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Magnetic field quantization in pulsars

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Magnetic field quantization is an important issue for degenerate environments such as neutron stars, radio pulsars and magnetars etc., due to the fact that these stars have a magnetic field higher than the quantum critical field strength of the order of 4.4×10^{13} G, accordingly, the cyclotron energy may be equal to or even much more than the Fermi energy of degenerate particles. We shall formulate here the exotic physics of strongly magnetized neutron stars, known as pulsars, specifically focusing on the outcomes of the quantized magnetic pressure. In this scenario, while following the modified quantum hydrodynamic model, we shall investigate both linear and nonlinear fast magnetosonic waves in a strongly magnetized, weakly ionized degenerate plasma consisting of neutrons and an electron–ion plasma in the atmosphere of a pulsar. Here, linear analysis depicts that sufficiently long, fast magnetosonic waves may exist in a weakly dispersive pulsar having finite phase speed at cutoff. To investigate one-dimensional nonlinear fast magnetosonic waves, a neutron density expression as a function of both the electron magnetic and neutron degenerate pressures, is derived with the aid of Riemann's wave solution. Consequently, a modified Korteweg–de Vries equation is derived, having a rarefractive solitary wave solution. It is found that the basic properties such as amplitude, width and phase speed of the fast magnetoacoustic waves are significantly altered by the electron magnetic and the neutron degenerate pressures. The results of this theoretical investigation may be useful for understanding the formation and features of the solitary structures in astrophysical compact objects such as pulsars, magnetars and white dwarfs etc.

Key words: astrophysical plasmas, plasma nonlinear phenomena, quantum plasma

1. Introduction

Neutron stars (NS) have been an attractive research area (Baade & Zwicky 1934) due to their exotic environments and being one of the important ingredients in stellar evolution. NS comprising of iron/oxygen/carbon and helium nuclei (Massey 1976; Shukla, Mamun & Mendis 2011) are dense enough ($n_s \gtrsim 10^{30}$ cm^{-3}) to be treated as a degenerate plasma system (Lai 2001; Chabrier, Saumon & Potekhin 2006), thus quantum corrections become quite important for such stars (Abdikian & Mahmood 2016). The physics of the highly degenerate NS crust involves several

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applications, ranging (Chamel & Haensel 2008) from nuclear and condensed matter physics to general relativity. The initial signals detected from neutron stars emerging from radio pulsars were found to spin many times in one second. Then, strongly magnetized NS, known as pulsars, were observed to spin down and their periods were found to increase continuously. In this context, (Hewish *et al.* 1968) the rotating dipole model was applied to find the spin-down rate of the pulsars, prescribing that the required surface magnetic fields should be in the range (10^{11}–10^{13}) G for the first detected pulsars. It was depicted in Chabrier, Douchin & Potekhin (2002), Harding & Lai (2006) that the magnetic field for radio pulsars is $B_o = (10^{11}\text{–}10^{14})$ G, whereas for anomalous X-ray pulsars the superstrong magnetic field may be higher i.e. $B_o \sim (10^{14}\text{–}10^{15})$ G (Lewin & Van Der Klis 2005). At such a high magnetic field the cyclotron energy $\hbar\omega_{ce}(= eB/\omega_{ce})$ may be of the order of or much larger than the electron Fermi energy (Tsintsadze & Tsintsadze 2012), $E_F(= ((3\pi^2)^{2/3}\hbar^2/2m_e)n_e^{2/3})$ i.e. $\hbar\omega_{ce} \gg E_F$.

