PART IV.

Considerations on Localized Velocity Fields in Stellar Atmospheres: Prototype — The Solar Atmosphere.

A. - Convection and Granulation.

Summary-Introduction.

E. BÖHM-VITENSE

Institute of Theoretical Physics - Kiel

Since there is the excellent summary on stellar convection given by J.-C. PECKER at the Liège symposium on stellar evolution in 1959, where he gives a complete account of the existing literature, I shall in this summary not try to do justice to the history of stellar convection research nor shall I try to mention all the existing papers even if they are quite important. I shall rather attempt to give an objective picture of the present state of affairs in order to be able to discuss our problems with the aerodynamicists. I shall cite only those papers that have been explicitly used for the results presented here. Additional references to literature may be found in the article by J.-C. PECKER.

Introduction.

The origin for turbulence in the atmosphere of stars with effective temperature $T_{\text{eff}} \leqslant 8\,000°$—spectral types later than about A6—may be found in the convectively unstable zone which occurs a few hundred km below the surface of the stars due to the ionization of hydrogen (UNSÖLD, 1930). For stars with higher temperature this instability zone is too flat for convection with any measurable velocities to occur. But in connection with rapid rotation of the stars it may still cause some turbulent motion, as KIPPENHAHN has pointed out (KIPPENHAHN, 1950).

A) Observations.

1. - Solar photosphere.

The velocity fields caused by the hydrogen convection zone can be best observed on the solar surface, which shows the well-known phenomenon of the solar granulation.

The main items that observers tried to investigate are the following:

I. Size and shape of granules.
II. The lifetime of granules.
III. Size of the temperature fluctuations between granular and intergranular regions.
IV. Velocity fluctuations between granular and intergranular regions.
V. Correlation between temperature and velocity fluctuations.
VI. Magnetic fields of granules.

In spite of the excellent observational work on which we heard reports yesterday, we can hardly provide definite answers to any of these problems.

The main difficulty for observations is the bad seeing, which means the small scale turbulence in the atmosphere of the earth that disturbs the wave front of the light and in this way prohibits excellent image quality. Even under best seeing conditions the resolution for observations from the ground is usually limited to about 1″ of arc, which corresponds to about 700 km distance on the solar surface.

Many attempts have been made to escape bad seeing. RÖSCH took pictures at the Pic du Midi Observatory; we saw one of his films yesterday. BLACKWELL, DEWHIRST, and DOLLFUSS took pictures from a manned balloon going up to 6 km height. SCHWARZSCHILD even took pictures from an unmanned balloon, reaching a height of 27 km. We also saw one of his films. Taking pictures from balloons has the advantage of escaping bad seeing conditions to a high degree, but it has the disadvantage of limited size of telescopes which can be borne by a balloon. The telescopes that have been used so far have an aperture of 12 in. = 30 cm, which gives a *theoretical resolving power of* 0″.4 *of arc*, which obviously is a lower limit to the actual resolving power.

We shall now proceed to discuss the observational results concerning items I to VI.

1·1 *Size and shape of granules.* – SCHWARZSCHILD (1959) found diameters of granules reaching from 300 to 1800 km, in agreement with Rösch's pictures, corresponding to about 0″.4 to 2″.5, with a mean size of about 700 km. The granules are separated by dark, narrow lanes. The structure of the granules is irregular but very often polygonal. These findings are confirmed by the other observers. SCHWARZSCHILD does not think that the granules could be Bénard cells, because

1) The polygonal structure of the granules is irregular.
2) The granules have varying diameters.

3) The solar granulation is a distinctly non-stationary phenomenon, as we saw yesterday.

Schwarzschild compares the solar granulation to a type of fluid motion which is called « non-stationary convection » (Siedentopf, 1948; Prandtl, 1942). It is a type between the regular Bénard cells and a completely irregular turbulent convection, which occurs for Rayleigh numbers of the order 10^5 ($R = (\Delta \operatorname{grad} T/T)\cdot(l^4\cdot g/\nu\cdot\varkappa)$, with ν = kinematic viscosity and $\varkappa$ = conductivity, being radiative conductivity in stellar atmospheres). For the upper solar convection zone Schwarzschild calculates $R = 10^{10}$.

