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The dust ion acoustic waves propagation in collisional dusty plasmas with dust charge fluctuations: Effect of ion loss and ionization

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Abstract. In the present research paper, the nonlinear propagation of dust ion acoustic solitary waves in a collisional dusty plasma, which consists of negatively charged small dust grains, positively charged ions and isothermal electrons with background neutral particles, is investigated. The low rates compared to the ion oscillation frequency, of the charge-fluctuation dynamics of the dust grains, the ionization, ion-neutral and dust-neutral collisions (*i.e.* weak dissipations) are considered. Using the reductive perturbation theory, a damped Korteweg-de Vries (DKdV) equation is derived. On the other hand, the dynamics of solitary waves at a critical phase velocity is governed by a damped modified Korteweg-de Vries (DMKdV) equation. The nonlinear properties of dust ion acoustic waves in the presence of weak dissipations in the two cases are discussed.

1. Introduction

Dusty plasmas (DPs) are different from the ordinary plasmas (electrons and ions). The DPs are the mixtures of ordinary plasma, a finite size of charged dust grains and the neutral particles occurring both in the space and the laboratory [1–4]. The ionization, recombination and electron-ion loss due to attachment on the dust grain processes play a significant role to define the stationary state in a DP. Accordingly, the laboratory gas-discharged weakly ionized DP is a strong non-equilibrium plasma medium [5]. The ionization instability was examined for ion-acoustic (IA) in a DP-containing positive ions, electrons, neutral gas and negatively charged dust grains at a sufficiently low pressure. It was found that the presence of negatively charged dust increases both the frequency and the growth rate of the IA wave excited through the ionization instability [6]. In fact, charged dust grains immersed in ambient plasmas are electrically charged due to the current flux of electrons (I_e) and ions (I_i) on the dust grain surfaces. Varma et al. [7] studied the dust charge fluctuation effect for the first time. Furthermore, several authors [8–10] showed that the charge fluctuation of the dust grains plays an important role in the study of collective effects of the DP. The presence of highly negatively charged and massive grains of dust particles in electron and ion plasmas is responsible for the appearance of new types of waves, depending on whether the dust grains are considered to be static or mobile. One type of these waves is the dust ion acoustic waves (DIAWs), which are the usual ion acoustic waves modified by the presence of dust grains. Shukla and Silin [11] predicted the existence of small amplitude DIAWs in an unmagnetized DP. Barkan et al. [12] observed these waves in a laboratory experiment. On the other hand, the nonlinear DIAW characteristics in a DP have been widely investigated [8, 13, 14]. For example, the nonlinear propagation of the DIAWs accounting for the charge-fluctuation dynamics of stationary dust grains in a DP consisting of a cold ion fluid and Boltzmann distribution electrons has been studied by Mamun and Shukla [10]. They demonstrated that the dust grain charge fluctuations reduce the speed of compressive DIAWs. All these investigations considered the propagation of the DIAWs without ionization source model. Ghosh [15, 16] has studied the nonlinear DIAWs propagation characteristics in the presence of ionization, ion-dust and ion-neutral collisions in a DP consisting of positively charged ions, isothermal electrons and immobile fixed [15] and variable [16] negatively charged dust grains. He illustrated that the ionization instability leads to the exponential growth of the DIAW amplitude with time, whereas ion-dust and ion-neutral collisions reduce the growth rate. Also, his analytical solution reveals that the ionization has a destabilizing effect, whereas ion loss and dust-charge variation play a stabilizing role to control the ionization instability. In fact, dust grains participate in the motion in the case of DIAWs. Therefore, in several papers, DIAWs are studied in the presence of the dynamics of charged dust particles. For example, Tiwari and Mishra [17] studied IA-dressed solitons in a collisionless DP having positively/negatively charged dust grains using the reductive perturbation method [18]. They found that the presence of positively charged dust grains in the system supports only compressive solitons. However, the plasma with negatively charged dust grains could support compressive solitons only up to a certain concentration of dust. Above this critical concentration of negative charge, the DP can support rarefactive solitons. A little work is concerned with the effects of ionization, ion-neutral, ion-dust and dust-neutral collisions on DIAWs in the presence of the dynamics of negatively charged dust grains. For example, Moslem and El-Taibany [19] have studied the propagation of nonlinear DIAWs in a DP consisting of warm positive ions, warm negatively charged dust fluid and low temperature-trapped electrons. The ionization and collisions between ions and dust grains are considered. They found that the wave amplitude is exponentially decaying with time and admits only compressive solitons. Recently, Shukla and Eliasson [20] have presented an updated knowledge of fundamentals of collective DP interactions and several novel phenomena that have been observed in laboratories and in space DPs. It should be mentioned here that both shocks and solitons in DPs could be formed by different means. These are not necessarily restricted to the mode excitation due to instabilities, or an external forcing, but can be a regular collective process analogous to the shock wave generation in gas dynamics. The anomalous dissipation in DPs, which originates from the dust particles' charging process, ionization, ion-neutral collisions and dust-neutral collisions, makes possible the existence of a new kind of shocks related to this dissipation [20,21]. If the dissipation is weak at the characteristic dynamical time scales of the system (*i.e.* for the case in hand), the balance between nonlinear and dispersion effects can result in the formation of symmetrical solitary waves – a soliton.

