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Control of nonholonomic mechanical systems using reduction and adaptation

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SUMMARY

This paper considers the problem of controlling the motion of nonholonomic mechanical systems in the presence of uncertainty regarding the system model and state. It is proposed that a simple and effective solution to this problem can be obtained by first using a reduction procedure to obtain a lower dimensional system which retains the mechanical system structure of the original system, and then adaptively controlling the reduced system in such a way that the complete system is driven to the goal configuration. This approach is shown to be easy to implement and to ensure accurate motion control despite measurement and model uncertainty. The efficacy of the proposed control strategy is illustrated through computer simulations and preliminary hardware experiments with nonholonomic mechanical systems arising from both explicit kinematic constraints and symmetries of the system dynamics.

KEYWORDS: Nonholonomic systems; Reduction; Adaptation; Motion control.

1. INTRODUCTION

There is significant interest in controlling the motion of mechanical systems in the presence of uncertainty regarding the system model and state. This challenging problem becomes even more difficult when the system is “underactuated”, that is, possesses fewer actuators than configuration degrees of freedom. A particularly interesting class of underactuated systems consists of those systems for which only a proper subspace of the space of generalized velocities is accessible at each configuration. This situation occurs, for instance, when a system is subject to nonholonomic (nonintegrable) constraints on its kinematics, and can also arise in systems with symmetric dynamics. As we shall see, (classical) nonholonomic systems and symmetric systems possess a very similar structure, and we will find it convenient in what follows to refer to both classes of systems as nonholonomic systems. Observe that nonholonomic systems are of considerable importance in applications. For example, nonholonomic constraints arise in systems with rolling contact, such as wheeled (and other) mobile robots and multifingered robotic hands, and in symmetric systems when the symmetry leads to nonintegrable constraints, such as space robots with angular momentum conservation.

One consequence of the practical importance of nonholonomic mechanical systems is that the problem of controlling the motion of these systems in an effective manner has attracted considerable attention in recent years. Most of the work reported to date on controlling nonholonomic systems has focused on the *kinematic control* problem, in which it is assumed that the system velocities are the control inputs and that the system can be adequately represented using the system kinematic model. However, there are important reasons for formulating the nonholonomic system control problem at the *dynamic control* level, where the control inputs are those produced by the system actuators and the system model contains the mechanical system dynamics. For example, since this is the level at which control actually takes place in practice, designing controllers at this level can lead to significant improvements in performance and implementability and can help in the early identification and resolution of difficulties. It is interesting to consider the problem of controlling holonomically constrained robotic manipulators in this regard: the kinematic control problem in this case is trivial, but the dynamic control problem is nevertheless quite challenging. Another motivation for considering the dynamic control problem is that certain classes of nonholonomic systems are most naturally studied at this level, so that such an approach broadens the scope of potential applications. Recognizing the importance of addressing the nonholonomic system control problem at the dynamic control level, several researchers have considered this problem in recent years [e.g. references 1–5]. Significant progress has been made in understanding the fundamental characteristics of these systems, and several useful dynamic controllers have been proposed. However, virtually all of the dynamic control strategies proposed to date have been developed by assuming that the full dynamic model is precisely known and that the entire system state is measurable. Exceptions to this trend include the papers,[6–8] which propose control strategies that can compensate for the effects of dynamic model uncertainty, and the paper,[8] which presents a controller that can be implemented without system velocity measurements.

Our own work in the area of uncertain nonholonomic mechanical system control consists of the papers[9,10] and the present contribution. The main result in reference [9] is a class of algorithms for controlling a nonholonomic system to a desired configuration by first planning and then tracking an appropriate trajectory, while the work in reference [10] extends this result by providing an algorithm for *stabilizing* a nonholonomic system to a goal configuration. Each of the controllers given in references [9,10] can be implemented without knowledge of the system dynamic model and each is verified through an extensive computer simulation study. The present paper proposes that a simple and effective solution to the problem of controlling uncertain nonholonomic mechanical systems can be obtained by employing a combination of *reduction* and *adaptation*. The process of reduction, taken from geometric mechanics [e.g. reference 11], permits the original mechanical system dynamics to be represented in terms of another mechanical system of reduced dimension plus an associated kinematic map. This reduced system is then made to track any desired trajectory in the space of the "reducing" coordinates, using only configuration measurements and without knowledge of the reduced system model, by employing the *performance-based adaptive control* methodology recently developed by the authors [e.g. references 12, 13]. Techniques based on differential flatness[14] and geometric phase[11] are presented for choosing a trajectory for the reducing coordinates which produces the desired motion for the complete system, and a method is given for accomplishing this objective even if there is (linearly parameterizable) uncertainty in the nonholonomic constraints. The efficacy of the proposed approach is illustrated through both computer simulations and preliminary hardware experiments with two classes of nonholonomic mechanical systems: those with explicit constraints on the system kinematics, such as arise in systems with rolling contact, and those with constraints which result from the presence of a symmetry of the system dynamics, such as occur in systems for which angular momentum is conserved.

2. PROPOSED CONTROL METHOD

In this section we consider the motion control problem for uncertain nonholonomic mechanical systems. We begin by applying the process of reduction [e.g. reference 11] to the original nonholonomic mechanical system model to obtain a new system model consisting of the reduced order mechanical system plus an associated kinematic map. This reduction process is carried out for nonholonomic mechanical systems arising from both explicit kinematic constraints and symmetries of the system dynamics, and indeed this reformulation clearly exhibits the structural similarity of these two classes of systems. We then show how adaptation can be used to permit these (reduced) nonholonomic mechanical systems to be accurately controlled despite the presence of model and measurement uncertainty.

2.1 Reduction

Consider first the class of nonholonomic mechanical systems arising from the presence of explicit constraints on the system kinematics; these systems can be modeled as [e.g. reference 1]

$$M(\mathbf{x})\mathbf{T} = H^*(\mathbf{x})\ddot{\mathbf{x}} + V^*_{cc}(\mathbf{x}, \dot{\mathbf{x}})\dot{\mathbf{x}} + \mathbf{G}^*(\mathbf{x}) + A^T(\mathbf{x})\lambda \qquad (1a)$$

$$A(\mathbf{x})\dot{\mathbf{x}} = \mathbf{0} \qquad (1b)$$

where $\mathbf{x} \in \Re^n$ is the vector of system generalized coordinates, $\mathbf{T} \in \Re^p$ is the vector of actuator inputs, $M : \Re^n \rightarrow \Re^{n\times p}$ is bounded and of full rank, $H^* : \Re^n \rightarrow \Re^{n\times n}$ is the system inertia matrix, $V^*_{cc} : \Re^n \times \Re^n \rightarrow \Re^{n\times n}$ quantifies Coriolis and centripetal acceleration effects, $\mathbf{G}^* : \Re^n \rightarrow \Re^n$ arises from the system potential energy, $A : \Re^n \rightarrow \Re^{m\times n}$ is a bounded full rank matrix quantifying the nonholonomic constraints, $\lambda \in \Re^m$ is the vector of constraint multipliers, and all functions are assumed to be smooth. The mechanical system dynamics (1) possesses considerable structure. For example, for any set of generalized coordinates $\mathbf{x}$, the matrix H^* is symmetric and positive definite, the matrix V^*_{cc} depends linearly on $\dot{\mathbf{x}}$, and the matrices H^* and V^*_{cc} are related according to $\dot{H}^* = V^*_{cc} + V^{*T}_{cc}$. Additionally, we will assume in what follows that the inertia matrix H^* and potential energy gradient $\mathbf{G}^*$ are bounded functions with bounded first partial derivatives; these latter properties hold for virtually all mechanical systems of practical interest.

The rows of A, say $\mathbf{a}_i \in \Re^{1\times n}$ for $i = 1, 2, \ldots, m$, are smooth covectors on the configuration space $\Re^n$ which quantify explicit kinematic constraints imposed on the system velocities. These constraints could arise from rolling contact, for example. We will assume that these constraints cannot be integrated to yield constraints on the configuration coordinates $\mathbf{x}$; this assumption is made more precise below. It is well-known that the presence of these nonholonomic constraints complicates the control problem considerably. For instance, in this case the basic problem of stabilizing the system (1) to some goal configuration $\mathbf{x}_d$ cannot be solved using standard techniques.[15,16] This difficulty is only increased in the case of control in the presence of measurement and model uncertainty. One means of simplifying the problem of controlling these systems is to employ a reduction procedure to decrease the dimension of the dynamics (1). Observe that the assumption that A is full rank implies that the codistribution spanned by the rows $\mathbf{a}_i$ has dimension m. The annihilator of this codistribution is then an $r = n - m$ dimensional smooth distribution $\Delta = \mathrm{span}[\mathbf{r}_1(\mathbf{x}), \mathbf{r}_2(\mathbf{x}), \ldots, \mathbf{r}_r(\mathbf{x})]$, where the $\mathbf{r}_i$ are smooth vector fields on the configuration space which satisfy $A\mathbf{r}_i = \mathbf{0}\ \forall \mathbf{x}$. Defining $R = [\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_r] \in \Re^{n\times r}$ permits this relationship to be expressed more concisely as $AR = 0$. As an example, let the matrix A be partitioned as $A = [A_1\ A_2]$, with $A_1 \in \Re^{m\times m}$ and $A_2 \in \Re^{m\times r}$ and where A_1 is nonsingular (this is always possible, possibly with a reordering of the configuration coordinates). Then R can be constructed as follows:

$$R = \begin{bmatrix} -A_1^{-1}A_2 \\ I_r \end{bmatrix} \qquad (2)$$

where I_r is the $r \times r$ identity matrix. Consider now the involutive closure of Δ, denoted Δ^* and defined as the smallest involutive distribution containing Δ. We will assume in what follows that Δ^* has constant rank n on the configuration space. In this case, Frobenius' theorem [e.g. reference 17] shows that the constraints are nonintegrable and there is no explicit constraint on the configuration space; thus the dimension of the space of realizable velocities is smaller than the dimension of the configuration space.

