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Accurate calculation of radiation damping parameters in the interaction between very intense laser beams and relativistic electron beams

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Abstract

We prove that the radiation damping force and the rate of change of the damping energy, in the Landau-Lifshitz forms, in interactions between very intense laser beams and relativistic electron beams, are periodic functions of only one variable, that is the phase of the electromagnetic field. The property is proved without using any approximation, in the most general case, when the degree of polarization of the electromagnetic field, the initial phase of the incident field and the initial energy of the electron have arbitrary values. This property leads to a strong simplification of the calculation of the radiation reaction parameters and of their dependence on the initial electron energy and angular frequency of the laser beam. Our analysis is performed in the proper inertial system of the electron. The radiation reaction is significant for laser beam intensities of the order 10^{22} W/cm^2, and for electron energy greater than 1 GeV. The calculations reveal limitations of the method of generating hard radiations by interactions between laser beams and relativistic electron beams.

Keywords: Electron beams; Laser beams; Radiation reaction; Relativistic electrodynamics

1. INTRODUCTION

The generation of radiation by motion of the electrons in electromagnetic field and the existence of a damping force, due to the emission of energy, are two complementary effects, which have studied since the beginning of the previous century (Thomson, 1881; Lorentz, 1916; Abraham, 1932; Dirac, 1938). The number of the papers treating these effects increased exponentially, starting in the 1970s, after the development of high-power lasers (Sarachik, 1970; Esarey, 1993; Harteman, 1995). More recently, the emergence of the ultra-intense laser pulses, generated by the technique of chirped pulse amplification, has made it possible to obtain beam intensities greater than 10^{18} W/cm^2. At such intensities, the study of the effect of radiation reaction becomes increasingly important, due to new applications, such as the generation of very energetic radiations (Eden, 2004), and the acceleration of particles in very intense electromagnetic fields (Mourou, 2006; Faure, 2006). At the same time, these applications are limited to ultrahigh laser beam intensities, due to the effect of radiation damping (Mao, 2010; Hadad, 2010; Deng, 2012).

In previous papers (Popa, 2011; 2012), we proved that the electrical field which results from the Liènard-Wiechert relation, due to the motion of the electron in the laser field, is a periodic function of only one variable, that is, the phase of the incident electromagnetic field. This property simplifies significantly the calculation model for the generation of hard radiations, in interactions between very intense laser fields and particles. Indeed all the functions which are involved in calculations are composite functions of only one variable, and their mathematical properties can be analyzed accurately. A synthesis of the applications of this method is presented by Popa (2013*a*; 2013*b*).

We show now that this method can be used also for accurately modeling the radiation reaction in interactions between very intense laser beams and relativistic electron beams. Our analysis is made in the inertial system S' in which the initial electron velocity is zero. We use the Landau-Lifschitz form of the radiation reaction force (Landau, 1987), which is a version of the Lorentz-Abraham-Dirac damping force (Lorentz, 1916; Abraham, 1932; Dirac, 1938). We prove that the radiation reaction force, as well as the rate of change of the damping energy, resulted by the action of this force, are periodic

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functions of only one variable, which is the phase of the laser field. With the aid of this property, the calculations are strongly simplified, because the average values of the damping and external forces, on one hand, and the average kinetic energy, on other hand, can be easily calculated over one period. At the same time, the damping energy, can be obtained without calculating the electron trajectory. Our approach is made in the most general case, when the degree of polarization of the electromagnetic field, the initial phase of the incident field, and the initial energy of the electron have arbitrary values.

More specifically, we calculate the maximum intensity of the laser beam at which the radiation reaction can be neglected, its dependence on the angular frequency of the field and on the initial electron energy, and the effect of the lowering of the maximum intensity value, due to increasing the initial electron energy. At the same time, the increasing of laser beam intensity in the ultrarelativistic regime has some collateral effects, such as the electron acceleration and the decrease in the variation of the field phase at the point where the electron is situated. These effects are used in the calculation of the radiation reaction parameters. Our results are compared with a series of data reported in recent papers. The equations are written in the International System.

2. INITIAL HYPOTHESES

We analyze the radiation damping effect in the case of a system composed of a very intense laser beam, interacting with a relativistic electron beam. We consider the following initial hypotheses:

(h1) In a Cartesian system of coordinates, the intensity of the electric field and of the magnetic induction vector of the laser beam, denoted, respectively, by $\bar{E}_L$ and $\bar{B}_L$, are elliptically polarized in the xy-plane, while the wave vector, denoted by $\bar{k}_L$, is parallel to the oz-axis. The expressions of the electric field and of the corresponding magnetic induction vector are

$$\bar{E}_L = E_{M1} \cos\eta \bar{i} + E_{M2} \sin\eta \bar{j}, \tag{1}$$

$$\bar{B}_L = -B_{M2} \sin\eta \bar{i} + B_{M1} \cos\eta \bar{j}, \tag{2}$$

where η is the phase of the electromagnetic field, $\bar{i}$, $\bar{j}$, and $\bar{k}$ are versors of the ox, oy, and oz axes, E_{M1}, E_{M2} are the amplitudes of the electric field oscillations in the ox and oy directions, and B_{M1} and B_{M2} are the amplitudes of the magnetic field oscillations in the oy and ox directions.

The following relations are also valid:

$$\eta = \omega_L t - |\bar{k}_L| z + \eta_i, \quad |\bar{k}_L| c = \omega_L, \tag{3}$$

and

$$E_{M1} = cB_{M1}, \quad E_{M2} = cB_{M2}, \quad c\bar{B}_L = \bar{k} \times \bar{E}_L, \tag{4}$$

where ω_L is the angular frequency of the laser electromagnetic field, c is the speed of light, η_i is an arbitrary initial phase, and t is the time in the xyz system in which the motion of the electron is studied.

It follows that the electromagnetic field described by Eqs. (1) and (2) is obtained by the superposition of two electromagnetic fields, linearly polarized along the ox and oy directions. We denote by I_L the average value of the intensity of the laser beam and by I_{L1} and I_{L2}, respectively, the average intensities of the components linearly polarized along the ox and oy directions. Since $I_L = \varepsilon_0 c/(2\pi) \int_0^{2\pi} \bar{E}_L^2 \, d\eta$ and so on, where ε_0 is the vacuum permitivity, we obtain the following relations:

$$I_L = \frac{1}{2}\varepsilon_0 c\left(E_{M1}^2 + E_{M2}^2\right), \quad I_{L1} = \frac{1}{2}\varepsilon_0 c E_{M1}^2,$$
$$I_{L2} = \frac{1}{2}\varepsilon_0 c E_{M2}^2. \tag{5}$$

(h2) We suppose that $E_{M1} > E_{M2}$ and the degree of the polarization of the field is given by Crawford (1999), written for the average intensities of the two components of the field. Taking into account (5), this relation can be written

$$P = \frac{I_{L1} - I_{L2}}{I_{L1} + I_{L2}} = \frac{E_{M1}^2 - E_{M2}^2}{E_{M1}^2 + E_{M2}^2}. \tag{6}$$

We see that when $P = 1$, the field is linearly polarized, and when $P = 0$, the field is circularly polarized.