The motive of this article is to study the propagation characteristics of weakly dissipative DIAWs with the effect of the dust charge fluctuation in a collisional DP in the presence of ionization, ion-neutral collisions and dust-neutral collisions taking into account the dynamics of a relatively small fraction of negatively charged small dust grains ($a \approx 0.1\,\mu$m). We focus our attention to the effects of the dynamics of charged dust particles, ion-neutral collisions and critical phase velocity on the nature of DIAWs. Therefore, we assume that the ion-loss frequency, ion-neutral collision frequency and dust-neutral collision frequency are smaller than the ion-plasma frequency ($\omega_{pi} \approx 10^7\,\mathrm{s}^{-1}$)(*i.e.* weak dissipations) [16]. In Sec. 2, we present the equations governing one-dimension dynamics of nonlinear DIAWs. The nonlinear propagation of DIAWs is investigated through the derivation of a damped Korteweg-de Vries (DKdV) equation. In Sec. 3, at the critical phase velocity, we derived a damped modified Korteweg-de Vries (DMKdV) equation using new stretched variables for describing the DIAWs. Numerical results and discussion in these two cases are given in Sec. 4.

2. Basic assumptions, equations and derivation of evolution equation

Before going to the basic equations, we consider the basic assumptions that will help us to formulate the physical problem. Let us consider a DP consisting of negatively variable-charged cold dust fluid, positively charged ion fluid, isothermal electrons and immobile background neutral particles. Also, there are ionizing (fast) electrons, whose density is much smaller than the density of the thermal electrons, which produce new ions due to the ionization of neutral fluid. Therefore, the ion-creation term is given by $\tilde{Q}_i = \sigma(\varphi) n_n \Psi$ [15], where $\sigma(\varphi)$ is an ionization cross section, n_n is the neutral gas density and Ψ is the flux of ionizing electrons. On the other hand, ions are lost from the ion fluid because of the attachment on the grain within the plasma. Hence, the ion-loss term is given by $\tilde{Q}_l = I_i n_d^{(0)}/e$, where I_i/e is the ion current per unit charge. The ions and the dust grains undergo elastic collision with the background neutral particles. According to the above-mentioned assumptions, the system of basic fluid equations, which governs the dynamics of one-dimensional DIAWs in a DP in the existence of the charge-fluctuation dynamics of the dust grains, the ionization and ion-neutral and dust-neutral collisions is given by the following normalized equations [5, 6, 16, 17, 19]

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d V_d) = 0, \tag{1}$$

$$\frac{\partial V_d}{\partial t} + V_d \frac{\partial V_d}{\partial x} + \beta_d (Q-1) \frac{\partial \varphi}{\partial x} + \nu_{dn} V_d = 0, \tag{2}$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i V_i) - Q_i + Q_\ell = 0, \tag{3}$$

$$n_i \left(\frac{\partial V_i}{\partial t} + V_i \frac{\partial V_i}{\partial x} \right) + n_i \frac{\partial \varphi}{\partial x} + \sigma_i \frac{\partial n_i}{\partial x} + \nu_{in} n_i V_i + \mathrm{Q}_i V_i = 0, \tag{4}$$

$$\delta \frac{\partial^2 \varphi}{\partial x^2} - n_e + \delta n_i + (\delta - 1)(Q-1) n_d = 0, \tag{5}$$

$$n_e = exp(\varphi) \tag{6}$$

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and

$$\left(\frac{\partial}{\partial t}+V_d\frac{\partial}{\partial x}\right)Q=\nu(I_e+I_i)/\nu_{ch}z_d^{(0)}e, \tag{7}$$

where the normalized expression for the ion-creation term Q_i, the ion loss term Q_ℓ, the electron current term I_e, the ion current I_i for spherical dust grains with radius a are, respectively, given by

$$Q_i=\frac{\tilde{Q}_i}{\omega_{pi}n_i^{(0)}}=Q_{i0}\left[1+\frac{\Delta\sigma}{\sigma_0}\varphi+\frac{1}{2\sigma_0}\left(\frac{d^2\sigma}{d\varphi^2}\right)\varphi^2+\cdots\right] \tag{8}$$

$$Q_\ell=\frac{I_i n_d^{(0)}}{\omega_{pi}n_i^{(0)}e}=\nu_L n_i\left(1-\frac{zQ}{\sigma_i+z}\right), \tag{9}$$

$$I_e=-e\pi a^2\left(\frac{8T_e}{\pi m_e}\right)^{1/2}n_e^{(0)}\exp\left[\varphi+z(Q-1)\right] \tag{10}$$

and

$$I_i=e\pi a^2\left(\frac{8T_i}{\pi m_i}\right)^{1/2}n_i^{(0)}n_i\left[\left(1+\frac{z}{\sigma_i}\right)-\frac{z}{\sigma_i}Q\right]. \tag{11}$$

