PEF, **17** (1): 1–22, January, 2018.
doi:10.1017/S1474747216000147 First published online 3 October 2016

*Does retirement flexibility provide a hedge against macroeconomic risk?**

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Abstract

Retirement flexibility is often seen as a hedge against macroeconomic risks such as capital market risks, which justifies more risky asset portfolios. This paper analyses the robustness of this claim in both a partial equilibrium and general equilibrium setting. We show that this positive relationship between risk taking and retirement flexibility is weakened and under some conditions even turned around if not only capital market risks, but also productivity risks are considered. Productivity risk in combination with a high elasticity of substitution between consumption and leisure creates a positive correlation between asset returns and labour income, reducing the willingness of consumers to bear risk.

JEL CODES: E21, G11, J26

Keywords: Retirement (in)flexibility, portfolio choice, general equilibrium, intratemporal substitution elasticity.

1 Introduction

In recent years, many Western countries have made their pension systems more flexible by moving from contracts with high implicit taxation and predominantly inflexible payout periods towards more actuarially neutral arrangements with flexible payout periods (OECD, 2011). This move to flexible pension schemes is partly induced by population ageing and the financial crisis, which put the traditional pension systems under financial pressure. Another important factor is the ongoing process

* We are grateful to Casper van Ewijk, Devis Geron, Jochen Mierau, Ward Romp and Ed Westerhout for their comments on earlier versions of this paper. We also wish to thank the participants of the Netspar pension workshop in Utrecht (November 10, 2010), the participants of the Netspar International Pension Workshop in Turin (June 16, 2011) and the participants of the Annual Congress of the International Institute of Public Finance in Ann Arbor, Michigan (August 8, 2011) for helpful comments and suggestions.

of individualization and the resulting acknowledgement that individuals differ in their tastes for leisure, earnings capacities, wealth positions, and therefore have different preferences for retirement. One of the implications of this move towards more flexible pension schemes is that individuals have much more flexibility in their decision when to retire.[1]

An advantage of retirement flexibility that is often mentioned in the literature is that it provides insurance against all types of risks, like disability risk (Diamond and Mirrlees, 1978), stock market risk (Bodie *et al.*, 1992) or productivity risk (Pestieau and Possen, 2010).[2] It gives individuals the ability to adjust the length of their working life in response to shocks and thus to mitigate the effects of such shocks on consumption. In this way, flexible retirement is a means to hedge against future shocks and basically serves as a fourth pillar of the pension system. Moreover, as flexible retirement serves as a hedge against adverse investment outcomes it allows for more risk taking in pension savings (Bodie *et al.*, 1992). In this paper, we analyse the robustness of this result in both a partial equilibrium and a general equilibrium setting. The main question we address in this paper is whether retirement flexibility can really serve as a hedge against macroeconomic risks and increases the willingness of individuals to carry risk. Our analysis shows that the answer to this question crucially depends on factors such as the type of risk, the willingness of consumers to substitute consumption for leisure, and general equilibrium effects and that these factors may change existing views from the literature.[3]

The number of studies that focus on the interaction between portfolio, consumption and retirement decisions is rather limited. Starting point is the paper of Bodie *et al.* (1992), which analyses this interaction assuming that labour can be adjusted continuously. Subsequent studies, such as Choi and Shim (2006); Farhi and Panageas (2007) and Choi *et al.* (2008) model optimal retirement as a discretionary stopping problem. Although all these studies differ in many respects, they have in common that they use partial equilibrium models and mainly stick to capital market risks. In addition, they all find that more flexibility in the retirement decision increases the portfolio share invested in stocks.

[1] Throughout the paper the term "retirement flexibility" is used to indicate the flexibility to choose the date of retirement, where retirement is a full withdrawal from the labour market. We do not analyse gradual retirement, i.e., a stepwise reduction of the number of hours of labour supplied.

[2] There are also two related papers that study the hedging of idiosyncratic risk using savings or work effort (Pijoan-Mas, 2006) or extending labour supply late in the life cycle (Erosa *et al.*, 2012). Our paper, however, focuses on the hedging of macroeconomic risks instead of idiosyncratic risk. Moreover, these papers do not model portfolio choice, while the main focus of our paper is on the interaction between portfolio choice and retirement flexibility.

[3] We do not address the question whether the retirement age should be flexible or fixed. We are primarily interested whether the existing view in the literature, that retirement flexibility can serve as a hedge and increases the willingness of individuals to take risk, is robust in a more general set-up which allows for general equilibrium effects and more types of macroeconomic risk factors. Therefore, we focus on the interaction between portfolio choice, consumption and retirement decisions. We do not consider the welfare effects of retirement flexibility. Actually, in our set-up retirement flexibility is always preferable to inflexibility in terms of welfare. To study the question whether retirement should be flexible or fixed would require a different set-up of the model including factors like myopia or other market failures that would justify a fixed retirement age. As these features would complicate the model to a large extent and are not necessary to address the main question of this paper, we leave this for future research.

Compared with the existing literature, we add three important elements to the analysis on portfolio choice and retirement. First, we complement the partial equilibrium approach with a general equilibrium one.[4] A general equilibrium perspective seems a natural road to take for large economies, but also for smaller economies because the move to flexibility in the retirement date clearly is a global phenomenon. In a general equilibrium framework, we explicitly recognize that consumption and labour supply decisions affect factor prices which, in turn, influence the insurance effect of retirement flexibility. To illustrate, if every older worker decides to work longer after an adverse shock, wages will decline making the insurance of retirement flexibility less effective. Second, we distinguish between productivity and depreciation (or capital market) risk and these risk factors are directly linked to production. This distinction is important because both risk factors constitute a rather different effect on income and substitution effects in labour supply. As will be shown, the relative strength of income and substitution effects determines whether retirement flexibility indeed serves as a hedge against poor asset returns. Third, following Choi *et al.* (2008), we allow for more general preferences, which are characterized by a constant elasticity of substitution (CES) function of consumption and leisure. This specification allows the relative strength of income and substitution effects to take any positive number.

There is also a related strand of literature that analyses the intergenerational risk-sharing properties of pension schemes using stochastic general equilibrium overlapping-generations (OLG) models (e.g., Krueger and Kubler, 2006; Bohn, 2009; Gottardi and Kubler, 2011). Although the models used in these papers are somewhat comparable, the main purpose of our paper is completely different. These papers are more of a normative nature and focus on the question whether it is optimal to have a pay-as-you-go (PAYG) pension system, while our paper takes a positive perspective and analyses a different question, i.e., whether retirement flexibility can serve as hedge against macroeconomic risks and how this changes the willingness of individuals to take risk.

