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Electron acceleration in a rectangular waveguide filled with unmagnetized inhomogeneous cold plasma

H.K. MALIK,[1] S. KUMAR,[2] AND K.P. SINGH[3]

[1]Plasma Waves and Particle Acceleration Laboratory, Department of Physics, Indian Institute of Technology Delhi, New Delhi, India
[2]Department of Physics, Pohang University of Science and Technology, Pohang, Korea
[3]Simutech, Gainesville, Florida

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Abstract

This paper deals with the study of propagation of electromagnetic wave in a rectangular waveguide filled with an inhomogeneous plasma in which electron density varies linearly in a transverse direction to the mode propagation. A transcendental equation in ω (microwave frequency) is obtained that governs the mode propagation. In addition, an attempt is made to examine the effect of density inhomogeneity on the energy gain acquired by the electron (electron bunch) when it is injected in the waveguide along the direction of the mode propagation. On the basis of angle of deflection of the electron motion we optimize the microwave parameters so that the electron does not strike with the waveguide walls. Conditions have been discussed for achieving larger energy gain. The plasma density inhomogeneity is found to play a crucial role on the cutoff frequency, fields and dispersion relation of the TE_{10} mode as well as on the acceleration gradient in the waveguide.

Keywords: Electron acceleration; Rectangular waveguide; Inhomogeneous plasma; Microwave

INTRODUCTION

The subject particle acceleration has been fascinating due to its diverse applications in the field of nuclear physics, high energy physics, harmonic generation, nonlinear phenomena observed during laser plasma interaction, etc. In this direction, various experimental and theoretical investigations (Balakirev *et al.*, 2001; Reitsma & Jaroszynski, 2004; Baiwen *et al.*, 2004; Kawata *et al.*, 2005; Koyama *et al.*, 2006; Lifschitz *et al.*, 2006; Kumar *et al.*, 2006; Sakai *et al.*, 2006; Flippo *et al.*, 2007; Gupta & Suk, 2007; Malik *et al.*, 2007; Nickles *et al.*, 2007; Zhou *et al.*, 2007) have been made along with the use of high peak power lasers that could be developed with the help of chirped pulse amplification (CPA) technique. The lasers have proved to be an effective tool to produce ultrahigh acceleration gradients using plasmas as the accelerating media as they can sustain large electric fields being in ionized state. Baiwen *et al.* (2004) have investigated the electron acceleration by an intense laser pulse in a low density plasma and observed a short high quality well collimated relativistic electron beam in the direction opposite to the laser propagation. Lifschitz *et al.* (2006) have proposed a design of a two stage compact GeV electron accelerator. Flippo *et al.* (2007) have shown in a laser driven ion accelerator that the spectral shape of the accelerated particles can be controlled to yield a range of distribution, from Maxwellian to ones possessing a monoenergetic peak at high energy. Nickles *et al.* (2007) have reviewed the ultrafast ion acceleration experiments in laser plasma and discussed the interaction of laser pulse with intensities above 10^{19} W/cm^2 with water and heavy water droplets as well as with thin foils.

In addition, efforts have been made related to wakefield excitation by relativistic electron bunch (Balakirev *et al.*, 2001; Zhou *et al.*, 2007), by different shapes of laser pulses (Kumar *et al.*, 2006; Malik *et al.*, 2007), and coupling of longitudinal and transverse motion of accelerated electrons in laser wakefield (Reitsma & Jaroszynski, 2004). Also Koyama *et al.* (2006) have experimentally generated monoenergetic electron beam by focusing 2 TW laser pulses of 50 fs on supersonic gas jet targets. Lotov (2001) has analytically studied the laser wakefield acceleration in narrow plasma filled channels. With regard to the importance of polarization effects, Kado *et al.* (2006)

Address correspondence and reprint requests to: Hitendra K. Malik, Plasma Waves and Particle Acceleration Laboratory, Department of Physics, Indian Institute of Technology Delhi, New Delhi – 110 016, India. E-mail: hkmalik@physics.iitd.ac.in

have observed strongly collimated proton beam from Tantalum targets when irradiated with circularly polarized laser pulses. With the help of radially polarized ultra relativistic laser pulses, Karmakar and Pukhov (2007) have shown that collimated attosecond GeV electron bunches can be produced by ionization of high-Z material. They also compared the results with the case of Gaussian laser pulses and found that the radially polarized laser pulses are superior both in the maximum energy gain and in the quality of the produced electron beams. Xu *et al.* (2007) made a comparison between circularly polarized (CP) and linearly polarized (LP) fields with regard to the laser driven electron acceleration in vacuum and found that the CP field can give rise to greater acceleration efficiency. In such schemes of wakefield acceleration, a large amplitude plasma wave is generated which is used for the particle acceleration. However, when this wave achieves a sufficiently large amplitude it becomes susceptible to the oscillating two stream instability (OTSI), which is an important issue in nonlinear plasma physics and has been studied in different plasma models (Nicholson, 1981; Kumar & Malik, 2006*a*; Malik, 2007).