Moreover, due to the fact that bound state species such as hydrogen and molecules are present in the photosphere of neutron stars, pulsars etc., the degenerate thermodynamic properties of protons, free electrons and bound species in these environments are significantly modified by the magnetic field. These modified thermodynamic properties of the plasma and the propagation of proper waves is an important research area (Tsintsadze 2010) in supernovae, pulsars, the convective zone of the Sun, white dwarfs, brown dwarfs, the early prestellar period of the evolution of the universe as well as in laboratory situations such as laser–matter interaction experiments. On the theoretical front, a lot of effort has been devoted in the direction (Abrahams & Shapiro 1991; Kouveliotou, Ventura & Van Den Heuvel 2001; Lai 2001; Tsintsadze & Tsintsadze 2009*a*,*b*, 2010) of describing the thermodynamic properties of strongly magnetized matter under NS conditions. However, the impact of this modified thermodynamics, particularly magnetic field quantization, has not received much attention in the literature (Shah *et al.* 2012), although it may be of great interest to formulate both linear and the nonlinear modes in quantum astrophysical environments. On the other hand, it is well known that the nonlinear effects appearing due to large amplitude oscillations in the plasma system may introduce a shock wave, whereas in the presence of dispersive effects, being balanced by the nonlinearity of the system, one may have a soliton structure. In this context, the Korteweg–de Vries (KdV) equation is well known for classical ion acoustic waves (Sagdeev 1966; Chen 1984), and solitary waves in a weakly ionized classical gas have been reported in Stenflo, Tsintsadze & Buadze (1989). Low frequency magnetosonic solitons are investigated in a magnetized spin-1/2 degenerate plasma, while opting for the Sagdeev potential approach (Marklund, Eliasson & Shukla 2007), where the authors found that rarefractive magnetosonic solitons may exist due to a balance between nonlinearities and the quantum diffraction term. Nonlinear magnetosonic waves in quantum dissipative magnetized plasmas are investigated in Masood *et al.* (2014). Linear and weak nonlinear propagation of magnetosonic waves in a degenerate plasma using perturbation theory was formulated in Haas & Mahmood (2018), modified KdV was derived having coefficients which are a strong function of quantum effects. Various linear and nonlinear aspects of magnetoacoustic waves are investigated (Masood *et al.* 2009; Masood, Jehan & Mirza 2010; Lui *et al.* 2011; Lui, Wang & Yang 2013; Iqbal *et al.* 2019). However, in all of the above mentioned literature regarding magnetosonic waves, magnetized plasmas are considered without taking into account the quantized magnetic pressure, which is the subject matter of the present study.

In this work, we aim to investigate the impact of quantized magnetic pressure on the linear and nonlinear properties of low frequency fast magnetosonic waves in a weakly ionized, dispersive degenerate neutron, electron–ion plasma in the atmosphere of a pulsar. The paper is organized as follows: in § 2, we derive the linear dispersion relation of fast magnetosonic waves using new set of quantum hydrodynamic equations. Section 3 presents the nonlinear behaviour of fast magnetosonic waves, and a brief summary is presented in § 4.

2. Basic formalism

Here, we consider a weakly ionized, quantum neutron–electron ion plasma in the presence of a super strong magnetic field $\hat{z}H_0$, in the atmosphere of a pulsar, where H_0 is the strength of the magnetic field and $\hat{z}$ is the unit vector along the z-axis in a Cartesian coordinate system. We shall formulate here a new quantum magnetohydrodynamic set of equations for the weakly ionized pulsars under study, where all components of the neutron star are in a degenerate state. For our theoretical description, the neutron dynamics is governed by the neutron momentum equation

$$n_N \frac{\mathrm{d}\boldsymbol{p}_N}{\mathrm{d}t} = -\nabla P_{FN} + \frac{\hbar^2 n_N}{2m_N}\nabla \frac{1}{\sqrt{n_N}}\Delta\sqrt{n_N} + \boldsymbol{f}_N, \tag{2.1}$$

and the neutron continuity equation

$$\frac{\partial n_N}{\partial t} + \operatorname{div} n_N \boldsymbol{u}_N = 0, \tag{2.2}$$

where p_N is the neutron momentum, $f_N = m_N n_N \sum_i \nu_{Ni}(\boldsymbol{u}_i - \boldsymbol{u}_N)$ is the neutron collisional frequency, $P_{FN} = (3\pi^2)^{2/3}\hbar^2/5m_N(n_N)^{5/3}$ is the neutron Fermi pressure $\hbar$ is Planck's constant divided by 2π, m_N, n_N, u_N are the mass, number density and velocity of the neutrons respectively, υ_{Ni} is the neutron–ion collisional frequency and u_i is the velocity of ions. Here, we suppose that the neutron–electron collision frequency ν_{Ne} is comparatively small. It was shown in Tsintsadze *et al.* (2018) that in the presence of a quantized magnetic pressure, the plasma becomes anisotropic, and the associated momentum equations in the directions perpendicular and parallel to the applied superstrong magnetic field are respectively