To analyse the interaction between portfolio choice, consumption and retirement decisions, we develop a two-period OLG model of a closed economy in the spirit of Samuelson (1958) and Diamond (1965). The model includes government debt and incorporates endogenous retirement. In our framework, the young working generation decides upon consumption and the allocation of the asset portfolio. Agents can either invest in risk-free government bonds or in risky firm stocks. It should be noted that one period in this type of model represents about thirty years, so the risks in our model should be interpreted as long-term shocks like economic or financial crises and not as business cycle risks. Our model is related to the model of Adema (2008) which is also a stochastic two-period OLG model of a closed economy with government debt. There, however, the return on bonds is subject to inflation risk, while retirement is exogenous. In our model, retirement is endogenous and we compare two different retirement settings: under *flexible* retirement, the old generation can freely postpone or advance retirement in the second period after a realization

[4] The partial equilibrium setting coincides with a small open economy, while the general equilibrium framework corresponds to a closed economy.

of shocks; under *fixed* retirement, this generation has to make this decision already before shocks are revealed. Once set, this decision cannot be changed when new information becomes available. The fixed-retirement setting can be interpreted as a country with a fixed statutory retirement age. Such a mandatory retirement age is often observed in countries with a PAYG pension scheme. As mentioned above, many of these countries now move to a system with more flexibility with actuarially neutral flexible payout periods. According to Bodie *et al.* (1992) this would allow savings for old age, either individual or via funded pension schemes, to be invested more in risky assets. In this paper, we analyse the robustness of this claim. In order to focus on this issue, we abstract from modelling a PAYG scheme. Although such a scheme is important for the interpretation of the shift towards more retirement flexibility, modelling it would only complicate the analysis and distract from the main message of the paper, but it would not change the main results.[5] It should be noted that we assume that the statutory retirement age is set at an ex ante optimal level. That is, the statutory retirement level is optimal before the realization of shocks is revealed. However, as it is institutionalized as part of the PAYG scheme, it cannot be adjusted when new information about the shocks becomes available. Adjustments to PAYG schemes almost always lead to intra- or intergenerational redistribution (see, e.g., Adema *et al.*, 2016) and therefore normally require a lengthy process of political decision making.

We use log-linearization techniques to characterize the main insights of the model. As a first step, we analyse the case that is 'standard' in the literature: a partial equilibrium setting with only depreciation (capital market) risk. We derive an analytical expression for the optimal portfolio and the retirement decision, both with flexible and inflexible retirement, in this case. Next, we present numerical simulation experiments to analyse the optimal portfolio and retirement in a more general setting. In the first set of experiments, we stick to the partial equilibrium setting, but add productivity risk to the model. In the second set of experiments, we study both depreciation risk and productivity risk in a general equilibrium setting. In these experiments, we study the steady state, but we also explicitly look at the dynamics using impulse-response functions. In addition, special attention is given to the role of the elasticity of substitution between consumption and leisure. It should be noted, however, that the simple set-up of our model does not allow for reproducing realistic data on portfolio composition and retirement decisions. It is well known that in order to realize that in a general equilibrium setting, a much more complicated model is required. However, closely reproducing real-world data are not the main purpose of our paper. This paper primarily aims at showing qualitatively that the well-known result that retirement flexibility provides a hedge against macroeconomic risks is not as robust as the literature seems to suggest. In particular, the paper shows that existing views from the literature may change if other important elements are incorporated into the analysis.

[5] As shown in Adema (2008), introducing a PAYG scheme would lead to an investment portfolio with more risky assets in the setting with fixed retirement. However, as long as the benefits are actuarially-neutral adjusted, the effect of introducing retirement flexibility on the portfolio composition would not be qualitatively different in a model with a PAYG scheme.

First, the positive relation between retirement flexibility and a higher demand for risky assets is weakened and under some conditions even turned around if not only depreciation shocks but also productivity shocks are considered. Depreciation shocks mainly affect the return on capital and through the income effect these shocks contribute to the traditional view that retirement flexibility increases risk-taking behaviour. Productivity shocks, in contrast, do not only affect capital returns, but also influence wages. Consequently, productivity shocks also induce substitution effects in labour supply, which work in the opposite direction. These substitution effects generate a positive correlation between asset returns and labour income, thereby reducing the willingness to bear risk of consumers.

Second, confining the analysis to Cobb–Douglas utility, as most of the existing studies do, ignores the essential role of the elasticity of substitution between consumption and leisure in studying retirement flexibility. This elasticity of substitution governs the relative strength of income and substitution effects in labour supply and, hence, determines the insurance provided by retirement flexibility. Our analysis clearly shows that flexible retirement amplifies consumption volatility if substitution effects are important, a notion also put forward by Basak (1999).

Finally, we find that general equilibrium effects play an important role in the interaction between portfolio choice and retirement. It is mainly the degree of substitution between consumption and leisure that determines the direction of these effects. For higher values of the elasticity of substitution, agents choose to supply less labour after a negative productivity shock. In general equilibrium, this labour supply response exacerbates the direct fall in the return on capital due to the productivity contraction. Compared with partial equilibrium, this higher sensitivity of the capital return for productivity risk therefore reduces the effectiveness of retirement flexibility as a hedging tool even more. Of course, for low elasticities of substitution the opposite holds: then the insurance effect is more effective in general than in partial equilibrium.

The results we derive in this paper are relevant for private or public pension institutions, like corporate pension funds, trust funds or life-insurance companies, to which individuals have dedicated or will dedicate their saving and investment decisions in the future. On the basis of the existing literature, one would argue that these pension institutions could take more risk with their investments when the retirement age becomes more flexible. Our analysis shows however that it is not always in the interest of individuals to have more risky investment strategies with flexible retirement. This is in particular the case if shocks to pension wealth and wages are positively correlated or if consumers view leisure and consumption as close substitutes. Indeed, many authors argue that the stock market and human capital are highly positively correlated (Baxter and Jermann, 1997; Benzoni *et al.*, 2007). Moreover, many empirical studies exploring the impact of financial incentives on the retirement decision typically find modest wealth effects (Krueger and Pischke, 1992; French, 2005; Bloemen, 2011) but large substitution effects (Gruber and Wise, 1999; Coile and Gruber, 2001; Asch *et al.*, 2005).

This paper is organized as follows. Section 2 sets out the basic equations of the stochastic OLG model, for the flexible retirement case as well as for the fixed retirement case. Section 3 presents analytical results for a specific model setting, which reproduces the main findings of the current literature. In Section 4, we present and compare numerical results for the partial equilibrium model and for the general equilibrium model, both with depreciation risk and productivity risk. Finally, Section 5 concludes.

2 The model

In this section, we develop a two-period OLG model of a closed economy.[6] In order to analyse the interaction between retirement and portfolio choice, we include government debt in the model as an alternative investment vehicle for future consumption and introduce endogenous retirement in the second period of life. The economy is subject to productivity risk and depreciation risk.

At each point in time, the young individual determines consumption of a single good and the proportion of financial wealth to invest in firm stocks. The old generation decides, which fraction of the second period it will spend on working and on enjoying retirement. We interpret this as a decision on the date of retirement. Alternatively, it could be interpreted as a decision on the number of hours of labour supplied during the second period of life with a fixed retirement date. As our results primarily concern the interaction between portfolio choice and the flexibility of labour supply later in life, it is merely a matter of taste which interpretation is used. Following Bodie *et al.* (1992), we consider two different retirement settings: under *flexible* retirement, the old generation can freely postpone or advance retirement in the second period after a realization of shocks; under *fixed* retirement, the retirement decision has to be made before shocks are revealed. Once set, the retirement age cannot be subsequently changed after new information has become available. Whatever the retirement setting (flexible or fixed), an individual sets his decision variables optimally, conditional on his information to date: his current financial wealth, the future dynamics of the asset returns and his uncertain future wage.