In the above equations the variables n_d, n_i and n_e are the dust, ion and electron number densities, $V_i(V_d)$ is the ion (dust grain) flow velocity and φ is the electric potential. x and t are the space coordinate and time variable respectively. ν, ν_L, ν_{in} and ν_{dn} is the dust-charging frequency, the ion-loss frequency, the ion-neutral and dust-neutral collision frequencies, respectively. $Q=(Q_d/z_d^{(0)}e)$ is the normalized dust-charge fluctuating, where Q_d is the fluctuating charge on the dust grains. At equilibrium, *i.e.* at $\varphi=0$, $n_e=n_e^{(0)}$, $n_i=n_i^{(0)}$, $n_d=n_d^{(0)}$ and $q_d=-z_d^{(0)}e$, the charge neutrality condition is given by $n_e^{(0)}+z_d^{(0)}n_d^{(0)}-z_i n_i^{(0)}=0$, where $n_i^{(0)}$, $n_d^{(0)}$ and $n_e^{(0)}$ are the unperturbed number densities of ions, dust grains and electrons respectively, $z_d^{(0)}$ is the equilibrium number of charges residing on the dust grain, and singly charged ions for which $z_i=1$. $\sigma=\sigma(\varphi)$ is the ionization cross section, where σ_0 represents the value of $\sigma(\varphi)$ at $\varphi=0$. Also, at equilibrium, (3) illustrates that $Q_{i0}=Q_{\ell 0}$, therefore $Q_{i0}=\nu_L$. We introduce the following notations:

$$\beta_d=\frac{z_d^{(0)}m_i}{m_d},\sigma_i=\frac{T_i}{T_e},\delta=\frac{n_i^{(0)}}{n_e^{(0)}},\nu=\frac{\nu_{ch}}{\omega_{pi}},\nu_{ch}=\frac{a}{\sqrt{2\pi}}\frac{\omega_{pi}^2}{V_{ti}}[\sigma_i+(1+z)],$$

$$\nu_L=\frac{a}{\sqrt{2\pi}}\frac{\omega_{pi}^2}{V_{ti}}\frac{[\sigma_i+z]}{z\delta}(\delta-1),\ \Delta\sigma=\left(\frac{d\sigma}{d\varphi}\right)_0,z=\frac{z_d^{(0)}e^2}{4\pi\varepsilon_0 a T_e}.$$

Here $T_e(T_i)$ is the electron (ion) temperature in energy units, $m_i(m_d)$ is the ion (dust) mass, ε_0 denotes the free space permittivity, $4\pi\varepsilon_0 a$ is the capacitance of a spherical dust grain and $V_{ti}(=\sqrt{T_i/m_i})$ is the ion thermal velocity. All physical quantities are normalized as follows: n_d, n_i and n_e are normalized by $n_d^{(0)}$, $n_i^{(0)}$ and $n_e^{(0)}$, respectively, V_i and V_d by the ion acoustic speed $\left(T_i/m_i\right)^{1/2}$, φ by (Te/e), x by the ion Debye length $\lambda_{Di}=(\varepsilon_0 T_e/n_i^{(0)}e^2)^{1/2}$, t by the ion plasma period $\omega_{\rm pi}^{-1}=(n_i^{(0)}e^2/\varepsilon_0 m_i)^{-1/2}$, ν, ν_L, ν_{in} and ν_{dn} by ω_{pi} and Q_i and Q_ℓ by $\omega_{pi}n_{i0}$.

In order to derive the nonlinear dynamical equation for DIAWs from (1)–(7), we introduce the stretched space-time coordinates [16, 17]

$$\xi = \varepsilon^{1/2}(x - V_0 t),\ \tau = \varepsilon^{3/2} t, \tag{12}$$

where V_0 is the phase velocity of the linear DIAWs to be determined later, and ε is a smallness parameter measuring the weakness of the nonlinearity. Now we can expand the plasma physical quantities in a power series of ε around their corresponding equilibrium values as follows:

$$\begin{aligned} n_d &= 1 + \varepsilon n_d^{(1)} + \varepsilon^2 n_d^{(2)} + \varepsilon^3 n_d^{(3)} + \cdots, \\ n_i &= 1 + \varepsilon n_i^{(1)} + \varepsilon^2 n_i^{(2)} + \varepsilon^3 n_i^{(3)} + \cdots, \\ V_d &= \varepsilon V_d^{(1)} + \varepsilon^2 V_d^{(2)} + \varepsilon^3 V_d^{(3)} + \cdots, \\ V_i &= \varepsilon V_i^{(1)} + \varepsilon^2 V_i^{(2)} + \varepsilon^3 V_i^{(3)} + \cdots, \\ \varphi &= \varepsilon \varphi^{(1)} + \varepsilon^2 \varphi^{(2)} + \varepsilon^3 \varphi^{(3)} + \cdots, \\ Q &= \varepsilon Q^{(1)} + \varepsilon^2 Q^{(2)} + \varepsilon^3 Q^{(3)} + \cdots. \end{aligned} \tag{13}$$

Before using the usual procedure of the reductive perturbation theory [18], let us make the nonlinear perturbation consistent with (12) and (13). Therefore, the following assumptions and scaling are made [15,16]

(i) The dust-charging frequency ν_{ch} is low compared to the ion-oscillation frequency ω_{pi}, *i.e.* $\nu 10^{-3}$ and hence it is assumed that $\nu \sim O(\varepsilon\, 3/2)$.

