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RATIONALITY, COMPARABILITY AND MAXIMIZATION

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INTRODUCTION

James Griffin (1986, 1997, 2000) and Ruth Chang (1997) have argued that alternatives (and values) can be comparable when it is neither true that one is better than the other, nor true that they are exactly equal in value. The relation which holds between them has gone under various names: the alternatives are 'roughly equal in value' (Griffin) or 'on a par' (Chang). In this paper, I give a formal analysis of this relation. This analysis allows us to distinguish between two slightly different notions of 'at least as good as'. It is argued that the distinction between these notions is important for discussions of rationality, as is the distinction between 'rough equality' or 'parity'and incomparability.

The paper has four sections. I motivate the discussion with examples, and define various relations in Section 1. In Section 2, I suggest that cases of parity do not necessarily promote the case Amartya Sen (1994, 1995, 1997 and 2000) makes for thinking of rationality in terms of 'maximization' – which involves choosing an alternative which is no worse than any other in the set from which choice is made – rather than 'optimization' – which involves selecting an alternative which is 'at least as good as' any other alternative in the relevant set. In Section 3, I focus on John Broome's recent discussion of incommensurable values and practical reason (Broome, 2000). Section 4 concludes.

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1. PARITY AND COMPARABILITY: MOTIVATION AND SOME DEFINITIONS

James Griffin and Ruth Chang have both argued that sometimes we are faced with alternatives for choice which, while 'comparable', are neither better than, nor worse than, nor exactly equal in value to, each other. Griffin uses the term 'roughly equal in value to' to refer to the evaluative relation which holds in such cases. Chang refers to it as 'on a par with'. While I shall develop some of Griffin's views in what follows, the use of the term 'roughly equal in value' has lead to confusion in this literature. When 'rough equality' is understood as a distinct relation, it cannot hold between items when one is better than the other. However, in ordinary language our use of 'rough equality' allows for the possibility that options are roughly equal in value while one is better than the other. There are also some who think that the very concept of equality involves precision. If it does, the idea of 'rough equality' is flawed. Chang's term 'on a par with' is perhaps less problematic, and so I shall use it to refer to the evaluative relation with which Griffin is concerned.[1]

This relation remains the subject of some controversy, and I shall try to motivate the use of it through the analysis of examples. I shall delay formal definitions for the moment, but will use P for 'on a par with' and B for 'better than'. The examples which are invoked (see, e.g., Griffin, 1986, pp. 80–1) when P is used often involve alternatives which are qualitatively different. In his discussion Griffin (1986, p. 80) uses an example involving two novelists with very diverse achievements. They bring us different values – insight and amusement. When these values come close, he thinks that the values are hard to rank. He writes: '[w]e might wonder whether it is because it is hard to discriminate the differences in value that are really there, or that there are no fine differences really there to discriminate. I think that it is not at all easy to determine which of these alternatives is true' (Griffin, 1986, p. 80). Griffin (1986, pp. 80–1) goes on to say: 'some values are only roughly equal, and the roughness is not in our understanding but ineradicably in the values themselves'. In what follows I shall not focus on the first alternative – where the roughness is in our understanding and where we have difficulty discriminating fine differences – but on the second

[1] It is worth noting that while the use of 'parity' avoids the second potential difficulty associated with 'rough equality' (i.e., that precision is part of the concept of equality), it does not avoid the first problem. Options can be 'on a par', in the sense of 'in the same league' or 'in the same class of goodness' while one is better than the other. So, as will become clear, I use 'on a par with' and 'parity' as terms of art, just as Griffin uses 'rough equality' as a term of art. It is worth noting that Griffin himself sometimes uses 'rough equality' so that it is compatible with 'exact equality'. On this see Qizilbash (2000, pp. 233–4).

alternative where 'the roughness is . . . in [the relation between] the values themselves'. It is this case which implies that there may be a little understood relation between values, which, Griffin thinks, is problematic for well-being rankings. My account of this relation is related to Chang's (1997, pp. 23–6) account of 'parity', but I will, for the most part, follow Griffin's intuition.