There remains the question whether the observed sizes of granules are real or whether they are counterfeit by the limited resolving power of the instrument, so that the observed granules are really conglomerates of small granules which cannot be observed.

Schwarzschild tries to answer this question by calculating the autocorrelation functions for the brightness fluctuations on his images of the solar surface:

$$Q(y) = \frac{1}{N-m}\sum_{n=1}^{n=N-m} \Delta I_n \cdot \Delta I_{\nu+m}\,, \qquad \text{with} \qquad y = m\ .138\ \text{km}\,.$$

He then calculates the same function for what he calls a point model, which means he assumed that all granules are really very small bright spots distributed randomly and that the instrumental profile is given by the diffraction pattern of the 12 in. aperture. This theoretical $Q(y)$ does not agree with the observed one. Schwarzschild also calculates $Q(y)$ with the assumption that the granules are bright lines distributed randomly and finds much better agreement with the observations. From this he concludes that he really resolves the individual granules.

To me it seems that the possibility has not been excluded that the granules are really point-like, with a tendency to congregate. In this connection I should like to draw attention to the experimental investigation of Fellgett (1959). He copied plate grains in a special way so as to show congregations of grains and then took out of focus photographs of these plate grains. It is amazing how much this blurred picture resembles the structure of a photograph of solar granulation, although it must be realized that the image quality of the best solar photographs is better that the one of this picture. But the main result is that by blurring the original picture the original plate grain is completely lost and, instead, there appears a network which was not visible on the original.

1·2. *The lifetime of granules.* – Rösch and Spiegel yesterday discussed this question. They obtained lifetimes of 6 to 8 minutes. It seems somewhat dif-

1170

ficult to define a lifetimes of granules because they change their appearance rather quickly but do exist a rather long time. On the other hand Leighton's investigations of line shifts and intensities indicated a lifetime of about 20 minutes but with an oscillation period of 5 minutes for the velocities. Since he obtains a correlation of brigthness and velocities this period should also be present in the brightness of granules. Rösch's and Spiegel's pictures certainly did not show these 5 minute oscillations. All these different observations have been made with so much care and skill that I personally hate to doubt any one of them. So, I am faced with the very difficult problem of bringing these observations together into a consistent picture which we can place before the aerodynamicists. I must admit that I find it extremely hard to fit in these oscillatory motions with the other observations.

If we have to accept the oscillating motions as being real, the only way could be that these oscillations occur in such high layers that they do not show up at all in the continuous spectrum. As I pointed out earlier those parts of the line profile of the CaI line 6130 which are used to determine the Doppler shifts can be expected to be formed in layers with optical depths $\tau_{\text{continuum}} = 0.05$ to 0.01 (corresponding to about 150 to 100 km below the sun's surface) or possibly even higher, while the continuous spectrum originates in $\tau = 1$. But then due to the correlation of brightness and velocity, which was observed by LEIGHTON, we should expect a correlation between line intensity and velocity which I do not think has been observed so far. Moreover, as WADDEL already pointed out some days ago, there has been derived a circulating motion just by investigating line profiles originating in high layers. We shall come back to this later.

In any case, for the granules seen in the continuum I am inclined to accept the lifetime of 6 to 8 minutes.

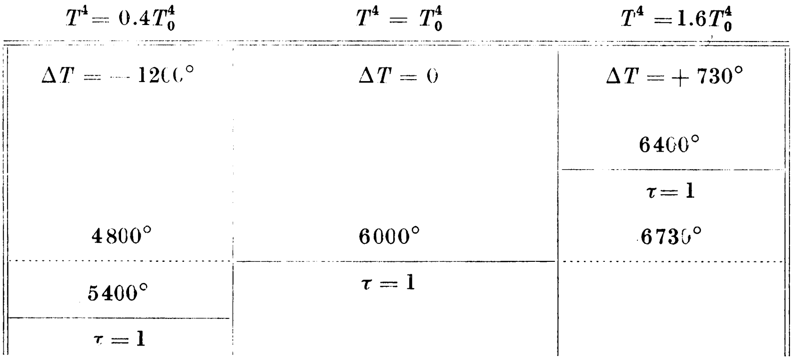
1.3. *Size of the temperature fluctuations between granular and intergranular regions.* – THIESSEN'S (1955) visual observations with the 60 cm refractor in Bergedorf (resolving power 0″.17 of arc) indicated mean brightness fluctuations of 35%, corresponding to mean temperature differences of 370° (±185°).

SCHWARZSCHILD on one side and BLACKWELL, DEWHIRST, and DOLLFUSS on the other side measured the brightness fluctuations on their new plates and both found $\sqrt{\overline{\Delta I^2}} = \pm 0.046$. The English-French group then pointed out that, due to the resolving power of the instrument of 0″.4, features of 1″ diameter lose much of their contrast; in fact, they measure the contrast transmission function for their instrument as a function of size of observed features with an object of known contrast. They found that the measured brightness fluctuations correspond to real temperature differences of at least 520° (±260°). This appears to be a lower limit, because for the correction the seeing conditions were not taken into account, and the mean granular and intergranular diameters were assumed to be $1''.4$, while the intergranular regions might be much narrower. For smaller features the correction to the measured contrast must be much greater.

This is the only direct method to determine temperature fluctuations independently of all model calculations for the solar atmosphere.

There are other indirect methods to derive temperature fluctuations in the solar photosphere, which indicate values in agreement with the one determined from the brightness fluctuations. First one can investigate the center-to-limb variations of the line intensities. This has been done so far under the assumption of L.T.E., which to me still seems to be a fairly adequate assumption. For each line there is a certain well-known temperature-dependence of intensity. In addition the regions with higher temperature contribute more to the observed mean intensity than do the colder regions. Therefore the observed line intensity is different from the one which we would expect to observe in a homogeneous photosphere.

K. H. Böhm was the first to derive values for ΔT by studying the center-to-limb variation of FeI line wings. He found that he got relatively good agreement with observations using a three-stream model with $\Delta T \sim \pm 1000°$ in $\bar{\tau}=1$ for equal geometrical depths t; corresponding to $\Delta T = \pm 500°$ in equal optical depths τ. (Observations of contrast always refer to equal optical depths, because radiation always escapes from $\tau = \cos\vartheta$.) Since the absorption coefficient $\varkappa$ is larger for higher T, the apparent ΔT for equal optical depths, $\tau = \int \varkappa \, dt$, are smaller than ΔT in equal geometric depths. The situations for a three-stream model is sketched in the following draft.



H. H. Voigt investigated in 1956 the center-to-limb variation of the infrared oxygen triplet being formed in rather deep photospheric layers where one would certainly not expect serious deviations from L.T.E. He found best agreement with observations assuming a three-stream model with $\Delta T = \pm 1000°$ in deep layers but decreasing with height, ΔT being 0 in $\tau_{\lambda=7775} = 0.03$.

Schröter (1957) finds agreement with measured red shifts and center-to-limb variations of weak FeI lines, assuming a two-stream model with $\Delta T = \pm 450°$ (in equal t) for $\bar{\tau} = 0.6$, decreasing in both directions upwards and downwards.

This is also in rough agreement with the observations of the Balmer lines. Using the new theory of hydrogen line broadening evaluated by GRIEM, KOLB and SHEN (1959) and taking into account the resonance broadening, TRAVING and CAYREL (1960) found that temperature fluctuations of $\leqslant \pm 300°$ in equal optical depths are still permitted by observations.

All these indirect determinations of temperature fluctuations are of course subject to great uncertainties, but they show that all investigations of line profiles suggest temperature fluctuations ΔT of the same magnitude as were derived by the direct photographs of the sun's surface, which means $\Delta T \geqslant \pm 260°$ in equal optical depths.

1.4. *Velocity fluctuations between granular and intergranular regions.* – Again we shall first deal with a direct method not making use of any assumptions about the source function $S(\tau)$.