(ii) The ratio of the ion-loss frequency to ion-plasma frequency is small, *i.e.* $\nu_L \approx 10^{-3}$ and hence it is assumed that $\nu_L {\sim} O(\varepsilon^{3/2})$.

(iii) The ratio of the ion-neutral frequency to ion-plasma frequency is small, *i.e.* $\nu_{in} \approx 10^{-3}$ and hence it can be assumed that $\nu_{in} \sim O(\varepsilon^{3/2})$.

(iv) Also, the ratio of the dust-neutral frequency to ion-plasma frequency is small, *i.e.* $\nu_{dn} \approx 10^{-6}$ and hence it is assumed that $\nu_{dn} \sim O(\varepsilon^3)$.

Substituting (12) and (13) into the basic set of (1)–(7), with the aid of the above assumptions, then collecting terms of like powers of ε, in the lowest order we obtain the following relations:

$$n_d^{(1)} = -\frac{\beta_d}{V_0^2}\varphi^{(1)}, \quad n_i^{(1)} = \frac{1}{V_0^2 - \sigma_i}\varphi^{(1)}, \quad Q^{(1)} = 0, \tag{14}$$

$$V_d^{(1)} = -\frac{\beta_d}{V_0}\varphi^{(1)}, \quad V_i^{(1)} = \frac{V_0}{V_0^2 - \sigma_i}\varphi^{(1)}, \tag{15}$$

$$\varphi^{(1)} = (\delta - 1)\left[Q^{(1)} - n_d^{(1)}\right] + \delta n_i^{(1)}. \tag{16}$$

Substituting (14) into (16), we get the linear dispersion relation

$$\frac{\delta}{V_0^2 - \sigma_i} + \frac{\beta_d(\delta - 1)}{V_0^2} - 1 = 0. \tag{17}$$

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The last equation gives the normalized linear-phase velocity of DIAWs, at $\beta_d^2 \approx 0$, as

$$V_0 = \left[\frac{(\sigma_i+\delta)}{2}\left(\left(1+\frac{\beta_d(\delta-1)}{(\sigma_i+\delta)}\right)+\sqrt{1+\frac{2\beta_d(\delta-1)(\delta-\sigma_i)}{(\sigma_i+\delta)}}\right)\right]^{1/2}, \tag{18}$$

where $V_0 = \sqrt{\delta+\sigma_i}$ is the linear-phase velocity of DIAWs at stationary dust grains, *i.e.* $\beta_d = 0$ [16]. Now, if we consider the next order in ε, we obtain a system of equations in the second-order perturbed quantities. Solving this system, we finally obtain the following DKdV equation,

$$\frac{\partial\varphi^{(1)}}{\partial\tau} + A\varphi^{(1)}\frac{\partial\varphi^{(1)}}{\partial\xi} + B\frac{\partial^3\varphi^{(1)}}{\partial\xi^3} + D\varphi^{(1)} = 0, \tag{19}$$

where

$$B = \frac{\delta}{2\left[\dfrac{\delta V_0}{\left(V_0^2-\sigma_i\right)^2} + \dfrac{\beta_d}{V_0^3}(\delta-1)\right]},$$

$$A = 2B\left[\frac{1}{\left(V_0^2-\sigma_i\right)^2}\left(\frac{V_0^2}{V_0^2-\sigma_i}+\frac{1}{2}\right)-\frac{1}{2\delta}\right],$$

$$D = \frac{V_0 B}{\left(V_0^2-\sigma_i\right)^2}\left[\nu_{in}+\nu_L\left(2-\left(\frac{\Delta\sigma}{\sigma_0}\right)\left(V_0^2-\sigma_i\right)\right)\right] + \nu\alpha B\frac{(\delta-1)\left(V_0^2-\sigma_i-1\right)}{\delta V_0\left(V_0^2-\sigma_i\right)}$$

and

$$\alpha = \frac{\sigma_i+z}{z\left[\sigma_i+(1+z)\right]}.$$

To obtain a solitary wave solution of (19) we introduce the variable

$$\eta = \xi - U(\tau)\tau,$$

where η is the transformed coordinate with respect to a frame moving with velocity $U(\tau)$. Integrating (19) with respect to the variable η and using the vanishing boundary conditions for $\varphi^{(1)}$ and its derivatives up to the second order for $|\eta| \to \infty$, we obtain the time evolution solitary waveform approximate solution as

$$\varphi^{(1)} = \varphi_m^{(1)}(\tau)\, sech^2\left(\frac{\eta}{W(\tau)}\right), \tag{20}$$