The initial data for our calculations are I_L, λ_L, P, E_0, and η_i, where λ_L is the wavelength of the electromagnetic field and E_0 is the initial energy of the electron.

With the aid of the Eqs. (5) and (6), we obtain:

$$E_{M1} = \sqrt{\frac{I_L}{\varepsilon_0 c}(1 + P)} \text{ and } E_{M2} = \sqrt{\frac{I_L}{\varepsilon_0 c}(1 - P)}. \tag{7}$$

(h3) We consider the following initial conditions, when the components of the electron velocities, denoted by v_x, v_y, and v_z, have the following values, in the laboratory system, which is denoted by S:

$$t = 0, \quad x = y = z = 0, \quad v_x = v_y = 0, \quad v_z = -|\bar{V}_0|$$
$$\text{and} \quad \eta = \eta_i, \tag{8}$$

where the value $|\bar{V}_0|$ results from the following relations:

$$E_0 = \gamma_0 mc^2, \quad \gamma_0 = \frac{1}{\sqrt{1 - \bar{\beta}_0^2}} \text{ and } \bar{\beta}_0 = -\frac{|\bar{V}_0|}{c}\bar{k}, \tag{9}$$

where m is the electron mass, while β_0 and γ_0 are the well-known symbols used in the relativistic theory.

(h4) We use the conventions of Landau and Lifshitz (Landau, 1987) to write the four-vectors and the electromagnetic field tensor. So, the contravariant and covariant components of the four-vector of the coordinates are denoted, respectively, by x^i and x_i. These components are, respectively, (ct, x, y, z) and $(ct, -x, -y, -z)$. Similarly, the contravariant components of the four-velocity vector, which are given by the relation $u^i = d\,x^i/ds$, and the covariant components of the same vector, given by $u_i = d\,x_i/ds$, are, respectively, the $(\gamma, \gamma\beta_x, \gamma\beta_y, \gamma\beta_z)$ and $(\gamma\ -\gamma\beta_x, -\gamma\beta_y, -\gamma\beta_z)$. In these relations, $\beta_x = v_x/c$, $\beta_y = v_y/c$ and $\beta_z = v_z/c$, where v_x, v_y, and v_z are the components of the electron velocity, while $\gamma = 1/\sqrt{1-\overline{v}^2/c^2}$ and $ds = (c/\gamma)dt$ are the infinitesimal interval in the four dimensional space.

The contravariant and the covariant electromagnetic field tensors, where the index $i = 0, 1, 2, 3$ labels the rows, and the index $k = 0, 1, 2, 3$ the columns, are given, respectively, by the following relations:

$$F^{ik} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -cB_z & cB_y \\ E_y & cB_z & 0 & -cB_x \\ E_z & -cB_y & cB_x & 0 \end{bmatrix};$$

$$F_{ik} = \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -cB_z & cB_y \\ -E_y & cB_z & 0 & -cB_x \\ -E_z & -cB_y & cB_x & 0 \end{bmatrix},$$

where E_x, E_y, E_z, B_x, B_y, and B_z are the components of the electromagnetic field in the Cartezian system xyz.

(h5) We use the Landau-Lifschitz (LL) form (Landau, 1987) of the radiation reaction force. The equation of motion of the electron, in four-dimensional form, written in the International System (LL use the Gaussian System), is

$$mc^2 \frac{du^i}{ds} = -eF^{ik}u_k + g^i, \tag{10}$$

where e is the absolute value of the electron charge and g^i is the radiation damping correction term, which is given by the contravariant vector (Landau, 1987)

$$g^i = \frac{e^2}{6\pi\varepsilon_0}\left(\frac{d^2u^i}{ds^2} - u^i u^k \frac{d^2u_k}{ds^2}\right). \tag{11}$$

By expressing d^2u^i/ds^2 in terms of the tensor of the external field, namely

$$\frac{du^i}{ds} = -\frac{e}{mc^2}F^{ik}u_k, \tag{12}$$

together with

$$\frac{d^2u^i}{ds^2} = -\frac{e}{mc^2}\frac{\partial F^{ik}}{\partial x^l}u_k u^l + \frac{e^2}{m^2c^4}F^{ik}F_{kl}u^l, \tag{13}$$

and substituting these relations in (11), the following relation results (Landau, 1987):

$$g^i = -\frac{e^3}{6\pi\varepsilon_0 mc^2}\frac{\partial F^{ik}}{\partial x^l}u_k u^l - \frac{e^4}{6\pi\varepsilon_0 m^2c^4}F^{il}F_{kl}u^k + \frac{e^4}{6\pi\varepsilon_0 m^2c^4}\left(F_{kl}u^l\right)\left(F^{km}u_m\right)u^i. \tag{14}$$

This relation is used for many evaluations of radiation corrections (Zhidkov, 2002; Hadad, 2010; Bulanov, 2011). The first terms on the right-hand side of Eqs. (11) and (14) stand for the nonrelativistic expression of the damping force, while the second and, respectively, the second and the third terms on the right-hand side of the same equations, correspond to the relativistic component of the damping force (Landau, 1987).

We observe that Eq. (12) is identical to the motion equation of the electron, written in the tensorial form, when the damping force is very small, compared to the external force. Since (12) has been used to obtain (14), it follows that this last equation is rigorously valid only for the domain the intensity values I_L of the laser beam for which the damping force, denoted by $\overline{F}_d$, is very small compared to the external electromagnetic force, denoted by $\overline{F}$. We will show that this condition is fulfilled throughout our calculations.

The tensorial relation (10) corresponds to four equations, written in three dimensional space. The three space components, for $i = 1, 2, 3$, are the equations of motion of the electron, while the time component, for $i = 0$, is the equation of the rate of change of the electron energy. The relation (10) corresponds to the following motion equations, which are written in three-dimensional notation.

$$mc\frac{d}{dt}(\gamma\beta_x) = \left(-eE_x - ec\beta_y B_z + ec\beta_z B_y\right) + \frac{1}{\gamma}g^1, \tag{15}$$

$$mc\frac{d}{dt}\left(\gamma\beta_y\right) = \left(-eE_y + ec\beta_x B_z - ec\beta_z B_x\right) + \frac{1}{\gamma}g^2, \tag{16}$$

$$mc\frac{d}{dt}(\gamma\beta_z) = \left(-eE_z - ec\beta_x B_y + ec\beta_y B_x\right) + \frac{1}{\gamma}g^3, \tag{17}$$

It is easy to see that $F_x = (-eE_x - ec\beta_y B_z + ec\beta_z B_y)$ and so on, are the spatial components of the external force, and $F_{dx} = (1/\gamma)g^1$ and so on, are the spatial components of the damping force.

Similarly, Eq. (10) corresponds to the following time component equation. This equation can be written in the following form using three-dimensional notation:

$$\frac{d}{dt}(mc^2\gamma) = -ec\left(\beta_x E_x + \beta_y E_y + \beta_z E_z\right) + \frac{c}{\gamma}g^0, \tag{18}$$

We see that $-ec(\beta_x E_x + \beta_y E_y + \beta_z E_z)$ is the rate of change of the electron energy, due to the interaction with the electromagnetic field, while $(c/\gamma)g^0$ is the rate of change of the damping energy, due to the action of the damping force.

(h6) The integral of the four-force, given by (11), over the world line of the motion of the electron, leads to the following relation of the total momentum ΔP^i (Landau, 1987):

$$\Delta P^i = -\frac{e^2}{6\pi\varepsilon_0}\int \frac{du_k}{ds}\frac{du^k}{ds}dx^i, \tag{19}$$

The temporal term of the contravariant vector ΔP^i, namely ΔP^0, is the damping energy, that is the energy radiated by the particle under the action of the electromagnetic field. This energy is denoted by E_d.

We assume that the radiative effects are negligible when the damping energy is much smaller than the kinetic energy of the electron. Since the damping energy increases continuously, when the time increases from 0 to τ_L, the length of the laser pulse, and the kinetic energy varies periodically in this interval, we assume that the maximum value of the laser beam intensity, denoted by I_{LM}, at which the effect of the damping reaction can be neglected, corresponds to the relation

$$\frac{E_d}{E_{kav}} = 0.1, \tag{20}$$

where E_{kav} is the average value of the kinetic energy.

(h7) The LL approach is based on the assumption that the damping force is smaller than the external force. This assumption leads to two requirements, given by the relations (75.11) and (75.12) from Landau (1987). These relations, written in the International System, are, respectively, $|\overline{E}_L| \ll 6\pi\varepsilon_0 m^2c^4/e^3 = 2.722\times10^{20}\ V/m$ and $\lambda_L \gg e^2/(3\varepsilon_0 mc^2) = 11.8\times10^{-14}\ m$. These conditions are overhelmingly satisfied in our calculations.

Since the LL relations are classical, another condition has to be satisfied. This is the requirement that the Compton relation be satisfied at the classical limit, written in the proper reference system of the electron (Bamber, 1999; Boca, 2009). This condition is

$$C_q = \frac{\hbar\ \omega'_L}{mc^2} \ll 1, \tag{21}$$

where C_q is the coefficient of the second term in the denominator of the Compton relation, ħ is the normalized Planck constant and ω'_L is the angular frequency of the laser field, in the proper system S' of the electron.

We will show that the relation (7) is fulfilled throughout the paper. From this point on, no approximation is made in order to calculate the damping force and damping energy, and their variation with the laser beam parameters.

3. SPACE AND TIME COMPONENTS OF THE DAMPING FORCE AND TOTAL MOMENTUM FOUR-VECTORS

We calculate now the space and time components of the contravariant vectors g^i and ΔP^i, which will be used in this paper. We note that the calculations are simplified because the components of the electromagnetic field depend only of two spatio-temporal coordinates, t and z, due to the formula for η, and the z components of the field are zero. For example, we have:

$$\begin{aligned}\frac{\partial F^{1k}}{\partial x^l}u_k u^l &= \sum_{l=0}^{3}\left[\left(\sum_{k=0}^{3}\frac{\partial F^{1k}}{\partial x^l}u_k\right)u^l\right]\\ &= \left(\frac{\partial F^{10}}{\partial x^0}u_0 + \frac{\partial F^{13}}{\partial x^0}u_3\right)u^0 + \left(\frac{\partial F^{10}}{\partial x^3}u_0 + \frac{\partial F^{13}}{\partial x^3}u_3\right)u^3\\ &= \left(-E_{M1}\sin\eta\frac{\omega_L}{c}\gamma + cB_{M1}\sin\eta\frac{\omega_L}{c}\gamma\beta_z\right)\gamma\\ &\quad + \left(E_{M1}\sin\eta|\overline{k}_L|\gamma - cB_{M1}\sin\eta|\overline{k}_L|\gamma\beta_z\right)\gamma\beta_z.\end{aligned}$$

The space components of the contravariant vector g^i, given by Eq. (11), lead to the following components of the radiation damping force

$$\begin{aligned}F_{dx} &= \frac{e^2\omega_L^2\gamma}{6\pi\varepsilon_0c^2}a_1\sin\eta\left(1-\beta_z\right)^2\\ &\quad - \frac{e^2\omega_L^2\gamma^2}{6\pi\varepsilon_0c^2}(a_1^2\cos^2\eta + a_2^2\sin^2\eta)\beta_x\left(1-\beta_z\right)^2,\end{aligned} \tag{22}$$

$$\begin{aligned}F_{dy} &= -\frac{e^2\omega_L^2\gamma}{6\pi\varepsilon_0c^2}a_2\cos\eta\left(1-\beta_z\right)^2\\ &\quad - \frac{e^2\omega_L^2\gamma^2}{6\pi\varepsilon_0c^2}\left(a_1^2\cos^2\eta + a_2^2\sin^2\eta\right)\beta_y\left(1-\beta_z\right)^2,\end{aligned} \tag{23}$$

$$\begin{aligned}F_{dz} &= \frac{e^2\omega_L^2\gamma}{6\pi\varepsilon_0c^2}\left(a_1\beta_x\sin\eta - a_2\beta_y\cos\eta\right)\left(1-\beta_z\right)\\ &\quad + \frac{e^2\omega_L^2}{6\pi\varepsilon_0c^2}\left(a_1^2\cos^2\eta + a_2^2\sin^2\eta\right)\left[1-\gamma^2\beta_z\left(1-\beta_z\right)\right]\left(1-\beta_z\right),\end{aligned} \tag{24}$$

where we have separated the term due to the relativistic effect, in the second term of the second hand member.

The damping energy E_d is equal to the temporal component of the total momentum ΔP^i, given by (19). It easy to show that ΔP^0, written with the aid of the three-dimensional notation, is given by the following relation.

$$\begin{aligned}E_d &= \Delta P^0\\ &= \frac{e^2\omega_L^2}{6\pi\varepsilon_0c}\int_{t_1}^{t_2}\gamma^2\left(a_1^2\cos^2\eta + a_2^2\sin^2\eta\right)\left(1-\beta_z\right)^2dt,\end{aligned} \tag{25}$$

where t_1 and t_2 are the time moments between which the energy is calculated.

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4. PERIODICITY PROPERTIES IN THE INERTIAL SYSTEM S'

4.1. Periodicity Property of External Force and Kinetic Energy

Our analysis is performed in the inertial system S' in which the initial velocity of the electron is zero. The Cartesian axes in the systems $S(t, x, y, z)$ and $S'(t', x', y', z')$ are parallel, and the system S' moves with velocity $-\left|\bar{V}_0\right|$ along the oz axis. Since our analysis is performed in the S' system, we have to calculate the parameters of the laser field, denoted by $\overline{E}'_L$, $\overline{B}'_L$, $\overline{k}'_L$ and ω'_L, in the S' system.

The contravariant wave vectors have, respectively, the following components in the S and S' systems: (ω_L/c, k_{Lx}, k_{Ly}, k_{Lz}) and (ω'_L/c, $k'_{Lx'}$, $k'_{Ly'}$, $k'_{Lz'}$). Using the Lorentz transformation, given by relations (11.22) in Jackson (1999), we have

$$\frac{\omega'_L}{c} = \frac{\omega_L}{c}\gamma_0\left(1 + |\bar{\beta}_0|\right), \tag{26}$$

$$k'_{Lz'} = \left|\bar{k}'_L\right| = |\bar{k}_L|\gamma_0\left(1 + |\bar{\beta}_0|\right), \tag{27}$$

$$k'_{Lx'} = k_{Lx} = k'_{Ly'} = k_{Ly} = 0, \tag{28}$$

where
$$\frac{\omega'_L}{|\bar{k}_L|} = \frac{\omega_L}{|\bar{k}_L|} = c. \tag{29}$$

Since the scalar product between the four-dimensional wave vector and the space-time four vector is invariant, it follows that the phase of the electromagnetic wave is invariant (Jackson, 1999), and we have

$$\eta = \omega_L t - |\bar{k}_L|z + \eta_i = \omega'_L t' - |\bar{k}'_L|z' + \eta_i = \eta', \tag{30}$$

where $\bar{r}$ and $\bar{r}'$ are the position vectors of the electron in the two systems.

We use equations (11.149) from Jackson (1999), which give the Lorentz transformation of the fields. We write these relations in the International System, and using (4) and (29), we obtain the following expressions for the components of the electromagnetic field in the S' system:

$$\bar{E}'_L = \gamma_0\left(\bar{E}_L + \bar{\beta}_0 \times c\bar{B}_L\right) = \gamma_0\left(1 + |\bar{\beta}_0|\right)\bar{E}_L, \tag{31}$$

$$\bar{B}'_L = \gamma_0\left(\bar{B}_L - \bar{\beta}_0 \times \bar{E}_L/c\right) = \gamma_0\left(1 + |\bar{\beta}_0|\right)\bar{B}_L. \tag{32}$$

By virtue of the relation (12), the external force can be calculated from the following system of equations for the electron motion, written in the S' system:

$$m\frac{d}{dt'}\left(\gamma' v'_{x'}\right) = \gamma_0\left(1 + |\bar{\beta}_0|\right) \times\left(-eE_{M1}\cos\eta + ev'_{z'}B_{M1}\cos\eta\right), \tag{33}$$

$$m\frac{d}{dt'}\left(\gamma' v'_{y'}\right) = \gamma_0\left(1 + |\bar{\beta}_0|\right)\left(-eE_{M2}\sin\eta + ev'_{z'}B_{M2}\sin\eta\right), \tag{34}$$

$$m\frac{d}{dt'}\left(\gamma' v'_{z'}\right) = \gamma_0\left(1 + |\bar{\beta}_0|\right) \times\left(-ev'_{x'}B_{M1}\cos\eta - ev'_{y'}B_{M2}\sin\eta\right), \tag{35}$$

where $v'_{x'}$, $v'_{y'}$ and $v'_{z'}$ are the components of the electron velocity in S' and

$$\gamma' = \left(1 - {\beta'_{x'}}^2 - {\beta'_{y'}}^2 - {\beta'_{z'}}^2\right)^{-\frac{1}{2}}, \tag{36}$$

with $\beta'_{x'} = v'_{x'}/c$, $\beta'_{y'} = v'_{y'}/c$, and $\beta'_{z'} = v'_{z'}/c$.

The solution of the system of equations (33)–(35), which leads to the expressions of $\beta'_{x'}$, $\beta'_{y'}$, γ', $d\gamma'/dt'$ and $d\eta/dt'$, is presented in our previous papers (Popa, 2011; 2012). For the completeness of the presentation, we will give it briefly below.

Using (4) and (26), the equations of motion become

$$\frac{d}{dt'}\left(\gamma'\beta'_{x'}\right) = -a'_1\,\omega'_L\left(1 - \beta'_{z'}\right)\cos\eta, \tag{37}$$

$$\frac{d}{dt'}\left(\gamma'\beta'_{y'}\right) = -a'_2\omega'_L\left(1 - \beta'_{z'}\right)\sin\eta, \tag{38}$$

$$\frac{d}{dt'}\left(\gamma'\beta'_{z'}\right) = -\,\omega'_L\left(a'_1\,\beta'_{x'}\cos\eta + a'_2\,\beta'_{y'}\sin\eta\right), \tag{39}$$

where a'_1 and a'_2 are the relativistic parameters

$$a'_1 = \frac{\gamma_0\left(1 + |\bar{\beta}_0|\right)eE_{M1}}{mc\omega'_L} \text{ and } a'_2 = \frac{\gamma_0\left(1 + |\bar{\beta}_0|\right)eE_{M2}}{mc\omega'_L}. \tag{40}$$

These parameters are relativistic invariants, because, by virtue of (1), (26), (31), and (40), we have

$$a_1 = a'_1 = \frac{eE_{M1}}{mc\omega_L} \text{ and } a_2 = a'_2 = \frac{eE_{M2}}{mc\omega_L}. \tag{41}$$

Note. Despite the fact that a relativistic electron beam interacts with the laser beam in the S system, the electron motion in the S' system can be non-relativistic, if $a_1 \ll 1$ and $a_2 \ll 1$.

The initial conditions in the system S' are the following:

$$t' = 0,\ x' = y' = z' = 0,\ v'_{x'} = v'_{y'} = v'_{z'} = 0 \text{ and } \eta = \eta' = \eta_i. \tag{42}$$

We multiply (37), (38), and (39), respectively, by $\beta'_{x'}$, $\beta'_{y'}$ and $\beta'_{z'}$. Taking into account that $\beta'^2_{x'} + \beta'^2_{y'} + \beta'^2_{z'} = 1 - 1/\gamma'^2$, their sum leads to

$$\frac{d\gamma'}{dt'} = -\omega'_L\left(a_1\beta'_{x'}\cos\eta + a_2\beta'_{y'}\sin\eta\right). \tag{43}$$

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From (39) and (43) we obtain $d(\gamma'\beta_{z'}')/dt' = d\gamma'/dt'$. We integrate this relation with respect to time between 0 and t', taking into account the initial conditions (42), and obtain $\gamma' - 1 = \gamma'\beta'_{z'}$. In virtue of (3), we have

$$1 - \beta'_{z'} = \frac{1}{\omega'_L}\frac{d\eta}{dt'} = \frac{1}{\gamma'}. \tag{44}$$

We integrate (37) with respect to time between 0 and t', taking into account (42) and (44), and obtain $\gamma'\beta_{x'}' = -a_1$-(sin η − sin η_i), or

$$\beta'_{x'} = \frac{f'_1}{\gamma'} \text{ where } f'_1 = -a_1(\sin\eta - \sin\eta_i). \tag{45}$$

Similarly, integrating (38) and taking into account (42) and (44), we obtain

$$\beta'_{y'} = \frac{f'_2}{\gamma'} \text{ where } f'_2 = -a_2(\cos\eta_i - \cos\eta). \tag{46}$$

We substitute the expressions of $\beta'_{x'}$, $\beta'_{y'}$ and $\beta'_{z'}$, respectively, from (45), (46), and (44) into $\beta'^2_{x'} + \beta'^2_{y'} + \beta'^2_{z'} = 1 - 1/\gamma'^2$ and obtain the expression of γ':

$$\gamma' = \frac{1}{2}\left(2 + {f'_1}^2 + {f'_2}^2\right). \tag{47}$$

From (44) we obtain

$$\beta'_{z'} = \frac{f'_3}{\gamma'} \text{ where } f'_3 = \gamma' - 1. \tag{48}$$

With the aid of the above relations, we obtain the components of the external force:

$$F'_{x'} = -mca_1\omega'_L(1 - \beta'_{z'})\cos\eta, \tag{49}$$

$$F'_{y'} = -mca_2\omega'_L(1 - \beta'_{z'})\sin\eta, \tag{50}$$

$$F'_{z'} = -mc\omega'_L\left(a_1\beta'_{x'}\cos\eta + a_2\beta'_{y'}\sin\eta\right). \tag{51}$$

The kinetic energy of the electron is

$$E'_k = mc^2(\gamma' - 1). \tag{52}$$

We observe that, by virtue of Eqs. (45)–(48), γ', $\beta'_{x'}$, $\beta'_{y'}$, and $\beta'_{z'}$ are periodic functions of η. From the relations (49)–(52) it follows that $F'_{x'}$, $F'_{y'}$, $F'_{z'}$ and E'_k are also periodic functions of only one variable, namely η.

4.2. Periodicity Property of the Damping Force and of the Rate of Change of the Damping Energy

The components of the radiation damping force, given by Eqs. (22)–(24), can be written in the system S', as follows:

$$F'_{dx'} = \frac{e^2{\omega'_L}^2\gamma'}{6\pi\varepsilon_0 c^2}a_1\sin\eta(1 - \beta'_{z'})^2 - \frac{e^2{\omega'_L}^2\gamma'^2}{6\pi\varepsilon_0 c^2}(a_1^2\cos^2\eta + a_2^2\sin^2\eta)\beta'_{x'}(1 - \beta'_{z'})^2, \tag{53}$$

$$F'_{dy'} = -\frac{e^2{\omega'_L}^2\gamma'}{6\pi\varepsilon_0 c^2}a_2\cos\eta(1 - \beta'_{z'})^2 - \frac{e^2{\omega'_L}^2\gamma'^2}{6\pi\varepsilon_0 c^2}(a_1^2\cos^2\eta + a_2^2\sin^2\eta)\beta'_{y'}(1 - \beta'_{z'})^2, \tag{54}$$

$$\begin{aligned} F'_{dz'} = {} & \frac{e^2{\omega'_L}^2\gamma'}{6\pi\varepsilon_0 c^2}\left(a_1\beta'_{x'}\sin\eta - a_2\beta'_{y'}\cos\eta\right)(1 - \beta'_{z'}) \\ & + \frac{e^2{\omega'_L}^2}{6\pi\varepsilon_0 c^2}(a_1^2\cos^2\eta + a_2^2\sin^2\eta) \\ & \times\left[1 - \gamma'^2\beta'_{z'}(1 - \beta'_{z'})\right](1 - \beta'_{z'}). \end{aligned} \tag{55}$$

The damping energy, given by Eq. (25), has the following expression in the S' system:

$$E'_d = \frac{e^2\omega'^2_L}{6\pi\varepsilon_0 c}\int_{t'_1}^{t'_2}\gamma'^2(a_1^2\cos^2\eta + a_2^2\sin^2\eta)(1 - \beta'_{z'})^2 dt', \tag{56}$$

where t'_1 and t'_2 are the time moments between which the energy is calculated. By virtue of the initial conditions (42), we have $t'_1 = 0$ and $t'_2 = \tau'_L$. Using the Lorentz relations, it follows that the length of the laser pulse, in the system S', denoted by τ'_L, is given by the following relation:

$$\tau'_L = \frac{\tau_L}{\gamma_0}. \tag{57}$$

The rate of change of the damping energy is

$$R'_{de} = \frac{dE'_d}{dt'} = \frac{e^2\omega'^2_L}{6\pi\varepsilon_0 c}\gamma'^2(a_1^2\cos^2\eta + a_2^2\sin^2\eta)(1 - \beta'_{z'})^2. \tag{58}$$

We have proved above that γ', $\beta'_{x'}$, $\beta'_{y'}$, and $\beta'_{z'}$ are periodic functions of η. It follows that the components of the damping force, namely $F'_{dx'}$, $F'_{dy'}$, $F'_{dz'}$, and the rate R'_{de}, are also periodic functions of only one variable, that is, the phase of the incident field.

Using (44), we change variables $dt' = 1/[\omega'_L(1-\beta'_{z'})]d\eta$, in the integral in Eq. (56), and obtain:

$$E'_d = \frac{e^2\omega'_L}{6\pi\varepsilon_0 c}\int_{\eta_i}^{\eta_i+\Delta\eta} \gamma'^2\left(a_1^2\cos^2\eta + a_2^2\sin^2\eta\right)\left(1-\beta'_{z'}\right)d\eta, \quad (59)$$

where $\Delta\eta$ is the variation of the phase of the electromagnetic field, at the point where the electron is situated, which corresponds to the time variation, equal to the length of the laser pulse. The relation (44) leads to the following equation, which can be used to calculate $\Delta\eta$:

$$\tau'_L = \frac{1}{\omega'_L}\int_{\eta_i}^{\eta_i+\Delta\eta}\gamma' d\eta = \frac{1}{\omega'_L}\int_{\eta_i}^{\eta_i+\Delta\eta}\frac{1}{1-\beta'_{z'}}d\eta. \quad (60)$$

4.3. Connection between the Electron Acceleration and the Decreasing of $\Delta\eta$, at Ultrarelativistic Values of I_L

All the physical quantities involved in our analysis, namely γ', $\overline{\beta}'$, E'_k, $\overline{F}'$, $\overline{F}'_d$ and R'_{de} are functions of only one variable, that is η, the phase of the field at the point in which the electron is situated. It follows that our analysis can be made in the domain of η. Using Eqs. (29), (30), (42), and (44), we can calculate the time t' and the coordinate z' which correspond to a certain value of η, with the aid of the following relations:

$$t' = \frac{1}{\omega'_L}\int_{\eta_i}^{\eta}\gamma' d\eta \text{ and } z' = ct' - \frac{\eta}{|\overline{k}'_L|}. \quad (61)$$

Now we keep constant the values of τ_L and ω_L, and vary I_L starting from values corresponding to the non-relativistic regime, up to values corresponding to the ultrarelativistic regime. All the computations are made using a MATHEMATICA 7 program.

We consider first the non-relativistic regime, when I_L has medium values, on the order 10^{16} W/cm^2. In this case, in Figures 1a, 1b, and 1c we show, respectively, typical variations of $\beta'_{x'}$, $\beta'_{y'}$, and $\beta'_{z'}$, calculated with the aid of the relations (45), (46), and (48). From these figures we see that the motion in the plane xy is dominant, namely $\beta'_{z'}$ is smaller compared to $\beta'_{x'}$ and $\beta'_{y'}$, and $\beta'_{y'} \ll 1$. It follows that, in virtue of (42) and (44), the following relations are valid

$$\frac{d\eta}{dt'} = \omega'_L\left(1-\beta'_{z'}\right) \cong \omega'_L,\ \eta \cong \omega'_L t' + \eta_i \text{ and } \Delta\eta = \omega'_L\tau'_L. \quad (62)$$

Typical variations of $\beta'_{x'}$, $\beta'_{y'}$, and $\beta'_{z'}$, in the relativistic regime, for high values of I_L, on the order 10^{20} W/cm^2, are shown, respectively, in Figures 2a, 2b and 2c. From these figures we see that the motion in the direction of propagation of the laser wave is dominant, namely $\beta'_{x'}$ and $\beta'_{y'}$ are smaller, compared to $\beta'_{z'}$ and $\beta'_{z'}$ is very close to 1. In this case, the ratio $1/(1-\beta'_{z'})$ is very big and it increases more, for bigger values of I_L. From Eq. (60) it follows that $\Delta\eta$ decreases, when $1/(1-\beta'_{z'})$ increases, in order to keep τ'_L constant.

The limit case is when $\beta'_{z'} \cong 1$, namely, when the electron moves almost together with the front of the electromagnetic wave, and the phase of the wave in the point where the electron is situated, remains approximately constant.

We denote by n the integer part of $\Delta\eta/2\pi$, and taking into account the periodicity property of R'_{de}, the expression of the damping energy becomes:

$$E'_d = n\int_{\eta_i}^{\eta_i+2\pi}\frac{R'_{de}}{\omega'_L\left(1-\beta'_{z'}\right)}d\eta + \int_{\eta_i+2n\pi}^{\eta_i+\Delta\eta}\frac{R'_{de}}{\omega'_L\left(1-\beta'_{z'}\right)}d\eta. \quad (63)$$

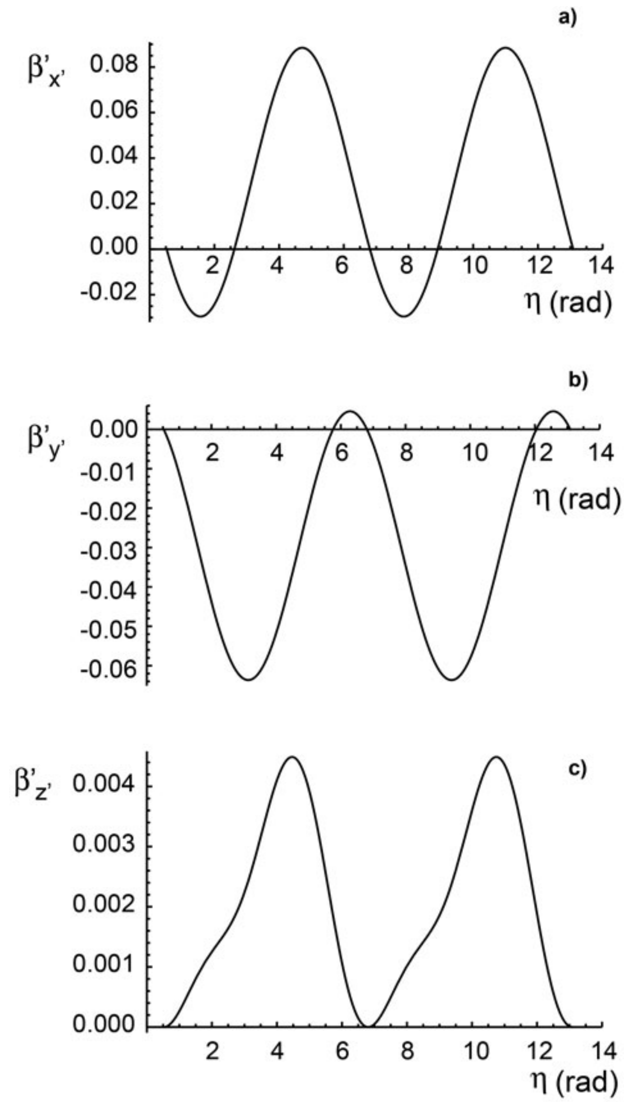

Fig. 1. Typical variations of $\beta'_{x'}$, $\beta'_{y'}$, and $\beta'_{z'}$ when η varies in the interval $[\eta_i, \eta_i + 4\pi]$, in the non-relativistic regime, shown, respectively, in figures (**a**), (**b**), and (**c**). Calculations are made for $I_L = 10^{16}\ W/cm^2$, $\lambda_L = 0.800 \times 10^{-6}\ m$, $\tau_L = 50 \times 10^{-15}\ s$, $P = 0.5$, $E_0 = 10^9\ eV$, and $\eta_i = 30^0$.

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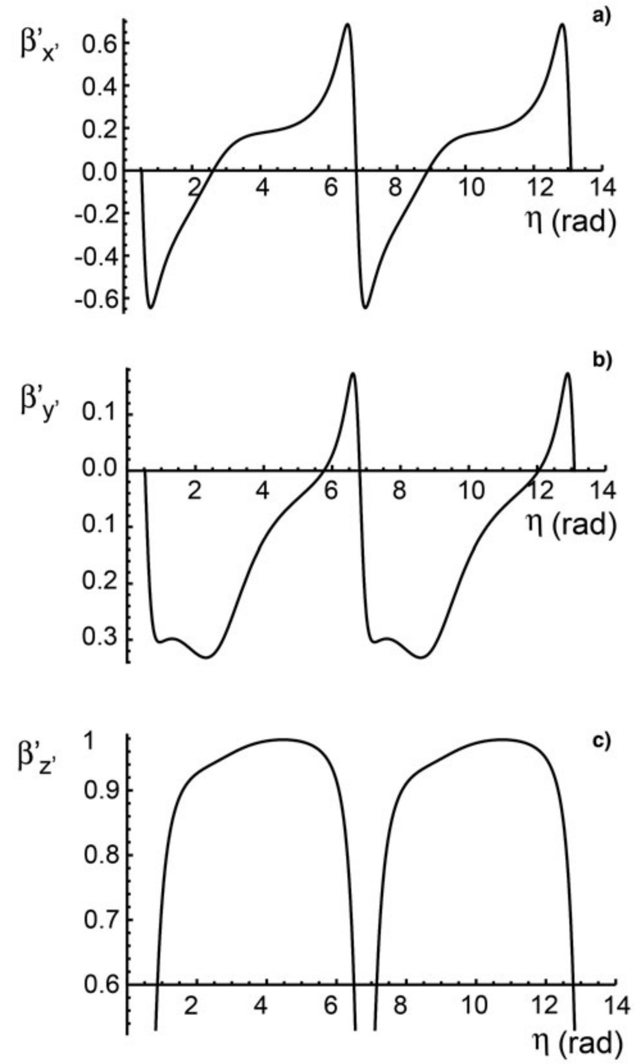

Fig. 2. Typical variations of $\beta'_{x'}$, $\beta'_{y'}$, and $\beta'_{z'}$ when η varies in the interval $[\eta_i, \eta_i + 4\pi]$, in the relativistic regime, shown, respectively, in figures (**a**), (**b**), and (**c**). Calculations are made for $I_L = 10^{20}\ W/cm^2$, $\lambda_L = 0.800 \times 10^{-6}\ m$, $\tau_L = 50 \times 10^{-15}\ s$, $P = 0.5$, $E_0 = 10^9\ eV$, and $\eta_i = 30^0$.

The average value of the kinetic energy is

$$E'_{kav} = \frac{1}{2\pi}\int_{\eta_i}^{\eta_i+2\pi} mc^2(\gamma' - 1)d\eta. \tag{64}$$

We note that, in the ultrarelativistic case, when $\Delta\eta < 2\pi$, the relations (63) and (64) become: $E'_d = \int_{\eta_i}^{\eta_i+\Delta\eta} R'_{de}/\left[\omega'_L\left(1 - \beta'_{z'}\right)\right]d\eta$ and $E'_{kav} = \left(1/\Delta\eta\right)\int_{\eta_i}^{\eta_i+\Delta\eta} mc^2\left(\gamma' - 1\right)d\eta$.

The ratio between the damping energy and average kinetic energy is

$$R_E = \frac{E'_d}{E'_{kav}}. \tag{65}$$

Since the average components of the external force are sometimes relatively close to zero, in order to compare the external and damping forces, it is more suitable to use the root mean square (Hoehn, 1985 of $\bar{F}'$ and $\bar{F}'_d$. The mean values of these forces, denoted, respectively, by $\left|\bar{F}'_{rms}\right|$ and $\left|\bar{F}'_{drms}\right|$, are as follows:

$$\left|\bar{F}'_{rms}\right| = \left[\frac{1}{2\pi}\int_{\eta_i}^{\eta_i+2\pi}\left(F'^2_{x'} + F'^2_{y'} + F'^2_{z'}\right)d\eta\right]^{1/2}, \tag{66}$$

$$\left|\bar{F}'_{drms}\right| = \left[\frac{1}{2\pi}\int_{\eta_i}^{\eta_i+2\pi}\left(F'^2_{dx'} + F'^2_{dy'} + F'^2_{dz'}\right)d\eta\right]^{1/2}, \tag{67}$$

with the note that, in the ultrarelativistic case, when $\Delta\eta < 2\pi$, these relations become:

$$\left|\bar{F}'_{rms}\right| = \left[\left(1/\Delta\eta\right)\int_{\eta_i}^{\eta_i+\Delta\eta}\left(F'^2_{x'} + F'^2_{y'} + F'^2_{z'}\right)d\eta\right]^{1/2}$$

and

$$\left|\bar{F}'_{drms}\right| = \left[\left(1/\Delta\eta\right)\int_{\eta_i}^{\eta_i+\Delta\eta}\left(F'^2_{dx'} + F'^2_{dy'} + F'^2_{dz'}\right)d\eta\right]^{1/2}.$$

The ratio between the mean values of the damping and external forces is

$$R_F = \frac{\left|\bar{F}'_{drms}\right|}{\left|\bar{F}'_{rms}\right|}. \tag{68}$$

5. CALCULATION OF RADIATION DAMPING PARAMETERS AND COMPARISON WITH RESULTS FROM LITERATURE

We use an algorithm similar to that presented in the papers by Popa (2011; 2012) to calculate the ratio between the damping energy and the average kinetic energy, and the ratio between the damping force and external force. This algorithm has the following stages. In the first stage, we use the initial data, which are I_L, λ_L, τ_L, P, η_i, and E_0, to calculate γ_0, β_0, E_{M1}, E_{M2}, E'_{M1}, E'_{M2}, ω_L, ω'_L, τ'_L, a_1, and a_2. In the second stage, we start with a value for the variable η and calculate successively $f'_1, f'_2, \gamma', f'_3$, $\beta'_{x'}$, $\beta'_{y'}$, $\beta'_{z'}$, R'_{de}, E'_k, $F'_{x'}$, $F'_{y'}$, $F'_{z'}$, $F'_{dx'}$, $F'_{dy'}$, and $F'_{dz'}$. In the third stage, we calculate with the aid of simple software the integrals which lead to $\Delta\eta$, E'_d, E'_{kav}, $\left|\bar{F}'_{rms}\right|$ and $\left|\bar{F}'_{drms}\right|$. Finally, we calculate the ratios R_E and R_F.

In Figure 3, we show typical variations of the ratio R_E with respect to the electron energy E_0, for different values of λ_L and I_L. Similar variations are shown in Figures 4 for the ratio R_F. The variations shown in Figures 3 and 4 are linear. The linearity is explained, in the case of the curves

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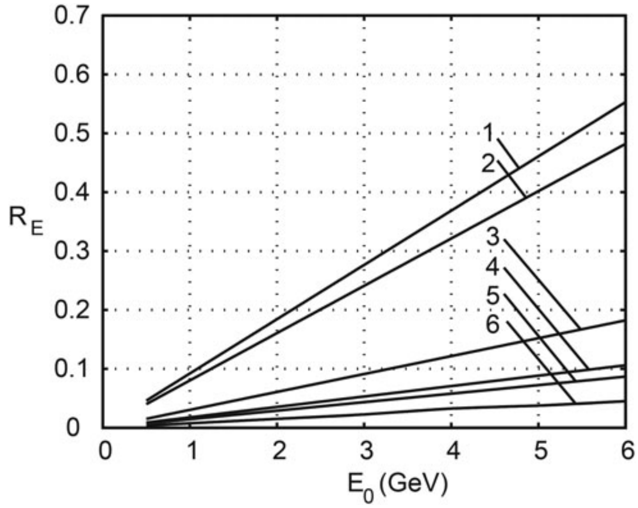

Fig. 3. Variation of the ratio between the damping energy and average kinetic energy, R_E, with initial electron energy, E_0, for different values of I_L and λ_L, for $\tau_L = 50 \times 10^{-15}$ s, $P = 0.5$ and $\eta_i = 30^0$. Curve 1 is for $I_L = 10^{22}$ W/cm^2 and $\lambda_L = 1.064$ μm, curve 2 is for $I_L = 10^{22}$ W/cm^2 and $\lambda_L = 0.800$ μm, curve 3 is for $I_L = 10^{21}$ W/cm^2 and $\lambda_L = 0.800$ μm, curve 4 is for $I_L = 10^{21}$ W/cm^2 and $\lambda_L = 1.064$ μm, curve 5 is for $I_L = 10^{20}$ W/cm^2 and $\lambda_L = 0.800$ μm and curve 6 is for $I_L = 10^{20}$ W/cm^2 and $\lambda_L = 1.064$ μm.