$$\frac{\mathrm{d}\boldsymbol{p}_{xj}}{\mathrm{d}t} = e_j\left(\boldsymbol{E} + \frac{1}{c}(\boldsymbol{u}_j \times \boldsymbol{H})\right)_x - \frac{1}{n_j}\nabla_x P_{xj} + \frac{\hbar^2}{2m_j}\nabla_x \frac{1}{\sqrt{n_j}}(\nabla^2\sqrt{n_j}) + \frac{1}{n_j}\boldsymbol{f}_{xj}, \tag{2.3}$$

$$\frac{\mathrm{d}\boldsymbol{p}_{zj}}{\mathrm{d}t} = e_j\left(\boldsymbol{E} + \frac{1}{c}(\boldsymbol{u}_j \times \boldsymbol{H})\right)_z - \frac{1}{n_j}\nabla_z P_{zj} + \frac{\hbar^2}{2m_j}\nabla_z \frac{1}{\sqrt{n_j}}(\nabla^2\sqrt{n_j}) + \frac{1}{n_j}\boldsymbol{f}_{zj}, \tag{2.4}$$

with the continuity equation

$$\frac{\partial n_j}{\partial t} + \nabla\cdot(n_j\boldsymbol{u}_j) = 0. \tag{2.5}$$

We shall also make use of the quasineutral condition

$$\sum_j e_j n_j = 0 \tag{2.6}$$

and Maxwell's equation

$$\nabla\cdot\boldsymbol{H} = 0, \tag{2.7}$$

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where j represents plasma species, $j=(e, i)$, x and z are the directions perpendicular and parallel to the external field respectively, e is the magnitude of electron charge, c is the speed of light in a vacuum, n_j, m_j and u_j are the number density, mass and velocity of plasma species j respectively, E is the electrostatic field. The collisional force of each plasma particle f_j in (2.3) and (2.4) in the presence of magnetic pressure inhomogeneity is defined as

$$\left.\begin{aligned} \boldsymbol{f}_{xj} &= m_j n_j \sum_{\alpha} \upsilon_{j\alpha}(\boldsymbol{u}_\alpha - \boldsymbol{u}_j)_x \\ \boldsymbol{f}_{zj} &= m_j n_j \sum_{\alpha} \upsilon_{j\alpha}(\boldsymbol{u}_\alpha - \boldsymbol{u}_j)_z \end{aligned}\right\}. \tag{2.8}$$

Here, $\upsilon_{j\alpha}$ is the collisional frequency of particle α with j and u_α, u_j are their relative velocities respectively. We note here that when collisions between particles are very frequent (Tsintsadze *et al.* 2018) the velocities of different plasma species must in fact be almost equal. It is quite evident from (2.1)–(2.8) that, if υ_{ei}, υ_{eN} and υ_{iN} are significantly large, the frictional forces can be balanced by other terms only if the relative velocities of different species, u_i-u_e, u_N-u_e and u_N-u_i are small i.e. $u_i \simeq u_e$, $u_N \simeq u_e$ and $u_N \simeq u_i$, which in turn leads to the relation (Tsintsadze *et al.* 2008).

$$\frac{n_e}{n_{0e}} \simeq \frac{n_N}{n_{N0}} \simeq \frac{n_i}{n_{0i}}. \tag{2.9}$$

The influence of a super strong magnetic field on the thermodynamic properties of a Fermi gas was presented in Tsintsadze (2010), Tsintsadze & Tsintsadze (2015) to emphasize that strongly magnetized systems hold the inequality $E_F \ll \hbar\omega_{ce}$, hence the consequent perpendicular, P_{xe}, and parallel, P_{ze}, components of the magnetic pressure are

