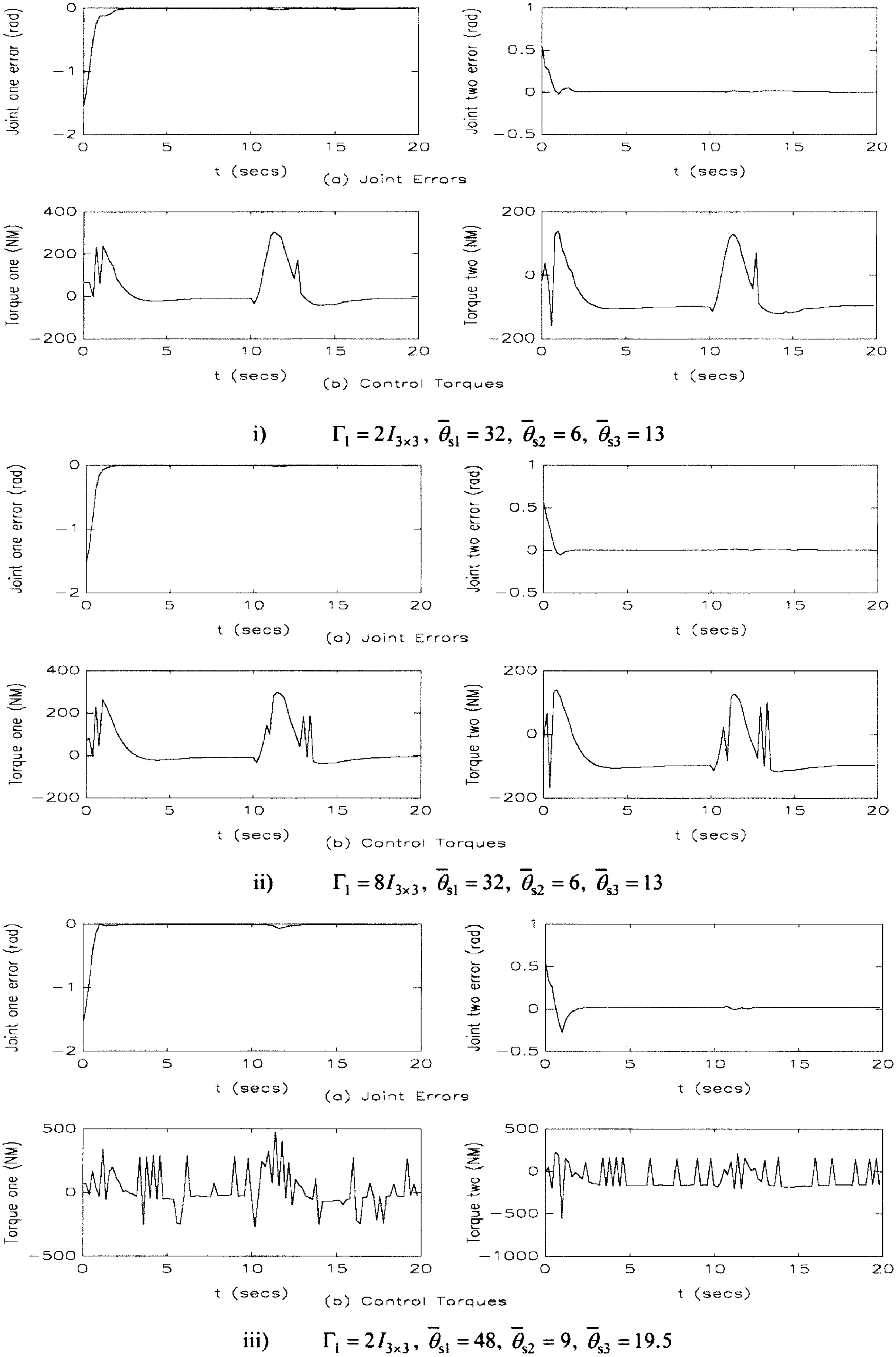

Fig. 9. VSA—Algorithm 2 witrh Input Disturbances ($P_s = I_{3\times3}$, $\delta = 0.00$)

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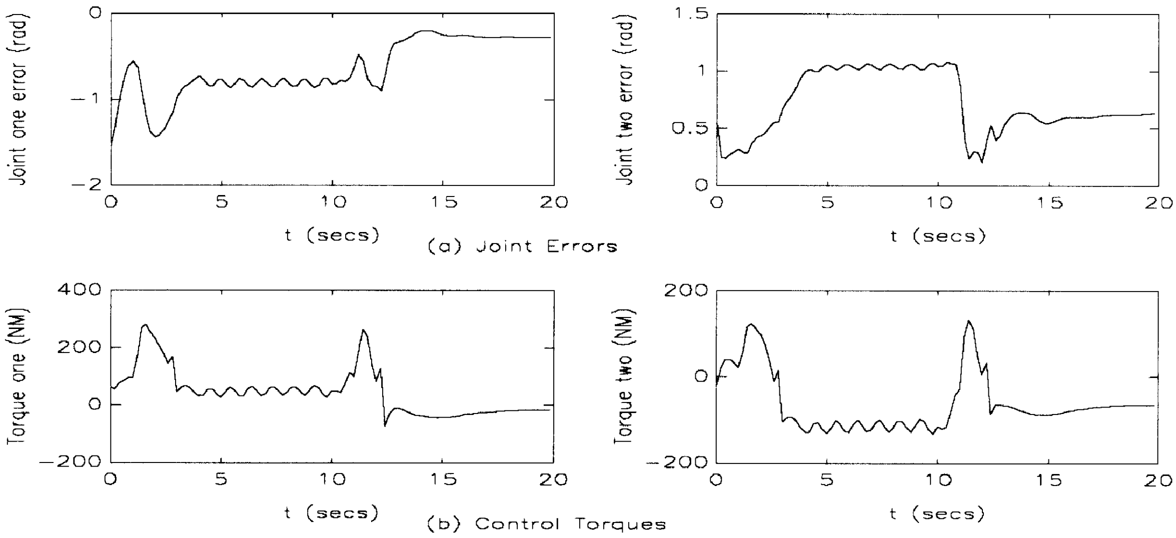
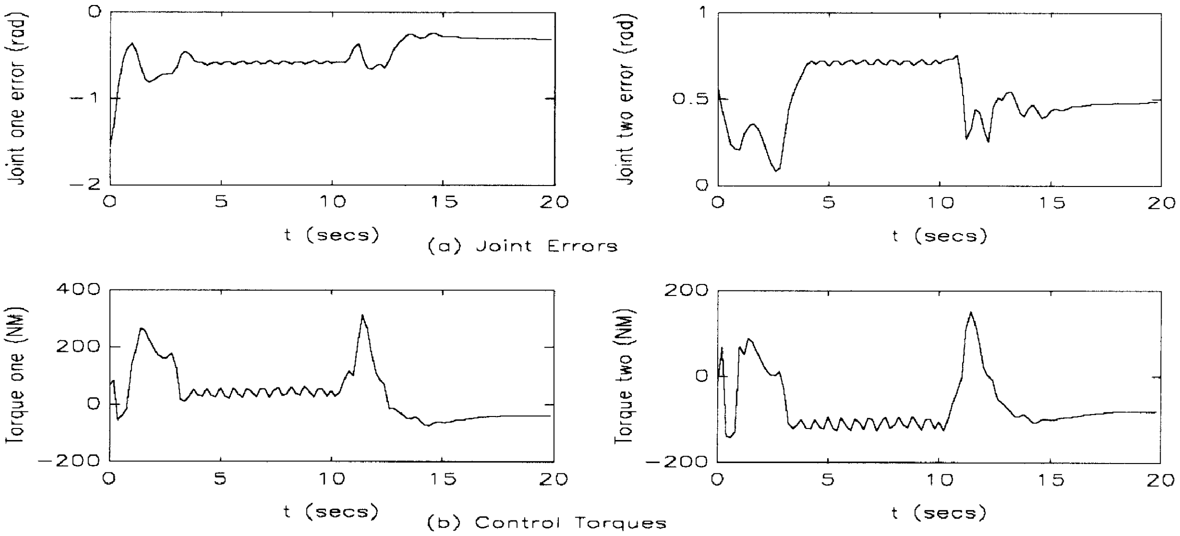
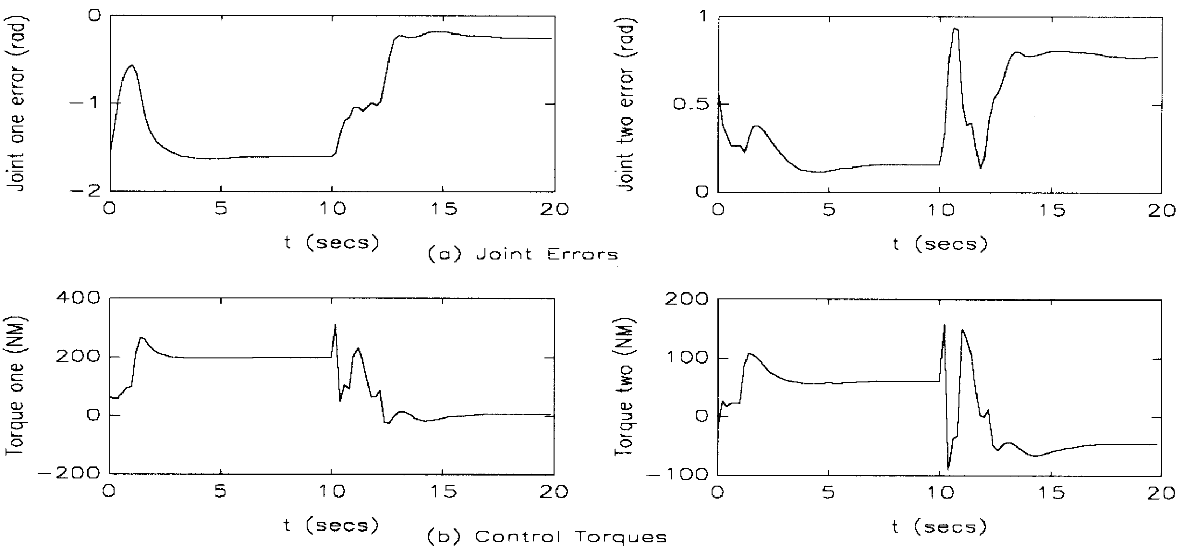


