Robotica (2000) volume 18, pp. 535–543. Printed in the United Kingdom © 2000 Cambridge University Press

Kinematics and dynamics of a parallel manipulator with a new architecture

A. Fattah and G. Kasaei

Department of Mechanical Engineering, Isfahan University of Technology, Isfahan (Iran) 84154
e-mail: fattah@cc.iut.ac.ir

(Received in Final Form: September 29, 1999)

SUMMARY

In this paper, the kinematics and dynamics of a parallel manipulator with a new architecture supposed to be used as a moving mechanism in a flight simulator project is studied. This manipulator with three independent degrees of freedom consists of a moving platform connected to a based platform by means of three legs. Kinematic solutions for this manipulator at position, velocity and acceleration levels are obtained. Moreover, the dynamical equations of motion of the manipulator are determined using Newton-Euler's equations and applying the natural orthogonal complement (NOC) method. Using kinematics and dynamics and also performing simulation for different manoeuvres of moving platform, the motion and the actuator forces of the legs are obtained.

KEYWORDS: Parallel manipulators; Forward and inverse kinematics; Inverse dynamics; Natural orthoganal complement

1. INTRODUCTION

Parallel manipulators have many applications in space structures and flight simulators. The main advantages of parallel manipulators, as compared with their counterparts, are greater rigidity, lower intertia load, higher accuracy due to the lack of cantilever structures, and higher load-carrying capacity.[1,2] In flight simulators comprise parallel manipulators, it is possible to have manoeuvres with desired frequencies for the cockpit. However, these manoeuvres are impossible with serial manipulators. Therefore, parallel manipulators are used as moving mechanism in most flight simulators. In this paper we address the kinematics and dynamics of a parallel manipulator with a new architecture supposed to be used for the moving mechanism of a flight simulator. The merits of this type of architecture, as compared with conventional type like Stewart platform,[3] are independent and low degrees of freedom (DOF), easy to control, advantage of manufacturing, chance to increase DOF by changing the type of joints. This parallel manipulator is composed of a moving platform (MP), a base platform (BP), and four legs, as depicted in Figure 1. Three legs are connected to MP by spherical joints and coupled to BP by universal joint. The fourth or central leg is connected to centroid of MP by universal joint and is fixed to BP. Each leg contains two links coupled by a prismatic joint. The DOF for the system at hand is obtained to be 3 using Chebyshev-Grübler-Kutzbach formula. Thus three linear hydraulic actuators are used to derive the prismatic joints of three legs with idle prismatic joint for the central leg. This new design with the central leg provides three independent DOF for the MP, namely, heave h, vertical displacement of MP along Z_0; pitch ψ, rotation of MP about y axis; and roll ϕ, rotation of MP about x axis, as shown in Figure 1. The independent DOF for MP is necessary for moving mechanisms in flight simulators in which any desired and independent motion for the MP should be available. This is a main advantage of this new type of manipulator as compared to its counterpart with 3 DOF.[4]

In this paper the kinematics and dynamics of the manipulator at hand are studied. Kinematics and dynamics of parallel manipulators have been considered by numerous investigators.[5–13] Inverse and forward kinematics are two basic problems arise in kinematics of the manipulator at hand. Inverse kinematics has been solved to obtain the motion of linear actuators of the legs for the desired motion of MP at position, velocity and acceleration levels. In forward kinematics, having the motion of three actuators of the legs, the motion of MP has been determined. Here, having position and velocity of three linear actuators and writing the kinematic constraint equations for the system at hand, the dependent or passive generalized coordinates and speeds can be expressed in terms of independent counterparts, i.e. position and velocity of three actuators. Then the position and velocity of MP can be obtained by having the positions and velocities of three noncollinear points of MP.

The governing equations of motion of this manipulator can be expressed in terms of nonlinear differential equations by modelling the manipulator as a mechanical system with kinematic loops. The independent governing equations of motion of the system can be determined using NOC method. This method is based on determining the orthogonal complement of the kinematic constraint velocity matrix. The form of this matrix depends on whether the system is being formulated in joint space or in Cartesian space. It has been shown that using the methodology of NOC leads to the elimination of the nonworking kinematic-constraint wrenches and also to the derivation of the minimum number of equations. A basic problem related to dynamics of the manipulator at hand is the study of inverse dynamics. Inverse dynamics has potential applications in obtaining the power of actuators and in controlling the system of the manipulator at hand. To this end, having the motion of MP

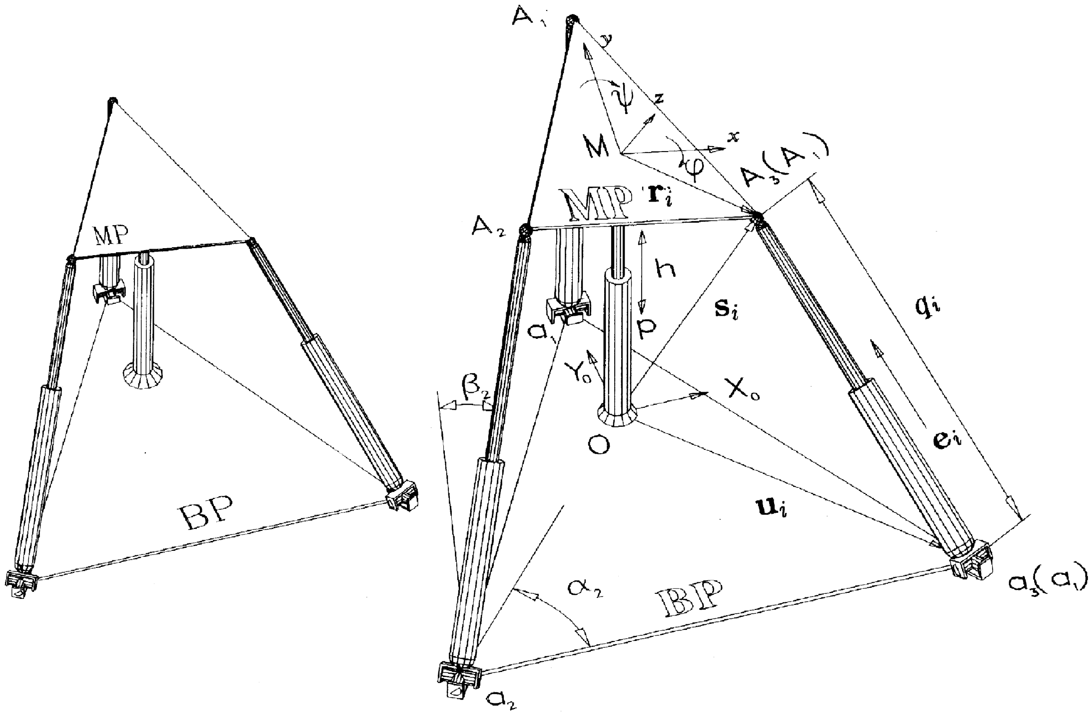

Fig. 1. The parallel manipulator with the new architecture.

and using the inverse kinematics of the system, the time history of legs motion and their time derivatives are determined. Thereafter, the actuator forces of the legs can be obtained such that it produces the desired motion of MP.

Several numerical examples are solved for different manoeuvres of MP to obtain the motion and forces of linear actuators of the legs. The results are validated using the forward kinematic and dynamics solution.

2. KINEMATICS

The forward and inverse kinematics of the manipulator at hand are studied here.

2.1. Inverse kinematics

Inverse kinematics is determined for position, velocity and acceleration.