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Now define a partition of $\mathbf{x}$ corresponding to the partition specified for A, so that $\mathbf{x}=[\mathbf{x}_1^T\ \mathbf{x}_2^T]^T$ with $\mathbf{x}_1\in\Re^m$ and $\mathbf{x}_2\in\Re^r$. Observe that the definition (2) and the constraint equation (1b) imply that the system velocities are determined by $\dot{\mathbf{x}}_2$ via $\dot{\mathbf{x}}=R(\mathbf{x})\dot{\mathbf{x}}_2$. This parameterization then permits (1) to be reformulated as

$$\dot{\mathbf{x}}_1=A^*(\mathbf{x})\dot{\mathbf{x}}_2 \tag{3a}$$

$$\mathbf{F}=H(\mathbf{x})\ddot{\mathbf{x}}_2+V_{cc}(\mathbf{x},\dot{\mathbf{x}}_2)\dot{\mathbf{x}}_2+\mathbf{G}(\mathbf{x}) \tag{3b}$$

where $A^*=-A_1^{-1}A_2$, $\mathbf{F}=R^TM\mathbf{T}$, $H=R^TH^*R$, $V_{cc}=R^T(H^*\dot{R}+V_{cc}^*R)$, and $\mathbf{G}=R^T\mathbf{G}^*$. In what follows, it is assumed that $p\geq r$ and $R^T M$ is full rank, so that any desired $\mathbf{F}$ can be realized through proper specification of $\mathbf{T}$ and the system (3b) is fully actuated. Note that (3) consists of a "reduced" dynamic model (3b), which defines the evolution of the "reducing outputs" $\mathbf{x}_2$, together with a purely kinematic relationship (3a). Therefore the representation (3) provides a simpler description of the nonholonomic mechanical system than that given in (1). Moreover, as shown in the next lemma, the dynamics (3b) retains much of the mechanical system structure of the original system (1).

Lemma 1: The dynamic model terms H, $\mathbf{G}$ are bounded functions of $\mathbf{x}$ whose time derivatives $\dot{H}$, $\dot{\mathbf{G}}$ are also bounded in $\mathbf{x}$ and depend linearly on $\dot{\mathbf{x}}_2$, the matrix H is symmetric and positive definite, and the matrices H and V_{cc} are related according to $\dot{H}=V_{cc}+V_{cc}^T$. Additionally, $V_{cc}(\mathbf{x},\dot{\mathbf{x}}_2)\mathbf{y}=V_{cc}(\mathbf{x},\mathbf{y})\dot{\mathbf{x}}_2$ for any vector $\mathbf{y}$, and if $\mathbf{y}$ and $\dot{\mathbf{y}}$ are bounded then $V_{cc}(\mathbf{x},\mathbf{y})$ is bounded and $\dot{V}_{cc}(\mathbf{x},\mathbf{y})$ grows linearly with $\dot{\mathbf{x}}_2$.

Proof: All of the properties can be established through direct calculation using the definitions of H, V_{cc}, and $\mathbf{G}$ and the properties of H^*, V_{cc}^*, and $\mathbf{G}^*$ (see reference [9]). ■

We now turn our attention to those nonholonomic mechanical systems which arise from the presence of a symmetry in the system dynamics. More specifically, consider the class of mechanical systems for which the system Lagrangian is G-invariant for some Lie group G (see, for example, reference [11] for a discussion of G-invariant Lagrangian systems), and suppose for concreteness that $G=SO(2)$ (more general situations can be treated using techniques similar to those developed here, although there may be technical complications). By decomposing the configuration space into irreducible representations of $SO(2)$, it is always possible to choose (local) configuration coordinates $\mathbf{x}$ so that each component transforms as $x_i\rightarrow x_i+n_i\alpha$ for some integer n_i and $\alpha\in[0,2\pi)$. The coordinates for which $n_j=0$ (i.e., the invariant coordinates) are then local coordinates for $\Re^n/G$. We collect these together and write $\mathbf{x}=[\mathbf{x}_1^T\ \mathbf{x}_2^T]^T$, where $\mathbf{x}_2\in\Re^r$ are the invariants and $\mathbf{x}_1\in\Re^m$ transform nontrivially. Choosing coordinates in this way permits the G-invariant system Lagrangian to be written in the form $L(\mathbf{x},\dot{\mathbf{x}})=\dot{\mathbf{x}}^T H^*(\mathbf{x}_2)\dot{\mathbf{x}}/2-U(\mathbf{x}_2)$ for some potential U and inertia matrix

$$H^*=\begin{bmatrix} J_1(\mathbf{x}_2) & Q(\mathbf{x}_2)\\ Q^{\mathrm{T}}(\mathbf{x}_2) & J_2(\mathbf{x}_2)\end{bmatrix} \tag{4}$$

with submatrices J_1, J_2, Q which are independent of $\mathbf{x}_1$.

Let us restrict our attention to those systems for which the control input $\mathbf{T}$ does not break the symmetry of the dynamics; no generality is lost with this assumption because, if this is not the case, then the $\mathbf{x}_1$ variables can be controlled directly and the (controlled) system is not nonholonomic by our definition. The fact that L is independent of $\mathbf{x}_1$ means that in this case the Euler-Lagrange equations corresponding to the $\mathbf{x}_1$ coordinates have the character of a velocity constraint:

$$\frac{\partial L}{\partial \dot{\mathbf{x}}_1}=J_1(\mathbf{x}_2)\dot{\mathbf{x}}_1+Q(\mathbf{x}_2)\dot{\mathbf{x}}_2=\mathbf{1} \tag{5}$$

where $\mathbf{1}\in\Re^m$ is constant. If the system starts from rest then $\mathbf{1}=\mathbf{0}$ and (5) can be used to parameterize the system velocities via $\dot{\mathbf{x}}=R(\mathbf{x}_2)\dot{\mathbf{x}}_2$, with R defined as

$$R=\begin{bmatrix} -J_1^{-1}Q\\ I_r\end{bmatrix} \tag{6}$$

We again assume that the smallest involutive distribution containing the span of the columns of R has constant rank n, in which case Frobenius' theorem [e.g. reference 17] shows that the constraints (5) are nonintegrable and the system is nonholonomic.

Now an analysis which exactly parallels the one given above for classical nonholonomic systems can be applied to reduce the original $2n$-dimensional symmetric mechanical system to a $2r$-dimensional mechanical system together with m kinematic equations:

$$\dot{\mathbf{x}}_1=A^{**}(\mathbf{x}_2)\dot{\mathbf{x}}_2 \tag{7a}$$

$$\mathbf{F}=H(\mathbf{x}_2)\ddot{\mathbf{x}}_2+V_{cc}(\mathbf{x}_2,\dot{\mathbf{x}}_2)\dot{\mathbf{x}}_2+\mathbf{G}(\mathbf{x}_2) \tag{7b}$$

where $A^{**}=-J_1^{-1}Q$ and $\mathbf{F}=B(\mathbf{x}_2)\mathbf{T}$ for some matrix $B\in\Re^{r\times p}$ which depends only on $\mathbf{x}_2$ (because the inputs do not break the system symmetry). It is assumed that $p\geq r$ and B is full rank, so that any desired $\mathbf{F}$ can be realized through proper specification of $\mathbf{T}$ and the system (7b) is fully actuated. Note that (7b) is a $2r$ order differential equation which defines the evolution of the $2r$ states $(\mathbf{x}_2,\dot{\mathbf{x}}_2)$, and that the behavior of the remaining configuration coordinates $\mathbf{x}_1$ is completely determined by the kinematic relationship (7a). Moreover, an analysis virtually identical to the one summarized in Lermma 1 can be used to show that the reduced system (7b) retains the mechanical system structure of the original system.

Reduced representations have now been obtained for both classical nonholonomic and symmetric nonholonomic mechanical systems. Examination of these reduced models reveals that the model (3) contains the model (7) as a special case (corresponding to the situation in which the Lagrangian $L(\mathbf{x},\dot{\mathbf{x}})$, distribution Δ, and input matrix are independent of the configuration coordinates $\mathbf{x}_1$). Thus, in what follows, we focus on developing control algorithms for the nonholonomic mechanical system (3), with the implicit understanding that all such schemes are directly implementable with the system (7) as well.

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2.2 Adaptation

We now consider the problem of controlling the motion of nonholonomic mechanical systems in the presence of measurement and model uncertainty. More specifically, we address the problem of driving the system (3) from any initial state $(\mathbf{x}(0), \dot{\mathbf{x}}(0))$ to any desired equilibrium state $(\mathbf{x}_d, \mathbf{0})$ in the presence of uncertainty regarding the dynamic model (3b) and the kinematic model (3a); additionally, we will assume in much of our development that only configuration measurements are available for feedback. It is shown in reference [1] that the system (1) is small time locally controllable at any equilibrium state $(\mathbf{x}_d, \mathbf{0})$, so that this problem is solvable. However, as mentioned above, it is also known that the problem cannot be solved using standard methods such as stabilization via continuous static feedback.[15,16] Here we propose that this control problem be addressed by dividing the problem into two subproblems: (1.) generate a trajectory for the "reducing output" $\mathbf{x}_2$ which will take the entire system to the goal configuration, and (2.) track the desired trajectory for $\mathbf{x}_2$ using the control input $\mathbf{F}$. We begin our implementation of this approach by introducing a strategy for control in the presence of uncertainty associated with the system dynamics (3b), and then show how this strategy can be generalized to handle (linearly parameterizable) uncertainty in the kinematic model (3a).

