The Bulletin of Symbolic Logic
Volume 24, Number 4, December 2018

2018 NORTH AMERICAN ANNUAL MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

Western Illinois University, Macomb, IL, USA
May 16–19, 2018

The 2018 North American Annual Meeting of the Association for Symbolic Logic was held at Western Illinois University in Macomb, Illinois from May 16–19, 2018. There were eight plenary talks, two tutorials, five special sessions, seven sessions of contributed talks and the presentations of the 2018 Karp Prize and the 2016 Shoenfield Prize.

A welcoming reception was held on the evening of Wednesday, May 16, and a banquet was held on the evening of Friday, May 18.

The program committee consisted of Tim Carlson, Barbara Csima, Todd Eisworth, Michael Glanzberg, Iraj Kalantari, David Marker (Chair) and Mariya Soskova. The local organizing committee consisted of John Chisholm, Rumen Dimitrov, Clifton Ealy, David Haugen, Iraj Kalantari (Chair), Doug LaFountain, Jana Marikova, Susan Martinelli, Mojtaba Moniri, Gordan Pettit, Bryan Powell, Christopher Pynes, Nader Vakil, Galen Weitkamp, and Larry Welch.

There were 137 registered participants at the meeting, including 48 graduate students. Generous financial support was provided by the Association for Symbolic Logic, the National Science Foundation, and the Department of Mathematics & Philosophy of Western Illinois University at Macomb, Illinois.

The plenary addresses at the meeting were as follows:

JC Beall (University of Connecticut), *Logic from a subclassical point of view*.

Artem Chernikov (University of California, Los Angeles), *Local distality and distal expansions of stable theories*.

Bradd Hart (McMaster University), *In defence of ultraproducts*.

Julia Knight (University of Notre Dame), *Roots of polynomials in fields of generalized power series*.

Joel Nagloo (Bronx Community College), *Model theory and classical differential equations*.

Dima Sinapova (University of Illinois, Chicago), *Stronger tree properties and the SCH*.

Slawomir Solecki (Cornell University), *Fraïssé limits and compact spaces*.

Andreas Weiermann (Ghent University), *Generalized Goodstein principles and notation systems for finite numbers*.

There were two tutorials, each with two one-hour sessions.

Theodore Slaman (University of California, Berkeley), *Recursion theory and Diophantine approximation*.

Andrew Marks (University of California, Los Angeles), *Descriptive set theory and geometrical paradoxes*.

The 2018 Karp Prize was presented to Matthias Aschenbrenner, Lou van den Dries and Joris van der Hoeven for their work on asymptotic differential algebra and the model theory of transseries. David Marker (University of Illinois, Chicago) lectured on the prize winners work.

1079-8986/18/2404-0006
DOI:10.1017/bsl.2018.60

The 2016 Shoenfield Prizes were presented at the banquet to Rod Downey and Denis Hirschfeldt for their book *Algorithmic randomness and complexity* and to Lou van den Dries for his article *Lectures on the model theory of valued fields*.

The program included five special sessions, held in parallel, each consisting of three two-hour periods. With organizers listed in parentheses, these were: Computability Theory (Laurent Bienvenu and Karen Lange), twelve talks; Logic and Philosophy (Curtis Franks), four talks (two cancelations after the schedule was printed); Model Theory (James Freitag and Jana Marikova), twelve talks; Proof Theory (Henry Towsner), seven talks (one cancelation after the schedule was printed) and Set theory (Dima Sinapova and Anush Tserunyan), nine talks; There were 26 contributed talks delivered at the meeting (one cancellation after the schedule was printed), and 4 additional abstracts presented by title.

Abstracts of the invited talks and the contributed talks (given in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
David Marker

Abstracts of invited tutorials

► ANDREW MARKS, *Descriptive set theory and geometrical paradoxes*.
Department of Mathematics, University of California, Los Angeles, Box 951555, Los Angeles, CA 90095-1555, USA.
E-mail: marks@math.ucla.edu.

In the last few years, a number of results have been proved showing that the "paradoxical" sets in many classical geometrical paradoxes can surprisingly be much "nicer" than one would naively expect. For example, a classical generalization of the Banach Tarski paradox states that any two bounded subsets A, B of R^3 with nonempty interior are equidecomposable by isometries. A recent result of Grabowski, Máthe and Pikhurko states that if additionally A and B are assumed to have the same Lebesgue measure, then A and B can be equidecomposed using Lebesgue measurable pieces.

These new results rely on progress made in two major research projects in descriptive set theory, ergodic theory, and related fields. The first project is to understand the "complexity" of actions of countable groups: deep results motivated by classification programs in these fields are vital ingredients these proofs. The second project is to understand the "definable combinatorics" of definable graphs. At their heart, geometrical paradoxes are matching problems and recently discovered techniques for finding definable matching problems have played a key role in these proofs. We'll sketch some of these mathematical developments and how they led to a completely constructive solution to Tarski's circle squaring problem.

Acknowledgment. This is joint work with Spencer Unger.

► THEODORE A. SLAMAN, *Recursion theory and diophantine approximation*.
Department of Mathematics, The University of California, Berkeley, 719 Evans Hall #3840, Berkeley, CA 94720-3840, USA.
E-mail: slaman@berkeley.edu.

We will discuss similarities between questions raised and crossovers between methods applied within Recursion Theory and Diophantine Approximation. We will give special attention to randomness/normality and Kolmogorov complexity/irrationality exponents.

Abstracts of invited plenary lectures

► JC BEALL, *Logic from a subclassical point of view*.
Philosophy, University of Connecticut, 334 Manchester Hall, Storrs, CT 06269-1054, USA.
E-mail: jc.beall@uconn.edu.
URL Address: entailments.net.

This talk gives a big-picture sketch of logical consequence from a particular subclassical point of view. Logic, in this talk, is taken to be so-called first-degree entailment (a natural subclassical logic discussed at length in the 60s and 70s). The role of logic, in this talk, is taken to be the universal closure relation on all of our true (hence, nonempty) theories. An obvious challenge to such a view is the apparent ubiquity of classically closed true theories (e.g., in maths and many other areas)—that is, true theories closed under classical logic. How can we have so many classically closed true theories if logic is so weak? After responding to the challenge I apply the overall picture to the ongoing Tarski-inspired issue of whether there is a 'universal truth predicate' (a predicate true of all the truths). To this issue I propose a marriage: truth is universal but it remains inexpressible in many of our true theories. The aim of this talk is to fill out some of these ideas in a simple and clear fashion.

► ARTEM CHERNIKOV, *Local distality and distal expansions of stable theories*.
Department of Mathematics, University of California Los Angeles, Los Angeles, CA 90095-1555, USA.
E-mail: chernikov@math.ucla.edu.
URL Address: http://www.math.ucla.edu/∼chernikov/.

I will give an overview of some recent interactions between Shelah's classification in model theory and extremal combinatorics for restricted families of graphs. The class of distal theories captures those NIP theories which have no nontrivial stable parts (examples are given by the o-minimal theories, p-adic fields and the field of transseries). While distality of a theory is not preserved under reducts, my recent work with Starchenko and others demonstrates that graphs definable in a reduct of a distal theory satisfy many strong combinatorial properties previously known in the special case of semialgebraic graphs (e.g., the strong Erdos–Hajnal property and strong Szemerédi-Trotter type bounds). Motivated by this phenomenon, we develop a notion of local distality and discuss the question of which stable theories admit distal expansions.

[1] A. Chernikov, D. Galvin, and S. Starchenko, *Cutting lemma and Zarankiewicz's problem in distal structures*, preprint, 2016, arXiv:1612.00908.

[2] A. Chernikov and P. Simon, *Externally definable sets and dependent pairs II*. ***Transactions of the American Mathematical Society***, vol. 367 (2015), no. 7, pp. 5217–5235.

[3] A. Chernikov and S. Starchenko, *Regularity lemma for distal structures*, preprint, 2015, arXiv:1507.01482.

[4] P. Simon, *Distal and non-distal NIP theories*. ***Annals of Pure and Applied Logic***, vol. 164 (2013), no. 3, pp. 294–318.

► BRADD HART, *In defence of ultraproducts*.
Mathematics and Statistics, McMaster University, Hamilton ON, Canada.
E-mail: hartb@mcmaster.ca.

The ultraproduct has been an important construction in model theory since the work of Łoś in the 1950's. Makkai explained the role of the ultraproduct in the reconstruction of a theory from its category of models in the 1980's via ultracategories. Nevertheless the role of the ultraproduct lessened in the eyes of some model theorists and was seen as a surrogate for the compactness theorem. With the advent of continuous model theory, the ultraproduct can be seen in a new light and its role in applied model theory is essential. I will give some examples involving the ultraproduct from abstract model theory and applications in model theoretic functional analysis.

► JULIA KNIGHT, KAREN LANGE, AND REED SOLOMON, *Roots of polynomials in fields of generalized power series*.
Mathematics Department, University of Notre Dame, 255 Hurley Hall, Notre Dame, IN 46556, USA.
E-mail: knight.1@nd.edu.
E-mail: klange2@wellesley.edu.
E-mail: solomon@math.uconn.edu.

https://doi.org/10.1017/bsl.2018.60 Published online by Cambridge University Press

We consider roots of polynomials in fields of generalized power series. Newton [4] and Puiseux [5], [6] showed that if K is an algebraically closed field of characteristic 0, then the field $K\{\{t\}\}$ of *Puiseux series* over K is algebraically closed. Maclane [3] generalized the Newton–Puiseux Theorem, showing that if K is an algebraically closed field of characteristic 0 and G is a divisible ordered Abelian group, then the *Hahn field* $K((G))$ is algebraically closed. Our goal is to measure the recursion-theoretic complexity of the root-taking process in these fields. Puiseux series have length at most ω, and we have a uniform effective procedure for computing a root of a nonconstant polynomial, given K and the coefficients. Hahn series may be longer, and the complexity of the root goes up with the length. We use results from [1], bounding the lengths of roots of a polynomial over $K((G))$ in terms of the lengths of the coefficients.

[1] J. F. Knight and K. Lange, *Lengths of developments in $K((G))$*, pre-print.

[2] ———, *Truncation-closed subfields of a Hahn field*, pre-print.

[3] S. MacLane, *The universality of formal power series fields*. ***Bulletin of the American Mathematical Society***, vol. 45 (1939), pp. 888–890.

[4] I. Newton, *Letter to oldenburg dated 1676 Oct 24*, ***The Correspondence of Isaac Newton II***, Cambridge University Press, 1960, pp. 126–127.

[5] V. A. Puiseux, *Recherches sur les fonctions algébriques*. ***Journal de Mathématiques Pures et Appliquées***, vol. 15 (1850), pp. 365–480.

[6] ———, *Nouvelles recherches sur les fonctions algébriques*. ***Journal de Mathématiques Pures et Appliquées***, vol. 16 (1851), pp. 228–240.

► JOEL NAGLOO, *Model theory and classical differential equations*.
Department of Mathematics and Computer Science, CUNY Bronx Community College, Bronx, NY 10453, USA.
E-mail: joel.nagloo@bcc.cuny.edu.

The study of differential fields using model theory has a long and rich history. The theory of differentially closed fields of characteristic 0, DCF_0, has been studied intensively and has played an important role in the development of geometric stability theory. Furthermore, many new results in number theory and differential Galois theory have been obtained using the model theoretic approach to differential algebra.

Nevertheless, only very recently has the techniques from geometric stability been used to study well-known differential equations. In this talk we highlight some of the contributions of model theory to the classification and study of the classical equations such as the Painlevé and Schwarzian equations. We also highlight the main challenges and open problems.

► DIMA SINAPOVA, *Stronger tree properties and the SCH*.
Department of Mathematics, Statistics, and Computer Science, University of Illinois at Chicago, 322 Science and Engineering Offices (M/C 249), 851 S. Morgan Street, Chicago, IL 60607-7045, USA.
E-mail: dimasinapova@yahoo.com.

Stronger tree properties capture the combinatorial essence of large cardinals. More precisely, for an inaccessible cardinal κ, κ has the strong, resp. super, tree property if and only if κ is strongly compact, resp. supercompact. An old project in set theory is to get the tree property at every regular cardinal greater than ω_1. Even more ambitiously, can we get stronger tree properties at all regular cardinals above ω_1? A positive answer would require many violations the singular cardinal hypothesis (SCH). This leads to the question whether the strong tree property implies SCH above. A positive answer would be an analogue of Solovay's theorem that SCH holds above a strongly compact cardinal.

We will show that consistently we can have the super tree property (ITP) at some λ together with failure of SCH above λ, for a non limit singular cardinal. The case of a limit singular cardinal is still open. We will also show that there is a model where ITP holds at the double successor of a singular and there are club many non internally unbounded models. This is another result in the direction of showing that ITP does not imply SCH above. Finally, we will discuss the situation for smaller cardinals like $\aleph_{\omega+2}$.

https://doi.org/10.1017/bsl.2018.60 Published online by Cambridge University Press

Acknowledgment. This is joint work with Sherwood Hachtman, University of Illinois at Chicago.

▶ SLAWOMIR SOLECKI, *Fraïssé limits and compact spaces*.
Department of Mathematics, 310 Malott Hall, Cornell University, Ithaca, NY 14853, USA.
E-mail: ss3777@cornell.edu.

Fraïssé theory is a method in classical Model Theory of producing canonical limits of certain families of finite structures. It turns out that this method can be dualized, with the dualization producing projective Fraíssé limits, and applied to the study of compact metric spaces. I will describe recent results, due to several people, on connections between projective Fraïssé limits and the structure of some canonical compact spaces and their homeomorphism groups (the pseudoarc, the Menger curve, the Lelek fan, simplexes with the goal of developing a projective Fraïssé homology theory).

▶ ANDREAS WEIERMANN, *Generalized Goodstein principles and notation systems for finite numbers*.
Department of Mathematics, Ghent University, Krijgslaan 281, S25, 9000 Ghent, Belgium.
E-mail: Andreas.Weiermann@UGent.be.

The Goodstein principle [1] is (besides some recent principles suggested by Harvey M. Friedman) arguably the most elementary statement about the natural numbers which is true but not provable from the Peano axioms [1, 3]. We are going to consider strong extensions of Goodstein's principle [2] which are defined relative to the Ackermann function and the Schwichtenberg Wainer hierarchy of fast growing functions [4]. These principles lead to canonical independence results for ATR_0 and $(\Pi^1_1 - \mathrm{CA})^-_0$.

We single out canonical properties of the underlying systems of notations for natural numbers and relate them to properties of ordinal notations for the proof-theoretic ordinals in question.

Acknowledgment. This is in part joint work with T. Arai and S. Wainer.

[1] R. L. Goodstein, *On the restricted ordinal theorem*. ***The Journal of Symbolic Logic***, vol. 9 (1944), pp. 33–41.

[2] ———, *Transfinite ordinals in recursive number theory*. ***The Journal of Symbolic Logic***, vol. 12 (1947), pp. 123–129.

[3] L. Kirby and J. Paris, *Accessible independence results for Peano arithmetic*. ***Bulletin of the London Mathematical Society***, vol. 14 (1982), no. 4, pp. 285–293.