where $\varphi_m^{(1)}(\tau) = \varphi_0^{(1)}\exp(-D\tau)$ and $W(\tau) = \sqrt{12B\exp(D\tau)/A\varphi_0^{(1)}}$ are the amplitude and width of DIAWs respectively, and $U(\tau) = (A\varphi_0^{(1)}/6)exp(-D\tau)$. To determine the value of $\varphi_0^{(1)}$, we let $D = 0$ in (19) and then it becomes the standard KdV equation and its solution is given by

$$\varphi^{(1)} = \varphi_0^{(1)} sech^2\left(\sqrt{12B/A\varphi_0^{(1)}}\,\eta_0\right), \tag{21}$$

where η_0 is the transformed coordinates with respect to a frame moving with velocity U_0 of $\varphi_m^{(1)} = \varphi_0^{(1)} = 6U_0/A$ at $\tau = 0$.

3. Derivation of the evolution equation at critical phase velocity

It should be mentioned here that there exists a special value of $V_0 = V_{0c}$, which may be called critical phase velocity, where the coefficient of the nonlinear term of (19) becomes zero and the amplitude for DIAWs becomes very large. This implies that the stretching used in Sec. 2 is not valid for the case $V_0 = V_{0c}$. Therefore, one has to look for another equation suitable for describing the evolution of the system. The critical phase velocityV_{0C} is given by

$$V_{0c} = \left((\sigma_i + 1) + \sqrt{1 + 2\sigma_i + \delta - 2\beta_d(\delta - 1)}\right)^{1/2}. \tag{22}$$

Now, let us introduce the new stretched coordinates, defined by [22]

$$\xi = \varepsilon(x - V_0 t),\ \tau = \varepsilon^3 t. \tag{23}$$

Using the above-mentioned assumptions, one can assume that ν_L, ν_{in} and $\nu \sim O(\varepsilon^3)$. Substituting (13) and (23) into the basic (1)–(7), we obtain to the lowest order in ε, the linearized solutions (14) and (15) and the linear dispersion relation (17).

If we continue to the next order in ε for continuity and momentum equations, we get

$$n_d^{(2)} = -\frac{\beta_d}{V_0^2}\varphi^{(2)}, \tag{24}$$

$$n_i^{(2)} = \frac{1}{\left(V_0^2 - \sigma_i\right)^2}\left(\frac{V_0^2}{V_0^2 - \sigma_i} + \frac{1}{2}\right)\left(\varphi^{(1)}\right)^2 + \frac{\varphi^{(2)}}{V_0^2 - \sigma_i}, \tag{25}$$

$$V_d^{(2)} = -\frac{\beta_d}{V_0}\varphi^{(2)}, \tag{26}$$

$$V_i^{(2)} = \frac{V_0}{\left(V_0^2 - \sigma_i\right)^2}\left(\frac{V_0^2}{V_0^2 - \sigma_i} - \frac{1}{2}\right)\left(\varphi^{(1)}\right)^2 + \frac{V_0}{V_0^2 - \sigma_i}\varphi^{(2)}. \tag{27}$$

Furthermore, the next order for the charging and Poisson's equations with the aid of the lowest order in ε, leads to the following equation:

$$\left\{\frac{\delta}{\left(V_0^2 - \sigma_i\right)^2}\left(\frac{V_0^2}{V_0^2 - \sigma_i} + \frac{1}{2}\right) - \frac{1}{2}\right\}\left(\varphi^{(1)}\right)^2 + \left(\frac{\delta}{V_0^2 - \sigma_i} - \frac{(\delta - 1)\,\beta_d}{V_0^2} - 1\right)\varphi^{(2)} = 0. \tag{28}$$

The coefficient of $\phi^{(2)}$ is identically zero because of the linear dispersion relation (17), while the coefficient of $\left(\varphi^{(1)}\right)^2$ is precisely A/B, which vanishes in the case at hand. Thus, Poisson's equation is automatically satisfied.

For the next order in ε, we obtain a system of equations in the third-order perturbed quantities. Solving this system, we finally obtain the DMKdV equation

$$\frac{\partial\varphi^{(1)}}{\partial\tau} + A_c\left(\varphi^{(1)}\right)^2\frac{\partial\varphi^{(1)}}{\partial\xi} + B\frac{\partial^3\varphi^{(1)}}{\partial\xi^3} + D\varphi^{(1)} = 0, \tag{29}$$

where

$$A_c = \frac{6V_0^2 B}{\left(V_0^2 - \sigma_i\right)^4}\left[\frac{V_0^2}{\left(V_0^2 - \sigma_i\right)} + \frac{\left(V_0^2 - \sigma_i\right)}{12V_0^2} + \frac{1}{6}\right] - \frac{B}{2\delta},$$

and B and D have the same forms as before.