i) $P_s = I_{3\times3},\ \Gamma_1 = I_{3\times3}$

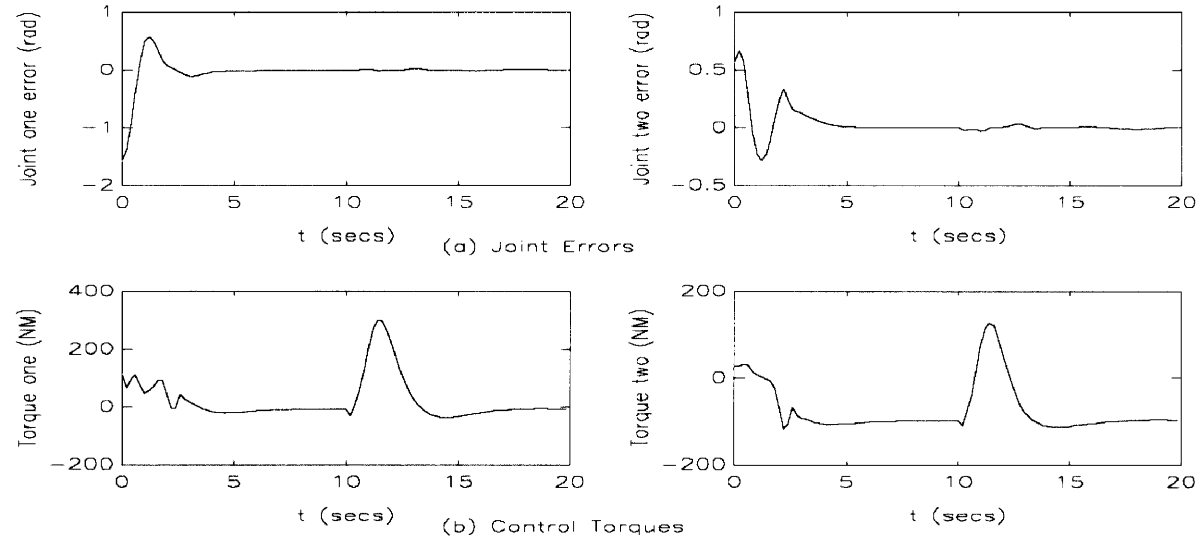


ii) $P_s = 4I_{3\times3},\ \Gamma_1 = 2I_{3\times3}$

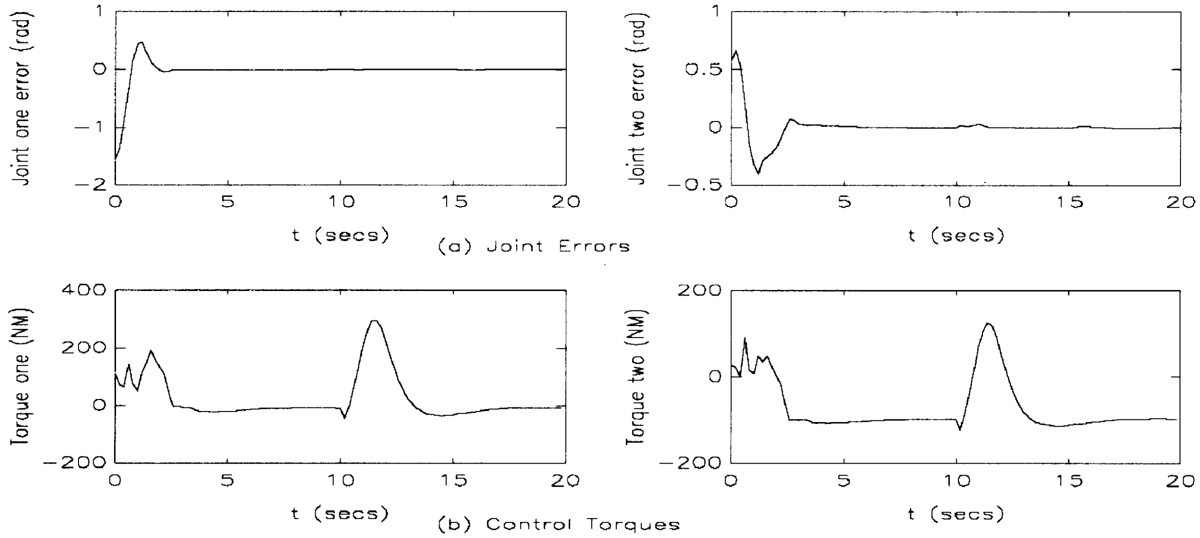


iii) $P_s = 4I_{3\times3},\ \Gamma_1 = 8I_{3\times3}$

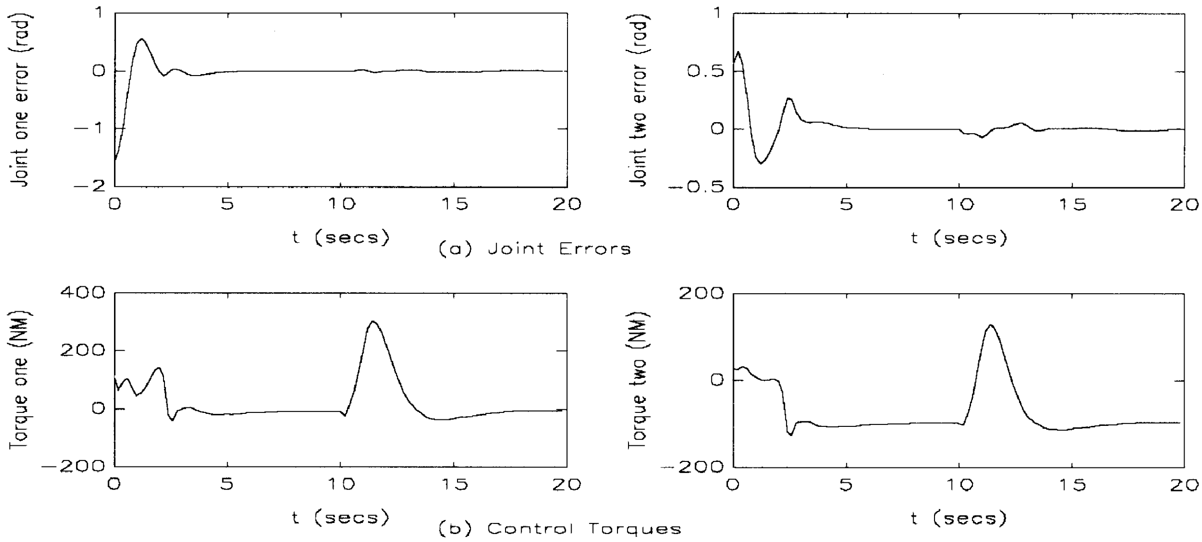
Fig. 10. VSA—Algorithm 3 with Input Disturbances ($\delta = 0.00$)



i) $P_s = I_{3\times3},\ \Gamma_1 = I_{3\times3}$