2.1 Production

The young and old generation are composed of the same large number of individuals and this number is normalized to unity. Production per young worker is described by a standard neoclassical constant-returns-to-scale Cobb–Douglas production function:

$$y = f(k_t, z_t) = A_t k_t^{\alpha}(1 + z_t)^{1-\alpha}, \tag{1}$$

with A_t is the stochastic total productivity parameter, α the capital share in production and k_t the capital stock per young worker. Total labour supply, $1 + z_t$, consists of young workers inelastically supplying one unit of labour and old workers, each spending a fraction $0 \leq z_t \leq 1$ of time on working. Profit maximization and perfect competition among producers results in the standard equilibrium conditions:

$$w_t = (1 - \alpha)A_t k_t^{\alpha}(1 + z_t)^{-\alpha}, \tag{2}$$

[6] In this section we only present the basic model equations. For more detailed derivations and information on how we solve the model using a log-linearization technique around the stochastic steady state, we refer to the technical appendix, Section A.2.

$$r_{k,t} + \delta_t = \alpha A_t k_t^{\alpha-1}(1+z_t)^{1-\alpha}, \tag{3}$$

where w_t is the real wage, $r_{k,t}$ the return on capital and δ_t can be interpreted as the stochastic depreciation rate of capital.

Productivity risk[7] and capital market risk are important in this context since we are interested in the return on long-term investment for retirement. Introducing both productivity- and capital market risk endogenizes the (positive) correlation between capital and labour income. Recent empirical evidence indeed suggests that aggregate labour income and stock returns are positively correlated, at least in the longer run (see, e.g., Baxter and Jermann, 1997; Bohn, 2009 and Benzoni *et al.*, 2007).

Productivity risk directly affects the capital return and the wage rate, while depreciation risk only directly affects the return on capital. Of course, there is an indirect link between the wage rate and depreciation risk, to the extent that labour supply behaviour affects factor prices in general equilibrium. Stochastic depreciation not only breaks down the (perfect) correlation between wages and capital returns, it also increases return volatility and may give capital returns a higher one-period-ahead variance than wages. The two stochastic processes for total factor productivity and capital depreciation are:

$$\log A_t = \log A + \omega_{A,t}, \tag{4}$$

$$\log \delta_t = \log \delta + \omega_{\delta,t}, \tag{5}$$

with $\omega_{A,t}$ and $\omega_{\delta,t}$ independently and identically distributed with mean zero and variance σ_A^2 and σ_δ^2.

2.2 Consumers

Individuals derive utility from consumption and leisure. Expected lifetime utility of a representative individual born at t is given by the following constant-relative-risk-aversion (CRRA) utility function:

$$U_t = \frac{c_{y,t}^{1-\gamma} - 1}{1-\gamma} + \beta \frac{\mathrm{E}_t u(c_{o,t+1}, 1 - z_{t+1})^{1-\gamma} - 1}{1-\gamma}, \tag{6}$$

where $c_{y,t}$ is the consumption when young at time t, $c_{o,t+1}$ is the consumption when old at $t+1$, β is the time discount factor and γ is the coefficient of relative risk aversion, which is identical to the inverse of the intertemporal elasticity of substitution. The per-period utility function $u(\cdot)$ has a CES specification and is defined as:

$$u(c_o, 1-z) = \left[(1-\eta)c_o^{1-\rho} + \eta(1-z)^{1-\rho}\right]^{\frac{1}{(1-\rho)(1-\eta)}}, \tag{7}$$

where η defines the relative preference for leisure and ρ represents the inverse of the elasticity of substitution between consumption and leisure in the second period. This specification includes the familiar Cobb–Douglas period utility function $u(c_o, 1-z) = c_o(1-z)^{\eta/(1-\eta)}$, if $\rho = 1$.

[7] Throughout the paper total factor productivity risk/shocks are referred to as productivity risk/shocks.

People can either invest in firm stocks which yield the stochastic return $r_{k,t+1}$ or in government bonds with the risk-free return $r_{b,t+1}$. The share of savings that is invested in stocks is denoted by λ_t, so that the return on the asset portfolio can be defined as:

$$r_{t+1} \equiv (1-\lambda_t)r_{b,t+1} + \lambda_t r_{k,t+1}. \tag{8}$$

Consumption in the first and second periods of life are respectively given by:[8]

$$c_{y,t} + s_t = w_t - \tau_t, \tag{9}$$

$$c_{o,t+1} = (1+r_{t+1})s_t + z_{t+1}w_{t+1}, \tag{10}$$

where τ_t are lump-sum taxes to finance the interest obligations on the government debt.

Maximizing lifetime utility with respect to consumption ($c_{y,t}$ and $c_{o,t+1}$) and the portfolio allocation (λ_t) subject to the budget constraints gives the following Euler condition:

$$c_{y,t}^{-\gamma} = \beta \mathrm{E}_t\Big[(1+r_{j,t+1})c_{o,t+1}^{-\rho}u(c_{o,t+1}, z_{t+1})^{\rho-\varphi}\Big], \tag{11}$$

for $j = b, k$ and with $\varphi \equiv \gamma - \eta(1-\rho)$.

The first-order condition with respect to labour supply (z_{t+1}) differs between flexible and inflexible retirement.[9] In the first case, the optimality condition is:

$$\left(\frac{c_{o,t+1}}{1-z_{t+1}}\right)^{\rho} = \frac{w_{t+1}}{\theta}, \tag{12}$$

with $\theta \equiv \eta/(1-\eta)$. In the optimum, the marginal rate of substitution between leisure and consumption is equal to the wage rate. Since agents can freely adjust labour supply in period $t+1$, this decision is conditional on the shocks that affect consumption and the wage rate in that period, i.e., $\omega_{A,t+1}$ and $\omega_{\delta,t+1}$ (see equations (4) and (5)). With inflexible retirement, though, the first-order condition is:

$$\theta(1-z_{t+1})^{-\rho}\mathrm{E}_t\big[u(c_{o,t+1}, z_{t+1})^{\rho-\varphi}\big] = \mathrm{E}_t\Big[w_{t+1}c_{o,t+1}^{-\rho}u(c_{o,t+1}, z_{t+1})^{\rho-\varphi}\Big]. \tag{13}$$

Since agents are not able to condition the retirement decision on the state of the economy in $t+1$, they have to form (rational) expectations. Obviously, z_{t+1} is known at time t.

2.3 Government

The government debt per young worker, b_{t+1}, is equal to the amount of debt in the previous period plus the interest obligations on the outstanding debt minus the collected tax receipts. That is,

$$b_{t+1} = (1+r_{b,t})b_t - \tau_t. \tag{14}$$

[8] So we assume that individuals save for their old age income and bear the full investment risk. That is, we abstract from pension schemes that share risks or redistribute income between individuals or generations. Individual savings in our model can thus be interpreted as a fully funded individual defined contribution pension scheme.

[9] Throughout the analysis, z_{t+1} indicates labour supply in the second period of life. Under fixed retirement, however, z_{t+1} is chosen in the first period and therefore known at time t.