On the other hand, the researchers have studied the propagation of electromagnetic waves in circular waveguides (Alexov & Ivanov, 1993; Maraghechi *et al.*, 1994; Watanabe *et al.*, 1995; Ivanov & Nikolaev, 1998; Ding *et al.*, 2001, 2004) considering different models, *viz.* partially plasma filled waveguides, completely plasma filled waveguides, unmagnetized, magnetized plasma filled waveguide etc. in view of their applications to a variety of sources including backward wave oscillators (BWO), traveling wave tube (TWT) amplifiers, gyrotrons, etc. Microwaves have also been used for the purpose of particle acceleration (Palmer, 1972; Hirshfield *et al.*, 1996; Park & Hirshfield, 1997; Zhang *et al.*, 1997; Yoder *et al.*, 2001; Jing *et al.*, 2003; Malik, 2003; Jawla *et al.*, 2005; Kumar & Malik, 2006*b*). On the basis of interaction of relativistic particles with free electromagnetic waves in the presence of a static helical magnet, Palmer (1972) has shown that the particles can be accelerated when they move in the direction of propagation of the circularly polarized radiation. Hirshfield *et al.* (1996) have discussed cyclotron autoresonance accelerator (CARA) using radiofrequency (rf) gyroresonant acceleration where the maximum energy achieved by the electron beam is up to 2.82 MeV. Zhang *et al.* (1997) have attempted for a wakefield accelerator using a dielectric lined waveguide structure and showed that the acceleration gradient for electrons or positrons can be achieved in the range of 50–100 MV/m for a few nC driving bunches. Using a uniform circular waveguide with a helical wiggler and axial magnetic field, Yoder *et al.* (2001) have measured the energy gain of about 360 keV for a 6 MeV electron bunch, and an accelerating gradient of 0.43 MV/m in a microwave inverse free electron laser accelerator. A theory was given by Park and Hirshfield (1997) for wakefield in a dielectric lined waveguide, and a

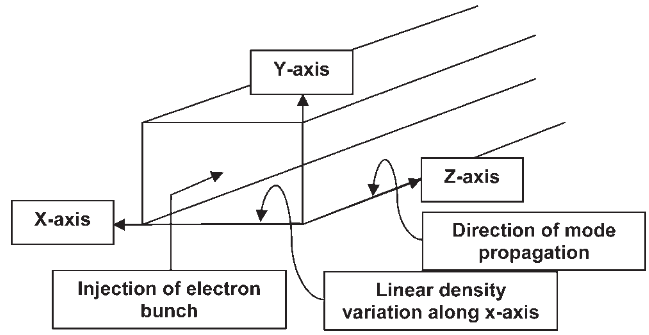

Fig. 1. Geometry of the plasma filled rectangular waveguide with a linear density variation along the x-axis and the mode propagation along the z-axis.

peak acceleration gradient of 155 MeV/m was predicted for a 2 nC rectangular drive bunch. Jing *et al.* (2003) have calculated dipole-mode wakefields in dielectric-loaded rectangular waveguide accelerating structure and predicted transverse wakefields of about 0.13 MeV/mnC (0.2 MeV/mnC) due to X-dipole modes (Y-dipole modes) in an X-band structure generated by an electron bunch. Jawla *et al.* (2005) have shown that the modified mode excited in a plasma filled waveguide under the effect of an external magnetic field can be quite useful for the electron acceleration using high intensity microwaves. Kumar and Malik (2006*b*) have discussed the importance of obliquely applied magnetic field to an electron acceleration and obtained that the larger acceleration is possible when the condition $\omega_p > \omega_c$ (ω_p is the electron plasma frequency and ω_c is the electron cyclotron frequency) is achieved in the plasma filled waveguide.

Since invariably we encounter with the inhomogeneous plasmas (Kovalenko & Kovalenko, 1996; Cho, 2004) and the waveguide can guide the electromagnetic radiations for longer distances, it would be of great interest to evaluate the effect of plasma density inhomogeneity on the electron motion and its acceleration in a rectangular waveguide when it is injected in the waveguide along the direction of mode propagation (schematic is shown in Fig. 1). In this paper we therefore discuss this problem by considering a weak gradient in the plasma density in the transverse direction to the mode propagation.

MODE FIELD: ANALYTICAL APPROACH

We focus on the transverse electric (TE) mode that is excited by a high intensity microwave in a lossless inhomogeneous unmagnetized plasma filled rectangular waveguide of dimensions $a \times b$ cm^2. For calculating the fields of the mode, we proceed with the following Maxwell's equations for the time dependence as $e^{-i\omega t}$

$$\vec{\nabla} \cdot \vec{D} = 0, \qquad \vec{\nabla} \cdot \vec{B} = 0,$$
$$\vec{\nabla} \times \vec{E} = i\omega\mu_0 \vec{H}, \quad \vec{\nabla} \times \vec{H} = -ne\vec{v} - i\omega\varepsilon_0 \vec{E}. \tag{1}$$

Here the velocity $\vec{v}$ is obtained from the electron equation of motion as $\vec{v} = (e\vec{E}/m_e i\omega)$. With the use of this expression in the above equations we get

$$\vec{\nabla} \times \vec{H} = -\frac{ne^2\vec{E}}{m_e i\omega} - i\omega\varepsilon_0\vec{E} = -i\omega\varepsilon_0\varepsilon\vec{E}, \tag{2}$$

where ε is the plasma dielectric function, given by $\varepsilon\,(\omega, x) = 1 - (\omega_p^2\,(x)/\omega^2)$ together with the plasma density $n = n_0\,(x)$, as we consider the density variation only in the x-direction. The curl of Eq. (2) together with the use of other Maxwell's equations and the standard identity $\vec{\nabla} \times \varepsilon\vec{E} = \varepsilon\vec{\nabla} \times \vec{E} + \vec{\nabla}\varepsilon \times \vec{E}$ yields

$$\nabla^2\vec{H} + \frac{\omega^2}{c^2}\varepsilon\vec{H} + \frac{1}{\varepsilon}\vec{\nabla}\varepsilon \times (\vec{\nabla} \times \vec{H}) = 0. \tag{3}$$

The above equation for H_z can be written as

$$\nabla^2 H_z + \frac{\omega^2}{c^2}\varepsilon H_z - \frac{1}{\varepsilon}\frac{d\varepsilon}{dx}\frac{\partial H_z}{\partial x} = 0. \tag{4}$$