$$P_{xe} = \frac{\hbar\omega_{ce}}{3} n_e, \tag{2.10}$$

$$P_{ze} = \gamma \left(\frac{n_e}{H}\right)^2 n_e, \tag{2.11}$$

where $n_e = \hbar\omega_{ce} m_e P_{Fe}/\pi^2\hbar^3$ with $P_{Fe} = (3\pi^2)^{1/3}\hbar n_e^{1/3}$ as the Fermi momentum at the Fermi level and $\gamma = \pi^4\hbar^4 c^2/3m_e e^2$. It is clear from the above expressions that for strongly magnetized NS the dependence of magnetic pressure on magnetic field intensity is quite different in the perpendicular and parallel directions. It may be noted here that the proton motion is also quantized into Landau levels due to the high magnetic field, but their corresponding cyclotron energy is much smaller than the electron cyclotron energy, i.e. $\hbar\omega_{cp} = \hbar\omega_{ce}(m_e/m_p)$ (here ω_{cp}, ω_{ce}, m_p, m_e are the cyclotron frequencies and masses of protons and electrons respectively.)

3. Linear dispersion equation

We shall follow Riemann's solution, to formulate the linear dispersive properties of one-dimensional fast magnetosonic waves propagating across the magnetic field, while assuming that $m_e \ll m_i \simeq m_N$ and $n_e \simeq n_i \ll n_N$. In this context, by adding (2.1)–(2.6), we may obtain

$$\begin{aligned} n_N \frac{\mathrm{d}p_{xN}}{\mathrm{d}t} &= \frac{1}{c}(\boldsymbol{J}\times\boldsymbol{H})_x - \nabla_x\left(P_{FN} + \sum_{j=e,i} P_{xj}\right) + \frac{\hbar^2 n_N}{2m_N}\nabla_x \frac{1}{\sqrt{n_N}}\Delta\sqrt{n_N} \\ &\quad + \frac{\hbar^2 n_e}{2m_e}\nabla_x \frac{1}{\sqrt{n_e}}\Delta\sqrt{n_e} + \frac{\hbar^2 n_i}{2m_i}\nabla_x \frac{1}{\sqrt{n_i}}\Delta\sqrt{n_i}. \end{aligned} \tag{3.1}$$

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We shall also make use of Magneto-hydrodynamic (MHD) equations

$$\frac{\partial \boldsymbol{H}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{H}) \tag{3.2}$$

$$\boldsymbol{J} = \frac{c}{4\pi}(\nabla \times \boldsymbol{H}), \tag{3.3}$$

where $J = -en_eu_e + Z_ien_iu_i$ is the plasma current density defined in the MHD approximation, P_{xj} is the quantized pressure of species j in a direction perpendicular to the external field. Let us rewrite (3.1) by making use of (3.3)

$$\frac{\mathrm{d}p_{xN}}{\mathrm{d}t} = -\frac{1}{n_N}\nabla_x\left(P_{FN} + P_{xe} + \frac{H^2}{8\pi}\right) + \frac{1}{4\pi n_N}(H\cdot\nabla)H_x + \frac{\hbar^2}{2m_e}\frac{n_e}{n_N}\nabla_x\frac{1}{\sqrt{n_e}}\Delta\sqrt{n_e}. \tag{3.4}$$

It may be noticed here that the diffraction term of neutrons and ions can be ignored as compared to the electron diffraction term due to different possible inequalities $n_e/n_N \ll m_e/m_N$ or $n_e/n_N \gg m_e/m_N = m_e/m_i$, consequently, the electrons may play an important role in exciting fast, degenerate magnetosonic waves. Using (2.9), we may re-write (3.4) as

$$\frac{\mathrm{d}p_{xN}}{\mathrm{d}t} = -\frac{1}{n_N}\nabla_x\left(P_{FN} + P_{xe} + \frac{H^2}{8\pi}\right) + \frac{1}{4\pi n_N}(H\cdot\nabla)H_x + \frac{\hbar^2}{2m_e}\frac{n_e}{n_N}\nabla_x\frac{1}{\sqrt{n_N}}\Delta\sqrt{n_N}, \tag{3.5}$$

