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Hegel's 'Bad Infinity' as a Logical Problem

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Abstract

The paper analyses the concept of 'bad infinity' in connection with Hegel's critique of infinitesimal calculus and with the belittling of Hegel's mathematical notions by the representatives of modern logic and the foundations of mathematics. The main line of argument draws on the observation that Hegel's difference is only derivatively a mathematical one and is primarily of a broadly logico-epistemological nature. Because of this, the concept of bad infinity can be fruitfully utilized, by way of inversion, in an analysis of the conceptual shortcomings of the most prominent foundational attempts at dealing with infinite quanta, such as Cantor's set theory and Hilbert's axiomatism. As such, the paper is an attempt at reconstructing Hegel's philosophy of mathematics and its role in his philosophical system and, more importantly, as a contribution to logic in the more general and radical sense of the word.

Hegel's notion of 'bad infinity', as it appears in connection with his critique of Newton's infinitesimal calculus in *Science of Logic*, has not been much appreciated nor incorporated into the tradition of logic or the philosophy of mathematics. In fact, it shares the fate of most of Hegel's logical distinctions, which are considered obscure, unscientific and best left to Hegelians.[1] However, and despite their generosity toward the idiosyncrasies of Hegel's style, Hegelians too generally doubt his competence in mathematical matters, bracketing his remarks on the 'bad' nature of infinitesimals with those on 'negative electricity' or phrenology.[2]

Only recently, following Pinkard's programmatic paper (Pinkard 1981), has a re-assessment of Hegel's philosophy of mathematics been established as a philosophically fruitful exercise for the philosophy of mathematics and, just as importantly, for the logical analysis of language. That such a re-assessment is taking place within the analytical movement might be interpreted as a by-product of analytical philosophy's ongoing recollection of its idealist roots. A further contributing factor is the increasing recognition that Hegel's knowledge of contemporary mathematics was anything but marginal (see Wolf 1986: 200 and Lacroix 2000: 298). Finally, as regards logical analysis, recent interpretations have shown that Hegel is not at variance with the methods of modern formal logic;

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rather, he is more radical than them.[3] In light of these considerations, the revision of the concept of bad infinity is of particular importance both for the study of Hegel's thought and for the philosophy of mathematics. The main thesis of this paper is that 'bad infinity' is a pivotal logical concept, introduced in a specific mathematical context for the sake of a more radical logical analysis of knowledge and of its evolutionary as well as social nature. Because of this generality, 'bad infinity' can be applied to all fields of knowledge, including mathematical development after Hegel in which the concept of infinity played a significant role.

To show this I start by challenging a widespread view, expressed by authors from Bolzano to Pinkard, that Hegel's notions of *bad* and *true* infinity replicate the traditional distinction between *potential* and *actual* infinity, and that Hegel favours the latter, though in some mysterious, qualitative form. This view, as I argue, misses the *logical* function of the concept which can be seen in the following reconstruction of the relevant portion of Hegel's developmental account of knowledge as given in his *Science of Logic* (*WL*),[4] particularly within the so-called doctrine of being. The first step is a move from the undifferentiated realm of *phenomenal qualities* to the *quantitative* determination of objects and to the higher order concepts of *discretion* and *continuity*. This allows for the introduction of 'bad infinity' as a rather simplistic way of concept-making by means of 'bad negation' in which—as in the introduction of infinitely small quanta—the lack of determination is presented as a new kind of positive difference. In the second step, the previous step is made explicit (*ent-äußert*): the given concepts, such as discretion, continuity or 'bad infinity', are considered not only as mere (methodo)-logical tools for describing the development of knowledge, but as parts of this knowledge itself. This yields the concept of the pure quantum divided into the categories of *natural* and *real* numbers, on the one hand, and the mathematical problem of the infinite quanta, on the other hand. In this way, I will argue, the concept 'bad infinity' can fruitfully serve both logical and mathematical purposes.

