Robotica (2006) volume 24, pp. 523–525. © 2006 Cambridge University Press
doi:10.1017/S0263574705002523 Printed in the United Kingdom

Modelling of a logarithmic parameter adaptation law for adaptive control of mechanical manipulators

Recep Burkan

Erciyes University, Vocational College of Kayseri, 38039 Kayseri (Turkey)
E-mail: burkanr@erciyes.edu.tr

(Received in Final Form: October 6, 2005. First published online: January 23, 2006)

SUMMARY

In the paper,[1] a new adaptive control law for controlling robot manipulators is derived based on the Lyapunov theory; trigonometric functions are used for the derivation of the parameter estimation law. In this note, we have derived a logarithmic parameter estimation law based on a previous paper[1], and the boundedness of tracking error has been shown.

I. INTRODUCTION

Some of adaptive control laws for rigid robot manipulators have been introduced by Kelly et al.,[2] Craig et al.,[3] Middleton and Goodwin,[4] Spong and Ortega,[5] Slotine and Li,[6] Spong et al.[7] and Egeland and Godhavn.[8] Parameter estimation laws depending on exponential functions are given in various studies.[9,10]

In this note, a new adaptive control law is derived for n-link robot manipulators based on the Lyapunov theory, paper[1] providing the basis of this study. Apart from similar adaptive control algorithms, robot parameters are estimated as a logarithmic function depending on manipulator kinematics, inertia parameters and tracking error, and the adaptive gain matrix is updated depending on robot kinematic parameters and tracking error.

II. DERIVATION OF THE ADAPTIVE CONTROL LAW

The dynamic model of an n-link manipulator can be written as.[11]

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Y(q,\dot{q},\ddot{q})\pi \qquad (1)$$

In the known parameter case, the proposed and the known adaptive controller[11] are identical. The control law is given as[11]

$$\begin{aligned}\tau &= \hat{M}(q)\ddot{q}_r + \hat{C}(q,\dot{q})\dot{q}_r + \hat{G} + K\sigma \\ &= Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\hat{\pi} + K\sigma \qquad (2)\end{aligned}$$

where $\hat{\pi}$ represents the estimate on the parameters and, accordingly, $\hat{M}$, $\hat{C}$, $\hat{G}$ denote the estimated terms in the dynamic model. The other quantities are given by

$$\begin{aligned}&\tilde{q} = q_d - q \quad \dot{q}_r = \dot{q}_d + \Lambda\tilde{q} \\ &\ddot{q}_r = \ddot{q}_d + \Lambda\dot{\tilde{q}} \quad \sigma = \dot{q}_r - \dot{q} = \dot{\tilde{q}} + \Lambda\tilde{q} \qquad (3)\end{aligned}$$

Substituting Equation (2) into (1) gives

$$\begin{aligned}M(q)\dot{\sigma} + C(q,\dot{q})\sigma + K\sigma &= -\tilde{M}(q)\ddot{q}_r - \tilde{C}(q,\dot{q})\dot{q}_r - \tilde{G} \\ &= -Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\tilde{\pi} \qquad (4)\end{aligned}$$

where $\tilde{\pi} = \hat{\pi} - \pi$ is the the error parameter vector.

Theorem:

Let $(\beta_i \int Y^T\sigma dt)_i > 0$, $i = 1, 2 \ldots\ldots p$, and the estimate of parameter $\hat{\pi}$ is defined as

$$\hat{\pi} = \beta^{-1} x \begin{bmatrix} \dfrac{\ln((\beta_1 \int Y^T\sigma dt)_1 + 1)}{\int(\beta_1 Y^T\sigma dt)_1 + 1} \\ \dfrac{\ln((\beta_2 \int Y^T\sigma dt)_2 + 1)}{\int(\beta_2 Y^T dt)_2 + 1} \\ \cdots \\ \dfrac{\ln((\beta_p \int Y^T\sigma dt)_p + 1)}{\int(\beta_p Y^T\sigma dt)_p + 1} \end{bmatrix} + \begin{bmatrix} \pi_1 \\ \pi_2 \\ \ldots\ldots \\ \pi_p \end{bmatrix}; \qquad (5)$$

Where β is a pxp dimensional diagonal matrix, $\beta_1, \beta_2 \ldots\ldots, \beta_p$ are real numbers. If the control input (5) is substituted into the control law (2) for the trajectory control of the model manipulator, then the tracking errors $\tilde{q}$ and $\dot{\tilde{q}}$ will be bounded.