We consider weak inhomogeneity, i.e., n is treated as a slow varying function of x as long as its variation length scale is far longer than the wavelength of the wave, or in other words, $(1/k_g n)\,(dn/dx) << 1$ together with k_g as the guide propagation constant. In view of this, the last term of Eq. (4) can be dropped. Further we assume that the plasma density is a linear function of space, i.e., $n = n_{cr}\,(x/L)$, where $n_{cr}\,(\equiv \varepsilon_0 m_e\,\omega^2/e^2)$ is the critical density, which we will call as reference plasma density n_0. Therefore, Eq. 4 reads

$$\nabla^2 H_z + \frac{\omega^2}{c^2}\left(1 - \frac{x}{L}\right)H_z = 0. \tag{5}$$

Using separation of variable method Eq. (5) can be solved by taking $H_z\,(x, y, z) = F(x)G(y)I(z)$. With this we obtain

$$\frac{1}{F}\frac{\partial^2 F}{\partial x^2} + \frac{1}{G}\frac{\partial^2 G}{\partial y^2} + \frac{1}{I}\frac{\partial^2 I}{\partial z^2} = -k^2, \tag{6}$$

where $k^2 = (\omega^2/c^2(1 - (x/L))$. Taking $-k^2 = k_x^2 + k_y^2 + k_z^2$ we may write the following from the above equation

$$\frac{1}{F}\frac{d^2F}{dx^2} = k_x^2, \quad \frac{1}{G}\frac{d^2G}{dy^2} = k_y^2 \quad \frac{1}{I}\frac{d^2I}{dz^2} = k_z^2 = k_g^2. \tag{7}$$

With a change of variables to $\eta = (\omega^2/c^2 L)^{1/3}(x - L)$, the x-dependent part takes the following form

$$\frac{d^2F}{d\eta^2} - \eta F = 0. \tag{8}$$

This differential equation defines the well-documented Airy functions A_i and B_i together with the general solution as

$$F(\eta) = \alpha A_i(\eta) + \beta B_i(\eta), \tag{9}$$

where α and β are constants that can be determined by matching to the boundary conditions. Since it is seen that $B_i(\eta) \to \infty$ as $\eta \to \infty$, we choose $\beta = 0$. Then the constant α is obtained by matching the magnetic field with the field of the incident wave at the interface between the vacuum and the front of plasma filled waveguide at $x = 0$, i.e., at $\eta = -(\omega L/c)^{2/3}$. Further if we assume that $(\omega L/c) >> 1$ and use the asymptotic representation $A_i(\eta) = (1/\sqrt{\pi}\eta^{1/4})\cos((2/3)\eta^{2/3} - (\pi/4))$, we obtain $\alpha = 2\sqrt{\pi}(\omega L/c)^{1/6}A_0$. Here A_0 is the value of magnetic field in the free space corresponding to the microwave intensity I_0. With this we write the expression for the $F(x)$ component as

$$F(x) = 2A_0\left(\frac{L}{x - L}\right)^{1/4}\cos\left[\frac{2}{3}\left(\frac{\omega^2}{Lc^2}\right)^{2/9}(x - L)^{2/3} - \frac{\pi}{4}\right].$$

With the use of solutions for the y- and z- dependence of H_z one can write the complete solution as

$$H_z(x, y, z) = 2A_0\left(\frac{L}{x - L}\right)^{1/4}\cos \times \left[\frac{2}{3}\left(\frac{\omega^2}{Lc^2}\right)^{2/9}(x - L)^{2/3} - \frac{\pi}{4}\right]\cos\left(\frac{n\pi y}{b}\right)e^{ik_g z}. \tag{10}$$

Here $k_g = \sqrt{k^2 - k_c^2}$ together with $-k_c^2 = k_x^2 + k_y^2$. One can obtain the following expressions for H_x, H_y, E_x, and E_y in terms of H_z from the Maxwell's equations for the space dependence as $e^{ik_g z}$

$$H_x = \left(\frac{ik_g}{n_1^2}\right)\frac{\partial H_z}{\partial x}, \qquad H_y = \left(\frac{ik_g}{n_1^2}\right)\frac{\partial H_z}{\partial y},$$
$$E_x = \left(\frac{i\omega\mu_0}{n_1^2}\right)\frac{\partial H_z}{\partial y}, \quad E_y = \left(\frac{-i\omega\mu_0}{n_1^2}\right)\frac{\partial H_z}{\partial x},$$

where $n_1^2 = [k_g^2 - (\omega^2/c^2)\,(1 - (\omega_p^2/\omega^2))]$.

The above relations are general expressions. The contribution of waveguide can be entered through the property of a pure conductor in view of which the tangential component of the electric field (e.g., E_y) vanishes at the conducting waveguide walls, i.e., at $x = 0$ and a. This condition is equivalent to $(\partial H_z/\partial x)_{x=0,a} = 0$, which yields the following

$$(-L)^{2/3}\left(\frac{16}{9}\right)\left(\frac{\omega^2}{Lc^2}\right)^{2/9}\tan\left[\frac{2}{3}\left(\frac{\omega^2}{Lc^2}\right)^{2/9}(-L)^{2/3} - \frac{\pi}{4}\right] = -1,$$

$$\text{for } x = 0, \tag{11a}$$

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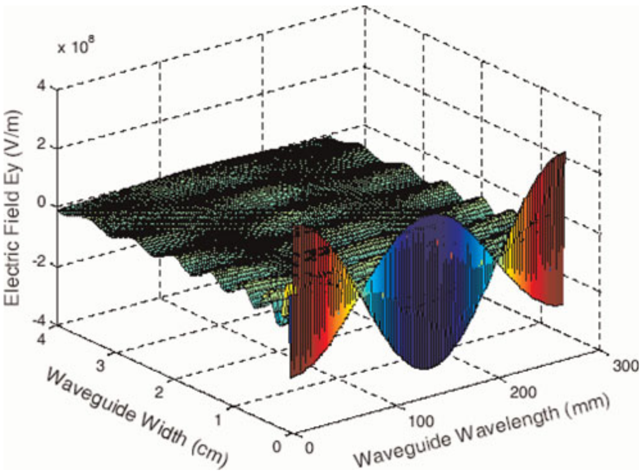

Fig. 2. (Color online) 3D variation of the resultant electric field E_y in the waveguide for one guide wavelength when $I_0 = 1 \times 10^{10}$ W/cm^2, $f = 11.6$ GHz, and $n_0 = 5 \times 10^{16}$/m^3 in a 4.0 cm × 2.5 cm rectangular waveguide.