Here are two cases of parity. Both involve a set of three alternatives, [x, x+, y], with the alternatives interpreted in different ways according to the specific case. First suppose that x is an excellent French meal, and x+ a very slightly better excellent French meal, while y is an excellent Italian meal, and suppose that any excellent French meal is neither better, nor worse, than any excellent Italian meal. Are they exactly as good? The relation 'exactly as good as' is, I suggest, an *equivalence relation*. Like any equivalence relation, it is transitive: that is, for any alternatives a, b and c, if a and b are exactly as good, and b and c are exactly as good, then a and c are exactly as good. However, the relation 'exactly as good as' does not hold between any excellent French meal and any excellent Italian meal. To see this, suppose that it did. If it did, then we would have: x is exactly as good as y and y is exactly as good as x+. By transitivity of 'exactly as good as' it then follows that x and x+ are exactly as good. However, we know that x+Bx, so that x and x+ are not exactly as good. We thus have a contradiction. This sort of case is not rare either. Consider another example with the same structure: x is a distinguished career in the law, x+ a very slightly better (distinguished) career in the law, and y is a distinguished career as a painter. We are drawn to the idea that distinguished careers in the two professions are 'just as good as' each other, though we cannot say, without contradiction, that they are exactly as good. If x and x+ are neither better than, nor worse than, nor exactly as good as y, are x and x+ incomparable with y? Some (like Joseph Raz, 1986, pp. 330–31) think so. However, there is a case for saying that x and x+ are nonetheless both 'just as good as' y, even if they are not exactly as good as y. It is this sort of case which fits best with the second alternative in Griffin's discussion of 'rough equality': there is 'equality of value' of a sort, but the options are not 'exactly as good'. Why did Griffin use the term 'rough equality' for this case? I suspect that it was because when options are on a par tiny changes in value do not tilt the balance in favour of one of the alternatives, as they do in cases of exact equality. However, some larger, or 'significant' change in the value of options would tilt the balance. A choice between rough equals is balanced, but not in the fine way that a choice between exact equals is. Griffin (1986, p. 81) reserved the term 'incomparability' for cases when, while one alternative or value is not better than the other, neither exact nor rough equality holds between them. If there are cases of parity, it looks as if we have two sorts of case where alternatives are *just as good*: in one sort of case they are exactly as good, and in the other they are on a par. In fact, we can define 'just as good as' to mean 'either exactly as good as, or on a par with'.[2] My use of 'just as good' is equivalent to Griffin's use of 'equal in value', since, for Griffin, options which are equal in value can be either *exactly (or 'strictly' in his terms) or roughly equal*.[3]

The first thing to note about P, which distinguishes it from 'exactly as good as', is that it is not transitive. In the above examples, xPy and yPx+ and x+Bx, which implies that P is not transitive. If alternatives which are on a par are just as good, this suggests that 'just as good as' is not transitive. It is worth distinguishing this violation of transitivity from another case where some think that 'just as good as' is not transitive. It is often discussed in the literature on the non-transitivity of the indifference relation (Armstrong, 1939 and Fishburn, 1988, *inter alia*) and it relates to the limits of our powers of discrimination. Suppose that excellent French meal x might improve with more salt. Adding a tiny amount of salt produces a better excellent French meal, x+. Adding a tiny amount more still would again produce a better excellent meal, x++. Now suppose my palate does not pick up a tiny change, but can pick up two tiny changes. Then I might express the following judgements: 'x and x+ are just as good'; 'x+ and x++ are just as good'; and 'x++ is better than x'. Here it can be argued that 'just as good as' is not transitive. However, this non-transitivity arises from an inability to discriminate fine differences: the first alternative Griffin mentioned. It is distinct from the alternative which implies a novel relation. Someone with perfect powers of discrimination would see that x++Bx+, x+Bx and x++Bx, which is compatible with transitivity (of B).

Having motivated the case for P, I need to show that it can be defined and contrasted with 'exactly as good as'. I shall use 'G' to mean *'better than or exactly as good as'*. I shall call this relation 'narrowly at least as good as', and treat it as a primitive relation in terms of which other relations are defined. I shall also use the following symbols in the rest of the paper: '&' for conjunction, '$\sim$' for negation, '$\vee$' for the inclusive 'or', '$\in$', for 'is a member of', '$\Rightarrow$' for the conditional 'if . . . then', '$\subseteq$' for set inclusion, 'is included in', '$\exists$' for the existential quantifier ('for some'), '$\forall$' for the universal quantifier ('for all'), $\Leftrightarrow$ for 'if and only if', $\neq$ 'is not identical to'. It is also worth noting some properties which relations may or may not have. Let X be the set of 'alternatives' (or, equivalently, 'options'). Alternatives (or options) are conceivable objects of choice. Choice is made from a set of options S, where $S \subseteq X$.[4] Any binary

[2] My use of 'just as good' here is specialized. Sometimes people use 'just as good' to mean something like 'at least as good'.