or in linearized form we may write

$$\frac{\mathrm{d}\delta p_{xN}}{\mathrm{d}t} = -\frac{1}{n_{0N}}\nabla_x\left(\delta P_{FN} + \delta P_{xe} + \frac{H_0^2}{4\pi}\frac{\delta H_z}{H_0}\right) + \frac{H_0}{4\pi n_{0N}}\nabla_z H_x + \frac{\hbar^2}{4m_e}\frac{n_{0e}}{n_{N0}}\nabla_x\nabla^2\frac{\delta n_N}{n_{0N}}, \tag{3.6}$$

where H_z, H_x, are the components of the magnetic field along and perpendicular to the external magnetic field respectively. Similarly, we can write the oscillations of the pressure expressions form (2.10), while taking into account equation (2.9)

$$\delta P_{xe} = P_{0ex}\left(\frac{\delta H_z}{H_0} + \frac{\delta n_N}{n_{0N}}\right), \tag{3.7}$$

where $P_{0ex} = (\hbar\omega_{ce0}/3)n_{0e}$. Further, let us now write down the components of the magnetic field from (3.2) as

$$\frac{\partial H_x}{\partial t} = H_0\nabla_z u_x. \tag{3.8}$$

Then, by making use of (3.6)–(3.8), (2.2), we may obtain the dispersion relation of fast, degenerate, neutron magnetosonic waves that are propagating perpendicular to the applied superstrong magnetic field

$$\omega^2 = k_x^2(V_{FN}^2 + C_x^2 + V_A^2)^{1/2} + \frac{\hbar^2k_x^4}{4m_em_N}\frac{n_{0e}}{n_{0N}}, \tag{3.9}$$

where $V_{FN} = (3\pi^2)^{1/3}\hbar/m_N(n_{0N})^{1/3}$ is the neutron Fermi speed, $C_x = \sqrt{2P_{x0e}/m_Nn_{0N}} = \sqrt{2/3(\hbar\omega_{ce0}/m_N)(n_{0e}/n_{0N})}$ is the magnetic neutron acoustic speed and $V_A = H_0/\sqrt{4\pi m_Nn_{0N}}$ is the neutron Alfvén speed. For a weakly dispersive medium, equation (3.9) may be written as

$$\omega = k_x(V_{FN}^2 + C_x^2 + V_A^2)^{1/2} + \lambda k_x^3, \tag{3.10}$$

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where $\lambda = (\hbar^2/8m_e m_N)(n_{0e}/n_{0N})(1/\sqrt{V_{FN}^2 + C_x^2 + V_A^2})$ is the dispersive parameter. The above equation describes that sufficiently long waves may exist in a weakly dispersive pulsar having a finite limit on the phase velocity at $k=0$. In the following section, we shall show that the dispersive properties of nonlinear fast magnetosonic waves, in the presence of a quantized magnetic pressure, are governed by a modified KDV equation.

4. Solitary fast magnetosonic waves in a weakly dispersive pulsar

To show that nonlinear stationary waves can be formed in the dispersive pulsar atmosphere, we shall follow the method presented in Whitham (1974), Landau & Lifshitz (1987), Tsintsadze, Hussain & Murtaza (2011) and Tsintsadze *et al.* (2018). For this purpose, in order to consider the x-dimensional propagation of fast magnetosonic waves travelling across the quantized magnetic field, let us re-write equations (2.2), (3.2), (3.5) respectively as

$$\frac{\partial n_N}{\partial t} = -n_N \frac{\partial u_{xN}}{\partial x} \tag{4.1}$$

$$\frac{\mathrm{d}H_z}{\mathrm{d}t} = -H_z \frac{\partial u_x}{\partial x} \tag{4.2}$$

$$\frac{\partial u_{xN}}{\partial t} + u_{xN}\frac{\partial}{\partial x}u_{xN} = -\frac{1}{m_N n_N}\frac{\partial}{\partial x}\left(P_{FN} + P_{xe} + \frac{H_z^2}{8\pi}\right) + \frac{\hbar^2}{2m_e m_N}\frac{\partial}{\partial x}\frac{1}{\sqrt{n_N}}\frac{\partial^2}{\partial x^2}\sqrt{n_N} \tag{4.3}$$