[4] A. Weiermann, *Ackermannian Goodstein principles for first order Peano arithmetic*, ***Sets and Computations, vol. 33*** (S. D. Friedman, D. Raghavan, and Y. Yang, editors), World Scientific, Singapore, 2017, pp. 157–181.

Abstracts of invited talks in the Special Session on Computability

▶ ERIC ASTOR, DAMIR DZHAFAROV, ANTONIO MONTALBÁN, REED SOLOMON, AND LINDA BROWN WESTRICK, *Determined Borel codes in reverse math*.
University of Connecticut, 341 Mansfield Rd U-1009, Storrs, CT, 06269.
E-mail: westrick@uconn.edu.

The standard definition of a Borel code in reverse math doesn't require the model to believe that each real is either in the coded set or in its complement. In fact, the statement "for every Borel coded set, either it or its complement is nonempty" already implies ATR_0. We define a *determined Borel code* to be a Borel code with the property that every real is contained either in the coded set or in its complement. While the statement "every Borel set has the property of Baire" is equivalent to ATR_0 due to the above-mentioned technicality, the statement "every determined Borel set has the property of Baire" is not. We discuss the location of this statement relative to ATR_0, theories of hyperarithmetic analysis, and the existence of hyperarithmetic generics.

▸ BARBARA F. CSIMA, DAMIR D. DZHAFAROV, DENIS R. HIRSCHFELDT, CARL G. JOCKUSCH, JR., REED SOLOMON, AND LINDA BROWN WESTRICK, *The reverse mathematics of Hindman's Theorem for sums of exactly two elements*.
Department of Pure Mathematics, University of Waterloo, Waterloo, ON N2L 3G1, Canada.
E-mail: csima@uwaterloo.ca.
Department of Mathematics, University of Connecticut, 196 Auditorium Road, Storrs, CT 06269, USA.
E-mail: damir@math.uconn.edu.
Department of Mathematics, University of Chicago, 5734 S. University Avenue, Chicago, IL 60637, USA.
E-mail: drh@math.uchicago.edu.
Department of Mathematics, University of Illinois at Urbana–Champaign, 1409 W. Green Street, Urbana, IL 61801, USA.
E-mail: jockusch@math.uiuc.edu.
Department of Mathematics, University of Connecticut, 196 Auditorium Road, Storrs, CT 06269, USA.
E-mail: david.solomon@uconn.edu.
E-mail: linda.westrick@uconn.edu.

Hindman's Theorem (HT) states that for every coloring of $\mathbb{N}$ with finitely many colors, there is an infinite set H such that every sum of distinct elements of H has the same color. The investigation of restricted versions of HT from the computability-theoretic and reverse-mathematical perspectives has been a productive line of research recently. In particular, $\mathsf{HT}^{\leqslant n}$ is the restriction of HT to sums of at most n many elements, and $\mathsf{HT}^{=n}$ is the restriction of HT to sums of *exactly* n many elements. Even $\mathsf{HT}^{\leqslant 2}$ is a strong principle, and may in fact be as strong as HT itself (which is known to imply ACA_0 but not known to be provable in ACA_0). By contrast, $\mathsf{HT}^{=2}$ is provable from Ramsey's Theorem for pairs, and it was not known even whether it is provable in RCA_0.

We show that $\mathsf{HT}^{=2}$, and a related version of the Ramseyan Factorization Theorem, are not provable in RCA_0, or even WKL_0, and in fact imply the existence of a function that is diagonally noncomputable relative to $\emptyset'$. The most interesting aspect of our argument is the use of an effective version of the Lovász Local Lemma due to Rumyantsev and Shen.

▸ FRANÇOIS G. DORAIS, *Reverse mathematics of countable second-countable spaces*.
Department of Mathematics and Statistics, University of Vermont, 16 Colchester Avenue, Burlington, VT 05405, USA.
E-mail: francois.dorais@uvm.edu.

We study the reverse mathematics of the theory of countable second-countable topological spaces (CSC spaces). The main advantage of CSC spaces for reverse mathematics is that they can be handled second order arithmetic without any coding. The general theory of CSC spaces works mostly as expected in the subsystem ACA_0 of second-order arithmetic, but many interesting pathologies occur in weaker subsystems. In particular, RCA_0 does not prove that every compact discrete CSC space is finite, nor does it prove that the product of two compact CSC spaces is compact. We will discuss these and other aspects of CSC spaces as well as some open questions.

▸ DAMIR D. DZHAFAROV, *Joins in the strong Weihrauch degrees*.
Department of Mathematics, University of Connecticut, 341 Mansfield Road, Storrs, Connecticut 06269-1009, USA.
E-mail: damir@math.uconn.edu.

Weihrauch reducibility is a tool for comparing the difficulty of various mathematical problems that has been widely applied in computable analysis, and more recently, also in computable combinatorics. In many ways, it is a refinement both of effective mathematics and reverse mathematics, and over the past few years it has seen a surge of interest. Many open problems remain about the basic algebraic structure of the Weihrauch degrees. We

https://doi.org/10.1017/bsl.2018.60 Published online by Cambridge University Press

answer a question of Brattka and Pauly by showing that the so-called strong Weihrauch degrees, which are a natural and well-studied subclass of the Weihrauch degrees, form a lattice. Previously, these were known only to form a lower semilattice. I will present a general introduction to this problem and give a sketch of the proof.

► RACHEL EPSTEIN, *Computable reducibility and agreement on a set A*.
Department of Mathematics, Georgia College, Milledgeville, GA, USA.
E-mail: rachel.epstein@gcsu.edu.

An equivalence relation E_1 on the set of all computably enumerable (c.e.) sets is *computably reducible* to an equivalence relation E_2 on the c.e. sets, written $E_1 \leq E_2$, if there is a computable function f such that $W_n\ E_1\ W_m$ if and only if $W_{f(n)}\ E_2\ W_{f(m)}$. Coskey, Hamkins, and R. Miller have explored the hierarchy of equivalence relations on the c.e. sets. Here we look at a natural class of equivalence relations and fit them into the hierarchy. The equivalence relation E_A on the c.e. sets is the equivalence relation of agreement on the set A, given by $W_n\ E_A\ W_m$ if and only if $W_n \cap A = W_m \cap A$. The set A is Σ^0_2 if and only if E_A is computably reducible to the equality equivalence relation on the class of c.e. sets, which we call $=^{ce}$. If $E_A \leq E_B$, then A is Σ^B_2. However, we see that the converse is not true. We also construct a set A such that E_A is incomparable with $=^{ce}$.

Acknowledgment. This is joint work with Karen Lange.

► DAVID FERNÁNDEZ-DUQUE, PAUL SHAFER, HENRY TOWSNER, AND KEITA YOKOYAMA, *Reverse mathematics, Ekeland's variational principle, and Caristi's fixed point theorem*.
Department of Mathematics, Ghent University, Krijgslaan 281 S23, B-9000 Ghent, Belgium.
E-mail: David.FernandezDuque@UGent.be.
School of Mathematics, University of Leeds, Leeds LS2 9JT, UK.
E-mail: p.e.shafer@leeds.ac.uk.
Department of Mathematics, University of Pennsylvania, 209 South 33rd Street, Philadelphia, PA 19104-6395, USA.
E-mail: htowsner@math.upenn.edu.
School of Information Science, Japan Advanced Institute of Science and Technology, 1-1 Asahidai, Nomi, Ishikawa 923-1292, Japan.
E-mail: y-keita@jaist.ac.jp.

We investigate the reverse mathematical strengths of two related theorems from analysis: *Ekeland's variational principle* [2] and *Caristi's fixed point theorem* [1].

Let $\mathcal{X}$ be a complete separable metric space, and let $V\colon \mathcal{X} \to \mathbb{R}^{\geq 0}$ be a lower semicontinuous function. Ekeland's variational principle states that V has a *critical point*, which is a point x_* such that $d(x_*, y) > V(x_*) - V(y)$ whenever $y \neq x_*$. This theorem has a variety of applications in analysis. For example, it implies that certain optimization problems have approximate solutions, and it implies a number of interesting fixed point theorems, including Caristi's fixed point theorem.

Let a *Caristi system* be a triple $(\mathcal{X}, V, f)$, where $\mathcal{X}$ is a complete separable metric space, $V\colon \mathcal{X} \to \mathbb{R}^{\geq 0}$ is a lower semicontinuous function, and $f\colon \mathcal{X} \to \mathcal{X}$ is an arbitrary function satisfying $(\forall x \in \mathcal{X})[d(x, f(x)) \leq V(x) - V(f(x))]$. Caristi's fixed point theorem states that if $(\mathcal{X}, V, f)$ is a Caristi system, then f has a fixed point.

We show that the strengths of these theorems vary from WKL_0 and ACA_0 in certain special cases, to strictly between ATR_0 and Π^1_1-CA_0 in certain fairly general cases of Caristi's fixed point theorem, to Π^1_1-CA_0 in the general case of Ekeland's variational principle, to beyond Π^1_1-CA_0 in certain extensions of Caristi's fixed point theorem.

[1] J. Caristi, *Fixed point theorems for mappings satisfying inwardness conditions*. ***Transactions of the American Mathematical Society***, vol. 215 (1976), pp. 241–251.

[2] I. Ekeland, *On the variational principle*. ***Journal of Mathematical Analysis and Applications***, vol. 47 (1974), no. 2, pp. 324–353.

https://doi.org/10.1017/bsl.2018.60 Published online by Cambridge University Press

▶ MATHIEU HOYRUP, *Extending computable real functions*.
Université de Lorraine, CNRS, Inria, LORIA, F 54000 Nancy, France.
Inria, 615 rue du jardin botanique, 54600 Villers-lès-Nancy, France.
E-mail: mathieu.hoyrup@inria.fr.
URL Address: https://members.loria.fr/MHoyrup/.

We study the computable aspects of an elementary problem from real analysis: extending a continuous function from a given interval to a larger domain. More precisely, if f is defined and computable on some interval $[0, a)$, with $0 < a < 1$, when can it be extended to a computable function over $[0, 1]$? Although this question has a very simple formulation, it does not have a simple answer. We present several results showing how the answer depends on a and on the way f converges at a.

We show how the properties of f on $[0, a)$ must propagate to all their computable extensions on $[0, 1]$, when a is not computable.

We give a complete answer for a restricted class of functions, called the *sawtooth* functions, defined in terms of computable enumerations of c.e. sets.

We identify sufficient and necessary conditions on f, and characterize the real numbers a for which the sufficient condition is necessary and the necessary condition is sufficient, and separate these two classes. It happens that they coincide with previously defined classes of real numbers from computability theory.

Along the way, we demonstrate the convenience of recently defined notions of genericity for various classes of enumerable objects (c.e. sets, left-c.e. reals, Π^0_1 classes, etc.) [1].

Acknowledgment. This work is joint with Walid Gomaa [2].

[1] M. Hoyrup, *Genericity of weakly computable objects*. ***Theory of Computing Systems***, vol. 60 (2017), no. 3, pp. 396–420.

[2] M. Hoyrup and W. Gomaa, *On the extension of computable real functions*, ***32nd Annual ACM/IEEE Symposium on Logic in Computer Science (LICS)***, IEEE Computer Society, Reykjavik, Iceland, 2017, pp. 1–12.

▶ JULIA KNIGHT, DAN TURETSKY, AND ROSE WEISSHAAR, *Which Harrison-type orderings look most like ordinals*?
University of Notre Dame, Department of Mathematics, 255 Hurley, Notre Dame, IN 46556, USA.
E-mail: dturetsk@nd.edu.

By a Harrision-type ordering, we mean a computable presentation of an ill-founded linear order with no descending Δ^1_1 sequences. The order-types of such orderings are well understood, but different presentations can have different levels of 'niceness'—that is, ways in which they resemble computable presentations of ordinals. Computable presentations of ordinals admit jump structures, and there are models of KP or even ZFC in which they are isomorphic to an ordinal. Harrison-type orderings can have some, all or none of these properties.

▶ GRÉGORY LAFITTE, *Higher order computability musings and absoluteness*.
LIRMM, CNRS, Université de Montpellier, 161 rue Ada, 34090 Montpellier, France.
E-mail: lafitte@lirmm.fr.

The primitive recursive functions were generalized to sets by Ronald Jensen and Carol Karp in their 1971 article. Other higher order computability settings and models, in particular E-recursion and infinite time computation models, were later introduced by Dag Normann, Gerald Sacks, *et alii*. Various closure and reachability properties emerge from these settings and models, giving rise to the identification and study of specific ordinals linked to computability results and variants of the Lévy-Shoenfield absoluteness theorem.

▶ STEFFEN LEMPP, *The complexity of countable models of strongly minimal theories*.
Department of Mathematics, University of Wisconsin-Madison, 480 Lincoln Drive, Madison WI 53706-1325, USA.
E-mail: lempp@math.wisc.edu.
URL Address: http://www.math.wisc.edu/~lempp/.

In joint work with Uri Andrews and Noah Schweber, we present an (almost complete) answer to an old question of mine from the mid-1990's: How complicated can all the countable models of an $\aleph_0$-categorical theory be if one of them is known to be computable (i.e., to have a copy with universe ω such that all functions, relations and constants are uniformly computable): For the case of strongly minimal theories (over a computable language), the answer is that a degree **d** can compute all countable models of a strongly minimal theory with a computable model iff **d** is "high over $\mathbf{0}''$", i.e., $\mathbf{d} \geq \mathbf{0}''$ and $\mathbf{d}' = \mathbf{0}^{(4)}$.

▸ CHRISTOPHER P. PORTER, *Aspects of Bernoulli randomness*.
Department of Mathematics and Computer Science, Drake University, Des Moines, IA 50311, USA.
E-mail: `cp@cpporter.com`.

This project brings together two strands of research in algorithmic randomness: (1) randomness with respect to computable measures and (2) randomness with respect to random measures. In particular, we examine which sequences are random with respect to a Bernoulli measure with a parameter p that is itself random with respect to some computable measure.

As anticipated by work of Vovk and V'yugin [3], Freer and Roy [1], and Hoyrup [2], all such sequences are, in fact, random with respect to a computable measure; such measures are obtained by taking a mixture of a collection of random measures. We further investigate the extent to which randomness with respect to a noncomputable Bernoulli measure is compatible with randomness with respect to some computable measure, as well as which random Bernoulli measures can be mixed to obtain a computable measure.

Acknowledgment. This is joint work with Quinn Culver.

[1] C. E. Freer and D. M. Roy, *Computable de finetti measures*. ***Annals of Pure and Applied Logic***, vol. 163 (2012), no. 5, pp. 530–546.

[2] M. Hoyrup, *Computability of the ergodic decomposition*. ***Annals of Pure and Applied Logic***, vol. 164 (2013), no. 5, pp. 542–549.

[3] V. G. Vovk and V. V. V'yugin, *On the empirical validity of the bayesian method*. ***Journal of the Royal Statistical Society. Series B (Methodological)***, (1993), pp. 253–266.

▸ HENRY TOWSNER, *Disentangling the complexity of Ramsey's Theorem, the first-order part*.
Department of Mathematics, University of Pennsylvania, 209 South 33rd Street, Philadelphia, PA 19104-6395, USA.
E-mail: `htowsner@math.upenn.edu`.
URL Address: `http://www.math.upenn.edu/~htowsner`.