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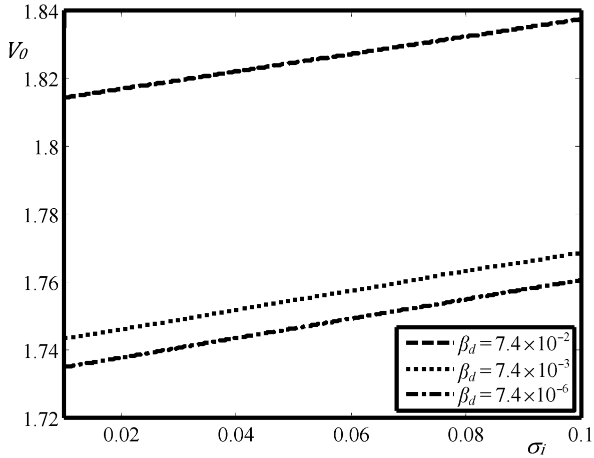

Figure 1. Plot of V_0 against σ_i for different values of β_d,$\delta = 3$ and $Z_d^{(0)} = 1000$.

The time evolution solitary waveform approximate solution of (29) is given by

$$\varphi^{(1)} = \varphi^{(1)}_{mc}(\tau)\sec h\left(\frac{\eta}{W_c(\tau)}\right), \tag{30}$$

where the amplitude $\varphi^{(1)}_{mc}(\tau)$ and the width W_c are given by $\varphi^{(1)}_{0c}\exp(-D\tau)$ and $W_c(\tau) = \sqrt{B exp(D\tau)/U_0}$, respectively, and $\varphi^{(1)}_{mc} = \varphi^{(1)}_{0c} = \pm\sqrt{6U_0/A_c}$ at $\tau = 0$.

4. Numerical results and discussion

In this paper we have considered the propagation of weakly dissipative DIA solitary waves in a DP composed of a relatively small fraction of negatively charged small dust grains with charge fluctuation, positively charged ions fluid, isothermal electrons, immobile background neutral particles in the presence of weak dissipations arising due to the low rates of ionization, dust charging and ion-neutral collision. For nonlinear DIAWs, the reductive perturbation theory is used to reduce the basic set of (1)–(7) to the DKdV equation, (19), and the DMKdV equation, (29), for critical phase velocity (V_{0c}). The coefficient of the linear damping term in (19) and (29) arises from the ionization, ion-neutral collisions and dust charge-fluctuation dynamics. It should be mentioned here that for our numerical analyses, we have carried a numerical investigation over a wide range of plasma parameters. These numerical values of the parameters are frequently used in the literature [17,23], $\sigma_i(= T_i/T_e)\sim 0.01 - 0.1, \beta_d(= z_d m_i/m_d)\sim 10^{-6} - 10^{-2}$ and $\delta(= n_i^{(0)}/n_e^{(0)})\sim 2.0 - 3.0$. As the coefficients A, B, C and A_c of DKdV and DMKdV equations play a crucial role in determining the existence criteria and the nature of the DIA solitary waves, it is instructive to investigate these coefficients in terms of their dependence quantities. Figure 1 demonstrates the variation of the linear-phase velocity V_0 of the DIA solitary waves with $\sigma_i$for different values of β_d. It is obvious from Fig. 1 that V_0 increases as σ_i and β_d increase. Figure 2 indicates that A (the coefficient of nonlinearity of (19)) increases with the increase of V_0, whereas it decreases as δ increases. Figure 3 shows that B (the coefficient of dispersion) decreases as V_0 increases, but increases as δ increases. In the absent of the ion-neutral collision

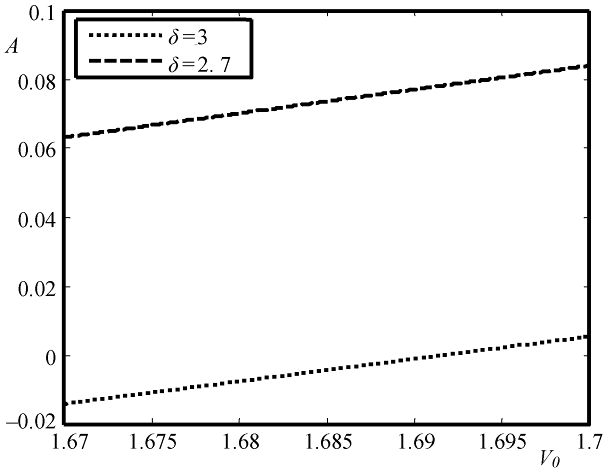

Figure 2. Plot of A against V_0 for different values of δ, $\beta_d = 10^{-6}$, and $Z_d^{(0)} = 1000$.