ii) $P_s = 4I_{3\times3},\ \Gamma_1 = 2I_{3\times3}$



iii) $P_s = 4I_{3\times3},\ \Gamma_1 = 8I_{3\times3}$

Fig. 11. Combined Direct Adaptive and VSA Control with Input Disturbances (K_d=$4I_{3\times3}$)

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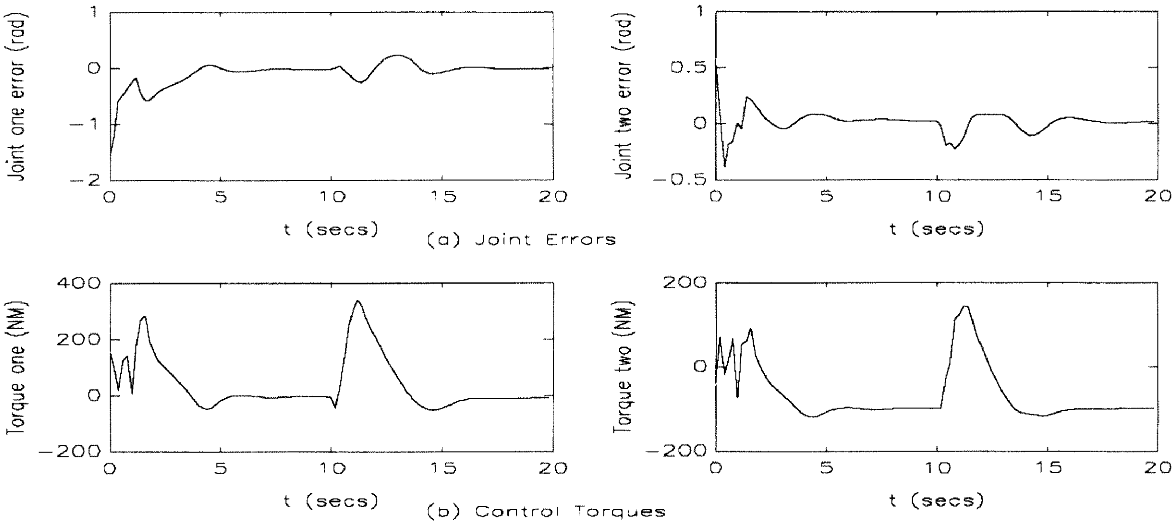
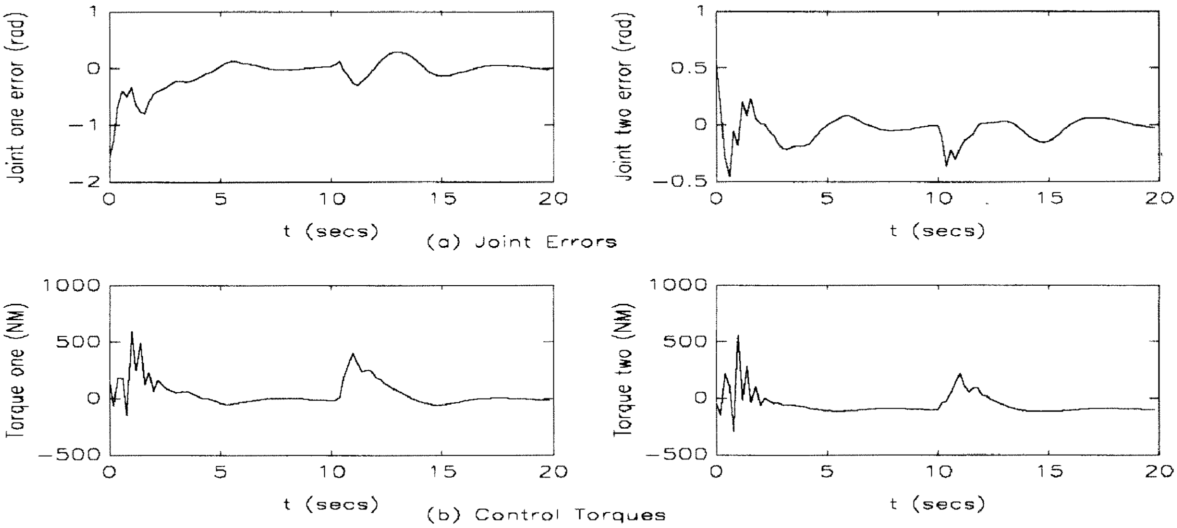
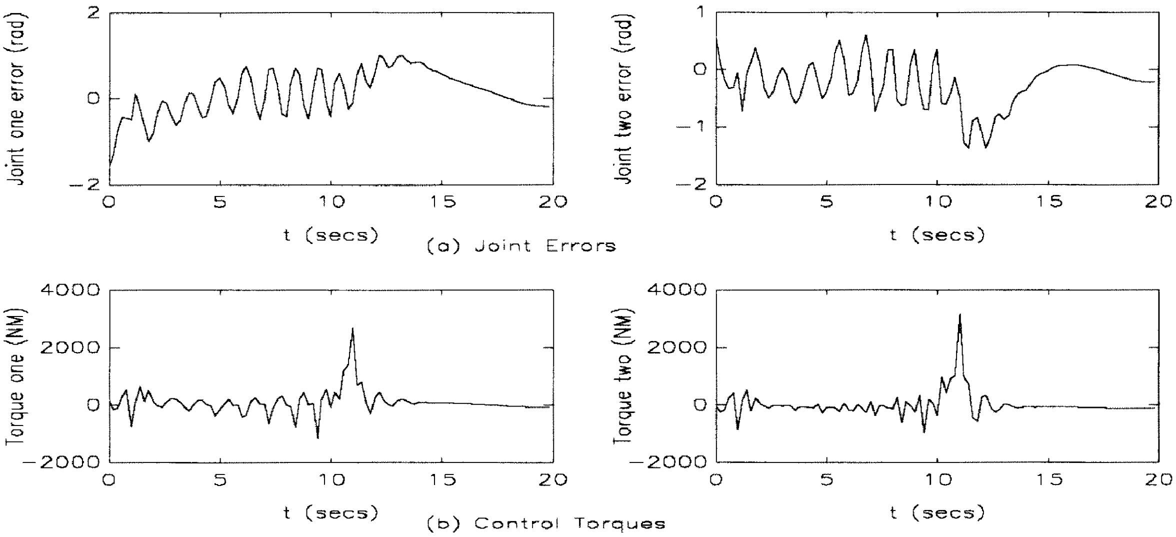