Throughout the paper we assume that the government keeps debt per young worker constant, i.e., $b_{t+1} = b_t = b$. This implies that the lump-sum taxes to finance the interest obligations on the government debt, τ, should be equal to:

$$\tau_t = r_{b,t} b. \tag{15}$$

Like the capital stock and labour supply (in case of fixed retirement), the bond return $r_{b,t}$ is a predetermined variable: it denotes the interest that is paid at time t on the government debt that is issued one period before, in $t-1$.[10]

2.4 Equilibrium

The capital market (and the goods market as well) is in equilibrium when savings at time t finance the capital stock and the government debt in the next period:

$$s_t = k_{t+1} + b. \tag{16}$$

Moreover, the portfolio allocation has to be such that the right amount of private savings goes to the capital stock and the government debt:

$$\lambda_t s_t = k_{t+1}. \tag{17}$$

This implies that there are two equilibrium conditions and k_{t+1} and $r_{b,t+1}$ adjust to make sure that these equilibrium conditions are satisfied. Obviously, equation (16) and equation (17) imply that $(1-\lambda_t)s_t = b$.

3 Retirement as hedge: the benchmark result

In this section, we illustrate the standard result in the literature[11] that flexibility in the retirement decision increases the fraction of wealth invested in equity in the context of our model. In order to do so, we take a partial equilibrium perspective (factor prices are exogenous) and assume that there is only capital market risk (depreciation risk) implying that wages are non-stochastic. To keep the analysis as simple as possible, we impose that expected life-time utility is log-linear in first-period consumption, second-period consumption and leisure (i.e., $\rho = \gamma = 1$). We follow the approach of Hansen and Singleton (1983) and Campbell and Viceira (2002) and assume that the joint distribution of consumption and asset returns is lognormal. Then the optimal solution for portfolio choice in case of flexible retirement is given by:[12]

$$\lambda_t^F = \left[1 + \frac{w_{t+1}}{(1+r_{b,t+1})s_t}\right] \frac{\log \mathrm{E}_t(1+r_{k,t+1}) - \log(1+r_{b,t+1})}{\mathrm{Var}_t \log(1+r_{k,t+1})}. \tag{18}$$

[10] Changes in the bond return $r_{b,t}$ will change the debt tax τ_t. As this tax is levied on the young, this will affect the consumption possibilities when young. As the main purpose of the paper is to analyse the willingness of consumers to take risk and the retirement decision, we will not discuss the (second-order) effects of the change in the bond return, and the corresponding change in the debt tax, on consumption in too much detail.

[11] See, e.g., Bodie *et al.* (1992); Choi and Shim (2006); Farhi and Panageas (2007) or Choi *et al.* (2008).

[12] See the technical appendix, Section A.3.1, for the derivation of equation (18).

In case of inflexible retirement, the optimal equity share equals:[13]

$$\lambda_t^I = \left[1 + \frac{w_{t+1} z_{t+1}}{(1 + r_{b,t+1}) s_t}\right] \frac{\log \mathrm{E}_t(1 + r_{k,t+1}) - \log(1 + r_{b,t+1})}{\mathrm{Var}_t \log(1 + r_{k,t+1})}. \tag{19}$$

Obviously, these differences in portfolio allocation have consequences for the retirement decision. With flexible labour supply, the optimal solution for retirement is equal to:[14]

$$z_{t+1}^F = 1 - \frac{\theta\beta(1 + r_{T,t+1})}{1 + \beta(1 + \theta)} \left(\frac{w_t - \tau_t}{w_{t+1}} + \frac{1}{1 + r_{b,t+1}}\right), \tag{20}$$

with,

$$r_{T,t+1} \equiv (1 - a_t) r_{b,t+1} + a_t r_{k,t+1}, \tag{21}$$

$$a_t \equiv \frac{\lambda_t^F s_t}{s_t + \left(w_{t+1}/1 + r_{b,t+1}\right)}. \tag{22}$$

Note that a_t is the fraction of an individual's total wealth (financial wealth plus human wealth) invested in the risky asset. Hence, $r_{T,t+1}$ is the effective return on the individual's total portfolio when human wealth (i.e., the discounted value of future labour income) is also taken into account. In case of a positive equity shock, i.e., r_T is high, agents will retire earlier due to a positive wealth effect, and *vice versa*. With inflexible retirement, the optimal retirement decision equals:[15]

$$z_{t+1}^I = 1 - \frac{\theta\beta(1 + r_{b,t+1})}{1 + \beta(1 + \theta)} \left(\frac{w_t - \tau_t}{w_{t+1}} + \frac{1}{1 + r_{b,t+1}}\right). \tag{23}$$

Note that the risk-free return $r_{b,t+1}$ now enters the retirement function rather than the stochastic return $r_{T,t+1}$. Given these equations, it is possible to derive the following result:

Proposition 1. *The investment allocation to the risky asset is larger and the expected retirement age is lower in the case of flexible retirement compared to the inflexible retirement case, i.e.,* $\lambda_t^F > \lambda_t^I$ *and* $\mathrm{E}_t z_{t+1}^F < z_{t+1}^I$.

Proof. The optimal solution for s_t (derived in the technical appendix, Section A.3) shows that private savings s_t is equal in the flexible and fixed retirement case. From equations (18) and (19) it then immediately follows that $\lambda_t^F > \lambda_t^I$ as $z_{t+1} < 1$.

Using the optimal solution for s_t, it follows from equation (18) that $\lambda_t s_t > 0$. Using equation (22), this implies that $a_t > 0$ and, hence, $\mathrm{E}_t r_{T,t+1} > r_{b,t+1}$ and thus $E_t z_{t+1}^F < z_{t+1}^I$. □

Proposition 1 indeed reproduces the well-known result from the literature that was first derived by Bodie *et al.* (1992): When people can adjust their retirement decision, they will invest more in the risky asset. Since the risky asset has a higher expected return, these people can on average afford to retire earlier.

[13] See the technical appendix, Section A.3.2, for the derivation of equation (19).
[14] See the technical appendix, Section A.3.1, for the derivation of equation (20).
[15] See the technical appendix, Section A.3.2, for the derivation of equation (23).

4 Productivity risk and general equilibrium effects

In the previous section, we reproduced the well-known result that retirement provides a hedge against capital market risk in a partial equilibrium setting where this capital market risk is the only risk factor. This section explores to what extent this benchmark result can be generalized to a more general setting, i.e., to what extent retirement provides a hedge against capital market risks when we introduce wage risk (productivity risk) and allow for general equilibrium effects. In this more general setting, the model cannot be solved analytically and we have to use numerical simulation experiments. After discussing the parameterization of the model, we explore the quantitative properties of the model and solve for the steady state[16] and the reaction of the various variables to productivity and depreciation shocks with retirement flexibility and retirement inflexibility. As a first step, this is done in a partial equilibrium setting. Subsequently, we introduce general equilibrium effects and compare the results to the partial equilibrium results.

It should be noted that the results of our numerical simulation experiments are not always in line with the empirics. For example, the results for the equity premium and the share of equity in the portfolio deviate from what is generally observed in reality. This is a well-known problem in the macro-finance literature. Producing realistic outcomes for these variables would require a much more elaborate model. That would, however, make it very difficult to interpret the results. Given that the aim of this paper is to show to what extent the well-known result in the literature that retirement flexibility provides a hedge against macroeconomic risks is robust to changes in the assumptions, and not to provide very precise numerical results, we chose to use a simple model that is close the existing literature. This implies that the numerical experiments presented below should merely be seen as an illustration of the effects of a change in the assumptions, and not as a precise estimate of the size of this effect. When interpreting the results, one should also bear in mind that savings in our model can be interpreted as savings for old age via pension funds. In contrast to individual households, these funds typically invest a substantial share of their portfolio in equity.