Informally speaking, there are two distinct features of Ramsey's Theorem for Pairs which make it difficult to construct a solution:

- the need to "play the colors against one another": to use the difficulty of finding a solution (or making progress towards finding a solution) in one color to make progress in a different color, and
- each point introduces its own partition of the other points, with each of these partitions completely distinct.

These features correspond to the division of Ramsey's Theorem for Pairs into two weaker properties—the Ascending/Descending Sequence (ADS) principle (which has the first difficulty but not the second) and the Erdős–Moser (EM) principle (which has the second difficulty but not the first).

We show that the former property is entirely responsible for the first-order strength of Ramsey's Theorem for Pairs with an arbitrary number of colors: that the analog of ADS for arbitrarily many colors has the same first-order strength as full Ramsey's Theorem for Pairs with arbitrarily many colors (namely, $B\Sigma^0_3$, as shown by Slaman and Yokoyama) while the first-order strength of the analog of EM for arbitrarily many colors is at most $I\Sigma^0_2$.

Acknowledgment. This is joint work with Keita Yokoyama.

Abstracts of invited talks in the Special Session on Logic & Philosophy

► WALTER DEAN, *Undecidability and intensionality via arithmetized completeness*.
Department of Philosophy, University of Warwick, Social Sciences Building, Coventry CV4 7AL, UK.
E-mail: W.H.Dean@warwick.ac.uk.

This talk will refine and generalize a method for obtaining formally undecidable arithmetical statements via the formalization of familiar paradoxes originally developed by Georg Kreisel and Hao Wang. A central tool will be the arithmetized completeness theorem which will be employed to obtain first-order interpretations of second-order theories in which various "paradoxical notions" may be formalized. Connections to the treatment of the paradoxes by Hilbert and Bernays (1939) and to the intensionality of arithmetization in the sense of Feferman (1960) will also be explored.

► FARZANEH DERAKHSHAN, *Computational interpretation of substructural proofs*.
Philosophy Department, Carnegie Mellon University, Pittsburgh, PA 15213, USA.
E-mail: fderakhs@andrew.cmu.edu.

Substructural Logics are mostly about governing how a collection of information given as assumptions can be used. Linear logic, for example, is a substructural logic that requires each assumption to be used exactly once, but there is no restriction on the order in which they are used. In the sequent $A_1, \ldots, A_n \vdash C$, for example, each A_i is a linear assumptions that must be used exactly once in an arbitrary order [1]. Session types is a type system that corresponds to the sequent calculus of the intuitionistic linear logic [2]. In this system, proofs are interpreted as concurrent processes that are communicating through the so-called typed session channels. The proof of the above sequent, for example, can be interpreted as a process labeled P that is using services of types A_i provided by the channels x_i, and offers its own service of type C along the channel z. This is written as $x_1 : A_1, \ldots, x_n : A_n \vdash P :: z : C$ [1]. Computation in this system corresponds to cut reduction. We can extend this correspondence by adding the least and greatest fixed point propositions to the language, so that μ- and ν-types corresponds, respectively, to the types of inductively and co-inductively defined stream of messages, and circular proofs corresponds to the recursive and corecursive calls. Fortier et al. described a cut elimination procedure for the so called valid proofs in a fragment of linear logic called subsingleton logic augmented with fixed points [3]. In this setting, we can interpret valid proofs described by Fortier et al. as corresponding to the terminating and productive processes.

[1] S. Balzer and F. Pfenning, *Manifest sharing with session types*. ***Proceedings of the ACM on Programming Languages, Association for Computing Machinery***, vol. 1, (2017), no. ICFP, p. 37.

[2] L. Caires and F. Pfenning, *Session types as intuitionistic linear propositions*, ***International Conference on Concurrency Theory***, Springer, Berlin, Heidelberg, 2010, pp. 222–236.

[3] J. Fortier and L. Santocanale, *Cuts for circular proofs: Semantics and cut-elimination*, ***LIPIcs-Leibniz International Proceedings in Informatics, vol. 23***, Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik, 2013.

► ROSALIE IEMHOFF, *The elimination of strong quantifiers in nonclassical logics*.
Department of Philosophy, Utrecht University, Janskerkhof 13, 3512 BL Utrecht, The Netherlands.
E-mail: r.iemhoff@uu.nl.

The Skolemization method is a well-known translation on formulas used in mathematics and computer science. It is a computable and uniform method, that, when applied to a formula, produces a formula without strong quantifiers that is derivable exactly when the original formula is. In combination with Herbrand's Theorem this method provides a striking connection between predicate and propositional logic, at least in the case of classical logic. In many other logics, the Skolemization method behaves quite differently. In intuitionistic logic,

https://doi.org/10.1017/bsl.2018.60 Published online by Cambridge University Press

for example, the relation, in terms of derivability, between a formula and its Skolemization is still not completely understood. In recent years there has appeared quite some work on the extent to which Skolem's method can be transferred to nonclassical logics, and several alternative approaches have been developed. This talk is a survey of these results.

► GRAHAM LEACH-KROUSE, *Burali–Forti as a purely logical paradox*.
Philosophy, Kansas State University, 116 Mid Campus Dr. North, Dickens Hall, Manhattan, Kansas 66506, USA.
E-mail: gleachkr@ksu.edu.

Russell's paradox is *purely logical* in the following sense: a contradiction can be formally deduced from the proposition that there is a set of all non-self-membered sets, in pure first-order logic—the first-order logical form of this proposition is inconsistent. This explains why Russell's paradox is *portable*—why versions of the paradox arise in contexts unrelated to set theory, from propositions with the same logical form as the claim that there is a set of all non-self-membered sets.

Burali–Forti's paradox, like Russell's paradox, is portable. I offer the following explanation for this fact: Burali–Forti's paradox, like Russell's, is purely logical. Concretely, I show that if we enrich the language $\mathcal{L}$ of first-order logic with a well-foundedness quantifier $\mathcal{W}$ and adopt certain minimal inference rules for this quantifier, then a contradiction can be formally deduced from the proposition that there is a greatest ordinal.

Moreover, a proposition with the same logical form as the claim that there is a greatest ordinal can be found at the heart of several other paradoxes that resemble Burali–Forti's. The reductio of Burali–Forti can be repeated verbatim to establish the inconsistency of these other propositions. Hence, the portability of the Burali–Forti's paradox is explained in the same way as the portability of Russell's: both paradoxes involve an inconsistent logical form—Russell's involves an inconsistent form expressible in $\mathcal{L}$ and Burali–Forti's involves an inconsistent form expressible in $\mathcal{L} + \mathcal{W}$.

► SEBASTIAAN A. TERWIJN, *Constructive logic and sets of reals*.
Radboud University Nijmegen, PO Box 9010, 6500 GL Nijmegen, The Netherlands.
E-mail: terwijn@math.ru.nl.

In this talk we survey results connecting the Medvedev lattice $\mathfrak{M}$ and the Muchnik lattice $\mathfrak{M}_w$ from Computability Theory to constructive logic. In recent times there has been a renewed interest in these lattices and the notions of reducibility that define them. These structures are generalizations of the Turing degrees, and they are interesting for purely computability-theoretic reasons, but they also provide an algebraic semantics for various propositional logics. In particular, we discuss the logics corresponding to factors of these lattices. It has been long known, by results of Medvedev and Sorbi, that the logic of $\mathfrak{M}$ and $\mathfrak{M}_w$ themselves is the logic of the weak law of the excluded middle $\neg p \vee \neg\neg p$. A classic result of Skvortsova showed that there is a factor of $\mathfrak{M}$ corresponding to intuitionistic propositional logic IPC. Sorbi and Terwijn showed that the same result holds for $\mathfrak{M}_w$. We discuss a family of formulas generalizing the weak law of the excluded middle, and discuss the relation to previous results by Gabbay and Smorynski. We prove a completeness theorem, and connect this with factors of $\mathfrak{M}$. By considering the ideal generated by a set of reals in $\mathfrak{M}$ or $\mathfrak{M}_w$, every set of reals has a corresponding propositional logic. This prompts the question what this logic is for various given sets of reals. In this vein, we discuss some recent results of Kuyper about natural factors of $\mathfrak{M}$ and $\mathfrak{M}_w$.

► FAN YANG, *Analyzing Arrow's Theorem through dependence and independence logic*.
PL 68 (Gustaf Hällströmin katu 2b), 00014 University of Helsinki, Finland.
E-mail: fan.yang.c@gmail.com.

The groundbreaking theorem in social choice theory, *Arrow's Impossibility Theorem* [1], states that a few natural requirements of aggregation functions are inconsistent with the fundamental condition that aggregation functions should exclude dictatorship. In this talk, we provide a new perspective on this theorem through the lens of *dependence and independence logic* (DIL).

Introduced by Väänänen [6] and Grädel and Väänänen [2], DIL is a logical formalism that characterizes the notions of "dependence" and "independence" in sciences. DIL adopts the so-called *team semantics*, which was introduced by Hodges [3, 4]. The basic idea of this semantics is that properties of dependence and independence cannot be manifested in single assignments, therefore unlike the usual semantics, formulas are evaluated on sets of assignments (called *teams*). In the case of Arrow's Theorem, a set of profiles or ballots together with the corresponding aggregated decisions forms a team. On the basis of this observation, we will formalize Arrow's Theorem in DIL as a theorem highlighting an interesting conflict between certain dependence and independence conditions, and we also sketch a formal proof of the theorem in the system of DIL. Finally, we will also argue that Arrow's Theorem can be understood as a type of dependency strengthening theorem.

Acknowledgment. This talk is partly based on a joint work with Eric Pacuit [5].

[1] K. J. Arrow, ***Social Choice and Individual Values***, Yale University Press, 1951.

[2] E. Grädel and J. Väänänen, *Dependence and independence*. ***Studia Logica***, vol. 101 (2013), no. 2, pp. 399–410.

[3] W. Hodges, *Compositional semantics for a language of imperfect information*. ***Logic Journal of the IG Petrochemicals Limited***, vol. 5 (1997), pp. 539–563.

[4] ———, *Some strange quantifiers*, ***Structures in Logic and Computer Science: A selection of Essays in Honor of A. Ehrenfeucht***, Lecture Notes in Computer Science, vol. 1261, Springer, 1997, pp. 51–65.

[5] E. Pacuit and F. Yang, *Dependence and independence in social choice: Arrow's theorem*, ***Dependence Logic: Theory and Application***, Birkhauser, 2016, pp. 235–260.

[6] J. Väänänen, ***Dependence Logic: A New Approach to Independence Friendly Logic***, Cambridge University Press, 2007.

Abstracts of invited talks in the Special Session on Model Theory

► VAHAGN ASLANYAN, *Ax-Schanuel and strongly minimal sets in reducts of differentially closed fields*.
Department of Mathematical Sciences, Carnegie Mellon University, 5000 Forbes Ave, Pittsburgh, PA 15213, USA.
E-mail: `vaslanya@andrew.cmu.edu`.
URL Address: `http://www.math.cmu.edu/~vahagn/`.

We will discuss Ax-Schanuel type inequalities and their importance for understanding the model theory of abstract differential equations. Our main examples are Ax's original theorem on the exponential differential equation which is an analogue of Schanuel's conjecture, and a similar theorem for the differential equation of the modular j-function established recently by Pila and Tsimerman. We will show how the above inequalities can be used to study strongly minimal sets related to the differential equation under consideration. In particular, we will see that we can get a Zilber style classification of certain strongly minimal sets.

► SAUGATA BASU, *O-minimal versions of Szemeredi–Trotter and Elekes–Ronyai theorems*.
Department of Mathematics, Purdue University, Rm. 742, 150 N. University Street, West Lafayette, IN 47907, USA.
E-mail: `sbasu@math.purdue.edu`.

I will describe some recent results giving o-minimal versions of certain theorems in incidence combinatorics that were already known in the algebraic/semialgebraic setting. I will also pose some open questions.

Acknowledgment. This is joint work with O. Raz.

► HUNTER CHASE AND JAMES FREITAG, *Machine learning and stability*.
Department of Mathematics, Statistics, and Computer Science, University of Illinois at Chicago, 851 S. Morgan Street, Chicago, IL 60607, USA.

E-mail: hchase2@uic.edu.
E-mail: freitagj@gmail.com.

In [2] Laskowski observed the connection between NIP theories and set systems of finite VC-dimension. Set systems of finite VC-dimension are exactly those which are PAC-learnable, a notion of machine learning carried out through random sampling [1]. The study of both NIP theories and PAC-learning have benefited from this connection.

We observe a similar connection between stable theories and online learning. Set systems of finite Littlestone dimension (also known as thicket dimension or, in model theory, Shelah 2-rank) are exactly those which are online learnable, in which samples are viewed as being chosen antagonistically rather than randomly [3]. We review known results and consider paths for further study.

[1] A. Blumer, A. Ehrenfeucht, D. Haussler, and M. K. Warmuth, *Learnability and the Vapnik-Chervonenkis dimension*. ***Journal of the Association for Computing Machinery***, vol. 36 (1989), no. 4, pp. 929–965.

[2] M. C. Laskowski, *Vapnik–Chervonenkis classes of definable sets*. ***Journal of the London Mathematical Society***, vol. s2–45 (1992), no. 2, pp. 377–384.

[3] N. Littlestone, *Learning quickly when irrelevant attributes abound: A new linear-threshold algorithm*. ***Machine Learning***, vol. 2 (1988), no. 4, pp. 285–318.

► GABRIEL CONANT, *Independence in generic incidence structures*.
University of Notre Dame, Notre Dame, IN 46556, USA.
E-mail: gconant@nd.edu.

Given integers $m, n \geq 1$, we consider $K_{m,n}$-free bipartite graphs as incidence structures in which the two parts of the partition are “points” and “lines”, and the edge relation is “incidence”. For the case, $m = n = 2$, these structures are combinatorial projective planes. In the general case, these incidence structures are relatives of combinatorial designs, in which the size of blocks is unrestricted. We construct the model completion $T_{m,n}$ of this theory, and study its behavior. When $m = 1$ or $n = 1$, the theory is $\aleph_0$-categorical and ω-stable of Morley rank $\max\{m, n\}$, but also represent canonical examples of the dimensional order property when $\max\{m, n\} \geq 3$. For $m, n \geq 2$, $T_{m,n}$ has continuum many countable models. For $T_{2,2}$, (the generic theory of projective planes), we show that the existence of a prime model is equivalent to a notoriously difficult open question from incidence geometry, posed by Erdős in 1974. Moreover, for $m, n \geq 2$, we show $T_{m,n}$ is not simple, but is NSOP_1 and exhibits a natural and especially well-behaved characterization of Kim independence.

Acknowledgment. This is joint work with A. Kruckman.

► PHILIPP HIERONYMI, *Properties of structures with o-minimal open core*.
Department of Mathematics, University of Illinois at Urbana-Champaign, 1409 W. Green Street, Urbana, IL 61801, USA.
E-mail: phierony@illinois.edu.

Let $\mathcal{R}$ be an expansion of a dense linear order $(R, <)$ without endpoints. The **open core of** $\mathcal{R}$ is the structure $(R, (U))$, where U ranges over all open sets of all arities definable in $\mathcal{R}$. Miller and Speissegger introduced this notion of an open core for expansions of $(\mathbb{R}, <)$ in [2]. In this talk I will present answers to questions by Dolich, Miller and Steinhorn [1] about rather basic properties of structures that have an o-minimal open core. Among other things I will show that there is essentially no restriction on what kind of structures can be interpreted in a structure with o-minimal open core, and I will give an example of a structure with o-minimal open core that has definable Skolem functions, but is not o-minimal itself.