i) $\eta = 0.1I_{2\times 2}$

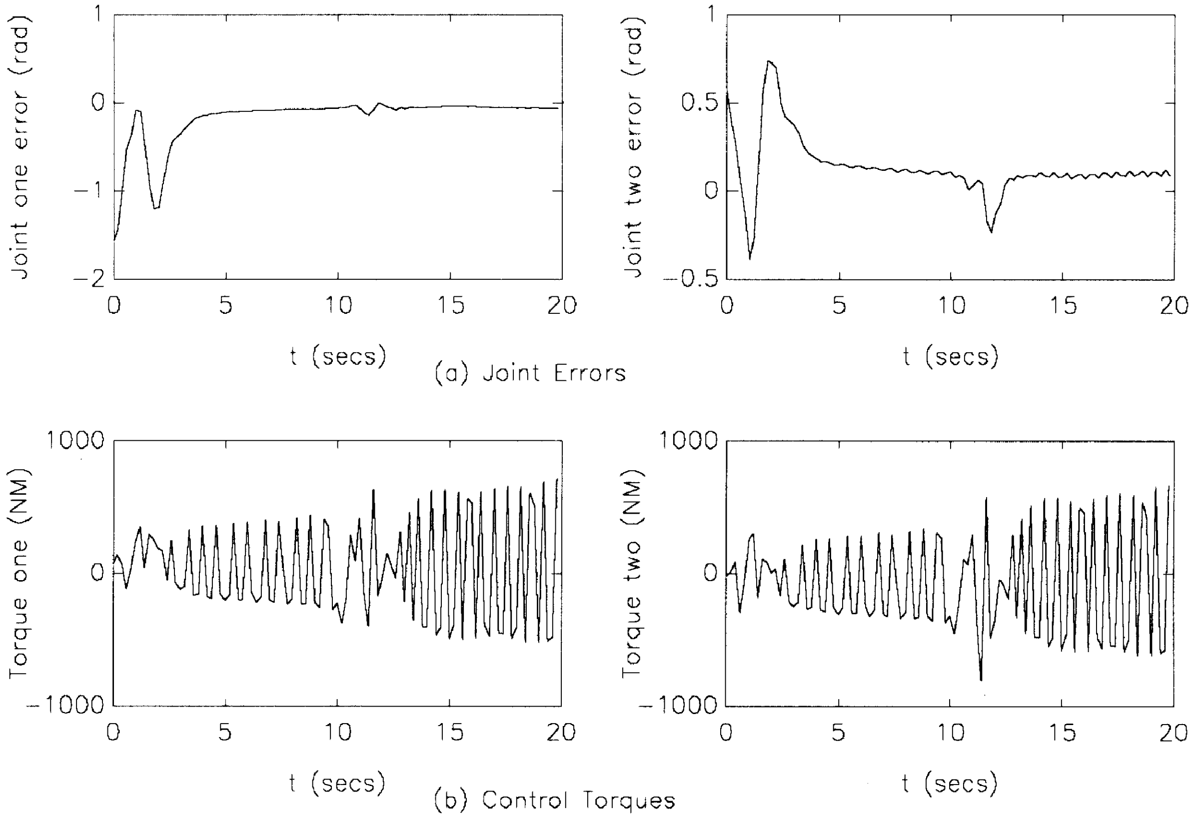


ii) $\eta = 0.15I_{2\times 2}$

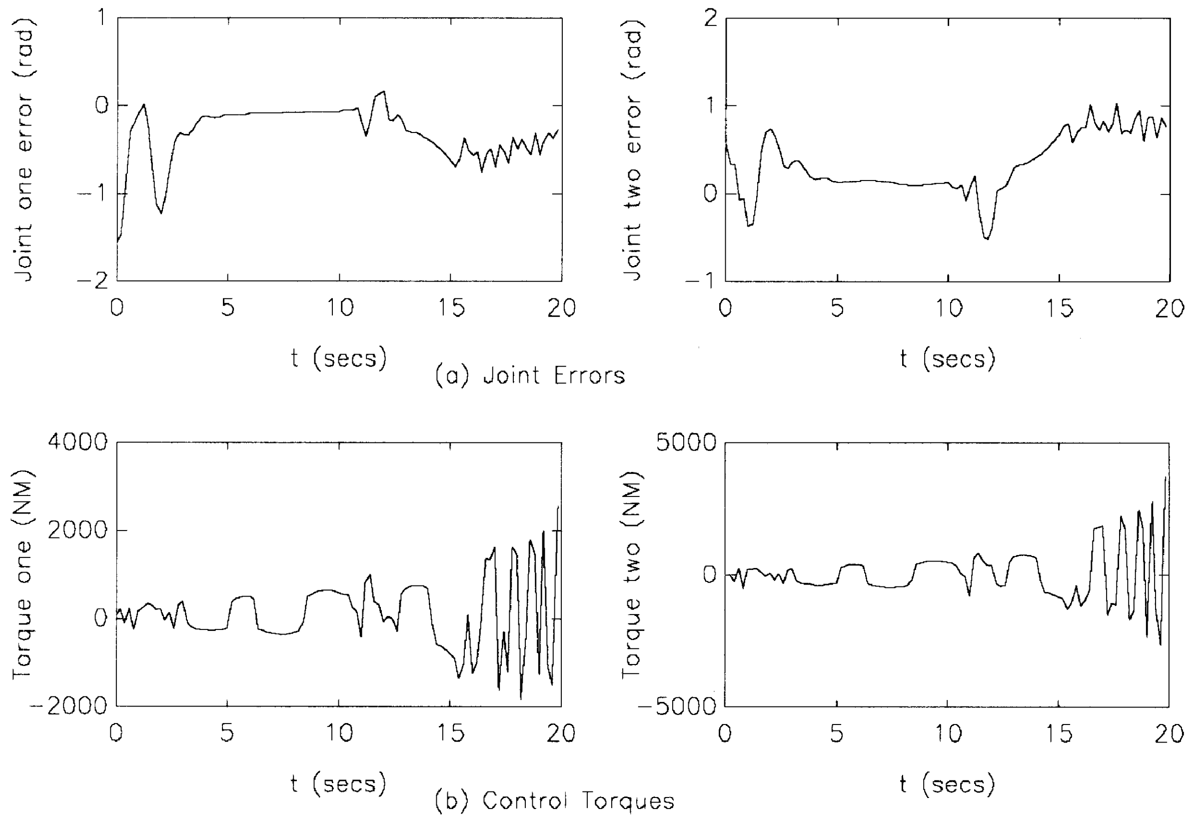


iii) $\eta = 0.2I_{2\times 2}$

Fig. 12. Direct Adaptive Control with Unmodelled Dynamics ($K_d = I_{3\times 3}$)



i) $\eta = 0.1 I_{2\times2}$



ii) $\eta = 0.15 I_{2\times2}$

Fig. 13. VSA Control—Algorithm 1 with Unmodelled Dynamics ($P_s = I_{3\times3}$, δ0.1)