With this argument in place, I then seek to show that bad infinity can be applied not only to the problem of infinitesimal calculus but also, outside Hegel's logic, to Cantor's concept of *infinitely large* quanta and the *axiomatic* foundations of infinity. In the final section of the paper I attempt a comparison of Hegel's discussion of 'bad infinity' to Wittgenstein's remarks on rule-following, also introduced in the mathematical context, to draw certain parallels in their respective treatment of the problem of the sociality of knowledge.

I. 'Bad' concept-making

In what follows, I build on an important observation made by Pirmin Stekeler-Weithofer in his exegesis of Hegel's 'mathematical' chapters from the *Science of*

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Logic (Stekeler-Weithofer 2005: chap. 7). In it Stekeler-Weithofer claims that the word 'bad' ('*schlecht*') is, at least sometimes, used by Hegel in the sense of 'mere' ('*schlicht*')—as reflected in words still in current usage such as '*schlechtweg*' or '*schlechthin*'—so as to refer to the sloppy introduction of certain concepts by a 'mere' negation of some established differences. Though probably surprising and controversial at first glance, this reading might be vindicated in a straightforward manner by its consequences, i.e., by making Hegel's use of the respective concepts immediately intelligible.

Take Hegel's general attack on the concept of *infinitely small*, as utilized in the infinitesimal calculus (*WL*: 258/355): 'The infinitely small signifies in the first instance the negation of the quantum as such, that is, of the so-called *finite* expression, of the completed determinateness that quantum as such possesses'. The suggested reading would go as follows: At first, the 'mere' and thus problematic introduction of the infinitely small consists in the idea originating in the practice of the approximation processes. In these the estimated value is determined by means of a difference smaller than every finite boundary and, in this sense, *non-finite*. In a similar way, one can treat the sequences 1, 2, 3, 4, … etc., or 2, 4, 6, 8, … as determining the infinitely large quantum. What Hegel rightfully draws our attention to is the fact that the concept of *quantity* is by definition connected to its finiteness in the sense of conceptual *determination*. In human practices of measuring or counting, what one wants to know is some determinate and in this sense 'finite' information, an answer to the original question 'how many' or 'how big (with respect to the unit)'. The mere denial that the answer is finite does not by any means amount to a positive answer of a new kind, just as true perfection or justice cannot be arrived at by mere observation that nobody is completely perfect or just. 'Bad' negation of the inherent imperfection or injustice of human beings leads only to the 'bad' concept of justice placed in the beyond (*Jenseits*) where it is, at least in its entirety, unattainable to our minds. As Hegel puts it:

> Only the bad infinite is the *beyond*, since it is *only* the negation of the finite posited as *real* and, as such, it is abstract first negation; thus determined *only* as negative, it does not have the affirmation of *existence* in it; held fast only as something negative, it *ought not to be there*, it ought to be unattainable. However, to be thus unattainable is not its grandeur but rather its defect, which is at bottom the result of holding fast to the finite as such, *as existent*. It is the untrue which is the unattainable, and what must be recognized is that such an infinite is the untrue. (*WL*: 119/164)

Now, as far as mathematics is concerned, such unattainability does not stand for the potentiality of the approximation process but for the tendency to treat it as

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specifying directly the new, infinite quantity. What the problem of such simple conceptual formations consists in is exactly the unclarified relation to the realm of the original differences, i.e., to the other so far introduced numbers in the case of the infinitely small (or large), and to our earthly legal system in the case of eternal justice.

In the case of the infinitely small, all these argumentative points were already made by Berkeley in his *Analyst*: Newton's (and Leibniz's) practices of infinitesimal calculus do not, e.g., enable one to decide whether the alleged infinitesimally small 'quanta' dx do or do not equal zero and, in fact, systematically entertain both possibilities. In calculating the derivative the algorithm presupposes, first, that $dx \neq 0$ so as to be able to divide by it, and, second, that $dx = 0$ so that the elements of the development having dx as a factor could be annihilated. The reason that calculus, despite this conceptual failure, usually leads to the correct results is famously explained by Berkeley with reference to the 'compensation of errors', the first one making up for the second (Berkeley 1734: §22).