Acknowledgment. This talk is based on joint work with Erin Caulfield, Travis Nell, and Erik Walsberg.

[1] A. Dolich, C. Miller, and C. Steinhorn, *Structures having o-minimal open core*. ***Transactions of the American Mathematical Society***, vol. 362 (2010), no. 3, pp. 1371–1411.

[2] C. Miller and P. Speissegger, *Expansions of the real line by open sets: O-minimality and open cores*. ***Fundamenta Mathematicae***, vol. 162 (1999), no. 3, pp. 193–208.

▸ TOBIAS KAISER AND PATRICK SPEISSEGGER, *Analytic continuation of functions definable in* $\mathbb{R}_{an,\exp}$.
Mathematics and Statistics, McMaster University, 1280 Main Street West, Hamilton ON L8S 4K1, Canada.
E-mail: speisseg@mcmaster.ca.
Fakultät für Informatik und Mathematik, Universität Passau, Innstraße 33, 94032 Passau, Germany.
E-mail: tobias.kaiser@uni-passau.de.

I will state a recent result on analytic continuation properties of unary functions definable in the o-minimal structure $\mathbb{R}_{\text{an,exp}}$ and outline a couple of applications.

▸ CHRIS MILLER AND ATHIPAT THAMRONGTHANYALAK, *Component-closed expansions of the real line* (*preliminary report*).
Department of Mathematics, The Ohio State University, 231 West 18th Avenue, Columbus, OH 43210, USA.
E-mail: miller@math.osu.edu.
Department of Mathematics and Computer Science, Faculty of Science, Chulalongkorn University, Bangkok 10330, Thailand.
E-mail: athipat.th@chula.ac.th.

We say that a structure $\mathfrak{R}$ with underlying set $\mathbb{R}$ is *component closed* if, for every E (of any arity) definable in $\mathfrak{R}$, every connected component (with respect to the usual topology) of E is definable in $\mathfrak{R}$. Easy, or previously known, facts: (a) the intersection of all component-closed expansions (in the sense of definability) of $\mathfrak{R}$ is component closed (so $\mathfrak{R}$ has a natural "component closure"); (b) if $\mathfrak{R}$ is component closed, then it defines $<$; (c) every o-minimal expansion of $(\mathbb{R}, <)$ is component closed; (d) $(\mathbb{R}, +, \cdot, \mathbb{Z})$ is component closed. The question arises: What can be said about component-closed expansions of $(\mathbb{R}, <)$ and their theories? We are particularly interested in expansions of $(\mathbb{R}, <)$ by boolean combinations of open sets. I will discuss our progress.

▸ ALF ONSHUUS, *Definably amenable groups and invariant means*.
Departamento de Matemáticas, Universidad de los Andes, Carrera 1 No 18A-10, Bogotá, Colombia.
E-mail: aonshuus@uniandes.edu.co.

Topologically amenable groups have been known to be characterized by the existence of invariant means on continuous and uniformly right continuous functions from the groups into the real numbers. In this talk I will show possible analogues of such connections for the definable amenable groups. The search for this connections will touch on sigma-continuity and continuous logic.

▸ NICHOLAS RAMSEY, *Measures in simple theories*.
Group in Logic, UC Berkeley, Berkeley, CA 94720, USA.
E-mail: nickramsey@berkeley.edu.

Keisler measures were introduced in the late 80's by Keisler but they became central objects in model theory only recently with the development of NIP theories. This led naturally to the question of whether there might be a parallel theory of measures in other tame classes, especially in the simple theories where pseudofinite counting measures supply natural and interesting examples. We will describe some first steps toward establishing such a theory, based on Keisler randomizations and the theory of independence for NSOP_1 theories in continuous logic.

Acknowledgment. This is joint work with Itaï Ben Yaacov and Artem Chernikov.

▸ CAROLINE TERRY, *Dividing lines and jumps in growth rates of hereditary properties*.
Department of Mathematics, University of Maryland, College Park, MD 20742, USA.
E-mail: cterry@umd.edu.

A hereditary graph property is a class of finite graphs closed under isomorphism and induced subgraphs. Given a hereditary graph property $\mathcal{H}$, the *speed* of $\mathcal{H}$ is the function which sends n to the number of distinct elements in $\mathcal{H}$ with underlying set $\{1, \dots, n\}$. Not just any function can occur as the speed of hereditary graph property. Specifically, there are discrete "jumps" in the possible speeds. Study of these jumps began with work of Scheinerman and Zito in the 90's, and culminated in a series of articles from the 2000's by Balogh, Bollobás, and Weinreich, in which essentially all possible speeds of a hereditary graph property were characterized. In contrast to this, many aspects of this problem in the hypergraph setting have remained unknown. In this talk we present new hypergraph analogues of many of the jumps from the graph setting, specifically those involving the polynomial, exponential, and factorial speeds. The jumps in the factorial range turned out to have surprising connections to model theory, which we also discuss.

Acknowledgment. This is joint work with Chris Laskowski.

▶ MINH CHIEU TRAN, *The groups of* $\mathbb{Z}$ *and* $\mathbb{Q}$ *with predicates for being square-free*.
Department of Mathematics, University of Illinois at Urbana-Champaign, 1409 W. Green St., Urbana, IL 61801, USA.
E-mail: `mctran2@illinois.edu`.
URL Address: `https://math.illinois.edu/directory/profile/mctran2`.

Inspired by results of Bateman, Jockusch, Woods, Kaplan, and Shelah, we study the model-theoretic properties of a number of structures concerning with the additive groups of $\mathbb{Z}$ and $\mathbb{Q}$ and suitable predicates for being square-free. We notably manage to avoid the use of Dickson's conjecture in earlier studies and in particular obtain the first unconditional natural example of a simple unstable expansion of $(\mathbb{Z}; +)$.

Acknowledgment. This is a joint work with Neer Bhardwaj.

▶ ERIK WALSBERG, *First order expansions of the additive group of reals*.
University of Illinois, Urbana–Champaign, 1409 W. Green St., Urbana, IL 61801, USA.
E-mail: `erikw@illinois.edu`.

I will discuss my work with Philipp Hieronymi on first order expansions of the ordered additive group of real numbers. Crucial to this is a classification of such structures into three types: expansions resembling o-minimal expansions, expansions which define all compact sets, and a mysterious third type closely connected to the monadic second order theory of one successor.

Abstracts of invited talks in the Special Session on Proof Theory

▶ LIRON COHEN, *Cycles for the sake of induction*.
Computer Science Department, Cornell University, Ithaca, NY 14850, USA.
E-mail: `lironcohen@cornell.edu`.

A core technique in mathematical reasoning is that of induction. Formal systems for mathematical reasoning usually capture the notion of inductive reasoning via one or more inference rules that express the general induction schemes, or principles, that hold for the elements being reasoned over. Increasingly, we are concerned with not only being able to formalize as much mathematical reasoning as possible, but also with doing so in an effective way. For this we employ *Transitive closure* logic, which is obtained by a modest addition to first-order logic that affords enormous expressive power. Most importantly, it provides a uniform way of capturing inductive principles. Thus, particular induction principles do not need to be added to, or embedded within, the logic; instead, all induction schemes are available within a single, unified language.

This expressiveness of the logic renders any finitary proof system for it incomplete for the standard semantics. Nevertheless, we develop an infinitary proof theory for transitive closure logic which is complete for the standard semantics. This system captures implicit induction, and its soundness is underpinned by the principle of *infinite descent*. While a full infinitary proof theory is clearly not effective, such a system can be obtained by restricting consideration to only regular infinite proofs (those representable by finite, possibly cyclic,

https://doi.org/10.1017/bsl.2018.60 Published online by Cambridge University Press

graphs). The uniformity of the transitive closure operator allows semantically meaningful complete restrictions to be defined using simple syntactic criteria. Consequently, the restriction to regular proofs provides the basis for more focussed proof-search strategies, further enhancing the potential for automation.

► VIJAY GANESH, *On the unreasonable effectiveness of boolean SAT solvers*.
Electrical and Computer Engineering, University of Waterloo, 200 University Avenue West, Ontario, Canada.
E-mail: vganesh@uwaterloo.ca.

Modern conflict-driven clause-learning (CDCL) Boolean SAT solvers routinely solve very large industrial SAT instances in relatively short periods of time. This phenomenon has stumped both theoreticians and practitioners since Boolean satisfiability is an NP-complete problem widely believed to be intractable. It is clear that these solvers somehow exploit the structure of real-world instances. However, to-date there have been few results that precisely characterize this structure or shed any light on why these SAT solvers are so efficient.

In this talk, I will present results that provide a deeper empirical and theoretical understanding of why CDCL SAT solvers are so efficient. Our results can be divided into two parts. First, I will talk about structural parameters that can characterize industrial instances and shed light on why they are easier to solve even though they may contain millions of variables. Second, I will talk about internals of CDCL SAT solvers, and describe why they are particularly suited to solve industrial instances.

► TAKAYUKI KIHARA, *Weihrauch counterparts of reverse mathematical principles*.
Department of Mathematical Informatics, Nagoya University, 1 Furo-cho, Chikusa-ku, Nagoya 464-0814, Japan.
E-mail: kihara@i.nagoya-u.ac.jp.
URL Address: http://www.math.mi.i.nagoya-u.ac.jp/∼kihara.

In this talk, we discuss an application of Weihrauch degrees to show separation results in constructive reverse mathematics over the intuitionistic base system EL_0. In particular, we consider several nonconstructive principles between LLPO and WKL. We also discuss candidates for Weihrauch counterparts of ATR_0.

► ULRICH KOHLENBACH, *Proof-theoretic methods in convex optimization*.
Department of Mathematics, Technische Universität Darmstadt, Schlossgartenstraße 7, D-64289 Darmstadt, Germany.
E-mail: kohlenbach@mathematik.tu-darmstadt.de.

We will report on some recent uses of proof-theoretic transformations for the extraction of explicit rates of convergence and bounds (‘proof mining’) in convex optimization: a polynomial rate of convergence in Bauschke’s solution of the zero-displacement conjecture [2], rates of convergence and metastability for general forms of the proximal point algorithm and related methods [3, 4], effective moduli of continuity for proximal maps in uniformly convex Banach spaces [1].

[1] M. Bačák and U. Kohlenbach, *On proximal mappings with young functions in uniformly convex Banach spaces*. ***Journal of Convex Analysis***, vol. 25 (2018), no. 4.

[2] U. Kohlenbach, *A polynomial rate of asymptotic regularity for compositions of projections in Hilbert space*. ***Foundations of Computational Mathematics***, to appear.

[3] U. Kohlenbach, G. López-Acedo, and A. Nicolae, *Moduli of regularity and rates of convergence for Fejér monotone sequences*, arXiv:1711.02130.

[4] L. Leuştean, A. Nicolae, and A. Sipoş, *An abstract proximal point algorithm*, ***Global Optimization***, to appear, arXiv:1711.09455.

► ANGELIKI KOUTSOUKOU-ARGYRAKI, *Proof mining mathematics, formalizing mathematics*.
Computer Laboratory, University of Cambridge, 15 JJ Thomson Avenue, Cambridge CB3 0FD, UK.
E-mail: ak2110@cam.ac.uk.

Proof mining is a research program in applied proof theory involving the extraction of quantitative, computable information from (even nonconstructive) mathematical proofs of statements of a certain logical form, via a pen-and-paper, i.e., *not* automated logical analysis. The program originated as "unwinding of proofs" in the ideas of Georg Kreisel from the fifties, and has been developed by Ulrich Kohlenbach and his collaborators during the past two decades. A great deal of applications for proofs in different research directions in Mathematics has been achieved. ALEXANDRIA is a new ERC project at the University of Cambridge under the leadership of Lawrence Paulson aiming at the creation of a proof development environment for working mathematicians through a collaboration of mathematicians and computer scientists. This will be achieved by formalizing mathematical proofs with the proof assistant *Isabelle*. The focus of the project is the management and use of large-scale mathematical knowledge, both as theorems and as algorithms. In addition to the obvious importance of proof verification for Mathematics and the usefulness of libraries of formalized proofs for (the future generations of) mathematicians, the formalization of mathematical proofs could possibly shed light on interesting proof theoretic questions. Moreover, enriching the libraries with formalized proof-mined proofs could open the way for the exciting prospect of automating proof mining itself.

▸ COLIN MCLARTY, *Class field theory in Exponential Function Arithmetic (EFA).*
Department of Philosophy, Case Western Reserve University, Clark Hall 211, 11130 Bellflower Road, Cleveland, OH 44106, USA.
E-mail: colin.mclarty@case.edu.

Class field theory was the center of algebraic number theory from Dedekind and Hilbert right up until it led to, and merged with, current cohomological methods. While it is widely considered 'abstract' in the sense of being difficult to beginners to relate to arithmetic intuition, and many of its tools look to a naive glance to be third order arithmetic, it is logically quite concrete. This talk presents work on formalizing classical results and methods of class field theory in Exponential Function Arithmetic. This is so weak it lies below the threshold of current Reverse Mathematics. So this talk contributes to Harvey Friedman's Grand Conjecture, saying all mainstream concrete mathematics lies inside EFA, and also to Angus Macintyre's saying that (full) "Peano Arithmetic is far too strong for mathematics."

▸ YURY SAVATEEV AND DANIYAR SHAMKANOV, *Cut-elimination for the modal logic of transitive closure via non-well-founded proofs.*
Faculty of Mathematics, National Research University, Higher School of Economics, Usacheva str. 6, 119048 Moscow, Russia.
E-mail: yury.savateev@gmail.com.
Department of Mathematical Logic, Steklov Mathematical Institute of the Russian Academy of Sciences, Gubkina str. 8, 119991 Moscow, Russia.
Faculty of Mathematics, National Research University, Higher School of Economics, Usacheva str. 6, 119048 Moscow, Russia.
E-mail: daniyar.shamkanov@gmail.com.

The modal logic of transitive closure K^+ is a bimodal logic, which can be characterized by Kripke frames with two accessibility relation R and S, where S is the transitive closure of R. We consider a sequent calculus for the logic K^+ allowing cyclic and arbitrary non-well-founded proofs and, directly for the given system, obtain the cut-elimination theorem. A non-well-founded proof is as a possibly infinite tree of sequents that is constructed according to inference rules and, in addition, that satisfies a particular condition on infinite branches. A cyclic, or circular, proof can be defined as a finite pointed graph of sequents which unraveling yields a non-well-founded proof. These proofs turn out to be an interesting alternative to traditional proofs for logics with inductive and co-inductive definitions, fixed-point operators and similar features. In order to avoid nested co-inductive and inductive reasoning in the proof of cut-elimination, we adopt an approach from denotational semantics of computer languages, where program types are interpreted as ultrametric spaces and fixed-point combinators are encoded using the Banach fixed-point theorem. We consider the set of non-well-founded proofs of K^+ and various sets of operations acting on theses proofs as generilized ultrametric spaces and define our cut-elimination operator using the Prieß–Crampe fixed-point theorem, which is a strengthening of the Banach's one.