4.1 Parameterization

In order to quantify the interaction between portfolio choice and the retirement decision, we first have to parameterize the model. We normalize the average productivity parameter at $A = 1$. The capital share in the Cobb–Douglas production function is taken to be $\alpha = 0.3$, as in Krueger and Kubler (2006) and Olovsson (2010). We set δ, the average depreciation rate, to 0.75. Assuming that one model period lasts about 30 years, this corresponds with a depreciation rate of 5% per year, like in

[16] Following Juillard and Kamenik (2005); Bovenberg and Uhlig (2008) and Beetsma and Bovenberg (2009), we use the concept of a *stochastic* steady state. This concept is defined as a situation in which each period shocks are equal to their expectations but agents are not aware of this (i.e., conditional variances are not zero). This point is solved from a non-linear system, and hence the solution does not generally correspond to the expected values of the variables involved. The steady state is therefore adjusted for risk. See the technical appendix, Section A.2 for the details.

Table 1. *Benchmark parameterization*

Parameter	β	η	ρ	γ	α	A	δ	b	σ_A	σ_δ
Values	0.71	0.5	1	2	0.3	1	0.75	0.016	0.31	1.26

Olovsson (2010). We choose as benchmark an intertemporal elasticity of substitution of one half, i.e., $\gamma = 2$, and an intratemporal substitution of $\rho = 1$. An intertemporal elasticity of substitution of one half lies well within the range of available estimates (see, e.g., Attanasio and Weber, 1995) and is commonly used in the macro and public finance literature (it implies a coefficient of relative risk aversion of 2). We choose as time discount factor $\beta = 0.71$, or a time discount rate of 1.1% per year, which lies between the values taken by Krueger and Kubler (2006) and Olovsson (2010). The leisure parameter is set at $\eta = 0.5$ and the supply of government debt is set at $b = 0.016$, these parameter values provide plausible values for the retirement age and the risk-free return on government bonds.

Since productivity risk directly affects all factor prices in the economy (wages and asset returns) and depreciation risk only influences capital returns, the two risk factors certainly have a different effect on retirement and portfolio decisions. We will therefore analyse the model for depreciation and productivity risk separately. We calibrate the standard deviation of the exogenous shock (i.e., σ_A in case of productivity risk and σ_δ in case of depreciation risk) in such a way that the annualized standard deviation of the return on capital is equal to 8.2%.[17] This leads to $\sigma_A = 0.31$ and $\sigma_\delta = 1.26$. All parameter values used in the benchmark model are summarized in Table 1.

4.2 Partial equilibrium

By definition, in the partial equilibrium model factor prices are exogenous and only influenced by exogenous shocks. For reasons of comparability, the exogenous factor prices (the capital return, the return on bonds and the wage rate) are set at the steady-state values that result from the general equilibrium model with flexible labour supply (see the next section). Table 2 compares the steady-state results for fixed and flexible labour supply. The table distinguishes between depreciation (capital market) risk and productivity risk. Note that, in case of depreciation risk, our model reproduces the traditional view that retirement flexibility increases risk exposure. A positive depreciation shock (i.e., a negative wealth shock) causes marginal utility of working to increase and, hence, agents increase labour supply (or postpone retirement). Consequently, income effects generate a negative correlation between asset returns and labour income. Retirement flexibility thus serves as a hedge against depreciation risk inducing individuals to invest more in the risky asset. The result of this investment strategy is that retirement flexibility allows agents to retire earlier on average compared with retirement inflexibility. Given our parameterization, agents choose to

[17] We follow Campbell and Viceira (2005) who show that returns on stocks are significantly less volatile when the investment horizon is long.

Table 2. *Steady state of partial equilibrium models*

	Depreciation risk		Productivity risk	
	Fixed	Flexible	Fixed	Flexible
c_y/y	37.74	37.61	37.47	37.44
c_o/y	51.03	49.70	49.63	49.76
s/w	32.79	33.00	33.57	33.62
z	20.75	17.00	16.27	16.83
λ	60.60	78.17	97.56	91.28

Note: all figures are expressed in percentages.

retire after 66.2 years in case of inflexible retirement while they retire on average after 65.1 years in case of flexible retirement, a difference of about 14 months.[18]

If productivity risk is the sole risk factor, however, the results turn around. In that case, retirement flexibility may instead amplify the productivity shocks absorbed into consumption, leading to less risk exposure and a higher retirement age compared with fixed retirement. The reason is that productivity shocks do not only induce an income effect in labour supply via the capital return, but also a substitution effect via the wage rate which works in the opposite direction. When productivity goes down, both the return on capital and the wage rate decrease. When people can freely adjust retirement, they will respond to this lower wage rate by reducing labour supply, which decreases labour income even further. This substitution effect exacerbates the positive correlation between labour income and capital returns. Hence, under retirement flexibility labour supply behaviour is subject to procyclical pressure which reduces the willingness of consumers to bear risk. As a result, people will invest less in equity and will work longer on average. Given our parameterization, this additional work span amounts to almost 2 months.

Figure 1 shows the change of the relative equity share (i.e., the equity share in case of flexible retirement divided by the equity share in case of inflexible retirement) for different values for σ_A and σ_δ. The two standard deviations are varied between 0.1 at the lower end and 0.9 at the upper end. If depreciation risk is high and productivity risk low, agents invest relatively much in equity when the retirement decision is flexible in the second period of life and *vice versa*.

4.3 General equilibrium

Now we turn to the general equilibrium solution. Table 3 shows the steady-state results in case of general equilibrium and again distinguishes between depreciation and productivity risk. The first column with numbers shows the results for the deterministic steady state, i.e., when the conditional variances are zero.

[18] We assume that the last period of life starts at the age of 60 and lasts 30 years. The first part of this period is devoted to working and lasts $30z$ years. The retirement age is thus $60 + 30z$.

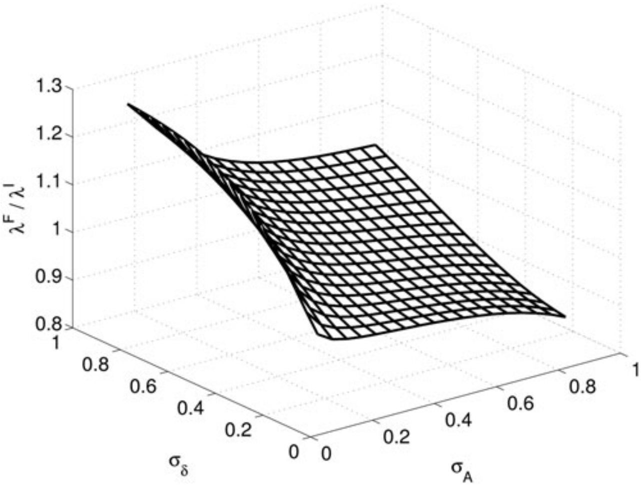

Figure 1. Equity share: fixed versus flexible retirement

Table 3. *Steady state of general equilibrium models*

	No risk	Depreciation risk		Productivity risk	
		Fixed	Flexible	Fixed	Flexible
c_y/y	37.61	36.72	37.67	37.56	37.42
c_o/y	50.23	51.55	49.76	49.49	49.66
s/w	31.96	32.11	32.89	33.58	33.65
μ	0.00	0.52	0.32	0.33	0.37
r_b	2.47	2.07	2.06	1.96	1.92
z	16.44	21.12	16.90	16.52	17.00
k/y	16.22	15.63	16.75	17.26	17.23
λ	84.41	84.23	85.06	85.55	85.57

Note: the equity premium (i.e., $\mu \equiv r_k - r_b$) and the return on government debt are annualized figures. All figures are expressed in percentages.