Taking into account Hegel's famous fondness for contradiction, one could expect he would approve of this conclusion as well. But it is not so.[5] The fact that the talk about the infinitely small based on a simple negation of the (finite) determination led to the contradiction indicates, in the first place, that the conceptual means chosen are still insufficient fully to determine the new quantity. Such a negation leaves a lot of things, particularly the relations $=$ and $<$ to other quanta, indeterminate, which is at variance with the very concept of a quantum. Along these lines I read Hegel's statements such as this: 'since the infinitely great or small is such that it can no longer be increased or diminished, it is no longer in fact a *quantum* as such' (*WL*: 206/282).

This conclusion should lead us to a simple rejection of talk about infinitesimals, as the subsequent tradition of calculus chose. It resorted instead to talk about limits by assigning to dx the syncategorematic role within complex expressions such as dy/dx or $f(x)dx = dy$. Similar negative decisions were adopted by Galilei and Bolzano with respect to infinitely large quanta, the comparability of which they rejected as impossible. The second choice we have is the adjustment of infinite talk so as to clear up the status of the infinitesimals as regular quanta. The set theory of Cantor and non-standard analysis of Robinson did exactly this for the cases of infinitely large and small quanta, respectively. From Hegel's or a Hegelian's point of view, the original contradiction becomes sublated and thus vindicated in its basic regulative function. This explains also what Hegel meant by saying that 'contradiction est regula veri, non-contradiction, falsi' (*JS*: 533).[6] In order to make all of this more transparent, let me give another example.

In ancient measuring practice, the determination claim concerning the question 'how big is some quantity with respect to the other?' led to the natural

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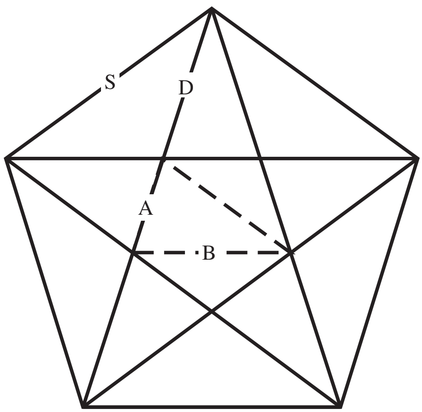
expectation that such a comparison of two quantities, or definition of their *ratio*, must produce the definitive answer. Such an answer was supposed to consist in a finite sequence of natural numbers to be achieved by a method known as reciprocal subtraction or *anthyphairesis*. Given two quantities $A > B$, the smaller one (B) is subtracted from the bigger one (A) as many times (M_1) as possible with the rest R_1. Since, by definition $B > R_1$, the process repeats itself, i.e., R_1 is subtracted from B (M_2 times) and the rest R_2 is obtained for which, again, $R_1 > R_2$ occurs. By focusing on the rest sequence $(A > B >)\ R_1 > R_2 > R_3 < \ldots$, one gets the procedure known as *Euclidian algorithms* for finding the greatest common divisor D of A, B, which is to be prospectively the last member of the described sequence. By focusing, on the other hand, on the sequence of natural numbers M_1, M_2, … describing how many times B was obtained in A, in general, R_{N+1} in R_N, one arrives at the so-called *anthyphairetic expansion*. Now, the concept of the (*anthyphairetic*) ratio is defined by means of the following definition, known also as *definition by abstraction*: The pairs of quantities A, B and C, D of the same kind are said to have the same *anthyphairetic ratio* if and only if they have the same anthyphairetic expansion. On this very ground one can state, e.g., that 24, 14 and 48, 28 determine the *same* ratio or proportion as opposed to the different common divisors 2 and 4. The reason is that the pairs 24, 14 and 48, 28, though having the different rest sequences 10, 4, 2 and 20, 8, 4, lead to the *same* anthyphairetic expansion 1, 1, 2, 2, which provides for *their* identity in the given respect.