▸ NOAH SCHWEBER, *Topological games and reverse mathematics*.
Department of Mathematics, University of Wisconsin — Madison, 480 Lincoln Dr., Madison, WI 53703, USA.
E-mail: `schweber@wisc.edu`.

In this talk I'll present some results on the reverse mathematical properties of Banach–Mazur games. For example, Borel Banach–Mazur determinacy is equivalent to ATR_0 in a "level by level" way; however, there is a subtlety in this proof which results in a surprising weakness in the context of *lightface* principles. I'll then talk about higher reverse mathematics and topological games beyond the Banach–Mazur game.

Abstracts of invited talks in the Special Session on Set Theory

▸ OMER BEN-NERIA, *Ordinal definable sets and singular cardinals*.
University of California, Los Angeles, 520 Portola Plaza, Los Angeles, CA 90095, USA.

The extent to which ordinal definable sets can capture essential information about the universe V has been extensively studied in the last few years. One line of study in this vein has been initiated by Shelah, who proved that for every singular strong limit cardinal κ, of uncountable cofinality, there exists a single subset $x \subseteq \kappa$ such that HOD_x contains the entire powerset of x. Using supercompact cardinals, Cummings, Friedman, Magidor, Rinot, and Sinapova, proved a consistency result which shows that Shelah's theorem cannot be extended to cardinals κ of countable cofinality. The purpose of the talk is to discuss the consistency strength of the failure of Shelah's theorem and present both upper and lower bounds.

Acknowledgment. This is joint work with Gitik, Neeman, and Unger.

▸ WILLIAM CHAN AND STEPHEN JACKSON, $L(\mathbb{R})$ *with determinacy satisfies the Suslin hypothesis*.
Department of Mathematics, University of North Texas, 1155 Union Circle #311430, Denton, Texas 76203, USA.
E-mail: `William.Chan@unt.edu`.
E-mail: `Stephen.Jackson@unt.edu`.

A Suslin line is an unbounded, nonseparable, complete dense linear ordering with the countable chain condition. $\mathsf{ZF} + \mathsf{AD}^+ + \mathsf{V} = \mathsf{L}(\mathcal{P}(\mathbb{R}))$ proves that there are no Suslin lines. In particular, if $L(\mathbb{R}) \models \mathsf{AD}$, then $L(\mathbb{R})$ has no Suslin lines.

▸ JAMES CUMMINGS, *Iteration theorems old and new*.
Department of Mathematical Sciences, Carnegie Mellon University, Pittsburgh PA 15213-3890, USA.
E-mail: `jcumming@andrew.cmu.edu`.

I will discuss some classical forcing iteration theorems and the possibilities for extending them.

▸ NATASHA DOBRINEN, *Ramsey theory of homogeneous k-clique-free graphs*.
Department of Mathematics, University of Denver, C.M. Knudson Hall, Room 300, 2390 S. York St., Denver, CO 80209, USA.
E-mail: `natasha.dobrinen@du.edu`.

In [1], we proved that the homogeneous triangle-free graph $\mathcal{H}_3$ has finite big Ramsey degrees: given any finite triangle-free graph A, there is a number $T(A)$ such that for any finite coloring of the copies of A in $\mathcal{H}_3$, there is a subgraph $\mathcal{H}'$, again homogeneous triangle-free, in which the copies of A take no more than $T(A)$ colors. This work involved developing a new notion of trees coding $\mathcal{H}_3$; using forcing techniques to prove, in ZFC, Ramsey theorems

https://doi.org/10.1017/bsl.2018.60 Published online by Cambridge University Press

for these trees, proving new Halpern–Läuchli and Milliken-style theorems; and deducing bounds $T(A)$ from the tree structures.

The methods developed in [1] are quite robust and seem to provide general means for investigating big Ramsey degrees of other homogeneous structures omitting some nontrivial substructure. In this talk, we extend the notion of strong coding tree to code the homogeneous k-clique free graph, $\mathcal{H}_k$, for any $k \geq 3$. Work in progress is to prove that each $\mathcal{H}_k$ has finite big Ramsey degrees.

[1] N. Dobrinen, *The universal homogeneous triangle-free graph has finite big Ramsey degrees*, submitted, p. 48.

► GARRETT ERVIN, *Sierpiński's cube problem for linear orders*.
Department of Mathematical Sciences, Carnegie Mellon University, 5000 Forbes Avenue, Pittsburgh, PA 15213, USA.
E-mail: gervin@andrew.cmu.edu.

In his 1958 book *Cardinal and Ordinal Numbers*, Sierpiński posed five questions concerning linear orders and their lexicographical products. One of these, the so-called cube problem for linear orders, asks whether there exists a linear order that is isomorphic to its lexicographically ordered cube but is not isomorphic to its square. The corresponding question has been answered positively for many different classes of structures, including groups, rings, graphs, Boolean algebras, and various kinds of topological spaces. However, the answer to Sierpiński's question turns out to be negative: every linear order X that is isomorphic to its cube is already isomorphic to its square. More generally, if X is isomorphic to any one of its finite powers X^n, $n > 1$, it is isomorphic to all of them.

The proof relies on a general representation theorem that characterizes, for a fixed structure A from a class of structures $\mathfrak{C}$, those structures $X \in \mathfrak{C}$ that satisfy the isomorphism $A \times X \cong X$. This characterization is based on an analysis of an arbitrary bijection $f\colon A \times X \to X$, and is closely connected to the tail-equivalence relation on the Baire space A^ω. I will give a brief outline of the proof, and discuss some related results, including my solutions to two of the remaining four problems.

► ARISTOTELIS PANAGIOTOPOULOS, *Menger compacta and projective Fraïssé limits*.
Mathematics Department, Caltech, 1200 E. California Blvd, MC 253-37 Pasadena, CA 91125, USA.
E-mail: panagio@caltech.edu.

In every dimension n, there exists a canonical compact, metrizable space called the n-dimensional Menger space. For $n = 0$ it is the Cantor space and for $n = \infty$ it is the Hilbert cube. In this talk we will show how projective Fraïssé methods can be used in the study of Menger spaces. In particular, we will provide a canonical construction of the 1-dimensional Menger space and illustrate how various homogeneity and universality properties of this space reduce to standard Fraïssé theory and basic combinatorics.

Acknowledgment. This is a joint work with Sławomir Solecki.

► NAM TRANG, *Large cardinals, determinacy, and forcing axioms*.
Department of Mathematics, 340 Rowland Hall, UC Irvine, Irvine 92530-3875, USA.
E-mail: ntrang@uci.edu.

Forcing and elementary embeddings are central topics in set theory. Most of what set theorists have focused on are the study of forcing and elementary embeddings over models of ZFC.

In this talk, we focus on forcing and elementary embeddings over models of the Axiom of Determinacy (AD). In particular, we focus on answering the following questions: work in V which models AD. Let $\mathbb{P}$ be a forcing poset and $g \subseteq \mathbb{P}$ be V-generic.

1. Does $V[g] \models \mathsf{AD}$?
2. Is there an elementary embedding from V to $V[g]$?

Regarding question (1), we want to classify what forcings preserve AD. We show that forcings that add Cohen reals, random reals, and many other well-known forcings do not preserve

https://doi.org/10.1017/bsl.2018.60 Published online by Cambridge University Press

AD. We, however, give an example of a forcing that preserves AD. Regarding question (2), an analogous statement to the famous Kunen's theorem for models of ZFC, can be shown: suppose $V = L(X)$ for some set X and $V \vDash \mathsf{AD}$, then there is no elementary embedding from V to itself. We conjecture that there are no elementary embeddings from V to itself.

We present some of the results discussed above. There is still much work to do to completely answer questions 1 and 2.

Acknowledgment. This is an ongoing joint work with D. Ikegami.

▸ ROBIN TUCKER-DROB, *Conjugation invariant means on groups and inner amenability*.
Department of Mathematics, Mailstop 3368, Texas A&M University, College Station, TX 77843-3368, USA.
E-mail: `rtuckerd@math.tamu.edu`.

A discrete group is said to be inner amenable if it admits a diffuse conjugation-invariant mean. I will give an overview of inner amenability and discuss some recent joint work with Bruno Duchesne and Phillip Wesolek in which we give a complete characterization of inner amenability for (a) generalized wreath products, and (b) groups acting nondegenerately on trees and, more generally, groups acting nondegenerately on finite dimensional CAT(0)-cube complexes.

▸ JOSEPH ZIELINSKI, *Roelcke precompact sets and locally Roelcke precompact Polish groups*.
Department of Mathematical Sciences, Carnegie Mellon University, Pittsburgh, PA 15213, USA.
E-mail: `zielinski@cmu.edu`.

A subset of a Polish group is Roelcke precompact if, given any open subset V of the group, it can be covered by finitely many sets of the form VfV. Such sets form an ideal and the familiar Roelcke precompact groups are those for which this ideal is improper. A group is said to be locally Roelcke precompact when this ideal countains an open set. Examples of such groups—in addition to all Roelcke precompact groups and all locally compact groups—include the isometry group of the Urysohn metric space and the automorphism group of the countably regular tree.

All locally Roelcke precompact groups are locally bounded in the sense of the coarse geometry of topological groups developed by C. Rosendal. Indeed, we characterize them as those locally bounded Polish groups for which every coarsely bounded subset is Roelcke precompact. We also characterize them as those groups whose completions with respect to their Roelcke (or lower) uniformities are locally compact. In this talk we discuss the above and other aspects of these groups from the perspectives of abstract topological dynamics and large scale geometry.

Abstracts of Contributed talks

▸ JOHN BALDWIN, *The paradigm shift in model theory*.
Department of Mathematics, Statistics, and Computer Science, M/C 249, 851 S. Morgan, Chicago, IL 60607, USA.
E-mail: `jbaldwin@uic.edu`.

Traditionally, logic was thought of as 'principles of right reason'. Early twentieth century philosophy of mathematics focused on the problem of a general foundation for all mathematics. By contrast, the last 70 years have seen model theory develop as the study and comparison of formal theories for studying specific areas of mathematics. While this shift began in work of Tarski, Robinson, Henkin, Vaught, and Morley, the decisive step came with Shelah's stability theory. After this paradigm shift there is a systematic search for a finite set of syntactic conditions which divide first order theories into disjoint classes such that models of different theories in the same class have similar mathematical properties. This classification of theories makes more precise the idea of a 'tame structure'. Thus, logic (specifically model theory) becomes a tool for organizing and doing mathematics with consequences for combinatorics, diophantine geometry, differential equations and other fields. In particular, I

https://doi.org/10.1017/bsl.2018.60 Published online by Cambridge University Press

will focus on *axiomatic* and *definable* analysis as two schools which allow first order logic to solve problems from classical analysis and number theory. This reports material in my recent book published by Cambridge: Formalization without Foundationalism: Model Theory and the Philosophy of Mathematical Practice.

► TEJAS BHOJRAJ, *Quantum Solovay randomness*.
Department of Mathematics, University of Wisconsin–Madison, 480 Lincoln Dr., Madison, WI 53706, USA.
E-mail: bhojraj@math.wisc.edu.

Martin-Löf randomness and Solovay randomness are equivalent notions of algorithmic randomness for infinite sequences of bits: elements of Cantor space, 2^{ω}. This article is about extending these notions to sequences of quantum bits (qubits). The main result is that these notions remain equivalent in the quantum setting. A sequence of n qubits is modelled by a density matrix on $\mathbb{C}^{2^n}$. Nies and Scholz modelled an infinite sequence of qubits by a *state*. A state is a sequence, $\rho = (\rho_n)_{n\in\omega}$ where for each n, ρ_n is a density matrix on $\mathbb{C}^{2^n} = \mathbb{C}^2 \otimes \mathbb{C}^{2^{n-1}}$ and the partial trace of ρ_n over $\mathbb{C}^2$ is ρ_{n-1}. They defined quantum Martin-Löf randomness (q-MLR), a notion of randomness for states and suggested that one could also define a notion of Solovay randomness for states. They asked if such a notion is equivalent to q-MLR. We define a notion of quantum Solovay randomness and show that it is equivalent to q-MLR.

► ALEXI BLOCK GORMAN, PHILIPP HIERONYMI, AND ELLIOT KAPLAN, *Pairs of theories satisfying a Mordell-Lang condition*.
Department of Mathematics, University of Illinois at Urbana-Champaign, 1409 West Green Street, Urbana, IL 61801, USA.
E-mail: atb2@illinois.edu.

In this article we consider the expansion of geometric theories (i.e., theories for which the algebraic closure operator is a pregeometry in every model, and the theory eliminates "$\exists^{\infty}$") by a predicate that satisfies certain properties. Namely, the subset which the predicate defines is the universe of a model for some theory T_{α} that interacts with the theory T_{β} of the larger structure in desirable ways. This framework generalizes the work of van den Dries in [2] on dense pairs of models of an o-minimal theory, the work of Hieronymi, Nell, and Walsberg in [3] on o-minimal structures with a dense predicate interpreting an arbitrary theory, and the notion of lovely pairs of geometric structures in [1]. The key results of this article are the near model completeness and completeness of these theories of pairs. We remark that there are many natural examples of pairs which would satisfy the conditions we impose, and discuss consequences of near model completeness for these examples. Such consequences include a characterization of the open core when T_{β} has a well-behaved definable topology. In particular, the theory of structure $(\overline{\mathcal{R}}, K)$, where $\overline{\mathcal{R}}$ is a real closed field and K is a pseudo-real closed subfield, is near model complete, and its open core is interdefinable with the open core of $\overline{\mathcal{R}}$. The theory of structure $(\mathcal{R}_K, \mathcal{Q})$ where $\mathcal{R}_K$ is the reals as an ordered K-vector space and $K \subseteq \mathcal{R}$ is a subfield, and $\mathcal{Q}$ is an ordered vector space over itself, also falls under this framework, and from its near-model completeness we obtain a characterization of when the theory of such a structure is decidable. We also extend much of the work done in [3] to p-adically closed fields with a dense, independent predicate whose theory is "wild."

[1] A. Berenstein and E. Vassilev, *On lovely pairs of geometric structures*. ***Annals of Pure and Applied Logic***, vol. 161 (2010), no. 7, pp. 866–878.

[2] L. van den Dries, *Dense pairs of o-minimal structures*. ***Fundamenta Mathematicae***, vol. 157 (1998), no. 1, pp. 61–78.

[3] P. Hieronymi, T. Nell, and E. Walsberg, *Wild theories with o-minimal open core*. ***Annals of Pure and Applied Logic***, vol. 169 (2018), no. 2, pp. 146–163.

► BERND BULDT, *On properties sufficient for Gödel's second incompleteness theorem*.
Department of Mathematical Sciences, Purdue University Fort Wayne, 2101 E. Coliseum Blvd., Fort Wayne, IN 46805, USA.
E-mail: bbuldt@purdue.edu.

It may be advisable to split complex assumptions into simpler ones when tracking of assumptions is important. E.g., in incidence geometry we may want to split the assumption of exactly one line going through any two points into two: one assumption about the existence and one about the uniqueness of the line. We argue for a similar split in respect to Gödel's second incompleteness theorem.