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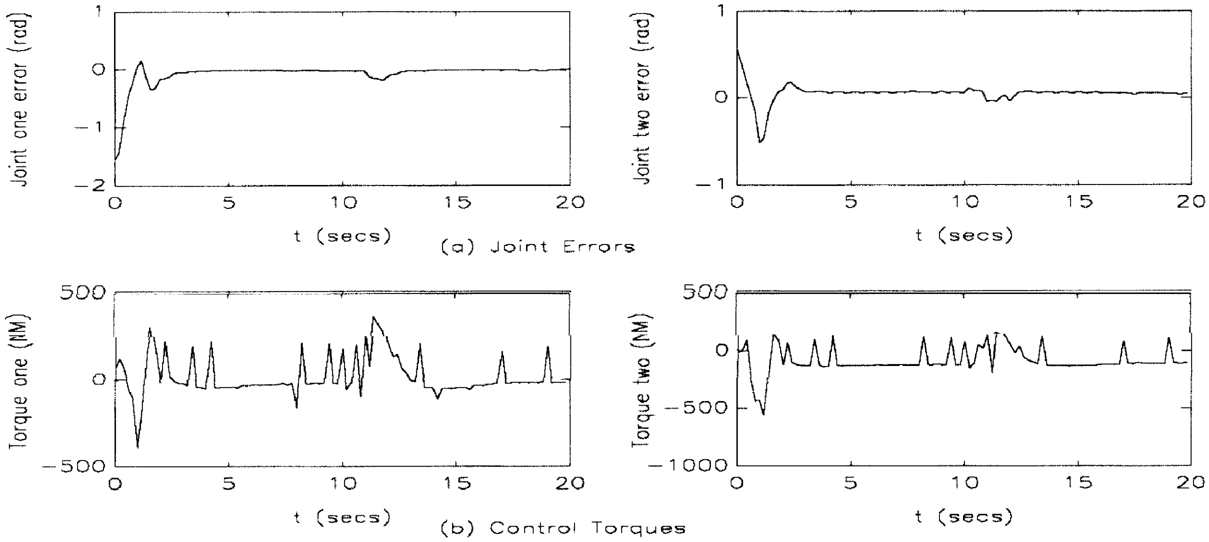
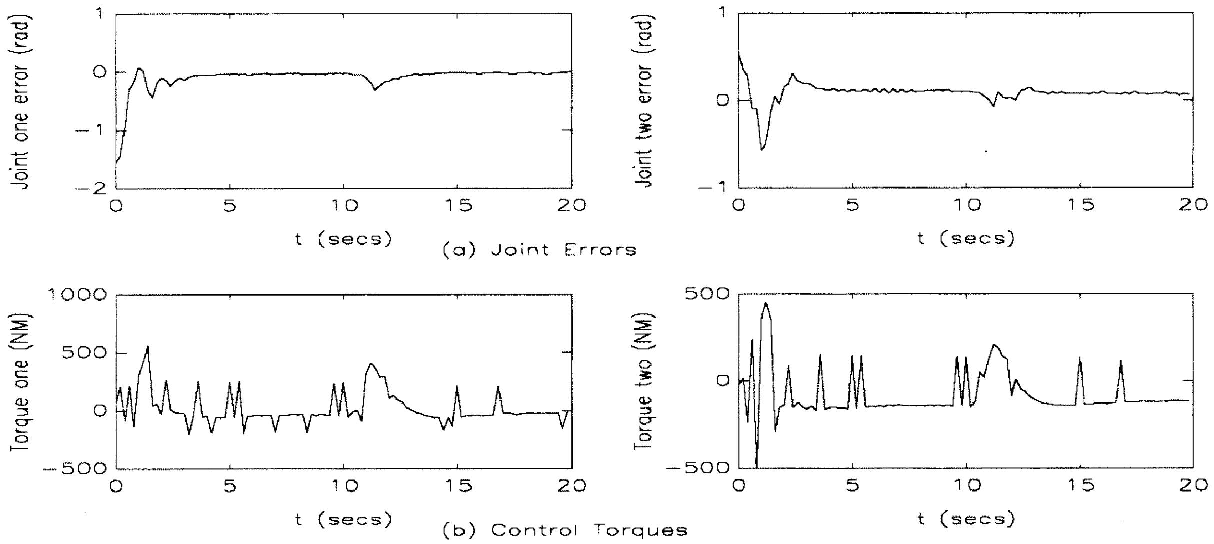
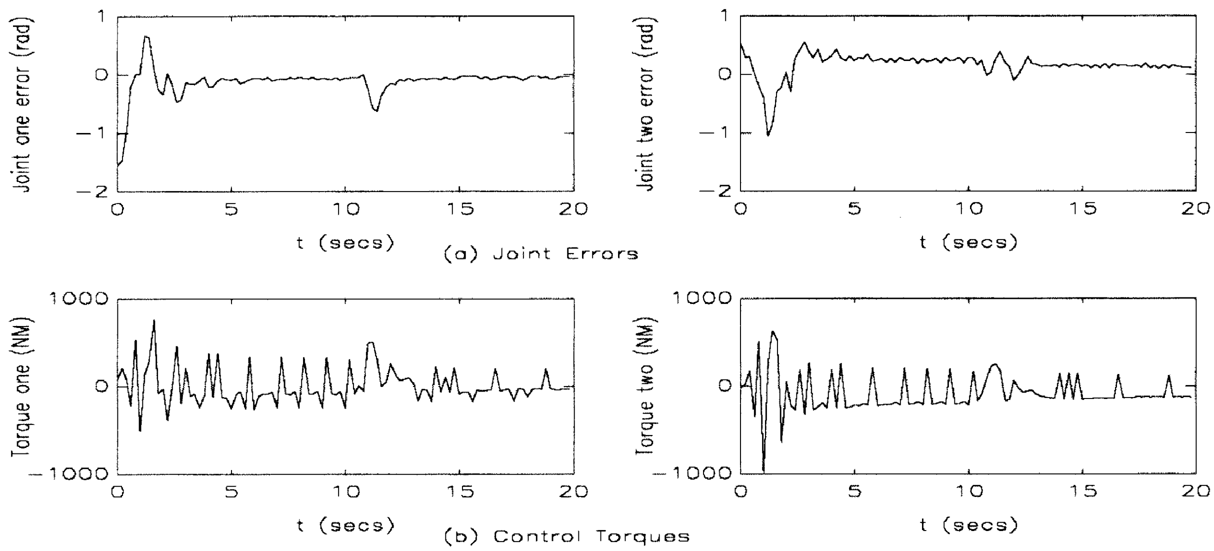


i) $\eta = 0.1 I_{2\times 2}$



ii) $\eta = 0.15 I_{2\times 2}$



iii) $\eta = 0.2 I_{2\times 2}$

Fig. 14. VSA Control—Algorithm 2 with Unmodelled Dynamics ($P_s = I_{3\times 3}$, $\Gamma_1 = I_{3\times 3}$, $\delta = 0.001$, $\bar{\theta}_{s1} = 32$, $\bar{\theta}_{s2} = 6$, $\bar{\theta}_{s3} = 13$)

