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MODELING MULTIPLE REGIMES IN THE BUSINESS CYCLE

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The interest in business-cycle asymmetry has been steadily increasing over the past 15 years. Most research has focused on the different behavior of macroeconomic variables during expansions and contractions, which by now is well documented. Recent evidence suggests that such a two-phase characterization of the business cycle might be too restrictive. In particular, it might be worthwhile to decompose the recovery phase in a high-growth phase (immediately following the trough of a cycle) and a subsequent moderate-growth phase. The issue of multiple regimes in the business cycle is addressed using smooth-transition autoregressive (STAR) models. A possible limitation of STAR models as they currently are used is that essentially they deal with only two regimes. We propose a generalization of the STAR model such that more than two regimes can be accommodated. It is demonstrated that the class of multiple-regime STAR (MRSTAR) models can be obtained from the two-regime model in a simple way. The main properties of the MRSTAR model and several issues that are relevant for empirical specification are discussed in detail. In particular, a Lagrange multiplier-type test is derived that can be used to determine the appropriate number of regimes. A limited simulation study indicates its practical usefulness. Application of the new model class to U.S. real GNP provides evidence in favor of the existence of multiple business-cycle phases.

Keywords: Business-Cycle Asymmetry, Multiple Regimes, Smooth-Transition Autoregression, Lagrange Multiplier Test

1. INTRODUCTION

The notion of business-cycle asymmetry has been around for quite some time. For example, Keynes (1936, p. 314) already observed that “the substitution of a downward for an upward tendency often takes place suddenly and violently, whereas there is, as a rule, no such sharp turning point when an upward is substituted for a downward tendency.” Following Burns and Mitchell (1946), conventional wisdom has long held that “contractions are shorter and more violent than expansions.”

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Starting with Neftçi (1984), interest in the subject of business-cycle asymmetry has revived and many macroeconomic variables [output and (un)employment series in particular] since have been examined for asymmetry. The statistical procedures that have been employed can be divided into two main categories.[1] First, various nonparametric techniques have been used. For example, Neftçi (1984), Falk (1986), Sichel (1989), Rothman (1991), and McQueen and Thorley (1993), among many others, test for asymmetry between expansions and contractions by using Markov chain methods to examine whether the transition probabilities from one regime to the other differ. Second, parametric nonlinear time-series models have been employed to render insight into the differing dynamics over the business cycle. Regime-switching models have been particularly popular in this line of research. Typically, these models consist of a set of linear models of which, at each point in time, only one or a linear combination of the models is active to describe the behavior of a time series, where the activity depends on the regime at that particular moment.

Within the class of regime-switching models, two main categories can be distinguished, depending on whether the regimes are determined exogenously, by an unobservable state variable, or endogenously, by a directly observable variable. The most prominent member of the first class of models is the Markov-switching (MS) autoregressive model, which has been applied to modeling business-cycle asymmetry by Hamilton (1989), Boldin (1996), and Diebold and Rudebusch (1996), among others. From the second class of models, the (self-exciting) threshold autoregressive [(SE)TAR] model [see Beaudry and Koop (1993), Tiao and Tsay (1994), Potter (1995), Peel and Speight (1996, 1998), and Clements and Krolzig (1998)] and the smooth-transition autoregressive (STAR) model have been applied most frequently [see Teräsvirta and Anderson (1992), Teräsvirta (1995), Skalin and Teräsvirta (1996), and Jansen and Oh (1996)]. Filardo (1994) and Filardo and Gordon (1998) consider a mixture of models, by allowing the transition probabilities between the states in an MS model to depend on observable (leading-indicator) variables.

It is now well understood that recessions are different from booms, and there seem to be possibilities for even further refinement. Ramsey and Rothman (1996) and Sichel (1993) discuss concepts such as “deepness,” “steepness” and “sharpness,” which relate to different aspects of asymmetry. A cycle is said to exhibit steepness if the slope of the expansion phase differs from the slope of the contraction phase. Deepness occurs when the distance from the mean of the cycle to the peak is not equal to the distance from the mean to the trough. Sharpness focuses on the relative curvature around peaks and troughs. Sichel (1993) argues that most research has focused exclusively on the possibility of steepness, neglecting other forms of asymmetry. The evidence presented by Sichel (1993) suggests however that deepness might be a more important characteristic of macroeconomic variables. This is confirmed by the analysis by Verbrugge (1997), which demonstrates that depth is a feature of numerous economic time series, whereas steepness is a feature of (un)employment-related variables but is absent from real GDP and aggregate industrial production. Concerning sharpness, peaks generally are thought to be "rounder" than troughs; see Emery and Koenig (1992) and McQueen and Thorley (1993) for some evidence in favor of this premise.

Intuitively, if a macroeconomic variable exhibits different types of asymmetry simultaneously, the distinction between expansion and contraction might not be sufficient to characterize its behavior over the business cycle completely. Sichel (1994) observes that real GNP tends to grow faster immediately following a trough than in the rest of the expansion phase. Wynne and Balke (1992) and Emery and Koenig (1992) present additional evidence in favor of this "bounce-back" effect. This suggests the possibility of three business-cycle phases—contractions, high-growth recoveries that immediately follow troughs of the cycle, and subsequent moderate growth phases.

The nonlinear time-series models mentioned above mainly focus on two regimes, i.e., expansions and contractions. The MS and SETAR models can be extended easily to multiple regimes, at least conceptually. For example, Boldin (1996) presents a three-regime MS model in which the expansion regime is split into separate regimes for the posttrough rapid recovery period and the moderate-growth period for the remainder of the expansion. In a similar vein, Pesaran and Potter (1997) and Koop et al. (1996) use principles of SETAR models to construct a "floor and ceiling" model that allows for three regimes corresponding to low, normal, and high growth rates of output, respectively. Tiao and Tsay (1994) develop a four-regime SETAR model for U.S. real GNP in which the regimes are labeled worsening/improving recession/expansion, thus allowing for variation in dynamics during different phases of the business cycle. In contrast, extending the number of possible regimes in STAR models does not seem to be straightforward. Therefore, the objective of our paper is to explore how STAR models can be modified to allow for more than two regimes, with the purpose of examining whether a multiple-regime STAR (MRSTAR) model can be used to describe the behavior of postwar U.S. real GNP.

The outline of our paper is as follows: In Section 2, we discuss the STAR model and a simple yet elegant way to generalize this model to accommodate more than two regimes. In Section 2.2, we give a theoretical account of this MRSTAR model and in Section 2.3, we focus on a simple example to demonstrate the main features of the MRSTAR model. In Section 3, we discuss some of the issues that are involved in specifying these models. Emphasis in that section is put on developing a test statistic that can be used to test a two-regime model against a multiple-regime alternative. Simulations are used to examine its empirical performance. In Section 4, we discuss previous research on modeling business-cycle asymmetry in somewhat more detail and apply the MRSTAR model to characterize the behavior of the growth rate of postwar U.S. real GNP. Finally, Section 5 contains some discussion.

2. EXTENDING THE STAR MODEL

In this section we describe an extension of the STAR model that allows for more than two regimes. We start with a brief description of the basic STAR model; for more elaborate discussions of these models we refer to Granger and Teräsvirta (1993) and Teräsvirta (1994, 1998). We next argue that, irrespective of the particular transition function that is used, this basic STAR model essentially allows for only two regimes. To overcome this limitation, the class of MRSTAR models is introduced. The potential usefulness of this class of models is illustrated by a simple example.

2.1. Basic STAR Model

Consider the following STAR model for a univariate time series y_t:

$$y_t = \phi_1' y_t^{(p)}[1 - F(s_t;\gamma,c)] + \phi_2' y_t^{(p)} F(s_t;\gamma,c) + \varepsilon_t, \qquad \mathbf{(1)}$$

where $y_t^{(p)} = (1, \tilde{y}_t^{(p)})'$, $\tilde{y}_t^{(p)} = (y_{t-1},\ldots,y_{t-p})'$, $\phi_i = (\phi_{i0},\phi_{i1},\ldots,\phi_{ip})'$, $i = 1, 2$, and ε_t is a white-noise error process with mean zero and variance σ^2. The so-called transition function $F(s_t;\gamma,c)$ is a continuous function bounded between zero and one. The transition variable s_t can be a lagged endogenous value ($s_t = y_{t-d}$ for certain $d > 0$), an exogenous variable ($s_t = x_t$), or a (possibly nonlinear) function of lagged endogenous and exogenous variables [$s_t = g(\tilde{z}_t)$ for some function $g(\cdot)$ with $\tilde{z}_t = (y_{t-1},\ldots,y_{t-p},x_{1t},\ldots,x_{kt})'$]. One of the most often applied choices for $F(s_t;\gamma,c)$, which is also central in this paper, is the logistic function[2]

$$F(s_t;\gamma,c) = \{1+\exp[-\gamma(s_t - c)]\}^{-1}, \qquad \gamma > 0, \qquad \mathbf{(2)}$$

where γ and c are scalars. The requirement that γ be positive is an identifying restriction. The model consisting of (1) with (2) is called a logistic STAR (LSTAR) model.

The way the model is written in equation (1) highlights the basic characteristic of the LSTAR model, which is that, at any given point in time, the evolution of y_t is determined by a weighted average of two different linear autoregressive (AR) models. The weights assigned to the two models depend on the value taken by the transition variable s_t. For small (large) values of s_t, $F(s_t;\gamma,c)$ is approximately equal to zero (one) and, consequently, almost all weight is put on the first (second) model. The parameter γ determines the speed at which these weights change as s_t increases; the higher γ, the faster is this change. If $\gamma \to 0$, the weights become constant (and equal to 0.5) and the model becomes linear, whereas, if $\gamma \to \infty$, the logistic function approaches a Heaviside function, taking the value 0 for $s_t < c$ and 1 for $s_t > c$. In that case, the LSTAR model reduces to a two-regime SETAR model; see Tong (1990) for an extensive discussion.

Teräsvirta (1994) outlines a specification procedure for STAR models. Because this will be part of the specification procedure for MRSTAR models to be discussed later, we briefly sketch the different steps in this procedure here. After estimating a suitable AR model for y_t, linearity is tested against the alternative of a two-regime STAR model (1) using the tests developed by Luukkonen et al. (1988). The testing problem suffers from what has become known as the Davies problem; that is, the model is not identified under the null hypothesis of linearity, which can be formulated as $H_0 : \gamma = 0$. This problem of nuisance parameters that are not identified under the null hypothesis was first considered in some depth by Davies (1977, 1987) and occurs in many testing problems; see Hansen (1996) for a recent account. The tests by Luukkonen et al. (1988) are based on replacing the transition function in (1) with a suitable approximation, which leads to a reparameterized model in which auxiliary regressors $y_{t-j}s_t^i$, $j = 1, \ldots, p$, $i = 1, \ldots, r$, appear (where r depends on the particular approximation that is used) and the identification problem is no longer present. Linearity is tested by examining the joint significance of the coefficients corresponding to these auxiliary regressors. For details, we refer to Luukkonen et al. (1988).