Since [1] it has become customary to require a formal system $\mathcal{F}$, beyond consistency and the existence of fixed points, to satisfy two more properties. First, (D1) If $\mathcal{F} \vdash \varphi$, then $\mathcal{F} \vdash Pr(\ulcorner\varphi\urcorner)$; and, second, (D2) $\mathcal{F} \vdash Pr(\ulcorner\varphi\urcorner) \rightarrow Pr(\ulcorner Pr(\varphi)\urcorner)$. If we sufficiently limit the arithmetical complexity of the expression 'Pr,' then (D1) follows if the formal system $\mathcal{F}$ is Σ_1-complete, and (D2) does if $\mathcal{F}$ is provably so. Observations will be shared why (in certain contexts) it may be advisable to split (D2) into two assumptions from which it follows, namely, (D3) $\mathcal{F} \vdash Pr(\ulcorner\varphi \rightarrow \psi\urcorner) \rightarrow (Pr(\ulcorner\varphi\urcorner) \rightarrow Pr(\ulcorner\psi\urcorner))$, i.e., provable closure under modus ponens, and $(I\Sigma_1)$, i.e., induction for Σ_1-expressions.

[1] R. G. Jeroslov, *Redundancies in the Hilbert-Bernays derivability conditions for Gödel's second incompletness theorem*. ***The Journal of Symbolic Logic***, vol. 38 (1973), no. 3, pp. 359–367.

▸ WESLEY CALVERT, SERGEI S. GONCHAROV, VALENTINA HARIZANOV, ANDREI MOROZOV, ALEXANDRA SOSKOVA, AND DANIEL TURETSKY, *Semigroups of partial automorphisms*.
Department of Mathematics, Southern Illinois University, Mail Code 4408, 1245 Lincoln Drive, Carbondale, IL 62901, USA.
E-mail: `wcalvert@siu.edu`.
Sobolev Institute of Mathematics, Siberian Branch Russian Academy of Sciences, Novosibirsk, Russia.
Department of Mathematics, George Washington University, Washington, DC 20052, USA.
Sobolev Institute of Mathematics, Siberian Branch Russian Academy of Sciences, Novosibirsk, Russia.
Faculty of Mathematics and Informatics, Kliment Ohridski University of Sofia, Sofia, Bulgaria.
Department of Mathematics, University of Notre Dame, Notre Dame, IN 46556, USA.

Let $\mathcal{M}$ be a structure in a finite relational language. We then consider the semigroup $\mathcal{S}_{\mathcal{M}}$ of finite partial automorphisms of $\mathcal{M}$ under composition. The central question is what features of $\mathcal{M}$ are retained by $\mathcal{S}_{\mathcal{M}}$.

We show that $\mathcal{M}$ is Δ^0_1 definable in $\mathcal{S}_{\mathcal{M}}$. We also give results (and limitations) on the transfer of Scott rank, index set, categoricity, fragmentary decidability, and computable embeddability. Moreover, we show that there is no computable enumeration of all inverse semigroups that arise as $\mathcal{S}_{\mathcal{M}}$ for some $\mathcal{M}$.

▸ SANTIAGO CAMACHO, *The exponential field of transseries with a monomial set is undecidable*.
Mathematics, Illinois Wesleyan University, 201 E. Beecher St., Bloomington, IL 61701, USA.
E-mail: `scamacho@iwu.edu`.

We study the theory of $(\mathbb{T}; +, \times, \exp, G^{\mathrm{LE}})$, where $\mathbb{T}$ is the field of logarithmic exponential transseries and G^{LE} is the monomial group of transmonomials. The structure $(\mathbb{T}; +, \times, \exp)$ is tame, being a nonstandard model of the real field with the exponential function. We show via an application of truncation in transseries how the structure $(\mathbb{T}; +, \times, \exp, G^{\mathrm{LE}})$ is undecidable. We will begin the talk by giving out examples of transseries, we will define the notion on truncation and show how in a reasonable language extending the language of rings the operation of truncation can be interpreted. We will then show how truncation is undecidable, and finally we will explain how to interpret truncation in the structure $(\mathbb{T}; +, \times, \exp, G^{\mathrm{LE}})$.

[1] L. van den Dries, A. Macintyre, and D. Marker, *Logarithmic-exponential series*. ***Annals of Pure and Applied Logic***, vol. 111 (2001), pp. 61–113.

https://doi.org/10.1017/bsl.2018.60 Published online by Cambridge University Press

[2] Y. Gurevich, *Chapter XIII: Monadic second-order theories*, ***Model-Theoretic Logics***, Springer-Verlag, New York, 1985, pp. 479–506. Available at `http://projecteuclid.org/euclid.pl/1235417279`.

[3] J. Robinson, *Definability and decision problems in arithmetic*. ***The Journal of Symbolic Logic***, vol. 14 (1949), pp. 98–114.

► YONG CHENG, *Analysis of Martin–Harrington theorem in higher order arithmetic*.
School of Philosophy, Wuhan University, Wuhan 430072 Hubei, Peoples Republic of China.
E-mail: `world-cyr@hotmail.com`.

Martin–Harrington theorem says that $Det(\Sigma^1_1)$ is equivalent to $0^\sharp$ exists. Boldface Martin–Harrington theorem is provable in Z_2. "$0^\sharp$ exists implies $Det(\Sigma^1_1)$" is provable in Z_2. All known proofs of "$Det(\Sigma^1_1)$ implies $0^\sharp$ exists" are done in two steps: first show that $Det(\Sigma^1_1)$ implies *Harrington's Principle* (HP) and then show that HP implies $0^\sharp$ exists. The first step is provable in Z_2. I prove that the second step is neither provable in Z_2 nor provable in Z_3, but it is provable in Z_4. In the joint work with Ralf Schindler, we prove that "$\mathsf{Z}_2 + \mathsf{HP}$" is equiconsistent with ZFC, and "$\mathsf{Z}_3 + \mathsf{HP}$" is equiconsistent with "$\mathsf{ZFC} +$ there exists a remarkable cardinal."

► REID DALE, *Is there a really good definition of mass*?
Group in Logic and the Methodology of Science, University of California, Berkeley, Berkeley, CA 94720, USA.
E-mail: `reiddale@math.berkeley.edu`.

Guided by a desire to eliminate language that refers to unobservable structure from mechanics, Ernst Mach proposed a definition of mass in terms of more directly observable data. A great deal of literature surrounds the question of whether this proposed definition accomplishes its stated goal, or even whether it constitutes a definition. In this talk we aim to bring clarity to this debate by using methods from model theory and from modal logic to classify, reconstruct, and evaluate these arguments. In particular, we exhibit a general construction of first-order modal frames for appropriately presented scientific theories with epistemic constraints. These frames allow us to characterize which properties are "modally definable" in the sense of Bressan.

► SEAN C. EBELS-DUGGAN, *Identifying cardinal abstracts via embeddings into induced models*.
Department of Philosophy, Northwestern University, Kresge 3512, 1880 Campus Drive, Evanston, IL 60208, USA.
E-mail: `s-ebelsduggan@northwestern.edu`.

Abstractionists about arithmetic are concerned to determine the reference of numerical terms with abstraction principles in second-order logic. However, there are many candidate principles for determining these referents, as noted in [2] and [3], for example,

$$HP : (\forall X, Y)(\#X = \#Y \leftrightarrow |X| = |Y|)$$
$$FHP : (\forall X, Y)(\#_F X = \#_F Y \leftrightarrow (|X|, |Y| \geq \omega \vee |X| = |Y|)).$$

This plurality motivates *cross-sortal identity axioms*, which identify objects determined by distinct abstraction principles. Among the extant identity axioms,

$$(\forall X, Y)(@_1 X = @_2 Y \leftrightarrow (\forall Z)(E_1(X, Z) \leftrightarrow E_2(Y, Z)),$$

called $ECIA_2$ in [1], seems to do best. However, $ECIA_2$ has implications out of line with the arguments underwriting it. For example, it denies that modular numbers are natural numbers, but affirms as natural numbers some abstracts that don't arise in a counting sequence. This article presents an alternative axiom based on embeddings between the induced models of abstracts treated in [4] and [5]. According to this axiom, abstracts $@_1 X$ and $@_2 Y$ should be identified if embeddings of the induced model of an initial segment of the $@_1$-abstracts into the induced model of the $@_2$-abstracts determine the value of $@_1 X$ to be $@_2 Y$, or vice versa. This principle matches $ECIA_2$'s success in identifying various natural number candidates (e.g., from HP and FHP), and gives compelling answers to many of the cases where $ECIA_2$'s determinations are more problematic. However, it does so at the cost of not being applicable to all abstracts.

[1] R. Cook and P. Ebert, *Abstraction and identity*. ***Dialectica***, vol. 59 (2005), no. 2, pp. 121–139.

[2] R. G. Heck, Jr., *Finitude and Hume's principle*. ***Journal of Philosophical Logic***, vol. 26 (1997), no. 6, pp. 589–617.

[3] P. Mancosu, *In good company? On Hume's principle and the assignment of numbers to infinite concepts*. ***Review of Symbolic Logic***, vol. 8 (2015), no. 2, pp. 370–410.

[4] S. Walsh, *Comparing Hume's principle, basic law V, and peano arithmetic*. ***Annals of Pure and Applied Logic***, vol. 163 (2012), no. 11, pp. 1679–1709.

[5] S. Walsh and S. C. Ebels-Duggan, *Relative categoricity and abstraction principles*. ***The Review of Symbolic Logic***, vol. 8 (2015), no. 3, pp. 572–606.

► LANDON D. C. ELKIND, *A theorem of infinity for Principia Mathematica*.
Department of Philosophy, University of Iowa, 250 English-Philosophy Building, 251 West Iowa Avenue, Iowa City, IA 52242, USA.
E-mail: `landon-elkind@uiowa.edu`.
URL Address: `https://sites.google.com/site/landondcelkind/`.

We prove a theorem of infinity for *Principia Mathematica* [3]. The theorem's proof requires altering the meta-theory of [3]. In [3] we have a simple type theory with a lowest type: the type hierarchy is well-founded. Call this 'simple $\mathbb{N}$-type theory'. Our key idea is to allow for infinitely descending types just as there are infinitely ascending types: we allow our type hierarchy to be foundationless. Call this 'simple $\mathbb{Z}$-type theory'. Given the acceptableness of well-founded simple type theory, adjusting the meta-theoretic specification of types to produce a foundationless type theory is acceptable. This adjustment is reinforced also by suggestive remarks of Whitehead and Russell. By so-adjusting [3], a core objection to Logicism—that Logicism cannot recover Peano arithmetic without an axiom of infinity—dissipates.

Two other sorts of modifications to the well-founded simple $\mathbb{N}$-type theory have been attempted: (1) transfinite type theories where we allow quantification over type indices and (2) foundationless simple $\mathbb{Z}$-type theories like ours. We give considerations against (1)-style proposals, focusing on the transfinite type theory **Q** in [1]. Now [2] appears to be the first work to have explored foundationless types. In [2] we find the claim that foundationless type theories like the one proposed here are inconsistent. But the argument given is flawed: it assumes incorrectly that foundationless type theories are conservative extensions of well-founded simple type theories. We show that the simple $\mathbb{Z}$-type theory we propose is not a conservative extension of simple $\mathbb{N}$-type theory.

[1] P. B. Andrews, ***A Transfinite Type Theory with Type Variables***, North-Holland, 1965.

[2] H. Wang, *Negative types*. ***Mind***, vol. 61 (1952), no. 243, pp. 366–388.

[3] A. Whitehead and B. A. W. Russell, ***Principia Mathematica, vol. I–III***, Cambridge University Press, 1925, 1927.

► GABRIEL GOLDBERG, *The Ultrapower Axiom*.
213 Summer St., Somerville, MA 02143, USA.
E-mail: `goldberg@math.harvard.edu`.

The unifying feature of the known hierarchies of canonical inner models for large cardinals is their *comparison process*, which is the key to the inductive construction and analysis of the known inner models. For the analysis of L, the comparison process reduces to Gödel's condensation lemma. For $L[U]$, it reduces to Kunen's method of iterated ultrapowers. As far as inner model theory is known to succeed (i.e., to Woodin limits of Woodin cardinals, and conditionally to the finite levels of supercompactness), the comparison process is its central component. Therefore a fundamental problem for the inner model program is whether comparison is compatible with supercompact cardinals.

To attack this problem, we isolate an abstract comparison principle that follows from the comparison lemma by such a general argument that it must hold in any canonical inner model built by today's technology or anything like it. The principle, called the Ultrapower

https://doi.org/10.1017/bsl.2018.60 Published online by Cambridge University Press

Axiom (or UA), roughly asserts that any two wellfounded ultrapowers of the universe have a common ultrapower; that is, ultrapowers can be compared.

Refuting UA from any large cardinal hypothesis would constitute a strong anti-inner model theorem. Instead we present some results that suggest that one can develop a compelling theory of large cardinals assuming UA.

▶ DANUL K. GUNATILLEKA, *Counting countable models of Baldwin–Shi hypergraphs*.
Department of Mathematics, University of Maryland in College Park, 4176 Campus Drive, William E. Kirwan Hall, College Park, MD 20742-4015, USA.
E-mail: danulg@math.umd.edu.

We take a closer look at generic structures built from classes of weighted finite hypergraphs $(K_{\overline{\alpha}}, \leq)$. Extending a result of Laskowski's, we show that the *ab initio* generic structures, which we will call Baldwin–Shi hypergraphs, have theories that admit a certain level of quantifier elimination. Furthermore, restricting our attention to the case where the weights $\overline{\alpha}$ are rational, we show that a natural (rational valued) notion of dimension is sufficient to determine any given countable model of a fixed Baldwin–Shi hypergraph up to isomorphism. We further show that for any given dimension, we may construct a countable model of a fixed Baldwin–Shi hypergraph as the generic structure for a suitable class $(K, \leq)$ with $K \subseteq K_{\overline{\alpha}}$ and hence we establish that each countable model of a fixed Baldwin–Shi hypergraph is itself a generic structure of a suitably selected subclass of $K_{\overline{\alpha}}$ provided that each of the weights $\overline{\alpha}$ are rational.

▶ JAMES HANSON, *Separable and inseparable Gromov–Hausdorff categoricity in continuous logic*.
Department of Mathematics, University of Wisconsin–Madison, Madison, WI 53706, USA.
E-mail: jehanson2@wisc.edu.

Gromov–Hausdorff distance is a notion of the similarity of metric spaces defined in terms of common embeddings, with natural analogs for metric structures with Lipschitz languages [2] such as the Kadets distance in Banach spaces. The equivalence relation it induces is strictly coarser than isometry for noncompact spaces. There are theories which are categorical with respect to Gromov–Hausdorff distance (GH-categorical) while failing to be isometrically categorical.

The separable case has a weak Ryll–Nardzewski characterization almost exactly as in [1], although it doesn't fit into the formalism presented there and is in fact slightly better behaved. We'll examine progress towards the Gromov–Hausdorff form of Morley's theorem, that is if a theory is GH-categorical in any uncountable density character, then it is GH-categorical in every uncountable density character.

[1] I. B. Yaacov, *On perturbations of continuous structures*. ***Journal of Mathematical Logic***, vol. 8 (2008), no. 2, pp. 225–249.