This determination works as expected for the case of discrete quantities such as sets since—having by definition the discrete unit as their common divisor—their comparison by means of reciprocal subtraction must end up after finitely many steps. In the case of continuous quanta, such as lengths or volumes, this does not have to be so however. The discovery of incommensurability within relatively simple forms, such as square or pentagon, showed that the anthyphairetic process does not have to terminate. As a result, one is at variance with the original assumption, concluding that some quantities cannot be consistently measured, i.e., cannot be quantities at all. The case of the pentagon is particularly illuminative in this respect, since it leads to the anthyphairetic expansion of 1, 1, 1, 1, … as might easily be read from the given figure, mainly due to the fact that the sides of the pentagon are



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always parallel to (some of) its diagonals. As a result, one gets the relations D = S + B and S = B + A between the diagonals D or B and the sides S or A of the big and inscribed pentagon, respectively. Now, the respective rest sequence obviously consists of these magnitudes alone, i.e., starts with D > S > B > A and, as such, must continue with the sides and diagonals of the further inscribed pentagons, which means: *indefinitely*.

The concept of the *alogoi logoi*, or *irrational ratio*, can be seen as capturing this contradictory situation which, at first, initiated the abandonment of the original concept of the real number (or 'proportion') so as later to become a sign of the dialectical *sublation* of both antithetic claims by forging a new kind of quantity, *irrational* numbers, out of them. This can be done in the original way of Eudoxos, who left the arithmetical means of *anthyphairesis* aside in favour of the definition of proportion built on the multiplicative comparison of geometrically defined quantities. In this he anticipated the modern idea of the Dedekind cut. The other way, anticipating Cantor's definition of reals as fundamental (or Cauchy) sequences, takes the *anthyphairetic* definition as justified even in the infinite case, thus making its non-finite quality of a mere negation of the finiteness of some development into the true infiniteness of what this non-finiteness aims at. In light of this, it would be reasonable to replace the recommended picture of Hegel—the circle as a symbol of closed and completed infinity (*WL*: 119/164)—with the more subtle figure of a pentagon with the inscribed diagonals. These circumscribe the smaller pentagon with its own diagonals, and so on, *ad infinitum*, visualizing—within the limits, i.e., sides of the circumscribed pentagon—the reason why the anthyphairetic expansion cannot end.

The fact that the problem of determinateness is not of graphical but rather conceptual origin is demonstrated by Hegel by way of decimal expansion 0.285714…. In this form, as represented by three dots, it is only negatively determined via the 'mere' possibility of being always extendable and, thus, non-finite (*WL*: 209–11/287–89). In order to arrive at the true infinity of the sequence one has to negate this indeterminacy by means of the second negation, to be found in the subsequent finite specification of what the sequence aims at, in this case the ratio 2/7 = 0.285714…. In the irrational case, the same can be achieved by taking into account that the expansion in the above-mentioned cases such as the proportion of diagonal and side of square or pentagon proceeds according to some determinate rule. It is this very rule that allows the capturing of the expansion in its existence. As a result, one is able to establish suitable relations (=, <) between the adequate representations (of the fundamental sequence, or, alternatively, Dedekind cut) and to look at the result as sublation of two extremes in one concept of the *irrational ratio*. These extremes are the representation's non-finiteness (given by the mere possibility of further expansion), on the one hand, and its finiteness, or rather

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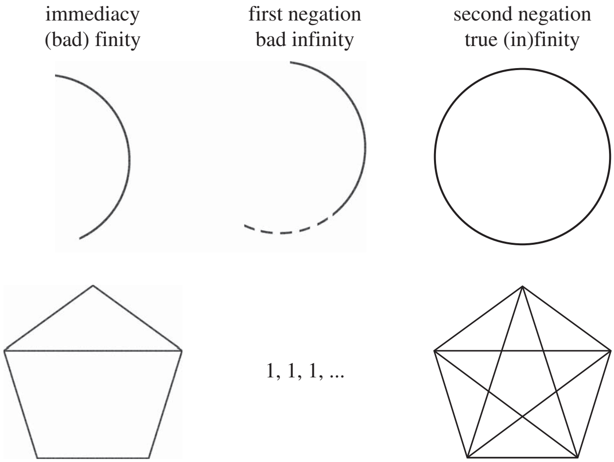
non-non-finiteness (given by the finite prescription as denial of the indefinite expandability), on the other hand. In the following figure, this dialectical



movement is schematized by the discussed visualizations of true infinity: the circle and the pentagon.