One type of nonholonomic system for which trajectory generation is straightforward is the class of so-called *differentially flat* systems [e.g. reference 14]. A system with states $\mathbf{s} \in \Re^{2n}$ and inputs $\mathbf{u} \in \Re^{p}$ is differentially flat if we can find outputs $\mathbf{y} \in \Re^{p}$ of the form

$$\mathbf{y} = \mathbf{h}_1(\mathbf{s}, \mathbf{u}, \dot{\mathbf{u}}, \ldots, \mathbf{u}^{(q)}) \tag{8a}$$

which are differentially independent and satisfy

$$\mathbf{s} = \mathbf{h}_2(\mathbf{y}, \dot{\mathbf{y}}, \ldots, \mathbf{y}^{(s)}) \tag{8b}$$

$$\mathbf{u} = \mathbf{h}_3(\mathbf{y}, \dot{\mathbf{y}}, \ldots, \mathbf{y}^{(s)}) \tag{8c}$$

for functions $\mathbf{h}_i$ and integers q, s. Differentially flat systems have attracted considerable attention recently and, while there are no general methods for determining whether or not a particular system is flat, it is known that many systems of interest in applications are flat. For example, all of the following are flat[5]: wheeled mobile robots and cars, a tractor pulling N trailers, hopping robots, underwater vehicles, various electromechanical drives, and (planar) satellite/manipulator systems. In view of the useful structural properties exhibited by flat systems and their importance in nonholonomic applications, we will devote some attention to these systems in what follows.

While differentially flat systems possess many interesting properties, our objective here will be simply to exploit flatness to solve the problem of finding a feasible trajectory taking the system (3) from its initial configuration to the final desired configuration $\mathbf{x}_d$. With flat systems there is a one to one correspondence between trajectories $(\mathbf{s}(t), \mathbf{u}(t))$ of the system and curves $\mathbf{y}(t)$ in the flat output space. As a consequence, the trajectory generation problem is easily solved for flat systems: given an initial state and desired final state for the system, determine (from (8b)) the values for the outputs and derivatives of outputs $(\mathbf{y}(0), \dot{\mathbf{y}}(0), \ldots, \mathbf{y}^{(s)}(0))$ and $(\mathbf{y}(T), \dot{\mathbf{y}}(T), \ldots, \mathbf{y}^{(s)}(T))$ corresponding to these initial and final states. Then any curve in the flat output space with the required initial and final location, slope, curvature, and so on defines a feasible trajectory for the full system (3) which produces the requisite motion. Note that this curve fitting problem is standard and can be efficiently solved in a number of ways.

Given this trajectory generation scheme for flat nonholonomic system, we now turn to the problem of causing (3) to track a given trajectory. Observe that the evolution of the system is completely determined by the behaviour of the reducing outputs $\mathbf{x}_2$ (because the evolution of $\mathbf{x}_1$ is related to that of $\mathbf{x}_2$ through a kinematic map). The behaviour of $\mathbf{x}_2$ is, in turn, governed by the reduced system (3b), and this system is shown in Lemma 1 to inherit all of the "nice" mechanical system structure of the original system. Thus the trajectory tracking problem for the complete system (3) can be solved by applying to (3b) any adaptive or robust control law for mechanical systems which does not require model information or rate measurements; for example, any of the control schemes presented in references [18–21] could be used. In what follows, we observe that the *performance-based adaptive control* methodology recently proposed by the authors [e.g. references 12, 13] can be utilized to design a controller for the (reduced) mechanical system (3b), and give one such tracking strategy. More specifically, we note that the following adaptive control scheme can be used to track any desired trajectory $\mathbf{x}_{2d}(t)$ for $\mathbf{x}_2$ without rate measurements or knowledge of the system dynamic model:

$$\mathbf{F} = A(t)\ddot{\mathbf{x}}_{2d} + B(t)\dot{\mathbf{x}}_{2d} + \mathbf{f}(t) + k_1\gamma^2\mathbf{w} + k_2\gamma^2\mathbf{e}$$
$$\dot{\mathbf{w}} = -2\gamma\mathbf{w} + \gamma^2\dot{\mathbf{e}} \tag{9}$$

where $\mathbf{e} = \mathbf{x}_{2d} - \mathbf{x}_2$ is the trajectory tracking error, $\mathbf{w}$ provides a means of injecting damping into the closed-loop system without using rate measurements, k_1, k_2, γ are positive scalar constants, and $\mathbf{f}(t) \in \Re^{r}$, $A(t) \in \Re^{r\times r}$, $B(t) \in \Re^{r\times r}$ are (feedforward) adaptive gains which are adjusted according to the following simple update laws:

$$\begin{aligned}\dot{\mathbf{f}} &= -\sigma_1\mathbf{f} + \beta_1\mathbf{q}\\ \dot{A} &= -\sigma_2 A + \beta_2\mathbf{q}\ddot{\mathbf{x}}_{2d}^T\\ \dot{B} &= -\sigma_3 B + \beta_3\mathbf{q}\dot{\mathbf{x}}_{2d}^T\end{aligned} \tag{10}$$

where $\mathbf{q} = \dot{\mathbf{e}} + k_2\mathbf{e}/k_1\gamma - \mathbf{w}/\gamma$ represents a weighted and filtered error term and the σ_i and β_i are positive scalar adaptation gains. Observe that this control law is composed of a linear first order compensator for stabilization together with an adaptive feedforward component for tracking. The feedforward terms $A\ddot{\mathbf{x}}_{2d}$, $B\dot{\mathbf{x}}_{2d}$, and $\mathbf{f}$ are intended to compensate for the dynamic model terms $H\ddot{\mathbf{x}}^2$, $V_{cc}\dot{\mathbf{x}}_2$, and $\mathbf{G}$, respectively, and this compensation permits accurate tracking control to be achieved in an efficient manner. Inspection of the scheme (9), (10) reveals that the proposed controller is implementable without velocity information because, although $\dot{\mathbf{w}}$, $\dot{\mathbf{f}}$, $\dot{A}$, and $\dot{B}$ depend on $\dot{\mathbf{e}}$, the control law terms $\mathbf{w}$, $\mathbf{f}$, A, and B can be integrated so as to depend only on $\mathbf{e}$ and the desired trajectory. Moreover, this adaptive control strategy does not require knowledge of the system "regressor matrix" or any other information concerning the dynamics (3b); note that this model independence is

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particularly attractive for the present application because the mechanical system (3b) is the result of a reduction procedure and is often quite difficult to compute explicitly.

The suitability of the adaptive tracking controller (9), (10) is indicated by the following lemma.

Lemma 2: The adaptive controller (9), (10) ensures that (3b) evolves in such a way that $\mathbf{e}, \dot{\mathbf{e}}, \mathbf{w}, \mathbf{f}, A, B$ are semiglobally uniformly bounded and the tracking error $\mathbf{e}, \dot{\mathbf{e}}$ converges exponentially to a neighborhood of the origin which can be made arbitrarily small.

Proof: The proof follows immediately from the proof of Theorem 2 in reference [13] once it is observed that (3b) is a fully actuated mechanical system. ■

Observe that we now have a means of defining a trajectory for the r-dimensional flat output $\mathbf{y}$ which ensures that the mechanical system (3) evolves as desired, and we also have a strategy for controlling the r-dimensional vector of reducing outputs $\mathbf{x}_2$ associated with the reduced system (3b). It can be shown that the flat output $\mathbf{y}$ and the reducing output $\mathbf{x}_2$ can never be identical,[9] so that we cannot simply specify the desired trajectory $\mathbf{y}_d(t)$ for the flat output and then generate the control input $\mathbf{F}$ which ensures that $\mathbf{x}_2(=\mathbf{y})$ in (3b) tracks this trajectory. However, it is possible to adopt essentially this strategy by adding one additional step in the algorithm. Thus we have:

Algorithm 1:

(i) Given an initial state $(\mathbf{x}(0), \dot{\mathbf{x}}_2(0))$ and a goal state $(\mathbf{x}_d, \mathbf{0})$, determine the sets of flat outputs and derivatives of outputs $(\mathbf{y}(0), \dot{\mathbf{y}}(0), \ldots, \mathbf{y}^{(s)}(0))$ and $(\mathbf{y}(T), \dot{\mathbf{y}}(T), \ldots, \mathbf{y}^{(s)}(\mathrm{T}))$ corresponding to the initial and goal states. Find a smooth curve $\mathbf{y}_d(t)$ in the flat output space connecting these initial and final output values.

(ii) Use the relation $\mathbf{x}_{2d}(t)=\mathbf{h}_4(\mathbf{y}_d(t), \ldots, \mathbf{y}_d^{(s)}(t))$ (for $\mathbf{h}_4$ obtained from $\mathbf{h}_2$ in (8b)) to determine the desired trajectory for $\mathbf{x}_2$.

(iii) Track the desired trajectory $\mathbf{x}_{2d}(t)$ for the configuration $\mathbf{x}_2$ of the reduced system (3b) using the adaptive scheme (9), (10) (or any other suitable tracking strategy).

Note that **Algorithm 1** provides a simple and computationally efficient means of controlling the motion of the nonholonomic system (3), and that the scheme can be implemented without rate measurements or knowledge of the system dynamics (3b).

While differential flatness appears to be a useful system property to exploit when solving the trajectory generation problem for (flat) nonholonomic systems, there are at present no general methods for determining whether or not a given system is flat or, if a system is flat, what the flat outputs might be. Thus it is desirable to identify other structural properties of nonholonomic systems that can be used for control purposes, and to develop strategies for controlling systems which possess this structure.