It is common practice to carry out the linearity test for different choices of the transition variable s_t in order to select the most appropriate transition variable(s) prior to estimation of the STAR model. For example, if s_t is limited to (functions of) lagged endogenous variables, it usually is assumed that only a single lagged value acts as transition variable, i.e., $s_t = y_{t-d}$ for certain $d > 0$. An alternative that might be of interest is when a lagged first difference Δy_{t-d} is taken to be the threshold variable. Following Enders and Granger (1998), the resulting model might be called a momentum STAR (MSTAR) model because the regime is determined by the direction in which the time series is moving, that is, by its momentum; see also Skalin and Teräsvirta (1998). The choice of s_t for which linearity is rejected most convincingly is considered to render the most appropriate one. The argument that is used to justify this approach is that the linearity test might be expected to have maximum power when s_t is correctly specified.[3]

If linearity is rejected, the parameters in the STAR model can be estimated by nonlinear least squares[4]; see Teräsvirta (1994) for a discussion of the issues involved. One of the characteristic features of estimating STAR models that has emerged from previous applications is that often one obtains a large and apparently insignificant estimate of γ. The reason that it is difficult to obtain a precise estimate of this parameter is that, for large values of γ, the switching of the transition function is almost instantaneous at c. In that case, a large number of observations for which s_t is equal or close to c would be required to estimate γ with a fair degree of accuracy. Furthermore, when γ is large, the shape of F is hardly affected by (even relatively large) changes in γ. This implies that convergence of the estimates to the optimum is slow and the standard error of γ tends to be large when the point estimate of this parameter is large. See Bates and Watts (1988, p. 87) for more on this issue. The implication from all this is that an insignificant estimate of γ should not be interpreted as insignificance of the regime switching. Put differently, the estimate of γ cannot be employed to infer the adequacy of the STAR model. This should be assessed by other means, such as inspection of the number of observations in the different regimes, application of diagnostic checks, or the out-of-sample forecast accuracy of the model.

The final stage of building a STAR model is to subject the estimated model to some diagnostic tests to check whether it adequately captures the main features of the data. Eitrheim and Teräsvirta (1996) develop appropriate test statistics for serial correlation, constancy of parameters, and remaining nonlinearity. Needless to say, the model can be modified if these diagnostic tests indicate possible misspecification. In particular, in case there is evidence that the model cannot describe all nonlinear features that are present in the time series under scrutiny, one might consider the possibility of extending the model to allow for multiple regimes. It is to this topic that we now turn our attention.

2.2. MRSTAR Model

The LSTAR model seems particularly well suited to describe asymmetry of the type that is encountered frequently in macroeconomic time series. For example, the model has been successfully applied by Teräsvirta and Anderson (1992) and Teräsvirta et al. (1994) to characterize the different dynamics of industrial production indexes in a number of OECD countries during expansions and recessions [see also Teräsvirta (1995)]. As argued in the introduction, sometimes more than two regimes might be required to describe adequately the behavior of a particular time series.

The notation in (1) shows that the set of linear AR models of which the STAR model is composed contains only two elements. Hence, it is immediately clear that the STAR model cannot accommodate more than two regimes, irrespective of what form the transition function takes. It has been suggested, though, that a three-regime model is obtained by using the exponential function

$$F(s_t, \gamma, c) = 1 - \exp\left[-\gamma(s_t - c)^2\right], \qquad \gamma > 0, \tag{3}$$

as transition function in (1). According to Teräsvirta and Anderson (1992), if $s_t = y_{t-d}$ the resulting exponential STAR (ESTAR) model allows expansions and contractions to have different dynamics than the "middle ground," similar to the "floor and ceiling" model of Pesaran and Potter (1997). However, the models in the two outer regimes, associated with very small and large values of y_{t-d} (and, hence, corresponding with the expansions and contractions), are restricted to be the same, so that effectively there still are only two distinct regimes. Furthermore, the ESTAR model does not nest the SETAR model as a special case because, for either $\gamma \to 0$ or $\gamma \to \infty$, the model becomes linear. The latter can be remedied by using the quadratic logistic function

$$F(s_t; \gamma, c_1, c_2) = \{1 + \exp[-\gamma(s_t - c_1)(s_t - c_2)]\}^{-1}, \qquad c_1 \leq c_2,\ \gamma > 0\,, \tag{4}$$

as proposed by Jansen and Teräsvirta (1996). In this case, if $\gamma \to 0$, the model becomes linear, whereas if $\gamma \to \infty$, the function $F(s_t; \gamma, c_1, c_2)$ is equal to 1 for $s_t < c_1$ and $s_t > c_2$ and equal to 0 in between. Hence, the STAR model with this particular transition function nests a three-regime SETAR model, although the models in the outer regimes are still restricted to be the same.

We propose an alternative way to extend the basic STAR model to allow for more than two, genuinely different, regimes. Building upon the notation used in (1), we suggest encapsulating two different LSTAR models as follows:

$$\begin{aligned} y_t = &\left\{\phi_1' y_t^{(p)}[1 - F_1(s_{1t}; \gamma_1, c_1)] + \phi_2' y_t^{(p)} F_1(s_{1t}; \gamma_1, c_1)\right\} \\ &\times [1 - F_2(s_{2t}; \gamma_2, c_2)] + \left\{\phi_3' y_t^{(p)}[1 - F_1(s_{1t}; \gamma_1, c_1)]\right. \\ &\left. + \phi_4' y_t^{(p)} F_1(s_{1t}; \gamma_1, c_1)\right\} F_2(s_{2t}; \gamma_2, c_2) + \varepsilon_t, \end{aligned} \quad (5)$$

where both transition functions F_1 and F_2 are taken to be logistic functions as in (2). Because both functions can vary between zero and one, (5) defines a model with four distinct regimes, each corresponding to a particular combination of extreme values of the transition functions. We call the model given in (5) the MRSTAR model. The MRSTAR model considered here allows for a maximum of four different regimes, but it will be obvious that, in the notation of (5), extension to 2^k regimes with $k > 2$ is straightforward, at least conceptually. A model with three regimes can be obtained from (5) by imposing appropriate restrictions on the parameters of the autoregressive models that prevail in the different regimes. If in fact $s_{1t} = s_{2t} \equiv s_t$, i.e., a single variable governs the transitions between all regimes, it will be intuitively clear that it is not sensible to allow for four different regimes. For example, if $c_1 < c_2$, F_1 changes from zero to one prior to F_2 for increasing values of s_t and, consequently, the product $(1 - F_1)F_2$ will be equal to zero almost everywhere, especially if γ_1 and γ_2 are large. Hence, it makes sense to exclude the model corresponding to this particular regime by imposing the restriction $\phi_3 = 0$. Because, in that case, $F_1 F_2 \approx F_2$, the resulting model may be rewritten as

$$y_t = \phi_1' y_t^{(p)} + (\phi_2 - \phi_1)' y_t^{(p)} F_1(s_t; \gamma_1, c_1) + (\phi_4 - \phi_2)' y_t^{(p)} F_2(s_t; \gamma_2, c_2) + \varepsilon_t. \quad (6)$$

In fact, the model as given in (6) is the form of the multiple-regime model as discussed by Eitrheim and Teräsvirta (1996), although they do not restrict the transition variables s_{1t} and s_{2t} to be the same [see also Öcal and Osborn (1997)].

Note that the MRSTAR model nests several other nonlinear time-series models. For example, an artificial neural network (ANN) model [see Kuan and White (1994)] is obtained by imposing the restrictions $\phi_{ij} = 0, i = 1, \ldots, 4, j = 1, \ldots, p$ and $\phi_{40} = \phi_{20} + \phi_{30} - \phi_{10}$. The last restriction ensures that the interaction term $\phi_{40}^* F_1 F_2$, where $\phi_{40}^* = \phi_{10} - \phi_{20} - \phi_{30} + \phi_{40}$ drops out of the model, which now can be rewritten as

$$y_t = \phi_{10}^* + \phi_{20}^* F_1(s_{1t}; \gamma_1, c_1) + \phi_{30}^* F_2(s_{2t}; \gamma_2, c_2) + \varepsilon_t, \quad (7)$$

where $\phi_{10}^* = \phi_1$, $\phi_{20}^* = \phi_{20} - \phi_{10}$, and $\phi_{30}^* = \phi_{30} - \phi_{10}$.

Additionally, the MRSTAR model (5) might be extended to a “semimultivariate” model by including exogenous variables as regressors or transition variables. Granger and Teräsvirta (1993) discuss incorporating exogenous variables x_{it} in the STAR model (1) to obtain the smooth transition regression (STR) model; see also Teräsvirta (1998) for a more recent survey. Likewise, the MRSTAR model can be extended to a multiple-regime STR (MRSTR) model by defining $z_t = (1, \tilde{z}_t')'$, $\tilde{z}_t = (y_{t-1}, \ldots, y_{t-p}, x_{1t}, \ldots, x_{kt})'$, and substituting z_t for $y_t^{(p)}$ in (5), i.e.,

$$\begin{aligned} y_t = {} & \{\phi_1' z_t[1 - F_1(s_{1t}; \gamma_1, c_1)] + \phi_2' z_t F_1(s_{1t}; \gamma_1, c_1)\}[1 - F_2(s_{2t}; \gamma_2, c_2)] \\ & + \{\phi_3' z_t[1 - F_1(s_{1t}; \gamma_1, c_1)] + \phi_4' z_t F_1(s_{1t}; \gamma_1, c_1)\} F_2(s_{2t}; \gamma_2, c_2) + \varepsilon_t, \qquad \textbf{(8)} \end{aligned}$$

where now the vectors ϕ_i, $i = 1, \ldots, 4$ are of length $m + 1$, with $m = p + k$. In particular, Lin and Teräsvirta (1994) argue that polynomials of time are allowed as transition variables in STAR models; even though these are nonstationary variables, no problems occur because the transition function is bounded between zero and one. Lütkepohl et al. (1995) and Wolters et al. (1996) apply this idea to model time-varying parameters in German money demand. In the MRSTR model, time trends might be used as transition variables as well. This opens the interesting possibility of modeling nonlinearity and time-varying parameters simultaneously. A possible application in business-cycle research might be to examine whether or not the properties of expansions and contractions are time-invariant. For example, Lin and Teräsvirta (1994) demonstrate that the properties of the index of industrial production in the Netherlands have changed after the oil crisis in 1975. Sichel (1991) claims that expansions have become longer after World War II and have started to exhibit duration dependence, whereas recessions have become shorter and duration dependence has disappeared; see also Diebold and Rudebusch (1992), Watson (1994), Romer (1994), Parker and Rothman (1996), and Cooper (1998). An extensive discussion of this issue is beyond the scope of this paper and is left for further research.

The MRST(A)R model also nests the class of Nested TAR (NeTAR) models recently proposed by Astatkie et al. (1997) as an extension of conventional TAR models to allow for multiple regimes determined by multiple sources. A NeTAR model is obtained from (8) [or (5)] if the parameters γ_1 and γ_2 both tend to infinity (or, equivalently, the logistic functions are replaced by Heaviside functions), such that the different regimes are separated by sharply determined borders.

