Probability in the Engineering and Informational Sciences, **27**, 2013, 465–466.
doi:10.1017/S026996481300020X

COMMENTS ON THE SURVEY BY BALAKRISHNAN AND ZHAO

MARCO BURKSCHAT
RWTH Aachen University
Institute of Statistics
D-52056 Aachen, Germany
E-mail: burkschat@stochastik.rwth-aachen.de

At first, I would like to thank the authors for giving the detailed survey on the current state of stochastic orderings of order statistics from heterogeneous populations. Their review illustrates the fast development of research in this area during the recent years, in particular including many important contributions by the authors themselves. Besides the concrete open problems given in the different sections of the review, the following general topics may be also of interest for further studies.

It is apparent from the survey paper that the situation of discrete underlying distributions has received much less attention than that of continuous distributions. This gap, especially in comparison to the wealth of results for the exponential distribution, provides a large field of study. As Example 4.2 in the present survey illustrates, new phenomena already arise in the case of the geometric distribution in comparison to known results for its continuous counterpart. In particular, stochastic orderings of sample ranges, or generally spacings, remain to be explored.

In the case of continuous population distributions, usually the exponential distribution represents the starting point of the analysis. Corresponding ordering results are then extended to, for example, Weibull and gamma distributions. One could also study other extensions of the exponential distribution. Similar to the mentioned examples, these extensions could be obtained by introducing additional parameters. Obviously, there exist many different approaches to do so. For instance, a variety of commonly used methods and their interpretations is given by Marshall and Olkin [4].

Furthermore, a more distant aim may be the extension of results for order statistics to progressively Type-II censored order statistics. In a progressively Type-II censored life test, a fixed number of randomly chosen intact units may be removed from the experiment after every failure of a unit. The ordered lifetimes of the observed failed units are the progressively censored order statistics. A recent survey on progressive censoring is given by Balakrishnan [1]. Clearly, if no censoring takes place, then the observations in such an experiment coincide with usual order statistics. Moreover, given a fixed sample size, the distributions of the first progressively censored order statistic and the first order statistic are identical by construction and, therefore, corresponding results for stochastic comparisons can be immediately transferred. However, the distributions of the second and larger progressively censored order statistics clearly depend on the employed censoring scheme. The distribution theory of progressively censored order statistics from heterogeneous populations has

© Cambridge University Press 2013 0269-9648/13 $25.00

https://doi.org/10.1017/S026996481300020X Published online by Cambridge University Press

been studied in Balakrishnan and Cramer [2] and Fischer, Balakrishnan, and Cramer [3]. It would be interesting to find stochastic comparisons also in this extended setting. In the latter paper, first results in this direction are given for the usual (multivariate) stochastic order. In the other paper, questions of robust estimation in the single-outlier model with exponential distributions are addressed. Examining that particular model may serve as a first step in order to obtain stronger stochastic orderings.

References

1. Balakrishnan, N. (2007). Progressive censoring methodology: An appraisal (with Discussion). *TEST* 16: 211–259.
2. Balakrishnan, N. & Cramer, E. (2008). Progressive censoring from heterogeneous distributions with applications to robustness. *Annals of the Institute of Statistical Mathematics* 60: 151–171.
3. Fischer, T., Balakrishnan, N., & Cramer, E. (2008). Mixture representation for order statistics from INID progressive censoring and its applications. *Journal of Multivariate Analysis* 99: 1999–2015.
4. Marshall, A. W. & Olkin, I. (2007). *Life distributions*. New York: Springer.

https://doi.org/10.1017/S026996481300020X Published online by Cambridge University Press