[3] For a fuller discussion of Griffin's use(s) of 'rough equality' see Qizilbash (2000).

[4] I am, thus, allowing for hypothetical choices, which involve options that might not be 'available' or 'feasible', but only conceivable.

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relation, O, is a weak ordering if it has the following properties: *reflexivity*: $\forall x \in X$, xOx; *transitivity*: $\forall x, y, z \in X$, (xOy & yOz) $\Rightarrow$ xOz; and *completeness*: $\forall x, y \in X, x \neq y$, xOy $\vee$ yOx. Some binary relations also have the property of *symmetry*: $\forall x, y \in X$, xOy $\Rightarrow$ yOx. Equivalence relations have the properties of symmetry, reflexivity and transitivity.

B and 'exactly as good as', which I will write as E, can be defined in terms of G as follows:

Definition 1: xBy $\Leftrightarrow$ [xGy & $\sim$(yGx)]

Definition 2: xEy $\Leftrightarrow$ (xGy & yGx)

G is, I assume, reflexive, transitive but not complete. In the remainder of the paper, I assume that the following postulate holds:

Postulate 1: G is reflexive and transitive.

The examples of parity mentioned above suggest that G is not complete. In those examples, when two options are on a par, one option is not better than the other and they are not exactly as good. The options are, on some views, 'incommensurate with' each other. Since I think John Broome uses the term 'incommensurate with' in this sense, I shall say that such options are 'B-incommensurate', to contrast this sense with an alternative sense of 'incommensurable' which Griffin uses. I shall discuss Broome's account of incommensurability in Section 3. We can then define the following relation:

Definition 3: x is B-incommensurate with y $\Leftrightarrow$ $\sim$(xGy $\vee$ yGx).

If x and y are not B-incommensurate they are 'B-commensurate'. So x and y are B-commensurate means xGy $\vee$ yGx. Alternatives can be B-incommensurate without being 'incomparable' in Griffin's sense. If x is on a par with y, x and y are B-incommensurate, but they are comparable, on Griffin's view. So, to define parity, we need another primitive relation. I shall use 'comparable with' as a primitive relation.[5] It is the sense of 'comparable with' that Griffin uses. As we saw, what Griffin means by 'comparable with' is '*either better than, worse than, exactly as good as, or on a par with*'. I shall use 'C' for this relation. It is reflexive and symmetric but not transitive. In the remainder of the paper, I assume that the following postulate holds:

Postulate 2: C is reflexive and symmetric.

[5] I am very grateful to Wlodek Rabinowicz for suggesting this simple way of defining some of the relations which I use in this paper.

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We can now define P as follows:

Definition 4: xPy ⇔ xCy & ∼(xGy ∨ yGx).

Options which are on a par are comparable, but B-incommensurate. To say that x and y are comparable means that they are either B-commensurate or on a par. Griffin (1997, p. 35) associates genuine 'incommensurability' with cases which are not comparable, in this sense. He refers to these as cases of 'incomparability'. I shall use the relation 'incomparable with' in Griffin's sense and define it as follows:

Definition 5: x is incomparable with y ⇔ ∼(xCy).

While Griffin's notion of 'incomparability' is distinct from the sense of incommensurability which I have attributed to Broome, Griffin sometimes takes Broome to be discussing 'incomparability' when he is not (Griffin, 2000, p. 285). Clearly, if options are incomparable, they are B-incommensurate. Options can, nonetheless, be B-incommensurate without being incomparable. It is in this latter case that they are on a par. We can now define two further relations. We can define a sense of 'at least as good as' which is distinct from G. I shall refer to this relation as 'broadly at least as good as'or 'A', and write 'just as good as' as 'J'.

Definition 6: xAy ⇔ xCy & ∼(yBx).

Definition 7: xJy ⇔ xEy ∨ xPy

In words: x is broadly at least as good as y if and only if x and y are comparable and x is no worse than y (i.e., y is not better than x),[6] and x and y are just as good if and only if x and y are either exactly as good or on a par. I shall not focus here on establishing the connections which hold between all these relations, given the above definitions and postulates.[7] I shall focus instead on some differences between G and A.