2.3. A Simple Example

In this section, we focus on a simple example of a four-regime MRSTAR model to highlight some features of the model. We set $p = 1$, require all intercepts to be equal to zero, and take $s_{1t} = \Delta y_{t-1}$ and $s_{2t} = y_{t-2}$. The resulting model then is given by

$$\begin{aligned} y_t = {} & \{\phi_1 y_{t-1}[1 - F_1(\Delta y_{t-1}; \gamma_1, c_1)] + \phi_2 y_{t-1} F_1(\Delta y_{t-1}; \gamma_1, c_1)\} \\ & \times [1 - F_2(y_{t-2}; \gamma_2, c_2)] + \{\phi_3 y_{t-1}[1 - F_1(\Delta y_{t-1}; \gamma_1, c_1)] \\ & + \phi_4 y_{t-1} F_1(\Delta y_{t-1}; \gamma_1, c_1)\} F_2(y_{t-2}; \gamma_2, c_2) + \varepsilon_t. \qquad \textbf{(9)} \end{aligned}$$

For each combination of the transition variables $(\Delta y_{t-1}, y_{t-2})$, the resulting model is a weighted average of the four AR(1) models associated with the four extreme regimes. Figure 1 shows the weights given to each of these four models in the

(y_{t-1}, y_{t-2}) plane, with $\gamma_1 = \gamma_2 = 2.5$ and $c_1 = c_2 = 0$. For $(\Delta y_{t-1}, y_{t-2}) = (0, 0)$ or, equivalently, $(y_{t-1}, y_{t-2}) = (0, 0)$, all models are given equal weight. Along the lines $y_{t-1} = y_{t-2}$ and $y_{t-2} = 0$, which might be interpreted as representing the borders between the different regimes, the models receive equal weight pairwise. For example, along $y_{t-2} = 0$, the models in the first and third regimes receive equal weight (where the subscript of the autoregressive parameters is used to identify the regime number); the same holds for the models in the second and fourth regimes. Moving into a particular regime increases the weight of the corresponding model.

To illustrate the possible dynamics that can be generated by the MRSTAR model, Figure 2 shows some time series generated by the sample model (9). Two hundred pseudo-random numbers are drawn from the standard normal distribution to obtain a sequence of errors ε_t, while the necessary initial values y_{-1} and y_0 are set equal to zero. The thresholds c_1, c_2 and the parameters γ_1 and γ_2 are set equal to the values given above. In the upper panel of Figure 2, the autoregressive parameters are set as follows; $\phi_1 = \phi_2 = 0.3$ and $\phi_3 = \phi_4 = 0.9$. Hence, the model reduces to a basic LSTAR model (1) with y_{t-2} as transition variable. In all panels of Figure 2, a realization of an AR(1) model $y_t = \phi y_{t-1} + \varepsilon_t$ with autoregressive parameter $\phi = 0.6$, using the same errors ε_t, also is plotted for comparison. Although the time series generated by the LSTAR model has the same average autoregressive parameter as the linear AR(1) model, the behavior is markedly different: For positive values of y_{t-2}, the tendency of the series to return to its attractor (which is equal to zero) is much smaller than for negative values of the transition variable. The middle panel of Figure 2 shows the AR(1) series together with a realization of the MRSTAR model with $\phi_1 = \phi_3 = 0.3$ and $\phi_2 = \phi_4 = 0.9$. The resulting model is a momentum STAR (MSTAR) model because the autoregressive parameters only depend on the direction in which the series is moving. In our example, the memory of the series is longer for upward than for downward movements. The main difference between the AR and MSTAR models occurs in the peaks, the upward (downward) peaks being more (less) pronounced in the nonlinear model. Finally, the lower panel of Figure 2 shows the AR(1) series together with a realization of the MRSTAR model (9), with the autoregressive parameters taken to be the averages of the parameters in the LSTAR and MSTAR models; that is, ϕ_1 through ϕ_4 are set equal to 0.3, 0.6, 0.6, and 0.9, respectively. Obviously, the resulting time series combines the properties of the LSTAR and MSTAR models: Persistence is strongest for positive and increasing values, intermediate for positive and decreasing values and negative and increasing values, and smallest for negative and decreasing values of the time series.

3. SPECIFICATION OF MRSTAR MODELS

We suggest a specific-to-general approach to specify MRSTAR models, that is, to build up the number of regimes by iterating between testing for the desirability of additional regimes and estimating multiple-regime models.[5] The reason for

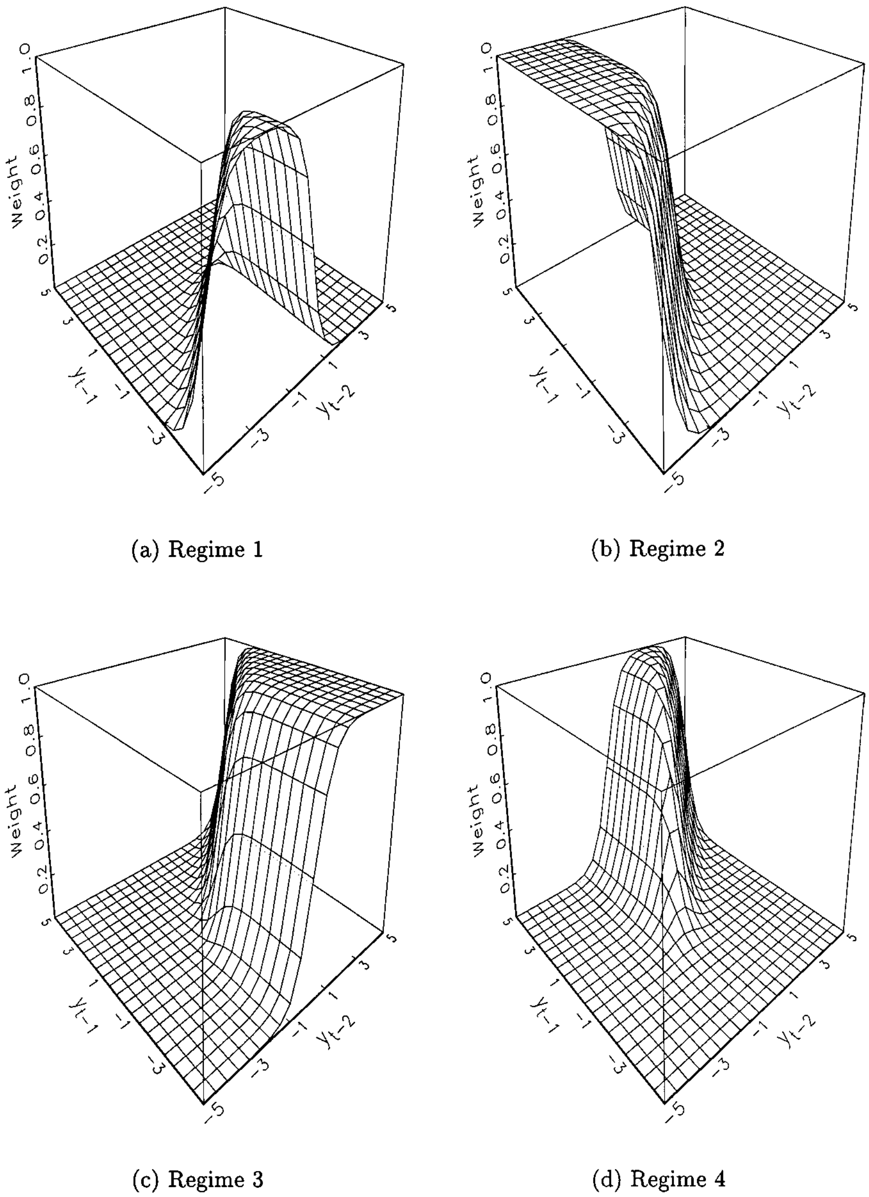

FIGURE 1. Weights in MRSTAR model [weights assigned to different AR models in the example MRSTAR model (9)].

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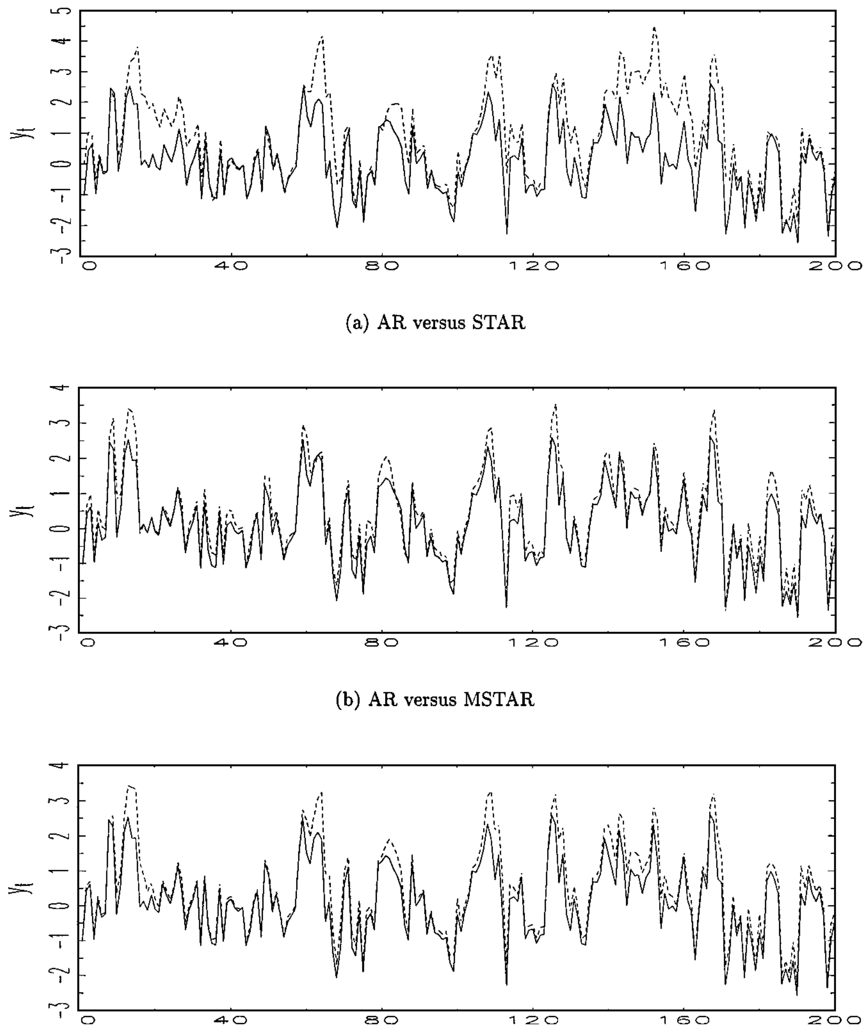

(a) AR versus STAR

(b) AR versus MSTAR

(c) AR versus MRSTAR

FIGURE 2. Realizations of sample MRSTAR model (9) with $\gamma_1 = \gamma_2 = 2.5$, $c_1 = c_2 = 0$, $\varepsilon_t \sim$ i.i.d. $N(0, 1)$ for different combinations of autoregressive parameters: (a) $\phi_1 = \phi_2 = 0.3$ and $\phi_3 = \phi_4 = 0.9$; (b) $\phi_1 = \phi_3 = 0.3$ and $\phi_2 = \phi_4 = 0.9$; (c) $\phi_1 = 0.3$, $\phi_2 = 0.6$, $\phi_3 = 0.6$, and $\phi_4 = 0.9$. The solid line is a realization of an AR(1) with autoregressive parameter 0.6, using the same errors ε_t.

preferring this approach rather than, for example, applying model selection criteria is that the latter approach requires the estimation of all candidate models. This may become very time-consuming if one wants to consider various choices for the transition variables s_{1t} and s_{2t} and combinations thereof. In Section 3.1, we outline the specification procedure in more detail and develop an LM-type test statistic that can be used to test a two-regime STAR model against a multiple-regime alternative. In Section 3.2, we investigate the small-sample properties of the test statistic by means of simulation experiments.