In 1950 RICHARDSON and SCHWARZSCHILD obtained a solar spectrogram on Mr. Wilson—with the slit put across the sun's image—which clearly showed a wiggling of the Fraunhofer lines attributed to different velocities on different points of the sun's disc. Miss MÜLLER will show us such a spectrogram this morning. RICHARDSON and SCHWARZSCHILD measured $\sqrt{2\overline{\Delta v^2}} = 0.37$ km/s. At the same time they found brightness fluctuations of 6.4%. THIESSEN (1955) compared this value with the 35% brightness fluctuations found by him and concluded that Richardson and Schwarzschild's values were very much reduced by scattered light. The scattered light must then also reduce the velocity fluctuations. He calculated that the measured value of $\sqrt{2\overline{\Delta v^2}} = 0.37$ km/s would mean true velocity fluctuations of 1.85 km/s.

Again there are indirect methods to determine mass motions. We can, for instance, measure the line profiles—integrated over several granules as one usually does when taking a spectrum of the sun. If we assume the non-thermal velocities to have a gaussian distribution, then the width of faint lines will be determined by $\Delta\lambda_D = \lambda\cdot(\xi/c)$ where $\xi = \sqrt{\xi^2_{\text{therm}} + \xi^2_{\text{turb}}}$. ξ is the velocity component in the line of sight. For stronger lines one has to correct for saturation effects.

By observing the center-to-limb variation of line profiles one can even determine the depth-dependence of the velocities. At the limb only the high layers contribute to the line profile, while in the center deeper layers also are seen.

The investigations of ALLEN (1949), REICHEL (1953), VOIGT (1956), SUEMOTO (1957), and WADDELL (1958) agree that in the center of the sun the mean line-of-sight velocity, which is there the radial component, is $(1.7 \div 1.8)$ km/s:

1173

while at the limb, the line-of-sight velocity, which is there mainly the horizontal component, is around 2.8 km/s. (The lines are observed to be broader at the limb.)

Voigt, taking into account also the temperature inhomogeneities, found agreement with observations, only by assuming an outward decreasing radial component ~ 3 km/s at $\tau = 1$ and an outward increasing tangential component ($v \sim$ km/s in $\tau = 0.02$, called by him microturbulence). The decreasing radial component was confirmed by Suemoto's paper, where he points out that weaker lines of the same multiplet have a larger Doppler width than to the stronger lines. The weaker lines are formed in deeper layers. On the other hand all lines are broadened toward the limb, indicating an increasing horizontal component of the velocities.

Unno (1959), studying various points in line profiles in the center of the disc being formed in various depths, also obtained an outward decreasing radial velocity component.

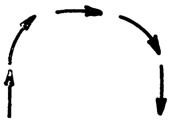


Fig. 1.

Altogether we obtain the picture of a circulating motion. There is only a slight discrepancy with a result of Miyamoto (1954) who obtained $v_{\rm rad} = 4.6$ km/s for those layers, where the cores of strong Fraunhofer lines are formed, which may however already be in the low chromosphere.

There is still the curve-of-growth method to determine non-thermal velocities. This, I think, is quite valuable for other stars, but for the sun the direct measurement of center-to-limb variation of line-profiles gives more accurate results. There seems to be a general tendency, however, toward somewhat smaller values for $\xi_{\rm turb}$ (~ 1.5 km/s) derived from curve-of-growth analysis than from line-profile method. If this is true there may be two reasons.

Curve of growth shows only small scale motions, while the line-profile includes also large scale motions. So the presence of large scale motions probably shows up (the same velocities over regions $> \tau = 1$ (> 400 km)). On the other hand, for curve-of-growth analysis strong lines originating in high layers have the highest statistical weight, while for measuring line-profiles one usually uses weaker lines originating in deeper layers. So a stratification effect may also be included in the different results.

1'5. *Correlation between temperature and velocity fluctuations.* – A correlation of ΔT and Δv has always been expected, but to my knowledge it seemed first to be proved to exist by Stuart and Rush in 1954. Their investigations were based on the spectrogram of Richardson and Schwarzschild. They found a correlation between the small scale fluctuations in brightness and velocities in the way that the brighter regions have outward velocities (correlation coefficient $r = -0.50$ to -0.68).