One key difference between G and A is that it can be true that xAy while x and y are B-incommensurate, since when xAy, x and y may be on a par. Clearly (from definition 3) this is not true when xGy. Furthermore, from definition 6 we know that xAy&yAx means that while neither x nor y is better than the other, they are comparable. Given Griffin's use of 'comparable with', this means that x and y must be either exactly as good, or on a par. Then (from definition 7) xAy & yAx is equivalent to

[6] In earlier work, I used 'no worse than' to refer to A, though 'x is no worse than y' is usually used to mean 'y is not better than x' as it is used here. See Qizilbash (2000, p. 236).

[7] Indeed, pursuing that task may require further postulates, which are more explicit than I have been in the text.

xJy (though not necessarily xEy). This is an important way in which A and G differ, since xGy & yGx means xEy.[8] Finally, it is clearly true that if A is complete, all alternatives are comparable, since $\forall$x, y $\in$X, x $\neq$y, xAy $\vee$ yAx implies that $\forall$x, y $\in$X, x $\neq$y, xCy $\vee$ yCx. From postulate 2, C is symmetric and completeness of C simply requires that $\forall$x, y $\in$X, x $\neq$y, xCy. Nonetheless, completeness of C does not imply completeness of G. Completeness of G is the property that $\forall$x, y $\in$X, x $\neq$y, xGy $\vee$ yGx: it is the property that all non-identical options in X are B-commensurate, while completeness of A requires that such options are comparable.

One motivation for distinguishing between P and E depended on the supposition that E is an equivalence relation. This is easily checked.

Theorem 1: E is an equivalence relation.

Proof: To be an equivalence relation E must be symmetric, transitive and reflexive. Is E reflexive? Suppose it is not. Then $\exists$x $\in$X: $\sim$(xEx). In which case, from definition 2, we have $\sim$(xGx). But from postulate 1, G is reflexive and thus xGx. So we have a contradiction: E is reflexive. Is E symmetric? Suppose not. Then $\exists$x, y $\in$X: xEy &$\sim$(yEx). But then, from definition 2, we have xGy & yGx & $\sim$(yGx & xGy), which yields (yGx & xGy) & $\sim$(yGx & xGy) which is a contradiction. So E is symmetric. Is E transitive? Suppose not. Then $\exists$x,y,z $\in$X: xEy & yEz & $\sim$(xEz). From definition 2, we then have: (xGy & yGx) & (yGz & zGy) & $\sim$(xGz &zGx).$\sim$(xGz & zGx) implies $\sim$(xGz)$\vee$ $\sim$(zGx). If $\sim$(xGz) then we have xGy & yGz & $\sim$(xGz), which violates transitivity of G. If $\sim$(zGx), then we have zGy & yGx & $\sim$(zGx), which also violates transitivity of G. But from postulate 1, G is transitive. So we have a contradiction. E is, thus, transitive, and is an equivalence relation.■

We can now move on to some recent discussions of rationality, and to potential problems which parity raises for rationality.

2. SEN ON OPTIMIZATION AND MAXIMIZATION

One way of thinking about the problem parity poses for rational choice focuses on G. It involves arguing that when there is parity between options the 'choice set' – which I will define in a moment – is empty. To understand this claim we need to understand what Amartya Sen (1997) calls the 'optimization' view of rationality. 'Optimization', as Sen uses the term, involves choosing an alternative which is 'at least as good as' any other in the relevant set and is, in that sense, a 'best element' of the set. A 'best element' must be defined with respect to a binary relation and a set of alternatives. Since Sen does not distinguish between G and

[8] In spite of these differences between G and A, it is easily checked that xBy $\Leftrightarrow$ [xGy & $\sim$(yGx)] $\Leftrightarrow$ [xAy & $\sim$(yAx)].

A, I shall start by assuming that it is G that is involved, and that choice is made from some set $S \subseteq X$. The set of best elements or 'choice set' can then be written as T(S,G) and Sen's definition (Sen, 1979, p. 10) of a 'best element' with respect to the binary relation 'at least as good as' for a nonempty S is equivalent to:

Definition 8: $x \in T(S,G) \Leftrightarrow \forall y: (y \in S \Rightarrow xGy)$.