[2] I. B. Yaacov, M. Doucha, A. Nies, and T. Tsankov, *Metric Scott analysis*. ***Advances in Mathematics***, vol. 318 (2017), pp. 46–87.

▶ LINDA LAWTON, *Decidability of the AE-theory of the lattice of Π^0_1 classes modulo finite differences*.
Department of Mathematics and Computer Science, Northern Michigan University, 1401 Presqe Isle, Marquette, MI 49855, USA.
E-mail: llawton@nmu.edu.

An AE-sentence is a sentence in prenex normal form with all universal quantifiers preceding all existential quantifiers, and the AE-theory of a structure is the set of all AE-sentences true in the structure. We show that the AE-theory of $(\mathcal{L}(\Pi^0_1), \cap, \cup, 0, 1)^*$ is decidable by giving a procedure which, for any AE-sentence in the language, determines the truth or falsity of the sentence in our structure.

▶ RUSSELL MILLER, *Hilbert's Tenth Problem does not respect Turing equivalence*.
Mathematics Department, Queens College & CUNY Graduate Center, 65-30 Kissena Blvd., Queens NY 11367, USA.

E-mail: Russell.Miller@qc.cuny.edu.
URL Address: qcpages.qc.cuny.edu/∼rmiller.

The *HTP-operator* is the map sending each subset W of the set $\mathbb{P}$ of prime numbers to the set

$$HTP(R_W) = \{f \in \mathbb{Z}[X_1, X_2, \dots]\colon f = 0 \text{ has a solution in } \mathbb{Z}[W^{-1}]\},$$

known as *Hilbert's Tenth Problem* for the subring $R_W = \mathbb{Z}[W^{-1}]$ of $\mathbb{Q}$. We show that this operator does not respect Turing equivalence, by producing complementary subsets W and $\overline{W}$ of $\mathbb{P}$ for which $HTP(R_{\overline{W}})$ computes the jump of $HTP(R_W)$. Using the technique of high permitting, we can also give an example where the HTP operator reverses the (strict) Turing reductions: $V <_T W$, yet $HTP(R_W) <_T HTP(R_V)$. Thus, deciding which rational numbers lie in the subring R_V of $\mathbb{Q}$ is easier than deciding which lie in the subring R_W, yet deciding which diophantine equations have solutions in R_V is harder than deciding which have solutions in R_W.

Acknowledgment. This is joint work with Ken Kramer.

▸ ITAY NEEMAN AND ZACH NORWOOD, *Remarkable cardinals and coding along trees*.
Department of Mathematics, University of California Los Angeles, CA 90095-1555, USA.
E-mail: ineeman@math.ucla.edu.
E-mail: znorwood@math.ucla.edu.

Neeman and Zapletal [2] showed that, assuming large cardinals in the region of infinitely many Woodin cardinals, proper forcing cannot change the theory of $L(\mathbb{R})$ with parameters, an assertion we will refer to as $L(\mathbb{R})$-*absoluteness for proper posets*. Schindler later showed [3] that the large-cardinal assumption of that theorem could be greatly reduced, computing the consistency strength of $L(\mathbb{R})$-absoluteness for proper posets to be exactly the existence of a *remarkable* cardinal, which is consistent even with $V = L$.

Schindler's theorem echoes a much earlier theorem of Kunen (see [1]), that $L(\mathbb{R})$-absoluteness for ccc posets is equiconsistent with a weakly compact cardinal. Schindler's proof does not resemble Kunen's, however, using almost-disjoint coding instead of Kunen's innovative method of coding along branchless trees. We show how to reconcile these two proofs, giving a new proof of Schindler's theorem that generalizes Kunen's methods and suggests further investigation of nonthin trees.

[1] L. Harrington and S. Shelah, *Some exact equiconsistency results in set theory*. ***Notre Dame Journal of Formal Logic***, vol. 26 (1985), no. 2, pp. 178–188.

[2] I. Neeman and J. Zapletal, *Proper forcing and* $L(\mathbb{R})$. ***The Journal of Symbolic Logic***, vol. 66 (2001), no. 2, pp. 801–810.

[3] R. Schindler, *Proper forcing and remarkable cardinals. II*. ***The Journal of Symbolic Logic***, vol. 66 (2001), no. 3, pp. 1481–1492.

▸ VICTORIA NOQUEZ, *Vaught's two-cardinal theorem and quasi-minimality in continuous logic*.
Department of Mathematics, Harvey Mudd College, 301 Platt Blvd., Claremont, CA 91711-5901, USA.
E-mail: noquez@math.hmc.edu.

We prove the following continuous analogue of Vaught's Two-Cardinal Theorem: if for some $\kappa > \lambda \geq \aleph_0$, a continuous theory T has a model with density character κ which has a definable subset of density character λ, then T has a model with density character $\aleph_1$ which has a separable definable subset. We also show that if we assume that T is ω-stable, then if T has a model of density character $\aleph_1$ with a separable definable set, then for any uncountable κ we can find a model of T with density character κ which has a separable definable subset. In order to prove this, we develop an approximate notion of quasi-minimality for the continuous setting. We apply these results to show a continuous version of the forward direction of the Baldwin–Lachlan characterization of uncountable categoricity: if a continuous theory T is uncountably categorical, then T is ω-stable and has no Vaughtian pairs.

► IVÁN ONGAY-VALVERDE, *Splitting localization and prediction numbers*.
Mathematics Department, University of Wisconsin-Madison, Van Vleck Hall, 480 Lincoln Drive, Madison, WI 53706, USA.
E-mail: ongay@math.wisc.edu.

In 1993, Newelski and Roslanowski (in [3]) studied some cardinal characteristics related to the unsymmetric game (I called them the localization numbers). While doing this, they found the n-localization property (later studied in [4] and [5]). When a forcing has this property, you can ensure that all new reals are 'tame' somehow (for example, you do not add Cohen or Random reals). In a different line of study, Andreas Blass worked with some cardinal characteristic related to the idea of guessing correctly a real number given certain amount of information (he called them evasion and prediction numbers). In 2010, in [1], he left an open question about identifying the possible variations of these numbers. Impressively, this two notions are related.

Using techniques analogue to Newelski and Roslanowski I was able to solve Blass's open question. To do it, it was necessary to use a forcing notion with accelerating trees and to define a variation of the k-localization property that I called the $(k+1)^{\omega}$-localization property. During the talk I will define all the related cardinal characteristics and the acceleration tree forcing, also I will explain what is the $(k+1)^{\omega}$-localization property.

[1] A. Blass, *Combinatorial cardinal characteristics of the continuum*, ***Handbook of Set Theory***, Springer, 2010, pp. 395–489.

[2] S. Geschke and M. Kojman, *Convexity numbers of closed sets in* $\mathbb{R}^n$. ***Proceedings of the American Mathematical Society***, vol. 130 (2002), no. 10, pp. 2871–2881.

[3] L. Newelski and A. Rosłanowski, *The ideal determined by the unsymmetric game*. ***Proceedings of the American Mathematical Society***, vol. 117 (1993), no. 3, pp. 823–831.

[4] A. Roslanowski, *n-localization property*. ***The Journal of Symbolic Logic***, (2006), pp. 881–902.

[5] J. Zapletal, *Applications of the ergodic iteration theorem*. ***Mathematical Logic Quarterly***, vol. 56 (2010), no. 2, pp. 116–125.

► FEDOR PAKHOMOV AND JAMES WALSH, *Reflection ranks of axiomatic theories*.
Steklov Mathematical Institute, Moscow 119991, Russia.
E-mail: pakhfn@mi.ras.ru.
Group in Logic and the Methodology of Science, University of California, Berkeley, Berkeley, CA 94720, USA.
E-mail: walsh@math.berkeley.edu.

It is a well-known empirical phenomenon that *natural* axiomatic theories are pre-well-ordered by proof-theoretic strength. Without a mathematical definition of "natural," it is unclear how to provide a general mathematical explanation of the apparent structure of the hierarchy of proof-theoretic strength. We develop a technique for showing that large classes of theories are well-founded by proof-theoretic strength. Our main theorem is that the $\prod_1^1$-sound extensions of ACA_0 are well-founded by the relation $<_{\prod_1^1}$, where $T <_{\prod_1^1} U$ if U proves the $\prod_1^1$ soundness of T. Accordingly, we can attach a well-founded rank—*reflection rank*—to $\prod_1^1$ sound extensions of ACA_0. What is the connection between the reflection rank of T and the $\prod_1^1$ proof-theoretic ordinal of T? Recall that ACA_0^+ is the statement "for every X, the ω^{th} jump of X exists." Our second main theorem is the following: For any $\prod_1^1$-sound extension T of ACA_0^+, the reflection rank of T is exactly the proof-theoretic ordinal of T.

► FRANCESCO PARENTE, *On regular ultrafilters, Boolean ultrapowers, and Keisler's order*.
School of Mathematics, University of East Anglia, Norwich NR4 7TJ, UK.
E-mail: f.parente@uea.ac.uk.

In this talk, we shall present some applications of the Boolean ultrapower construction to the study of Keisler's order.

Over the last decade, Malliaris and Shelah proved a striking sequence of results in the intersection between model theory and set theory, settled affirmatively the question of whether $\mathfrak{p} = \mathfrak{t}$, and developed surprising connections between classification theory and cardinal

https://doi.org/10.1017/bsl.2018.60 Published online by Cambridge University Press

characteristics of the continuum. The main motivation of their work is the study of Keisler's order, introduced originally in 1967 as a device to compare the complexity of complete theories by looking at regular ultrapowers of their models.

In this context, there has been a recent shift towards building ultrafilters on complete Boolean algebras. In particular, moral ultrafilters have emerged as the main tool to find dividing lines among unstable theories.

Motivated by this new Boolean-algebraic framework, in the first part of the talk we shall analyse and compare two different notions of regularity for ultrafilters on complete Boolean algebras. This analysis will show that most model-theoretic properties of κ-regular ultrafilters can be generalized smoothly to the context of $\langle\kappa, 2\rangle$-distributive Boolean algebras. On the other hand, we shall prove the existence of regular ultrafilters on the Cohen forcing algebra $\mathbb{C}_\kappa$ with unexpected model-theoretic features.

In the second part of the talk, the following question will be addressed: what kind of classification can arise when we compare theories according to the saturation of Boolean ultrapowers of their models? In order to provide an answer to this question, we shall introduce a new Boolean-algebraic analogue of Keisler's order and compare it with the usual one.

▸ NIGEL PYNN-COATES, *Differential-henselianity and maximality of asymptotic valued differential fields*.
Department of Mathematics, University of Illinois at Urbana–Champaign, 1409 West Green Street, Urbana, IL 61801, USA.
E-mail: `pynncoa2@illinois.edu`.
URL Address: `https://faculty.math.illinois.edu/~pynncoa2`.

The notion of differential-henselianity, introduced by Scanlon [2] and developed in greater generality by Aschenbrenner, van den Dries, and van der Hoeven [1], plays an important role in the study of valued differential fields. After reviewing this notion, we discuss its relation to differential-algebraic maximality. Next, we discuss the issue of uniqueness of maximal immediate extensions of valued differential fields, and our positive result in the case of asymptotic fields. Finally, we show the existence and uniqueness of differential-henselizations of such fields.

[1] M. Aschenbrenner, L. van den Dries, and J. van der Hoeven, ***Asymptotic Differential Algebra and Model Theory of Transseries***, Annals of Mathematics Studies, vol. 195, Princeton University Press, 2017.

[2] T. Scanlon, *A model complete theory of valued D-fields*. ***The Journal of Symbolic Logic***, vol. 65 (2000), no. 4, pp. 1758–1784.

▸ WIM RUITENBURG, *A constructive logic and Fregean set theory which avoids Russell's Paradox*.
Department of Mathematics, Marquette University, P.O. Box 1881, Milwaukee, WI 53201, USA.
E-mail: `wimr@mscs.mu.edu`.

We consider a proof interpretation corresponding with the basic predicate calculus of [1], a logic which has its origin in work by Albert Visser in [2]. Over this basic predicate logic we describe a Fregean style set theory over which the 'Russell Paradox' turns into a valuable theorem rather than inconsistency.

[1] W. Ruitenburg, *Basic predicate calculus*. ***Notre Dame Journal of Formal Logic***, vol. 39 (1998), no. 1, pp. 18–46.

[2] A. Visser, *A propositional logic with explicit fixed points*. ***Studia Logica***, vol. 40 (1981), pp. 155–175.

▸ DAMON SCOTT, *Modeling mathematical practice with well-formed mathematical contexts*.
Department of Mathematics, Francis Marion University, Florence, SC 29502, USA.
E-mail: `DScott@FMarion.edu`.

Like Structured Programming for algorithms, doing formal logic with well-formed mathematical contexts is simultaneously user-friendly and machine-parsably rigorous. A full treatment of contexts may be found in [2]. There the expressive power of well-formed contexts has been shown to be coextensive with that of conventional well-formed statements for presenting the results of conventional mathematics, and the calculus of manipulations on them has been shown to be theoretically sufficient for first-order proof. But more than that, contexts are *practical*, in that it is far, far easier to reduce conventional mathematics to machine-parsably formal forms with context-oriented methods than with currently standard statement-oriented methods. Even when machine-parsable rigor is not the goal, context-oriented logic allows one to peer into and understand the structure and practice of conventional mathematics in ways that statement-oriented methods do not.

We present three ways of understanding well-formed mathematical contexts: their *formal definition as syntax*, their *genus* (actually meta-genus), and most importantly their *meaning*. The full scope, utility, and meaning of contexts will become readily apparent when Euclid's well-known proof of the Pythagorean Theorem is presented as a composition of well-formed contexts. The proof of Euclid is thus *readily* reducible to machine-checkable form. In this way, well-formed mathematical contexts are shown to be what Jon Barwise so long sought but never found.

[1] J. Barwise, ***The Situation in Logic***, Center for the Study of Language and Information, Stanford, California, 1989.

[2] D. Scott, ***Well-Structured Mathematical Logic***, Carolina Academic Press, Durham, North Carolina, 2013.

► ASSAF SHANI, *Borel equivalence relations and weak choice principles*.
Department of Mathematics, University of California Los Angeles, CA 90095-1555, USA.
E-mail: assafshani@ucla.edu.

Using tools from [1] and [2], we develop a relationship between the study of certain Borel equivalence relations and weak choice principles. We show that separation of these choice principles is closely related to questions about equivalence relations, such as Borel irreducibility and ergodicity.

For example, given a countable Borel equivalence relation E we define the choice principle *countable choice for E-classes* as "every countable sequence of E-classes has a choice function". We show that for countable Borel equivalence relations E and F, if E is F-ergodic (with respect to some measure), then there is a model which satisfies countable choice for F classes but not for E classes. A main ingredient in the proof is showing that if E is F-ergodic with respect to μ, then E^ω is F-ergodic with respect to the product measure μ^ω.

Furthermore, we construct a model in which there is a countable sequence of countable sets of reals without a choice function, yet for every countable Borel equivalence relation E, countable choice for E-classes holds. This separation in turn gives rise to an interesting new Borel equivalence relation. This equivalence relation is pinned, below $=^+$, and strictly above $(E_\infty)^\omega$.

[1] V. Kanovei, M. Sabok, and J. Zapletal, ***Canonical Ramsey Theory on Polish Spaces***, Cambridge Tracts in Mathematics, vol. 202, Cambridge University Press, Cambridge, 2013.