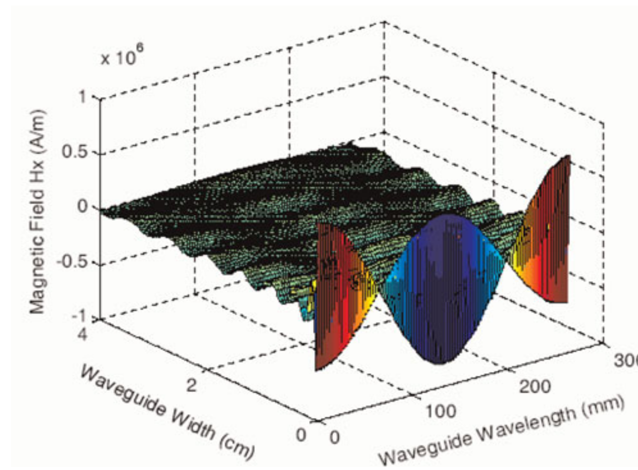

Fig. 3. (Color online) 3D variation of the resultant magnetic field H_x in the waveguide for one guide wavelength and the same parameters as in Figure 2.

$$(a-L)^{2/3}\left(\frac{16}{9}\right)\left(\frac{\omega^2}{Lc^2}\right)^{2/9}\tan\left[\frac{2}{3}\left(\frac{\omega^2}{Lc^2}\right)^{2/9}(a-L)^{2/3}-\frac{\pi}{4}\right] = -1,$$

for $x = a$. (11b)

For typical values of microwave frequency ω and waveguide width a, we solve these two coupled equations and obtain the common values of L for which the field components would be applicable to the waveguide. Keeping this in mind and taking $X_a = (1/(x-L))$ and $a_1 = (2/3)\ (\omega^2/Lc^2)^{2/9}\ (x-L)^{2/3} - (\pi/4)$, we write the solution of H_z in the simpler form as $H_z(x, y, z) = 2A_0\ (LX_a)^{1/4} \cos a_1 \cos (n\pi y/b) e^{ik_g z}$.

Finally one can obtain the complete set of field components of the fundamental TE_{10} mode as given below

$$H_x = \left(\frac{ik_g}{n_1^2}\right)2A_0(LX_a)^{1/4}\left[-\frac{X_a}{4}\cos a_1 - \frac{4}{9X_a^{1/3}}\left(\frac{\omega^2}{Lc^2}\right)^{2/9}\sin a_1\right]e^{ik_g z},$$

$$E_y = \left(\frac{i\omega\mu_0}{n_1^2}\right)2A_0(LX_a)^{1/4}\left[\frac{X_a}{4}\cos a_1 + \frac{4}{9X_a^{1/3}}\left(\frac{\omega^2}{Lc^2}\right)^{2/9}\sin a_1\right]e^{ik_g z},$$

$$H_z = 2A_0(LX_a)^{1/4}\cos a_1 e^{ik_g z}. \quad (12)$$

Figures 2 to 4 show the variation of field components E_y, H_x, and H_z, of the TE_{10} mode in a 4.0 cm × 2.5 cm waveguide for one guide wavelength when $I_0 = 1 \times 10^{10}$ W/cm^2, $f = 11.6$ GHz, and $n_0 = 5 \times 10^{16}$/m^3. Here we mention that due to some problem in programming, we are unable to show the variation until the wall $x = 0$. However, the field component E_y vanishes at both the walls at $x = 0$ and a, which confirms the appropriateness of field components to the TE_{10} mode in the waveguide. Further, a comparison of Figure 3 with Figure 4 reveals that the component H_x dominates over the component H_z.

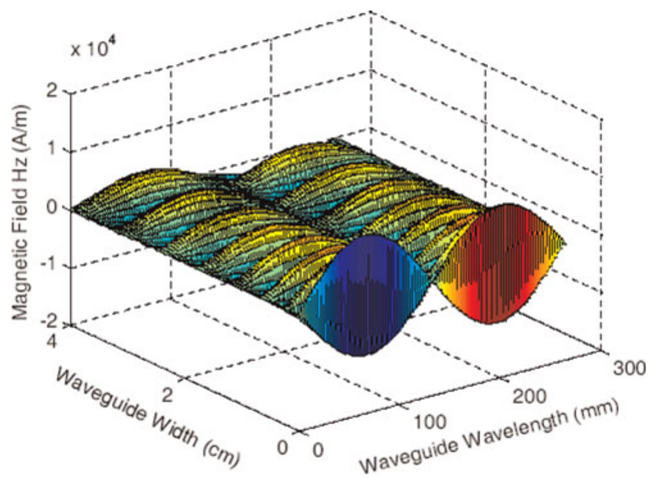

Fig. 4. (Color online) 3D variation of the resultant magnetic field H_z in the waveguide for one guide wavelength. Here, the parameters are the same as in Figure 2.