Someone who optimizes, when faced with a set of alternatives, chooses any alternative which is best. However, optimization always takes place with respect to some relation. I use the term 'G-optimization' for optimization with respect to G. Parity does cause difficulties for someone who optimizes with respect to G. Consider the set [x,y]. When xPy, there is no best element with respect to G and $T([x,y], G) = \emptyset$. By contrast, if it were the case that xEy then both x and y would be best elements with respect to G, and $T([x, y], G) = [x,y]$. However, this difference between P and E does not *seem* to have much basis in rationality. Surely when two options are just as good, and we can only choose one at some point in time,[9] we can rationally choose either, irrespective of whether they are exactly as good or on a par.[10] When options are just as good, the choice between them is balanced, though the balance need not be fine. It is the fact of, rather than the nature of, the balance which matters for rationality. The emptiness of the choice set when xPy follows from the nature of G, rather than some underlying notion of rationality. This looks like a weakness in the view of rationality which is based on G-optimization.

One alternative to G-optimization is 'maximization'. A 'maximizer' is someone who chooses a 'maximal element' with respect to G from any $S \subseteq X$ which she is confronted with.[11] A maximal element with respect to G in the set S is a member of the maximal set, M(S,G):

Definition 9: $x \in M(S,G) \Leftrightarrow \sim[\exists y: (y \in S \ \& \ yBx)]$.

Clearly maximization and G-optimization can diverge. If for a set [x,y], xPy, then both x and y are maximal elements, and $M([x,y], G) = [x,y]$. The same would be true if xEy. From the maximizer's point of view, there is

[9] Issues raised by non-transitivity are discussed in relation to Broome's work in Section 3, but they relate to the impact of choices over time, rather than at a point in time.

[10] This is actually one way of interpreting Griffin's thought that, as regards choice, we can treat rough equality 'as if' it is strict equality (see, Griffin, 1986, p.97 and Griffin, 2000, p. 287).

[11] Since X is the set of conceivable options, and there is (I think) always an alternative one can conceive of which is a little better than any other, X is not finite, and $T(X,G) = M(X,G) = \emptyset$.

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no significant difference between E and P, and this seems to capture the intuition that there is no significant difference between choices where all relevant options are on a par and choices where all the options are exactly as good: in both cases options are just as good and we can rationally choose any one of them.[12] To this degree maximization constitutes a more plausible account of rational choice than G-optimization. However, for the maximizer there is also no significant difference between exact equality of goodness and parity on the one hand *and* incomparability on the other. If ∼(xCy) then x and y are nonetheless maximal elements of the set [x,y]. We may want to distinguish this case, where options are incomparable, from cases where there is a balance involved in the choice between options. If so, we need to draw a line between those options which are on a par and those which are incomparable (or those B-incommensurate options which are and those which are not on a par). I return to this issue in Section 3. Before moving on, it is worth noting some well-known results from social choice theory. If G is reflexive and complete, M(S,G) = T(S,G), and, in general, for a reflexive G, T(S,G) ⊆ M(S,G) (see, Sen, 1979, pp. 9–12). The divergence between M(S,G) and T(S,G) occurs when G is incomplete. Incompleteness arises when there are cases of B-incommensurateness, and since parity is an example of B-incommensurateness, it causes a divergence between M(S,G) and T(S,G).

Sen (1994, 1995, 1997 and 2000) has recently argued that incompleteness may not imply a breakdown of rationality.[13] Sen argues in favour of thinking about rational choice in terms of maximization rather than 'optimization'. We know that neither parity nor incomparability leads to an empty maximal set, so the problems associated with G-optimization do not arise for maximization. Sen has suggested that, because of such problems, the 'reach' of maximization – which involves choosing maximal elements – is greater than that of 'optimization'.

Does Sen have G rather than A in mind when he makes his case for maximization? As I said earlier, he does not distinguish them. If he is using A, we need to define a best and maximal element, choice set and maximal set with respect to A rather than G. Replacing G with A in definitions 8 and 9, we have the following definitions of (respectively) a best and a maximal element with respect to A:

Definition 10: $x \in T(S,A) \Leftrightarrow \forall y: (y \in S \Rightarrow xAy)$.

Definition 11: $x \in M(S,A) \Leftrightarrow \sim[\exists y: (y \in S \;\&\; yBx)]$.

[12] A difference may emerge in a series of choices over time, but at present I am concerned with choices at one point in time.

[13] Sen's argument echoes Herzberger (1973).