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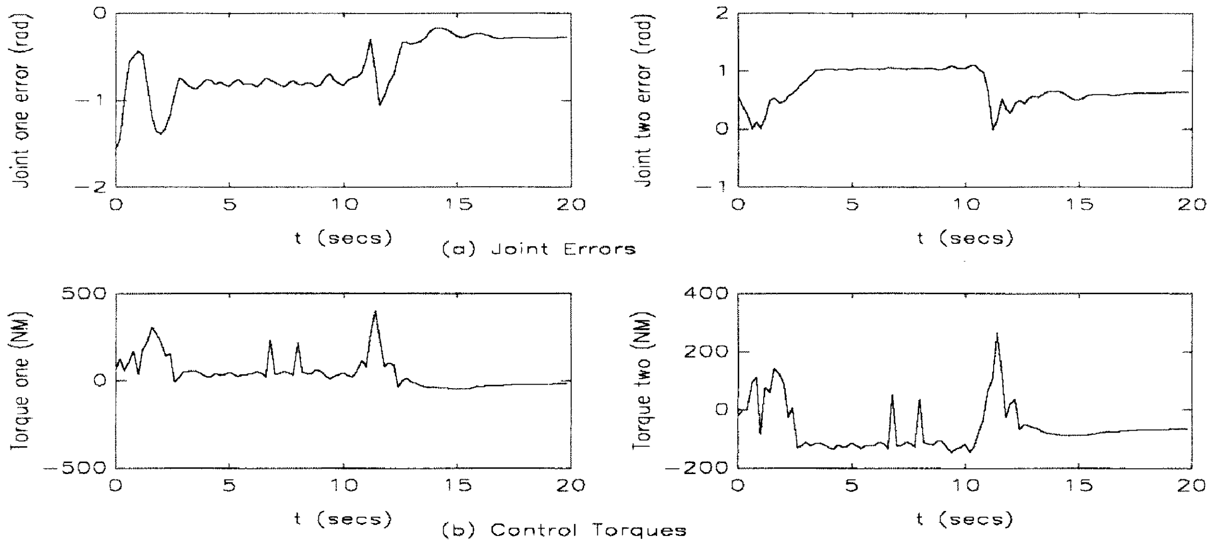
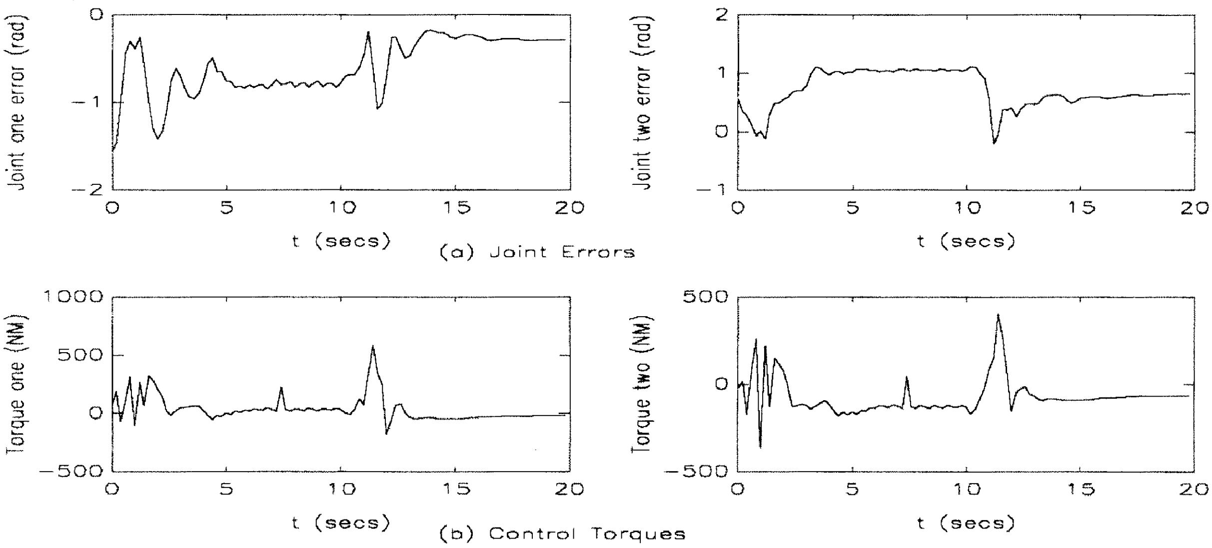
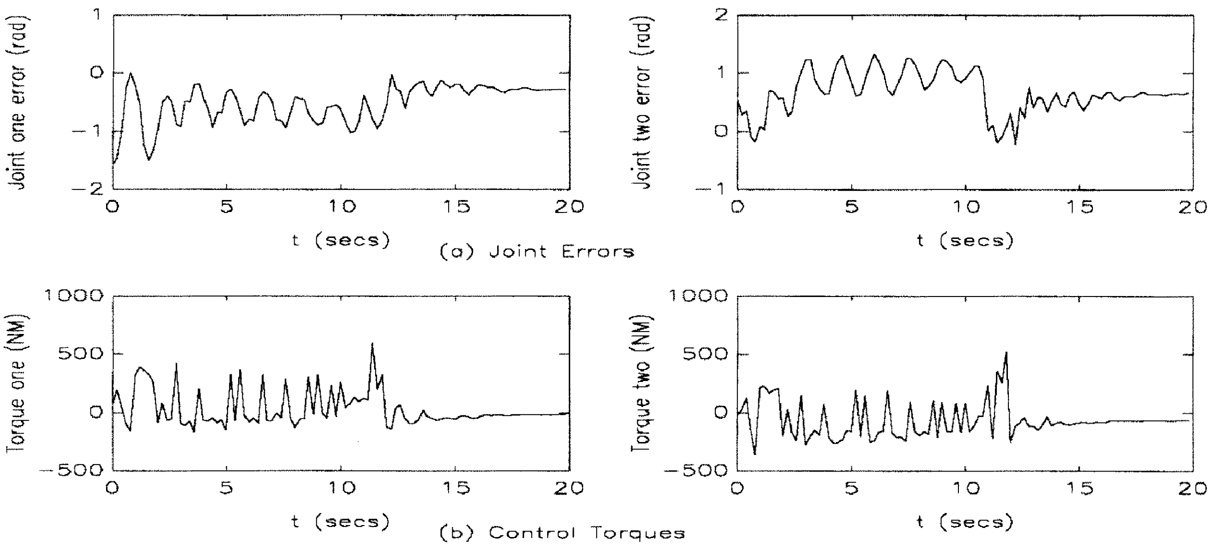


i) $\eta = 0.1I_{2\times2}$



ii) $\eta = 0.15I_{2\times2}$



iii) $\eta = 0.2I_{2\times2}$

Fig. 15. VSA Control—Algorithm 3 with Unmodelled Dynamics ($P_s = I_{3\times3}$, $\Gamma_1 = I_{3\times3}$, $\delta = 0.001$)