[2] J. Zapletal, ***Forcing Idealized***, Cambridge Tracts in Mathematics, vol. 174, Cambridge University Press, Cambridge, 2008.

► KAMERYN WILLIAMS, *Minimal models for second-order set theories*.
Department of Mathematics, The Graduate Center, CUNY, 365 Fifth Ave, New York, NY 10016, USA.
E-mail: kwilliams4@gradcenter.cuny.edu.

Shepherdson and, independently, Cohen showed that there is a least transitive model of ZFC, i.e., a transitive model of ZFC which is contained inside every transitive model of ZFC. An analogous question can be asked of other set theories. I will consider second-order set theories, those which have both sets and classes as their objects. It was known to Shepherdson that von Neumann–Bernays–Gödel set theory NBG has a smallest transitive model. I will show that this phenomenon fails for stronger second-order set theories: there is no least transitive model of Kelley–Morse set theory KM. Indeed, there is no least transitive model

https://doi.org/10.1017/bsl.2018.60 Published online by Cambridge University Press

of $\mathsf{NBG} + \Pi^1_1$-Comprehension, nor any computably enumerable extension thereof. On the other hand, fragments of NBG + Elementary Transfinite Recursion, which sit between NBG and Π^1_1-Comprehension in consistency strength, do have a least transitive model.

Abstracts of talks presented by title

▸ S. S. BAIZHANOV AND B. SH. KULPESHOV, *Expansions of models of weakly o-minimal theories by equivalence relations*.
Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan.
E-mail: sayan-5225@mail.ru.
International Information Technology University, Almaty, Kazakhstan; Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan.
E-mail: b.kulpeshov@iitu.kz.

The present talk is concerned with the notion of *weak o-minimality* originally deeply studied in [2]. A subset A of a linearly ordered structure M is *convex* if for any $a, b \in A$ and $c \in M$ whenever $a < c < b$ we have $c \in A$. A *weakly o-minimal structure* is a linearly ordered structure $M = \langle M, =, <, \dots \rangle$ such that any definable (with parameters) subset of the structure M is a finite union of convex sets in M.

It was proved in [1] that an expansion of a model a weakly o-minimal theory by a unary predicate distinguishing finitely many convex sets is weakly o-minimal. Here we study the question of preserving properties at expanding models of weakly o-minimal theories by an equivalence relation.

Consider the following example:

Example 1. Let $M := \langle \mathbb{Q}, < \rangle$ be a linearly ordered structure on the set of rational numbers $\mathbb{Q}$. Obviously, M is a countably categorical o-minimal structure. Expand the model M by a new binary relation $E(x, y)$ as follows: let $M' := \langle \mathbb{Q}, <, E^2 \rangle$ so that for any $a, b \in \mathbb{Q}$

$$E(a, b) \Leftrightarrow (2n-1)\sqrt{2} < a, b < (2n+1)\sqrt{2}$$

for some $n \in \mathbb{Z}$.

It is not difficult to understand that $E(x, y)$ is an equivalence relation partitioning $\mathbb{Q}$ into infinitely many infinite convex classes so that E-classes are ordered by $\omega^* + \omega$.

It can be proved that M' is a weakly o-minimal structure, but $Th(M')$ is not countably categorical.

Here we discuss necessary and sufficient conditions when an expansion of an 1-indiscernible countably categorical weakly o-minimal theory of finite convexity rank by an equivalence relation with infinitely many infinite convex classes is both weakly o-minimal and countably categorical.

[1] B. S. Baizhanov, *Expansion of a model of a weakly o-minimal theory by a family of unary predicates*. ***The Journal of Symbolic Logic***, vol. 66 (2001), no. 3, pp. 1382–1414.

[2] H. D. Macpherson, D. Marker, and C. Steinhorn, *Weakly o-minimal structures and real closed fields*. ***Transactions of the American Mathematical Society***, vol. 352 (2000), pp. 5435–5483.

▸ JOHN CORCORAN, *Variable-enhanced English and structural ambiguity*.
Philosophy, University at Buffalo, Buffalo, NY 14260-4150, USA.
E-mail: corcoran@buffalo.edu.

An English sentence that can be used to express propositions having different logical structures, or forms, is *structurally ambiguous* [1]. An example is (1).

(1) Every integer precedes some integer.

One use of *variable-enhanced English* [2, Section 4] is to exhibit the different logical structures. In one context, (1) expresses a true universal proposition.

(1-universal) Every integer x is such that for some integer y, x precedes y.

https://doi.org/10.1017/bsl.2018.60 Published online by Cambridge University Press

Every integer precedes its successor. In another context, (1) expresses a false existential proposition.

(1-existential) Every integer x is such that x precedes y, for some integer y.

This is an existential since the existential quantifier's scope is the clause it follows. The proposition expressed is "for some integer y, every integer x is such that x precedes y". Of course, there is no such integer: for no integer y, is every integer x such that x precedes y.

The existential implies the universal but not conversely. Sentence (1) exemplifies *quantifier-scope ambiguity*.

Even more familiar is the structural ambiguity called *negation-scope ambiguity* where one sentence can express either a negation or a universal: either the negation of a universal with an affirmative predicate or a universal with a negative predicate. In English sentences such as (2) 'not' can have broad scope—yielding a negation—or narrow scope—yielding a universal.

(2) Every integer does not precede some integer.

Negation-scope ambiguities can often be separated without using variables: In broad scope (2-not) 'not every integer precedes some integer', in narrow scope (2-every) 'every integer is one that doesn't precede some integer'. These two sentences, though no longer having negation-scope ambiguity, have quantifier-scope ambiguity resolvable as above using variable-enhanced English.

This article delves further into the syntax, semantics, and heuristics of variable-enhanced English and into its pedagogical and hermeneutic uses.

[1] J. Corcoran, *Ambiguity: Lexical and structural*, this Bulletin, vol. 15 (2009), p. 235.
[2] ———, *Logic teaching in the 21st century*. ***Quadripartita Ratio: Revista de Argumentación y Retórica***, vol. 1 (2016), pp. 1–34.

► JOHN CORCORAN, PIERRE JORAY, AND KEVIN TRACY, *Deduction, reduction, and eduction: Three proof-theoretic processes in prior analytics*.
University at Buffalo, Buffalo, NY 14260-4150, USA.
E-mail: corcoran@buffalo.edu.
Université de Rennes, 135042 Rennes, France.
Christendom College, Front Royal, VA 22630-6534, USA.

Mill, De Morgan, Łukasiewicz, and many others discuss Aristotle's *reductions* [1] [2]; references [3] and [4] treat Aristotle's *deductions*. Even without explicit contrast, the distinction is clear in Aristotle's work.

Deductions are *sentence*-sequences constructed using evident rules showing the premise-set implies the conclusion [4]. Aristotle's took seven rules: three one-premise "conversions" and four two-premise "perfect-syllogism" rules—four "moods" among 256, only 24 of which are consequence-preserving or "valid" [4, p. 83]. Deductions have epistemic force: Every deduction produces knowledge that its conclusion is a consequence of its premise-set.

Reductions are *argument*-sequences: each successive argument derives from its predecessor by "weakening" the premise-set, by "strengthening" the conclusion, or by "contraposition" [2]. The initial argument, chosen from the 256 two-premise moods or even from multipremise moods, is "reduced to" the final argument.

Some valid arguments reduce to invalid [2]. However, as Aristotle noticed, invalid arguments reduce only to invalid: reductions ending with valid arguments started with valid arguments, thereby providing marks of validity. Nevertheless, reduction isn't reasoning. In fact, reduction is invalidity-preserving.

Reduction is a computational process for studying relationships among arguments (29b26). What reductions "show", if anything, is an open question not addressed by Aristotle.

Besides deductions, Aristotle also considered sentence-sequences we call *eductions*. These are similar to deductions: deductions require "perfect-syllogism" two-premise rules, but eductions use arbitrary two-premise rules among the 256 moods. The eductions Aristotle constructed use as rules some of the 20 nonperfect two-premise valid moods (29b6–16, 62b38ff, 63a14–18).

https://doi.org/10.1017/bsl.2018.60 Published online by Cambridge University Press

[1] M. Cohen and E. Nagel, ***Introduction to Logic***, Hackett, 1993.
[2] J. Corcoran, *Deduction and reduction: Two proof-theoretic processes in prior analytics I*. ***The Journal of Symbolic Logic***, vol. 48 (1983), p. 906.
[3] ———, *Completeness of an ancient logic*. ***The Journal of Symbolic Logic***, vol. 37 (1972), pp. 696–702.
[4] ———, *Aristotle's demonstrative logic*. ***History and Philosophy of Logic***, vol. 30 (2009), pp. 1–20.

▸ JOHN CORCORAN AND JUSTIN LEGAULT, *Reformalizing Euclid's first axiom: Euclitivity*.
Philosophy, University at Buffalo, Buffalo, NY 14260-4150, USA.
E-mail: corcoran@buffalo.edu.

Euclid's ***Elements*** divides its ten nonlogical premises into two groups of five. No logical axioms or rules—governing identity or other logical notions—are stated or even mentioned.

The first five (*postulates*)—applying in geometry but nowhere else—are *specifically* geometrical. The first: "to draw a line from any point to any point"; the last: the parallel postulate.

The second five (*axioms*) apply in geometry *and* elsewhere. They are nonlogical principles governing *magnitude types* (e.g., angles, lines, surfaces, and volumes) [1]. The first axiom is:

Ta toi autoi isa kai allelois estin isa.
Things that equal the same thing equal one another.

One first-order translation in variable-enhanced English (1) was reported in [2].

(1) Given two things x, y, *if* for something z, x and y equal z, *then* x equals y.

Translation (1) is closer to Euclid's meaning than (2), which resembles but doesn't imply transitivity, as reported in [2].

(2) Given any things x, y, z *if* x equals z and y equals z, *then* x equals y.

We recognize its Euclidean origin—and its resemblance to and distinctness from transitivity—by calling it *Euclitivity*, specifically for reasons given below, *simple Euclitivity*. The idea of (2) is used widely in the literature [3, p. 121].

Translations (1) and (2) both overlook Euclid's reflexive construction 'equal one another'. Translation (3), *conjunctive Euclitivity*, avoids that objection.

(3) Given any things x, y, z *if* x equals z and y equals z, *then* x equals y and y equals x.

Translations (1), (2), and (3) are logically equivalent. Thus by [2] none of them imply reflexivity, symmetry, or transitivity.

We treat several other translations and formalizations.

[1] J. Corcoran and D. Novotný, *Formalizing Euclid's first axiom*, this Bulletin, vol. 20 (2014), pp. 404–405.
[2] J. Corcoran and J. M. Sagüillo, *Euclid's weak first axiom*, this Bulletin, vol. 20 (2014), p. 405.
[3] A. Tarski, ***Introduction to Logic***, Dover, 1995.

▸ JOACHIM MUELLER-THEYS, *Soundness of the provability predicate*.
Kurpfalzstr. 53, 69 226 Nußloch, Germany.
E-mail: mueller-theys@gmx.de.

Let $L^\Box$ be the extension of any first-order language L by formulæ $\Box\alpha$ in the way known from modal logic, Σ be any set of L-sentences, and $\mathcal{M}, \mathcal{M}'$ be L-models. The satisfaction relation $\mathcal{M}, V \models\!\!\models_\Sigma \alpha$ for the *Metalogical Extension* is defined: for all $\mathcal{M}' \models \Sigma$ and all V': $\mathcal{M}', V' \models\!\!\models_\Sigma \beta$, if $\alpha = \Box\beta$; and like $\mathcal{M}, V \models \phi$, otherwise.

The theory of $\mathcal{M}$ extended like this integrates and represents the theory of Σ: $\mathcal{M} \models\!\!\models_\Sigma \Box\sigma$ *iff* $\Sigma \models \sigma$ *iff* $\Sigma \vdash \sigma$ for all L-sentences σ, and the former is able to express the consistency of the latter: $\mathcal{M} \models\!\!\models_\Sigma \nabla$ *iff* Σ *is consistent*, where $\nabla := \neg\Box\bot$. Both is also true if $\Sigma := \mathrm{Th}(\mathcal{M})$.

One can try to encode the extended theory, which sets a standard for adequate formal incorporation. Accordingly, $\imath(x)$ is called *sound* if $\Sigma \vdash \mathrm{T}_\imath(\alpha)$ implies $\mathcal{M} \models\!\!\models_\Sigma \alpha$, and *complete*

https://doi.org/10.1017/bsl.2018.60 Published online by Cambridge University Press

if the converse is true too. Thereby, for the sake of simplicity, the scope of α be limited to $\sigma, \Box\sigma$, $\beta \rightarrow \gamma$, and T_ι rewrite α replacing all $\Box\sigma$ by $\langle\sigma\rangle$.

Now consider arithmetics: $\mathcal{N} \models \mathrm{Prov}_\Sigma(\ulcorner\sigma\urcorner)$ iff $\Sigma \vdash \sigma$. It can be shown that $\mathcal{N} \Vdash_\Sigma \mathrm{Prov}_\Sigma(\ulcorner\sigma\urcorner) \leftrightarrow \Box\sigma$, whence $\mathcal{N} \Vdash_\Sigma \mathrm{T}_{\mathrm{Prov}_\Sigma}(\alpha) \leftrightarrow \alpha$, whereby $\mathcal{N} \models \mathrm{T}_{\mathrm{Prov}_\Sigma}(\alpha)$ iff $\mathcal{N} \Vdash_\Sigma \mathrm{T}_{\mathrm{Prov}_\Sigma}(\alpha)$ iff $\mathcal{N} \Vdash_\Sigma \alpha$.

If $\mathcal{N} \models \Sigma$ and $\Sigma \vdash \mathrm{T}_{\mathrm{Prov}_\Sigma}(\alpha)$, $\mathcal{N} \models \mathrm{T}_{\mathrm{Prov}_\Sigma}(\alpha)$, whence $\mathcal{N} \Vdash_\Sigma \alpha$. Thus, $\mathrm{Prov}_\Sigma(x)$ *is sound*. However, Prov_Σ *is not complete*: this would entail that $\mathcal{N} \models \sigma$ implies $\Sigma \vdash \sigma$.

► TRAVIS NELL, *Distal and nondistal behavior in pairs*.
Department of Mathematics, University of Illinois at Urbana-Champaign, 1409 W. Green Street, Urbana, IL 61801, USA.
E-mail: tnell2@illinois.edu.

After proving in [1] that certain expansions of the real ordered additive group were nondistal in the sense of Simon from [2], we were asked whether one could determine the distal types from the nondistal types in terms of a family of generically stable types. We answer this in the positive in the case where the expansion is of the form $(R; +, <, \dots, Q)$, where the interpretation of Q is a dense $(+, <, \dots)$-elementary substructure.

[1] P. Hieronymi and T. Nell, *Distal and non-distal pairs*. ***The Journal of Symbolic Logic***, vol. 82 (2017), no. 1, pp. 375–383.

[2] P. Simon, *Distal and non-distal NIP theories*. ***Annals of Pure and Applied Logic***, vol. 164 (2013), no. 3, pp. 294–318.

https://doi.org/10.1017/bsl.2018.60 Published online by Cambridge University Press