Comparing the deterministic steady state with the stochastic steady states illustrates the role of uncertainty in the model. Obviously, if there is no uncertainty, the equity premium (denoted by μ) is equal to zero since capital investments and government bonds are perfect substitutes. In the stochastic steady states, the equity premia are positive reflecting the higher riskiness of capital investments.[19] Including the risk terms in the optimality conditions introduces a precautionary motive for more savings and later retirement, i.e., the saving rate (s/w) and labour supply (z) are higher in the stochastic steady states than in the deterministic steady state.

[19] Note that the reported risk premia are on the low side, which is a manifestation of the equity premium puzzle.

In general equilibrium, exactly the same risk features appear as in partial equilibrium but now they are operating through price adjustments rather than quantity adjustments. With exogenous factor prices, we saw that agents invest more in equity under flexible labour supply than under fixed labour supply if depreciation risk is the dominant source of uncertainty. When productivity risk is the dominant source, we found the opposite result, namely that agents invest less in equity under retirement flexibility than under retirement inflexibility. With endogenous factor prices and a fixed supply of government bonds, though, different risk attitudes affect the price of risk taking, i.e., the equity premium. If productivity risk is the sole risk factor, the equity premium is *higher* in case of flexible retirement than in case of inflexible retirement. The intuition for this lower demand for risk taking under flexible retirement is the same as before: the substitution effect related to labour market flexibility exacerbates the positive correlation between asset returns and labour income which decreases the willingness to bear risk of consumers. Hence, people are only willing to invest in the domestic capital stock if they receive a higher expected compensation. If there is only depreciation risk, however, the insurance mechanism related to the income effect dominates, resulting in a *lower* equity premium under labour market flexibility.

Like in the partial equilibrium model, steady-state labour supply is lower with flexible retirement than with inflexible retirement if there is only depreciation risk. With flexible retirement, people on average choose to retire after 65.1 years, while in the inflexible retirement case they retire after 66.3 years, a difference of about 15 months. When agents have no retirement flexibility and only face depreciation risk, labour supply is an attractive way to finance future consumption compared with private savings, because wages are not uncertain while the proceeds of savings are uncertain. On the contrary, with retirement flexibility equity savings are attractive because people will probably earn the equity premium, while they always have the option to postpone retirement if things go wrong. Hence, compared with the inflexible setting, agents save more and a higher fraction of these savings is allocated to firm equity. Since the supply of government debt is given in general equilibrium, the equity premium has to decline to make sure that enough savings are allocated to this debt. It turns out that the wealth effect (more savings) dominates the price effect (lower equity premium), resulting in lower labour supply under retirement flexibility.

If there is only productivity risk, instead, retirement flexibility is less interesting from an insurance perspective because capital returns are low in states in which wages are also low. Therefore, agents have a relative high demand for risk-free bonds, which drives down the interest rate on government debt. This negative wealth effect implies that agents on average retire about 2 months later with flexible labour supply.

Figure 2 shows the dependence of the equity premium and retirement decisions on the two risk factors in a more general way. These figures compare the equity premium (left panel) and labour supply (right panel) in case of retirement flexibility with those in case of retirement inflexibility. If depreciation risk is high and productivity risk low, the risk premium is lower under flexible retirement, reflecting the self-insurance role of voluntary retirement. When productivity risk becomes more important, the equity premium increases and ultimately passes the levels of the fixed retirement setting. A comparable pattern emerges for labour supply behaviour. For higher degrees of

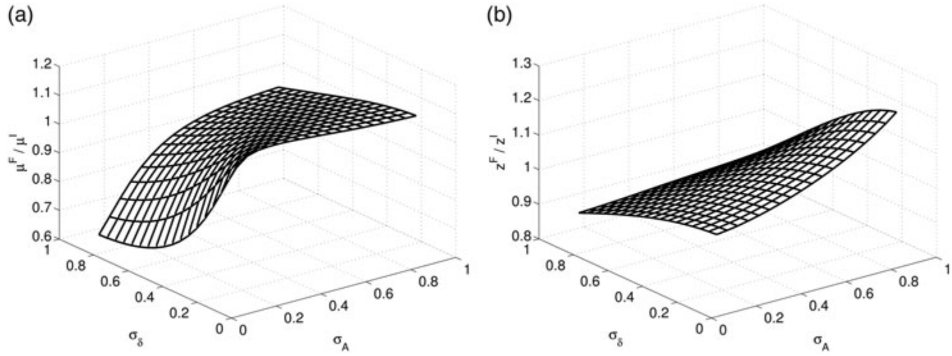

Figure 2. Equity premium (a) and labour supply (b): fixed retirement versus flexible retirement

productivity risk, the hedging effect of retirement flexibility decreases which leads to a higher demand for risk-free government bonds and, given the fixed level of government debt, to lower risk-free interest rates. This negative wealth effect induces agents to postpone retirement.

It should be stressed that from a welfare perspective, flexibility is always preferable to inflexibility. With retirement flexibility, expected lifetime utility is unambiguously higher, both in case of depreciation risk and productivity risk.[20] This result makes sense because the model does not include any distortion or externality.

4.4 Dynamics

The different roles of the interaction between retirement flexibility and portfolio allocation played by productivity and depreciation shocks over time can best be illustrated using impulse response functions. Figure 3 shows the response of the capital stock, the return on capital and bonds, the wage rate, labour supply and old-age consumption to a 10% positive depreciation shock (solid lines) and to a 10% negative productivity shock (dashed lines). The responses are expressed in per cent deviation from the steady state.

Note first that depreciation shocks lead to relative small responses compared with productivity shocks. After a depreciation shock of 10%, the capital return immediately decreases and, due to the income effect, labour supply increases. This negative correlation between the capital return and labour supply moderates consumption volatility and that is why flexibility provides insurance against adverse shocks. At impact, the decline of old-age consumption is small compared to the decline of the capital return. The capital stock is a predetermined variable and falls one period later. This lower level of the capital stock increases its marginal product, so that labour supply declines and, hence, wages and consumption gradually return to their pre-shock levels. The return on bonds moves in the opposite direction of the capital stock: a lower capital stock increases its marginal product leading to a higher demand for

[20] By simulating the derived recursive laws, we have calculated the unconditional means of most important model variables. It turns out that the unconditional mean of life-time utility in case of flexible retirement is always higher than that in case of inflexible retirement; see Section A.4 of the technical appendix.