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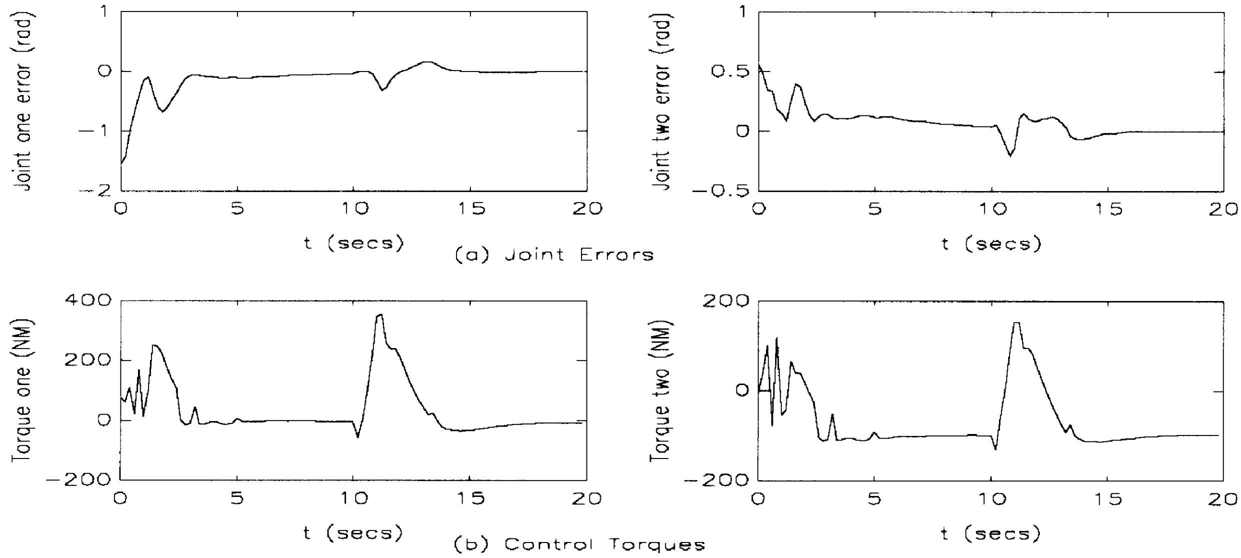
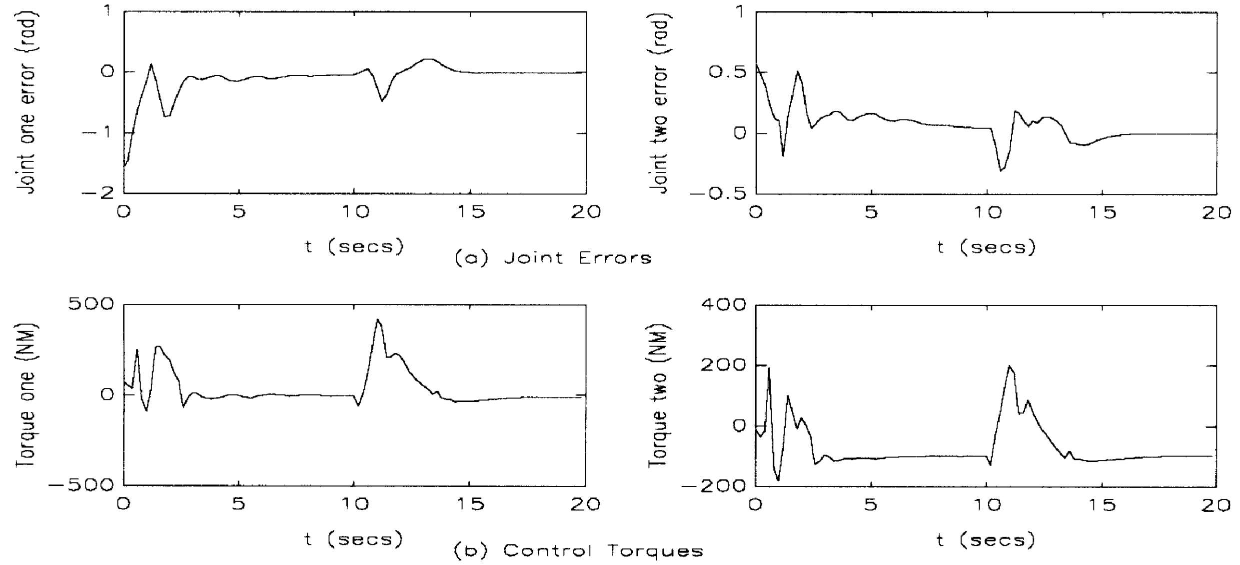
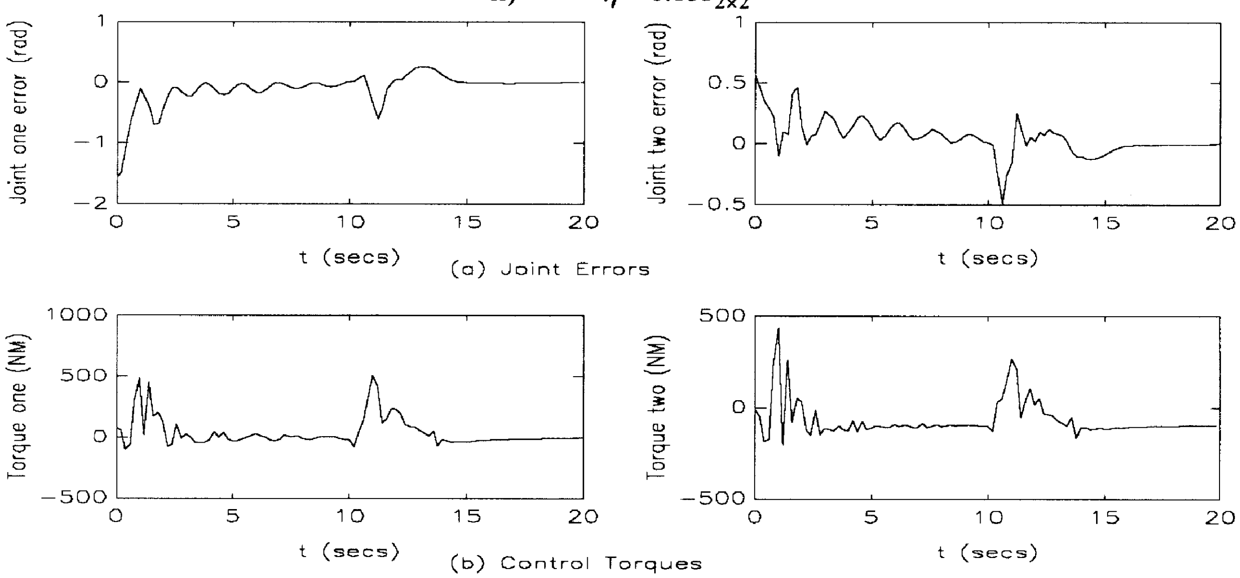


i) $\eta = 0.1 I_{2\times 2}$



ii) $\eta = 0.15 I_{2\times 2}$



iii) $\eta = 0.2 I_{2\times 2}$

Fig. 16. Combined Direct Adaptive and VSA Control with Unmodelled Dynamics ($K_d = I_{3\times 3}$, $P_s = I_{3\times 3}$, $\Gamma_1 = I_{3\times 3}$, $\delta = 0.001$)

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$$\tau_{d1}(t)=k_{11}\dot{q}_1+k_{12}sgn(\dot{q}_1)+d_1(t)\ \ \tau_{d2}(t) =k_{21}\dot{q}_2+k_{22}sgn(\dot{q}_2)+d_2(t) \quad (83)$$

where $k_{11}=k_{21}=d_1(t)=d_2(t)=0$, $k_{12}=k_{21}=0.5$, $0\le t<1[s]$; $k_{11}=k_{21}=k_{12}=k_{22}=0$, $d_1(t)=d_2(t)=1$, $1[s]\le t5[s]$; $k_{11}=k_{21}=0.5$, $k_{21}=k_{22}=d_1(t)=d_2(t)=0.5$, $5\le t<10$; $k_{11}=k_{21}=0.5$, $k_{12}=k_{22}=0$, $d_1(t)=d_2(t)=-1$, $10[s]\le t<15[s]$; $k_{11}=1$, $k_{21}=0.5$ $k_{12}=k_{22}=0$, $d_1(t)=1+sin(t)$, $d_2(t)=1+0.5cos(t)$, $10[s]\le t\ge 20[s]$. For the convenience of drawing the figures, the output data is selected once every two hundred sampling time intervals in the following figures.

1) Direct adaptive control: The simulation results are shown in Fig. 7, for the time interval from 0 [s] to 20 [s]. From the simulation results, we can see that the direct adaptive control is robust to reasonable input disturbances, and the smaller integral parameter gains give a better performance when the reference model changes over.

2) VSA control — algorithm 1: Figure 8 shows the simulation results under input disturbances. From Fig. 8 ii), we see that the input chattering becomes worse than the ideal case (Fig. 3).

3) VSA control — algorithm 2: For the different $\bar{\Theta}_s$, the simulation results are shown in Fig. 9 under same input disturbances as in the direct adaptive control.

4) VSA Control — Algorithm 3: Figure 10 shows the simulation results under input disturbances. From Fig. 10, we see that the large Γ_1 increases the robustness of the control system to the external input disturbances.

5) Combined direct adaptive and VSA control: Figure 11 shows the simulation results for three groups of parameters of P_s and Γ_1 under input disturbances. The results show that when $P_s=4I_{3\times 3}$ and $\Gamma_1 = 2I_{3\times 3}$ lead to the better performance.

We stimulate various external disturbances to compare the robustness properties of the five kinds of adaptive control laws used. The VSA control algorithm 3 has the best robustness, while the combined adaptive control gives good performance for both transient and steady state tracking performances. Increasing Γ_1 can further increase the robustness for the VSA control, but the steady state performance becomes worse.