Toward this end, consider the nonholonomic mechanical system (3) and suppose that the system constraint matrix A^* is independent of $\mathbf{x}_1$. Such systems are known as (controlled) *Caplygin* systems and are common in applications;[1] for example, the symmetric systems (7) are Caplygin systems. Observe that, as before, the evolution of the reducing coordinates $\mathbf{x}_2$ can be controlled by properly specifying the input $\mathbf{F}$, and that the resulting trajectory for $\mathbf{x}_2$ completely determines the behaviour of the remaining configuration coordinates $\mathbf{x}_1$. In this case, however, the relationship between the evolution of $\mathbf{x}_1$ and $\mathbf{x}_2$ is simplified. This observation is used in reference [1] to develop an algorithm for controlling the motion of the full system configuration $\mathbf{x}$. The algorithm proposed in reference [1] is based on the use of geometric phase, which is the extent to which a closed path in the $\mathbf{x}_2$ space fails to be closed in the configuration space, to maneuver both $\mathbf{x}_1$ and $\mathbf{x}_2$ to their desired values. We now present a version of this control algorithm which is implementable without model information or rate measurements. Consider the problem of transferring the Caplygin system from an arbitrary initial state $(\mathbf{x}_1^0, \mathbf{x}_2^0, \dot{\mathbf{x}}_2^0)$ to a user specified goal configuration, and suppose for simplicity of notation that the goal configuration is the origin.

Algorithm 2:

(i) Given an initial state $(\mathbf{x}_1^0, \mathbf{x}_2^0, \dot{\mathbf{x}}_2^0)$, drive the system (3b) to the origin in the $(\mathbf{x}_2, \dot{\mathbf{x}}_2)$ reduced state space in finite time using the trajectory tracking scheme (9), (10). Observe that this can be accomplished using any smooth path between $(\mathbf{x}_2^0, \dot{\mathbf{x}}_2^0)$ and $(\mathbf{0}, \mathbf{0})$ and that, in general, the resulting state of the system will be $(\mathbf{x}_1^1, \mathbf{0}, \mathbf{0})$ for some nonzero $\mathbf{x}_1^1$.

(ii) Find a closed path (or series of closed paths) in the $\mathbf{x}_2$ space which produces the desired geometric phase in the configuration space, so that $(\mathbf{x}_1^1, \mathbf{0}, \mathbf{0})$ is transferred to $(\mathbf{0}, \mathbf{0}, \mathbf{0})$.

(iii) Track the desired trajectory $\mathbf{x}_{2d}(t)$ for $\mathbf{x}_2$ using the adaptive scheme (9), (10).

In order to successfully implement this approach to controlling (3), we must determine a closed path P in the $\mathbf{x}_2$ space which satisfies the geometric phase condition

$$\mathbf{x}_1^1=-\int_P A^*(\mathbf{x}_2)d\mathbf{x}_2$$

Explicit characterization of such closed paths P can be achieved for several systems of practical interest (see, for instance references [1,5]); however, some systems may require a more general computational approach [e.g. reference 5]. In any case, once such a closed path in the $\mathbf{x}_2$ space is found it is clear that **Algorithm 2** will produce the desired evolution of (3).

Implicit in the development of the control algorithms given above is the assumption that there is no uncertainty associated with the kinematic relationship (3a), so that a trajectory can be specified for $\mathbf{x}_2$ which produces the requisite $\mathbf{x}_1$ motion. This can be a reasonable assumption for many nonholonomic systems but is certainly not always the case. Consider, for example, the situation in which the nonholonomic constraints are a consequence of a symmetry of the system dynamics, as in (7a). In this case, the constraints depend on the inertial parameters of the system, and it is often desirable to permit these parameters to be uncertain.

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In view of these considerations, we now turn our attention to the case in which there is uncertainty in the constraint (3a), and present control algorithms for driving the system to the goal configuration $\mathbf{x}_d$ despite this uncertainty. Observe first that any uncertainty in the kinematic constraints which is associated with *inertial* parameters (such as occur with symmetric systems) can always be linearly parameterized by slightly modifying (3a) as follows:

$$\dot{\mathbf{x}}_1 = P\mathbf{g}(\mathbf{x}, \dot{\mathbf{x}}_1, \dot{\mathbf{x}}_2) \qquad (11)$$

where the map $\mathbf{g}$ is known and the matrix P is unknown. Indeed, the existence of such a linear parameterization is a direct consequence of the well-known property that the inertial parameters appear linearly in the mechanical system dynamic model [e.g. reference 22]. Additionally, this linear parameterization property also holds for many systems with uncertain *kinematic* parameters; one such example is described in Section 3 below.

Suppose that we have an estimate for P, say $\hat{P}$, and that this estimate is used (as in **Algorithms 1** and **2**) to determine a desired trajectory for the reducing output which is designed to cause both $\mathbf{x}_1$ and $\mathbf{x}_2$ to evolve to their desired values $\mathbf{x}_{1d}$ and $\mathbf{x}_{2d}$, respectively. That is, suppose $\mathbf{x}_{2d}(t)$ is specified so that $\mathbf{x}_{2d}(0) = \mathbf{x}_2(0)$, $\mathbf{x}_{2d}(T) = \mathbf{x}_{2d}$, and

$$\Delta\mathbf{x}_{1d} = \int_P \hat{P}\mathbf{g}(\mathbf{x}, \dot{\mathbf{x}}_1, \dot{\mathbf{x}}_2)dt$$

where P is the path corresponding to $\mathbf{x}_{2d}(t)$ and $\Delta\mathbf{x}_{1d} = \mathbf{x}_{1d} - \mathbf{x}_1(0)$. Now, since $P \neq \hat{P}$ in general, this trajectory will not actually drive the system to the goal configuration. However, we can use the error associated with this process to improve our estimate for P and ultimately ensure convergence of $\mathbf{x}$ to $\mathbf{x}_d$. Toward this end, define the *predicted value* of $\dot{\mathbf{x}}_1$, denoted $\hat{\dot{\mathbf{x}}}_1$, as follows:

$$\hat{\dot{\mathbf{x}}}_1 = \hat{P}\mathbf{g}(\mathbf{x}, \dot{\mathbf{x}}_1, \dot{\mathbf{x}}_2) \qquad (12)$$

Comparing (11) and (12) we see that if $P = \hat{P}$ then $\dot{\mathbf{x}}_1 = \hat{\dot{\mathbf{x}}}_1$ and the $\mathbf{x}_{2d}(t)$ specified in the trajectory generation process will ensure that the entire system evolves as desired. If $\dot{\mathbf{x}}_1 \neq \hat{\dot{\mathbf{x}}}_1$ then we can adjust our estimate $\hat{P}$, and the corresponding desired trajectory $\mathbf{x}_{2d}(t)$, so that $\hat{\dot{\mathbf{x}}}_1(t)$ converges to $\hat{\dot{\mathbf{x}}}_1(t)$ and the entire system converges to the goal configuration $\mathbf{x}_d$. It turns out that it is useful to update the parameter estimate $\hat{P}$ at discrete instants t_k, chosen so that $t_{k+1} - t_k = T$, since this permits convenient recalculation of the appropriate reducing output trajectory $\mathbf{x}_{2d}(t)$ and also enhances the robustness of the estimation process. One such estimation scheme is the subject of the next lemma.

Lemma 3: Suppose that the parameter estimate $\hat{P}$ is updated according to

$$\hat{P}(\mathrm{t}_{k+1}) = \hat{P}(\mathrm{t}_k) + \Delta P_k$$

$$\Delta P_k = -\frac{\alpha}{T}\int_{t_k}^{t_k+T} \frac{\mathbf{E}\mathbf{g}^T}{(1+\|\mathbf{g}\|^2)}dt$$

where $\mathbf{E} = \hat{\dot{\mathbf{x}}}_1 - \dot{\mathbf{x}}_1$ and α is a positive constant. If $\mathbf{x}_{2d}(t)$ is computed based on this estimate and α is not chosen too large then $\mathbf{x} \rightarrow \mathbf{x}_d$ as $t \rightarrow \infty$.

Proof: Let $\Phi = \hat{P} - P$ and $\Phi_k = \Phi(t_k)$. Then the Lyapunov function candidate

$$V_k = \frac{1}{2}\,\mathrm{tr}[\Phi_k\Phi_k^T]$$

leads to

$$\Delta V_k = -\frac{\eta}{2}\,tr[\Phi_k R_k \Phi_k^T] \leq 0$$

provided α is not chosen too large, where R_k is a positive semidefinite matrix obtained through routine manipulation and η is a positive constant. Thus V_k is a Lyapunov function, and we have shown that all signals are bounded. Moreover, since the sequence V_k is non-increasing and bounded below (by zero), it can be concluded that ΔV_k converges to zero. Standard arguments can then be used to show that $\mathbf{E}$ converges to zero,[23] and this in turn implies convergence of $\mathbf{x}$ to $\mathbf{x}_d$. ■

The adaptation strategy proposed in Lemma 3 allows us to give algorithms for controlling mechanical systems with uncertainty in the nonholonomic constraints which parallel **Algorithms 1** and **2**. It should be noted, however, that these algorithms will in general require rate measurements because this information is utilized in the adaptation scheme given in Lemma 3.

Algorithm 3:

(i) Given an initial state $(\mathbf{x}(0), \dot{\mathbf{x}}_2(0))$ and a goal state $(\mathbf{x}_d, \mathbf{0})$, determine estimates of the sets $(\mathbf{y}(0), \ldots, \mathbf{y}^{(s)}(0))$ and $(\mathbf{y}(T), \ldots, \mathbf{y}^{(s)}(T))$ corresponding to the initial and goal states. Find a smooth curve $\mathbf{y}_d(t)$ in the flat output space connecting these initial and final output values.
(ii) Use the relation $\mathbf{x}_{2d}(t) = \mathbf{h}_4(\mathbf{y}_d(t), \ldots, \mathbf{y}_d^{(s)}(t))$ (for $\mathbf{h}_4$ obtained from $\mathbf{h}_2$ in (8b)) to determine the desired trajectory for $\mathbf{x}_2$.
(iii) Track the desired trajectory $\mathbf{x}_{2d}(t)$ using the adaptive controller (9), (10).
(iv) Use the adaptation strategy given in Lemma 3 to adjust the estimate for the desired flat output trajectory $\mathbf{y}_d(t)$ and repeat steps (ii) and (iii).
(v) Repeat step (iv) until the system converges to the goal state $(\mathbf{x}_d, \mathbf{0})$.

Algorithm 4:

(i) Given an initial state $(\mathbf{x}_1^0, \mathbf{x}_2^0, \dot{\mathbf{x}}_2^0)$, drive the system (3b) to the origin in the $(\mathbf{x}_2, \dot{\mathbf{x}}_2)$ reduced state space in finite time using the trajectory tracking scheme (9), (10). Observe that, in general, the resulting state of the system will be $(\mathbf{x}_1^1, \mathbf{0}, \mathbf{0})$ (for some nonzero $\mathbf{x}_1^1$).
(ii) Determine an estimate for a closed path P in the $\mathbf{x}_2$ space which produces the desired geometric phase in the configuration space, so that $(\mathbf{x}_1^1, \mathbf{0}, \mathbf{0})$ is transferred to $(\mathbf{0}, \mathbf{0}, \mathbf{0})$.
(iii) Track the desired $\mathbf{x}_{2d}(t)$ using the adaptive controller (9), (10).
(iv) Use the adaptation strategy given in Lemma 3 to adjust the estimate for the path P, and repeat step (iii).

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(v) Repeat step (iv) until the system converges to the goal state ($\mathbf{x}_d$, $\mathbf{0}$).

3. CASE STUDIES

We now apply the proposed approach to controlling uncertain nonholonomic mechanical systems to three such systems: a two wheel mobile robot, a three wheel mobile robot, and a "free-flying" space robot. The mobile robots are representative of the class of nonholonomic systems which result from explicit constraints on the system kinematics, while the free-flying space robot is an example of a system with a nonholonomic constraint arising from the presence of symmetry in the system dynamics. To provide a basis for comparison, an adaptive dynamic feedback linearizing controller is also implemented in the simulations with the three wheel mobile robot. The case study with the free-flying space robot includes the results of both computer simulations and preliminary hardware experiments.

3.1 Two wheel mobile robot

The first system considered in this series of case studies is the simple two wheel mobile robot (with front castor for balance) shown in Figure 1 and described in reference [9]. For this system, the dynamic model (1) has the following form:

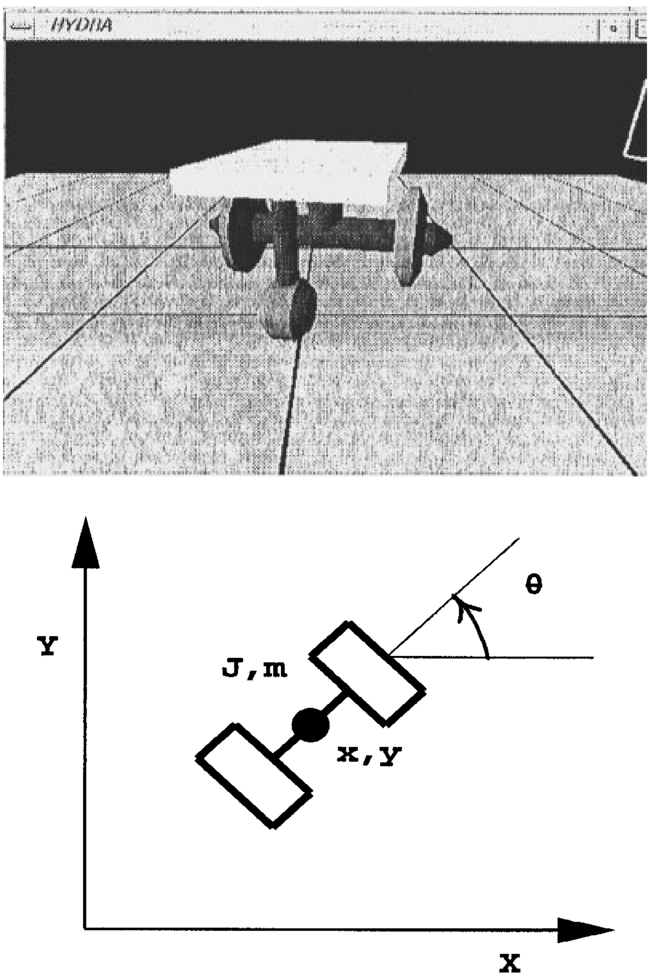

Fig. 1. Illustration of two wheel mobile robot.

$$
\begin{aligned}
m\ddot{x} &= \lambda \cos\theta - (T_1 + T_2)\sin\theta \\
m\ddot{y} &= \lambda \sin\theta + (T_1 + T_2)\cos\theta \\
J\ddot{\theta} &= (T_1 - T_2) \\
0 &= \dot{x}\cos\theta + \dot{y}\sin\theta
\end{aligned} \tag{13}
$$

where x, y, θ are the position and orientation coordinates of the (axle of the) mobile robot, m, J are the system inertial parameters, λ is the constraint multiplier, and T_1, T_2 are the torques provided at the wheels. This system is differentially flat with flat outputs $\mathbf{y} = [x\ y]^T$ and reducing outputs $\mathbf{x}_2 = [x\ \theta]^T$. Since the system is flat and there is no uncertainty associated with the kinematic constraints, **Algorithm 1** can be applied to ensure that the mobile robot moves from any initial state to any desired final configuration.

The simulation considered here illustrates the capability of Algorithm 1 to drive the system to the origin from various initial configurations with no model information or rate measurements. The trajectory generation phase of **Algorithm 1** is accomplished by fitting a fifth order polynomial for each flat output coordinate to the necessary end point conditions. This output curve is then used to specify a desired trajectory for the reducing output $\mathbf{x}_{2d}(t)$, and the adaptive tracking scheme (9), (10) is utilized to track this trajectory. The algorithm is applied to the mathematical model of the mobile robot through computer simulation with a sampling period of two milliseconds. The system model parameters are defined as $m = J = 10$, and the controller parameters k_1, k_2, γ are set as follows: $k_1 = 10$, $k_2 = 20$, $\gamma = 5$. The controller terms **w, f,** A, and B are set to zero initially, and the adaptation parameters are set as follows: $\sigma_i = 0.01$ and $\beta_i = 100\ \forall i$. It is noted that no attempt was made to "tune" the controller gains to obtain the best possible performance. The control strategy given in **Algorithm 1** was tested using a wide range of initial conditions; sample results are given in Figure 2 and indicate that the motion of the system is accurately controlled.

3.2 Three wheel mobile robot

The proposed approach to nonholonomic system control is now applied via computer simulation to the three wheel mobile robot shown in Figure 3. The dynamic model (1) has

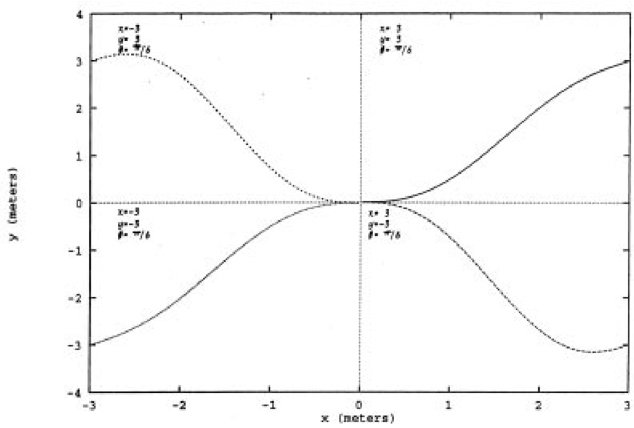

Fig. 2. Response of x, y coordinates of two wheel mobile robot using **Algorithm 1** for four sample initial conditions.

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the following form for this system:

$$\begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ l\sin\phi\cos\phi & 0 \\ 0 & 1 \end{bmatrix} \mathbf{T} = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & (I_r+I_f) & I_f \\ 0 & 0 & I_f & I_f \end{bmatrix} \ddot{\mathbf{x}} + \begin{bmatrix} -\sin\theta & -\sin(\theta+\phi) \\ \cos\theta & \cos(\theta+\phi) \\ 0 & l\cos\phi \\ 0 & 0 \end{bmatrix} \lambda \qquad (14)$$

$$\begin{bmatrix} -\sin\theta & \cos\theta & 0 & 0 \\ -\sin(\theta+\phi) & \cos(\theta+\phi) & l\cos\phi & 0 \end{bmatrix} \dot{\mathbf{x}} = \mathbf{0}$$

where (x, y) is the rear axle position, θ and ϕ are the body angle and steering angle, respectively, $\mathbf{x}=[x\ y\ \theta\ \phi]^T \in \Re^4$ is the system configuration vector, I_r, I_f, m are the system inertia parameters, l is the wheel base length, $\mathbf{T} \in \Re^2$ is the control input, and $\lambda \in \Re^2$ is the vector of constraint multipliers.

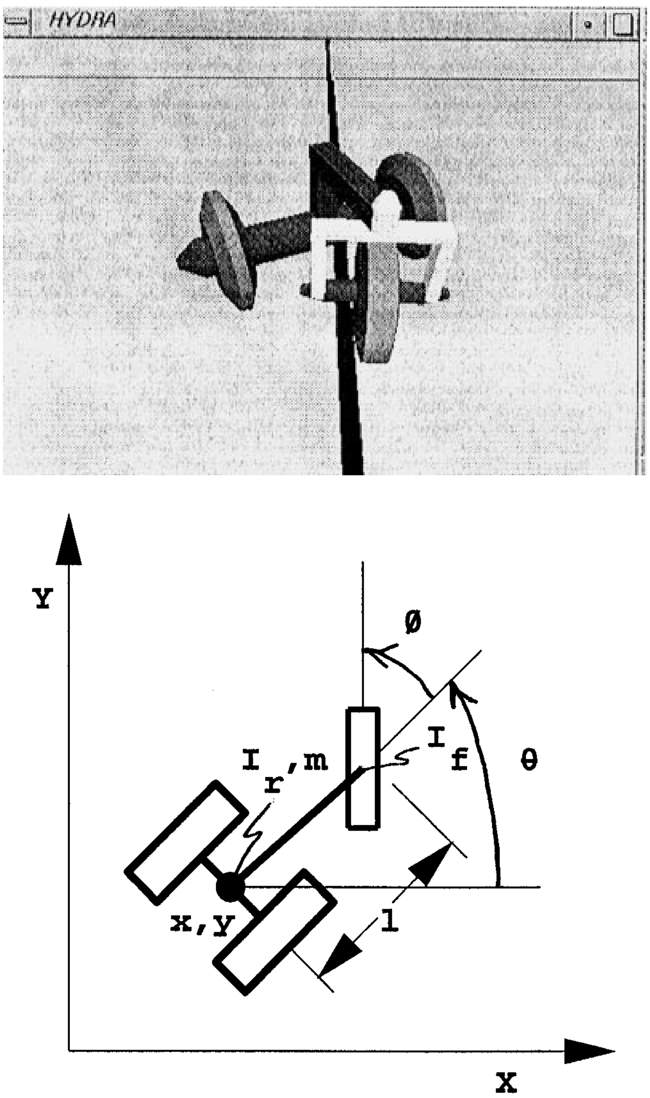

Fig. 3. Illustration of three wheel vehicle.

This system is differentially flat with flat outputs $\mathbf{y}=[x\ y]^T$ and reducing outputs $\mathbf{x}_2=[\theta\ \phi]^T$. Observe that in this case the kinematic constraints contain the parameter l, which may or may not be uncertain. Thus in what follows we first assume that l is accurately known and apply **Algorithm 1** for motion control of the mobile robot, and we then consider the possibility that l is uncertain and utilize **Algorithm 3** to move the mobile robot to the goal configuration despite this uncertainty.

Consider first the case in which l is assumed to be accurately known and **Algorithm 1** is used to drive the system to the origin from various initial configurations with no model information or rate measurements. The trajectory generation phase of **Algorithm 1** is accomplished by fitting a fifth order polynomial for each flat output coordinate to the necessary end point conditions. This output curve is then used to determine the desired trajectory for the reducing output $\mathbf{x}_{2d}(t)$, and the adaptive tracking scheme (9), (10) is utilized to track this trajectory. The algorithm is applied to the mathematical model of the mobile robot through computer simulation with a sampling period of two milliseconds. The system model parameters are defined as $l=1$, $m=I_f=I_r=10$. All controller parameters are set to the values used in the previous simulation, despite the fact that the two mobile robots have quite different properties. This choice for the controller terms is made to demonstrate that these gains need not be tuned for a particular system to obtain good performance.

To provide a basis for evaluating the performance of the proposed approach to motion control, an adaptive controller developed using the dynamic feedback linearization approach is also used to control the mobile robot. More specifically, a dynamic feedback linearizing control scheme [e.g. reference 17] is designed for *kinematic control* of the mobile robot, assuming velocities are inputs, and these velocities are then tracked using an adaptive velocity tracking controller for mechanical systems which closely resembles the position tracking scheme (9), (10). Note that such a controller can always be developed for flat nonholonomic systems because flatness implies dynamic feedback linearizability.[14] Sample results obtained using Algorithm 1 and the dynamic feedback linearizing controller just described are given in Figure 4a. These results indicate that both schemes accomplish the desired motion control, but that Algorithm 1 achieves noticeably better accuracy. It is worth mentioning that considerable effort was required to tune the gains for the dynamic feedback linearizing scheme in order to obtain acceptable performance, while very little effort was needed to determine gains for Algorithm 1 (indeed, we simply used the gains from the previous simulation); this provides an indication of the ease with which the proposed approach can be implemented relative to other methods.

We next consider the case in which our estimate of l is assumed to contain some uncertainty, and use Algorithm 3 to control the system. The trajectory generation phase of Algorithm 3 is accomplished by fitting a fifth order polynomial for each flat output coordinate to the given end conditions. This output curve is then used to specify the desired trajectory for the reducing output $\mathbf{x}_{2d}(t)$, and the

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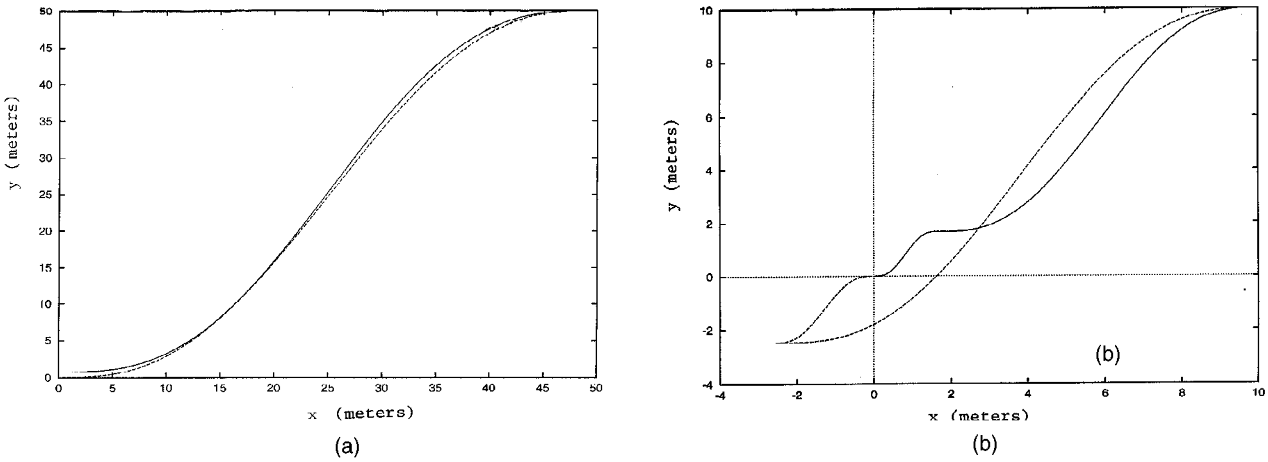

Fig. 4. (a) Response of *x*, *y* coordinates of three wheel vehicle using **Algorithm 1** (dashed) and adaptive dynamic feedback linearization controller (solid). (b) Response of *x*, *y* coordinates of three wheel vehicle using **Algorithm 3** with $\hat{l}(0) = 1.2$ (solid) and $\hat{l}(0) = 0.8$ (dashed).

adaptive tracking scheme (9), (10) is utilized to track this trajectory. The algorithm is applied to the mathematical model of the mobile robot using exactly the procedure described above. The estimation strategy given in Lemma 3 is implemented with $\alpha = 10$ and with two different initial estimates for l: $\hat{l}(0) = 1.2$ and $\hat{l}(0) = 0.8$. In this simulation, the mobile robot is initially at rest with configuration $x(0) = 10$, $y(0) = 10$, $\theta(0) = 0°$, $\phi(0) = 0°$, and is commanded to smoothly move to the origin. As specified in **Algorithm 3**, this objective is attained by repeatedly planning and tracking trajectories for the reducing outputs until convergence to the goal is achieved. As seen in Figure 4b, in the present application convergence to the goal requires only two trajectories. It is interesting to note that when l is (initially) underestimated the system overshoots the goal and must back up in the second trajectory, while the opposite situation occurs when l is overestimated.

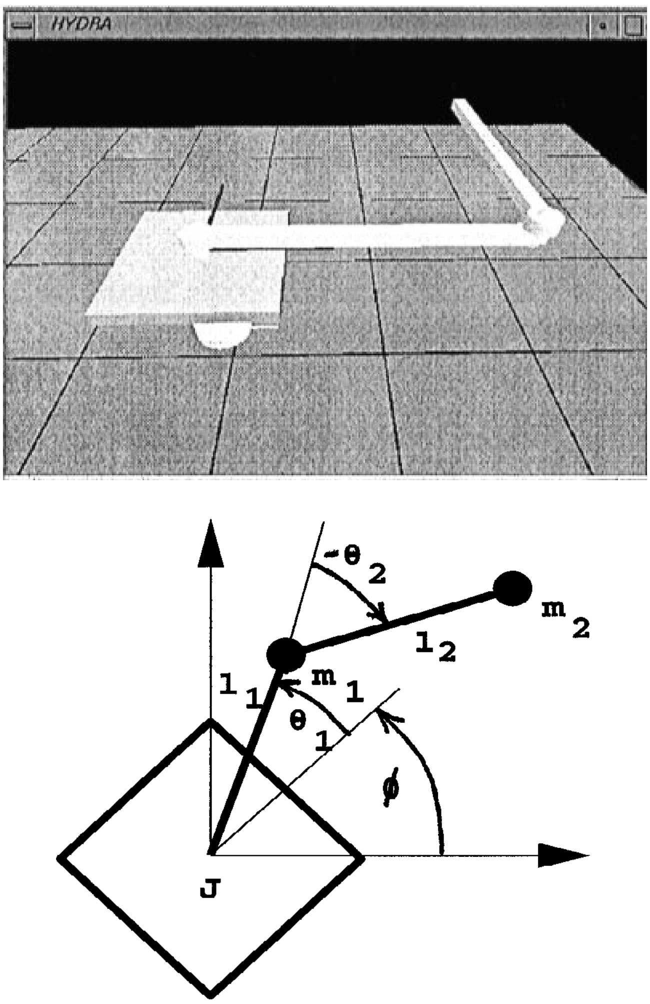

Fig. 5. Illustration of free-flying space robot.

3.3 *Free-flying space robot*

Finally, we turn our attention to a symmetric mechanical system: a simple model of a "free-flying" space robot (see Figure 5). The system is modeled as a rigid "vehicle" with inertia J pinned to the ground at its center of mass, and a two link planar "manipulator" with link lengths l_1, l_2 and link masses m_1, m_2, assumed for simplicity to be concentrated at the distal ends of the two links. The manipulator has two actuators, one at each joint, while the vehicle's pinned connection to the ground is unactuated. Note that pinning the vehicle in this way permits the body to rotate freely but prevents translation. Thus the nonholonomic constraint arising from angular momentum conservation is retained, while the holonomic constraints arising from linear momentum conservation in a truly "free-flying" space system are replaced with holonomic pinned constraints; observe that this simplifies the subsequent analysis but removes none of the essential structure of the system. Let ϕ denote the angle of the vehicle and (θ_1, θ_2) be coordinates for the manipulator. It is easily verified that in this case the system model is of the form (7), where the mechanical system dynamics (7b) is standard and the kinematic map (7a) can be obtained from the nonholonomic constraint corresponding to angular momentum conservation.

In symmetric coordinates the Lagrangian for this system can be written

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$$L=\frac{1}{2}J\dot{\phi}^2+\frac{1}{2}(m_1+m_2)l_1^2(\dot{\phi}+\dot{\theta}_1)^2+\frac{1}{2}m_2l_2^2(\dot{\phi}+\dot{\theta}_1+\dot{\theta}_2)^2$$
$$+m_2l_1l_2\cos\theta_2(\dot{\phi}+\dot{\theta}_1)(\dot{\phi}+\dot{\theta}_1+\dot{\theta}_2)$$

where ϕ is the orientation of the platform relative to a fixed axis in the plane, θ_1 is the angle of the first link relative to the platform, and θ_2 the angle of the second link relative to the first. It is clear that (θ_1, θ_2) are reducing outputs for this system. Additionally, it can be seen that the Lagrangian L is independent of ϕ and that this coordinate is unactuated. Because of these facts the Euler-Lagrange equation corresponding to ϕ leads to a conservation law, which if the system starts at rest can be written

$$[J+(m_1+m_2)l_1^2+m_2l_2^2]\dot{\phi}+[(m_1+m_2)l_1^2+m_2l_2^2]\dot{\theta}_1+m_2l_2^2\dot{\theta}_2$$
$$=-m_2l_1l_2\cos\theta_2(2\dot{\phi}+2\dot{\theta}_1+\dot{\theta}_2)$$

Examination of this constraint reveals that

$$y_1=[J+(m_1+m_2)l_1^2+m_2l_2^2]\phi+[(m_1+m_2)l_1^2+m_2l_2^2]\theta_1+m_2l_2^2\theta_2$$
$$y_2=m_2l_1l_2(2\phi+2\theta_1+\theta_2)$$

are a set of flat outputs for the system. Note that in this case the flat output curves cannot be specified arbitrarily, as was the case with the mobile robots above, and instead must be chosen so that the differential inequalities $|\dot{y}_1/\dot{y}_2|\leq 1$ are satisfied; this situation is not uncommon with symmetric flat systems.

For this system, we first present simulation results for the case in which all of the parameters are assumed known and **Algorithm 1** is used to drive the system to the goal configuration. The path in flat output space is computed by requiring that $\dot{y}_1/\dot{y}_2=\pm 1$ at the endpoints of the accessible range of θ_1. Additionally, we assume the existence of practical limits on the apparatus due to imperfect construction which will generally occur in real systems. In particular, in this simulation, we assume that $-35°<\theta_1<0°$ which corresponds to a real robotic platform in our laboratory. Figure 6a shows the path in flat output space and illustrates the effect of the differential inequalities on the flat trajectory. Figure 6b shows the convergence of the angle of the platform ϕ to the goal $\phi_d=20°$.

Next we consider the case in which there is uncertainty in the kinematic constraints, and use **Algorithm 3** to control the system. A linearly parametrized form of the conservation law is

$$\dot{\phi}=-\frac{1}{J}\{[(m_1+m_2)l_1^2+m_2l_2^2]\dot{\phi}+[(m_1+m_2)l_1^2+m_2l_1^2]\dot{\theta}_1+m_2l_2^2\dot{\theta}_2$$
$$+2m_2l_1l_2\cos\theta_2(\dot{\phi}+\dot{\theta}_1+\dot{\theta}_2)\}$$

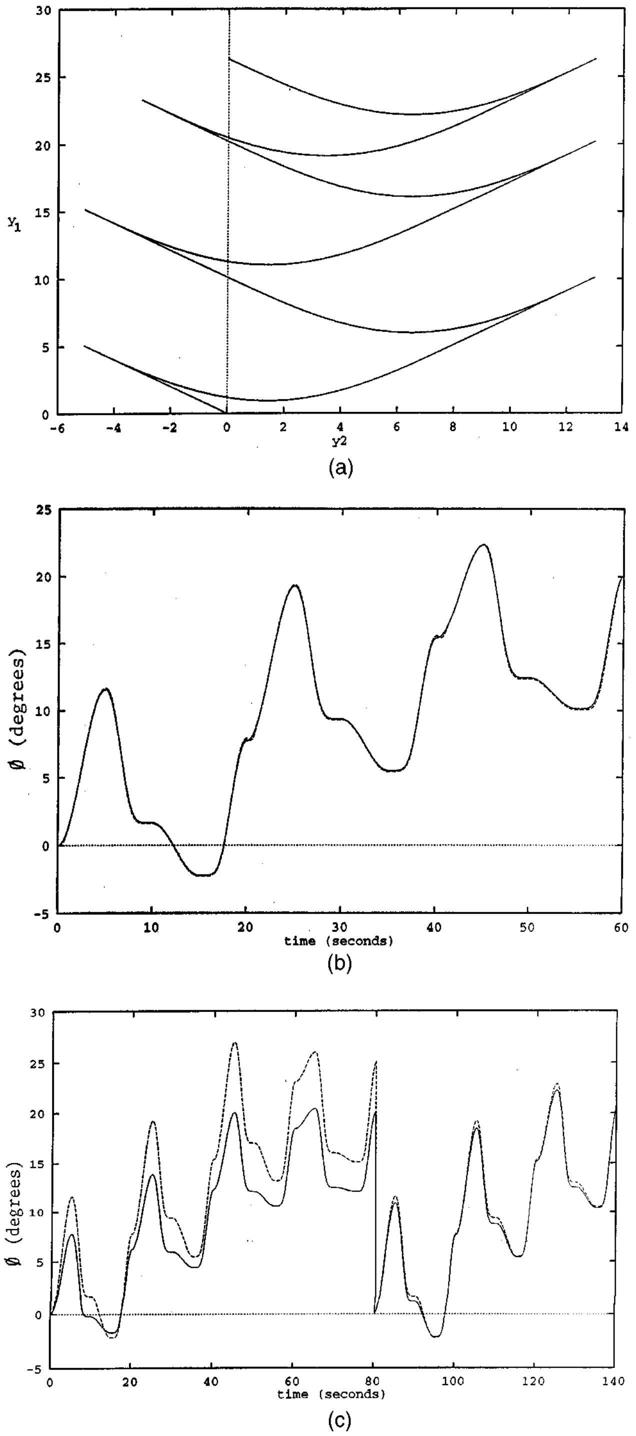

Fig. 6. (a) Response of y_1, y_2 coordinates of free-flying space robot using **Algorithm 1** (b) Desired (solid) and actual (dotted) evolution of ϕ coordinate of free-flying space robot using **Algorithm 1** (c) Desired (solid) and actual (dotted) evolution of ϕ coordinate of free-flying space robot using **Algorithm 3**.

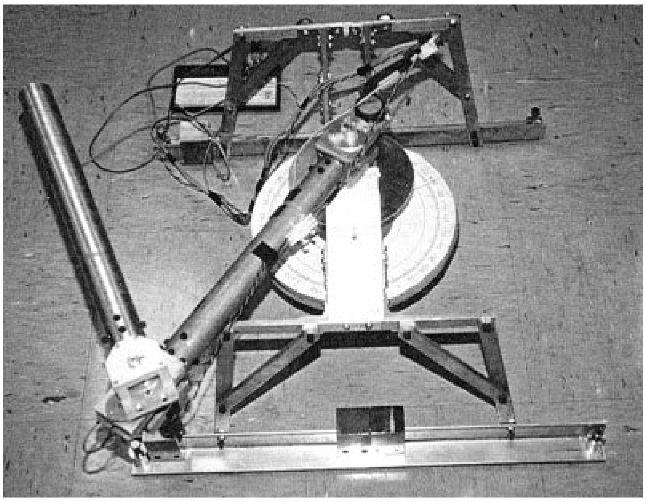

Fig. 7. Free-flying space robot experimental testbed.