The Δv were determined as deviations from a running mean. This gave $\sqrt{2\,\overline{\Delta v^2}} = 0.17$ km/s.

The discussion of FELLGETT (1959), however, shows that this value of r is not so significant as has been supposed thus far. The subtraction of a running mean is dangerous. Without using running means one obtains $r \approx -0.3$, and this obviously does not mean very much, because FELLGETT in one case even obtained $r = -0.41$ when comparing velocity fluctuations on one spectrogram with the brightness fluctuations on another one taken 20 minutes later. But FELLGETT also states that exceedingly bright granules are always correlated with negative velocities (rising matter).

It may not be surprising that the observed correlation between ΔT and Δv is rather weak. First there are the difficulties of accurate measurements because of scattered light, and second we have to keep in mind that brightness and velocity variations refer to quite different depths, as was pointed out earlier.

1·6. *Magnetic fields of granules.* – Magnetic fields of granules have not been observed so far, although they have been looked for. It seems that magnetic fields larger than 50 gauss correlated with granules do not exist.

2. – Summary of granule observations.

Finally we may summarize the observational results concerning solar granules. Mean diameters are $\leqslant 700$ km. The observed brightness fluctuations on the solar surface, observed as the phenomenon of granules, correspond to temperature fluctuations $\overline{\Delta T} \geqslant 520°$ in equal optical depths. ΔT is probably decreasing with height. The velocity fluctuations are about $(2.5 \div 3.0)$ km/s, changing their direction from vertical motions in deeper layers $(\tau = 1)$ to horizontal motions in high photospheric layers. Very bright granules certainly rise, less bright ones probably do. Magnetic fields connected with granules, if present, must be smaller than 50 gauss.

3. – Velocities in sunspots.

BRAY and LOUGHEAD (1959) have published photographs of granules in sunspots which possibly have somewhat smaller diameters than the ones in the photosphere. HOWARD (1958) found from curve-of-growth analysis velocities $\bar{v} = (2.9 \div 3.7)$ km/s. These are larger than the ones derived for the surrounding photosphere.

There are also the line shifts in the sunspots penumbra corresponding to a nearly horizontally outwards streaming gas with maximum velocity of about 3 km/s for a large spot—usually called the Evershed effect.

4. – Velocities in the chromosphere.

The observations of velocities in the solar chromosphere have been summarized by DE JAGER in his article in the *Handbuch der Physik* (1959). Velocities increase with height up to about 15 km/s in about 3 000 km above the solar limb.

5. – Velocities arising from convection in other stars.

The observational data are reviewed in the article of WRIGHT in the *IAU Transactions* (1955). Miss UNDERHILL has included the latest observations in her summary talk. The measured velocities increase with increasing effective temperatures of the stars and with decreasing surface gravity. You still have the table of Miss UNDERHILL.

B) Theory of the Hydrogen Convection Zone.

Convection occurs when

$$\nabla = \frac{\mathrm{d}\log T}{\mathrm{d}\log P_{\mathrm{g}}} > \frac{\mathrm{d}\log T}{\mathrm{d}\log P_{\mathrm{g}\,\mathrm{adiabatic}}} = \nabla_{\mathrm{ad}} .$$

In the high photospheric layers of a star this is not fulfilled; they are in radiative equilibrium, meaning that the whole energy transport is performed by radiation. In such an atmosphere the temperature distribution is given approximately by

$$(1) \qquad T^4 = \tfrac{3}{4} T^4_{\mathrm{eff}}(\bar{\tau} + \tfrac{2}{3}) \quad \text{with} \quad \sigma T^4_{\mathrm{eff}} = \pi F , \quad \pi F = \text{net flux} ,$$

while the distribution of the gas pressure P_{g} obeys the hydrostatic equation

$$(2) \qquad \frac{\mathrm{d}P_{\mathrm{g}}}{\mathrm{d}\bar{\tau}} = \frac{g}{\bar{\varkappa}/gr} , \qquad g = \text{gravitational acceleration}$$

For the sun $T_{\mathrm{eff}} = 5\,800^{\circ}$, $g = 2.82 \cdot 10^4$.