3.1. A Specification Procedure for MRSTAR Models

We suggest that specification begins with specifying and estimating a basic LSTAR model (1), using the specification procedure of Teräsvirta (1994) as discussed in Section 2.2. The two-regime model then should be tested against the alternative of a general MRSTAR as given in (5). The principle of approximating the transition function as applied by Luukkonen et al. (1988) to develop LM-type tests against STAR nonlinearity can be used to obtain a test against the MRSTAR alternative (5). For this purpose, it is convenient to rewrite the model as follows:

$$\begin{aligned} y_t = {} & \phi_1^{*\prime} y_t^{(p)} + \phi_2^{*\prime} y_t^{(p)} F_1(s_{1t};\gamma_1,c_1) + \phi_3^{*\prime} y_t^{(p)} F_2(s_{2t};\gamma_2,c_2) \\ & + \phi_4^{*\prime} y_t^{(p)} F_1(s_{1t};\gamma_1,c_1) F_2(s_{2t};\gamma_2,c_2) + \varepsilon_t, \end{aligned} \tag{10}$$

where $\phi_1^* = \phi_1$, $\phi_2^* = \phi_2 - \phi_1$, $\phi_3^* = \phi_3 - \phi_1$, and $\phi_4^* = \phi_1 - \phi_2 - \phi_3 + \phi_4$. The two-regime model that has been estimated is assumed to have $F_1(\cdot)$ as transition function. Hence, we wish to test whether the addition of the regimes determined by $F_2(\cdot)$ is appropriate. Subtracting 1/2 from the logistic function F_2 does not alter the model but it allows expression of the null hypothesis to be tested as $H_0 : \gamma_2 = 0$. Because the model is not identified under the null hypothesis, a test statistic cannot be derived directly. We proceed by replacing the transition function $F_2(s_{2t};\gamma_2,c_2)$ in (10) with a third-order Taylor expansion[6] around the point $\gamma_2(s_{2t} - c_2) = 0$. After rearranging terms, the model becomes

$$\begin{aligned} y_t = {} & \theta_1' y_t^{(p)} + \theta_2' y_t^{(p)} F_1(s_{1t};\gamma_1,c_1) + \beta_1' y_t^{(p)} s_{2t} + \beta_2' y_t^{(p)} s_{2t}^2 + \beta_3' y_t^{(p)} s_{2t}^3 \\ & + \left(\beta_4' y_t^{(p)} s_{2t} + \beta_5' y_t^{(p)} s_{2t}^2 + \beta_6' y_t^{(p)} s_{2t}^3\right) F_1(s_{1t};\gamma_1,c_1) + e_t, \end{aligned} \tag{11}$$

where the parameter vectors $\beta_i = (\beta_{0i}, \beta_{1i}, \ldots, \beta_{pi})'$, $i = 1, \ldots, 6$ are defined in terms of ϕ_i^*, $i = 1, \ldots, 4$, γ_2, and c_2, whereas the error term e_t is the sum of ε_t and the approximation error that arises from replacing the transition function F_2 with a finite-order Taylor expansion. The null hypothesis now can be reformulated as $H_0^* : \beta_i = 0,\ i = 1, \ldots, 6$. Note that, under the null hypothesis, $\theta_1 = \phi_1^* = \phi_1$, $\theta_2 = \phi_2^* = \phi_2 - \phi_1$, and $e_t = \varepsilon_t$. It also should be remarked that if $s_{2t} = y_{t-d}$ for certain $d \le p$ or if $s_{2t} = \alpha' \tilde{y}_t^{(p)}$ for certain α, the terms $\beta_{0i} s_{2t}^i$, $i = 1, 2, 3$ and $\beta_{0i} s_{2t}^{i-3} F_1(s_{1t};\gamma_1,c_1)$, $i = 4, 5, 6$ in (11) are redundant and should be omitted. Assuming the errors to be normally distributed, it follows that the conditional

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log-likelihood for observation t is given by

$$l_t = -\frac{1}{2}\ln 2\pi - \frac{1}{2}\ln \sigma^2 - \frac{e_t^2}{2\sigma^2}. \tag{12}$$

Because the information matrix is block diagonal, the error variance σ^2 can be assumed to be fixed. The remaining partial derivatives evaluated under the null hypothesis are given by

$$\left.\frac{\partial l_t}{\partial \theta_1}\right|_{H_0} = \frac{1}{\sigma^2}\hat{e}_t y_t^{(p)}, \tag{13}$$

$$\left.\frac{\partial l_t}{\partial \theta_2}\right|_{H_0} = \frac{1}{\sigma^2}\hat{e}_t y_t^{(p)} F_1(s_{1t}; \hat{\gamma}_1, \hat{c}_1), \tag{14}$$

$$\left.\frac{\partial l_t}{\partial \beta_i}\right|_{H_0} = \frac{1}{\sigma^2}\hat{e}_t y_t^{(p)} s_{2t}^i, \qquad i = 1, 2, 3, \tag{15}$$

$$\left.\frac{\partial l_t}{\partial \beta_i}\right|_{H_0} = \frac{1}{\sigma^2}\hat{e}_t y_t^{(p)} F_1(s_{1t}; \hat{\gamma}_1, \hat{c}_1) s_{2t}^{i-3}, \qquad i = 4, 5, 6, \tag{16}$$

$$\left.\frac{\partial l_t}{\partial \gamma_1}\right|_{H_0} = \frac{1}{\sigma^2}\hat{e}_t \hat{\theta}_2' y_t^{(p)} \frac{\partial F_1(s_{1t}; \hat{\gamma}_1, \hat{c}_1)}{\partial \gamma_1}, \tag{17}$$

$$\left.\frac{\partial l_t}{\partial c_1}\right|_{H_0} = \frac{1}{\sigma^2}\hat{e}_t \hat{\theta}_2' y_t^{(p)} \frac{\partial F_1(s_{1t}; \hat{\gamma}_1, \hat{c}_1)}{\partial c_1}, \tag{18}$$

where

$$\begin{aligned}\frac{\partial F_1(s_{1t}; \hat{\gamma}_1, \hat{c}_1)}{\partial \gamma_1} &= \{1 + \exp[-\hat{\gamma}_1(s_{1t} - \hat{c}_1)]\}^{-2} \exp[-\hat{\gamma}_1(s_{1t} - \hat{c}_1)](s_{1t} - \hat{c}_1) \\ &= F_1(s_{1t}; \hat{\gamma}_1, \hat{c}_1)[1 - F_1(s_{1t}; \hat{\gamma}_1, \hat{c}_1)](s_{1t} - \hat{c}_1), \end{aligned} \tag{19}$$

$$\begin{aligned}\frac{\partial F_1(s_{1t}; \hat{\gamma}_1, \hat{c}_1)}{\partial c_1} &= \hat{\gamma}_1\{1 + \exp[-\hat{\gamma}_1(s_{1t} - \hat{c}_1)]\}^{-2} \exp[-\hat{\gamma}_1(s_{1t} - \hat{c}_1)] \\ &= \hat{\gamma}_1 F_1(s_{1t}; \hat{\gamma}_1, \hat{c}_1)[1 - F_1(s_{1t}; \hat{\gamma}_1, \hat{c}_1)]. \end{aligned} \tag{20}$$

The partial derivatives (19) and (20) are denoted as $\hat{F}_{\gamma_1}(t)$ and $\hat{F}_{c_1}(t)$, respectively; we also use the shorthand notation $\hat{F}_1(t)$ to denote $F_1(s_{1t}; \hat{\gamma}_1, \hat{c}_1)$.

The above suggests that an LM-type test statistic to test H_0^* can be computed in a few steps as follows:

1. Estimate the two-regime LSTAR model (1) with (2) by nonlinear least squares, obtain the residuals $\hat{e}_t \equiv y_t - \hat{\phi}_1 y_t^{(p)}[1 - \hat{F}_1(t)] - \hat{\phi}_2 y_t^{(p)} \hat{F}_1(t)$, and compute the sum of squared residuals under the null hypothesis, $\mathrm{SSR}_0 = \sum \hat{e}_t^2$.
2. Regress the residuals $\hat{e}_t$ on $[y_t^{(p)}, y_t^{(p)}\hat{F}_1(t), \hat{\theta}_2' y_t^{(p)} \hat{F}_{\gamma_1}(t), \hat{\theta}_2' y_t^{(p)} \hat{F}_{c_1}(t)]$ and the auxiliary regressors $[y_t^{(p)} s_{2t}^i, y_t^{(p)} \hat{F}_1(t) s_{2t}^i, i = 1, 2, 3]$ and compute the sum of squared residuals under the alternative, SSR_1.

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3. Compute the LM-type test statistic as

$$\mathrm{LM}_{\mathrm{MR}} = \frac{(\mathrm{SSR}_0 - \mathrm{SSR}_1)/6(p+1)}{\mathrm{SSR}_1/[T - 6(p+1) - 2(p+1)]}, \tag{21}$$

where T denotes the sample size.

In step 2, the estimates of the autoregressive parameters in the LSTAR model are used to obtain an estimate of θ_2, that is, $\hat{\theta}_2 = \hat{\phi}_2 - \hat{\phi}_1$, which is consistent under the null hypothesis. Under the null hypothesis, the statistic $\mathrm{LM}_{\mathrm{MR}}$ is F distributed with $6(p+1)$ and $T-6(p+1)-2(p+1)$ degrees of freedom. As usual, the F version of the test statistic is preferable to the χ^2 variant in small samples because its size and power properties are better. The remarks made by Eitrheim and Teräsvirta (1996) concerning potential numerical problems are relevant for our test as well. If $\hat{\gamma}_1$ is very large, such that the transition between the two regimes in the model under the null hypothesis is fast, the partial derivatives of the transition function F_1 with respect to γ_1 and c_1, as given in (19) and (20), approach zero functions [except for $F_{c_1}(t)$ at the point $s_{1t} = \hat{c}_1$]. Hence, the moment matrix of the regressors in the auxiliary regression becomes near-singular. However, because the terms in the auxiliary regression involving these partial derivatives are likely to be very small for all $t = 1, \dots, T$, they contain very little information. It is therefore suggested that these terms simply be omitted under such circumstances, which will not harm the test statistic. Furthermore, the residuals $\hat{e}_t$ obtained from estimating the two-regime LSTAR model may not be exactly orthogonal to the gradient matrix [which may also result from omitting the terms involving $\hat{F}_{\gamma_1}(t)$ and $\hat{F}_{c_1}(t)$]. Following Eitrheim and Teräsvirta (1996), we suggest accounting for this by performing the following additional step in calculating the test statistic

1′. Regress $\hat{e}_t$ on $y_t^{(p)}$ and $y_t^{(p)}\hat{F}_1(t)$ [and $\hat{\theta}_2' y_t^{(p)}\hat{F}_{\gamma_1}(t)$ and $\hat{\theta}_2' y_t^{(p)}\hat{F}_{c_1}(t)$ if these terms are not excluded], compute the residuals $\tilde{e}_t$ from this regression, and the residual sum of squares $\mathrm{SSR}_0 = \sum \tilde{e}_t^2$.