Proof:

In order to derive a new adaptive control law, the same Lyapunov function[1] is used.

$$\begin{aligned}V(\sigma,\tilde{q},\tilde{\pi}) = &\frac{1}{2}\sigma^T M(q)\sigma + \frac{1}{2}\tilde{q}^T B\tilde{q} \\ &+ \frac{1}{2}\tilde{\pi}^T H^2\tilde{\pi} \quad \forall\sigma, q, \tilde{\pi} \neq 0 \qquad (6)\end{aligned}$$

Taking $B = 2\Lambda K$ and using the property $\sigma^T[\dot{M}(q) - 2C(q,\dot{q})]\sigma = 0 \, \forall\sigma \in R^n$, the time derivative of V along the trajectory of system (4) is

$$\begin{aligned}\dot{V} = &-\dot{\tilde{q}}^T K\dot{\tilde{q}} - \tilde{q}^T\Lambda K\Lambda\tilde{q} + \tilde{\pi}^T(HH\dot{\tilde{\pi}} + H\dot{H}\tilde{\pi} \\ &- Y^T(q,\dot{q},\dot{q}_r,\ddot{q}_r)\sigma) \qquad (7)\end{aligned}$$

The remaining terms in Equation (7) are given as

$$HH\dot{\tilde{\pi}} + H\dot{H}\tilde{\pi} - Y^T\sigma = 0 \qquad (8)$$

If multiply both side of Equation (8) by H^{-1}, the result will be

$$H\dot{\hat{\pi}} + \dot{H}\hat{\pi} = H^{-1}Y^T\sigma + \dot{H}\pi \quad (9)$$

Since $\dot{\tilde{\pi}} = \dot{\hat{\pi}}$ (π is a constant). Equation (9) is written as

$$\frac{d}{dt}(H\hat{\pi}) = H^{-1}Y^T\sigma + \dot{H}\pi \quad (10)$$

Integration both side of Equation (10) yields

$$H\hat{\pi} = \int H^{-1}Y^T\sigma\, dt + \int \dot{H}\pi\, dt \quad (11)$$

Then, the equation (11) is arranged as

$$H\hat{\pi} = \int H^{-1}Y^T\sigma\, dt + H\pi + C \quad (12)$$

In order to derive $\hat{\pi}$ that ensures stability of the uncertain system, H must be defined. Before definition of H, we define α and β matrices as

$$\alpha = \begin{bmatrix} (\int Y^T\sigma\, dt)_1 & 0 & \cdots & 0 \\ 0 & (\int Y^T\sigma\, dt)_2 & \cdots & 0 \\ \cdots\cdots & & \cdots & \cdots \\ 0 & & \cdots & (\int Y^T\sigma\, dt)_p \end{bmatrix} \text{ and}$$

$$\beta = \begin{bmatrix} \beta_1 & 0 & \cdots & 0 \\ 0 & \beta_2 & \cdots & 0 \\ \cdots & & \cdots & \cdots \\ 0 & 0 & \cdots & \beta_p \end{bmatrix} \quad (13)$$

where α and β are diagonal pxp dimensional diagonal matrices. Then, H and H^{-1} are defined as

$$H = \beta x \alpha + I_{pxp}$$

$$= \begin{bmatrix} (\int \beta_1 Y^T\sigma\, dt)_1 + 1 & 0 & \cdots & 0 \\ 0 & (\int \beta_2 Y^T\sigma\, dt)_2 + 1 & \cdots & 0 \\ \cdots\cdots & & \cdots & \cdots \\ 0 & & \cdots & (\int \beta_p Y^T\sigma\, dt)_p + 1 \end{bmatrix} \quad (14)$$

$$H^{-1} = \begin{bmatrix} \frac{1}{(\int \beta_1 Y^T\sigma\, dt)_1 + 1} & 0 & \cdots & 0 \\ 0 & \frac{1}{(\int \beta_2 Y^T\sigma\, dt)_2 + 1} & \cdots & 0 \\ \cdots & & \cdots & \cdots \\ 0 & & \cdots & \frac{1}{(\int \beta_p Y^T\sigma\, dt)_p + 1} \end{bmatrix} \quad (15)$$

Substitution of Equation (14) and (15) into Equation (12) yields

$$\begin{bmatrix} (\int \beta_1 Y^T\sigma\, dt)_1 + 1 & 0 & \cdots & 0 \\ 0 & (\int \beta_2 Y^T\sigma\, dt)_2 + 1 & \cdots & 0 \\ \cdots & & \cdots & \cdots \\ 0 & & \cdots & (\int \beta_p Y^T\sigma\, dt)_p + 1 \end{bmatrix}$$

$$\times \begin{bmatrix} \hat{\pi}_1 \\ \hat{\pi}_2 \\ \cdots\cdot \\ \hat{\pi}_p \end{bmatrix} = \int \begin{bmatrix} \frac{(Y^T\sigma)_1}{(\int \beta_1 Y^T\sigma\, dt)_1 + 1} \\ \frac{(Y^T\sigma)_2}{(\int \beta_2 Y^T\sigma\, dt)_2 + 1} \\ \cdots \\ \frac{(Y^T\sigma)_p}{(\int \beta_p Y^T\sigma\, dt)_p + 1} \end{bmatrix} dt$$

$$+ \begin{bmatrix} (\int \beta_1 Y^T\sigma\, dt)_1 + 1 & 0 & \cdots & 0 \\ 0 & (\int \beta_2 Y^T\sigma\, dt)_2 + 1 & \cdots & 0 \\ \cdots\cdots & & \cdots & \cdots \\ 0 & & \cdots & (\int \beta_p Y^T\sigma\, dt)_p + 1 \end{bmatrix}$$

$$\times \begin{bmatrix} \pi_1 \\ \pi_2 \\ \cdots\cdot \\ \pi_p \end{bmatrix} + C \left\| \begin{matrix} 1 \\ 1 \\ \cdots\cdot \\ 1 \end{matrix} \right\| \quad (16)$$

After integration, the result is

$$\begin{bmatrix} (\int \beta_1 Y^T\sigma\, dt)_1 + 1 & 0 & \cdots & 0 \\ 0 & (\int \beta_1 Y^T\sigma\, dt)_2 + 1 & \cdots & 0 \\ \cdots\cdots & & \cdots & \cdots \\ 0 & & \cdots & (\int \beta_p Y^T\sigma\, dt)_p + 1 \end{bmatrix}$$

$$\times \begin{bmatrix} \hat{\pi}_1 \\ \hat{\pi}_2 \\ \cdots\cdot \\ \hat{\pi}_p \end{bmatrix} = \beta^{-1} \times \begin{bmatrix} \ln((\int \beta_1 Y^T\sigma\, dt)_1 + 1) \\ \ln((\int \beta_2 Y^T\sigma\, dt)_2 + 1) \\ \cdots \\ \ln((\int \beta_p Y^T\sigma\, dt)_p + 1) \end{bmatrix}$$

$$+ \begin{bmatrix} (\int \beta_1 Y^T\sigma\, dt)_1 + 1 & 0 & \cdots & 0 \\ 0 & (\int \beta_2 Y^T\sigma\, dt)_2 + 1 & \cdots & 0 \\ \cdots\cdots & & \cdots & \cdots \\ 0 & & \cdots & (\int \beta_p Y^T\sigma\, dt)_p + 1 \end{bmatrix}$$

$$\times \begin{bmatrix} \hat{\pi}_1 \\ \hat{\pi}_2 \\ \cdots\cdot \\ \hat{\pi}_p \end{bmatrix} + C \left\| \begin{matrix} 1 \\ 1 \\ \cdots\cdot \\ 1 \end{matrix} \right\| \quad (17)$$

If we multiply both side of Equation (17) by H^{-1}, the result will be

$$\hat{\pi} = \beta^{-1} \times \begin{bmatrix} \frac{\ln((\beta_1 \int Y^T\sigma\, dt)_1 + 1)}{\int(\beta_1 Y^T\sigma\, dt)_1 + 1} \\ \frac{\ln((\beta_2 \int Y^T\sigma\, dt)_2 + 1)}{\int(\beta_2 Y^T dt)_2 + 1} \\ \cdots \\ \frac{\ln((\beta_p \int Y^T\sigma\, dt)_p + 1)}{\int(\beta_p Y^T\sigma\, dt)_p + 1} \end{bmatrix}$$

$$+ \begin{bmatrix} \pi_1 \\ \pi_2 \\ \cdots\cdot \\ \pi_p \end{bmatrix} + CH^{-1} \begin{bmatrix} 1 \\ 1 \\ \cdots\cdot \\ 1 \end{bmatrix} \quad (18)$$

https://doi.org/10.1017/S0263574705002523 Published online by Cambridge University Press

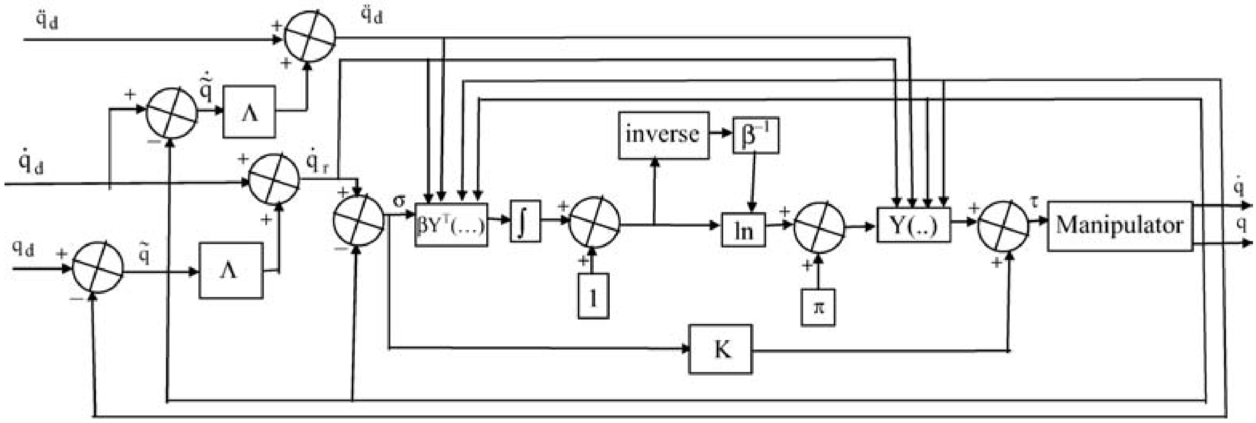

Fig. 1. Implementation of the proposed adaptive control law.

The initial estimation of error vector $\hat{\pi}$ is initially zero, that is $\hat{\pi}(0) = \pi$, then the constant C will be equivalent to zero. Hence, the parameter adaptation law is derived as

$$\hat{\pi} = \beta^{-1} \times \begin{bmatrix} \dfrac{\ln((\beta_1 \int Y^T \sigma dt)_1 + 1)}{\int(\beta_1 Y^T \sigma dt)_1 + 1} \\ \dfrac{\ln((\beta_2 \int Y^T \sigma dt)_2 + 1)}{\int(\beta_2 Y^T dt)_2 + 1} \\ \cdots \\ \dfrac{\ln((\beta_p \int Y^T \sigma dt)_p + 1)}{\int(\beta_P Y^T \sigma dt)_P + 1} \end{bmatrix} + \begin{bmatrix} \pi_1 \\ \pi_2 \\ \cdots \\ \pi_p \end{bmatrix} \quad (19)$$

By considering this control input, the resulting block diagram of the adaptive control law is given in Figure 1.

III. CONCLUSION

Equation (6) shows that V is a positive continuous function and $V(0) = 0$, that is lower bounded by zero when $\tilde{\pi}(0) = \hat{\pi}(0) - \pi = 0$ and at the equilibrium points $\dot{\tilde{q}} \equiv 0, \tilde{q} \equiv 0$. Since $\dot{V} \leq 0$, and V is a positive definite function and lover bounded by zero, V tends to a constant as $t \rightarrow \infty$ and therefore V remains bounded. Thus $\dot{\tilde{q}}$ and $\tilde{q}$ are bounded, that is, $\dot{\tilde{q}}$ and $\tilde{q}$ converge to zero and this implies that σ is bounded and converges to zero. As a result, $\int Y^T \sigma dt$ is bounded, and these imply that H, $\tilde{\pi}$ and $\hat{\pi}$ are bounded.

The previous study[1] also makes it possible to derive a new parameter estimation law. We have used system parameters and mathematical insight to search appropriate function of H, then we have derived a logarithmic parameter estimation law for adaptive control of mechanical manipulators.

References

1. R. Burkan, "Design of adaptive control law using trigonometric function for robot manipulators", *Robotica* **23**(1), 93–99 (2005).
2. R. Kelly, C. Carelli and R. Ortega, "Adaptive motion control design of robot manipulators: an input output approach", *International Journal of Control* **50**(6), 2563–2581 (1989).
3. J.J. Craig, P. Hsu and S.S. Sastry, "Adaptive control of robot manipulator", *Int. J. of Robotics Research* **6**, 16–28 (1987).
4. R.H. Middleton and G.C. Goodwin, "Adaptive computed torque control for rigid link manipulators", *System Control Letters* **10**, 9–16 (1988).
5. M.W. Spong and R. Ortega, "On adaptive inverse dynamics control of rigid robots", *IEEE Transactions on Automatic Control* **35**(1), 92–95 (1990).
6. J.J. Slotine and W. Li, "On the adaptive control of robotic manipulator", *Int. J. of Robotics Research* **6**(3), 49–59 (1987).
7. M.W. Spong, R. Ortega and R. Kelley, "Comment on adaptive manipulator control: A case study", *IEEE Transactions on Automatic Control* **35**(6), 761–762 (1990).
8. O. Egeland and J.M. Godhavn, "A note on Lyapunov stability for adaptive robot controller", *IEEE Transactions on Automatic Control* **39**(8), 1671–1673 (1994).
9. R. Burkan, "New approaches in controlling robot manipulators with parametric uncertainty", *Ph.D. Thesis* (Erciyes University, Institute of Science, Turkey, 2002).
10. R. Burkan and İ. Uzmay, "A model of parameter adaptive law with time varying function for robot control", *Applied Mathematical Modelling* **29**, 361–371 (2005).
11. L. Sciavicco and B. Siciliano, *Modeling and Control of Robot Manipulators* (The McGraw-Hall Companies, 1996).

https://doi.org/10.1017/S0263574705002523 Published online by Cambridge University Press