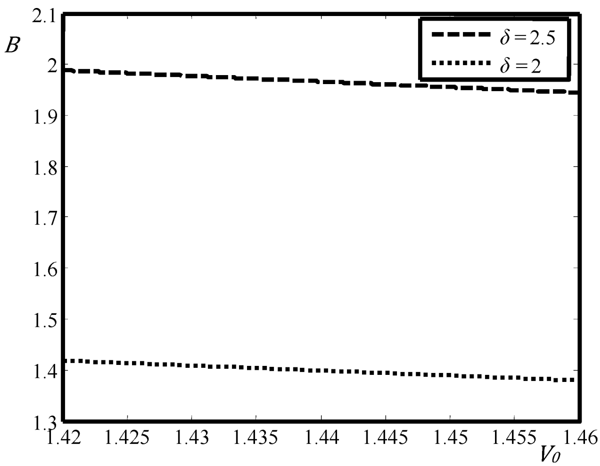

Figure 3. Plot of B against V_0 for different values of δ, $\beta_d = 10^{-6}$, and $Z_d^{(0)} = 1000$.

frequency, *i.e.* $v_{in} = 0$, and $\Delta\sigma/\sigma_0 = 2$, Figs. 4 and 5 clarifies that |D| increases as v and v_L increase, respectively, and it decreases with the decrease of δ. Figure 6 points out that D increases as V_0 and v_{in} increase. It is worth to notice from Fig. 4–6 that the values of v, v_L and v_{in} justify assumptions and scaling of (i), (ii) and (iii), respectively, in Sec. 2, on the basis of which the reductive perturbation analysis is used. Figure 7 shows how at critical phase velocity the coefficient of nonlinearity of (29) A_c changes with $V_0$for different values of δ. It is clear that A_c increases as V_0 increases, whereas it decreases with the increase of δ. Moreover, from the study of the behavior of DIAWs, the following interesting features are deduced:

- The phase velocity of the DIAWs in DPs depends on the dynamics of the small fraction of small dust grains (Fig. 1).
- Both compressive and rarefactive DIAWs are obtained in our system (Fig. 2).
- The coefficients of nonlinearities A and A_c decrease, whereas the coefficient of dispersion B increases with the increase of ion-electron number density ratio δ.

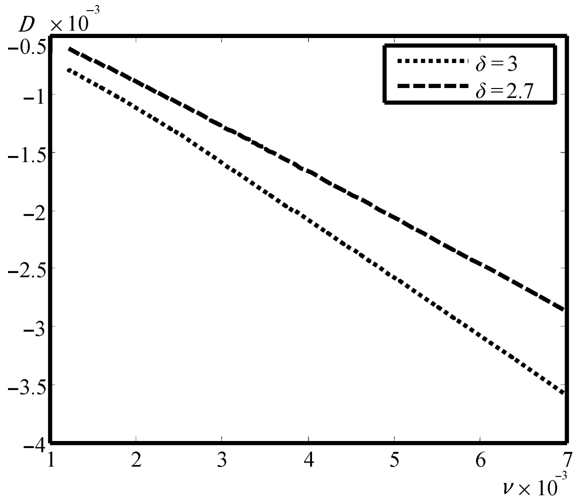

Figure 4. Plot of D against ν for different values of δ, $\nu_{in} = 0$, $\Delta\sigma/\sigma_0 = 2$, $\beta_d = 10^{-6}$ and $Z_d^{(0)} = 1000$.

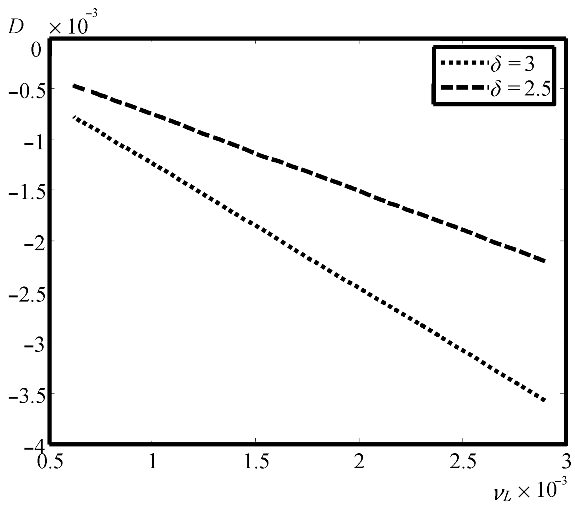

Figure 5. Plot of D against ν_L for different values of δ, $\nu_{in} = 0$, $\Delta\sigma/\sigma_0 = 2$, $\beta_d = 10^{-6}$ and $Z_d^{(0)} = 1000$.

Therefore, the DIA amplitude, which is proportional to $1/A$ and $1/\sqrt{Ac}$, respectively, increases, and the spatial width of the DIAWs, which is proportional to $\sqrt{B}$ also increases; *i.e.* the DIAWs grow as δ increases (Figs. 2, 3 and 7).

• The presence of the low rates of charge-fluctuation dynamics of the dust grains, ion-neutral collision, ionization and ion loss modifies the characteristics of nonlinear propagation of DIAWs, for which the nonlinear waves are governed by the DKdV equation, and whereas at critical phase velocity by DMKdV equation (19) and (29) respectively.