As far as the other examples, such as (in)justice or (im)perfection, are concerned, one can think of the mathematical entities and their imperfect earthly realizations as mentioned in Plato's *Seventh letter* and compare it with previous ones (Plato 2005: 342 a–e). (1) The imperfect *picture* of the circle is not able to represent the intended abstract distinction per se, but only via (2) a verbal *definition* which, being constituted from the conventionally chosen signs, does not refer to some independent heavenly entity but (3) to the *same* circle conceived in a *different*, normatively charged way. The main lesson is thus as follows. There are no circles, justice or numbers beyond this world but only their satisfactory realizations, their here's and now's, consisting in a different usage or attitude to the same worldly phenomena such as pictures (as representing the circle), particular people (as representing eternal justice and law) or objects (such as the Parisian meter representing the measuring etalon).

In this, the double negation consists in (1) denying it is the phenomenon in itself that one is interested in and (2) denying that there is something behind it (3) except its *relations to* other phenomena and the meaning it has for us. Here, the deeper conceptual plan of Hegel's narrative is already captured, including the dialectical transfers from (1) the *immediacy* through (2) the *medium* to (3) the *mediated immediacy*, and from (1) the *being-in-itself* through (2) the *being-for-itself* to (3) the *being-in-and-for-itself*, as will be discussed in the next section.

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II. The infinity of rule-following

After the various examples of the 'bad' concept formation have been sketched, I would like to generalize about them from a purely logico-epistemic point of view. The plan is to reconstruct the path of knowledge from *qualitative* differentiation to the point where *quantitative* speech, i.e., talk about the object, arises. To make clear which role the concepts of 'bad' and 'true' infinity play in this at the utmost generality, I suggest proceeding by way of an occasional comparison with the relatively new and seemingly unconnected problem known from Wittgenstein's *Philosophical Investigations* under the title of *rule-following*. Here, the mathematical examples connected to the expansion of number sequences are dealt with as problems of *meaning*, i.e., from the logico-epistemic point of view that we are after.

Let me, though, start with Hegel: In order to attain knowledge of what there objectively is, that is to say, to recognize something as something—e.g., the colour blue, the tone A or the numeral 5—one has to show the differences between it and the other members of the respective 'space'. In this context, Hegel uses the term 'negation', quoting Spinoza's '*omnis determinatio est negatio*' (although Spinoza only says '*determinatio negatio est*'). This first negation starts in the pre-linguistic space of phenomenal qualities, of what is immediately 'here' and 'now' (*Breite des Daseins*), be it the space of acoustic qualities, the haptic space or the visual one. To quote Hegel:

> Determinateness thus isolated by itself, as *existent* determinateness, is *quality*, something totally simple, immediate. […] Quality, in the distinct value of *existent*, is *reality*; when affected by a negating, it is *negation* in general, still a quality but one that counts as a lack and is further determined as limit, restriction. (*WL*: 85/118)

Such immediate difference is to be further mediated by other cases of the same, in proto-judgments such as

(α) 'this (here and now) is *A*, and this (here and now) is also *A*, but this (here and now) is not *A*,' etc.

In these, the attributions—contradictory from the immediate point of view (this is *A* and this is not *A*)—are unified into the mediated objects of our perception, or being here (*Dasein*). I called the instances of (α) 'proto-judgements', because they are not something which can be truly or falsely claimed but are only a means by which the original difference, e.g., that of being a cat as opposed to being a dog, is developed and established. The sequence of supporting as well as disproving cases

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(α’) ‘(proto-)Felix is a cat, (proto-)Furball is a cat, (proto-)Fritz is a cat’, etc.,

(α’’) ‘(proto-)Goofy is not a cat, (proto-)Gromit is not a cat, (proto-)Gaspard is not a cat’, etc.,

is always expandable by other cases and is, in this sense, non-finite. Such infinity, and here we will work in our Wittgensteinian parallel, is a ‘bad’ one in the sense circumscribed in what Kripke called Wittgenstein’s sceptical argument (Kripke 1982: 55). It may be phrased like this:

Because of its finiteness, the respective sequence (α) via (α’) and (α’’) is not able to guarantee *in itself* what will follow, i.e., one can look at it as the development of *indefinitely* many different (proto)concepts such as hairiness, four-leggedness, or the instance of rabies, depending on the cases met so-far. In a similar way, and contrary to what producers of IQ-tests think, any finite sequence of natural numbers can be extended indefinitely with the law or the rule of their development to be found subsequently. As a result: ‘no course of action could be determined by a rule, because every course of action can be made out to accord with the rule’ (Wittgenstein 1954: §201). The true infinity, and the basis of the true solution to this Wittgensteinian paradox, comes into being exactly in the moment when the sequence is negated in its indeterminateness. Thus, one obtains some determinate quality to be applicable to an indefinite range of instances (here’s and now’s) and, hence, the true infinity of some rule to be followed and used as a standard of the right and wrong uses.

The full separation of some difference (quality) and the cases it is independently applied to (objects), though, is yet to be developed. In judgements which can be true or false, such as ‘this is a cat’, the term ‘this’, at first, does not refer to anything in particular but only re-presents the form of the respective claim consisting in ‘cat’ being present. The full separation comes into being only later, in the basic form of the subject-predicate sentence in which the subject, e.g.,

> ‘this which is something particular, such as a cat’

acts as an independent instance ‘this cat’ of some fallible property, e.g., ‘this cat is black’. Hegel treats this separation under the title of *quantitative speech* in which, from some qualitative differences, the simple quanta—meaning discrete objects separated from each other—arise. From the ‘logical’ point of view, this happens roughly in the following way:

The original simple differentiation of the particular here’s and now’s, i.e., the first negation of the form

> ‘that cat (yesterday) ≠ this cat (now) ≠ …’,

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which in its possible continuation is infinite simply by definition, can be determined by means of the second negation, in which some of the original differences are denied. This happens in sentences like 'it is not true that this cat is not that cat' amounting to 'this cat is the same as that cat', or

'this cat = that cat'.

Such a contradiction is sublated in the very concept of *object* which these *representations* refer to. Here, through the double negation of the indefinite possibilities of what can be later described as specifying the *same* discrete *object* as opposed to other discrete objects, the *quantity* arises.

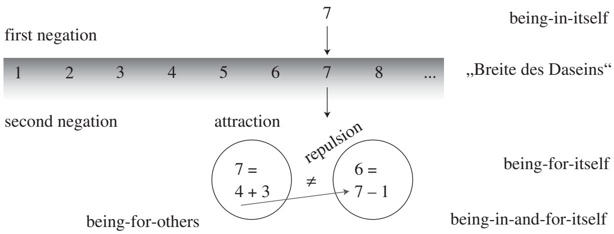
Hegel refers to this process via specific logical notions of the *attractive* and *repulsive forces* which capture the basic differences such as

(1) '$7 \neq 3+4 \neq 6$'

and their denial

(2) 'not $7 \neq 4+3$',

i.e., '$7 = 4+3$', i.e., the qualitative in-difference ('Gleich-gültigkeit').[7] In phase (1), these expressions, and the other qualitative differences, exist only *in themselves*, and, as such, they are basically different from each other. In phase (2), they are related to each other constituting mutually their *being-for-itself* ($7 = 4+3$) and *being-for-others* ($6 \neq 4+3$) in the sense that only then do they themselves—in their *internal* relation to the infinity of other objects—become independent and discrete objects. This is schematized in the following figure.



The figure also encapsulates the following points: Because of the internal connection, or continuity, by which 7 is contained in 6 because $6 = 7 - 1$ and vice versa, the numbers are not just a contingent totality of atoms but a necessary unity of citizens of the same realm. Every object is thus, by definition, a representation of true infinity constituting its *being-in-and-for-itself*:[8]

> The qualitative character, which in the One has reached the extreme point of its characterisation [*An-und-für-sich-Bestimmtsein*], has thus passed over into determinateness

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> suppressed [*aufgehoben*], i.e. into Being as Quantity. (*EPW*: 156/§98)

The emergence of the *quantum* from the sublation of some *quality* by means of its *double negation* happens in the context which is still qualitative in the following sense: The original quality becomes accidental as opposed to the sublated quality that is substantial as far as the essence of the object is concerned. This is to say that, in their constitution, the objects are pulled out of the qualitative realm of differences (this cat, that cat) by means of other qualitative differences and their predicative development bound by the indefinite 'also':

(β) 'this (A) is B and also C, but not D', etc.