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where we identify the quantities $p_1=(m_1+m_2)l_1^2/J$, $p_2=m_2l_2^2/J$, and $p_3=m_2l_1l_2/J$ as the uncertain parameters. For purposes of the simulation we set $J=20$, $l_1=l_2=1$, and $m_1=m_2=10$. The estimation strategy given in Lemma 3 is implemented with $\alpha=10$ and initial estimates $\hat{p}_i=0.5p_i$. The free-flying space robot is initially at rest with configuraion $\phi=0°$, $\theta_1=\theta_2=0°$, and is commanded to smoothly move to $\phi=20°$, $\theta_1=\theta_2=0°$. As specified in **Algorithm 3**, this objective is attained by repeatedly planning and tracking trajectories for the reducing outputs until convergence to the goal is achieved. As seen in Figure 6c, ϕ converges to the desired value of $\phi=20°$ after only two such trajectories.

The performance of the proposed approach for controlling nonholonomic mechanical systems was also verified through preliminary hardware experiments with a laboratory version of the simple "free flying" space robot described above. The facility utilized for this study is the New Mexico State University Robotics Laboratory. The experimental testbed consists of a three degree of freedom manipulator/vehicle system similar to the one used in the simulation study (see Figure 7), together with the associated controller electronics and control computer. All control software is written in 'C' and is hosted on an IBM-compatible 486 personal computer. The manipulator is constructed so that $l_1=l_2=0.38m$ and so that the arm's mass distribution can be accurately approximated as consisting of point masses at the distal ends of the two links. The end-effector of the arm is designed to accept a variety of payloads, and the vehicle's rotational inertia is also adjustable. In the present preliminary set of experiments the end-effector payload is chosen to be large relative to the remaining mass of the arm, and the vehicle inertia is adjusted to be roughly twice that of the arm when the arm is outstretched. For the experiment presented here the control law is applied to the robot with a sampling period of seven milliseconds. All controller parameters and adaptation gains for the tracking controller are set to values similar to those used in the simulation study described above.

In contrast to the other case studies, in this experiment we achieve motion control for the robotic system using ideas

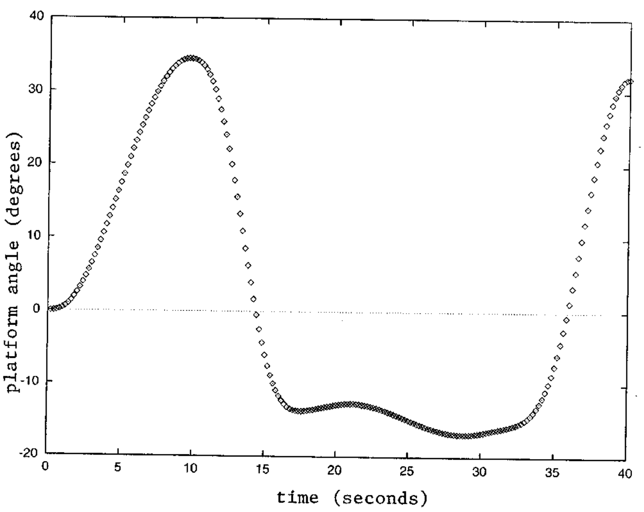

Fig. 8. Evolution of ϕ coordinate of free-flying space robot in preliminary hardware experiment.

based on geometric phase. More specifically, **Algorithm 4** is utilized to drive the manipulator/vehicle system from its initial position $\phi=\theta_1=\phi=\theta_2=0°$ to the goal configuration $\phi=30°$, $\theta_1=\theta_1=0°$. The control algorithm is implemented in a few trials first, to obtain an estimate for the uncertain parameter $\hat{P}$ (which in the present case is a scalar), and is then used to accomplish the motion control objective. The experiment was repeated ten times. A sample plot depicting the motion of the platform for one trial is given in Figure 8. In each trial the final manipulator configuration was achieved with good accuracy; this is to be expected, of course, since the manipulator coordinates are the reducing outputs for this system and are controlled directly. The average final vehicle configuration was $\phi=31.9°$ (Figure 8 shows a typical run), indicating that the proposed approach is capable of controlling the *entire* system configuration with reasonable accuracy.

4. CONCLUSIONS

This paper considers the problem of controlling nonholonomic mechanical systems in the presence of incomplete information concerning the system model and state, and proposes that a simple and effective solution to this problem can be obtained through the use of reduction and adaptation. It is shown that this approach is easy to implement and ensures accurate motion control despite limited measurements and uncertainty regarding the system model. The performance of the proposed method is illustrated through applications with nonholonomic mechanical systems arising from both explicit kinematic constraints and symmetries of the system dynamics. Future work will include a detailed study of the performance of the proposed control algorithms in applications which require high levels of reliability and autonomy, such as operations in remote and hazardous environments.

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References

1. A. Bloch, M. Reyhanoglu and N. McClamroch, "Control and Stabilization of Nonholonomic Dynamic Systems", *IEEE Transactions on Automatic Control* **37**, No. 11, 1746–1757 (1992).
2. G. Walsh and L. Bushnell, "Stabilization of Multiple Input Chained Form Control Systems", *Proc. 32nd IEEE Conference on Decision and Control*, San Antonio, TX, USA (December, 1993) pp. 959–964.
3. R. M'Closkey and R. Murray, "Extending Exponential Stabilizers for Nonholonomic Systems from Kinematic Controllers to Dynamic Controllers", *Proc. IFAC Symposium on Robot Control*, Capri, Italy (September, 1994) pp. 243–248.
4. H. Khennouf, C. Canudas de Wit and A. van der Schaft, "Preliminary Results on Asymptotic Stabilization of Hamiltonian Systems with Nonholonomic Constraints", *Proc. 34th IEEE Conference on Decision and Control*, New Orleans, LA, USA (December, 1995) pp. 4305–4310.
5. R. Murray, "Nonlinear Control of Mechanical Systems: A Lagrangian Perspective", *Proc. IFAC Symposium on Non-*

linear Control System Design, Lake Tahoe, CA, USA, (June, 1995) (preprint version).

6. G. Bastin and G. Campion, "Adaptive Control of Nonholonomic Mechanical Systems", *Proc. European Control Conference*, Grenoble, France (1991) pp. 1334–1338.
7. R. Fierro and F. Lewis, "Practical Point Stabilization of a Nonholonomic Mobile Robot Using Neural Networks", *Proc. 35th IEEE Conference on Decision and Control*, Kobe, Japan (December, 1996) pp. 1722–1727.
8. Y. Chang and B. Chen, "Adaptive Tracking Control Design of Nonholonomic Mechanical Systems", *Proc. 35th IEEE Conference on Decision and Control*, Kobe, Japan, (December, 1996) pp. 4739–4744.
9. R. Colbaugh, E. Barany and K. Glass, "Adaptive Control of Nonholonomic Mechanical Systems", *Proc. 35th IEEE Conference on Decision and Control*, Kobe, Japan (December, 1996) pp. 1428–1434; and *J. Robotic Systems* **15**, No. 7, 365—393 (1998).
10. R. Colbaugh, E. Barany and K. Glass, "Adaptive Stabilization of Uncertain Nonholonomic Mechanical Systems", *Robotica* **16**, Part 2, 181–192 (1998).
11. J. Mardsen and T. Ratiu, *Introduction to Mechanics and Symmetry* (Springer-Verlag, New York, 1994).
12. R. Colbaugh, H. Seraji and K. Glass, "Adaptive Compliant Motion Control for Dexterous Manipulators", *Int. J. Robotics Research* **14**, No. 3, 270–280 (1995).
13. R. Colbaugh, K. Glass and E. Barany, "Adaptive Regulation of Manipulators Using Only Position Measurements", *Int. J. Robotics Research* **16**, No. 5, 703–713 (1997).
14. M. Fliess, J. Levine, P. Martin and P. Rouchon, "Flatness and Defect of Nonlinear Systems: Introductory Theory and Examples", *Int. J. Control* **61**, No. 6, 1327–1361 (1995).
15. R. Brockett, "Asymptotic Stability and Feedback Stabilization", *Differential Geometric Control Theory*, (Birkhauser, Boston, 1983) pp. 181–191.
16. G. Campion, B. d'Andrea-Novel and G. Bastin, "Controllability and State Feedback Stabilizability of Nonholonomic Mechanical Systems", *Proc. Advanced Robot Control*, Lecture Notes in Control and Information Science, No. 162 (Springer, Berlin 1990) pp. 106–124.
17. H. Nijmeijer and A. van der Schaft, *Nonlinear Dynamical Control Systems* (Springer-Verlag, New York, 1990).
18. C. Canudas de Wit and N. Fixot, "Adaptive Control of Robot Manipulators via Velocity Estimated Feedback", *IEEE Transactions on Automatic Control* **37**, No. 8, 1234–1237 (1992).
19. H. Berghuis and H. Nijmeijer, "Robust Control of Robots via Linear Estimated State Feedback", *IEEE Transactions on Automatic Control* **39**, No. 10, 2159–2162 (1994).
20. T. Burg, D. Dawson and P. Vedagarbha, "A Redesigned DCAL Controller Without Velocity Measurements: Theory and Experimentation", *Proc. 33rd IEEE Conference on Decision and Control*, Orlando, FL, USA (December, 1994) pp. 824–828.
21. Z. Qu, D. Dawson, J. Dorsey and J. Duffie, "Robust Estimation and Control of Robot Manipulators", *Robotica* **13**, Part 3, 223–231 (1995).
22. F. Lewis, C. Abdallah and D. Dawson, *Control of Robot Manipulators* (Macmillan, New York, 1993).
23. K. Narendra and A. Annaswamy, *Stable Adaptive Systems*, (Prentice Hall, Englewood Cliffs, NJ, 1989).