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It is worth noting that a best element with respect to A is not necessarily a best element with respect to G. I shall call optimization with respect to A, 'A-optimization'. A-optimization unlike G-optimization treats parity and exact equality equivalently when it comes to choice: when two options are just as good and better than all others in S, the two options are best with respect to A, irrespective of whether they are on a par or exactly as good. This makes A-optimization more attractive than G-optimization as an account of rationality. The well-known results from social choice theory also hold for A: if A is reflexive and complete, T(S,A) = M(S,A), and if it is reflexive but not complete then T(S,A) ⊆ M(S,A). However, we can also show that:

Theorem 2: If A is reflexive and complete, T(S,A) = M(S,G).[14]

Proof: From definitions 9 and 11, a maximal element with respect to G is also maximal with respect to A. So M(S,G) = M(S,A). Furthermore, if A is reflexive and complete, T(S,A) = M(S,A), and so T(S,A) = M(S,G).■

One can express this simple result in terms of definitions 3 and 4 as follows: if the only options which are B-incommensurate are on a par (because A is complete and there are no cases of incomparability) then the 'reaches' of maximization and A-optimization are identical. If the most plausible version of the 'optimization' view involves A-optimization, then the case for maximization rests on the existence of incomparability, rather than the existence of cases which are B-incommensurate, as such. (It rests on the existence of options which are B-incommensurate but not on a par).

The claim that there are no alternatives which are incomparable is highly relevant here. If it were true, then A would be (reflexive and) complete, and the case for maximization would collapse. Griffin (1997, p. 40) has argued that there is not much, if any, incomparability in the prudential realm. He has not, however, made this claim for the realm of practical reason as a whole (which includes the moral realm). So he has not claimed that A is complete (though his views are sometimes interpreted in this way). Nonetheless, his arguments might be interpreted as advancing the claim that the 'optimization' – understood as A-optimization – view of rationality is adequate in the prudential realm, where, on his view, there is little if any incomparability.[15]

Thus far, the problem posed by parity has been conceived of in terms of incompleteness of G. If instead we focus on A, then, even if A is

[14] It is worth noting that if A is reflexive and complete it may be equivalent to what Sen calls the 'the completed extension' of 'at least as good as' (1997, p. 766), so that this theorem can be seen as much the same as Theorem 5.4 in Sen (1997, p. 764).

[15] Griffin (1997, p. 40) has mentioned the possibility of 'opaque choices' in the prudential realm. But these may not be cases of incomparability. See Qizilbash (2000, p. 239).

complete, another issue surfaces. We saw in Section 1 that P is not transitive. In fact, the cases that involve non-transitivity of P also imply that A is non-transitive. In those cases, xAy & yAx+ & ∼(xAx+). Non-transitivity can cause problems and I discuss some of them in the next section, in relation to John Broome's work.

3. BROOME ON INCOMMENSURABLE VALUES, ROUGH EQUALITY AND PRACTICAL REASON

In his paper 'Incommensurable values', John Broome defines relations in a different way to the way I have defined them in Section 1. I shall, for this reason, use his definitions as much as possible in this section, and sometimes relate these to those used in Sections 1 and 2. Broome uses 'better than' as a primitive relation. He also uses a device called the 'standard configuration'. In explaining this device (Broome, 2000, pp. 25–7) he asks us to consider the goodness of different careers (where there is no uncertainty about how these will turn out). We can construct a chain of careers by starting with one career and conceiving of a career which is slightly better than this one and another which is a little worse than it. So careers can be represented on a line which is continuous. We can go through this sort of exercise for any option. The chain of options constructed in this way is represented in Figure 1. In addition to this chain of options, a standard configuration involves a single option which is fixed: 'the standard'. I call this option 'Z'.

Broome defines 'equally as good as' in terms of this configuration. When an alternative is equally as good as Z, the slightest improvement in this option will make it better than Z, and the slightest worsening will make it worse than Z. This situation is described in Figure 2.

Broome also defines 'equally as good as' as follows: two options are equally as good if and only if, while one is not better than the other, any distinct alternative that is better or worse than one is also better or worse than the other (Broome, 2000, p. 26). This relation looks identical to E, as it was defined in Section 1. If x and y are equally as good, a choice between x and y would be very finely balanced (Broome, 2000, p. 26) just as it is when xEy.