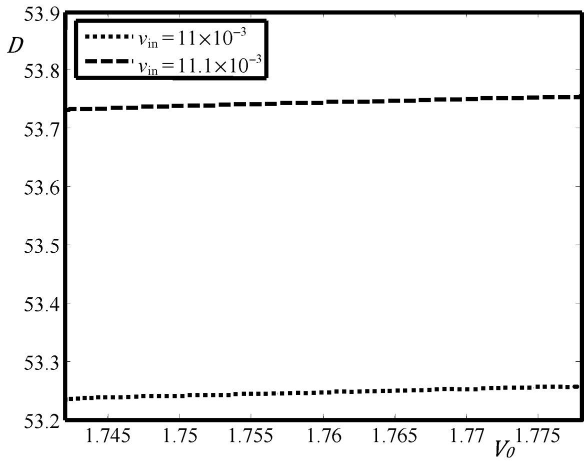

Figure 6. Plot of D against V_0 for different values of ν_{in}, $\Delta\sigma/\sigma_0 = 2$, $\beta_d = 10^{-6}$ and $Z_d^{(0)} = 1000$.

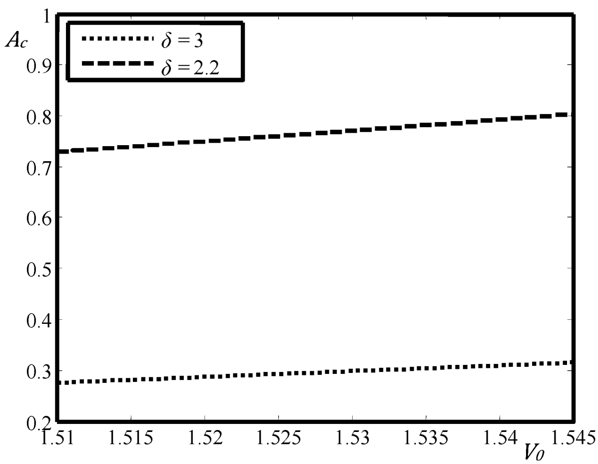

Figure 7. Plot of A_c against V_0 for different values of δ, $\beta_d = 10^{-6}$ and $Z_d^{(0)} = 1000$.

• The magnitude of the damping term D due to ion-neutral collisions is much greater than that due to charge fluctuation and ion loss. Also, ion-neutral collision dominates over ionization, therefore the DIAWs are always stable (Fig. 6).

• In the absence of the ion-neutral collision frequency, the ionization dominates over the charge variation and ion loss. Hence, the DIAWs become unstable due to ionization (Figs. 4 and 5).

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References

[1] Jana, M. R., Sen, A and Kaw, P. K. 1993 *Phys. Rev.* E **48**, 3930.
[2] Mendis, D. A. and Rosenberg, M. 1994 *Ann. Rev. Astron. Astrophys.* **32**, 419.
[3] Verheest, F. 2000 *Waves in Dusty Plasmas*. Dordrecht, Netherlands: Kluwer Academic Publishers.
[4] Shukla P. K. and Mamun, A. A. 2002 *Introduction to Dusty Plasma Physics*. Bristol, UK: Institute of Physics.
[5] D'Angelo, N. 1998 *Phys. Plasmas* **5**, 3155.
[6] Ivlev, A. V., Samsonov, D., Goree, J., Morfill, G. and Fortov, V. E. 1999 *Phys. Plasmas* **6**, 714.
[7] Varma, R. K., Shukla, P. K. and Krishan, V. 1993 *Phys. Rev.* E **47**, 3612.
[8] Nejoh, Y. N. 1997 *Phys. Plasmas* **4**, 2813.
[9] Ghosh, S., Sarkar, S. Khan, M. and Gupta, M. R. 2000 *Phys. Plasmas* **7**, 3594.
[10] Mamun, A. A. and Shukla, P. K. 2002 *IEEE Trans. Plasma Sci.* **30**, 720.
[11] Shukla, P. K. and Siline, V. P. 1992 *Phys. Scr.* **3**, 508.
[12] Barkan, A. Merlino, R. L. and D'Angelo, N. 1995 *Phys. Plasmas* **2**, 3563.
[13] Moslem, W. M. 2006 *Chaos, Solitons and Fractals* **28**, 994.
[14] Rahman, A. Sayed, F. and Mamun, A. A. 2007 *Phys. Plasmas* **14**, 034 503.
[15] Ghosh, S. 2005 *J. Plasma Phys.* **71**, 519.
[16] Ghosh, S. 2007 *J. Plasma Phys.* **73**, 515 (and references therein).
[17] Tiwari, R. S. and Mishra, M. K. 2006 *Phys. Plasmas* **13**, 062 112.
[18] Washimi, H. and Taniuti, T. 1966 *Phys. Rev. Lett.* **17**, 996.
[19] Moslem, W. M. and El-Taibany, W. F. 2005 *Phys. Plasmas* **12**, 122 309.
[20] Shukla, P. K. and Eliasson, B. 2009 *Rev. Mod. Phys.* **81**, 25.
[21] Popel, S. I., Yu, M. Y. and Tsytovich, V. N. 1996 *Phys. Plasmas* **3**, 4313.
[22] El-Labany, S. K. and El-Shamy, E. F. 2005 *Phys. Plasmas* **12**, 042 301.
[23] Wang, X. Bhattacharjee, X. Gou, S. K. and Goree, J. 2001 *Phys. Plasmas* **8**, 5018.