With reference to Figure 1, the geometrical parameters of the manipulator at hand are defined as follows: The reference frame $X_0Y_0Z_0$ located at O, the coordinate frame xyz located at M and attached to MP, the *ith* leg is connected to MP by A_i and to BP by a_i, position vector of a_i with respect to O by $\mathbf{u}_i$, the length of *ith* leg by q_i; its unit vector by $\mathbf{e}_i$ and position vector of A_i with respect to a_i by $\mathbf{l}_i$, angle of leg i with Z_0 by β_i and angle of the projection of leg i on BP with the axis X_0 by α_i, position vector of A_i with respect to M expressed in reference frame and frame xyz by $\mathbf{r}_i$ and ${}^M\mathbf{r}_i$, respectively ($i=1,2,3$). Moreover, position vector of M with respect to O is defined by $\mathbf{p}$ and the rotation matrix of MP with respect to reference frame by $\mathbf{R}$. The rotation matrix $\mathbf{R}$ can be readily determined by having the roll and pitch angle of MP.

The inverse position kinematics is defined as follows: Given $\mathbf{p}$ and $\mathbf{R}$, determine the motion of actuators of the legs, i.e. q_i and $\mathbf{e}_i$. The vector $\mathbf{l}_i=\overrightarrow{a_iA_i}$ can be written as

$$\mathbf{l}_i=q_i\mathbf{e}_i=-\mathbf{u}_i+\mathbf{p}+\mathbf{r}_i \qquad \text{for} \quad i=1,2,3 \tag{1}$$

The length of *ith* leg q_i can be expressed as

$$q_i^2=\mathbf{l}_i^T\mathbf{l}_i=(-\mathbf{u}_i+\mathbf{p}+\mathbf{r}_i)^T(-\mathbf{u}_i+\mathbf{p}+\mathbf{r}_i) \tag{2}$$

Having $\mathbf{u}_i$, $\mathbf{p}$ and $\mathbf{r}_i=\mathbf{R}^M\mathbf{r}_i$, length q_i can be easily determined from equation (2). Moreover, the unit vector $\mathbf{e}_i$ can be obtained from equation (1) as

$$\mathbf{e}_i=\frac{\mathbf{l}_i}{q_i}=\frac{-\mathbf{u}_i+\mathbf{p}+\mathbf{r}_i}{q_i} \tag{3}$$

In other words, the unit vector $\mathbf{e}_i$ can be written in terms of angles α_i and β_i as

$$\mathbf{e}_i=[\,\sin\beta_i\cos\alpha_i \quad \sin\beta_i\sin\alpha_i \quad \cos\beta_i]^T \tag{4}$$

The angles α_i and β_i are determined from equation (4) in terms of $\mathbf{e}_i$ for $i=1,2,3$.

The inverse velocity kinematics for the manipulator at hand can be defined as follows: Given the linear velocity of point M of MP, i.e., $\dot{\mathbf{p}}$ and $\dot{\mathbf{R}}$, determine the velocity of each leg, namely, $\dot{\mathbf{q}}_i$ and $\dot{\mathbf{e}}_i$. This can be done by differentiating both sides of equation (2) with respect to time, thus obtaining

$$(-\dot{\mathbf{u}}_i+\dot{\mathbf{p}}+\dot{\mathbf{r}}_i)^T(-\mathbf{u}_i+\mathbf{p}+\mathbf{r}_i) + (-\mathbf{u}_i+\mathbf{p}+\mathbf{r}_i)^T(-\dot{\mathbf{u}}_i+\dot{\mathbf{p}}+\dot{\mathbf{r}}_i)=2q_i\,\dot{q}_i \tag{5}$$

where $\mathbf{u}_i$ is constant and thus $\dot{\mathbf{u}}_i=0$. Moreover, defining $\boldsymbol{\omega}$ as the angular velocity of MP with respect to reference frame and expanding equation (5), one can obtain

$$(-\mathbf{u}_i+\mathbf{p}+\mathbf{r}_i)^T\dot{\mathbf{p}}+[\mathbf{r}_i\times(-\mathbf{u}_i+\mathbf{p}+\mathbf{r}_i)]^T\boldsymbol{\omega}=q_i\,\dot{q}_i \tag{6}$$

Equation (6) can be written in matrix form upon substituting $i=1,2,3$ as follows

$$\mathbf{A}\mathbf{t}_{MP}=\mathbf{B}\dot{\mathbf{q}} \tag{7}$$

https://doi.org/10.1017/S026357470000271X Published online by Cambridge University Press

where $\mathbf{t}_{MP}=[\boldsymbol{\omega}^T \quad \dot{\mathbf{p}}^T]^T$ is a 6-dimensional twist vector of MP comprising of $\boldsymbol{\omega}$ and the linear velocity of point M in reference frame. Moreover, $\mathbf{A}$ is a 3×6 matrix, $\mathbf{B}$ is a 3×3 diagonal matrix, and $\dot{\mathbf{q}}$ is a 3-dimensional vector of the legs velocities defined, respectively, as

$$\mathbf{A}=\begin{bmatrix} \{\mathbf{r}_1\times(-\mathbf{u}_1+\mathbf{p}+\mathbf{r}_1)\}^T & (-\mathbf{u}_1+\mathbf{p}+\mathbf{r}_1)^T \\ \{\mathbf{r}_2\times(-\mathbf{u}_2+\mathbf{p}+\mathbf{r}_2)\}^T & (-\mathbf{u}_2+\mathbf{p}+\mathbf{r}_2)^T \\ \{\mathbf{r}_3\times(-\mathbf{u}_3+\mathbf{p}+\mathbf{r}_3)\}^T & (-\mathbf{u}_3+\mathbf{p}+\mathbf{r}_3)^T \end{bmatrix} \tag{8}$$

$$\mathbf{B}=\begin{bmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{bmatrix}; \qquad \dot{\mathbf{q}}=\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}$$

The angular velocity $\boldsymbol{\omega}$ of MP can be determined by writing $\dot{\mathbf{R}}$ as $\boldsymbol{\Omega}=\dot{\mathbf{R}}^T\mathbf{R}$ where $\boldsymbol{\Omega}$ is the cross-product matrix of $\boldsymbol{\omega}$. The vector $\boldsymbol{\omega}$ can be readily determined from the off-diagonal arrays of the skew-symmetric matrix $\boldsymbol{\Omega}$. Next, from equation (7), $\dot{\mathbf{q}}$ can be written as

$$\dot{\mathbf{q}}=\mathbf{B}^{-1}\mathbf{A}\mathbf{t}_{MP} \tag{9}$$

Here, the inverse of $\mathbf{B}$ can be readily calculated because $\mathbf{B}$ is a diagonal matrix.

Thereafter, $\dot{\mathbf{e}}_i$ can be determined by differentiating both sides of equation (3) with respect to time, namely,

$$\dot{\mathbf{e}}_i=\frac{(-\dot{q}_i\mathbf{e}_i+\dot{\mathbf{p}}+\dot{\mathbf{R}}^M\mathbf{r}_i)}{q_i} \tag{10}$$

The time rate of changes for the angles α_i and β_i, i.e., $\dot{\alpha}_i$ and $\dot{\beta}_i$, are determined upon differentiating both sides of equation (4) with respect to time.

Finally, the inverse acceleration kinematics are solved to obtain the leg acceleration $\ddot{q}_i$ and angular acceleration of legs with the axes of the reference frame, i.e., $\ddot{\alpha}_i$ and $\ddot{\beta}_i$ for a given linear and angular acceleration of MP. To this end, upon differentiating both sides of equation (9) with respect to time, thus obtaining $\ddot{\mathbf{q}}$ as

$$\ddot{\mathbf{q}}=(\dot{\mathbf{B}})^{-1}\mathbf{A}\mathbf{t}_{\mathbf{MP}}+\mathbf{B}^{-1}\dot{\mathbf{A}}\mathbf{t}_{\mathbf{MP}}+\mathbf{B}^{-1}\mathbf{A}\dot{\mathbf{t}}_{\mathbf{MP}} \tag{11}$$

Then $\ddot{\mathbf{e}}_i$ can be determined from differentiation of equation (10) with respect to time. Finally, $\ddot{\alpha}_i$ and $\ddot{\beta}_i$ obtain from twice differentiation of equation (4) with respect to time.