The residuals $\tilde{e}_t$ instead of $\hat{e}_t$ then should be used in steps (2) and (3).

The LM test presented here is in fact a generalization of the diagnostic test of Eitrheim and Teräsvirta (1996) against time-varying coefficients, in which s_{2t} is taken equal to time, $s_{2t} = t$. Furthermore, their test for remaining nonlinearity can be regarded as a test against the restricted version of the MRSTAR model given in (6) with s_t in F_1 and F_2 replaced by s_{1t} and s_{2t}, respectively, which are not necessarily the same. Recall, however, that such a restricted specification may be convenient/appropriate (only) if the transition variables s_{1t} and s_{2t} are in fact the same. Obviously, then, our test also can be interpreted and used as a diagnostic tool to evaluate estimated two-regime STAR models.

If the LM-type test (21) rejects the two-regime model in favor of the four-regime alternative, one might proceed with estimation of the alternative model by nonlinear least squares. Once the general model has been estimated, restrictions on the autoregressive parameters to test, for example, equality of models in different regimes can be tested using likelihood ratio tests. Diagnostic tests for serial correlation, constancy of parameters, and remaining nonlinearity can be developed along the same lines as in Eitrheim and Teräsvirta (1996).

3.2. Small-Sample Properties of LM-Type Test for No Remaining Nonlinearity

Before we turn to our empirical application of the MRSTAR models and the specification procedure discussed above, we evaluate the small-sample properties of the LM-type test (21) by means of a limited simulation experiment.

To investigate the size of the $\mathrm{LM}_{\mathrm{MR}}$ test, a two-regime LSTAR model (1)–(2) is used as data-generating processes (DGP), with $p = 1$, $\phi_{10} = \phi_{20} = 0$, $s_t = y_{t-1}$, $\gamma = 2.5$, $c = 0$, and the errors ε_t standard normally distributed. The procedure that is followed in the simulation experiments mimics the setup of Eitrheim and Teräsvirta (1996). Each replication is subjected first to the LM-type linearity test that is used in the specification procedure for STAR models of Teräsvirta (1994), assuming that the true order of the model and the transition variable are known. The series is retained only if the null hypothesis is rejected at the 5% level of significance. The reason for doing this is to avoid estimating a STAR model on series in which very little or no evidence of nonlinearity is present. If the series is not discarded, a two-regime LSTAR model is estimated and, if the estimation algorithm converges, the $\mathrm{LM}_{\mathrm{MR}}$ test statistic is computed as discussed above for $s_{2t} = y_{t-1}$, y_{t-2}, and Δy_{t-1}. In computing the test statistic, the terms involving $\hat{F}_{\gamma_1}(t)$ and $\hat{F}_{c_1}(t)$ are always omitted, and the orthogonalization step $(1')$ is always applied. We fix the total number of accepted replications at 1000 for all DGP's. We consider series of $T = 200$ observations. The choice for this particular sample size is motivated by the length of our empirical time series on U.S. GNP in Section 4. In all experiments reported later, necessary starting values of the time series are set equal to zero. To eliminate possible dependencies of the results on this initialization, the first 100 observations of each series are discarded.

Table 1 shows the empirical size at 1, 5, and 10% significance levels, using the appropriate critical values from the F-distribution. It is seen that, for all combinations of ϕ_{11} and ϕ_{21} that are considered, the empirical size of the $\mathrm{LM}_{\mathrm{MR}}$ test statistic is below its nominal size. Especially if $s_{2t} = y_{t-1}$, which is the transition variable in the estimated LSTAR model, the test is very conservative. Unreported results for the $\mathrm{LM}_{\mathrm{MR}}$ test statistic based on a first-order Taylor approximation of the transition function F_2 and the test for no remaining nonlinearity of Eitrheim and Teräsvirta (1996) demonstrate that these tests also suffer from the same problem.

The power properties of the $\mathrm{LM}_{\mathrm{MR}}$ statistic are investigated in two different ways. First, we use a two-regime ESTAR model (1) with (4) as DGP, with p, ϕ_1, ϕ_2, and s_{1t} as above, $\gamma = 10$, $c_1 = -1$, $c_2 = 1$, and ε_t again standard normally distributed. For replications that pass the linearity test, we erroneously fit an LSTAR model to the series and, upon normal convergence of the estimation algorithm, apply the $\mathrm{LM}_{\mathrm{MR}}$ test for the same choices of s_{2t} as above. Second, we use the example MRSTAR model (9) as DGP, with $\gamma_1 = \gamma_2 = 2.5$ and $c_1 = c_2 = 0$.

TABLE 1. Empirical size of LM_{MR} test for MRSTAR nonlinearity[a]

		Transition variable s_{2t}								
		y_{t-1}			y_{t-2}			Δy_{t-1}		
ϕ_{11}	ϕ_{21}	0.010	0.050	0.100	0.010	0.050	0.100	0.010	0.050	0.100
−0.5	−0.9	0.006	0.027	0.053	0.008	0.029	0.064	0.005	0.027	0.049
	0.0	0.006	0.032	0.059	0.004	0.044	0.101	0.004	0.019	0.053
	0.4	0.001	0.018	0.051	0.008	0.038	0.084	0.008	0.034	0.081
	0.9	0.007	0.024	0.037	0.012	0.044	0.082	0.007	0.038	0.086
0.5	−0.9	0.002	0.013	0.030	0.008	0.026	0.061	0.008	0.027	0.061
	−0.5	0.003	0.014	0.036	0.006	0.033	0.072	0.003	0.031	0.068
	0.0	0.002	0.024	0.058	0.007	0.038	0.095	0.006	0.038	0.083
	0.9	0.006	0.029	0.055	0.012	0.030	0.073	0.012	0.046	0.090

[a] Empirical size of the LM_{MR} test (21) of no remaining STAR-type nonlinearity at 0.010, 0.050, and 0.100 significance levels for series generated by the two-regime LSTAR model (1) with (2) with $\phi_{10} = \phi_{20} = 0$, $\gamma = 2.5$, $c = 0$, and $\varepsilon_t \sim$ i.i.d. $N(0, 1)$. The table is based on 1000 replications for sample size $T = 200$.

TABLE 2. Empirical power of LM_{MR} test for MRSTAR nonlinearity[a]

		Transition variable s_{2t}								
		y_{t-1}			y_{t-2}			Δy_{t-1}		
ϕ_{11}	ϕ_{21}	0.010	0.050	0.100	0.010	0.050	0.100	0.010	0.050	0.100
−0.5	−0.9	0.035	0.114	0.200	0.017	0.067	0.133	0.019	0.093	0.171
	0.0	0.133	0.341	0.467	0.012	0.044	0.087	0.038	0.151	0.241
	0.4	0.502	0.749	0.851	0.005	0.045	0.088	0.057	0.184	0.289
	0.9	0.731	0.867	0.919	0.103	0.263	0.385	0.020	0.077	0.149
0.5	−0.9	0.673	0.838	0.887	0.163	0.346	0.464	0.386	0.598	0.700
	−0.5	0.654	0.859	0.926	0.020	0.078	0.133	0.200	0.428	0.567
	0.0	0.133	0.307	0.430	0.009	0.038	0.097	0.034	0.116	0.202
	0.9	0.025	0.095	0.176	0.012	0.052	0.099	0.009	0.050	0.083

[a] Empirical power of the LM_{MR} test (21) of no remaining STAR-type nonlinearity at 0.010, 0.050, and 0.100 significance levels when series are generated according to the two-regime ESTAR model (1) with (4), with $\phi_{10} = \phi_{20} = 0$, $\gamma = 10$, $c_1 = -1$, $c_2 = 1$, and $\varepsilon_t \sim$ i.i.d. $N(0, 1)$, but an LSTAR model is erroneously fitted to the data. The table is based on 1000 replications for sample size $T = 200$.

Only series for which the LM-type linearity test rejects the null hypothesis at the 5% significance level when both Δy_{t-1} and y_{t-2} are used as transition variable are retained. For these series, two different two-regime LSTAR models are estimated, with Δy_{t-1} and y_{t-2} as transition variables, respectively.

The results for the experiments with an ESTAR model as DGP are displayed in Table 2. It is seen that the power of the test is reasonably good, provided that the nonlinearity is fairly strong, that is, ϕ_{11} and ϕ_{21} are not too close.

The results for the experiments with the MRSTAR model (9) as DGP are shown in Table 3. The results in this table show that our test compares favorably with the test proposed by Eitrheim and Teräsvirta (1996). In the next section, we apply our specification procedure to U.S. GNP.

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TABLE 3. Empirical power of $\mathrm{LM}_{\mathrm{MR}}$ test for MRSTAR nonlinearity[a]

Transitions variable s_{1t}	ϕ_1	ϕ_2	ϕ_3	ϕ_4	Test with transitions variable s_{2t}: $\mathrm{LM}_{\mathrm{MR}}$ y_{t-2}	$\mathrm{LM}_{\mathrm{MR}}$ Δy_{t-1}	ET y_{t-2}	ET Δy_{t-1}
Δy_{t-1}	−0.7	0.1	0.1	0.9	0.196	0.007	0.186	0.006
		−0.4	0.6		0.790	0.006	0.111	0.005
		0.6	−0.4		0.681	0.011	0.134	0.007
	−0.3	0.3	0.3	0.9	0.608	0.004	0.594	0.002
		0.0	0.6		0.773	0.007	0.630	0.006
		0.6	0.0		0.517	0.008	0.524	0.010
	0.1	0.5	0.5	0.9	0.966	0.000	0.975	0.001
		0.3	0.7		0.982	0.006	0.985	0.003
		0.7	0.3		0.947	0.002	0.964	0.000
y_{t-2}	−0.7	0.1	0.1	0.9	0.022	0.145	0.066	0.103
		−0.4	0.6		0.015	0.860	0.027	0.031
		0.6	−0.4		0.019	0.714	0.034	0.057
	−0.3	0.3	0.3	0.9	0.023	0.098	0.037	0.042
		0.0	0.6		0.022	0.324	0.038	0.027
		0.6	0.0		0.045	0.128	0.081	0.127
	0.1	0.5	0.5	0.9	0.022	0.031	0.043	0.030
		0.3	0.7		0.062	0.063	0.081	0.044
		0.7	0.3		0.013	0.020	0.014	0.035

[a] Empirical power of the $\mathrm{LM}_{\mathrm{MR}}$ test (21) and the Eitrheim-Teräsvirta (ET) test of no remaining STAR-type nonlinearity at 5% significance level when series are generated according to the MRSTAR model (9) with F_1 and F_2 both equal to logistic functions (2) with $\gamma_1 = \gamma_2 = 2.5$, $c_1 = c_2 = 0$, and $\varepsilon_t \sim$ i.i.d. $N(0, 1)$. A two-regime LSTAR model with transitions variable s_{1t} is fitted to the data, and the tests for no remaining nonlinearity are applied with transition variables s_{2t} in the additional transition function. The table is based on 1000 replications for sample size $T = 200$.