The bad infinity of this specification is to be transformed into the true one at the moment when one already knows what the relevant qualities are and which of them apply to 'this A' and which do not. The fact, e.g., that $4+3$ is a prime number determines the validity of '$7=4+3$' and is, in this sense, in-different to this difference; the fact that $4+3$ contains the sign $+$ or that 7 is Arabic is not in-different in the same way and must be excluded by means of the first negation. The non-in-difference of the prime number quality with respect to the number 4 is a complement of the second negation, i.e., it is related to the constituted objects.

The true infinity of the determination of the object via the relevant qualities happens in the moment that it is generally settled which of these qualities are in-different and which of them are not, and under what conditions they belong to the specific object. Put differently: In order to specify what 7 as *number* contrary to mere *numeral* is, one has to specify, via the respective representations, the following. (1) What are its relations to the other objects of the relevant realm, i.e., the respective equalities ($7=4+3$) and non-equalities ($7\neq3+3$), and (2) what are the properties that it obviously has and that it can have without our knowing it yet, within the development of (β). The object is thus a result of (β)'s transformation from the bad (in)finity of always extendable but finite sequence to its true infinite version after the respective truth conditions are settled.

III. The discrete and the continuous aspects of quantity

In the previous section, I analysed the role that the concept of bad infinity plays in the constitution of the *quantitative difference*. In the steps to follow, this logical analysis leads to a state where the concept of the pure quantum, i.e., number, arises. As such, the science of arithmetic shows itself to be a discipline that makes

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the logical features of quantitative speech explicit and is thus, contrary to what Kant says, *analytical*.

Quantitative speech arises from the differentiation between (1) independent objects, defined by means of the qualitative in-difference or identity of certain representations or differences, and (2) those independent objects' qualities that these objects *can* but *do not have to* have. Against this background, the further degrees of explicitation are as follows: Starting with the subject-predicate form of

'(this) S is P'

one can proceed to the forms of

'some S are P' and 'all S are P'

in which the range of the qualitative determination P with respect to the quantitative basis S is more closely specified. Further specification involves the determination of the quantity in sentences such as

'there are 5 trees on the meadow' or 'the edge of the table measures 5.6 cm'

in which some specific quantity is ascribed to some being-here (*Dasein*). The traditional division of quantities into *discrete* and *continuous* ones originates from the basic subject-predicate sentential form, as it captures the constitution of the domains of objects and their being-in-and-for-itself. Let me focus on this.

At the bottom of this constitution is the dialectical process of setting and overcoming the limit as met in both parallel processes of (α) development of some property by means of its (counter-)instances and (β) development of some object by means of some properties. The sequence of natural numbers

1, 2, 3, 4, …

represents an obvious way of making this process explicit. This might be further explicated into the form of the recursive definition

(1) 1
(2) x + 1

in which both aspects of this development, i.e., (1) setting the limit and (2) overcoming it, are separated. Here, at the same time, the discreteness of the individual numbers is contrasted with the continuity of the process of the unlimited addition of the number 1 to the numbers construed so far.