2.2. Forward kinematics

The forward kinematics can be defined as follows: Given the time history of motion of the legs, i.e. position, velocity and acceleration of the legs, namely, q_i, $\dot{q}_i$ and $\ddot{q}_i$, determine the motion of moving platform.

The main problem in forward position kinematics is how to express the passive generalized coordinates in terms of independent ones. The system at hand has 3 DOF and thus it has three independent generalized coordinates. Here, the manipulator has nine generalized coordinates which are three independent generalized coordinates, the length of the legs q_i, and six dependent of passive ones, angles α_i and β_i; for $i=1,2,3$. Therefore, six kinematic constraint equations for the system are necessary in order to express the passive generalized coordinates in terms of independent ones. The first three equations are from the fact that Euclidean norms of the three sides of the MP are constant, namely,

$$\begin{aligned} \|\overrightarrow{A_1A_2}\|^2&=(\overrightarrow{A_1A_2})^T(\overrightarrow{A_1A_2})=Const. \\ \|\overrightarrow{A_2A_3}\|^2&=(\overrightarrow{A_2A_3})^T(\overrightarrow{A_2A_3})=Const. \\ \|\overrightarrow{A_3A_1}\|^2&=(\overrightarrow{A_3A_1})^T(\overrightarrow{A_3A_1})=Const. \end{aligned} \tag{12}$$

where $\overrightarrow{A_1A_2}$, $\overrightarrow{A_2A_3}$ and $\overrightarrow{A_3A_1}$ are readily written from Figure 1 in terms of $\mathbf{u}_i$ and $\mathbf{e}_i$.

Moreover, three more kinematics constraint equations exist since the architecture of the manipulator forces the central leg to be always vertical and fixed to BP. This implies that the point M has no movement in X_0 and Y_0 directions in addition to the fact that MP has no swive, i.e. rotation about z axis of MP that can be expressed as the vector $\mathbf{s}_1$ is always located in the Y_0Z_0 plane. These constraint equations can be written as

$$\mathbf{p}\cdot\mathbf{i}_{X_0}=0; \quad \mathbf{p}\cdot\mathbf{i}_{Y_0}=0; \quad \mathbf{s}_1\cdot\mathbf{i}_{X_0}=0 \tag{13}$$

where $\mathbf{i}_{X_0}$ and $\mathbf{i}_{Y_0}$ are the unit vectors along X_0 and Y_0 respectively. The vector $\mathbf{s}_i$ can be expressed from Figure 1 as

$$\mathbf{s}_i=\mathbf{u}_i+q_i\mathbf{e}_i \qquad \text{for} \quad i=1,2,3 \tag{14}$$

Moreover, the position vector $\mathbf{p}$ can be related in terms of $\mathbf{s}_i$ as

$$\mathbf{p}=\frac{\mathbf{s}_1+\mathbf{s}_2+\mathbf{s}_3}{3} \tag{15}$$

Upon substitution of $\mathbf{s}_i$ from equation (14) into equation (15), $\mathbf{p}$ can be expressed in terms of q_i and $\mathbf{e}_i$ as

$$\mathbf{p}=\frac{\mathbf{u}_1+\mathbf{u}_2+\mathbf{u}_3+q_1\mathbf{e}_1+q_2\mathbf{e}_2+q_3\mathbf{e}_3}{3} \tag{16}$$

Introducing $\mathbf{e}_i$ from equation (4) into above equation, $\mathbf{p}$ can be expressed in terms all dependent and independent generalized coordinates.

Equations (12) and (13) are the six constraint equations, which are in terms of q_i, α_i and β_i, that should be solved numerically in order to obtain the six passive legs angles in terms of q_i. Therefore, the position vector $\mathbf{p}$ can be expressed in terms of independent generalized coordinates, q_i using equation (16). Moreover, the rotation matrix of MP with respect to reference frame can be written as

$$\mathbf{R}=[\mathbf{e}_x \quad \mathbf{e}_y \quad \mathbf{e}_z] \tag{17}$$

where $\mathbf{e}_x$, $\mathbf{e}_y$ and $\mathbf{e}_z$ are unit vectors, along x, y and z axes of the coordinate frame attached to MP, expressed in reference frame with x axis parallel to $\overrightarrow{A_2A_3}$ and z axis perpendicular to plane $A_1A_2A_3$. These unit vectors are, in turn, calculated as

$$\mathbf{e}_x=\frac{\mathbf{s}_3-\mathbf{s}_2}{\|\mathbf{s}_3-\mathbf{s}_2\|};\mathbf{e}_z=\frac{\mathbf{e}_x\times\mathbf{e}_{A_2A_1}}{\|\mathbf{e}_x\times\mathbf{e}_{A_2A_1}\|};\ \mathbf{e}_y=\mathbf{e}_z\times\mathbf{e}_x \tag{18}$$

while $\mathbf{e}_{A_2A_1}$ is unit vector along A_2A_1.

The roll and pitch angles of MP, ϕ and ψ are readily obtained from the components of the rotation matrix $\mathbf{R}$.

https://doi.org/10.1017/S026357470000271X Published online by Cambridge University Press

Thus the forward position kinematics solution is complete by having $\mathbf{p}$ and $\mathbf{R}$ from equations (16) and (17), respectively.

Forward velocity kinematics is solved as follows: Differentiating both sides of six kinematic constraint equations with respect to time and expansion of the equations thus obtained, one derives the six dimensional vector of passive generalized speeds, $\dot{\boldsymbol{\beta}}$, symbolically in terms of vector of independent generalized speeds, $\dot{\mathbf{q}}$ as

$$\dot{\boldsymbol{\beta}} = \mathbf{J}\dot{\mathbf{q}} \tag{19}$$

where $\mathbf{J}$ is a 6×3 matrix which components are functions of legs length and legs angles with the axes of reference frame.[14] Here, $\dot{\boldsymbol{\beta}}$ is a 6-dimensional vector defined as

$$\dot{\boldsymbol{\beta}} = [\dot{\alpha}_1 \quad \dot{\beta}_1 \quad \dot{\alpha}_2 \quad \dot{\beta}_2 \quad \dot{\alpha}_3 \quad \dot{\alpha}_3]^T \tag{20}$$

After expressing all passive generalized speeds in terms of independent ones, one obtains the velocity of three noncollinear points of MP, A_1, A_2 and A_3 that are now expressed in terms of independent generalized speeds, $\dot{\mathbf{q}}$.

The linear velocity of point M of MP and angular velocity of MP can be calculated using the method presented in reference 15. The velocity of point M is expressed as

$$\dot{\mathbf{p}} = \frac{\dot{\mathbf{s}}_1 + \dot{\mathbf{s}}_2 + \dot{\mathbf{s}}_3}{3} \tag{21}$$

The angular velocity of MP with respect to reference frame can be calculated by having the velocity of three points of MP as

$$\boldsymbol{\omega} = \mathbf{T}^{-1}\mathrm{vect}(\dot{\boldsymbol{\Pi}}) \tag{22}$$

where $\mathbf{T}^{-1}$ is defined, under the condition that neither $\mathrm{tr}(\boldsymbol{\Pi})$ nor $\mathrm{tr}^2(\boldsymbol{\Pi})$-$\mathrm{tr}(\boldsymbol{\Pi}^2)$ vanish, as

$$\mathbf{T}^{-1} = \frac{2}{\mathrm{tr}\boldsymbol{\Pi}}\mathbf{1}_{33} + \frac{4}{\mathrm{tr}\boldsymbol{\Pi}\,[\mathrm{tr}^2(\boldsymbol{\Pi}) - \mathrm{tr}(\boldsymbol{\Pi}^2)]}\boldsymbol{\Pi}^2 \tag{23}$$

where $\mathbf{1}_{33}$ is the 3×3 indentity matrix. While $\boldsymbol{\Pi}$ and $\dot{\boldsymbol{\Pi}}$ are 3×3 matrices defined as

$$\boldsymbol{\Pi} = [\mathbf{s}_1 - \mathbf{p} \quad \mathbf{s}_2 - \mathbf{p} \quad \mathbf{s}_3 - \mathbf{p}] \tag{24}$$
$$\dot{\boldsymbol{\Pi}} = [\dot{\mathbf{s}}_1 - \dot{\mathbf{p}} \quad \dot{\mathbf{s}}_2 - \dot{\mathbf{p}} \quad \dot{\mathbf{s}}_3 - \dot{\mathbf{p}}]$$

with $\mathrm{tr}(\cdot)$ and $\mathrm{vect}(\cdot)$ are trace and vector of matrix $(\cdot)$, respectively.