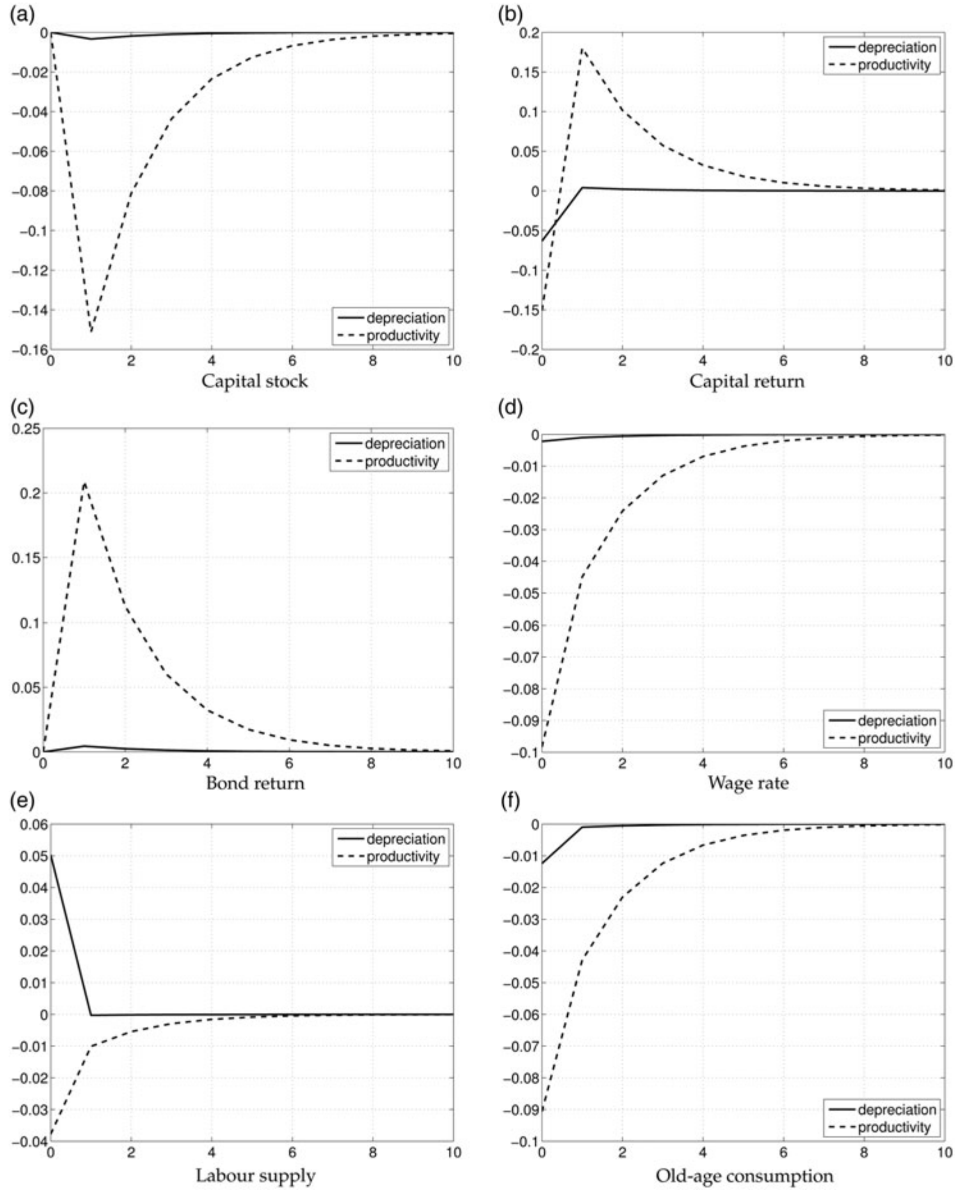

Figure 3. Impulse responses functions

capital investment and a lower demand for bond investments. As a result, the return on bonds should increase in order to ensure that the fixed supply of government debt will be financed each period.[21]

The economic responses after a productivity shock are much larger. In this case, the decrease in the capital return is even larger than the initial decline in productivity

[21] The higher return on bonds also means that the interest obligations on government debt rises and that the debt tax increases, which will affect the consumption possibilities when young negatively.

itself. Compared with a depreciation shock, a productivity shock does not only directly affect the return on capital but also the wage rate which falls at impact. As explained above, a productivity shock induces income and substitution effects in labour supply. In this benchmark parameterization, the substitution effect dominates the income effect and that is why labour supply decreases. Hence, productivity shocks result in pro-cyclical labour supply behaviour which exacerbates consumption volatility. Note that the initial decline in old-age consumption is almost as high as the relative decrease in productivity. From an investment point of view, the positive co-movement between capital returns and labour income reduces the demand for risk taking. Consequently, the equity premium will be relatively higher under retirement flexibility. Compared to partial equilibrium the decline in labour supply exacerbates the direct fall of the capital return on account of the productivity contraction. This implies that in general equilibrium the capital return is more sensitive to productivity risk than in partial equilibrium which decreases the effectiveness of retirement flexibility as a hedge even more in general equilibrium.

4.5 The role of the elasticity of substitution between consumption and leisure

The previous analysis has shown that the insurance effect of retirement flexibility very much depends on income and substitution effects in labour supply. In our benchmark parameterization, the substitution effect slightly dominates the income effect so that old-age consumption becomes more sensitive to productivity risk in case of retirement flexibility. As a result, agents ask for a higher risk compensation (in general equilibrium) or decrease the equity share in the total asset portfolio (in partial equilibrium).

The relative strength of income and substitution effects is governed by the elasticity of substitution between consumption and leisure. In this section, we study the role of the substitution elasticity in the hedging effect of retirement flexibility into more detail. Figure 4 shows the responses of labour supply and consumption to a negative productivity shock of (again) 10% for various degrees of substitutability between consumption and leisure. As shown, the labour-supply choice may amplify the productivity shocks absorbed into consumption if the elasticity of substitution of leisure for consumption ($1/\rho$) is high (i.e., a lower ρ). If this substitution elasticity is low, however, consumers may use their labour/leisure choice to mitigate the effect of the shock on consumption. It is a well-known macroeconomic finding that consumption and labour move positively (Blanchard and Fisher, 1989). Therefore, for high substitutability our model is consistent with the data.

A higher elasticity of substitution (i.e., a lower ρ) generates a more positive comovement of consumption and labour in case of retirement flexibility, which leads to higher equity premia. Figure 5a shows the reaction of the equity premium in case of retirement flexibility relative to the equity premium in case of inflexibility for different degrees of substitution between consumption and leisure.[22] For low values of ρ (high elasticity of substitution), the equity premium under flexible retirement exceeds

[22] In Figure 5, it is assumed that productivity risk is the sole risk factor, because substitution effects in labour supply are not relevant in case of depreciation risk.