6.3 Unmodelled Dynamics

Robot arm equations of motion are generally derived assuming rigid links and neglecting actuator dynamics and other flexibility effects, and most control laws are designed based on such models. However, in practice, the input to a manipulator driven by DC servo motors is the vector of armature voltages of the joint actuators, and the dynamic model from the armature voltages to the joint positions is at least third order. Although the adaptive control approach based on the third order dynamics has been proposed[20], generally, the controller will become more complicated. Hence, we need to consider the robustness of control methods due to perturbations from unmodelled dynamics. The dynamic model is described by the equations

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q}+G(q)=Z(t);\ \eta\dot{Z}(t)=-Z(t)+U(t) \quad (84)$$

where $U(t)\in R^n$ are the voltage control inputs and $Z(t)\in R^n$ are the motor states[20]. In the following simulations the robot dynamics (84) is used.

1) Direct adaptive control: Figure 12 shows the simulation results for adaptive control under unmodelled dynamics, with $\eta=0.1\,I_{2\times 2}$, $\eta = 0.15I_{2\times 2}$, and $\eta = 0.21I_{2x2}$. For the small unmodelled dynamic coefficient, η, the direct adaptive control gives an acceptable performance. But the performance deteriorates significantly with increasing η.

2) VSA control algorithm 1: For the same unmodelled dynamics effects, the simulation results are shown in Fig. 13 using VSA control algorithm 1. The simulation results show that this control algorithm has a weak robustness to the unmodelled effects.

3) VSA control algorithm 2: Figure 14 shows the simulation results using VSA control algorithm 2 under the same unmodelled effects. This control algorithm provides better robustness than direct adaptive control and VSA control algorithm 1. But information on Θ_s is required.

4) VSA control algorithm 3: Figure 15 shows the simulation results using VSA control algorithm 3 under the same unmodelled dynamics. This control algorithm has the best robustness properties compared to the previous algorithms. But tracking performance is not acceptable, due to the large tracking errors.

5) Combined direct adaptive and VSA control: For the same unmodelled dynamics effects, the simulation results are shown in Fig. 16 using the combined adaptive control algorithm.

Comparing all the simulation results under the same unmodelled dynamics effects, the combined adaptive control algorithm provides the best performance. The direct adaptive control becomes unacceptable when $\eta\geqslant 0.15I_{2\times 2}$ and the control inputs are larger than those of the combined adaptive control. The VSA control algorithm 1 shows the worst robustness to the unmodelled dynamics.

7 CONCLUSIONS

First, the direct and indirect adaptive control methods are briefly reviewed. Then, the combined direct and indirect adaptive control method is presented. The control systemmakes both the tracking (output) error and the prediction (input) error converge to zero. The computational efficient identifying algorithm of the indirect parameter estimation law is proposed. The method has been further extended to the position/force control[24].

After that, three types of variable structure adaptive (VSA) control algorithms are proposed. The methods have the form of the variable structure controller discussed in Spong[12] and Yu[13], but the switching gains are adjusted based on the gradient rule which can avoid the high gain control issue[12, 13]. The robustness to other uncertainties (except parameter uncertainty) is discussed and a modified scheme to overcome these uncertainties is proposed.

Finally, a new adaptive control scheme combining direct adaptive control and variable structure adaptive control for non-linear robot manipulators is presented. The method can be also extended to design combined direct, indirect and variable structure adaptive controllers. The combined adaptive control improves both the transient and robust

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properties for the robot system compared with adaptive control, and reduce the chattering compared with variable structure control. The various methods are tested and compared using extensive computer simulations.

References

1. J.J. Craig, P. Hsu and S. Sastry. "Adaptive control of mechanical manipulators", *Proc. of IEEE Conf. on Robotics and Automation*. San Francisco (1986) pp. 190–195.
2. J.J.E. Slotine and W. Li, "Adaptive manipulator control: a case study," *IEEE Trans.*, **AC-33**, p. 995–1003 1998).
3. R. Ortega and M.W. Spong, "Adaptive Motion Control of Rigid Robots: A Tutorial," *Proc. 27th Conf. on Decision and Control*, Austin, Texas (1988) pp. 1575–1584.
4. R. Johansson, "Adaptive control of robot manipulator motion," *IEEE Trans.*, **RA-6**, 483–490 (1990).
5. R.H. Middleton and G. C. Goodwin, "Adaptive Computed Torque Control for Rigid Manipulations," *System and Control Lett.*, **10**, 9–16 (1988).
6. W. Li and J.J.E. Slotine, "Indirect Adaptive Robot Control," *Proc. IEEE Int. Conf. on Robotics and Automation*, Philadelphia, (1988) **vol. 2**, pp. 704–709.
7. M.A. Duarte and K.S. Narendra, "Combined Direct and Indirect Approach to Adaptive Control," *IEEE Trans. Automat. Contr.* 1071–1075 (1989).
8. J.J.E. Slotine and W. Li, "Composite Control of Robot Manipulators," *Automatica*, **25**, 509–519 (1989).
9. M. Corless and G. Leitman, "Continuous state feedback guaranteeing uniforn ultimate boundedness for uncertain dynamic systems", *IEEE Trans.*, **AC-26**, 1139–1144 (1981).
10. K.K.D. Young, "Controller design for a manipulator using theory of variable structure systems," *IEEE Trans.*,**SMC-8**, 210–218 (1978)
11. J.J.E. Slotine, "The robust control of robot manipulators," *Int. J. Robot Research*, **4**, 49–64 (1985)
12 M. W. Spong,"On the robust control of robot manipulators," *IEEE Trans.*, **AC-27**, 1782–1786 (1992)
13. H. Yu, L.D. Seneviratne and S. W. E. Earles,"A robust model reference adaptive control for robut manipulators using sliding mode," *Proc. of the Sixth International Manufacturing Conf. with China, Hong Kong.* (1993)
14. K.S. Narendra and J. D. Boskovic,"A combined Direct, Indirect, and Variable Structure Method for Robust Adaptive Control," *IEEE Trans, Automat. Contr.*, **37**, 262–268 (1992)
15. H. Yu, L.D. Seneviratne and S. W. E. Earles,"Robust adaptive control for robut manipulators using a combined method," *Proc. of IEEE International Conf. on Robotic and Automation*, **No. 1**, 612–617 (1993)
16. H. Yu, L. D. Seneviratne and S. W. E. Earles,"Exponentially stable robot control law for robot manipulators," *IEE Proc. D.* **141**, 389–395, (1994)
17. C. Y. Su and T. P. Leung,"A sliding controller with bound estimation for robot manipulators," *IEEE Trans.* **RA-9**, 208–214 (1993)
18. K. M. Koo and J. W. Kim,"Robust control of robot manipulators with parametric uncertainty, *IEEE Trans.* **AC-39**, 1230–1233 (1994)
19. H. Yu,"Modeling and robust adaptive control of robot manipulators," *Ph. D. Thesis* University of London, UK (1994)
20. H. Yu and S. Lloyd,"Adaptive control of robot manipulators including motor dynamics," *Proc. Inst. Mech.Engrs.* **Part1, 209(2)**, 207–217 (1995)
21. K. S. Narendra and A. M. Annaswarmy, *Stable Adaptive Systems* (Prentice-Hall International Editions, 1989)
22. K. J. Astrom and B. Wittenmark, *Adaptive Control* (Addison-Wesley, New York, 1989)
23. M. W. Spong and M. Vidyasagar, *Robot Dynamics and Control* (John Wiley Sons, New York, 1989)
24. H. Yu and S. Lloyd,"Combined Direct and Indirect Adaptive Control of Constrained Robots," *Int J. Control*, **68**(5), 955–970 (1997)