Forward acceleration kinematics can be also solved by differentiating equations (21) and (22) with respect to time to obtain $\ddot{\mathbf{p}}$ and $\dot{\boldsymbol{\omega}}$.

3. DYNAMICS

The governing equations of motion can be determined as follows: First the dynamical equations of motion for each link of the legs and MP are written using Newton-Euler equations. These equations are expressed in terms of the twist vector of each link that is a six dimensional vector composed of angular and linear velocities of the link. Then by assembling all equations of motion of all links together, the governing equations of the whole system are obtained. The next step is to formulate the NOC matrix $\mathbf{N}$ and premultiplying the governing equations by $\mathbf{N}$ to obtain the minimum number of equations of motion of the system at hand.

3.1. *Modelling*

The Newton-Euler's formula for each link of the system can be written as

$$\mathbf{M}_i\dot{\mathbf{t}}_i + \boldsymbol{\Omega}_i\mathbf{M}_i\mathbf{t}_i = \mathbf{w}_i \tag{25}$$

$$\mathbf{w}_i = \mathbf{w}_i^E + \mathbf{w}_i^C \tag{26}$$

where $\mathbf{t}_i$ is twist vector of link i which can be expressed in terms of angular velocity of link i, i.e., $\boldsymbol{\omega}_i$ and linear velocity of center of mass of link i, i.e., $\dot{\mathbf{c}}_i$ as

$$\mathbf{t}_i = \begin{bmatrix} \boldsymbol{\omega}_i \\ \dot{\mathbf{c}}_i \end{bmatrix} \tag{27}$$

Moreover, $\mathbf{M}_i$ and $\boldsymbol{\Omega}_i$ are extended mass matrix and angular velocity matrix that can be defined as

$$\mathbf{M}_i = \begin{bmatrix} \mathbf{I}_i & 0_{33} \\ 0_{33} & m_i\mathbf{1}_{33} \end{bmatrix}; \qquad \boldsymbol{\Omega}_i = \begin{bmatrix} \boldsymbol{\omega}_i \times \mathbf{1}_{33} & 0_{33} \\ 0_{33} & 0_{33} \end{bmatrix} \tag{28}$$

Here $\mathbf{I}_i$ is inertia matrix of link i about its center of mass, 0_{33} and $\mathbf{1}_{33}$ are 3×3 zero and identity matrices, respectively, m_i is the mass of link i and $\boldsymbol{\omega}_i \times \mathbf{1}_{33}$ is cross product matrix of angular velocity. $\mathbf{w}_i$ is the wrench of link i comprises the resultant forces and moments $\mathbf{f}_i$ and $\mathbf{n}_i$ applied on link i. $\mathbf{w}_i^E$ is the external forces and moments as well as actuators forces applied on link i and $\mathbf{w}_i^C$ is nonworking kinematic constraint wrenches due to the coupling of the links together. The governing equations of motion of the whole system are determined by assembling the dynamics of all links, represented by equation (25), thereby obtaining

$$\mathbf{M}\dot{\mathbf{t}} + \boldsymbol{\Omega}\mathbf{M}\mathbf{t} = \mathbf{w}^E + \mathbf{w}^C \tag{29}$$

where $\mathbf{M}$ is the generalized extended mass matrix, $\boldsymbol{\Omega}$ is the generalized angular velocity matrix, $\mathbf{t}$ is the generalized twist vector and $\mathbf{w}$ is the generalized wrench vector of the system and can be written as

$$\mathbf{M} = diag(\mathbf{M}_1, \mathbf{M}_2, \ldots, \mathbf{M}_r) \tag{30}$$

$$\boldsymbol{\Omega} = diag(\boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2, \ldots, \boldsymbol{\Omega}_r) \tag{31}$$

$$\mathbf{t} = [\mathbf{t}_1^T \quad \mathbf{t}_2^T \quad \ldots \quad \mathbf{t}_r^T]^T \tag{32}$$

$$\mathbf{w} = [\mathbf{w}_1^T \quad \mathbf{w}_2^T \quad \ldots \quad \mathbf{w}_r^T]^T \tag{33}$$

Here, r is the number of moving links plus the MP of the system.

For an n-degree-of-freedom (DOF) system, the generalized twist vector $\mathbf{t}$ can be expressed as a linear transformation of $\dot{\mathbf{q}}$, which is an n-dimensional vector of independent generalized speeds, namely, $\mathbf{t} = \mathbf{N}\dot{\mathbf{q}}$, where $\mathbf{N}$ is orthogonal complement of kinematic constraint velocity matrix and because of its definition, it was named the natural orthogonal complement.[16] By definition, the power developed by the nonworking kinematic constraint wrench $\mathbf{w}^C$ vanishes and therefore by premultiplying both sides of equation (29) by $\mathbf{N}^T$ and inserting $\mathbf{t} = \mathbf{N}\dot{\mathbf{q}}$, the governing equations of motion of the system can be obtained as

$$\mathbf{N}^T\mathbf{M}\ddot{\mathbf{q}} + \mathbf{N}^T(\mathbf{M}\dot{\mathbf{N}} + \boldsymbol{\Omega}\mathbf{M}\mathbf{N})\dot{\mathbf{q}} = \mathbf{N}^T\mathbf{w}^E \quad (34)$$

$$\mathbf{w}^E = \mathbf{w}^M + \mathbf{w}^G + \mathbf{w}^D$$

where $\mathbf{w}^M$, $\mathbf{w}^G$ and $\mathbf{w}^D$ are actuator wrench, gravity wrench and damping wrench, respectively. The generalized angular velocity matrix $\boldsymbol{\Omega}$ can be determined by kinematic analysis of the manipulator at hand as explained in Section 2 and the generalized mass matrix is readily computed by having the inertia of each link. The effect of damping forces is neglected in this problem. Moreover, the gravity wrench for each link of the leg is a vector which all arrays are zero except the last one which is $m_i g$. Having the geometric and inertia properties of the manipulator at hand and solving its kinematics, it is possible to compute $\tau = \mathbf{N}^T\mathbf{w}^M$ from equation (34). It may be noted that the components of the vector τ are actuator forces of the three legs because the lengths of three legs have been chosen as the independent generalized coordinates for the system at hand. The computation of $\mathbf{N}$ is the most important part which will be described in next subsection.

3.2. Computation of NOC matrix N

The natural orthogonal complement (NOC) matrix $\mathbf{N}$ can be computed symbolically or numerically. Symbolic computation of $\mathbf{N}$ for parallel manipulators and in general for mechanical systems with kinematic loops is very cumbersome and it is sometimes impossible to express explicit relations in terms of independent generalized speeds. Therefore, numerical computation of $\mathbf{N}$ is an alternative method which can be used.[13,17] Here, $\mathbf{N}$ is computed symbolically as follows: With reference to Figure 2, q_i the length of legs are joint space variables; ϕ roll, ψ pitch and h heave of MP are cartesian space variables; and α_i and β_i the angles of legs with coordinate axes are passive joint variables. The twist vector of each leg can be expressed in terms of independent generalized speeds $\dot{\mathbf{q}} = [\dot{q}_1, \dot{q}_2, \dot{q}_3]^T$ and passive generalized speeds $\dot{\boldsymbol{\beta}} = [\dot{\alpha}_1, \dot{\beta}_1, \dot{\alpha}_2, \dot{\beta}_2, \dot{\alpha}_3, \dot{\beta}_3]^T$. Then expressing the kinematic constraint equations governing the system and in the light of their time derivatives, vector $\dot{\boldsymbol{\beta}}$ can be expressed in terms of $\dot{\mathbf{q}}$ and thereafter the generalized twist vector of the system can be expressed, in turn, in terms of $\dot{\mathbf{q}}$. Finally, having the above relations, matrix $\mathbf{N}$ can be derived symbolically.

The twist vector of the lower and upper links of each leg is written, respectively, as

$$\mathbf{t}_{1i} = \begin{bmatrix} \boldsymbol{\Lambda}_{1i} \\ l_{1i}\boldsymbol{\Gamma}_{1i} \end{bmatrix} \begin{bmatrix} \dot{\alpha}_i \\ \dot{\beta}_i \end{bmatrix} \quad i = 1, 2, 3 \quad (35)$$

$$\mathbf{t}_{2i} = \begin{bmatrix} \mathbf{0}_{31} \\ \mathbf{e}_i \end{bmatrix} \dot{\mathbf{q}}_i + \begin{bmatrix} \boldsymbol{\Lambda}_{2i} \\ (q_i - l_{2i})\boldsymbol{\Gamma}_{2i} \end{bmatrix} \begin{bmatrix} \dot{\alpha}_i \\ \dot{\beta}_i \end{bmatrix} \quad i = 1, 2, 3 \quad (36)$$

where $\mathbf{e}_i$ is the unit vector along the leg i and $\boldsymbol{\Lambda}_{1i}$, $\boldsymbol{\Lambda}_{2i}$, $\boldsymbol{\Gamma}_{1i}$, and $\boldsymbol{\Gamma}_{2i}$, are 3×2 matrices written as

$$\boldsymbol{\Lambda}_{1i} = \begin{bmatrix} 1 & -\sin\alpha_i \\ 0 & -\cos\alpha_i \\ 1 & 0 \end{bmatrix} \boldsymbol{\Gamma}_{li} = \begin{bmatrix} -\sin\alpha_i\sin\beta_i & \cos\alpha_i\cos\beta_i \\ -\cos\alpha_i\sin\beta_i & \sin\alpha_i\cos\beta_i \\ 0 & -\sin\beta_i \end{bmatrix} i = 1,2,3 \quad (37)$$

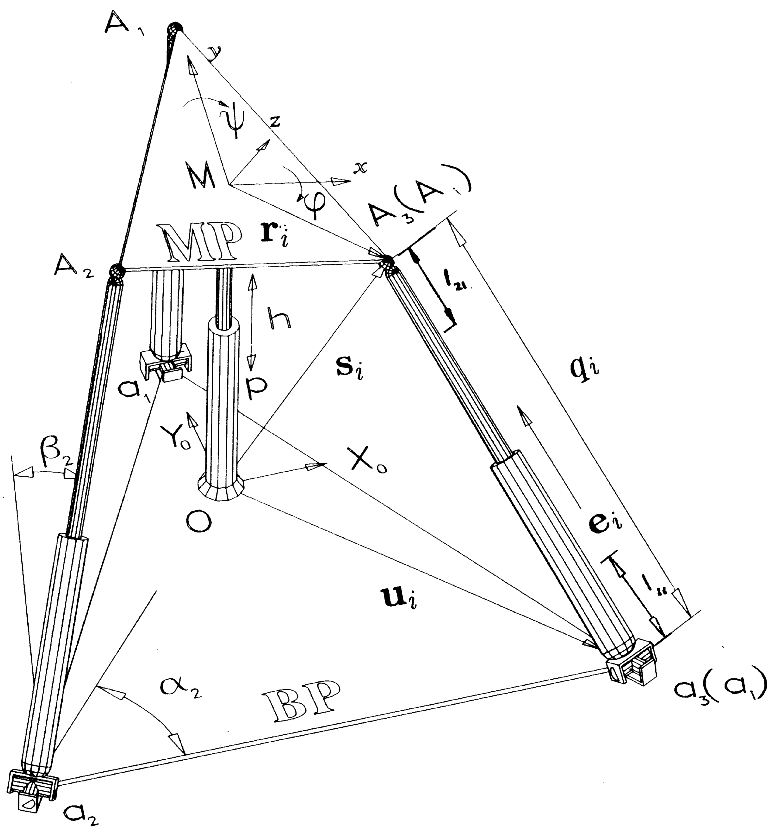

Fig. 2. Geometric properties of parallel manipulator with the new architecture.

https://doi.org/10.1017/S026357470000271X Published online by Cambridge University Press

Here, l_{1i} is the center of mass of lower link of leg i with respect to a_i and l_{2i} is the center of mass of upper link of leg i with respect to A_i. It is apparent from Figure 2 that $\mathbf{\Lambda}_{1i}=\mathbf{\Lambda}_{2i}$ and $\mathbf{\Gamma}_{1i}=\mathbf{\Gamma}_{2i}$ for $i=1,2,3$. The velocity of three non-collinear points a_i of the MP are expressed in terms of the independent generalized speed $\dot{\mathbf{q}}$ and passive joint speeds $\dot{\beta}$. Then the twist vector of the MP can be written as

$$\mathbf{t}_{MP}=\mathbf{N}_{1MP}\dot{\mathbf{q}}+\mathbf{N}_{2MP}\dot{\beta} \tag{38}$$

where $\mathbf{N}_{1MP}$ and $\mathbf{N}_{2MP}$ are 6×6 matrices. Therefore, the twist vector of the whole system can be defined as

$$\mathbf{t}=[\mathbf{t}_{11}^T \quad \mathbf{t}_{21}^T \quad \mathbf{t}_{12}^T \quad \mathbf{t}_{22}^T \quad \mathbf{t}_{13}^T \quad \mathbf{t}_{23}^T \quad \mathbf{t}_{MP}^T]^T \tag{39}$$

where $\mathbf{t}_{1i}$ and $\mathbf{t}_{2i}$ for $i=1,2,3$ are twist vectors of lower and upper links of leg i.

Upon substitution of twist vectors of each leg from equations (35) and (36) and MP into equation (39), one obtains,

$$\mathbf{t}=\mathbf{N}_1\dot{\mathbf{q}}+\mathbf{N}_2\dot{\boldsymbol{\beta}} \tag{40}$$

where $\mathbf{N}_1$ and $\mathbf{N}_2$ are 42×3 and 42×6 matrices. By substituting $\dot{\boldsymbol{\beta}}$ from equation (19) into equation (40) and factoring out $\dot{\mathbf{q}}$, one derives,

$$\mathbf{t}=(\mathbf{N}_1+\mathbf{N}_2\mathbf{J})\dot{\mathbf{q}} \tag{41}$$

Therefore, the matrix N obtains as

$$\mathbf{N}=\mathbf{N}_1+\mathbf{N}_2\mathbf{J} \tag{42}$$

Moreover, $\dot{\mathbf{N}}$ is also computed as

$$\dot{\mathbf{N}}=\dot{\mathbf{N}}_1+\dot{\mathbf{N}}_2\mathbf{J}+\mathbf{N}_2\dot{\mathbf{J}} \tag{43}$$

It may be noted that the forms of $\mathbf{N}_1$ and $\mathbf{N}_2$ are described by Kasaei.[14]

4. NUMERICAL EXAMPLE

The geometric properties of the manipulator at hand are as follows: The moving and based platforms are equilaterals with side of 1 meter and 2 meter, respectively. The minimum length of central leg is 1 meter and its stroke, heave of MP, is 0.4 meter. The pitch and roll angles of MP are varied from -30 deg to $+30$ deg.