Here, P_{FN} is the neutron Fermi pressure and P_{xe} is the electron magnetic pressure defined in (2.11). Since the magnetic field is favoured by conduction currents frozen in the interior part of NS (Chabrier *et al.* 2002), we shall apply here the frozen-in condition to reflect that the lines of magnetic force are frozen with the magnetic field lines to move with them i.e. $H_z/n_N = \text{constant}$, then we may have

$$P_{xe} = P_{0e}\left(\frac{n_N}{n_{0N}}\right)^2 \quad \text{and} \quad H_z^2 = H_0^2\left(\frac{n_N}{n_{0N}}\right)^2, \tag{4.4a,b}$$

where $P_{0e} = (\hbar\omega_{ce}(n_{0N})/3)n_{0e}$. For a weakly dispersive medium, let us linearize the last term of (4.3), to obtain from (4.1)–(4.3)

$$\frac{\partial n_N}{\partial t} + n_N \frac{\partial}{\partial x}u_{xN} = 0 \tag{4.5}$$

$$\frac{\partial u_{xN}}{\partial t} + u_{xN}\frac{\partial}{\partial x}u_{xN} = -\frac{\partial}{\partial x}(\alpha n_N^{2/3} + \beta n_N) + \frac{\hbar^2}{2m_e m_N}\frac{\partial^3}{\partial x^3}\frac{n_N}{n_{0N}} \tag{4.6}$$

where here $\alpha = (3\pi^2)^{2/3}\hbar^2/2m_N^2$, and $\beta = ((2\hbar\omega_{ce}/3)n_{0e} + H_0^2/4\pi)1/n_{0N}^2 m_N$. Now, if we ignore the last term in the above equation and look for Riemann's solution of (4.5) and (4.6) for a simple plane wave propagating in the positive x-direction, we have

$$\frac{\partial u_{xN}}{\partial n_N} = \sqrt{\frac{2}{3}\alpha\frac{1}{n_N^{4/3}} + \frac{\beta}{n_N}}, \tag{4.7}$$

or

$$n_N = \frac{1}{\beta^3}\left\{-\frac{\alpha}{3} + \left(\frac{\alpha}{3} + \beta n_{0N}^{1/3}\right)\left(1 + \frac{\beta u_N}{2\left(\frac{\alpha}{3} + \beta n_{0N}^{1/3}\right)^{3/2}}\right)^{2/3}\right\}^3. \tag{4.8}$$

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Substitution of (4.8) into (4.6) may lead to a modified KdV equation with complex nonlinearity. To get a simplified KdV equation, let us simplify (4.8) for two particular cases: (i) if the neutron degenerate pressure is much more than the electron magnetic pressure i.e. $\alpha \gg 3\beta n_{0N}^{1/3}$, equation (4.8) reduces to

$$n_N = \frac{m_N^3 u_N^3}{3^{5/2}\pi^2\hbar^3}, \tag{4.9}$$

and (ii) conversely, if $\alpha \ll 3\beta n_{0N}^{1/3}$, we may obtain from (4.8)

$$n_N = n_{0N}\left(1 + \frac{u_N}{\sqrt{\beta n_{0N}}}\right)^2. \tag{4.10}$$

To show that both cases will lead to a soliton solution, let us substitute the density expression (4.10) into (4.6) to obtain the KdV equation

$$\frac{\partial u_{xN}}{\partial t} + (u_{xN} + \eta)\frac{\partial}{\partial x}u_{xN} = \sigma\frac{\partial^3 u_{xN}}{\partial x^3} \tag{4.11}$$

having the solitary wave solution

$$u_{xN} = u_{x0N}\left\{\cosh\left[\left(\frac{u_{x0N}}{12\sigma}\right)^{1/2}(x - \eta t - \frac{1}{3}u_{x0N}t)\right]\right\}^{-2}. \tag{4.12}$$

Here, $\eta = \frac{3}{4}\alpha n_{0N}^{1/6}\sqrt{1/\beta} + \sqrt{\beta n_{0N}}/2$ and $\sigma = \hbar^2/4m_e m_N\sqrt{\beta n_{0N}}$. We want to emphasize here that, since the sign of the last term of (4.11) is positive, the obtained solitary waves will be rarefractive.