This basic differentiation—mirroring the process already met with in the development (α) of qualitative differences—is further specified in the higher

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process of establishing the quantum. Discreteness and continuity are thus only different names for the logical forces of repulsion and attraction and their role of attracting some qualitative representations as expressions of the same objects ($5 = 3+2$), or their being-for-itself, and their concurrent repulsion ($5 \neq 3+3$), constituting their being-for-others. Now, the logical function of these forces consists in relating all the underlying differences together by means of the higher type of in-difference which, e.g., allows one to recognize some tone as the false variant of A (false A), i.e., as identical with A, as opposed to other, qualitatively different tones which will be treated as something quantitatively different from A (e.g., B). When this is realized, one sees almost immediately that the continuity of the respective objects stems from their internal relations to each other. This is what makes these objects citizens of the same domain or society as opposed to a mere set of (social) atoms. Hegel comments on this as follows:

> *Continuity* is therefore simple, self-same reference to itself unbroken by any limit or exclusion—*not*, however, *immediate* unity but the unity of ones which have existence for themselves. [...] Quantity is the unity of these moments, of continuity and discreteness. At first, however, it is this continuity in the *form* of one of them, of *continuity*, as a result of the dialectic of the being-for-itself which has collapsed into the form of self-equal immediacy. Quantity is as such this simple result in so far as the being-for-itself has not yet developed its moments and has not posited them within it. (*WL*: 154–55/212–13)

Kant's antinomies, in which the world adopts the quality of being simple and that of being complex, finiteness and infinity, discreteness and continuity, freedom and determinism, are only another articulation of this emergence of what there is from the sublation of the opposites by means of the double negation. From Hegel's point of view, however, Kant stopped prematurely at the contradiction as something which reason can only forbid, not realizing that it belongs to reason's very nature and to the means by which reality is conceived (see *WL*: 200–202/275–77).

The further development of quantitative speech, in which discrete and continuous aspects of reality are made explicit in the concepts of number as the form of *pure quantity*, consists in establishing sentences such as

> '5 is prime' or 'π is transcendental'

with numbers being the very objects of talk, as opposed to their syncategorematic forms such as 'there are *5 trees*' or 'this is *3.2 m* long' mentioned above. The purity

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of the quantum is given exactly by the fact that no specific qualitative but only quantitative determinations are mentioned:

> Thus completely posited in these determinations, quantum is *number*. The complete positedness lies in the existence of the limit as a *plurality* and so in its being distinguished from the unity. Number appears for this reason as a discrete magnitude, but in unity it has continuity as well. It is, therefore, also quantum in complete *determinateness*, for in it the limit is the determinate *plurality* that has the one, the absolutely determined, for its principle. Continuity, in which the one is only implicitly present as a sublated moment—posited as unity—is the form of the indeterminateness. (*WL*: 169/232)

As objects, of course, numbers, including reals and other kinds, are things that are determined and thus *discrete* as far as conceptual separateness is concerned. And the other way around, as for their being-for-others, all objects, including natural numbers, are present in other members of the same domain, which provides for their *continuity*. As a result, the way one can conceive the respective numbers as the ultimate explication of the respective aspects of the dialectical process must consist in the peculiarity of their constitution which, in the case of real numbers, goes back to the specific way they were brought together by the logical forces of attraction and repulsion.

The uniqueness of real numbers, as opposed to the prototype of natural numbers, cannot lie only in the fact that they are *infinite* in their total (1, 2, 3, 4, ...) and in the total of their representations (3, 2+1, 4 – 1, $\sqrt{9}$, ...) because this, obviously, holds for the natural numbers as well as for other domains of reference. Although the true ground can be found in the insight that their representations are infinite in itself, in their inner structure, as demonstrated by the case of *alogoi logoi*, I would suggest finding a second and more interesting way of characterization of their *continuity* in the later argument of Cantor. According to it, every enumeration of real numbers $r_1, r_2, r_3, r_4, \ldots$, via their decimal representations

$$\begin{array}{ll} \mathrm{r}_1 & \boldsymbol{a_{11}}, a_{12}, a_{13}, a_{14} \ldots, \\ \mathrm{r}_2 & a_{21}, \boldsymbol{a_{22}}, a_{23}, a_{24} \ldots, \\ \mathrm{r}_3 & a_{31}, a_{32}, \boldsymbol{a_{33}}, a_{34} \ldots, \\ \mathrm{r}_4 & a_{41}, a_{42}, a_{43}, \boldsymbol{a_{44}} \ldots, \\ \vdots, & \end{array}$$

leads to the determination of the number which is not among them, simply by the deformation of the diagonal $d = a_{11}, a_{22}, a_{33}, a_{44} \ldots$. