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OPTIMAL TAXATION AND SOCIAL NETWORKS

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We study optimal taxation when jobs are found through a social network. The network determines employment, which workers may influence by engaging in social activities. The network parameters play an important role in determining the economy's employment level and the optimal income tax. The optimal labor income tax depends on both the traditional intensive margin of labor supply and a new extensive margin that depends on the structure of the social network. Social activities that promote social connections are instrumental to acquiring job information; taxation thus discourages both social activities and labor supply, reducing employment. Labor taxes vary positively with labor supply and negatively with employment. When networking is absent, taxes are higher and the economy's employment rate is lower. The optimal capital tax rate is zero, independent of labor market frictions. Social networking reduces job search frictions and is welfare-enhancing.

Keywords: Optimal Taxation, Social Networks, Labor Markets

1. INTRODUCTION

The importance of social networks in labor markets has long been understood. Networking plays a critical role in job search and in improving the quality of the match between firms and workers. Access to information about job opportunities is influenced by social structure and individuals use connections with others (e.g., relatives, friends, acquaintances) to build and maintain information networks.[1] Empirical research indicates that about half of jobs are obtained through networking and the other half are obtained through more formal methods [see Holzer (1988); Montgomery (1991); Gregg and Wadsworth (1996); Addison and Portugal (2001); Topa (2001)].[2] The job network literature has shown that social networks have important implications for the dynamics of employment, as well as the

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duration and persistence of unemployment [Calvó-Armengol and Jackson (2004)]. To the extent that networks can affect economic outcomes, the relevance of social networks for the design of government policies must be recognized and explored.

The literature on optimal income taxation, however, has neglected the role of social networks in the labor market and has mainly focused on competitive or search labor markets. Well-known results in the theory of optimal labor taxation are that tax rates on labor should be roughly constant [Barro (1979); Kyndland and Prescott (1980); Chari and Kehoe (1999)] and labor taxes should vary positively with employment [Zhu (1992); Scott (2007)]. Jones et al. (1997) show that optimal capital and labor income taxes are zero in the long run, in an economy where labor services are a combination of human capital and workers' time. Basu and Renström (2007) study the optimal dynamic taxation when labor supply is indivisible. The optimal labor tax is generally positive, except for some special cases when leisure is non-normal and the government can use debt as a policy instrument in addition to its tax instruments. They show that optimal paths of the labor tax differ between divisible- and indivisible-labor economies [see also Renström (1999)]. Regarding the optimal capital tax rate, the Chamley (1986)–Judd (1985) result of optimal zero capital tax is, in general, verified.

In this paper, we examine how different network structures affect the structure of optimal taxation. We study optimal tax policy in an economy where the informational structure of the job market follows the job offer model of Calvó-Armengol and Jackson (2004), adapted to large, complex networks, surveyed in Vega-Redondo (2007). Each agent is connected to others through a social network. Information about job opportunities arrives randomly; if an agent is unemployed, she will take the job. On the other hand, if the agent is already employed, then she may pass job information along to a friend, relative, or acquaintance. The strength of social ties among workers determines the probability that their peers pass job information along. Unemployment results when individuals are unsuccessful in hearing about job opportunities either directly or through their peers in a network, or when jobs exogenously break up.

This paper embeds the job network model into the general equilibrium framework, and the design of optimal tax policy follows the Ramsey approach [Lucas and Stokey (1983); Chari et al. (1991)]. We consider an economy with a representative infinitely lived household. Each household consists of a continuum of family members, who either work or are unemployed. Employed workers receive a wage that is determined competitively, whereas agents without a job receive an unemployment benefit. Unemployed workers do not search for a job but rather learn about job opportunities through peers in their social network.

Workers are endowed with these peers exogenously. The rate at which job information is passed from employed workers to his unemployed peers in any period depends on how much time agents spent socializing in the previous period. This "socializing effort" intensity represents an additional trade-off for the agents: it improves their chances of becoming employed but at a cost of leisure. Our model includes both intensive and extensive margins: employed family members

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decide how much they work (intensive), whereas family members without a job can engage in social activities that develop their social connections, increasing the strength of their ties to peers and, ultimately, influencing their chances of finding a job (extensive).

The labor market studied in this paper is analytically simple and allows us to calculate well the long-run average behavior of arbitrary networks. In particular, we are able to analyze power-law networks that exhibit the "small-worlds" properties of low average distances between agents—the "six degrees of separation" phenomenon—and dense "clusters" of connections. These properties have been identified by Watts (1999a, 1999b) as crucial components of empirical social networks, and have been found in many important contexts, including scientific collaboration [see Jackson (2008) for a survey], long-distance telephone calls received [Aiello et al. (2000)], e-mail contact networks [Ebel et al. (2002)], and the sizes of e-mail address books at a large university [Newman et al. (2002)].

We consider several different classes of networks and characterize their effect on optimal government policies. The introduction of labor market frictions through job networks implies that the optimal tax policy should feature some response to unemployment and, consequently, to the dynamics of labor networks. The economy's equilibrium unemployment rate is determined by the structure and properties of the job network. In our model, there are no frictions between workers and firms. Frictions arise solely from information transmission among workers, and firms have no active role in labor market search and cannot affect the employment rate. We show that regardless of the structure of the social network and the dynamics of the labor market, the optimal limiting capital tax rate is zero [Judd (1985); Chamley (1986)].

Regarding the labor income tax, the role of labor supply intensity versus participation in the labor market is key, and the optimal labor income tax responds differently to each of these two margins. Taxation discourages labor supply (the intensive margin) as well as those social activities that are instrumental to acquiring job information. And consequently, it reduces the employment rate (the extensive margin). Comparing numerical results across network structures, we observe that the higher the labor supply, the higher the optimal labor tax, and the labor income tax is decreasing in the economy's employment rate. The extensive effect is small in geometric and power-law networks, so the optimal tax is high in these networks. In the power-law network, the network with the lowest employment rate, the optimal tax is higher. In these networks, labor supply is highest and labor market participation is less responsive to effort than in any other network. The important element here is the ability agents have to influence their participation in the labor market and manipulate the extensive margin as well as the intensive margin. We also observe that social networking is welfare-enhancing in this sense, and it can be interpreted as a technology that uses time as its only input and reduces search frictions.

Finally, our approach differs from traditional search models of labor markets, and previous attempts to introduce networks into labor markets, in important ways. We assume that workers can only indirectly influence the rate at which they find each other; much of the labor market is exogenous.[3] We show that with endogenous link strength, complex networks are very different from regular networks. This highlights the role of social structure, rather than search, in the labor market.

The paper proceeds as follows. In Section 2, we characterize the labor market dynamics governed by social networks and exogenous job separation. In Section 3, we define the economy, the social planner's problem, and the steady-state behavior of the labor market network. We discuss how the unemployment rate is affected by social activities and the properties of the social network and derive the optimal labor tax. Section 4 presents a numerical exercise and Section 5 offers concluding comments.

2. DEMOGRAPHY, NETWORK STRUCTURE, AND EMPLOYMENT RATE

There is a continuum of infinitely lived agents whose total measure is normalized to one. The economy is populated by agents in a representative household who consume, save, and work and are connected to one another in a social network. Each agent is either employed or unemployed. Time evolves in discrete periods indexed by t, and information about job opportunities arrives randomly and depends on a fundamental job arrival process and each agent's position in the social network. All jobs are identical, and employed workers receive a wage that is determined competitively, whereas agents without a job receive an unemployment benefit. The labor market is mediated by the social network, and social ties between workers facilitate the transmission of job information.

Each agent hears about a job opening with probability $\gamma \in [0, 1]$. If the agent is unemployed, she will take the job. On the other hand, if the agent is already employed, then she may pass this information along to a friend, relative, or acquaintance. The rate at which an employed worker passes information to each of her unemployed peers is determined by the number of peers she has, z, the job arrival rate, γ, and a parameter $v \in [0, 1]$ that measures the strength of social ties. Let ρ be the exogenous job separation rate, which is independent across agents.

Each agent may have peers to whom she passes job information when employed, and from whom she may receive job information when unemployed. These peers are connected to one another in a social network. A *network* is described by a symmetric matrix M, where $m_{ij} \in \{0, 1\}$ denotes whether a link exists between agents i and j. That is, $m_{ij} = 1$ indicates that i and j know each other and $m_{ij} = 0$ otherwise. We assume that $m_{ij} = m_{ji}$, meaning that the relationship between i and j is reciprocal. The structure of this network m will determine how information flows through the network, and will have a large impact on each agent's employment status.

We are concerned with *large* networks, that is, a network among the continuum of agents, so that there are infinitely many nodes in this network. We identify each network by its *degree distribution* $\{D_z\}_{z=1}^{\infty}$, where D_z is the proportion of agents who have z peers. A network's degree distribution summarizes much of its structure: whether there are some workers with many links, or not, and the relative prevalence of highly connected workers.[4] In general, there may be many networks consistent with a particular degree distribution $\{D_z\}_{z=1}^{\infty}$. In large networks, much local information ceases to matter, so focusing on degree distributions is appropriate [Vega-Redondo (2007)]. In other words, we are not concerned about detailed local interactions but rather focus on large classes of networks sharing the same degree distribution. We may think of the actual network m as being a random draw from the set of networks having degree distribution $\{D_z\}_{z=1}^{\infty}$. This is called the *random network* approach.

To analyze the dynamics of employment, we apply the mean field approach to modeling complex systems, which assumes that there are no correlations or neighborhood effects in information transmission. Essentially, how many peers a worker has does not predict how many peers their own peers have nor how likely those peers are to be employed. These peers are heterogenous in their employment status, and the employment rate will differ among agents with different numbers of peers. The average employment rate in the household can be expressed as follows:

$$n_t = \int_{z=1}^{\infty} (n_{zt} D_z)\, dz,$$

where n_{zt} is the employment rate at time t among agents with z links. Agents who have more links can expect to hear about jobs from their peers more often, and their employment status will evolve differently than that of an unemployed agent with fewer links.

Household members without a job can spend time in social activities, which develops their social connections, increasing the strength of their ties to their peers. Stronger ties result in more job information from their employed peers, and will improve their chances of finding a job while unemployed. The structure of the network is exogenous, and we assume that the time devoted to social networking affects only a job transmission parameter v. That is, the rate at which job information is passed from employed workers to their unemployed peers in time t depends on how much effort (e_{t-1}) agents spent on social activities in the previous period; i.e., $v = v(e_{t-1})$. This introduces an additional trade-off for the agents, as they can improve their chances of becoming employed in the future, but at a cost in terms of current leisure.[5] The time spent on networking does not depend on the number of links an agent has. However, the rate at which an employed worker transmits job information to her unemployed peers will be declining in her number of peers.

The job transmission rate is determined according to the following decreasing-returns-to-scale technology:

$$v(e_{t-1}) = e_{t-1}^{1-\lambda},$$

where λ measures the efficacy of this technology. When λ is close to 1, workers are able to build strong relationships with relatively little (leisure) cost. When λ is close to 0, maintaining social relationships is more difficult, and requires a greater investment of time. Once a worker finds a job, he is beyond the social network dynamics and will devote no effort to improving his social contacts. Viewing v as a function of the investment in social ties implies that the entire long-run level of employment in the economy is also a function of it; i.e., $n_t = n(e_{t-1})$.

Our approach amounts to assuming that the *average* state of the network is replicated *locally*, so that the proportion of an agent's peers who are unemployed depends only on the fundamentals of the job process and the network structure [Vega-Redondo (2007)].[6] We do *not* assume that all agents have the same employment rate, or that there is no heterogeneity among agents; rather, we assume that *among each agent's peers*, there is no correlation in employment states, so that the probability that one peer is employed is independent of the probability that another is, and depends *only* on that peer's characteristics.

The mean field approach relies on an assumption of *homogenous mixing*, that there are no systemic differences between workers' local neighborhoods, beyond their size z. Although we can imagine that the local structure of a network may be important for how information is transmitted through the network, the degree distribution itself is not enough to determine this structure; many different networks are consistent with a given degree distribution, ranging from highly structured lattices and trees to more random networks with very different local architectures. Averaging over these many possible configurations, however, allows the use of more powerful tools to analyze the average state of the network, and simulations have shown that in the networks we consider, this method's results are often as accurate as those that use detailed local properties on the network. In considering long-run average behavior in these networks, we rely on this assumption and avoid needlessly complicating the model.

Following this approach, and suppressing the subscript t when there is no confusion, we can determine the law of motion for employment for workers with z peers as follows:

$$\dot{n}_z = -\rho n_z + (1 - n_z)\{\gamma + (1 - \gamma)[1 - (1 - \Theta)^z]\}. \tag{1}$$

The change in the level of employment has three main components. First, a fraction ρ of agents who are employed will lose their jobs. Second, a fraction γ of the unemployed agents will hear of a job themselves. Third, unemployed workers who do not hear of a job opportunity themselves are passed job information from each peer with probability Θ; the probability that at least one of their peers passes them a job is therefore $1 - (1 - \Theta)^z$.

We calculate Θ as follows. The probability that a given agent has s peers is $\psi_s = (sD_s)/\langle z\rangle$, where $\langle z\rangle = \int_{z=1}^{\infty}(zD_z)dz$ is the *average degree* in the network. Note that $\psi_s \neq D_s$; i.e., the probability that one of your peers has s links is not equal to the proportion of the population that has s links. This is because agents with many peers, and a large s, are disproportionately likely to be your peers, so we must scale D_s by $s/\langle z\rangle$. This gives the probability that a peer with s links passes you a job. The employment rate among those with s peers is n_s; these agents hear about a job with probability γ, and pass it on with probability (v/s). That is, the greater v is, the more likely this job is to be passed, but the greater s, the more competition for this job there is. Here we assume that workers do not know the employment status of their peers; a worker with s peers may pass it to any one of them. This leads to a potential inefficiency not present in the Calvó-Armengol and Jackson (2004) model of job transmission: workers may wastefully pass job information to a peer who already has a job. We believe our approach is more natural in an infinite network setting. It amounts to assuming that only a single contact between peers occurs for each piece of job information. The alternative assumption, that the worker learns each of her peers' status before passing any job information along, implies a large implicit cost of job information transmission. Rather than model the cost of learning about peers, we assume this information is not available.[7]

We interpret v as a "socializing" parameter, representing how strongly tied to one another peers in this network are. Integrating over all possible s, the probability a worker is passed job information from a peer is therefore

$$\begin{aligned}\Theta &= \int_{s=1}^{\infty} \gamma n_s \frac{v}{s} \psi_s ds \\ &= \gamma v \int_{s=1}^{\infty} n_s \frac{1}{s} \left(\frac{s}{\langle z \rangle} D_s \right) ds \\ &= \gamma v \frac{1}{\langle z \rangle} \int_{s=1}^{\infty} n_s D_s ds \\ &= \gamma v \frac{1}{\langle z \rangle} n = \frac{\gamma v n}{\langle z \rangle}.\end{aligned}$$

In words, the probability that one of your peers passes you job information depends on the job arrival rate, the socializing parameter v, the average unemployment rate n, and the average degree in the network $\langle z \rangle$. Hence, the average employment rate in the economy n_t will depend on each of these variables, as well as on the job separation rate ρ, the job arrival rate γ, the network D_z, and networking effort e.

3. OPTIMAL TAXES AND NETWORK EFFORT

3.1. The Economy

In a typical household there are a measure n_t of employed family members and a measure $1 - n_t$ of unemployed family members. Employed members supply labor hours l_t and unemployed family members can spend time e_t in social activities, which develops their social connections, increasing the strength of their ties to their peers.

Preferences of the household are represented by the utility function

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, h_t), \tag{2}$$

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where the instantaneous utility function u is increasing, concave, and differentiable and β is the discount rate, which lies in (0, 1). The variable c_t is family consumption and the time endowment is normalized to 1, so that average leisure in the household is $h_t = 1 - n_t l_t - (1 - n_t)e_t$. Here we implicitly assume that there is consumption sharing within the family, and the household cares only about average leisure.

The timing of the model is as follows. At the beginning of each period, employed family members—those that started the period with a job and those that just heard about and got a job—choose l. Family members without a job choose network effort e.[8] Then goods consumption c is determined. Households have two options with output they do not consume: they can invest in capital (k) or purchase government bonds (B). Next, employed family members are paid a wage (w) for the labor services, the unemployed receives unemployment benefits (b), and the family receives (tax-free) interest (R) earnings on bonds and rental rate (r) of capital. The household takes as given government-determined tax rates on labor (τ^l) and capital (τ^k) income. As in Calvó-Armengol and Jackson (2004), we interpret the timing as one where job break-up occurs, essentially, at the beginning of the period.

The sequence of real budget constraints reads as follows:

$$\begin{aligned} c_t + k_{t+1} + B_{t+1} &= n_t \left(1 - \tau_t^l\right) w_t l_t + (1 - n_t) b_t \\ &+ \left[1 + (1 - \tau_t^k)(r_t - \delta)\right] k_t + B_t R_t, \end{aligned} \quad (3)$$

where δ is the rate at which capital depreciates each period, r_t is the real rate of return on capital, and $[1 + (1 - \tau_t^k)(r_t - \delta)]$ is the gross return on capital after taxes. Total household income is divided evenly among all individuals, so that family members perfectly insure each other against variation in labor income. Or, alternatively, we can assume that agents can insure themselves against earnings uncertainty and, for this reason, wage earnings are interpreted as net of insurance costs [Merz (1995); Andolfatto (1996); Faia (2008)]. Employed and unemployed family members consume the same amount and capital allocation and bonds purchase is a family decision.

Firms produce a single good and maximize profit, taking factor prices as given. Production technology is a constant-returns-to-scale Cobb–Douglas specification, so that output y is

$$y_t = F(k_t, l_t) = (k_t)^{\alpha} (n_t l_t)^{1-\alpha}, \quad (4)$$

where $\alpha \in (0, 1)$ is the capital income share. Firms operate under perfect competition and factors of production are paid their marginal products; i.e., $F_k(t) = r_t = \alpha({n_t}^{1-\alpha}) k_t^{\alpha-1} l_t^{1-\alpha}$ and $F_l(t) = w_t = (1 - \alpha)({n_t}^{1-\alpha})(k_t)^{\alpha} l_t^{-\alpha}$, where $F_k(t)$ and $F_l(t)$ denote the marginal product of capital and labor, respectively, and w_t the wage rate for labor. Differently than models of search that allow firms to influence the number of workers to be hired through vacancies posted, we assume that the number of workers employed is not a choice variable of the firms.

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The government faces the budget constraint

$$g_t + (1-n_t)b_t = n_t \tau_t^l w_t l_t + \tau_t^k (r_t - \delta)k_t + B_{t+1} - B_t R_t, \tag{5}$$

where g_t denotes government consumption, which is assumed to be exogenously specified. The government finances its expenditures by levying taxes on labor and capital and issuing government bonds.

The economy as a whole faces the following aggregate resource constraint:

$$c_t + k_{t+1} + g_t = (k_t)^\alpha (n_t l_t)^{1-\alpha} + (1-\delta)k_t. \tag{6}$$

3.2. A Network Competitive Equilibrium

A representative household, taking prices, taxes, and the social network structure as given, chooses $\{c_t, k_{t+1}, l_t, e_t, B_{t+1}\}$ to solve

$$\max_{\{c_t, k_{t+1}, l_t, e_t, B_{t+1}\}} \sum_{t=0}^{\infty} \beta^t u(c_t, h_t), \tag{P.1}$$

subject to (3) and (1), with k_0, B_0, and n_0 given. Let $u(t) = u(c_t, h_t)$ and likewise $u_i(t) = u_i(c_t, h_t)$, where $i = 1$ for consumption and $i = 2$ for leisure. The equilibrium conditions for an interior solution of the family's maximization problem are given by

$$u_2(t) = u_1(t)(1-\tau_t^l)w_t, \tag{7}$$

$$R_t = [1 + (1-\tau_t^k)(r_t - \delta)], \tag{8}$$

$$(1-n_t)u_2(t) = \beta n_t' \left\{ u_1(t+1)\left[\left(1-\tau_{t+1}^l\right) w_{t+1} l_{t+1} - b_{t+1}\right] - u_2(t+1)(l_{t+1} - e_{t+1}) \right\}, \tag{9}$$

where $n_t' = \partial n(e_{t-1})/\partial e_{t-1}$. Equation (7) is the standard equation showing how the income labor tax affects the labor–leisure choice and equation (8) is the no-arbitrage condition for capital and bonds. Equation (9) states that the utility cost of socializing effort (LHS) equals the discounted (expected) gain from successfully finding a job, where the gain of one additional worker equals the additional consumption gain in period $t+1$ less the leisure cost of working and not spending time in social networking.

DEFINITION 1. *A network competitive equilibrium is a policy* $\Upsilon = \{\tau_t^l, \tau_t^k\}_{t=0}^{\infty}$, *government spending* $\bar{G} = \{g_t, b_t\}_{t=0}^{\infty}$, *household's allocations* $\hat{x} = \{c_t, k_{t+1}, l_t, e_t, B_{t+1}\}_{t=0}^{\infty}$, *a price system* $\hat{P} = \{w_t, r_t, R_t\}_{t=0}^{\infty}$, *and the state of the network variables* $\{\rho, \gamma, \upsilon, D_z\}$ *such that given the policy, government spending, the price system, and the state of the network, the resulting household's allocation choice maximizes the consumer's utility and satisfies the government's budget constraint, the economy's resource constraint, and market clearing conditions.*

3.3. Ramsey Equilibrium

At the beginning of each period, the government announces its program of tax rates and individuals behave competitively.[9] The objective of the social planner is to choose values of its fiscal instruments such that the agent's utility is maximized. The Ramsey problem is a programming problem of finding the optimum within a set of allocations that can be implemented as a competitive equilibrium with distortionary taxes. In other words, the Ramsey problem is to choose a process for tax rates $\{\tau_t^l, \tau_t^k\}$ that maximizes social welfare and satisfies (3) and an implementability constraint [see Chari and Kehoe (1999)], a second-best optimal tax problem. In this paper the unemployment benefit is exogenously given and the planner does not choose it optimally.

DEFINITION 2. *A Ramsey equilibrium is a policy $\digamma$, an allocation rule $\hat{x}$, and a price rule $\hat{P}$ that satisfy the following two conditions:* (i) *the policy $\digamma$ maximizes* (2) *subject to the government budget constraint* (5) *and the state of the network* $\{\rho, \gamma, \upsilon, D_z\}$ *with allocations and prices given by $\hat{x}$ and $\hat{P}$ and* (ii) *for every $\digamma'$, the allocation $\hat{x}(\digamma')$, the price rule $\hat{P}(\digamma')$, and the policy $\digamma'$ constitute a network competitive equilibrium.*

The fact that agents can spend time on social activities leads to two important changes for the formulation of the Ramsey problem with respect to (i) the implementability constraint and (ii) the intertemporal choice of labor supply. The key element here is the relationship between effort in the current period and payment of taxes in the future, in the case where an agent successfully finds a job.

We use the family's first-order conditions and the intertemporal budget constraint—problem (**P.1**)—to derive the following the implementability constraint (see Appendix A.1 for derivation details):

$$\sum_{t=0}^{\infty} \beta^t \left\{ u_1(t)\,(c_t - b_t) - u_2(t) \left[n_t(l_t - e_t) + (1 - n_t)\,\frac{n_{t+1}}{n_t'} \right] \right\} = Z_0, \qquad \textbf{(10)}$$

where $Z_0 = u_1(0)[n(e_{-1})(1 - \tau_0^l)w_0 l_0 - n(e_{-1})b_0 + T_0 k_0 + R_0 B_0] - u_2(0)n(e_{-1})(l_0 - e_0)$.

We first assume that taxes at time zero (τ_0^l, τ_0^k) are given. Next, equations (7) and (8) can be used to compute τ_t^l and τ_t^k, respectively. This process leaves (9) as a condition that must be imposed. Notice that the labor income tax affects both the static choice of labor supply (l_t) and the dynamic choice of networking effort (e_t). Hence, it is necessary to guarantee that, given an allocation, the taxes τ_t^l from (7) and (9) coincide [Jones et al. (1997); Domeij (2005); Reinhorn (2009)]. Imposing this equality is equivalent to requiring $\Phi(t) = 0$, where

$$\Phi(t) = \Phi(c, l, e, b) = (1 - n_t)\,u_2(t) - \beta n_t'\,[u_2(t+1)e_{t+1} - u_1(t+1)b_{t+1}]\,. \qquad \textbf{(11)}$$

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PROPOSITION 1. *The household's allocations and the date-*0 *policy* Υ_0*, in a network competitive equilibrium, satisfy the economy's resource constraint* (6)*, the law of motion for employed workers* (1)*, the implementability constraint* (10)*, and a constraint on labor income taxes, equation* (11)*. Furthermore, given the household's choices and* Υ_0*, prices and policies can be constructed for all dates, which together with the choices and date-*0 *policies constitute a network competitive equilibrium.*

Proof. See Appendix A.2. ■

3.4. Long-Run Employment in Networks and Optimal Taxation

Our analysis focuses on the steady state of the labor marketthat is, when the change in the level of employment for each type of worker is equal to zero, i.e., $\dot{n}_z = 0$ for all z. The number of newly employed agents of each type z is exactly equal to the number of newly unemployed agents, and the economy will remain at this level of employment indefinitely. This is the *long-run prevalence* of employment in the economy.[10]

Setting $\dot{n}_z = 0$ in equation (1), we find that the steady-state level of employment n_z^*, the employment rate for agents with z peers, satisfies

$$n_z^* = 1 - \left(\frac{\rho}{\gamma + \rho - (1-\gamma)\left[1 - \left(1 - \frac{\gamma v n^*}{\langle z \rangle}\right)^z\right]} \right), \tag{12}$$

which is averaged over z to get the average employment rate n^*:

$$n^* = \int_{z=1}^{\infty} (n_z^* D_z) dz. \tag{13}$$

Note that because n_z^* depends on n^*, this equation defines n^* only implicitly. For different degree distributions D_z, the long-run steady-state employment rate, equation (13), will have different solutions, with different characteristics and different implications for optimal taxation.

There are several well-known classes of large, complex networks that we analyze. We first consider *regular* networks where every agent has the same number of peers, k. For these networks, $D_z = 1$ for $z = k$, and $D_z = 0$ for all other z. Each worker is exactly the same, so that $\langle z \rangle = k$. For $k = 0$, this is the *empty* network (our baseline), and may be taken as a worst case scenario, where workers must hear of jobs themselves, at the exogenous arrival rate γ.

Empirical social networks have low average degree $\langle z \rangle$, compared to the size of the network as a whole, but this is coupled with great heterogeneity in degree across agents. For this reason, regular networks are not realistic. We consider two alternative models of large networks with heterogeneous workers: *geometric* and *power-law* degree distributions; these networks differ in how random their links

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are, and how many agents there are with a large number of peers. In the geometric network, links are close to random. Power-law networks are less random and have more agents with many peers. These networks exhibit the "small-worlds" properties identified by Watts (1999a).[11] The link formation processes that give rise to geometric and power-law networks are discussed in Appendix A.3.

3.5. The Ramsey Problem

The planner's maximization problem can thus be written as follows:

$$\max_{\{c_t, k_{t+1}, l_t, e_t\}} \sum_{t=0}^{\infty} \beta^t u[c_t, 1 - n_t l_t - (1 - n_t) e_t], \tag{P.2}$$

subject to (10), (11), (6), (1), and $\bar{g}$, τ_0^l, τ_0^k, k_0, n_0 given.

Introduce the auxiliary function for this problem:

$$\begin{aligned} Z(c_t, l_t, e_t, n_t; \hat{\eta}) &\equiv u[c_t, 1 - n_t l_t - (1 - n_t) e_t] \\ &+ \hat{\eta} \left\{ u_1(t)\,(c_t - b_t) - u_2(t) \left[n_t (l_t - e_t) + (1 - n_t)\, \frac{n_{t+1}}{n_t'} \right] \right\}, \end{aligned}$$

where $\hat{\eta}$ is the Lagrange multiplier on (10). The first-order conditions of the planner's problem are not the same in the first period and subsequent periods, which implies that this Ramsey problem is nonstationary. Because our goal is to study this economy in the steady state, we will focus our attention on the first-order conditions for period 1 and onward. Evaluating them at the steady state, and after some manipulation, we obtain

$$0 = Z_c^* + \mu^* [1 - n(e^*)] u_{21}^* - \varkappa^*, \tag{14}$$

$$0 = -1 + \beta [F_k(k^*, n^* l^*) + (1 - \delta)], \tag{15}$$

$$0 = Z_l^* + \mu^* [1 - n(e^*)] u_{22}^* - \varkappa^* F_l(k^*, n^* l^*), \tag{16}$$

$$\begin{aligned} 0 = Z_e^* &+ \mu^* \left(u_{22}^* \left\{ [1 - n(e^*)]^2 - \beta n'(e^*)(l^* - e^*)[1 - n'(e^*) e^* - n(e^*)] \right\} \right) \\ &+ \beta \mu^* \left\{ u_2^* \left[n''(e^*) e^* + n'(e^*) \right] - u_1^* n''(e^*) b + u_{12}^* n'(e^*)(l^* - e^*) b \right\} \\ &- \beta \varkappa^* F_l(k^*, n^* l^*) n'(e^*) l^*, \end{aligned} \tag{17}$$

where μ, $\varkappa$ are the Lagrange multipliers on the condition (11) and resource constraint (6), respectively and Z_c^*, Z_l^*, and Z_e^* are defined as follows:

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$$Z_c^* = (1+\hat{\eta}^*)u_1^* + \hat{\eta}^*\left(u_{11}^*(c^*-b) - u_{21}^*\left\{n(e^*)(l^*-e^*) + [1-n(e^*)]\frac{n(e^*)}{n'(e^*)}\right\}\right),$$

$$Z_l^* = (1+\hat{\eta}^*)u_2^* + \hat{\eta}^*\left(u_{12}^*(c^*-b) - u_{22}^*\left\{n(e^*)(l^*-e^*) + [1-n(e^*)]\frac{n(e^*)}{n'(e^*)}\right\}\right),$$

$$\begin{aligned} Z_e^* &= (1+\hat{\eta}^*)u_2^*\{[1-n(e^*)] + \beta n'(e^*)(l^*-e^*)\} \\ &- \hat{\eta}^* u_2^*[(1+\beta)n(e^*) + (1-n(e^*)]\frac{n(e^*)n''(e^*)}{[n'(e^*)]^2}) \\ &+ \hat{\eta}^*\Bigg[\{[1-n(e^*)] + \beta n'(e^*)(l^*-e^*)\} \\ &\times \left(u_{12}^*(c^*-b) - u_{22}^*\left\{n(e^*)(l^*-e^*) + [1-n(e^*)]\frac{n(e^*)}{n'(e^*)}\right\}\right)\Bigg]. \end{aligned}$$

In our model, there are no frictions between workers and firms. Frictions arise solely from information transmission among workers, and firms have no active role in labor market search and cannot affect the employment rate.[12] We show that regardless of the structure of the social network and the dynamics of the labor market, the optimal limiting capital tax rate is zero [Judd (1985); Chamley (1986)]. Capital taxation plays no role in reducing labor distortions and a zero tax on capital stimulates investment, raising output and consumption for all households in the long run.

PROPOSITION 2. *If the solution to the Ramsey problem converges to a steady state and the labor market is governed by social networks, then in a steady state, the tax rates are as follows:*

$$\tau^{*k} = 0,$$
$$\tau^{*l} = 1 - [(Z_c^*/u_1^*)/(Z_l^*/u_2^*)].$$

Proof. See Appendix A.4. ■

Recent papers have shown that the introduction of search frictions changes the Chamley–Judd result of zero capital taxation, e.g., Shi and Wen (1999) and Domeij (2005). Regarding the optimal labor income tax, our results are not directly comparable to results obtained in the search environment. The structure of the labor market and its frictions are very different, and we have to be cautious not to draw inappropriate comparisons.

4. NUMERICAL RESULTS

4.1. Parameterization and Solution Method

We now use numerical methods to simulate calibrated versions of our model. We use these results to illustrate our main points and to explore further the relationship between network structures and labor taxation. The model is calibrated so that the steady state is consistent with U.S. data. We assume the instantaneous utility function $U(c_t, h_t) = \ln c_t + \kappa \ln h_t$ and set the weight on leisure, κ, to match initial steady-state labor force participation, $n^*l^* - (1 - n^*)e^* = 0.68$. The time period is one year and we set the discount factor $\beta = 0.96$, which implies a rate of time preference of 4% on an annual basis. Production technology is a constant-returns Cobb–Douglas specification of the form $F(k_t, l_t) = (k_t)^\alpha (n_t l_t)^{1-\alpha}$, and we set $\alpha = 0.30$ and a depreciation rate of 0.04.

Regarding parameters related to the network structure, there are no available estimates in the literature. Our strategy is to choose the job arrival and the job break-up probability so that the steady-state employment rate in the empty network (our baseline), i.e., $n^R_{k=0} = \gamma/(\rho + \gamma)$, is consistent with the annual employment rate in the United States. The exogenous job separation probability ρ is set to 0.15 [Andolfatto (1996)] and the probability for a worker of finding a job γ is set equal to 0.4. We assume that $v(e) = e^{1-\lambda}$, where λ is initially set to 0.95. For what follows, the average number of peers in each network is the same. We consider two models of large complex networks with heterogeneous workers and set $a = 2.25$ and $\nu = 1.284$ for the *power-law* and *geometric* degree distributions, respectively. These numbers are calibrated so that all three networks (*regular*, *power-law,* and *geometric*) have the same average number of peers, i.e., $\langle z \rangle = 5$. We investigate the sensitivity of the results by considering a range of alternative values for the network parameters γ, ρ, and λ.

We calculate the solution of the optimal tax problem for the calibrated version of the model described in the preceding and follow Domeij (2005)'s calibration strategy and solution method. The initial capital and labor tax rates are set to i.e., $\tau_0^k = 0.30$ and $\tau_0^l = 0.30$ [Carey and Tchilingurian (2000)]. Initial government debt, B_0, is assumed to be zero and government purchases, g_t, are such that the steady-state ratio of government purchases to GDP generated by the model with initial policy is of GDP. Unemployment benefits b are also constant and set equal to 0.04. The method used to solve for the Ramsey equilibrium is standard in the literature and we briefly describe it. We assume that the economy converges to a steady state and the Ramsey equilibrium is characterized by a system of nonlinear equations. We solve for the Ramsey equilibrium by adjusting the multiplier $\hat{\eta}$ on the implementability constraint, equation (10), until this constraint is satisfied.

4.2. Network Structure and the Employment Rate

Before turning to the analysis of the labor taxation, it is instructive to consider how properties of the network labor market change with respect to network parameters. When γ increases, this increases the rate at which jobs arrive, and naturally will increase the employment rate n^*. In contrast, when the job separation rate ρ rises, the employment rate will fall. An increase in socializing effort e will increase

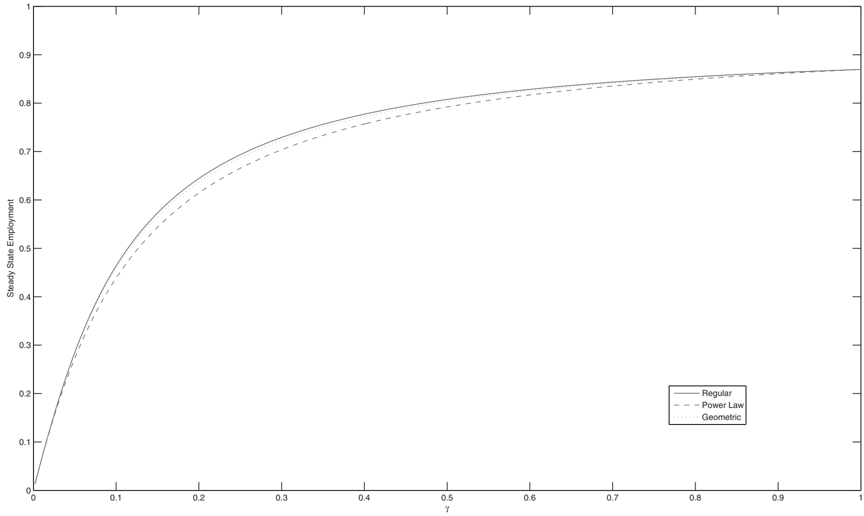

FIGURE 1. Job arrival rate and employment rate.

employment, whereas a rise in the average degree $\langle z \rangle$ increases competition for job information, and will reduce the employment rate. We prove these results formally in the Appendix A.5.

To illustrate how network structure affects the steady state employment rate, we present numerical results for our baseline set of parameters ($\gamma = 0.4$, $\rho = 0.15$, $\lambda = 0.95$, $\langle z \rangle = 5$). In all cases, the regular network has the highest employment rate, followed by the geometric network, the power-law network, and finally, the empty network, a network without any job transmission. This is depicted as γ and ρ vary in Figures 1 and 2. As $\langle z \rangle$ increases, we see that the employment rate in all networks falls, but falls most in the power-law network (Figure 3).

Figure 4 shows that, for each of these networks, the employment rate is increasing in agent's networking effort. The ordering of employment by network is unchanged; the regular network has the highest employment, followed by the geometric and power-law networks.

The differences in optimal tax rates among different networks are driven by differences in the efficacy of socializing effort, $n'(e)$; exerting effort to strengthen social ties is the means by which workers affect the employment rate in this model, and network structure determines how effective this effort is. We see that $n'(e)$ is the largest in the regular network, followed by the geometric network, whereas the power-law network has the least response to socializing effort (Figure 5). This implies that the trade-off between time spent socializing and leisure will be different in each network, with different implications for optimal taxation.

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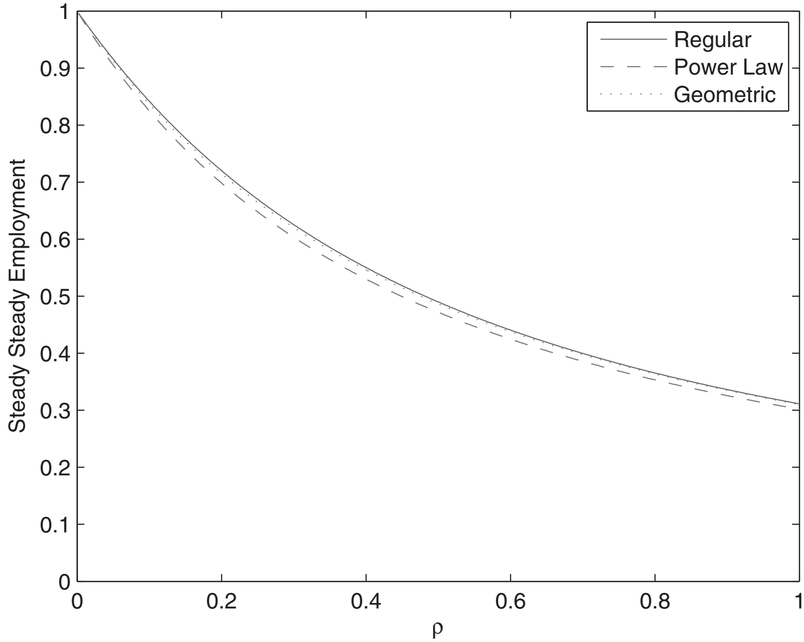

FIGURE 2. Job separation rate and employment rate.

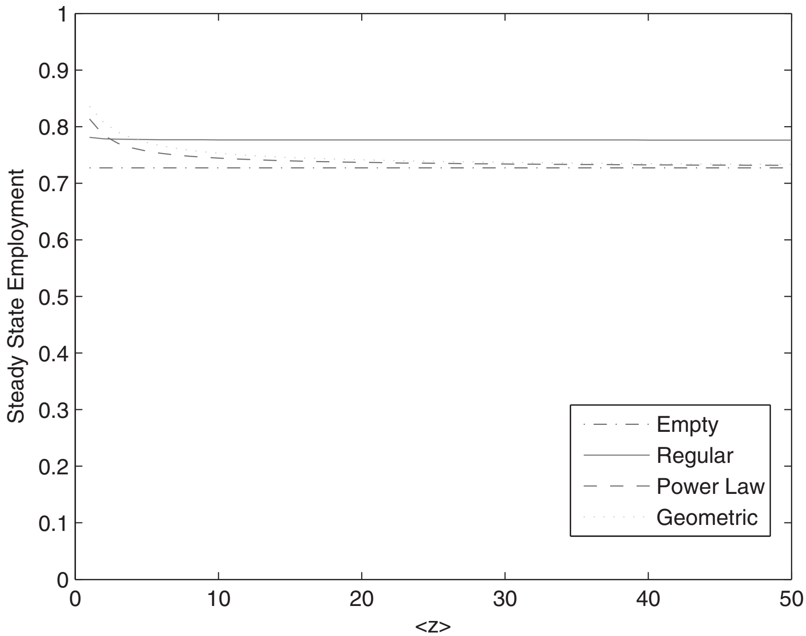

FIGURE 3. Average number of peers and employment rate.

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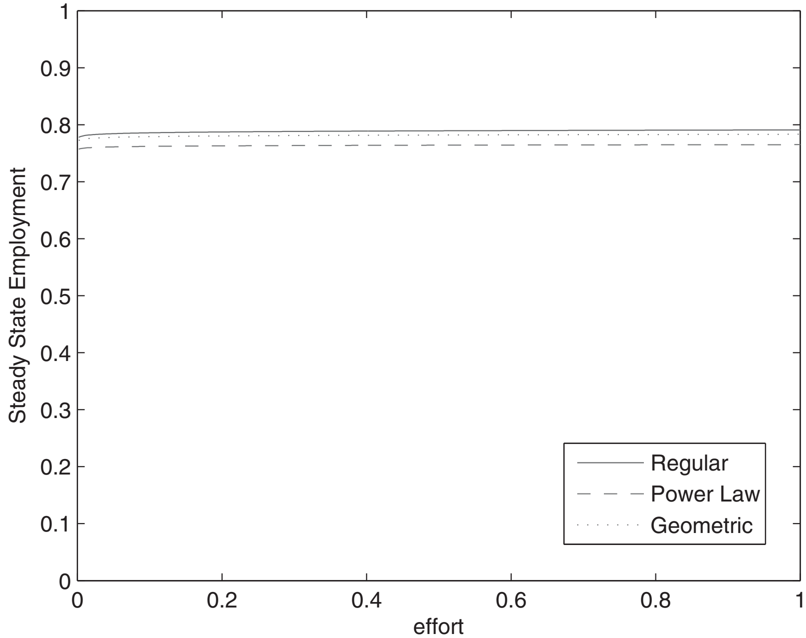

FIGURE 4. Effort and employment rate.

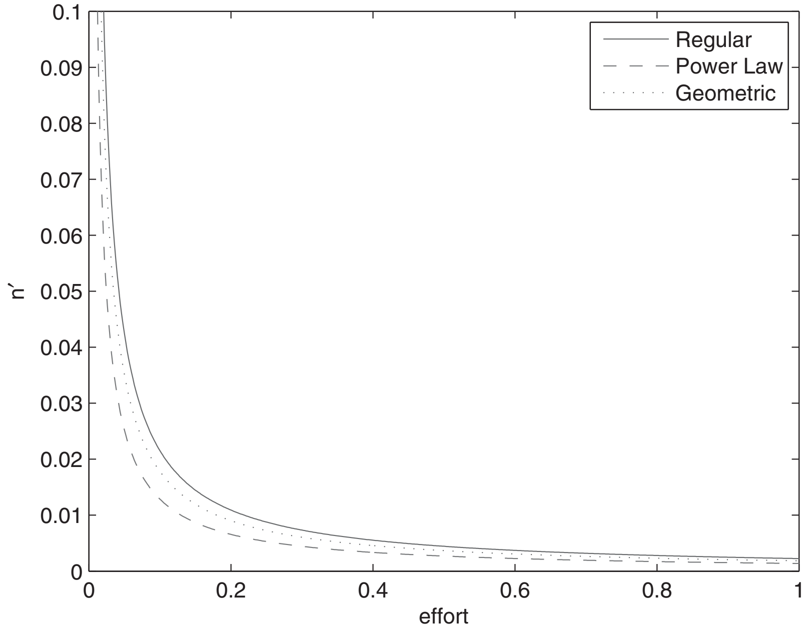

FIGURE 5. Effort and marginal employment rate $n'(e)$.

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TABLE 1. Social network, optimal tax and allocations average number of peers $\langle z \rangle = 5$, $\gamma^a = 0.40$, $\rho^a = 0.15$, $\lambda^a = 0.95$

	Empty	Regular	Geometric	Power-law
Labor income tax τ^{l*}	0.2231	0.1893	0.1897	0.1950
Welfare U^*	−17.2460	−14.8330	−14.8330	−14.7890
Consumption c^*	0.7784	0.8192	0.8196	0.8197
Labor l^*	0.9229	0.8119	0.8184	0.8329
Effort e^*	0.0000	0.0014	0.0011	0.0008
Leisure h^*	0.3288	0.3688	0.3684	0.3700
Employment rate n^*	0.7273	0.7770	0.7715	0.7561

[a]Baseline values.

4.3. Optimal Labor Tax Rates and Social Networks

The results for our benchmark parameterization are presented in Table 1. As discussed in Section 3, in the presence of social networks where unemployment arises because of frictions in information transmission among workers, the optimal capital tax rate is zero and labor tax revenue only finances government expenditures. The employment rate is lower in the empty network and employed family members work more. For other network structures, employed agents tend to work less in the regular network, whereas the family enjoys higher consumption and welfare, though the differences among networks are slight. Taxes are highest in the empty network.

As agents can influence the rate at which they learn about job opportunities, the labor tax rate will influence this decision. In addition to discouraging labor supply (the intensive margin), the tax will discourage social activities that are instrumental to acquiring job information, and reduce the employment rate (the extensive margin). Comparing results across network structures, we observe that the optimal labor income tax responds differently depending on the extensive and intensive margins. First, the higher the labor supply the higher the optimal labor tax. Second, the labor income tax is decreasing in the economy's employment rate.

The extensive effect is small in geometric and power-law networks—that is, the marginal employment rate $n'(e)$ is small in these networks—so the optimal tax is high, and varies negatively with n^*. In contrast, in the regular network $n'(e)$ is bigger. The intensive effect is smallest, whereas n^* is largest. In the power-law network, the network with the lowest employment rate, the optimal tax is the highest. In these networks labor supply is highest, and it is less responsive to effort than in any other network. This result illustrates the importance of the intensive and extensive margins in the determination of our results.

From our numerical exercise, we can also compute a measure of the value of the networks, in terms of welfare. It is clear that social networking is welfare-enhancing. Between the empty network and other network structures, the higher

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TABLE 2. Regular network, optimal tax and allocations average number of Peers $\langle z \rangle = 5$, $\lambda = 0.95^a$

	$\gamma^a = 0.40$ $\rho^a = 0.15$	Arrival rate $\gamma = 0.50$	$\gamma = 0.60$	$\gamma = 0.80$
Labor income tax τ^{l*}	0.1893	0.1800	0.1727	0.1651
Welfare U^*	−14.8330	−14.8900	−14.9580	−15.0010
Consumption c^*	0.8192	0.8196	0.8194	0.8185
Labor l^*	0.8119	0.7848	0.7682	0.7471
Effort e^*	0.0014	0.0010	0.0007	0.0003
Leisure h^*	0.3688	0.3660	0.3639	0.3622
Employment rate n^*	0.7770	0.8071	0.8277	0.8538

[a] Baseline values.

welfare comes from the fact that family members are enjoying more leisure; social networks reduce search frictions. Steady state welfare is higher in the power-law network than in the regular network, despite the higher employment rate in the regular network. This is because, at the low levels of effort that are optimal, the employment rate is more responsive to effort in the power-law networks. That is, $n'(e)$ is smaller in these networks than in the regular network. The household is able to devote less time to networking, and still enjoy a relatively high level of employment. This allows the household to obtain most of its leisure from unemployed family members, and have employed members supply more labor. Even though the optimal tax is higher, consumption and leisure are the highest, leading to higher welfare in the power-law than in the regular network. In other words, employed family members tend to work more, but the participation rate is lower, which requires the government to tax those who have jobs more. Nevertheless, the family's welfare is higher and agents are better off.

Tables 2–5 show how the optimal tax rates and allocations are affected by a variety of parameter changes. Although we could perform sensitivity analyses for all parameters, we restrict our attention to those pertinent to social networks. We present results for the regular network regarding the arrival and break-up probability, efficacy of effort, and average number of peers. The first column of each table shows the results for our benchmark parameterization, to allow comparisons. We then vary one parameter at a time while all others are kept at their benchmark levels. For other networks, results are similar, and are available upon request. As the probability of a worker finding a job increases, the economy's employment rate increases (Table 2). The case of $\gamma = 0.80$ is illustrative. At this rate, agents will become employed almost certainly—and this is actually a worse outcome for them than if γ were smaller. Unemployed members of the household exert very little effort in searching, whereas employed members supply less labor, but there are so many more employed members of the household that average leisure actually falls, and the fall in labor supply leads to a fall in consumption as

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TABLE 3. Regular network, optimal tax and allocations average number of peers $\langle z \rangle = 5$, $\lambda = 0.95^a$

	$\gamma^a = 0.40$ $\rho^a = 0.15$	Separation rate		
		$\rho = 0.10$	$\rho = 0.20$	$\rho = 0.25$
Labor income tax τ^{l*}	0.1893	0.1706	0.2058	0.2207
Welfare U^*	−14.8330	−14.9840	−14.7920	−14.827
Consumption c^*	0.8192	0.8181	0.8169	0.8119
Labor l^*	0.8119	0.7550	0.8696	0.9275
Effort e^*	0.0014	0.0011	0.0016	0.0018
Leisure h^*	0.3688	0.3645	0.3731	0.3777
Employment rate n^*	0.7770	0.8414	0.7204	0.6704

[a]Baseline values.

TABLE 4. Regular network, optimal tax and allocations average number of peers $\langle z \rangle = 5$, $\gamma = 0.40^a$, $\rho = 0.15^a$

	Efficacy of networking effort			
	$\lambda = 0.05$	$\lambda = 0.20$	$\lambda = 0.40$	$\lambda^a = 0.95$
Labor income tax τ^{l*}	0.4451	0.2227	0.2131	0.1893
Welfare U^*	−14.0070	−14.3470	−14.5700	−14.8330
Consumption c^*	0.6699	0.8166	0.8171	0.8192
Labor l^*	0.4571	0.8378	0.8498	0.8119
Effort e^*	5.0×10^{-9}	1.9×10^{-7}	0.0002	0.0014
Leisure h^*	0.6675	0.3906	0.3813	0.3688
Employment rate n^*	0.7272	0.7273	0.7279	0.7770

[a]Baseline values.

TABLE 5. Regular network, optimal tax and allocations baseline values: $\gamma = 0.40^a$, $\rho = 0.15^a$, $\lambda = 0.95^a$

	Average number of peers $\langle z \rangle$			
	$\langle z \rangle^a = 5$	$\langle z \rangle = 3$	$\langle z \rangle = 10$	$\langle z \rangle = 500$
Labor income tax τ^{l*}	0.1893	0.1887	0.1895	0.1895
Welfare U^*	−14.8330	−14.8420	−14.1483	−14.8300
Consumption c^*	0.8192	0.8193	0.8193	0.8194
Labor l^*	0.8119	0.8117	0.8123	0.8128
Effort e^*	0.0014	0.0014	0.0013	0.0013
Leisure h^*	0.3688	0.3685	0.3687	0.3687
Employment rate n^*	0.7770	0.7777	0.7765	0.7761

[a]Baseline values.

well. It would be better in this case if γ were lower, so that the household could specialize, allowing employed members to supply more labor, and unemployed members to enjoy leisure. Note also that because employment is higher, the larger tax base allows the government to reduce its labor income tax.

Compared to the benchmark case, a higher job separation rate ρ has two effects. First, the equilibrium employment rate n falls. This leads employed members of the household to supply more hours in labor. Second, employed members of the household increase their socializing effort, to attempt to increase n. This effort is less effective, however, and cannot compensate for the increased job separation. In equilibrium, leisure rises and consumption falls. With a small tax base and decreased sensitivity of labor hours, the optimal labor tax rises. Regarding the efficacy of the socialization effort, for a small λ, effort is less effective at affecting the transmission probability Θ; there are both income and substitution effects as λ falls. Because effort is less effective, more is required to affect n (income effect). The substitution effect is that time spent in socializing may now be better spent in leisure, if socializing is ineffective. Furthermore, as λ falls, n' falls, along with the tax base, leading to a higher optimal labor tax—the intensive margin effect of taxation decreases. As λ approaches 1, socializing effort becomes more effective, and these effects reverse. At equilibrium, the optimal labor tax is decreasing in λ; more unemployed workers find jobs, increasing the economy's employment rate and the family's consumption. The household experiences less leisure and welfare falls.

Finally, we analyze how the average number of peers affects our results. When $\langle z \rangle$ increases, there are two effects. First, the transmission probability Θ falls, as there are now on the average more competitors for each peer's job information, which will tend to decrease the employment rate. Second, from our baseline $\langle z \rangle = 5$, we observe that with more competition for job information, effort becomes less effective, unemployed workers dedicate less time to social networking, and effort falls. Employed workers work more, as the fraction of the family that is employed decreases and the optimal tax rises.[13]

5. CONCLUSION

This paper studies optimal labor income taxation in the presence of social networks. The unemployment rate is determined by the dynamics of the labor market, which is governed by social networks. The labor income tax is decreasing in the economy's employment rate (extensive margin) and increasing in hours worked (intensive margin). Social networking reduces job search frictions and it is welfare-enhancing; welfare gains are mainly due to the fact that agents can enjoy more leisure. The optimal limiting capital tax rate is zero, independent of the labor market frictions.

This paper also highlights the importance of worker heterogeneity, and the exercise we carry out providesnew insight into the relationship between taxes and labor market dynamics. It should be seen as an illustration of a much broader line of research. A natural extension of our model is to allow firms to have a more active role in the labor market, for instance, receiving information on the productivity of job applicants through current employees.

Another interesting research possibility is to analyze how agents with different numbers of peers choose their social networking effort. Allowing members of the household to exert varying levels of effort would potentially lead to more efficient time allocation and increase employment. This complicates the household problem substantially, however, and would require variational methods to characterize the household behavior in a way that could be integrated into the planner's problem. We pursue these extensions in future research.

NOTES

1. See Granovetter (1995) and Ioannides and Loury (2004) for a recent survey.

2. For instance, according to the National Longitudinal Survey of Youth (NLSY), more than 85% of workers use informal contacts when searching for a job [Holzer (1988)] and more than 50% of all workers found their jobs through their social networks, according to data from the Panel Study of Income Dynamics [Corcoran et al. (1980)]. On the firm side, between 37% and 53% of employers use the social networks of their current employees to advertise jobs, according to data from the National Organizations Survey [Mardsen (2001)] and the Employment Opportunity Pilot Project (EOPP), respectively. According to the EOPP, 36% of firms filled their last openings through a referral [Holzer (1987)].

3. Calvó-Armengol and Zenou (2005) extend the Calvó-Armengol and Jackson (2004) model to provide a micro foundation of the matching function. They allow job search by workers and firms, and explicitly model job information transmission between workers. Their approach differs from the present paper in that they do not consider complex network or workers who are heterogeneous in the number of peers they have, nor do they allow workers to invest in the strength of their social ties. The key network parameter for them is the number of links a worker has; essentially, they restrict their attention to regular networks.

4. Some important network properties, however, are not captured by the degree distribution, such as detailed local structures and clustering. For example, if workers who have a common peer are also likely to be connected themselves, this fact will not be captured by the degree distribution.

5. Although other means by which workers may affect their social network can be imagined, we believe this captures the role networks play in job matching; network formation models capture more the role networks play as an investment in social capital, which is beyond the scope of this paper.

6. This is not true, in general, for many network structures. Calvó-Armengol and Jackson (2004) showed that each worker's employment status is correlated with that of his peers, so an agent who remembers his past status could infer the expected employment rates of his peers, and this need not be equal to the average state of the network.

7. If workers did pass job information only to their unemployed peers, this would reduce the employment rate and increase the effectiveness of socializing effort. We believe it would have the same effect as an increase in λ, which we explore in Table 4. We believe all our of network comparison results would be unchanged.

8. We do not allow workers of various types z to choose different levels of effort. This is firstly because we consider the household to be the decision making unit, not the individual worker. Secondly, because we have a continuum of workers whose impact on job transmission depends on the entire state of the network, allowing effort to vary by z would greatly complicate our analysis, requiring the method of section 3.2 infeasible.

9. We follow the majority of the literature in assuming that the government can commit to follow a long-term program for taxing labor income. We assume that there are institutions that effectively solve the time inconsistency problem so that the government can commit to the tax plan it announces in the initial period.

10. This definition is in terms of the Susceptible–Infected–Susceptible model of Vega-Redondo (2007). That is, an agent has a job, and loses it, but could still get one again.

11. For more discussion of the structural properties of social networks, see Watts (1999a, 1999b) and Jackson (2008).

12. Firms could have a more active role in the labor market, for instance, receiving information on the productivity of job applicants through current employees. Krauth (2004) analyzes a model economy in which networking arises because firms have limited information on the skill of job applicants. A firm's current employees provide information on the job-specific skill of their friends, thus improving the likelihood of a productive match. See also Bramoullé and Saint-Paul (2010).

13. For our parameterization, there is little overall effect as $\langle z \rangle$ becomes very large; effort is already not very effective. This is mainly due to how we model the transmission probability Θ in section 2. If we were to considere an approach where the transmission probability was independent of the number of links, effort would be more effective, although still decreasing in $\langle z \rangle$. Results available upon request.

REFERENCES

Addison, John T. and Pedro Portugal (2002) Job search methods and outcomes. *Oxford Economic Papers* 54(3), 505–533.

Aiello, William, Fan Chung, and Linyuan Lu (2000) A random graph model for massive graphs. *Proceedings of the Thirty-Second Annual ACM Symposium on Theory of Computing, Portland, OR*, pp. 171–180. New York: Association for Computing Machinery.

Andolfatto, D. (1996) Business cycles and labor-market search. *American Economic Review* 86(1), 112–132.

Barro, R. (1979) On the determination of the public debt. *Journal of Political Economy* 87(5), 940–971.

Basu, P. and T. Renström (2007) Optimal dynamic labor taxation. *Macroeconomic Dynamics* 11(5), 567–588.

Bramoullé, Y. and G. Saint-Paul (2010) Social networks and labor market transitions. *Labour Economics* 17(1), 188– 195.

Calvó-Armengol, Antoni and Matthew Jackson (2004) The effects of social networks on employment and inequality. *American Economics Review* 94(3), 426–454.

Calvó-Armengol, Antoni and Yves Zenou (2005) Job matching, social network and word-of-mouth communication. *Journal of Urban Economics* 57(3), 500–522.

Carey, D. and H. Tchilingurian (2000) Average Effective Tax Rates on Capital, Labor and Consumption. Working paper 258, OECD.

Chamley, Christophe (1986) Optimal taxation of capital income in general equilibrium with infinite lives. *Econometrica* 54(3), 607–622.

Chari, V.V., L. Christiano, and P. Kehoe (1991) Optimal fiscal and monetary policy: Some recent results. *Journal of Money, Credit and Banking* 23, 519–539.

Chari V.V. and P. Kehoe (1999) Optimal fiscal and monetary policy. In J.B. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics*, vol. 1C, pp. 1671–1745. Amsterdam: North-Holland.

Corcoran, Mary, Linda Datcher, and Greg Duncan (1980) Information and influence networks in labor markets. In Greg Duncan and James Morgan (eds.), *Five Thousand American Families: Patterns of Economic Progress*, vol. 7, 1–37. Ann Arbor, MI: Institute for Social Research.

Domeij, D. (2005) Optimal capital taxation and labor market search. *Review of Economic Dynamics* 8, 623–650.

Ebel, H., L. I. Mielsch, and S. Bornholdt (2002) Scale-free topology of e-mail networks. *Physical Review E* 66, 035103.

Faia, E. (2008) Optimal monetary policy rules with labor market frictions. *Journal of Economic Dynamic and Control* 32, 1600–1621.

Granovetter, Mark (1995) *Getting a Job: A Study of Contacts and Careers*, 2nd ed. Chicago: University of Chicago Press.

Gregg, P. and J. Wadsworth (1996) It Takes Two: Employment Polarisation in the OECD. Technical report, Centre for Economic Performance, LSE, September.

Holzer, Harry (1987) Hiring Procedures in the Firm: Their Economic Determinants and Outcomes. NBER working paper 2185.

Holzer, Harry J. (1988) Search method use by unemployed youth. *Journal of Labor Economics* 6(1), 1–20.

Ioannides, Yannis and Linda Loury (2004) Job information networks, neighborhood effects, and inequality. *Journal of Economic Literature* 43, 1056–1093.

Jackson, Matthew (2008) *Social and Economic Networks*. Princeton, NJ: Princeton University Press.

Jones, Larry E., Rodolfo E. Manuelli, and Peter E. Rossi (1997) On the optimal taxation of capital income. *Journal of Economic Theory* 73(1), 93–117.

Judd, Kenneth L. (1985) Redistributive taxation in a simple perfect foresight model. *Journal of Public Economics* 28(1), 59–83.

Krauth, B. (2004) A dynamic model of job networking and social influences on employment. *Journal of Economic Dynamic and Control* 28, 1185–1204.

Kyndland, F. and E. Prescott (1980) A competitive theory of fluctuations and the feasibility of stabilization policy. In Stanley Fischer (ed.), *Rational Expectations and Economic Policy*, pp. 169–198. Chicago: University of Chicago Press.

Lucas, R. and N. Stokey (1983) Optimal fiscal and monetary policy in an economy without capital. *Journal of Monetary Economics* 12, 55–93.

Mardsen, Peter V. (2001) Interpersonal ties, social capital, and employer staffing practices. In Nan Lin, Karen Cook, and Ronald S. Burt (eds.), *Social Capital: Theory and Research*, pp. 105–125. New Brunswick, NJ: Transaction Publishers.

Merz, M. (1995) Search in the labor market and the real business cycle. *Journal of Monetary Economics* 36, 269–300.

Montgomery, James D. (1991) Social networks and labor-market outcomes: Toward an economic analysis. *American Economic Review* 81(5), 1408–1418.

Newman, M.E., S. Forrest, and J. Balthrop (2002) Email networks and the spread of computer viruses. *Physical Review E* 66(3), 035101.

Reinhorn, Leslie J. (2009) Dynamic optimal taxation with human capital. *B.E. Journal of Macroeconomics* 9(1) (Topics), Article 38.

Scott, A. (2007) Optimal taxation and OECD labor taxes. *Journal of Monetary Economics* 54, 925–944.

Shi, S. and Q. Wen (1999) Labor market search and the dynamic effects of taxes and subsidies. *Journal of Monetary Economics* 43, 457–495.

Topa, Giorgio (2001) Social interactions, local spillovers and unemployment. *Review of Economic Studies* 68(2), 261–295.

Vega-Redondo, Fernando (2007) *Complex Social Networks,* Econometric Society Monographs 44. New York: Cambridge University Press.

Watts, D.J. (1999a) Networks, dynamics, and the small-world phenomenon. *American Journal of Sociology* 105(2), 493–527.

Watts, D.J. (1999b) *The Dynamics of Networks between Order and Randomness*, Princeton Studies in Complexity. Princeton, NJ: Princeton University Press.

Zhu, X. (1992) Optimal fiscal policy in a stochastic growth model. *Journal of Economic Theory* 58, 250–589.

APPENDIX

A.1. DERIVATION OF THE IMPLEMENTABILITY CONSTRAINT

To derive the implementability constraint, equation (10), first premultiply the family's budget constraint in period t with the associated Lagrangian multiplier $\beta^t \varphi_t$ and sum over all periods $t \geq 0$.

$$\sum_{t=0}^{\infty} \beta^t u_1(t)\left(c_t + k_{t+1} + B_{t+1}\right)$$
$$= \sum_{t=0}^{\infty} \beta^t u_1(t)\left\{n(e_{t-1})\left(1-\tau_t^l\right) w_t l_t + [1-n(e_{t-1})]b_t + T_t k_t + B_t R_t\right\}. \quad \textbf{(A.1)}$$

Using first-order conditions with respect to capital and bonds to eliminate the after-tax return on capital and bonds, we obtain

$$\sum_{t=0}^{\infty} \beta^t u_1(t)\left[c_t\right] = \sum_{t=0}^{\infty} \beta^t u_1(t)\left\{n(e_{t-1})\left(1-\tau_t^l\right) w_t l_t + [1-n(e_{t-1})]b_t\right\} + A_{00}, \quad \textbf{(A.2)}$$

where $A_{00} = u_1(0)[T_0 k_0 + B_0 R_0]$. Multiplying equilibrium equation (9) by $\beta^{t+1} u_1(t+1)$, we get

$$\beta^{t+1} u_1(t+1)\left(1-\tau_{t+1}^l\right) w_{t+1} l_{t+1} = \beta^{t+1} u_1(t+1) b_{t+1} + \beta^{t+1} u_2(t+1)(l_{t+1} - e_{t+1})$$
$$+ \beta^t u_2(t) \frac{[1-n(e_{t-1})]}{n'(e_t)},$$

and then multiplying it by $n(e_t)$ yields

$$\sum_{t=0}^{\infty} \beta^{t+1} n(e_t) u_1(t+1)\left[\left(1-\tau_{t+1}^l\right) w_{t+1} l_{t+1} - b_{t+1}\right]$$
$$= \sum_{t=0}^{\infty} \beta^{t+1} n(e_t) u_2(t+1)(l_{t+1} - e_{t+1})$$
$$+ \sum_{t=0}^{\infty} \beta^{t+1} n(e_t) u_2(t) \frac{[1-n(e_{t-1})]}{n'(e_t)}. \quad \textbf{(A.3)}$$

Notice that the right-hand side of equation (A.2) can be written as

$$u_0(0)\left\{n(-1)\left(1-\tau_0^l\right) w_0 l_0 + [1-n(-1)]b_0\right\}$$
$$+ \sum_{t=0}^{\infty} \beta^{t+1} u_1(t+1) n(e_t)\left[\left(1-\tau_{t+1}^l\right) w_{t+1} l_{t+1} - b_{t+1}\right] + \sum_{t=0}^{\infty} \beta^t u_1(t) b_t.$$

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Substituting (A.3) into (A.2) and after some manipulation, we obtain the implementability constraint for this problem equation,

$$\sum_{t=0}^{\infty} \beta^t \left(u_1(t)\,(c_t - b_t) - u_2(t) \left\{ n(e_{t-1})(l_t - e_t) + [1 - n(e_{t-1})]\, \frac{n(e_t)}{n'(e_t)} \right\} \right) = A_0,$$

where $A_0 = u_1(0)[n(e_{-1})(1 - \tau_0^l) w_0 l_0 - n(e_{-1}) b_0 + T_0 k_0 + R_0 B_0] - u_2(0) n(e_{-1})(l_0 - e_0)$.

A.2. PROOF OF PROPOSITION 1

To show that any allocation that satisfies equations (6), (10), and (11) can be decentralized as a network competitive equilibrium, we use these allocations together with the family's and firm's first-order conditions to construct the corresponding prices and taxes. The rental rate r_t is given by the firm's first-order condition with respect to capital. The capital tax τ_t^k is determined using the family's and firm's first-order condition with respect to capital, and implicitly defined by

$$\frac{u_1(t)}{\beta u_1(t+1)} = \left\{ 1 + \left(1 - \tau_t^k\right) [F_1(t+1) - \delta] \right\}.$$

The wage rate w_t and the labor tax rate τ_t^l are determined by substituting equation (9) into the firm's first-order condition with respect to labor, obtaining

$$\frac{1}{\left(1 - \tau_{t+1}^l\right)} = u_1(t+1) F_2(t+1) l_{t+1} \times \left\{ u_1(t+1) b_{t+1} + u_2(t+1)(l_{t+1} - e_{t+1}) + u_2(t) \frac{[1 - n(e_{t-1})]}{n'(e_t)} \right\}^{-1}. \qquad \textbf{(A.4)}$$

The family's first-order condition with respect to labor for period $t+1$ is

$$\frac{1}{\left(1 - \tau_{t+1}^l\right)} = \frac{u_1(t+1)}{u_2(t+1)} F_2(t+1). \qquad \textbf{(A.5)}$$

Combining (A.4) and (A.5) and rearranging, we obtain

$$\Phi(c_t, l_t, e_t, c_{t+1}, l_{t+1}, e_{t+1}) = [1 - n(e_{t-1})]\, u_2(t) - \beta n'(e_t)\, [u_2(t+1) e_{t+1} - u_1(t+1) b_{t+1}],$$

which is equivalent to equation (11). The labor tax τ_t^l is implicitly defined by both $(A.4)$ and $(A.5)$, and to ensure that the labor taxes implied by these two conditions coincide, the constraint (11) is imposed in the Ramsey problem.

To show that any network competitive equilibrium allocations satisfy equations (6), (1), (10), and (11), we proceed as follows: (a) The resource constraint, equation (6), is implied by the family's and government's period-by-period budget constraints; thus feasibility is satisfied. (b) Premultiply the family's budget constraint in period t by the associated Lagrangian multiplier $\beta^t \varphi_t$ and sum over all periods $t \geq 0$. We proceed by solving for taxes and prices as a function of allocations, using the family's and firm's first-order conditions. This results in the implementability constraint, equation . (c) Because, by definition, the labor tax rate τ^l satisfies both (A.4) and (A.5), the allocations also satisfy the intertemporal constraint on labor taxes, equation (11). ■

A.3. LINK FORMATION PROCESS: GEOMETRIC AND POWER-LAW NETWORKS

We will describe the link formation process that gives rise to geometric and power-law networks. Suppose that agents arrive over time, forming links to those agents who arrived before. Then, among those agents who arrive early, there will be some with many, many links, whereas among agents who arrive later, there will be fewer. The geometric network results from the limit of this process, as $n \to \infty$. These networks are not qualitatively very different from networks in which links are formed entirely at random [Vega-Redondo (2007, p. 67)], and they exhibit relatively few agents with many links. This network has distribution $D_z = \nu^{1-z} \log \nu$, where ν is a parameter that controls the average number of links (among other properties).

The power-law network results from the case where, rather than forming links randomly, agents have a preference for being linked to agents who already have many links. It is easy to imagine why this bias might exist; workers with many links are more likely to be employed, and have access to job information. This network has distribution $D_z = (\alpha - 1)z^{-\alpha}$. Here, there are many more agents who have very many links, the "fat tail" of the power-law distribution. This leads to fundamentally different structural properties of the network than for geometric networks. These networks have a number of attractive features, which matchl many properties of empirical social networks well [Vega-Redondo (2007); Jackson (2008)].

The economy's long-run employment rate for a regular network n^R is equal to the employment rate of the agents with k links; i.e., $n^R = n_k^R$, where n_k^R is the solution of the following expression:

$$n_k^{*R} = 1 - \left(\frac{\rho}{\gamma + \rho - (1-\gamma)\left[1 - \left(1 - \frac{avn^*}{\langle z \rangle}\right)^k\right]} \right). \tag{A.6}$$

For the empty network, $k = 0$, and this expression simplifies to $n_{k=0}^E = \gamma/(\rho + \gamma)$, whereas for the regular network it is the root of a k-degree polynomial. Unfortunately, for the power-law and geometric networks, no analytical solution to equation (13) exists, and it must be characterized numerically.

A.4. PROOF OF PROPOSITION 2

The result of Judd (1985) and Chamley (1986) that capital taxes are zero in the limit follows directly from an evaluation of the first-order conditions of the family's and the Ramsey's problems with respect to capital. Once again, $\tau^{*k} = 0$ regardless of the labor network structure or dynamics. Next, we discuss the result for the labor income taxes.

Notice that the multipliers η, μ, and $\varkappa$ appear in equations (14), (16) and (17). First, we reduce these three expressions to one, where only the Lagrangian multiplier on the implementability constraint, η, appears. Manipulating (14) and (16), we obtain

$$\varkappa^* = Z_c^* + \frac{[Z_l^* - Z_c^* F_l(k^*, n^*l^*)]u_{21}^*}{F_l(k^*, n^*l^*)u_{21}^* - u_{22}^*}, \quad \textbf{(A.7)}$$

$$\mu^* = \frac{Z_l^* - Z_c^* F_l(k^*, n^*l^*)}{[1 - n(e^*)]\left[F_l(k^*, n^*l^*)u_{21}^* - u_{22}^*\right]}. \quad \textbf{(A.8)}$$

Substituting (A.7) and (A.8) into (17), we get

$$\frac{1}{F_l(k^*, n^*l^*)} = \frac{\beta[1 - n(e^*)]n'(e^*)l^*\left(Z_l^* u_{21}^* - Z_c^* u_{22}^*\right) - [1 - n(e^*)]Z_e^* u_{21}^* + Z_c^* \Gamma^*}{Z_l^* \Gamma^* - [1 - n(e^*)]Z_e^* u_{22}^*}, \quad \textbf{(A.9)}$$

where

$$\Gamma^* = u_{22}^* \left\{[1 - n(e^*)]^2 - \beta n'(e^*)(l^* - e^*)[1 - n'(e^*)e^* - n(e^*)]\right\}$$
$$+ \beta u_2^* \left[n''(e^*)e^* + n'(e^*)\right] - \beta u_1^* n''(e^*)b + \beta u_{12}^* n'(e^*)(l^* - e^*)b.$$

Manipulating $(A.9)$ further and using the fact that $Z_e^* = \beta n'(e^*)l^* Z_l^*$, we obtain

$$\frac{1}{F_l(k^*, n^*l^*)} = \frac{Z_c^*}{Z_l^*}. \quad \textbf{(A.10)}$$

Expression $(A.10)$, together with the family's problem (**P.1**) first-order conditions, implies that

$$\tau^{*l} = 1 - \frac{u_2^*}{u_1^*}\frac{Z_c^*}{Z_l^*}. \quad \textbf{(A.11)}$$

For a broad class of preferences, we argue that the labor income tax is nonzero. Labor taxes would be zero in the limit under two possibilities: either $\hat{\eta}^* = 0$, in which case $\tau^{*l} = 0$ and the solution is first best, or $\hat{\eta}^* \neq 0$, in which case $\tau^{*l} = 0$ if and only if

$$(u_{11}^* u_2^* - u_{12}^* u_1^*)(c^* - b) = (u_{21}^* u_2^* - u_{22}^* u_1^*)\left\{n(e^*)(l^* - e^*) + [1 - n(e^*)]\frac{n(e^*)}{n'(e^*)}\right\}. \quad \textbf{(A.12)}$$

In general, (A.12) will not be satisfied and the optimal labor income tax is given by (A.11). ■

A.5. COMPARATIVE STATICS: THE NETWORK AND THE EMPLOYMENT RATE

PROPOSITION 3. *The economy's employment rate is increasing in the job arrival probability ($\partial n/\partial \gamma > 0$) and in the socializing effort ($\partial n/\partial e > 0$), and it is decreasing in the job separation rate ($\partial n/\partial \rho < 0$) and in the network average degree z ($\partial n/\partial \langle z \rangle < 0$).*

Proof. We can write the employment rate among agents with z links as follows:

$$n_z = 1 - \frac{\rho}{\rho + \gamma + (1 - \gamma)X},$$

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where $X = 1 - (1 - \frac{\gamma n v}{\langle z \rangle})^z$. Therefore,

$$\frac{\partial n_z}{\partial \gamma} = \frac{\rho z\{1 - X + (1-\gamma)[\frac{zv}{\langle z \rangle}(1 - \frac{\gamma n v}{\langle z \rangle})^{z-1}(n + \gamma \frac{\partial n}{\partial \gamma})]\}}{[\gamma + \rho + (1-\gamma)X]^2}.$$

By the fundamental theorem of calculus, we have

$$\frac{\partial n}{\partial \gamma} = \int_1^\infty \frac{\partial n_z}{\partial \gamma} D_z dz.$$

Remember that $\partial n / \partial \gamma$ appears on both the left and right sides of this equation. Some algebra leads us to

$$\frac{\partial n}{\partial \gamma} = \frac{A - Bn}{1 - \gamma B},$$

where

$$A = \int_1^\infty \frac{\rho z (1 - X)}{[\gamma + \rho + (1-\gamma)X]^2} D_z dz,$$

$$B = \int_1^\infty \frac{(1-\gamma)\frac{zv}{\langle z \rangle}(1 - \frac{nv\gamma}{\langle z \rangle})^{z-1}}{[\gamma + \rho + (1-\gamma)X]^2} D_z dz.$$

We will show that $0 < B < 1$. Note that $0 < (1-\gamma)v(1 - \frac{nv\gamma}{\langle z \rangle})^{z-1} < 1$ for all $z \geq 1$, and that $[\gamma + \rho + (1-\gamma)X]^2 > 0$. Thus, $0 < B < \int_1^\infty \frac{z}{\langle z \rangle} D_z dz = 1$. A similar argument shows that $0 < A < 1$, and is also used in the following. Therefore $\partial n / \partial \gamma > 0$, as was to be shown. We now show that $\partial n / \partial \rho < 0$. We have

$$\frac{\partial n_z}{\partial \rho} = \frac{\rho[1 + (1\gamma)\frac{\partial X}{\partial \rho}]}{[\gamma + \rho + (1-\gamma)X]^2} - \frac{1}{\gamma + \rho + (1-\gamma)X},$$

$$\frac{\partial X}{\partial \rho} = \frac{\gamma z v}{\langle z \rangle}\left(1 - \frac{\gamma n v}{\langle z \rangle}\right)^{z-1} \frac{\partial n}{\partial \rho},$$

and

$$\frac{\partial n}{\partial \rho} = \int_1^\infty \frac{\partial n_z}{\partial \rho} D_z dz.$$

Let

$$E = \int_1^\infty \frac{\rho(1-\gamma)\frac{\gamma v z}{\langle z \rangle}(1 - \frac{\gamma n v}{\langle z \rangle})^{z-1}}{[\gamma + \rho + (1-\gamma)X]^2} D_z dz.$$

Solving for $\partial n / \partial \rho$, we have

$$\frac{\partial n}{\partial \rho} = \frac{\rho}{[\gamma + \rho + (1-\gamma)X]^2} + \frac{\partial n}{\partial \rho} D - \frac{1}{\gamma + \rho + (1-\gamma)X}$$

$$= \frac{\rho - [\gamma + \rho + (1-\gamma)X]}{(\gamma + \rho + (1-\gamma)X)^2} + \frac{\partial n}{\partial \rho} D$$

$$\Rightarrow \frac{\partial n}{\partial \rho} = \frac{(1-\gamma)X + \gamma}{(D-1)[\gamma + \rho + (1-\gamma)X]^2}.$$

Because $0 < D < 1$, we have $\partial n/\partial\rho < 0$, as was to be shown. Next, to show $\partial n/\partial e > 0$, we have

$$\frac{\partial n_z}{\partial e} = \frac{(1-\gamma)\rho\frac{\partial X}{\partial e}}{[\gamma+\rho+(1-\gamma)X]^2},$$

$$\frac{\partial X}{\partial e} = \frac{\gamma z}{\langle z\rangle}\left(1-\frac{\gamma n v}{\langle z\rangle}\right)^{z-1}\left(n\frac{\partial v}{\partial e}+v\frac{\partial n}{\partial e}\right).$$

So

$$\frac{\partial n}{\partial e} = \int_1^\infty \frac{\partial n_z}{\partial e}D_z dz.$$

We can solve this equation for $\partial n/\partial e$, and find that

$$\frac{\partial n}{\partial e} = \frac{E}{1-F},$$

$$E = \int_1^\infty \frac{(1-\gamma)\rho\frac{\gamma z}{\langle z\rangle}(1-\frac{\gamma n v}{\langle z\rangle})^{z-1}n\frac{\partial v}{\partial e}}{[\gamma+\rho+(1-\gamma)X]^2}D_z dz,$$

$$F = \int_1^\infty \frac{(1-\gamma)\rho\frac{\gamma z}{\langle z\rangle}(1-\frac{\gamma n v}{\langle z\rangle})^{z-1}v}{[\gamma+\rho+(1-\gamma)X]^2}D_z dz.$$

Because $0 < F < 1$ and $E > 0$, $\partial n/\partial e > 0$, as was to be shown.

Finally, we will show that $\partial n/\partial\langle z\rangle < 0$. We have

$$\frac{\partial n_z}{\partial\langle z\rangle} = \frac{(1-\gamma)\rho\frac{\partial X}{\partial\langle x\rangle}}{[\gamma+\rho+(1-\gamma)X]^2},$$

$$\frac{\partial X}{\partial\langle z\rangle} = \frac{v\gamma z}{\langle z\rangle}\left(1-\frac{\gamma n v}{\langle z\rangle}\right)^{z-1}\left(\frac{\partial n}{\partial\langle z\rangle}-\frac{n}{\langle z\rangle}\right).$$

So we must solve

$$\frac{\partial n}{\partial\langle z\rangle} = \int_1^\infty \frac{\partial n_z}{\partial\langle z\rangle}D_z dz.$$

Letting

$$G = \int_1^\infty \frac{(1-\gamma)\rho\frac{v\gamma z}{\langle z\rangle}\left(1-\frac{\gamma n v}{\langle z\rangle}\right)^{z-1}}{[\gamma+\rho+(1-\gamma)X]^2}D_z dz,$$

this simplifies to

$$\frac{\partial n}{\partial\langle z\rangle} = \left(\frac{\partial n}{\partial\langle z\rangle}-\frac{n}{\langle z\rangle}\right)G.$$

Solving, we have

$$\frac{\partial n}{\partial\langle z\rangle} = \frac{\frac{n}{\langle z\rangle}G}{G-1}$$

because $0 < G < 1$, $\partial n/\partial\langle z\rangle < 0$, as was to be shown. ■

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