DISCUSSION ON MODE PROPAGATION

By putting the solution of H_z in Eq. (5), we get the following dispersion relation for the TE_{10} mode

$$\omega^2 = \omega_p^2 + c^2\left[k_g^2 - \frac{5}{16}X_a^2 - \frac{10}{27}\left(\frac{2\omega^2}{ac^2}\right)^{2/9}X_a^{4/3}\tan\right.$$
$$\left.\times\left\{\frac{2}{3}\left(\frac{2\omega^2}{ac^2}\right)^{2/9}(x-L)^{2/3}-\frac{\pi}{4}\right\}+\frac{16}{81}\left(\frac{2\omega^2}{ac^2}\right)^{4/9}X_a^{2/3}\right]. \quad (13)$$

Since we have already put the expression of the density (which is the function of x), we obtain this transcendental equation in ω that describes the propagation of the mode in the waveguide filled with an inhomogeneous plasma. In this context, the lower cut-off frequency is calculated by putting $k_g = 0$ and $\omega = \omega_{cutoff} = 2\pi f_L$. With this one

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obtains the following equation

$$\omega_{cutoff}^2 = \omega_p^2 + c^2\left[-\frac{5}{16}X_a^2 - \frac{10}{27}\left(\frac{2\omega_{cutoff}^2}{ac^2}\right)^{2/9} X_a^{4/3}\tan \times\left\{\frac{2}{3}\left(\frac{2\omega_{cutoff}^2}{ac^2}\right)^{2/9}(x-L)^{2/3} - \frac{\pi}{4}\right\} + \frac{16}{81}\left(\frac{2\omega_{cutoff}^2}{ac^2}\right)^{4/9} X_a^{2/3}\right]. \tag{14}$$

The above equation can be solved for the typical values of x along the waveguide width, the reference plasma density n_0, and waveguide width a in order to obtain the lower cut-off frequency for the propagation of the mode. However, the upper cut-off frequency can be obtained by replacing the width a by the height b of the waveguide in the above equation. Since the plasma has a density gradient along the waveguide width, the cut-off frequency will vary along the width. For example, the values of lower cut-off frequency f_L when n_0 is taken as $5 \times 10^{16}/\mathrm{m}^3$ in a 4.0 cm × 2.5 cm rectangular waveguide come out to be 7.96 GHz (at $x = 2.4$ cm), 7.97 GHz (at $x = 2.8$ cm), 7.99 GHz (at $x = 3.2$ cm), and 8.0 GHz (at $x = 3.6$ cm).

In order to discuss the dispersion characteristics of the mode, Figure 5 shows the behavior of $ck_g/\omega p$ and ω/ω_p with respect to x for different values of plasma density. Here the solid graphs k_1, k_2, and k_3 correspond to ck_g/ω_p for three different plasma densities $5\times10^{16}/\mathrm{m}^3$, $8\times10^{16}/\mathrm{m}^3$, and $10\times10^{16}/\mathrm{m}^3$, respectively. Similarly the dotted graphs ω_1, ω_2, and ω_3 correspond to ω/ω_p for the same respective values of plasma density. This is clear from the figure that as we move away from $x = a/2$ toward $x = a$, ω/ω_p as well as ck_g/ω_p get reduced. For a fixed microwave frequency, the decrease of ck_g/ω_p means the propagation constant k_g gets smaller or the mode velocity gets larger. Moreover, the wavelength of the microwave, i.e., guide

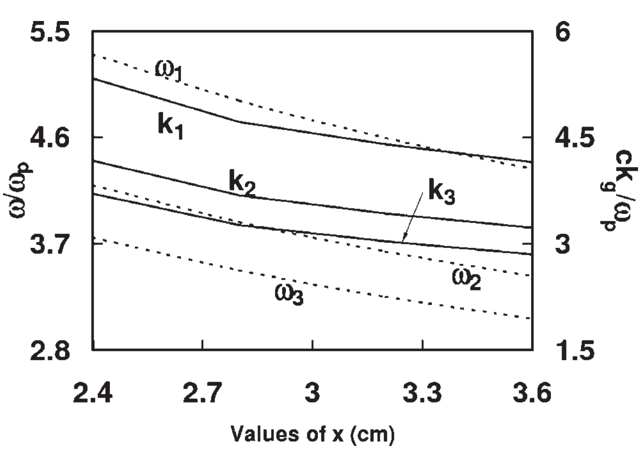

Fig. 5. Behavior of ω/ω_p and ck_g/ω_p of the TE_{10} mode along the waveguide width or the distance x in a 4.0 cm × 2.5 cm waveguide when $I_0 = 1 \times 10^{10}\mathrm{W/cm}^2$.

wavelength λ_g ($=2\pi/k_g$) becomes larger as we move toward $x = a$, i.e., toward the region of higher plasma density. On the other hand, for a fixed reference plasma density n_0, ω/ω_p decreases as we move toward $x = a$; this is because of the increased plasma oscillation frequency ω_p due to the density gradient.

ELECTRON MOTION IN THE WAVEGUIDE

Now we study the motion of the electron (or electron bunch) in the field of the mode, when it is injected along the direction of mode propagation. Here we assume that the electron bunch does not affect the fields of the mode and employ the same approach as used by Malik (2003) and Jawla *et al.* (2005) for calculating the expression of the angle of deflection of the electron motion from the direction of mode propagation (z-axis). The y-component of electron momentum equation under the effect of field components E_y, H_x, and H_z gives rise to the following expression for the angle θ

$$\tan\theta = \frac{eE_0}{m_e v_z \omega k_g^2 (T_2+T_3)^2}\left[k_g\left\{\left(\frac{\omega + v_{z_0}k_g}{v_{z_0}}\right) - \left(\frac{\omega + v_z k_g}{v_z}\right)\cos k_g z\right\}(T_2+T_3)^2 + n_1^4(\cos k_g z - 1)\cos^2 a_1\right], \tag{15}$$

where $T_2 = (1/8)\, X_a \cos a_1$, $T_3 = (4/9)\,(2\omega^2/ac^2)^{2/9}\,(X_a)^{1/3}\sin a_1$, v_{z0} is the initial velocity of electron corresponding to its energy with which it is injected in the waveguide and E_0 is the field corresponding to the microwave intensity I_0. It is clear from this relation that the angle of deflection is directly proportional to E_0 and it also has a dependence on the waveguide width a, microwave frequency ω and reference plasma density n_0. Here it is also evident that the electron will execute sinusoidal oscillations in the waveguide. Therefore, the microwave parameters and the waveguide dimensions should be optimized such that the amplitude of oscillations always remains less than half of the waveguide height. We will take this point into consideration, when analyze the acceleration of the electron in the next section.