4. MULTIPLE REGIMES IN THE BUSINESS CYCLE?

Business-cycle asymmetry has been investigated mainly by examining U.S. output series, such as GNP and industrial production, and U.S. (un)employment series. We follow this practice here and explore whether multiple regimes in the business cycle can be identified by applying MRSTAR models to U.S. real GNP.

Previous studies applying tests for asymmetry to U.S. real GNP have provided mixed results. In particular, the evidence obtained from nonparametric procedures has not been very compelling. For example, Falk (1986) cannot reject symmetry when examining U.S. real GNP for steepness; see also DeLong and Summers (1986) and Sichel (1993). Similarly, Brock and Sayers (1988) only marginally reject linearity, whereas Sichel (1993) finds only moderate evidence for deepness. An exception to the rule is Brunner (1992), who obtains fairly strong indications for asymmetry in GNP, which might be associated with an increase in variance during contractions. This is confirmed by Emery and Koenig (1992), who suggest that the variance of leading and coincident indexes increases as contractions proceed.

Additionally, Cooper (1998) finds very strong evidence for the existence of multiple regimes in industrial production series using a regression-tree approach.

The application of parametric nonlinear time-series models has been more successful. Hamilton (1989) and Durland and McCurdy (1994), for example, find that a two-state Markov switching model for the growth rate of postwar quarterly U.S. real GNP outperforms linear models. Boldin (1996) examines the stability of this model and demonstrates that the model is not robust to extension of the sample period. Tiao and Tsay (1994), Potter (1995) and Clements and Krolzig (1998) all estimate a two-regime SETAR model consisting of AR(2) models [although Potter (1995) adds an additional fifth lag]. The growth rate two periods lagged is used as the transition variable, and the threshold is either fixed at zero [Potter (1995)] or estimated to be equal to or close to zero [Tiao and Tsay (1994), Clements and Krolzig (1998)]. Hence, a distinction is made between periods of positive and negative growth. A common feature of all of these estimated models is that the dynamics in contractions are very different from those during expansions. In particular, the SETAR models, which are estimated on data from 1948 until 1990, all contain a large negative coefficient on the second lag in the contraction regime, suggesting that U.S. GNP moves quickly out of recessions. Notably, Clements and Krolzig (1998) find much less evidence of this property when they reestimate their model on a recent vintage of data ranging from 1960 until 1996. Beaudry and Koop (1993) estimate a linear AR model in which the "current depth of recession," which measures deviations from past highs in the level of real GNP, is added as regressor. This variable is discussed in more detail below. As shown by Pesaran and Potter (1997), the resulting model also can be interpreted as a SETAR model.

Whereas most attention focuses on the distinction between contractions and expansions, some indications for the existence of multiple regimes have been obtained as well. For example, Sichel (1994) demonstrates that growth in real GDP is larger immediately following a business-cycle trough than during later parts of the expansion, suggesting that the business cycle consists of three distinct phases: contractions, high-growth recoveries, and moderate-growth expansions. Wynne and Balke (1992) and Balke and Wynne (1996) also document this bounce-back effect in industrial production. Furthermore, they examine the relationship between growth during the first 12 months following a trough and the decline of the preceding contraction and show that deep recessions generally are followed by strong recoveries. Emery and Koenig (1992) also find that the mean growth rate in leading and coincident indexes is larger (in absolute value) in early (late) stages of the expansion (contraction). The three-regime Markov switching model estimated by Boldin (1996), the floor-and-ceiling model of Pesaran and Potter (1997), and the four-regime SETAR model of Tiao and Tsay (1994) explicitly model the existence of a strong-recovery regime because these models include a regime in which output is growing fast (following a recession).

Compared to the previous studies mentioned above, we use a relatively long span of data, which ranges from 1947:I to 1995:II. The data, which are at 1987 prices, are seasonally adjusted and are taken from the Citibase database. The growth rate

y_t is graphed in the upper panel of Figure 3. The solid circles indicate NBER-dated peaks and troughs, which are marked with P's and T's, respectively, as well. The lower graph of this figure shows the mean growth rates during contractions and different phases of expansions. It is seen that, in the first four quarters following a trough, growth is considerably higher than during the rest of this expansion, confirming the observation of Sichel (1994).

Following many of the previously mentioned authors, we use an AR(2) model as the basis for our model-building exercise. The estimated model over the period 1947:IV–1995:II is

$$\begin{aligned} y_t &= \underset{(0.091)}{0.430} + \underset{(0.073)}{0.345} y_{t-1} + \underset{(0.073)}{0.095} y_{t-2} + \hat{\varepsilon}_t, \end{aligned} \tag{22}$$

$\hat{\sigma}_\varepsilon = 0.917$, SK $= 0.01(0.48)$, EK $= 1.40(0.00)$, JB $= 15.58(0.00)$, ARCH(1) $= 3.03(0.08)$, ARCH(4) $= 9.27(0.06)$, LB(8) $= 5.05(0.41)$, LB(12) $= 14.00(0.12)$, AIC $= -0.142$, BIC $= -0.091$,

where standard errors are given in parentheses below the parameter estimates, $\hat{\varepsilon}_t$ denotes the regression residual at time t, $\hat{\sigma}_\varepsilon$ is the residual standard deviation, SK is skewness, EK is excess kurtosis, JB is the Jarque-Bera test of normality of the residuals, ARCH is the LM test of no autoregressive conditional heteroskedasticity (ARCH), LB is the Ljung-Box test of no autocorrelation, and AIC and BIC are the Akaike and Schwarz information criteria, respectively. The values in parentheses following the test statistics are p-values.

Normality of the residuals is rejected because of the considerable excess kurtosis. Closer inspection of the residuals reveals that this may be caused by large residuals in the first quarter of 1950 and the second quarter of 1980. These observations also may cause the ARCH tests to reject homoskedasticity. On the other hand, the LM test for ARCH is known to have power against alternatives other than ARCH as well, and, hence, it also may be that the significant values of this test statistic are caused by neglected nonlinearity.

This final conjecture is investigated further by applying the LM-type tests of Luukkonen et al. (1988) to test for the possibility of STAR-type nonlinearity. We only report results of their test S_2, which is obtained by replacing the transition function in (1) with a third-order Taylor approximation [similar to going from (10) to (11)], as well as the economy-version S_3, which is obtained from S_2 by omitting redundant terms and which therefore might have better power properties. Apart from lagged growth rates and changes therein, we also consider a measure of the current depth of recession (CDR) as possible transition variable, following Beaudry and Koop (1993). We define CDR_t as

$$\text{CDR}_t = \max_{j \geq 1}\{x_{t-j}\} - x_t, \tag{23}$$

with x_t the log of U.S. real GNP. As noted above, Beaudry and Koop (1993) include CDR_{t-1} as an additional regressor in an otherwise linear AR model for the

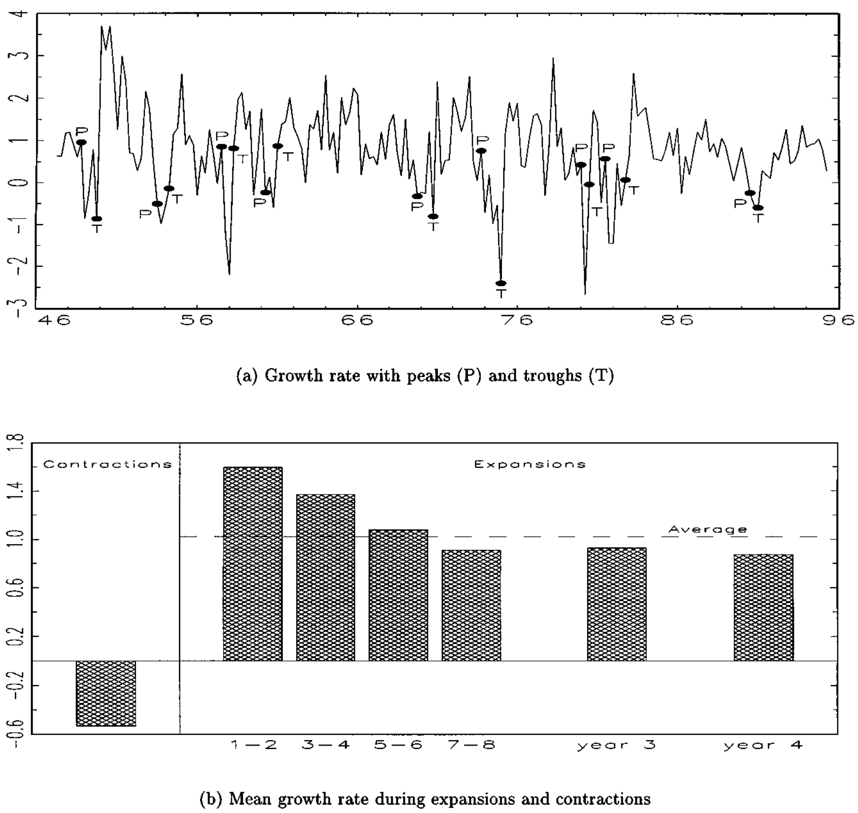

FIGURE 3. U.S. real GNP, quarterly growth rate. The upper graph shows quarterly growth rates of U.S. real GNP, 1947:II–1995:II. Solid circles indicate NBER-dated peaks (P) and troughs (T). The lower graph displays average growth over the business cycles.

GNP growth rate y_t. They claim that their CDR measure allows examination of the possibly different impact of positive and negative shocks. This is disputed by Elwood (1998), who argues that CDR_t only indicates (approximately) whether the economy is in recession or expansion, but does not measure the impact of negative shocks per se.[7] Following this argument, we only consider the CDR measure as a possible transition variable in STAR models.[8] Note that our definition of the CDR in (23) differs slightly from the original one of Beaudry and Koop (1993), which involves the maximum of past and *current* GNP. Hence, their CDR measure is equal to zero if real GNP is at an all-time high, and greater than zero otherwise. Because using such a truncated variable as the transition variable in STAR models is not very convenient, we only consider the maximum up to time t.

General versions of the LM-type tests for STAR nonlinearity, in which the transition variable only is assumed to be a linear combination of lagged endogenous

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values but is otherwise left unspecified, reject the null hypothesis of linearity quite convincingly; the p-values of the S_2 and S_3 tests are equal to 0.029 and 0.057, respectively. However, if s_t is specified in advance in order to get an impression of the most appropriate transition variable(s), the evidence for nonlinearity, in particular from the S_2 test, disappears somewhat.[9] As shown in Table 4, the p-values of the tests seem to suggest that y_{t-2}, Δy_{t-1}, Δy_{t-2}, CDR_{t-1}, and CDR_{t-2} might be considered as transition variables.