The mass of MP is 500 kg, the mass of each leg is 10 kg, and the moment inertia matrix of MP and each link of the leg expressed in the coordinate frame attached to their center of masses are expressed as

$$\mathbf{I}_{MP}=\begin{bmatrix}800 & 0 & 0\\ 0 & 800 & 0\\ 0 & 0 & 800\end{bmatrix};\ \mathbf{I}_{1i}=\mathbf{I}_{2i}=\begin{bmatrix}4 & 0 & 0\\ 0 & 4 & 0.02\\ 0 & 0.02 & 4\end{bmatrix};\ (kg-m^2) \tag{44}$$

while the vector of center of mass of MP with respect to point M the center of xyz coordinate is written as

$$\mathbf{r}^*=[0.0,\ 0.3,\ 0.4]^T(m) \tag{45}$$

For solving inverse position kinematics, position vector of point M of MP, $\mathbf{p}$, with respect to reference frame is defined by a prescribed *cycloidal manoeuvre* as

$$\mathbf{p}=[0 \quad 0 \quad h(t)]^T$$
$$h(t)=1+0.4\left[\frac{t}{T}-\frac{1}{2\pi}\sin\frac{2\pi t}{T}\right],\ (\mathrm{m})\ \mathbf{0}\le t\le T \tag{46}$$

where $h(t)$ shows the heave of MP and T is the period of the manoeuvre in second. The rotation matrix $\mathbf{R}$ of MP with respect to reference frame for the motion of heave is the 3×3 identity matrix. However, $\mathbf{R}$ can be defined, in turn, for the pitch and roll motion of MP as

$$\mathbf{R}=\mathbf{R}_p=\begin{bmatrix}\cos\psi(t) & 0 & \sin\psi(t)\\ 0 & 1 & 0\\ -\sin\psi(t) & 0 & \cos\psi(t)\end{bmatrix} \tag{47}$$

$$\mathbf{R}=\mathbf{R}_R=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\phi(t) & -\sin\phi(t)\\ 0 & \sin\phi(t) & \cos\phi(t)\end{bmatrix}$$

while in the combined motion of roll-pitch-heave(RPH) for the MP, the rotation matrix is written as $\mathbf{R}=\mathbf{R}_{RPH}=\mathbf{R}_R\mathbf{R}_P$

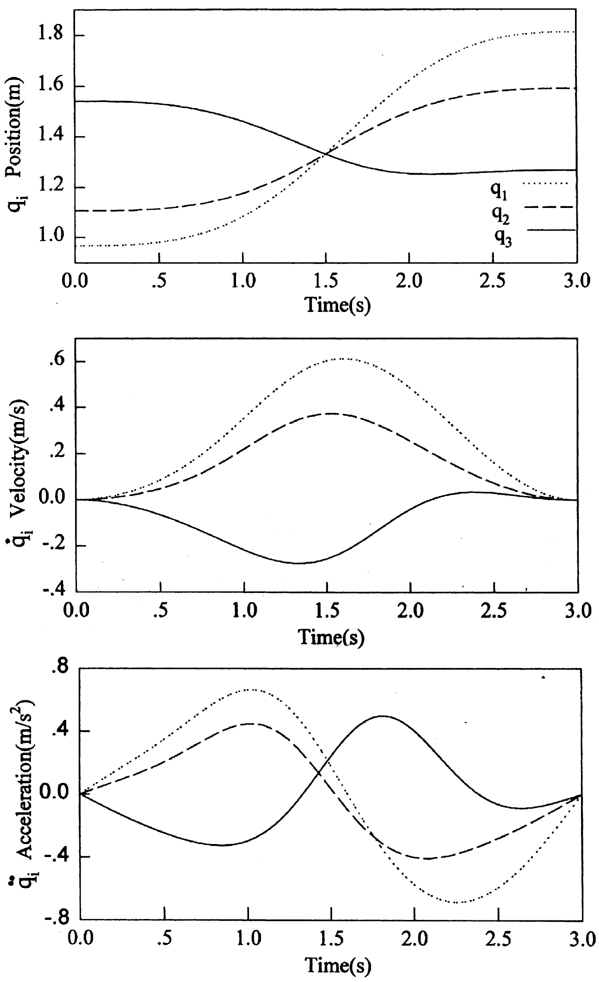

Fig. 3. Time history of legs length q_i, its speed $\dot{q}_i$ and its acceleration $\ddot{q}_i$ (q_1, $\dot{q}_1$, $\ddot{q}_1$, …; q_2, $\dot{q}_2$, $\ddot{q}_2$, - - -; q_3, $\dot{q}_3$, $\ddot{q}_3$, —).

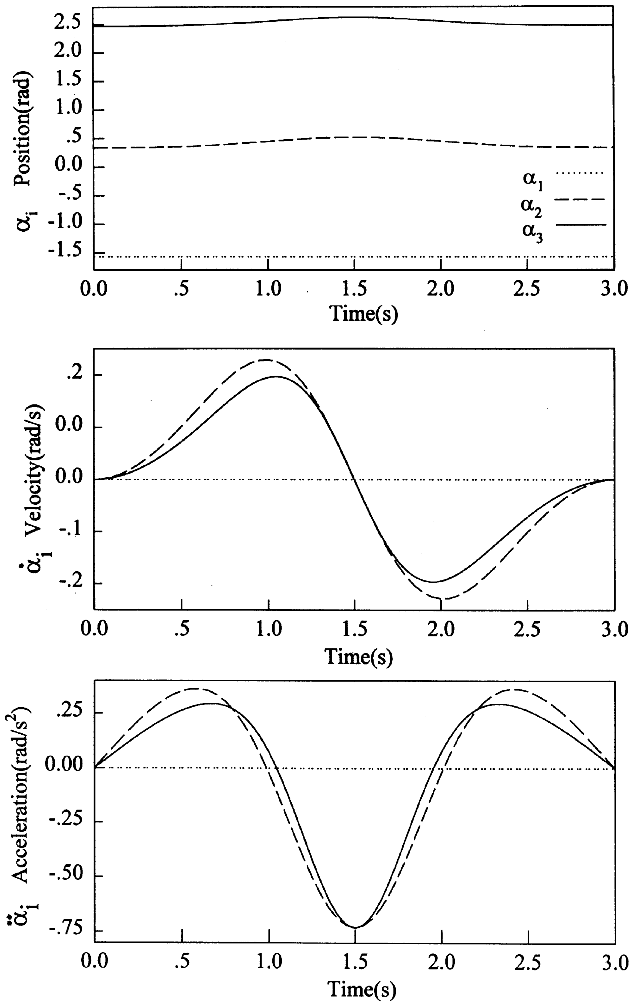

Fig. 4. Time history of legs angle and its first and second time rate of changes (α_1, $\dot{\alpha}_1$, $\ddot{\alpha}_1$, …; α_2, $\dot{\alpha}_2$, $\ddot{\alpha}_2$, - - -; α_3, $\dot{\alpha}_3$, $\ddot{\alpha}_3$, —).

with $\phi(t)$ and $\psi(t)$ expressed in terms of a cycloidal maneouvre as

$$\phi(t) = \frac{\pi}{6}\left[\frac{2t}{T} - \frac{1}{\pi}\sin\frac{2\pi t}{T} - 1\right]$$
$$\psi(t) = \frac{\pi}{6}\left[\frac{2t}{T} - \frac{1}{\pi}\sin\frac{2\pi t}{T} - 1\right] \tag{48}$$

here, the time t is defined as $0 \le t \le T$ while $\phi(t)$ and $\psi(t)$ are varied from -30 deg to $+30$ deg.

After defining **p** and **R** for any desired motion of MP and computing their time rate of changes, inverse kinematics of the problem is solved for any chosen motion of MP using the method presented in Secion 2.1. As an example, the results for the RPH motion of MP and for $T = 3$ s is shown in Figures 3, 4 and 5. The time history of legs length, linear velocity and linear acceleration of the legs are shown in Figure 3. Moreover, the angle of the projection of leg i on BP with the axis X_0 and their time rate of changes, i.e., α_i, $\dot{\alpha}_i$, $\ddot{\alpha}_i$ are depicted in Figure 4; and the legs angles with the vertical axis of reference frame and their time rate of changes, namely, β_i, $\dot{\beta}_i$, $\ddot{\beta}_i$ are shown in Figure 5. The forward kinematics is solved as follows: Given the time histories of independent generalized coordinates and generalized speeds q_i and $\dot{q}_i$. determine the leg angles and their time rate of changes in terms of independent generalized coordinates and generalized speeds using the kinematic constraint equations and then compute the motion of MP using the method presented in Section 2.2. It may be noted that all results from inverse kinematics have been validated by the forward kinematic solution and the results are in a very good agreement in the order of 10^{-9}.

The forces of the actuators of each leg for different manoeuvres of MP, i.e., roll, pitch, heave and the combined motion of RPH are determined using the result of Section 3. The results show that these forces depend highly on the kind of motion of MP, the mass of MP, position of center of mass of MP, and moment of inertia of the MP. As an example, the time history of leg forces for different manoeuvres of heave, roll, pitch and RPH motion are shown in Figure 6.

In order to highlight the effect of leg masses, the above mentioned example is also solved for the case of zero mass for the legs in RPH motion. The results for the leg forces with considering the leg masses and the force differences for two cases, one with considering the leg masses, and one without considering the leg masses are shown in Figure 7. The results show that there is not noticeable effect due to the masses of the legs and it is possible to neglect the masses of the legs in dynamic computation of this manipulator.

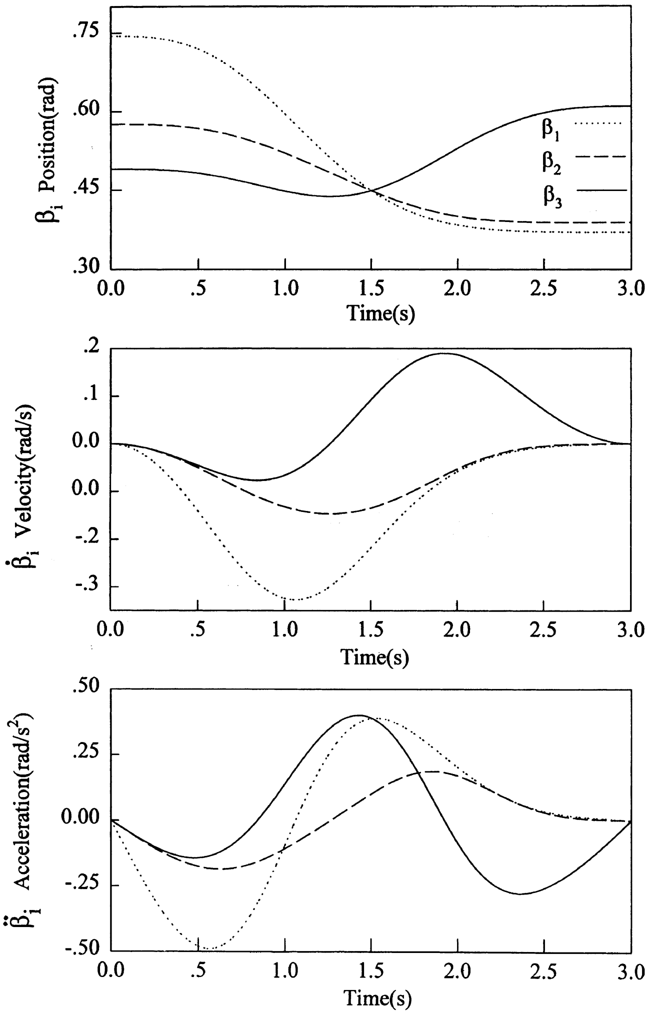

Fig. 5. Time history of legs angle and its first and second time rate of changes (β_1, $\dot{\beta}_1$, $\ddot{\beta}_1$, …; β_2, $\dot{\beta}_2$, $\ddot{\beta}_2$, - - -; β_3, $\dot{\beta}_3$, $\ddot{\beta}_3$, —).

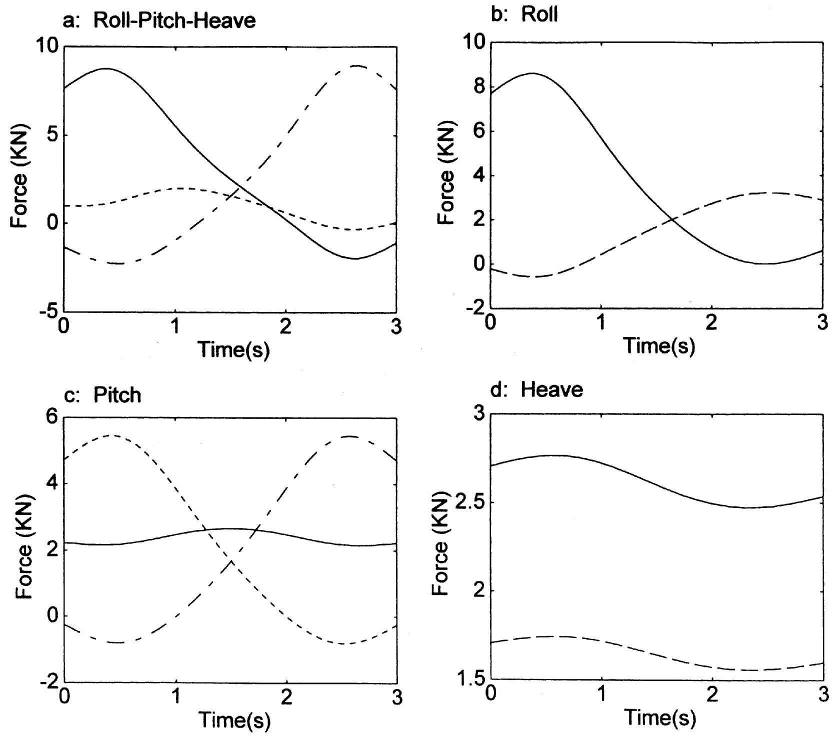

Fig. 6. Time history of leg forces for different manoeuvres of MP; a: RPH motion, b: Roll motion, c: Pitch motion, d: Heave motion Force in leg 1:—, Force in leg 2:- - -, Force in leg 3-.-.-.

Moreover, the effect of position of center of mass of MP is shown by solving the problem for two cases one with assuming $\mathbf{r}^* = \mathbf{0}$, and one with $\mathbf{r}^* = [0.0, 0.3, 0.4]^T$ (m) which the results are shown in Figure 8. It was noticed that, the position of center of mass of MP has an important role on the forces of the legs.

