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Conversion efficiency of even harmonics of whistler pulse in quantum magnetoplasma

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Abstract

Study of even harmonic generation resulting from propagation of whistler pulse in homogeneous high-density quantum plasma immersed in an externally applied magnetic field, using the recently developed quantum hydrodynamic model is presented. The effects of quantum Bohm potential, quantum statistical pressure, and electron spin have been taken into account. The field amplitude of even harmonic of the whistler with respect to fundamental wave and the conversion efficiency for phase-mismatch has been analyzed. The conversion efficiency of harmonic radiation depends on the plasma electron density, magnetic field strength as well as the intensity of whistler pulse. The efficiency increases significantly with an increase in plasma density, magnetic field and whistler wave intensity. Higher conversion efficiency is observed in degenerate plasma for lower values of the static magnetic field as compared with classical plasma.

Introduction

In recent years, there has been a rapid increase in the study of whistler waves. Having been discovered more than a century ago (Preece, 1894), these waves are still a subject of intense interest. Whistler (also known as helicon in solid-state plasma) is one of the most important electromagnetic waves in plasma, being circularly polarized guided along external magnetic field in the dense plasma. Whistler waves are observed in outer Earth's radiation belt, Earth's magnetopause (Stenberg *et al.*, 2007), beam-plasma systems (Volokitin *et al.*, 1995; Starodubstev *et al.*, 1999) immersed in external magnetic field and in several laboratory experiments (Stenzel, 1975, 1976; Compernolle *et al.*, 2014, 2015). These waves are excited essentially in plasma by collective electron oscillations in the presence of an external or self-consistent large-scale magnetic field. The nonlinear dynamics of whistler wave in magnetized plasma has been investigated (Baker & Hall, 1974; Sharma and Tripathi, 1993; Martino *et al.*, 2005; Streltsov *et al.*, 2006; Karavaev *et al.*, 2010; Gupta *et al.*, 2015).

The study of electromagnetic wave–plasma interaction is an active area of research in the context of harmonic generation. Over the last few years, the generation of harmonics of electromagnetic radiation in plasma has been a subject of extensive study (Krenz and kino, 1965; Tamaki, 1999; Banerjee, 2002; Mihailescu *et al.*, 2014; Tang *et al.*, 2017) because of wide range of applications. Harmonics provide valuable diagnostics for plasma parameters such as local electron density, electrical conductivity, and can also be used to detect the presence of large electric and magnetic fields and plasma waves. Plasma is an attractive medium for harmonic generation which can convert the fundamental frequency of whistler into linear and nonlinear harmonics. Nonlinear even harmonic especially second harmonic generation (SHG) has its unique place as it converts the fundamental frequency of whistler into twice that is, $\omega_2 = 2\omega_0$. In the case of second harmonic, the main mechanism is the presence of density gradient produced by electron plasma wave excited by propagation of whistler pulse. Even harmonic generation in plasma has been studied both experimentally and theoretically by many authors (Malka *et al.*, 1997; Agarwal *et al.*, 2001; Kant and Sharma, 2004; Kaur *et al.*, 2009; Aggarwal *et al.*, 2015; Iwai *et al.*, 2015).

In high-density plasma, where the de-Broglie wavelength of particles is of the order of or greater than the interparticle distance, the study of quantum effects becomes important. When the temperature of such plasma is less than the Fermi temperature degeneracy of particles comes into the picture. During the last decade, the focus is on investigating new aspects of quantum plasma due to its important applications in nanoscale and nanoelectronic devices (Abrahams *et al.*, 2001; Magnus and Schoemaker, 2002), in superdense astrophysical objects (Lai, 2001; Opher *et al.*, 2001; Chabrieret *et al.*, 2002) (such as white dwarfs, neutron stars, magnetostars and supernova), quantum plasma echoes (Manfredi and Fexi, 1996), quantum X-ray free electron laser (Piovella *et al.*, 2008), intense laser-solid density plasma experiments (Malkin *et al.*, 2007; Hartemann *et al.*, 2008). Most of the theoretical investigations on whistler waves have been focused on classical plasma, however, the nonlinear dynamics, collisional damping of the whistler mode (Watanabe *et al.*, 1967), dispersion of linear wave (Ren *et al.*, 2007), generation of Wakefield by whistler pulse (Mishra *et al.*, 2010*b*), circularly polarized modes (Mishra *et al.*, 2010*a*), whistler mode turbulence (Trukhanova, 2013) have been studied for quantum plasma. To the best of our knowledge, till now no attempt has been made to investigate excitation of SHG due to whistler propagation in strongly magnetized dense quantum plasma. Our objective in this paper is to present a theoretical study of the propagation of whistler pulse through quantum magnetoplasma embedded in an external magnetic field. When an external magnetic field is applied to quantum plasma, electron dynamics is modified and leads to nonlinear current modification. The density perturbation produced by magnetic field coupled with electron quiver motion leads to generation of harmonic radiation.

In the present paper, the interaction dynamics of whistler with plasma has been built in the mildly relativistic regime, using the recently developed quantum hydrodynamic (QHD) (Ren *et al.*, 2007; Mishra *et al.*, 2010*c*; Ghosh *et al.*, 2012; Hass and Eliasson, 2015) model which is generalization of classical fluid model for plasma with inclusion of quantum correction terms. It has been observed in the perturbative analysis that the relativistic effects become important in higher orders of perturbation only (Ghorbanalilu, 2012). The momentum equation has been modified to incorporate the effects of spin magnetic moment (Mishra *et al.*, 2010*a*, *b*, *c*) and the relativistic variation of mass (Ghosh *et al.*, 2012). The advantages of the QHD model over kinetic ones are its numerical efficiency, the direct use of macroscopic variables of interest such as momentum and energy and the easy way the boundary conditions are implemented. This allows considering the nonlinear phenomena relatively easier and so the QHD approach is preferred for describing such phenomena in quantum plasma (Shukla and Ali, 2006; Shukla and Eliasson, 2010; Vladimirov and Tyshetskiy, 2011). The effects of Fermi statistical pressure, the quantum Bohm potential and electron spin have been incorporated. The systematic organization of this paper is as follows: The section 'Whistler wave propagation' is organized by using perturbation technique in mildly relativistic regime to setup the oscillatory electron velocities, perturbed density and nonlinear current density for the propagation of whistler in high-density plasma. In mildly relativistic regime, the relativistic effect comes into play in third and higher order velocity components and higher order. In the section 'Second harmonic conversion efficiency' the nonlinear wave equation is solved and conversion efficiency of second radiation generation is estimated. The section 'Summary and discussion' is devoted to summary and discussion.

Whistler wave propagation

Consider a whistler beam propagating parallel to an externally applied static magnetic field in the uniform quantum plasma. The magnetic field is assumed to be along the z-axis($B_0 = b\hat{e}_z$). The electric and magnetic fields of the right circularly polarized whistler pulse are

$$\vec{E} = E(z,t)(\hat{x} - i\hat{y})e^{i(k_1 z - \omega_1 t)} + c.c., \tag{1}$$

$$\vec{B} = \frac{c(\vec{k}_1 \times \vec{E})}{\omega_1}, \tag{2}$$

where $\mathrm{E}(z, t)$ is the slowly varying amplitude of fundamental whistler pulse inside plasma, c is speed of light in vacuum and k_1 is propagating wave vector at whistler wave frequency ω_1. As the pulse propagates through magnetized plasma the current density at $2\omega_1$ arises and acts as a source for second harmonic.

The QHD equations governing the motion of an electron in the presence of whistler field and static external magnetic fields are given by

$$\left(\frac{\partial}{\partial t} + \vec{v}.\nabla\right)(\gamma\vec{v}) = -\frac{e}{m}[\vec{\mathrm{E}} + (\vec{v} \times \vec{\mathrm{B}})] - \frac{\nabla P}{mn} + \frac{\hbar^2}{2m^2\gamma^2}\vec{\nabla}\left(\frac{1}{\sqrt{n}}\vec{\nabla}^2\sqrt{n}\right) - \frac{2\mu_B}{m\gamma\hbar}\vec{S}.(\vec{\nabla}\vec{\mathrm{B}}), \tag{3}$$

$$\left(\frac{\partial}{\partial t} + \vec{v}.\nabla\right)\vec{S} = \frac{2\mu_B}{\hbar}(\vec{\mathrm{B}} \times \vec{S}), \tag{4}$$

and the continuity equation

$$\frac{\partial n}{\partial t} + \nabla(n\vec{v}) = 0, \tag{5}$$

where, n is electron density, m is the electron rest mass, $\hbar$ is the Planck's constant, $\vec{v}$ is the velocity of electron, $\vec{S}$ is the spin magnetic moment with its absolute value $|S_0| = (\hbar/2)$, γ is the relativistic factor and $\mu_B = ((e\hbar)/(2m))$ is the Bohr magneton. The second term on the left-hand side of Eq. (3) is the convective derivative of velocity field. On the other hand, the first term on the right-hand side of Eq. (3) is the Lorentz force, the second term denotes electron Fermi pressure ($P = mv_F^2 n^{5/3}/5n_0^{2/3}$), where $v_F = \hbar(3\pi^2 n_0)^{1/3}/m$ is the Fermi velocity, the third term is the Bohm force due to quantum correction in density fluctuations. The last term is the force due to the spin magnetic moment of plasma electrons. The classical equations may be recovered in the limit $\vec{J}$. The wave equation for source current $\hbar = 0$ is

$$\left(\vec{\nabla}^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = \frac{4\pi}{c^2}\frac{\partial\vec{J}}{\partial t}. \tag{6}$$

Assuming that the ions form a neutralizing background in dense plasma, the perturbative expansion of the set of QHD equations [Eqs (3)–(5)] governing interaction dynamics for first order of electromagnetic whistler field gives

$$\frac{\partial\vec{v}^{(1)}}{\partial t} = -\frac{e\vec{\mathrm{E}}^{(1)}}{m} - \frac{e(\vec{v}^{(1)} \times \vec{B}^{(1)})}{mc} - \frac{v_F^2}{n_0}\vec{\nabla}n^{(1)} + \frac{\hbar^2}{4m^2}\left(\frac{1}{n_0}\vec{\nabla}(\vec{\nabla}^2 n^{(1)})\right) - \frac{2\mu_B S_0}{m\hbar}(\vec{\nabla}\vec{\mathrm{B}}^{(1)}), \tag{7}$$

$$\frac{\partial\vec{S}^{(1)}}{\partial t} = \frac{2\mu_B}{\hbar}(\vec{B}^{(1)} \times S^{(0)}), \tag{8}$$

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$$\frac{\partial n^{(1)}}{\partial t} + n_0.\nabla\vec{v}^{(1)} = 0. \quad (9)$$

From Eq. (7) the first order perturbed equations for transverse quiver velocity components are obtained as

$$\frac{\partial v_x^{(1)}}{\partial t} = -\frac{e}{m}\mathrm{E}_x^{(1)} - \omega_c v_y^{(1)} - \frac{v_F^2}{n_0}\vec{\nabla} n_x^{(1)} + \frac{\hbar^2}{4m^2 n_0}\vec{\nabla}^3 n_x^{(1)} - \frac{2\mu_B S_0}{m\hbar}\vec{\nabla} B_x^{(1)}, \quad (10)$$

$$\frac{\partial v_y^{(1)}}{\partial t} = -\frac{e}{m}E_y^{(1)} - \omega_c v_x^{(1)} - \frac{v_F^2}{n_0}\vec{\nabla} n_y^{(1)} + \frac{\hbar^2}{4m^2 n_0}\vec{\nabla}^3 n_y^{(1)} - \frac{2\mu_B S_0}{m\hbar}\vec{\nabla} B_y^{(1)}. \quad (11)$$

In order to study the propagation of whistler pulse Eqs (10) and (11) are to be solved simultaneously. The plasma electrons interact with the whistler pulse to acquire the transverse quiver velocity,

$$\begin{pmatrix} v_x^{(1)} \\ v_y^{(1)} \end{pmatrix} = \begin{pmatrix} v_{x1} \\ v_{y1} \end{pmatrix} E(z,t)e^{i(k_1 z - \omega_1 t)} + c.c., \quad (12)$$

where

$$v_{x1} = \frac{1}{(\omega_c^2 - \omega_1^2)}\left[\frac{ie(\omega_1 - \omega_c)}{m} + \frac{k_1 Q(i\omega_c n_{y1} - \omega_1 n_{x1})}{n_0} + \frac{2i\mu_B k_1 S_0(\omega_c - \omega_1)}{m\hbar}\right],$$

$$v_{y1} = \left[\frac{-e}{m\omega_1} + \frac{i\omega_c v_{x1}}{\omega_1} + \frac{k_1 n_{y1} Q}{n_0\omega_1} + \frac{2\mu_B k_1 S_0}{m\hbar\,\omega_1}\right],$$

$$Q = \left[v_F^2 + \frac{\hbar^2 k_1^2}{4m^2}\right],$$

and $\omega_c = ((eb)/m)$ is the cyclotron frequency of plasma electrons.

The first order perturbed electron density is obtained by substituting the quiver velocities in the perturbed first order continuity equation,

$$\begin{pmatrix} n_x^{(1)} \\ n_y^{(1)} \end{pmatrix} = \begin{pmatrix} n_{x1} \\ n_{y1} \end{pmatrix} \mathrm{E}(z,t)e^{i(k_1 z - \omega_1 t)} + c.c., \quad (13)$$

where

$$n_{x1} = \frac{n_0 k}{\omega_1\{k_1^2 Q + (\omega_c^2 - \omega_1^2)\}}\left[\frac{ie(\omega_1 - \omega_c)}{m} + \frac{ik_1 Q\omega_c n_{y1}}{n_0} + \frac{2i\mu_B S_0 k_1(\omega_c - \omega_1)}{m\hbar}\right],$$

and

$$n_{y1} = \frac{n_0 k_1(\omega_c^2 - \omega_1^2)}{\{(\omega_1^2 - k_1^2)(\omega_c^2 - \omega_1^2) + k_1^2 Q\omega_c^2\}}\left[\frac{-e}{m} + \frac{e\omega_c}{m(\omega_c + \omega_1)} - \frac{ik_1 Q\omega_1\omega_c n_{x1}}{n_0(\omega_c^2 - \omega_1^2)} - \frac{2\omega_c\mu_B S_0 k_1}{m\hbar(\omega_c + \omega_1)} + \frac{2\mu_B S_0 k_1}{m\hbar}\right].$$

Spin is an important property of quantum degenerate plasma. It plays a crucial role in exposing the plasma to the external magnetic field, the effect of which can be ascertained in the perturbed spin magnetic moment for plasma electron through the spin angular momentum

$$\begin{pmatrix} S_x^{(1)} \\ S_y^{(1)} \\ S_z^{(1)} \end{pmatrix} = \begin{pmatrix} S_{x1} \\ S_{y1} \\ S_{z1} \end{pmatrix} \mathrm{E}(z,t)e^{i(k_1 z - \omega_1 t)} + c.c., \quad (14)$$

where

$$S_{x1} = \frac{2i\mu_B S_0\left\{\frac{2b\mu_B}{\hbar} - \omega_1\right\}}{\hbar\left\{\frac{4b^2\mu_B^2}{\hbar^2} - \omega_1^2\right\}},$$

$$S_{y1} = \frac{-2\mu_B(bS_{x1} - iS_0)}{i\hbar\,\omega_1} \text{ and } S_{z1} = \frac{-2\mu_B S_0(1 - i)}{\hbar\,\omega_1}.$$

By following similar steps for n^th^ harmonic, the quiver velocity, perturbed density and spin magnetic moment component can be obtained by substituting $\omega_1 \to n\omega_1$, $\mathrm{E}(z,t) \to \mathrm{E}_n(z,t)$, $(k_1 z - \omega_1 t) \to (k_n z - n\omega_1 t)$, in Eqs (12–14). Hence, the linear part of induced current density for n^th^ harmonic, $J^{(1)}(n\omega_1) = J_c^{(1)}(n\omega_1) + J_s^{(1)}(n\omega_1)$ can be written as,

$$J^{(1)}(n\omega_1) = [J_c^{(1)}(n\omega_1) + J_s^{(1)}(n\omega_1)]\mathrm{E}_n(z,t)e^{i(k_n z - n\omega_1 t)} + c.c. \quad (15)$$

where, $\vec{J_s} = -((2\mu_B)/\hbar)\nabla(n.S)$, is the current density due to spin magnetic moment and $\vec{J_c} = -e(n.v)$ is the conventional current. Substitution of Eq. (15) in the wave Eq. (6) for n^th^ harmonic yields the dispersion equation

$$c^2 k_n^2 = n^2\omega_1^2 + \omega_p^2\frac{imn\omega_1\{J_c^{(1)}(n\omega_1) + J_s^{(1)}(n\omega_1)\}}{n_0 e^2}. \quad (16)$$

Now, we proceed to obtain the second order perturbed velocity and particle density which are obtained using the same procedure as adopted for the first order fields,

$$\begin{pmatrix} v_x^{(2)} \\ v_y^{(2)} \\ v_z^{(2)} \end{pmatrix} = \begin{pmatrix} v_{x2} \\ v_{y2} \\ v_{z2} \end{pmatrix} E^2(z,t)e^{2i(k_1 z - \omega_1 t)} + c.c. \quad (17)$$

$$\begin{pmatrix} n_x^{(2)} \\ n_y^{(2)} \\ n_z^{(2)} \end{pmatrix} = \begin{pmatrix} n_{x2} \\ n_{y2} \\ n_{z2} \end{pmatrix} E^2(z,t)e^{2i(k_1 z - \omega_1 t)} + c.c. \quad (18)$$

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where

$$v_{x2} = \left[\frac{2i\mu_B k_1(\omega_c S_{y1} - 2\omega_1 S_{x1})}{m\hbar\,(\omega_c^2 - 4\omega_1^2)}\right],$$
$$v_{y2} = \left[\frac{i\omega_c v_{x2}}{2\omega_1} + \frac{\mu_B k_1 S_{y1}}{m\hbar\,\omega_1}\right],\; v_{z2} = \left[\frac{-ie(v_{x1} - iv_{y1})}{2m\omega_1 c}\right]$$

$$n_{x2} = \left[\frac{k_1 n_0 v_{x2}}{\omega_1} + \frac{k_1 n_{x1} v_{x1}}{\omega_1}\right],$$
$$n_{y2} = \left[\frac{k_1 n_0 v_{y2}}{\omega_1} + \frac{k_1 n_{y1} v_{y1}}{\omega_1}\right] \text{and } n_{z2} = \left[\frac{k_1 n_0 v_{z2}}{\omega_1} + \frac{k_1 n_{z1} v_{z1}}{\omega_1}\right].$$

Thus, the second order velocity components of plasma electrons oscillate with frequency twice that of fundamental whistler frequency. This effect arises from the coupling of propagating magnetic field of whistler pulse with the external magnetic field. The second order spin magnetic moment is obtained as

$$\begin{pmatrix} S_x^{(2)} \\ S_y^{(2)} \\ S_z^{(2)} \end{pmatrix} = \begin{pmatrix} S_{x2} \\ S_{y2} \\ S_{z2} \end{pmatrix} E^2(z,t) e^{2i(k_1 z - \omega_1 t)} + c.c. \tag{19}$$

where

$$S_{x2} = \frac{[((2b\mu_B)/\hbar)\{((2i\mu_B/)\hbar) + ik_1 s_{y1} v_{y1}\} - 2i\omega_1\{((2\mu_B S_{z1})/\hbar) - ik_1 v_{x1} S_{x1}\}]}{\{((4\mu_B^2 b^2)/\hbar^2) - 4\omega_1^2\}},$$

$$S_{y2} = \left[\frac{ib\mu_B S_{x2}}{\hbar\,\omega_1} + \frac{\mu_B S_{z1}}{\hbar\,\omega_1} + \frac{k_1 S_{y1} v_{y1}}{2\omega_1}\right] \text{and } S_{z2} = \frac{i\mu_B(iS_{y1} - S_{x1})}{\hbar\,\omega_1}.$$

The first order quiver velocity beats with the first order density to produce second harmonic nonlinear current density which leads to even second harmonic radiation generation at ($2\omega_1$, $2k_1$) as,

$$\vec{J}_{NL}^{(2)} = (J_{S2} + J_{c2})E^2(z,t)e^{2i(k_1 z - \omega_1 t)} + c.c.$$

where

$$J_{S2} = -\frac{4ik_0\mu_B}{\hbar}[n_0 S_{x2} + S_0 n_{x2} + n_{x1} S_{x1} + n_0 S_{y2} + S_0 n_{y2} + n_{y1} S_{y1} + n_0 S_{z2} + S_0 n_{z2} + S_{z1} n_{z1}]$$

and

$$J_{c2} = -e.[n_0 v_{x2} + n_{x1} v_{x1} + n_0 v_{y2} + n_{y1} v_{y1} + n_0 v_{z2} + n_{z1} v_{z1}].$$

There also exists a self-consistent even second harmonic field $\vec{E}^{(2)} = E_2(z,t)e^{i(k_2 z - 2\omega_1 t)} + c.c$ due to which the linear current density is given by

$$\vec{J}_L^{(2)} = \left(\frac{ie^2 n_0 E_2(z,t)}{2m\omega_1}\right) e^{i(k_2 z - 2\omega_1 t)} + c.c. \tag{20}$$

Second harmonic conversion efficiency

The normalized amplitude for phase mismatched second harmonic is obtained by substituting linear and nonlinear current densities in the wave equation [Eq. (6)] governing the second harmonic field $\vec{E}^{(2)}$, which on simplification yields

$$\frac{E_2(z,t)}{E(z,t)} = \frac{8\pi i a_0 mc(J_{s2} + J_{c2})(\omega_1/\omega_p)^2}{\{((2im\omega_1[J_c(2\omega_1) + J_s(2\omega_1)])/(en_0)) + e\}} e^{i\Delta kZ}, \tag{21}$$

where, $\Delta k = (k_2 - 2k_1)$ represents the phase difference between the generated even second harmonic and the fundamental frequency. We get the second harmonic conversion efficiency as

$$\eta_2 = \frac{64\pi^2 m^2 c^2 a_0^2 (J_{s2} + J_{c2})^2}{e^2(((\omega_p)/(\omega_1)))^4\{((2im\omega_1)/(n_0 e^2))[J_c(2\omega_1) + J_s(2\omega_1)] + 1\}^2}, \tag{22}$$

where

$$J_s(2\omega_1) = -\frac{2ik_1\mu_B}{\hbar}[n_0 S_{x1} + S_0 n_{x1} + n_0 S_{y1} + S_0 n_{y1} + n_0 S_{z1} + S_0 n_{z1}](2\omega_1)$$

$$J_c(2\omega_1) = -en_0[v_{x1} + v_{y1} + v_{z1}](2\omega_1) \text{ and } a_0 = \frac{eE_0}{mc\omega}.$$

The conversion efficiency is proportional to the plasma electron density, strength of the magnetic field and the intensity of whistler pulse.

Figure 1, shows the variation of conversion efficiency (η_2%) with normalized plasma electron density for different values of magnetic field strength. The figure shows that for a constant magnetic field, harmonic radiation grows with increase in the plasma density until saturation. The saturation value of plasma density depends on the applied magnetic field and is more for the weaker magnetic field.

Figure 2, shows the variation of second harmonic conversion efficiency (η_2%) with the magnetic field strength at the varying value of normalized plasma density. The dark line is for $\omega_p/\omega_1 = 0.3$ and the dashed line is for $\omega_p/\omega_1 = 0.5$. It is observed that the efficiency increases with the magnetic field and approaches to saturation. The saturation strength increases for the lower density of plasma.

Figure 3, shows the variation of second harmonic conversion efficiency as a function of the intensity of the whistler pulse for different values of normalized plasma density. The dark line is for $\omega_p/\omega_1 = 0.3$, while the dashed line is for $\omega_p/\omega_1 = 0.5$. The efficiency increases for lower values of intensity, however conversion efficiency saturates at large value of the intensity of whistler pulse.

Figure 4 shows the variation of harmonic conversion efficiency as a function of whistler pulse intensity for various values of

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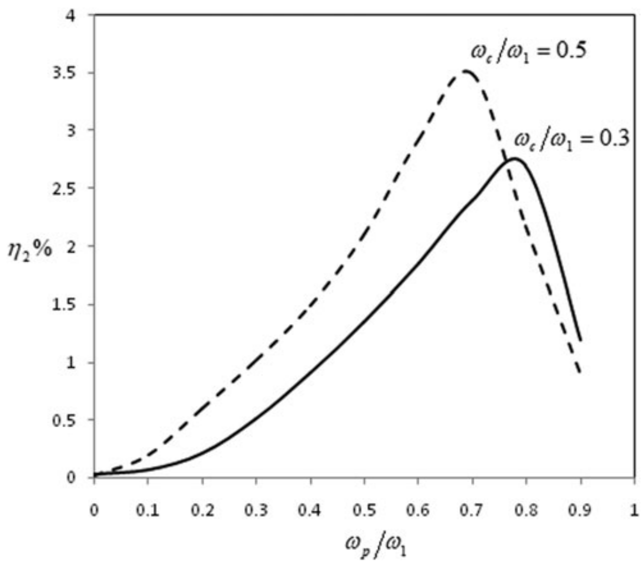

Fig. 1. Variation of conversion efficiency as a function of ω_p/ω_1 for $a_0 = 0.271$, $n = 10^{30}\ m^{-3}$ and different value of ω_c/ω_1.

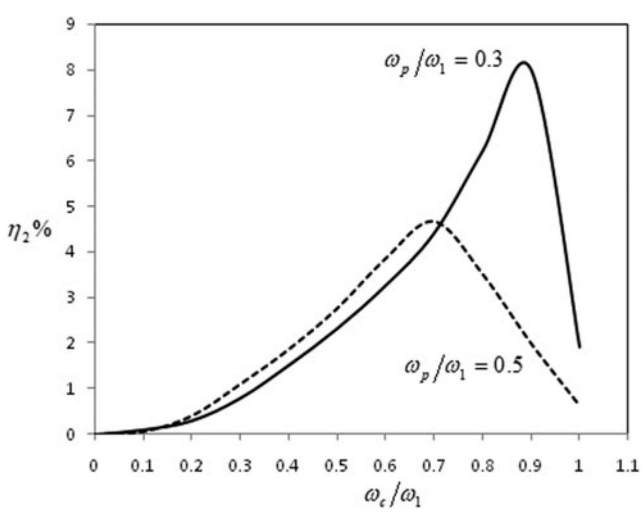

Fig. 2. Variation of conversion efficiency as a function of ω_c/ω_1 for $a_0 = 0.271$, $n = 10^{30}\ m^{-3}$ and different value of ω_p/ω_1.

magnetic field strength ($\omega_c/\omega_1 = 0.4$ for dark line and $\omega_c/\omega_1 = 0.6$ for dashed line). The figure depicts that the conversion efficiency of even second harmonic radiation generation increases with increase in whistler intensity and magnetic field. The increase in the conversion efficiency of second harmonic with the intensity of fundamental whistler pulse is due to the generation of strong nonlinear current. The efficiency tends to saturate for high value of a_0.

In order to compare the conversion efficiency of second harmonic with the classical case, Figure 5 has been plotted for parameters $\omega_c/\omega_1 = 0.3$, $a_0 = 0.271$ and $n = 10^{30} m^{-3}$. The upper solid line shows the variation of conversion efficiency in presence of quantum effects and the dashed line is in the absence of quantum effects (in the limit $\hbar = 0$). It is evident from the figure that the conversion efficiency of second harmonic is more by about 11% due to the presence of quantum effects in magentoplasma

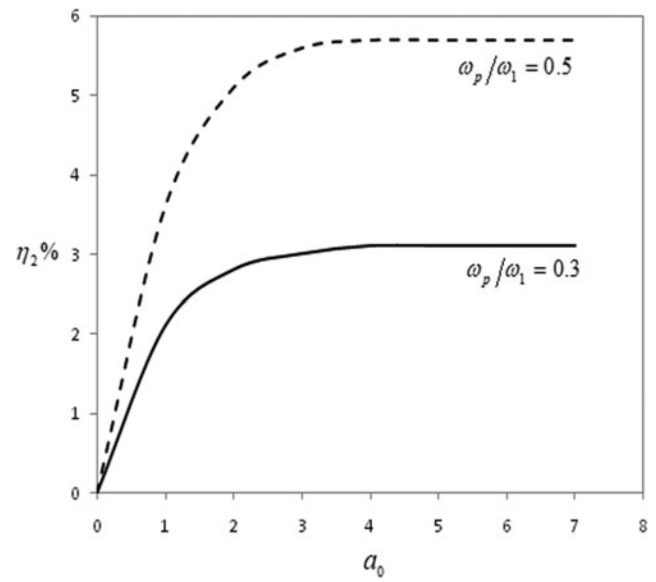

Fig. 3. Variation of conversion efficiency against normalized fundamental whistler intensity a_0 for various values of ω_p/ω_1.