5. Summary

We have presented a graphical analysis of (3.9), (4.10) and (4.12) to study the magnetic field quantization effects on the dispersive properties of fast, degenerate, magnetosonic neutron waves that are propagating perpendicular to an applied superstrong magnetic field. For the graphical analysis the typical parameters of an astrophysical degenerate plasma, present in the magnetosphere of a highly magnetized star such as a radio pulsar or magnetar, are chosen (Bailes 1989; Zhang & Harding 2000; Chabrier *et al.* 2002; Harding & Lai 2006), which in the cgs system of units are: $n_{0e} = (10^{22}\text{–}10^{23})\ \text{cm}^{-3}$, $n_{0N} = (10^{30}\text{–}10^{34})\ \text{cm}^{-3}$, $u_{x0N} = (1\text{–}2) \times 10^7\ \text{cm s}^{-1}$, $H_0 = (10^{10}\text{–}10^{14})$ G with the physical constants $c = 3 \times 10^{10}$, $m_e = 9.1 \times 10^{-28}$, $m_N = 1.6749286 \times 10^{-24}$, $e = 4.8 \times 10^{-10}$, $\hbar = 1.05 \times 10^{-27}$. We have displayed equation (3.9) in figure 1 to show that the angular frequency (ω) of a fast magnetosonic wave increases by increasing the strength of the magnetic field H_0. Figure 2 depicts that the neutron number density (n_N) decreases with the increase of magnetic field strength, as shown in (4.10). The impact of the quantized magnetic field and neutron density on the dispersive properties of solitons, given in (4.12), are displayed in figure 3(*a*,*b*), respectively, to show that the width of the soliton decreases by increasing the magnetic field intensity, while it increases by increasing the neutron density concentration.

To conclude, in this paper we have investigated both linear and weakly nonlinear propagation of fast magnetosonic waves in a strongly magnetized, degenerate weakly

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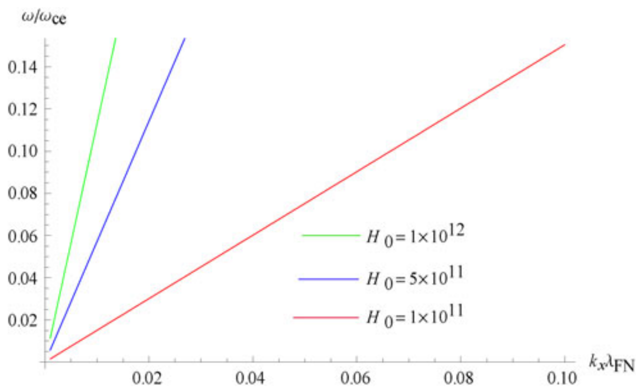

FIGURE 1. Normalized angular frequency (ω) of fast, degenerate, neutron magnetosonic waves is plotted against the wavenumber (k_x) (as given by (3.9)) for different values of the magnetic field, $H_0 = 6 \times 10^{10}$ G (green curve), $H_0 = 3 \times 10^{10}$ G (blue curve) and $H_0 = 8 \times 10^{10}$ G (red curve), while assuming $n_{0e} = 10^{22}$ cm^{-3} and $n_{0N} = 10^{30}$ cm^{-3}.

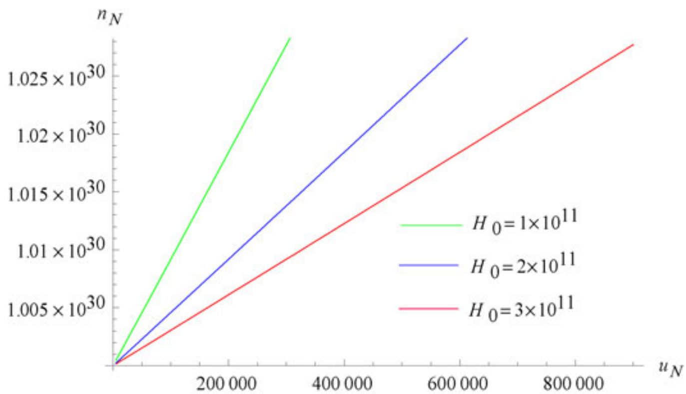

FIGURE 2. The neutrons number density (n_N) is plotted against the neutron velocity (u_N) (as given by (4.10)), for different magnetic field strengths $H_0 = 1 \times 10^{11}$ G (green curve), $H_0 = 2 \times 10^{11}$ G (blue curve) and $H_0 = 3 \times 10^{11}$ G (red curve), other parameters are the same as in figure 1.