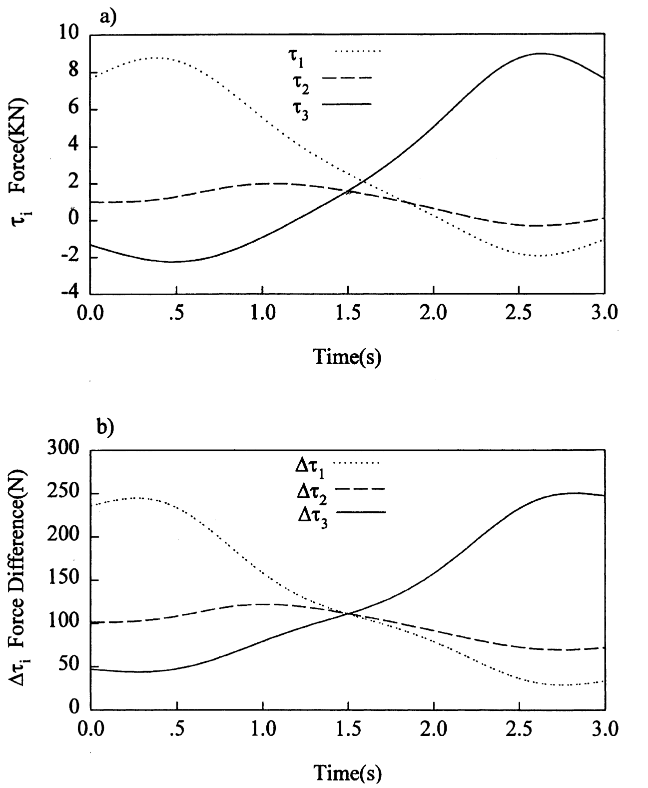

Fig. 7. The effect of leg masses on the leg forces in RPH motion
a) Leg forces with considering leg masses
b)Force differences for two cases.

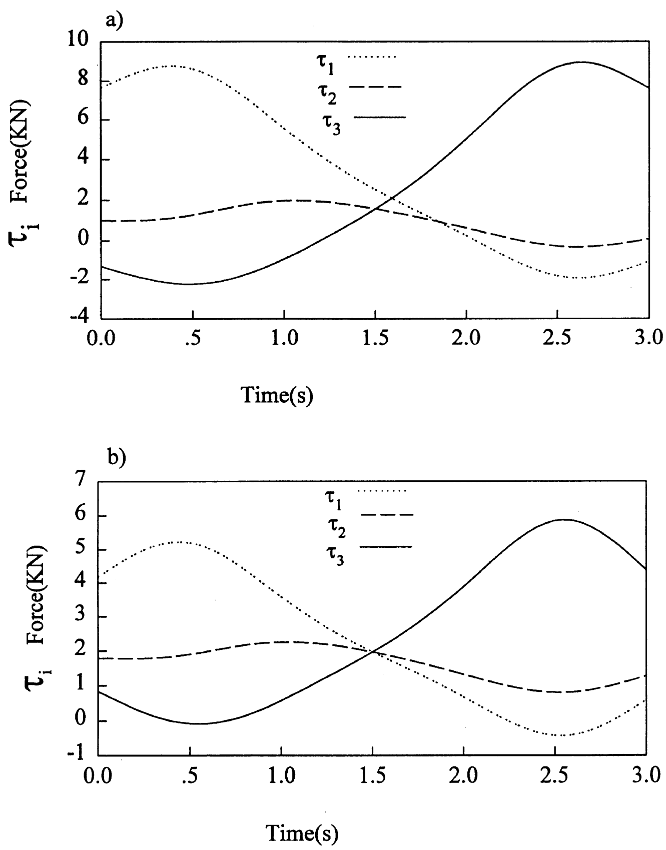

Fig. 8. The effect of position of center of mass of MP on the leg forces in RPH motion
a) Leg forces with $\mathbf{r}^* = [0.0, 0.3, 0.4]^T$ (m)
b) Leg forces with $\mathbf{r}^* = 0$.

5. CONCLUSION

The kinematic and dynamics analysis of a new architecture parallel manipulator has been studied in this paper. This manipulator consists of a base platform(BP) and a moving platform(MP) which are connected by means of three legs. Each leg is connected to BP by a universal joint and to the MP by a spherical joint. There is also a central leg connected to MP by a universal joint and fixed to the BP. The role of the central leg is to provide 3 independent DOF for the manipulator at hand. Inverse and forward kinematics problems were studied for the problem in position, velocity and acceleration. The minimum number of equations of motion of the manipulator have been derived using the natural orthogonal complement methodology.

Numerical examples are solved for the different motions of MP in order to obtain the motion of the legs and the actuator forces of the legs. The effects of leg masses and position of the center of mass of MP on the results have been examined. The present study can be used in design and control of this type of manipulator. Moreover, it can further display the potential applications of the proposed manipulator as a moving mechanism in flight simulators.

Acknowledgements

The work reported here was possible under financial support from Isfahan University of Technology. The first author is grateful to Mr. Abedini for their valuable helps to this work.

References

1. E.F. Fichter, "A Stewart platform-based manipulator: General theory and practical construction", *Int. J. Robotics Res.* **5**(2), 157–182 (1986).
2. K.H. Hunt, "Structural kinematics of in. parallel actuated robot arms", *ASME J. of Mech. Transm. and Automn. in Design* **105**, 705–712 (1983).
3. D. Stewart, "A platform with six degrees of freedom", *Proc. Inst. Mech. Engr.* **109**, No. 15, 371–378 (1965).
4. K.M. Lee and K. Shah, "Kinematic analysis of a three-degrees-of-freedom in-parallel actuated manipulator", *IEEE J. of Robotics and Automation* **4**(3), 354–360 (1988).
5. M.G. Mohammad and J. Duffy, "A direct determination of the instantaneous kinematics of fully parallel robot manipulators", *ASME J. Mech. Transm. and Automn. in Design* **107**, 226–229 (1985).
6. X. Shi and R.G. Fenton, "A complete and general solution to the forward kinematics problem of the platform-type robotic manipulators", *Proc. IEEE Int. Conf. on Robotics and Automation* (1994) **Vol 4**, pp. 3055–3062.
7. J.P. Merlet, "Direct kinematics and assembly model of parallel manipulators", *Int. J. Robotics Res.* **11**, 150–162 (1992).
8. K. Sugimoto, "Kinematics and dynamics analysis of parallel manipulators by means of motor algebra", *ASME J. Mech. Transm. and Auto. in Design* **109**, 3–7 (1987).
9. C. Gosselin, "Parallel computational algorithms for the kinematics and dynamics of parallel manipulators", *Proc. IEEE Int. Conf. on Robotics and Automation* (1993) **Vol. 1**, pp. 883–888.
10. G. Lebert, K. Liu and F.L. Lewis, "Dynamics analysis and control of a Stewart platform manipulator", *J. Robotic Systems* **10**(5), 629–655 (1993).
11. H. Pang and M. Shahinpour, "Analysis of static equilibrium of a parallel manipulator", *Robotica* **11**, Part 5, 433–443 (1993).
12. H. Pang and M. Shahinpour, "Inverse dynamics of a parallel manipulator", *J. Robotic Systems* **11**(8), 693–702 (1994).
13. K.E. Zanganeh, R. Sinatra and J. Angeles, "Kinematics and dynamics of a six-degree-of-freedom parallel manipulator with revolute legs", *Robotica* **15**, Part 4, 385–394 (1997).
14. G. Kasaei, "Kinematics and dynamics of a parallel manipulator with a new architecture", *M.S. Thesis*, (Department of Mechanical Engineering, Isfahan University of Technology, 1998).
15. J. Angeles, *Rational Kinematics* (New York: Springer-Verlag, 1988).
16. J. Angeles and S. Lee, "The formulation of dynamical equation of holonomic mechanical systems using a natural orthogonal complement", *ASME J. Applied Mechanics* **55**, 243–244 (1988).
17. O. Ma and J. Angeles, "Direct kinematics and dynamics of a planar parallel manipulator", *Proc. of the 15th ASME Design and Automation Conf.*, (1989) **DE-VOL.3**, pp. 313–320.