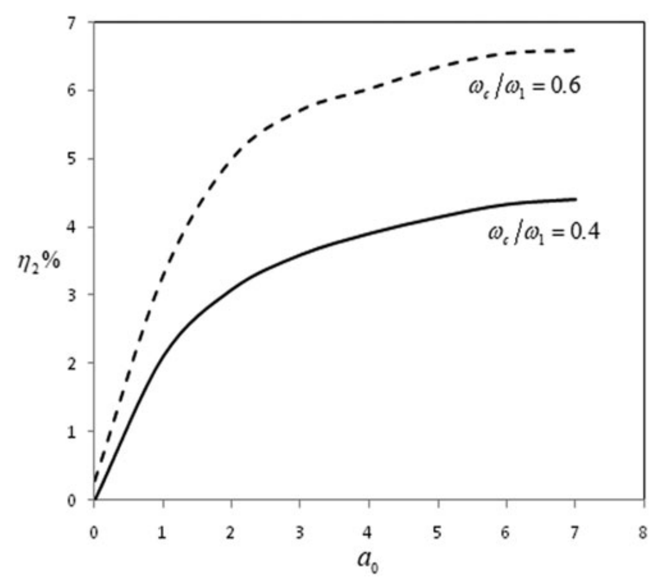

Fig. 4. Variation of conversion efficiency of second harmonic with normalized fundamental whistler intensity a_0 for various values of ω_c/ω_1.

because quantum diffraction effects play a crucial role by modifying the efficiency of second harmonic of whistler pulse.

Summary and discussion

A study of SHG resulting from propagation of whistler wave in homogenous high-density magnetized quantum plasma is presented. The static magnetic field applied for magnetization is in the longitudinal direction. The interaction mechanism has been built using the recently developed QHD model. The effects of Fermi statistical pressure, the quantum Bohm potential, and the spin of electron have been taken into account. The quiver and second order velocities along with electron densities and the spin angular momenta have been obtained through the perturbative

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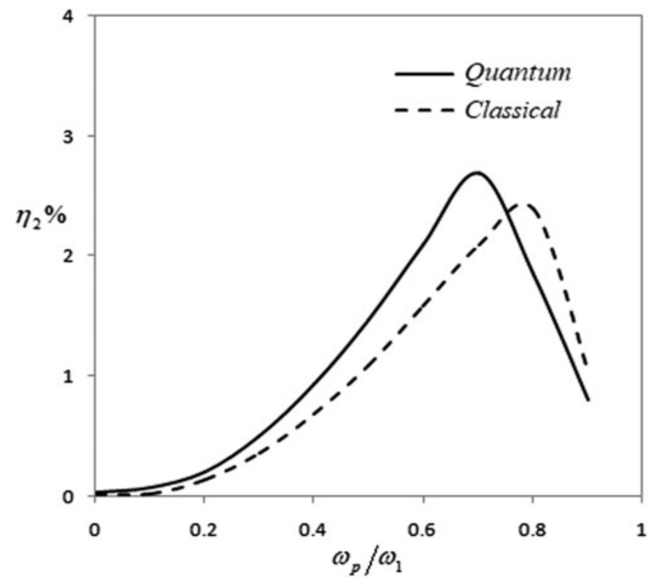

Fig. 5. Variation of conversion efficiency as a function of ω_p/ω_1, (i) solid line in the presence of quantum effects and (ii) dashed line in absence of quantum effects.

expansion of QHD equations. The nonlinear current density is the sum of conventional current and the current due to spin magnetic moment. The linear current results from the self-consistent field. The efficiency of SHG has been obtained for the phase mismatched case. It has been found that the SHG grows with the plasma density and the magnetic field up to the respective saturation values. The harmonic generation stops beyond saturation. The saturation of plasma density occurs earlier for increasing magnetic field. This is due to the polarization field effect in strongly magnetized dense plasma. The efficiency of SHG also increases with intensity of whistler pulse. The noteworthy observation is occurrence of enhanced SHG for high-density degenerate plasma at lower values of external magnetic field strength. The present study of second harmonic of whistler will be useful in acceleration of whistler pulse in astrophysical environments, fabrication of microelectronic devices, for high-quality plasma processing and to derive dc current in toroidal fusion devices.

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