Broome contrasts 'equally as good as' with 'incommensurate with'. In Broome's terms (Broome, 2000, p. 26, following Raz, 1986) x and Z are incommensurate if and only if it is neither true that one is better than the

← worse better →

Alternatives

FIGURE 1. The chain of alternatives

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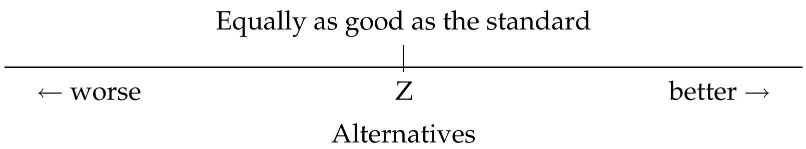

FIGURE 2. Equality of goodness

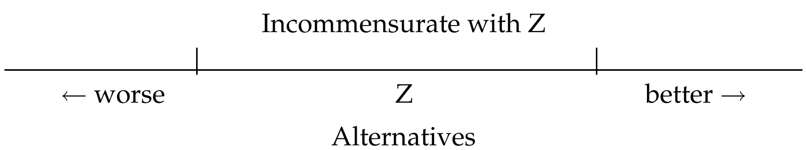

FIGURE 3. The zone of incommensurateness

other, nor true that x and Z are equally as good. If 'equally as good as' is the relation defined as E in Section 1, then Broome's definition of 'incommensurate with' fits with the definition of 'B-incommensurate with' in Section 1. If any alternative were incommensurate with Z, Broome thinks there would be a zone of incommensurate options in the standard configuration. This situation is illustrated in Figure 3.

Why is there a zone of incommensurateness? The options in the zone are neither better nor worse than Z. Are they equally as good as it? If any option in the zone were equally as good as Z, anything a little better than it would be in the better zone, and anything a little worse would be in the worse zone (Broome, 2000, p. 24). In effect, there would be no zone. So, Broome thinks there is no option in the zone which is equally as good as Z.

Broome tries to map Griffin's views onto this configuration. Broome uses the term 'rough equality' in something like its ordinary sense, and he thinks that options which are 'roughly equally as good' as Z would fall in a narrow band around Z in the standard configuration. However, he does not define rough equality or parity. One way one might do it is this. If Broome adds another primitive relation – 'significantly better than' – to his primitive 'better than', and his relation, 'incommensurate with', then one might define parity in his terms.[16] We need also to say that when x is significantly better than y, y is significantly worse than x. Now we can define parity as follows: two options are on a par if and only if they are incommensurate and anything that is significantly better than one is better than the other, and anything that is significantly worse than one is worse than the other. Incomparable options can then be

[16] I am very grateful to John Broome for suggesting this possibility to me.

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defined as options which are incommensurate and not on a par. If one adopted these definitions of parity and incomparability, the width of any zone of options which are on a par in the standard configuration would depend on what made a change in value 'significant'. One attraction of this way of defining parity is, as we shall see, that it helps to motivate the importance of this relation for practical reasoning.

On his reading of Griffin, Broome thinks that Griffin's view is that 'no incommensurate zone can be very wide' (Broome, 2000, p. 29). This is a way of interpreting the claim that all options which are incommensurate are on a par with each other, and that there are no incomparable options – though Broome does note that Griffin only makes this claim for the prudential realm (Broome, 2000, p. 21ff.). It is worth noting that, in terms of the relations defined in Section 1, this is the claim that A and C are complete. Broome goes on to say that:

> There seems to be no reason why it [some incommensurate zone] should not be [very wide]. We would generally expect the zone to be narrow when comparing similar types of things, and wide when we are comparing very different things. Between joining the Northumberland Rifles and the Fifth Lancers, the zone may be narrow, but between the church and the army it may be very wide. A mediocre career in the church may not be definitely worse than a career in the army. But a successful career in the church is much better than a mediocre one. It follows that not all careers in the incommensurate zone are roughly equally as good as a career in the army. (Broome, 2000, p. 29)

Broome concludes his discussion by saying that 'we need from Griffin an argument why this zone should be narrow' (Broome, 2000, p. 30). In responding to Broome, Griffin (2000, p. 287) observes that Broome's view that the zone can be *very* wide is based on Broome's intuition about examples, and indeed his own views are based on intuition about examples. He notes that if 'the zone turns out to be wide, there is after all a threat to practical rationality' (Griffin, 2000, p. 287). Why? Presumably because, if the zone is sufficiently wide, there must be incommensurate alternatives which are not on a par: they must be incomparable. Griffin thinks that such incomparability constitutes a threat to practical rationality, and he clearly does not think that parity constitutes a similar problem. So the distinction between parity and incomparability matters on his view.