We decide to estimate an LSTAR model with CDR_{t-2} as the transition variable because the p-value of the S_3 test is lowest for this variable. The parameters in this LSTAR model are estimated as

$$\begin{aligned} y_t = {} & (\underset{(0.138)}{0.160} + \underset{(0.090)}{0.346} y_{t-1} + \underset{(0.108)}{0.282} y_{t-2}) \times [1 - F(\mathrm{CDR}_{t-2})] \\ & + (\underset{(0.163)}{0.665} + \underset{(0.121)}{0.308} y_{t-1} + \underset{(0.148)}{0.048} y_{t-2}) \times F(\mathrm{CDR}_{t-2}) + \varepsilon_t, \end{aligned} \quad (24)$$

$$F(\mathrm{CDR}_{t-2}) = \left\{1 + \exp\left[-\underset{(-)}{200.0}(\mathrm{CDR}_{t-2} - \underset{(0.135)}{0.281})/\sigma_{\mathrm{CDR}_{t-2}}\right]\right\}^{-1}, \quad (25)$$

$\hat{\sigma}_\varepsilon = 0.899$, $\mathrm{SK} = -0.17(0.16)$, $\mathrm{EK} = 1.19(0.00)$, $\mathrm{JB} = 12.21(0.00)$, $\mathrm{ARCH}(1) = 2.74(0.09)$, $\mathrm{ARCH}(4) = 7.09(0.13)$, $\mathrm{LM_{SI}}(4) = 1.39(0.24)$, $\mathrm{LM_{SI}}(8) = 1.48(0.17)$, $\mathrm{LM}_{C1} = 1.12(0.35)$, $\mathrm{LM}_{C2} = 1.01(0.44)$, $\mathrm{LM}_{C3} = 0.87(0.62)$, $\mathrm{AIC} = -0.129$, $\mathrm{BIC} = 0.008$,

where $\sigma_{\mathrm{CDR}_{t-2}}$ denotes the standard deviation of the transition variable CDR_{t-2}, $\mathrm{LM_{SI}}(q)$ denotes the LM-type test for qth-order serial correlation in the residuals and LM_{Ci}, $i = 1, 2, 3$ denotes LM-type tests for parameter constancy. Both sets of diagnostic checks are developed by Eitrheim and Teräsvirta (1996), to whom we refer for details.

Table 4. LM-type tests for STAR nonlinearity in U.S. GNP growth rates[a]

Transition variable	Test	d = 1	2	3	4	5	6
y_{t-d}	S_2	0.211	0.120	0.646	0.602	0.242	0.376
	S_3	0.330	0.053	0.256	0.258	0.235	0.248
Δy_{t-d}	S_2	0.089	0.065	0.982	0.819	0.291	0.220
	S_3	0.074	0.248	0.971	0.840	0.287	0.460
CDR_{t-d}	S_2	0.023	0.083	0.157	0.758	0.835	0.664
	S_3	0.022	0.014	0.123	0.498	0.645	0.564
$\Delta\mathrm{CDR}_{t-d}$	S_2	0.777	0.059	0.714	0.712	0.296	0.587
	S_3	0.649	0.159	0.745	0.544	0.067	0.356

[a] p-values for LM-type tests for smooth-transition nonlinearity in quarterly growth rate of U.S. real GNP. CDR_t measures the current depth of a recession, $\mathrm{CDR}_t = \max_{j\geq 1}\{x_{t-j}\} - x_t$ with x_t the log of U.S. GNP.

The exponent in the transition function is scaled by the standard deviation of the transition variable in order to make γ scale-free. We do not report a standard error for $\hat{\gamma}$ for reasons discussed in Section 2.1. The sum of the autoregressive coefficients is considerably larger in the regime where $F(\text{CDR}_{t-2})$ is equal to zero, which corresponds to expansions. This confirms the findings of Beaudry and Koop (1993) and Potter (1995), among others, that contractions are less persistent than expansions. Also note the large constant in the upper regime, which might be taken as an additional indication of a quick recovery following contractions [cf. Sichel (1994) and Wynne and Balke (1992)].

Apart from the diagnostic checks reported below the LSTAR model (24), we also apply the LM-type test against the MRSTAR alternative, developed in Section 3.1, as well as the LM-type tests of Eitrheim and Teräsvirta (1996) for remaining nonlinearity. Table 5 shows the p-values of the different tests for various choices of transition variables in the second transition function. The same table also reports results of the same tests when the additional transition function is replaced by only a first-order Taylor expansion, which, in theory at least, should be sufficient if only the logistic function is considered. It can be seen from the entries in Table 5 that there is some evidence for the necessity of considering a more elaborate nonlinear model than the fitted standard LSTAR model, especially if the change in the growth rate lagged one period is taken to be the transition variable in the second transition function.

Hence we proceed with estimating a four-regime MRSTAR model, with CDR_{t-2} and Δy_{t-1} as transition variables in the two logistic functions. The estimated model is given below:

$$
\begin{aligned}
y_t = {} & \{(\underset{(0.195)}{0.394} + \underset{(0.138)}{0.460} y_{t-1} + \underset{(0.156)}{0.092} y_{t-2}) \times [1 - F(\Delta y_{t-1})] \\
& + (\underset{(0.322)}{-0.121} + \underset{(0.284)}{0.442} y_{t-1} + \underset{(0.344)}{0.346} y_{t-2}) \times F(\Delta y_{t-1})\} \times [1 - F(\text{CDR}_{t-2})] \\
& + \{(\underset{(0.283)}{0.360} - \underset{(0.362)}{0.530} y_{t-1} + \underset{(0.449)}{0.963} y_{t-2}) \times [1 - F(\Delta y_{t-1})] \\
& + (\underset{(0.283)}{-0.019} + \underset{(0.187)}{0.744} y_{t-1} - \underset{(0.215)}{0.235} y_{t-2}) \times F(\Delta y_{t-1})\} \times F(\text{CDR}_{t-2}) + \hat{\varepsilon}_t,
\end{aligned} \tag{26}
$$

$$
F(\Delta y_{t-1}) = \left\{1 + \exp\left[-\underset{(-)}{500}(\Delta y_{t-1} - \underset{(0.032)}{0.250})/\sigma_{\Delta y_{t-1}}\right]\right\}^{-1}, \tag{27}
$$

$$
F(\text{CDR}_{t-2}) = \left\{1 + \exp\left[-\underset{(-)}{500}(\text{CDR}_{t-2} - \underset{(0.259)}{0.064})/\sigma_{\text{CDR}_{t-2}}\right]\right\}^{-1}. \tag{28}
$$

$\hat{\sigma}_\varepsilon = 0.867$, SK $= -0.12(0.25)$, EK $= 0.55(0.06)$, JB $= 2.82(0.24)$, ARCH(1) $= 1.08(0.30)$, ARCH(4) $= 4.28(0.37)$, AIC $= -0.117$, BIC $= 0.155$.

TABLE 5. LM-type tests for multiple regimes in U.S. GNP growth rates[a]

Transition variable	Test			
	ET_1	ET_3	$LM_{MR,1}$	$LM_{MR,3}$
y_{t-1}	0.35	0.26	0.27	0.53
y_{t-2}	0.35	0.06	0.16	0.15
Δy_{t-1}	0.08	0.06	0.01	0.05
CDR_{t-1}	0.18	0.06	0.23	0.07
CDR_{t-2}	0.18	0.32	0.12	0.61
ΔCDR_{t-1}	0.56	0.56	0.22	0.41

[a] The entries in columns ET_1 and ET_3 are p-values for the LM-type tests of Eitrheim and Teräsvirta (1996) for remaining nonlinearity, based on first- and third-order Taylor approximations of the second transition function, respectively. The entries in columns $LM_{MR,1}$ and $LM_{MR,3}$ are p-values for the tests of a basic LSTAR model against an MRSTAR alternative as developed in Section 3.1, also using first- and third-order Taylor approximations, respectively.

The large estimates of γ_1 and γ_2 in (28) and (27) imply that for both $F(\Delta y_{t-1})$ and $F(\text{CDR}_{t-2})$ the transition from zero to one is almost instantaneous at the estimated thresholds. The model is thus very similar to a NeTAR model. The model distinguishes between four different regimes, depending on whether the level of real GNP is above or below its previous high and whether growth is increasing or decreasing, which suggests the following interpretation of the four regimes.

- $\Delta y_{t-1} < 0$, $\text{CDR}_{t-2} < 0$. The economy is in expansion (recall that CDR_t as defined in (23) measures the distance in the level of real GNP relative to the previous all-time high), but growth is declining.
- $\Delta y_{t-1} > 0$, $\text{CDR}_{t-2} < 0$. The economy is in a strengthening expansion, as growth is accelerating.
- $\Delta y_{t-1} < 0$, $\text{CDR}_{t-2} > 0$. The economy is in a worsening contraction.
- $\Delta y_{t-1} > 0$, $\text{CDR}_{t-2} > 0$. The economy is in a contraction, but is improving given the positive change in growth.

The fourth regime more or less corresponds with the recovery phase identified by Sichel (1994), in which growth is strong immediately following a trough.

Figure 4 shows the distribution of the observations across the different regimes. When we take model (26), it is seen that the bulk of the observations is in regime 1, followed by regime 2. The worsening-contraction regime (regime 3) contains only 19 observations, confirming that the U.S. economy tends to recover quickly from recessions.

The various diagnostic tests for the MRSTAR model demonstrate that the residuals are much better behaved than the residuals from the AR(2) and LSTAR models. For example, normality cannot be rejected anymore. On the other hand, comparing the residual standard deviations suggests that the additional regimes improve the fit of the model only slightly, whereas both information criteria clearly favor the parsimonious AR(2) model. As an alternative way to evaluate the potential usefulness of the elaborate MRSTAR model, we focus on the implied propagation

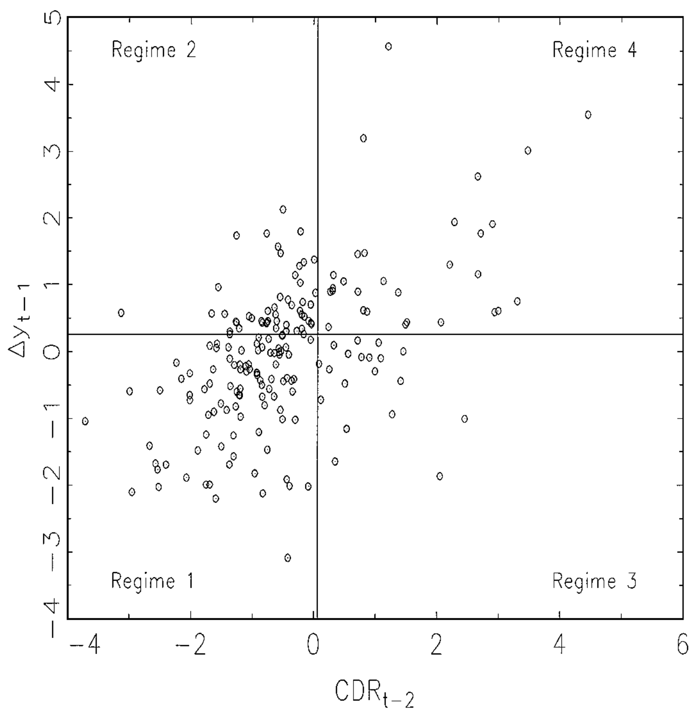

FIGURE 4. U.S. real GNP growth rates: Distribution of observations on quarterly growth rates of U.S. real GNP over the different regimes in the estimated MRSTAR model (26)–(28).