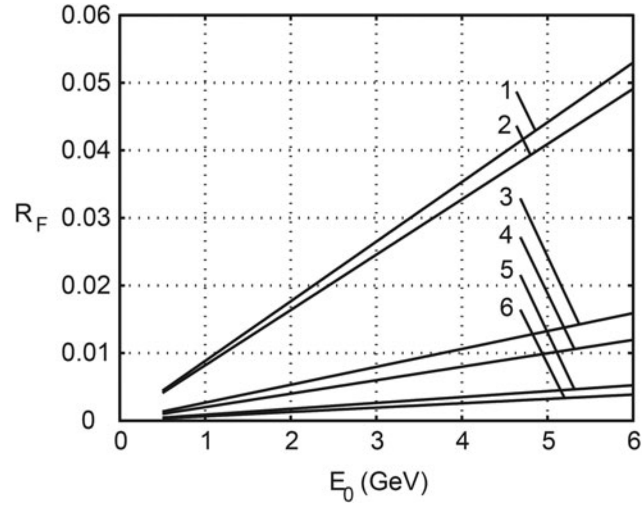

Fig. 4. Variation of the ratio between the mean values of damping force and external force, R_F, with initial electron energy, E_0, for different values of I_L and λ_L, for $\tau_L = 50 \times 10^{-15}$ s, $P = 0.5$ and $\eta_i + 30^0$. Curve 1 is for $I_L = 10^{22}$ W/cm^2 and $\lambda_L = 1.064$ μm, curve 2 is for $I_L = 10^{22\prime}W/cm^2$ and $\lambda_L = 0.800$ μm, curve 3 is for $I_L = 10^{21}$ W/cm^2 and $\lambda_L = 0.800$ μm, curve 4 is for $I_L = 10^{21}$ W/cm^2 and $\lambda_L = 1.064$ μm, curve 5 is for $I_L = 10^{20}$ W/cm^2 and $\lambda_L = 1.064$ μm and curve 6 is for $I_L = 10^{20}$ W/cm^2 and $\lambda_L = 0.800$ μm.

from Figure 3, by the fact that E'_d is proportional to ω'_L, which is, in its turn, proportional to γ_0 and E_0. On the other hand, E'_k does not depend on E_0. In the case of the curves from Figure 4, the linearity is explained by the fact that F'_d is proportional to ω'^2_L, while F' is proportional to ω'_L, resulting that their ratio is proportional to ω'_L, which is, in its turn, proportional to E_0.

The analysis of the curves from Figure 3 shows that, by virtue of relation (20), the radiation reaction effect has to be taken into account for laser beam intensities higher than 10^{21} W/cm^2, and for electron energies higher than 1 GeV, when $R_E = E'_d/E'_{kav} > 0.1$.

The data from Figure 4 show that the LL assumption from hypothesis (h5) is fulfilled for our calculations, namely the damping force is much smaller than the external force.

The condition for the validity of the classical treatment, resulted from Compton's relation and shown by Eq. (21), is also fulfilled by our calculations. In Table 1, we show typical results of the calculations, for relatively high values of laser beam intensity and initial electron energy, for which the C_q term from Compton's relation, is much smaller than unity.

Our results are in agreement with the data presented in the literature. As an example, our results are in agreement with the condition for significant damping, (Thomas, 2012). This condition is $\psi > 1$, where $\psi = 10\sqrt{2\pi^3}\omega_L\gamma_0\tau_0 a_0^2$. In this relation, τ_0 is a specific time, having the value 6.4 $\times 10^{-24}$ s and a_0 is the relativistic parameter, when initial field is linear polarized. This relation is verified by our data from Figure 3. For example, we consider the data from curve 3 of Figure 3, which correspond to the case when $R_E = 0.1216$ and the damping reaction has to be taken into account: $\lambda_L = 0.8$ μm, $I_L = 10^{21}$ W/cm^2, $a_1 = 18.7$ and $E_0 = 4$ GeV. Making the approximation $a_1 \approx a_0$, we obtain $\psi = 3.26$, which verifies the condition of the existence of damping reaction, from the paper (Thomas, 2012).

Also, in the paper by Hadad (2010), it is shown that the radiation reaction dominated regime starts when $\gamma_0 a_0^2 \approx 10^8$ (see Eq. (41) from that paper). This value is in agreement with our results, because in our case the radiation regime is dominating when E_0 is of the order 5 GeV, corresponding to γ_0 of the order 10000 and I_L is of the order 10^{22} W/cm^2, corresponding to a_0 on the order of 100 (see Fig. 3).

Compared to the approaches from the literature, which are very diverse, our treatment is based on the periodicity property proved in the paper, which leads to a solution which does not use any approximation.

Table 1. *Typical variations of* R_E, R_F *and* C_q *with* E_0, *for* $I_L = 10^{22}$ W/cm^2, $\lambda_L = 0.800$ μm, $\tau_L = 50 \times 10^{-15}$ s, $P = 0.5$ *and* $\eta_i = 30^0$.

E_0(GeV)	0.5	1	2	4	6
R_E	4.017×10^{-2}	8.033×10^{-2}	0.1607	0.3213	0.482
R_F	4.093×10^{-3}	8.185×10^{-3}	1.637×10^{-2}	3.274×10^{-2}	4.911×10^{-2}
C_q	5.935×10^{-3}	1.187×10^{-2}	2.374×10^{-2}	4.748×10^{-2}	7.122×10^{-2}

6. CONCLUSIONS

We have presented an accurate calculation of the radiation damping force and damping energy, in interactions between very intense laser beams and relativistic electron beams, in the proper inertial system S' of the electron. We have used the relations of Landau and Lifschitz, without making any approximation, to calculate the radiation reaction parameters. We have found that, for laser beam intensity on the order 10^{22} W/cm^2, and for electron energy greater than 1 GeV, the damping energy is comparable to the average energy of the electron, in the S' system. Our analysis is potentially important, because it shows that the applications of the interactions between laser beams and relativistic electron beams for the generation of very energetic radiations have limitations at the values of I_L and E_0 for which the radiation reaction is significant.

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Supplementary material

To view supplementary material for this article, please visit http://dx.doi.org/10.1017/S026303461400041X.

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