ELECTRON ACCELERATION: ENERGY GAIN

In order to analyze the electron acceleration in the waveguide, we employ the momentum equation $(d\vec{p}/dt) = -e(\vec{E} + \mu_0\vec{v}\times\vec{H})$ and energy equation $(d(m_e\gamma c^2)/dt) = -e(\vec{v}.\vec{E})$ of the electron (or electron bunch) under the effect of the mode field components. Since TE_{10} mode is a fast wave ($v_p > c$), a synchronous interaction of the moderate energy (few hundred keV) electrons with the mode is not possible. However, in the present paper, we are interested in accelerating such electrons and therefore analyze the simple interaction on the scale of velocity v_g.

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We transform the coordinates to $\varsigma = v_g t - z$ and obtain the following relations from the energy and momentum equations

$$\frac{(v_g - v_z)d(m_e\gamma v_y)}{d\varsigma} = -e[\vec{E}_y + \mu_0(v_z H_x - v_x H_z)], \tag{16}$$

$$\frac{(v_g - v_z)d(m_e\gamma c^2)}{d\varsigma} = -e(v_x H_x + v_y H_y + v_z H_z). \tag{17}$$

Also, the following relation between v_x and v_z is realized

$$\frac{d(\gamma v_x)}{d\varsigma} + \left(\frac{H_z}{H_x}\right)\frac{d(\gamma v_z)}{d\varsigma} = 0. \tag{18}$$

Then with the use of Eqs. (16) to (18) we obtain the following second order differential equation

$$\frac{d\gamma}{d\varsigma}\left(\frac{d(\gamma v_z)}{d\varsigma}\right) + \gamma\left(\frac{d^2(\gamma v_z)}{d\varsigma^2}\right) = -\frac{e^2\mu_0}{m_e^2 v_z(v_g - v_z)^2} \times [E_y H_x + \mu_0 v_z(H_x^2 + H_z^2)]. \tag{19}$$

Now we substitute the values of field components E_y, H_x, and H_z in the above equation and obtain the following solution by treating γ (or v_z) as a slowly varying function

$$\left(\frac{d\gamma}{dz}\right) = \frac{eE_0}{m_e\omega(T_2 + T_3)} \times \left[\begin{array}{c} k_g(T_2+T_3)^2\left\{\left(\frac{\gamma_0}{\gamma}\right)^2\left(\frac{\omega + v_{z0}k_g}{v_{z0}(v_g - v_z)^2}\right) \right. \\ \left. -\left(\frac{\omega + v_z k_g}{v_z(v_g - v_z)^2}\right)\cos(2k_g z)\right\} \\ +n_1^4\left\{\frac{\cos(2k_g z)}{(v_g - v_z)^2} - \left(\frac{\gamma_0}{\gamma}\right)^2\left(\frac{1}{(v_g - v_z)^2}\right)\right\}\cos^2 a_1 \end{array}\right]^{1/2}. \tag{20}$$

This relation gives the change in γ per unit length for the electron during its motion in the waveguide. Therefore, by multiplying it with the factor $(m_e c^2/e)$ we can obtain the energy gain or acceleration gradient in eV/m as

$$\left(\frac{\Delta U}{\Delta z}\right)_{ev/m} = \left(\frac{c^2E_0}{\omega(T_2+T_3)}\right) \times \left[\begin{array}{c} k_g(T_2+T_3)^2\left\{\begin{array}{c}\left(\frac{\gamma_0}{\gamma}\right)^2\left(\frac{\omega+v_{z0}k_g}{v_{z0}(v_g-v_{z0})^2}\right) \\ -\left(\frac{\omega+v_z k_g}{v_z(v_g-v_z)^2}\right) \\ \times\cos(2k_g z)\end{array}\right\} \\ +n_1^4\left\{\frac{\cos(2k_g z)}{(v_g-v_z)^2} - \left(\frac{\gamma_0}{\gamma}\right)^2\left(\frac{1}{(v_g-v_z)^2}\right)\right\}\cos^2 a_1 \end{array}\right]^{1/2}. \tag{21}$$

CONDITION FOR LARGER ENERGY GAIN

This is clear from Eq. (21) that the acceleration gradient is strongly affected by the plasma density inhomogeneity *via* a_1 and X_a terms (in the expressions of T_2 and T_3) and it increases for the higher field E_0 or the microwave intensity I_0. We can obtain the point of injection and position where the electron will achieve larger energy gain (or the acceleration gradient). For this an approximate estimation of the point of injection can be obtained from the term $\cos a_1$ for which a_1 should be zero. This reads

$$x = L \pm \left(\frac{3\pi}{8}\right)^{3/2}\left(\frac{Lc^2}{\omega^2}\right)^{1/3}. \tag{22}$$

It means larger energy gain depends on the microwave frequency ω and the density inhomogeneity scale length L. On the other hand, the value of x cannot be exactly 0 or a. This point leads to the following condition for the minimum value of L

$$L_{\min} = \left(\frac{c}{\omega}\right)\left(\frac{3\pi}{8}\right)^{9/4}. \tag{23}$$