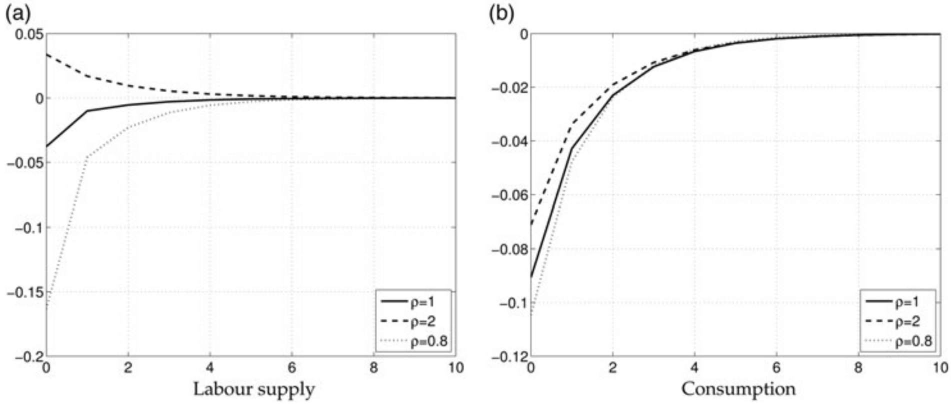

Figure 4. Impulse response functions with CES utility

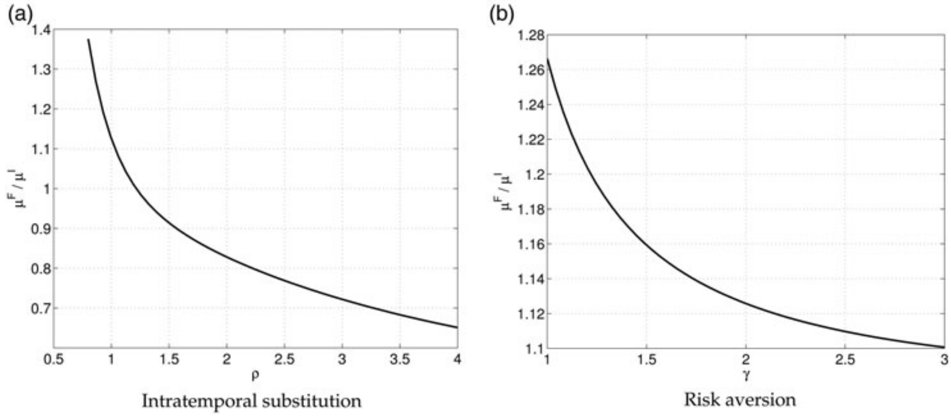

Figure 5. Relative equity premium for various degrees of intratemporal substitution and risk aversion

the equity premium under inflexible retirement. For higher values of ρ (lower elasticity of substitution), the income effect becomes more important and, hence, also the insurance effect of retirement flexibility increases. So when the elasticity of substitution is high, retirement flexibility acts in the direction of resolving the equity risk premium puzzle (Basak, 1999).

Figure 5b illustrates the sensitivity of the relative equity premium for different degrees of risk aversion (or intertemporal substitution). As one can see, for all values of γ considered, the ratio is decreasing in relative risk aversion but it never falls below unity.[23] This means that, contrary to the elasticity of intratemporal substitution, the

[23] Figure 5b shows that in *relative* terms (i.e., in percentage of the equity premium under fixed retirement) the equity premium under flexible retirement is decreasing in risk aversion. In *absolute* terms, however, the difference between the equity premia under flexible and fixed retirement is increasing in risk aversion, as would be expected: for higher degrees of risk aversion, agents ask for a higher expected return to compensate for the positive correlation between factor prices under retirement flexibility.

coefficient of relative risk aversion does not alter the order of the equity premium: the equity premium is higher with flexible retirement than with fixed retirement.

5 Conclusion

In this paper, we have developed a stochastic general equilibrium model with two overlapping generations. The model is used to analyse the interaction between consumption, portfolio choice and retirement decisions. In the literature, retirement flexibility is often seen as a way to insure against bad investment outcomes. This paper reviews the robustness of this benchmark result in a more general model. In particular, in our model the risk factors (productivity risk and depreciation risk) are directly linked to the production structure of the economy. Second, and more importantly, we combine a partial equilibrium approach with a general equilibrium approach thereby explicitly recognizing that correlations between productivity and depreciation shocks are endogenous. Finally, we allow for more general preferences which are characterized by a CES function of consumption and leisure.

Our main findings are as follows. First, the relevance of retirement flexibility as a hedging instrument strongly depends on the type of risk agents are subject to. Productivity risk affects wages and asset returns in the same direction. Under retirement flexibility, this positive correlation between wages and asset returns is reinforced by the substitution effect on labour supply resulting in a *lower* preference for risk taking. In partial equilibrium, this lower demand leads to lower equity shares in the total investment portfolio while in general equilibrium it leads to higher equity premia as the supply of assets is (partly) fixed. With depreciation risk, though, wages are only indirectly affected by general equilibrium effects. In this case, the income effect dominates implying that labour income and capital returns are negatively correlated, which leads to a *higher* preference for risk taking. In partial equilibrium, this higher demand leads to higher portfolio shares invested in equity, in general equilibrium it leads to lower equity premia.

Second, our analysis shows that the elasticity of substitution between consumption and leisure is of crucial importance in determining to what extent retirement flexibility protects retirees against bad investment returns. Indeed, this elasticity governs the relative strength of income and substitution effects in labour supply and therefore determines the hedging effect of retirement flexibility. Our analysis clearly shows that the advantage of flexible retirement as a hedging instrument is smaller if substitution effects are relatively important. Empirical studies indeed suggest that substitution effects are more important for the retirement decision than income or wealth effects.

Finally, we find that general equilibrium effects play an important role in the interaction between portfolio choice and retirement. It is mainly the degree of substitution between consumption and leisure that determines the direction of the general equilibrium effects. For high substitution elasticities, which seem empirically the most relevant case, labour supply behaviour amplifies the sensitivity of capital returns to productivity risk making retirement flexibility less effective as hedging tool in general equilibrium than in partial equilibrium.

This paper has shown that the main results of existing studies in the field of retirement flexibility and portfolio choice (e.g., Bodie *et al.*, 1992; Choi and Shim, 2006) may not be robust to alternative (i.e., more realistic) model settings. These studies argue that with labour flexibility much of the uncertainty is absorbed by the labour-supply decision, leaving consumption relatively smooth. This suggests a negative covariability between human capital and the equity market and between consumption and labour supply. These findings are consistent with our model as long as shocks do not directly affect wages (such as depreciation shocks). However, if shocks do affect wages (such as productivity shocks) and if consumers have a high elasticity of substitution of leisure for consumption, the comovement between consumption and labour supply becomes positive, resulting in a lower demand for risk. Empirical studies typically find that in the long run the stock market and human capital are highly positively correlated, while it is well known that observed consumption and labour supply move in the same direction. These empirical results imply that more weight should be given to productivity risk than to depreciation risk.

Although our model is consistent with empirical macro findings, it still raises a number of empirical questions that paves the way for future research. For example, will retirement (or labour) flexibility induce a greater or lower risk taking in an individual's asset portfolio? Answering this question requires identification of a measure of flexibility. A possibility is to compare job categories with a fixed amount of hours with job categories that offer opportunities for working extra hours. Another relevant empirical question is whether a higher riskiness of individual's human capital indeed leads to less risk taking, as our analysis suggests. One way to answer this question is by investigating whether there is a negative association between the correlation between wages and asset returns on the one hand and risk-taking investment behaviour on the other hand.

Supplementary material

To view supplementary material for this article, please visit https://doi.org/10.1017/S1474747216000147

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