Broome, on the other hand, has not made much of the distinction between parity and incomparability. He argues that the distinction which matters for practical reason is that between the two relations he defines: 'equally as good as' and 'incommensurate with'. He does not discuss the difference between those options which are incommensurate and on a par, and those which are incommensurate and incomparable.

Instead, he focuses on a puzzle. The puzzle goes like this. Broome asks us to consider two careers: a career in the army and a good career as a priest. These are incommensurate. So, according to Broome, choosing either would not be wrong, but choice between them must be made without the guidance of reason.[17] Suppose you choose the army and give up the chance of a good career in the church. In doing so you are doing nothing wrong. Then suppose another opportunity comes up to join the church, this time in much worse conditions. You now have a choice between the army and a much less good career as a priest. Again these are incommensurate. Choosing either would not be wrong. This time you choose the church career. The effect of the two choices together is that you end up with a much worse career in the church than you could have had. Surely, Broome tells us, 'rationality should be able to protect you from this sort of bad result; surely there is something irrational in what you have done. Yet apparently neither of your decisions was irrational. This is puzzling' (Broome, 2000, p. 33). Later Broome goes on to say that '[t]he puzzle only arises if, when you make the second choice of a poor church career rather than the army, you do not at the same time repudiate your first choice of the army rather than a good church career' (Broome, 2000, p. 34).[18] He explains that this sort of puzzle arises in cases of incommensurateness but not in cases where options are equally as good, because it requires non-transitivity, and 'incommensurate with' is not transitive whereas 'equally as good as' is, on his definition. Since non-transitivity leads to such unfortunate consequences in apparently reasonable pair wise choices, Broome draws a line in the sand between cases where alternatives are equally as good, and cases where they are incommensurate.

The example Broome discusses involves incommensurate careers, but it does not involve two church careers which are on a par with the army career. To see this, suppose that the good church career and the poor church career were on a par with the army career. Then, by definition, the poor (good) church career could not be significantly worse (better) than the good (poor) church career. But we know that the poor (better) career is much, and thus significantly, worse (better) than the good (poor) career. So at least one of the church careers is incomparable with the army career. It is because the much less good church career is significantly worse than the good church career that the pattern of choices described leaves the chooser particularly badly off.

Parity does, nonetheless, generate the same sort of puzzle, because

[17] In his paper, Broome (2000, pp. 32–3) is assuming teleology: what we ought to do depends entirely on the goodness of alternatives.

[18] Broome (forthcoming) analyses this puzzle further in terms of a sophisticated account of reasons and intentions.

'on a par with' is non-transitive. Consider a variation on Broome's puzzle. Suppose that the agent is faced with a choice between excellent careers. Firstly, there is the choice between a really excellent career in the army (c1), and an excellent career in the church (c2). c1 and c2 are on a par. Given a choice between them, a rational agent can choose either. Suppose that, having chosen c2, the agent is given a choice between c2 and an excellent career in the army (c3) which is somewhat less brilliant than c1, and c2 and c3 are on a par. Given a choice between c2 and c3, she chooses either, and if she chooses c3, she is worse off than if she had chosen c1 in the first place since c1 is better than c3. However, since c1 and c3 are on a par with c2 she cannot be *significantly* worse off.[19] If she were significantly worse off with c3 than with c1, then c1 and c2 would not, by definition, be on a par. The difference between this example and the one which Broome discusses arises because choices between options which are on a par are balanced, and no balance holds in cases of incomparability. That difference between parity and incomparability matters for practical reason.

4. CONCLUSIONS

In this paper, parity has been defined using two primitive relations, G and C. The system of relations which is defined in terms of these primitives helps to illuminate Griffin's account of incomparability. This system is not standard, and involves two notions of 'at least as good as', whereas mainstream welfare economics works with one. The analysis of parity is related to Sen's discussion of maximization, and suggests that A-optimization can allow for options which are on a par, in a way that G-optimization cannot. Finally, the system of relations is related to Broome's system in his discussion of incommensurable values. Parity can be defined in Broome's system, with the use of another primitive relation – 'significantly better than'. The distinction between parity, thus defined, and incomparability matters for practical reason.

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[19] She may be significantly worse off over time if she were presented with several variations of this puzzle in succession. However, that is not the case we are concerned with here.

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