Interestingly, the above equation infers that the waveguide filled with inhomogeneous plasma having lower scale of density gradient is more appropriate for getting effective acceleration if we use larger frequency microwave for exciting the fundamental TE_{10} mode, otherwise *vice-versa* is true. Now with Eq. (22) we obtain the following expression for larger acceleration gradient in eV/m

$$\left(\frac{\Delta U}{\Delta z}\right)_{(ev/m)_L} = \left(\frac{8c^2E_0}{\omega X_a}\right) \times \left[\begin{array}{c} \left(\frac{k_g}{64(x-L)^2}\right)\left\{\left(\frac{\gamma_0}{\gamma}\right)^2\left(\frac{\omega + v_{z0}k_g}{v_{z0}(v_g - v_z)^2}\right) \right. \\ \left. -\left(\frac{\omega + v_z k_g}{v_z(v_g - v_z)^2}\right)\cos(2k_g z)\right\} \\ +n_1^4\left\{\frac{\cos(2k_g z)}{(v_g - v_z)^2} - \left(\frac{\gamma_0}{\gamma}\right)^2\left(\frac{1}{(v_g - v_z)^2}\right)\right\} \end{array}\right]^{1/2}. \tag{24}$$

In addition to above Eqs. (22) and (23), we can see from the above equation that the maximum gradient is attained when the electron completes the distance $z = \lambda_g/4$ in the waveguide. In view of this for showing the results graphically we will restrict our calculations for energy gain and acceleration till $z = \lambda_g/4$.

RESULTS AND DISCUSSION

In view of the discussion made in the above sections, now we present the results by giving typical values to I_0, f, a, n_0, and initial electron energy E_{in}. Figure 6 shows the variation of the

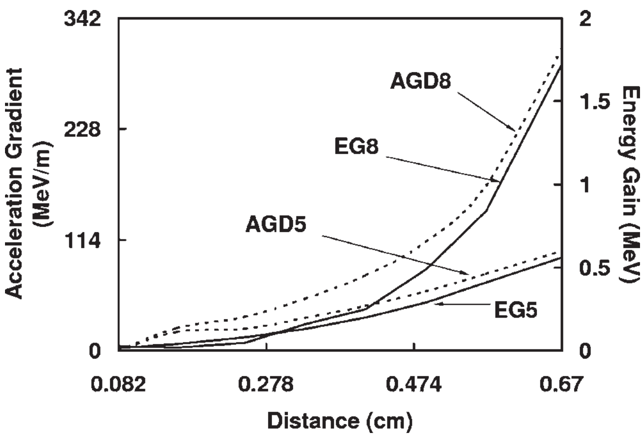

Fig. 6. Variation of the energy gain (EG) and acceleration gradient (AGD) up to $\lambda_g/4$ in a 4.0 cm × 2.5 cm waveguide for two different values of n_0 as $5 \times 10^{16}/\mathrm{m}^3$ (graphs AGD5 and EG5) and $8 \times 10^{16}/\mathrm{m}^3$ (graphs AGD8 and EG58), when $I_0 = 1 \times 10^{10}\mathrm{W/cm}^2$, $f = 11.66$ GHz, $E_{\mathrm{in}} = 500$ keV and point of injection $x = 3.2$ cm.

energy gain and the acceleration gradient along the z-axis in a 4.0 cm × 2.5 cm rectangular waveguide when $f = 11.66$ GHz, $I_0 = 1 \times 10^{10}$ W/cm^2 and $E_{\mathrm{in}} = 500$ keV. This is clear that the acceleration gradient as well as the energy gain achieved by the electron are increased during its motion in the waveguide.

Effect of Plasma Density and Waveguide Width

In Figure 7, the effect of plasma density n_0 on the total energy gain (TEG) and the maximum acceleration gradient (MGD) acquired by a 500 keV electron is shown for the parameters mentioned in the figure caption. Here, this can be seen that the effect of plasma density as well as the waveguide width is to enhance the energy gain and the acceleration gradient: the gain is increased from 0.62 MeV to 5.94 MeV whereas increase in the acceleration gradient is from 119 MeV/m to 248 MeV/m, when the plasma density is raised from $4.5 \times 10^{16}/\mathrm{m}^3$ to $8.0 \times 10^{16}/\mathrm{m}^3$ and the waveguide width remains fixed at $a = 4.0$ cm. However, for 4.2 cm width of the waveguide, the gradient is increased to 266 MeV/m and the gain to 8.20 MeV, which is quite a good amount of gain for a 500 keV electron. Also, we realize that these acceleration gradient and the energy gain are larger than those obtained by other investigators (Hirshfield *et al.*, 1996; Park & Hirshfield, 1997; Yoder *et al.*, 2001; Jing *et al.*, 2003).

The increase in gain with plasma density can be explained on the basis of the interaction of the electron with the mode fields via the waveguide wavelength λ_g. Since the propagation constant k_g decreases (hence λ_g increases) with the increasing plasma density (Fig. 5), an increase in the density finally enhances the interaction length for the effective electron acceleration.