ionized dispersive neutron, electron–ion plasma in the atmosphere of a pulsar. It is shown that the linear propagation frequency of a fast magnetosonic wave increases as a function of both the neutron degenerate pressure and electron magnetic pressure via the neutron acoustic speed. It is depicted that sufficiently long linear waves may exist in a weakly dispersive neutron star (pulsar), having a finite limit on the phase velocity at cutoff. Next, the weak nonlinear features of one-dimensional fast, degenerate magnetosonic waves are formulated with the aid of Riemann's solution for simple plane waves, to obtain a general expression for the neutron density as a function of both neutron degenerate pressure and electron magnetic pressure, which further was discussed for two special cases by comparing the neutron degenerate pressure with the electron magnetic pressures. The obtained neutron density expressions correspond to two different types of KdV equation, having coefficients which are a strong

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(*a*)

(*b*)

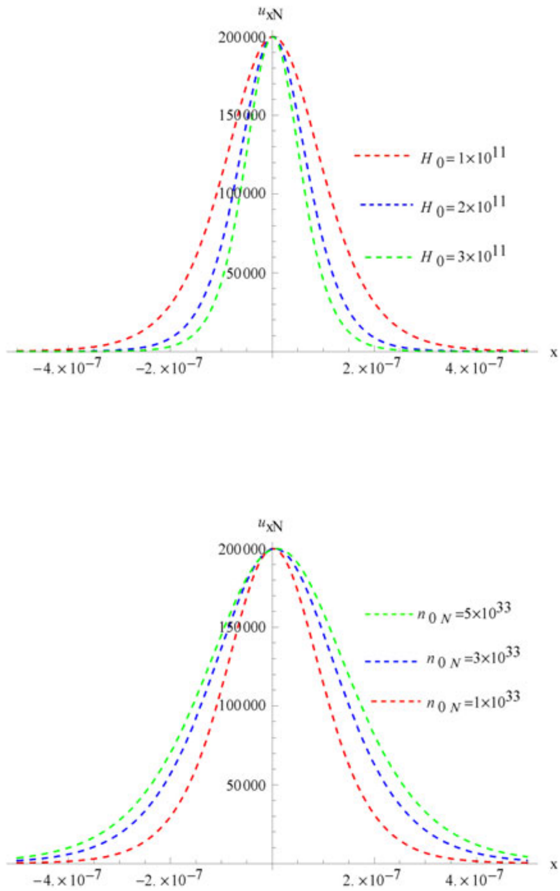

FIGURE 3. (*a*) The x-dimensional neutron velocity (u_{xN}) of fast magnetosonic neutron waves is plotted against the x-coordinate for different magnetic field values (as given by (4.12)), $H_0 = 1 \times 10^{11}$ G (red curve), $H_0 = 2 \times 10^{11}$ G (blue curve) and $H_0 = 3 \times 10^{11}$ G (green curve). Here we consider $n_{0e} = 10^{22}$ cm^{-3}, $n_{0N} = 10^{33}$ cm^{-3}, the neutron velocity $u_{x0N} = 1 \times 10^7$ cm s^{-1} and time $t = (10^{-15}$ s). (*b*) The x-dimensional neutron velocity (u_{xN}) of fast magnetosonic neutron waves is plotted against the x-coordinate (as given by (4.12)), for different neutron density concentrations; $n_{0N} = 10^{33}$ cm^{-3}, (red curve) $n_{0N} = 3 \times 10^{33}$ cm^{-3} (blue curve), $n_{0N} = 5 \times 10^{33}$ cm^{-3} (green curve), with the neutron velocity $u_{x0N} = 1 \times 10^7$ cm s^{-1}, $n_{0e} = 10^{22}$ cm^{-3}, $H_0 = 1 \times 10^{11}$ G and time $t = (10^{-15}$ s).

function of both quantized magnetic pressures and neutron density concentration, the consequent solitary wave solutions are found to be rarefractive.

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