of shocks occurring in different regimes. Toward this end we compute generalized impulse response functions (GIRF)'s discussed extensively by Koop et al. (1996). In nonlinear models, the impact of a shock ε_t on y_{t+n} depends on (i) the history of the process up to time t, (ii) the size of the shock occurring at time t, and (iii) the shocks occurring during intermediate time periods $t+1, \ldots, t+n$. The GIRF is designed to take these factors influencing the impulse response explicitly into account. For an arbitrary current shock $\varepsilon_t = e_t$ and history $\Omega_{t-1} = \omega_{t-1}$, where for the MRSTAR model $\omega_{t-1} = \{y_{t-1}, y_{t-2}, \text{CDR}_{t-2}\}$, the GIRF is defined as

$$\text{GIRF}_y(n, e_t, \omega_{t-1}) = E(y_{t+n} \mid \varepsilon_t = e_t, \Omega_{t-1} = \omega_{t-1}) - E(y_{t+n} \mid \Omega_{t-1} = \omega_{t-1}), \quad (29)$$

for $n = 0, 1, 2, \ldots$. The GIRF is defined as the difference between the expectation of the growth rate n periods ahead, y_{t+n}, conditional on the history and the current shock, and the expectation of y_{t+n} conditional only on the past. The future is dealt with by averaging out the effect of intermediate shocks such that the response is an average of what might happen, given the past and present. The GIRF given in (29) is a function of e_t and ω_{t-1} (and n, of course). Koop et al. (1996) strongly emphasize

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that, by treating e_t and ω_{t-1} as realizations of the same stochastic process that generates realizations of y_t, the GIRF can be considered to be a realization of a random variable defined by

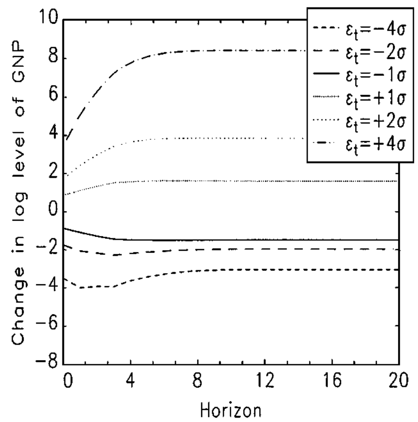
$$\mathrm{GIRF}_y(n, \varepsilon_t, \Omega_{t-1}) = E(y_{t+n} \mid \varepsilon_t, \Omega_{t-1}) - E(y_{t+n} \mid \Omega_{t-1}). \tag{30}$$

Various conditional versions of the GIRF might be of interest and can be defined by conditioning on particular subsets of the history and shocks, denoted A and B, respectively; that is,

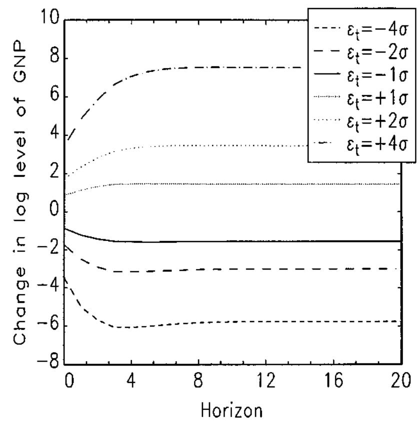
$$\mathrm{GIRF}_y(n, A, B) = E(y_{t+n} \mid \varepsilon_t \in A, \omega_{t-1} \in B) - E(y_{t+n} \mid \omega_{t-1} \in B). \tag{31}$$

We use a special case of (31) to obtain an impression of the dynamics in the different regimes of the estimated MRSTAR model by examining the GIRF for specific shocks, conditioning on all histories in a particular regime. That is, the set A is taken to consist of a single element e_t, while the set B consists of all histories belonging to one of the four regimes in the MRSTAR model. For the shock e_t we consider values equal to ± 1, ± 2, and ± 4 times the residual standard deviation. Because analytic expressions for the conditional expectations in (31) are not available, the GIRF's are estimated using the simulation procedure outlined by Koop et al. (1996). In particular, we use all observed histories in our estimation sample 1947:IV–1995:II and the corresponding residuals from the MRSTAR model to obtain the conditional expectations $E(y_{t+n} \mid \varepsilon_t = e_t, \Omega_{t-1} = \omega_{t-1})$ and $E(y_{t+n} \mid \Omega_{t-1} = \omega_{t-1})$ to obtain the shock- and history-specific GIRF as given in (29). The conditional GIRF's then are computed by averaging across histories in a particular regime. The resulting GIRF's for the log level of U.S. GNP (which are obtained by taking cumulative sums of the GIRF's for the growth rate) are shown in Figure 5.

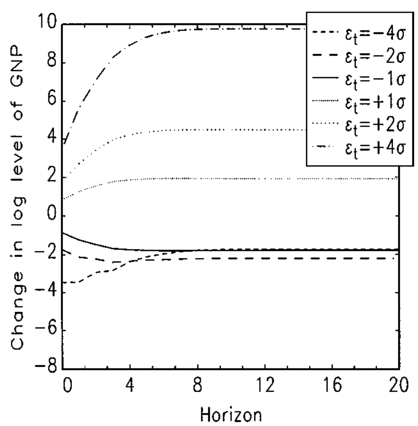
Several conclusions can be drawn from this figure. First, negative shocks appear to be less persistent than positive shocks, in the sense that in three out of the four regimes the average long-run response to negative shocks is smaller than the long-run response to positive shocks of equal size. This corresponds with the conclusions of Beaudry and Koop (1993) and Potter (1995), but contradicts the findings of Pesaran and Potter (1997). Second, whereas the response to positive shocks is quite similar in the different regimes, the response to negative shocks differs markedly. In the strengthening-expansion regime 2, negative shocks are magnified by a factor of 1.5 in the long run. In both the weakening-expansion and improving-contraction regimes, the long-run impact of negative shocks is approximately equal to the size of the shock. Finally, in the worsening-contraction regime 3, all negative shocks appear to generate approximately the same response, irrespective of their size. Inspection of the GIRF's for individual histories in this regime reveals that the long-run response to negative shocks can even be positive, while reversals also occur, that is, the largest (absolute value) negative shock has the largest positive response.



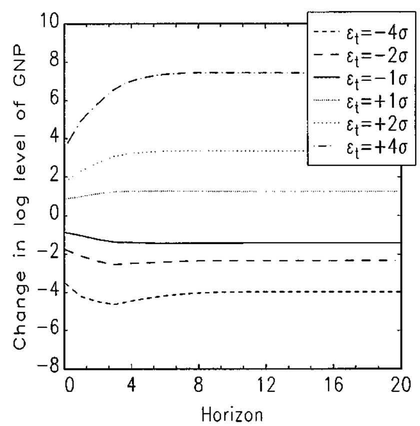
(a) Regime 1



(b) Regime 2



(c) Regime 3



(d) Regime 4

FIGURE 5. Generalized impulse response functions for the log level of U.S. real GNP for shocks ε_t equal to ± 1, ± 2, and ± 4 times the standard deviation based on the estimated MRSTAR model (26)–(28).

5. CONCLUDING REMARKS

We have explored possibilities of extending the basic STAR model to allow for more than two regimes. We have shown that this can be done by writing the model such that the different models that constitute the STAR model appear explicitly. A (specific-to-general) specification procedure was proposed and a new LM test for nonlinearity was developed, which can be used to test for the presence of multiple regimes. Alternatively, this test might be used as a diagnostic tool to test the adequacy of a fitted STAR model, complementing the tests of Eitrheim and Teräsvirta (1996). The application of the MRSTAR model to postwar U.S. real GNP demonstrates that a multiple-regime characterization of the business cycle might indeed be useful.

This paper offers some possibilities for further research. First, the effect of outliers on the detection of regimes seems to be of interest, as one does not want to fit spuriously a model that contains additional regimes only to capture some aberrant observations. It appears that a robust estimation method for STAR models needs to be developed to achieve proper protection against the influence of such anomalous observations. Alternative ways to compare different STAR models, possibly with a different number of regimes, also might be explored. It should be possible to use the techniques of Hess and Iwata (1997b) to examine explicitly whether the switching-regime models are capable of replicating basic stylized facts such as amplitude and duration of expansions and contractions. Finally, it might be worthwhile to extend the application to U.S. real GNP to a multivariate model, following the ideas of Koop et al. (1996), or to model nonlinearity and time-varying parameters simultaneously. All these issues are left for further research.

NOTES

1. See also Mittnik and Niu (1994) for a comprehensive overview.

2. Chan and Tong (1986) first proposed the STAR model as a generalization of the two-regime SETAR model, to alleviate the problem of estimating the threshold c in the latter model. They suggested the use of the standard normal cumulative distribution function as the transition function. The logistic function has become the standard choice, probably because of the existence of an explicit analytical form, which greatly facilitates estimation of the model.

3. It might be argued that it is not appropriate to choose the transition variable by comparing p-values as suggested above, because the models with different choices for s_t are nonnested. An alternative way to interpret and motivate this decision rule is the following: If the choice of the transition variable is made endogenous, one could estimate LSTAR models (1) for various choices of s_t and select the model that minimizes the residual variance (assuming the AR-order p is fixed). An obvious drawback of this procedure is, of course, the necessary estimation of nonlinear LSTAR models, which may be time-consuming. However, if the auxiliary regression model that is used in calculating the LM-type test statistic is considered to approximate the LSTAR model to a certain degree of accuracy, estimation of nonlinear models could be avoided by selecting s_t as the choice that minimizes the residual variance of this auxiliary model. Since the LM-type test is a monotonic transformation of the residual variance, this is equivalent to selecting s_t as the choice that maximizes the LM-type statistic. This is exactly what the minimum p-value rule employed here does; see also Caner and Hansen (1997).

4. To be precise, the specification procedure of Teräsvirta (1994) first proceeds by applying a sequence of nested tests to decide whether a logistic- or exponential-type transition function [given in equation (3)] is most appropriate. We omit details here because we focus on models with logistic transition functions to introduce the multiple-regime models. Note, however, that the same principles discussed below apply to models with different transition functions as well.

5. See Teräsvirta and Lin (1993) for a similar approach to determine the appropriate number of hidden units in ANN models.

6. Because we restrict attention to logistic transition functions, a first-order Taylor expansion would suffice. However, there might be certain alternatives against which the resulting test statistic has very little or no power; see Luukkonen et al. (1988) for details.

7. See Hess and Iwata (1997a) for another critical assessment of the model of Beaudry and Koop (1993).

8. Note that CDR_t resembles the growth rate y_t quite closely. Given that real GNP is upward trending, $\max_{j\geq 1} x_{t-j}$ will be equal to x_{t-1} most of the time. In that case, CDR_t equals $-y_t$. To be more precise, it is straightforward to show that $\mathrm{CDR}_t = \max(\mathrm{CDR}_{t-1}, 0) - y_t$. Hence, during expansions (i.e., when $\mathrm{CDR}_{t-1} > 0$), CDR_t and y_t coincide, whereas during contractions they might differ. The correlation between CDR_t and y_t equals -0.8, which confirms their similarity.

9. Jansen and Oh (1996) also report that tests for STAR-type nonlinearity do not reject the null hypothesis of linearity. Similarly, Hansen (1996) shows that tests for threshold-type nonlinearity do not provide very convincing evidence in favor of a threshold model.

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