Effect of Initial Electron Energy and Microwave Intensity

In Figure 8, we analyze the effect of electron's initial energy E_{in} on the total energy gain and the acceleration gradient for the parameters mentioned in the caption. It is clear that electrons injected with larger energy achieve larger energy gain and also higher gradient is realized. The same effect is obtained for the increasing microwave intensity: the gain (gradient) is increased from 1.05 MeV (260 MeV/m) to 3.52 MeV (440 MeV/m) when the intensity is raised from 1×10^{10} W/cm^2 to 1.5×10^{10} W/cm^2 and the electrons are injected with 550 keV energy. This is due to the increased amplitude of the electric field of the mode, which provides the accelerating force to the electron. Due to the increased force on the electron the velocity of the electron gets larger and hence its interaction with the mode fields becomes

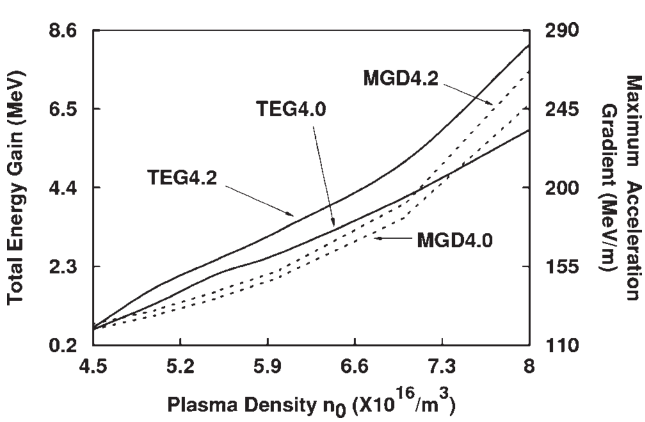

Fig. 7. Effect of the plasma density n_0 on the total energy gain (TEG) as well as on the maximum acceleration gradient (MGD) for two different widths of the waveguide when $I_0 = 1 \times 10^{10}\mathrm{W/cm}^2$, $f = 11.66$ GHz, and the electron is injected with energy of 500 keV in the waveguide at $x = 3.4$ cm. Graphs TEG4.0 (MGD4.0) and TEG4.2 (MGD4.2) are for the width $a = 4.0$ cm and 4.2 cm, respectively.

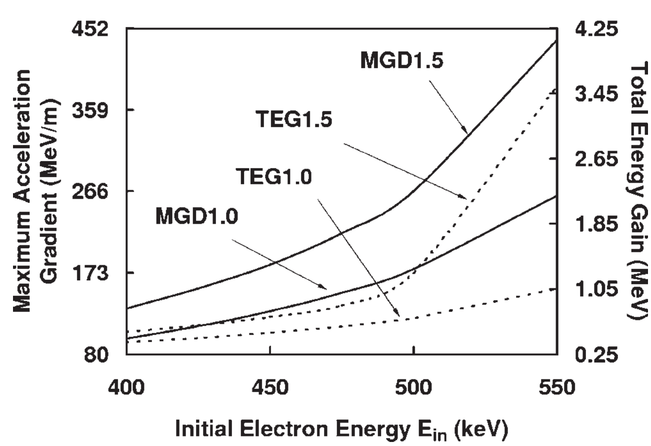

Fig. 8. Effect of initial electron energy on the maximum acceleration gradient (MGD) and total energy gain (TEG) for different values of microwave intensity $I_0 = 1 \times 10^{10}\mathrm{W/cm}^2$ (graphs MGD1.0 and TEG1.0) and $1.5 \times 10^{10}\mathrm{W/cm}^2$ (graphs MGD1.5 and TEG1.5) in a 4.0 cm × 2.5 cm rectangular waveguide when $n_0 = 5 \times 10^{16}/\mathrm{m}^3$, $f = 11.66$ GHz and point of injection $x = 3.4$ cm.

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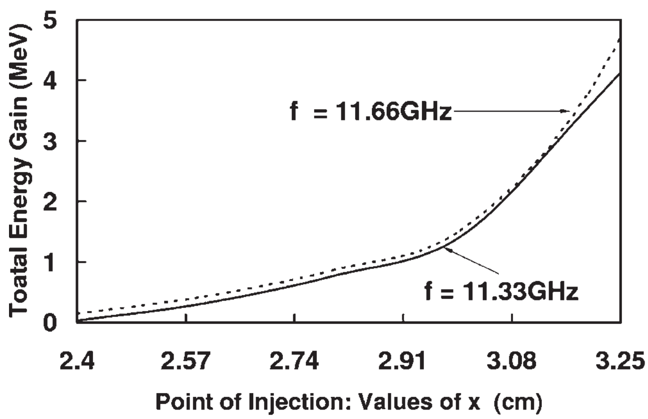

Fig. 9. Effect of point of injection on the total energy gain for a 500 keV electron at 11.33 GHz and 11.66 GHz microwave frequencies, when $I_0 = 1 \times 10^{10}$ W/cm^2 and $n_0 = 8 \times 10^{16}/\text{m}^3$.

stronger, which leads to the larger acceleration gradient and higher energy gain to the electrons.

Effect of Microwave Frequency and Point of Electron Injection

The effect of point of injection of the electrons and the microwave frequency on the total energy gain is shown in Figure 9 for a 500 keV electron at $I_0 = 1 \times 10^{10}$ W/cm^2 in a 4.0 cm $\times$ 2.5 cm rectangular waveguide. Here, it can be seen that the energy gain is enhanced from 3.92 MeV to 4.49 MeV when the microwave frequency is raised from 11.33 GHz to 11.66 GHz. It means the microwave frequency has a significant effect on the electron energy gain and the acceleration gradient. Since cut-off frequency of the microwave is decided by the plasma density in addition to the waveguide width, the plasma density plays a crucial role in selecting the microwave frequency for achieving larger electron acceleration. Moreover, the energy gain is larger when the electrons are injected at larger values of x, i.e., toward the side of higher plasma density region (Fig. 9). This reveals that a stronger density gradient would be more helpful to achieve larger energy gain.

CONCLUDING REMARK

By considering a more realistic case of inhomogeneous plasma in a rectangular waveguide, the present analysis shows that larger acceleration gradient and energy gain can be achieved at the larger microwave frequency, higher intensity, longer waveguide width and higher plasma density. Since both the waveguide wavelength and the cut-off frequency level are enhanced for the increasing plasma density and the gain is increased for the higher microwave frequency, a waveguide filled with higher density plasma and/or a plasma having a stronger density gradient are more suitable for the effective particle acceleration.

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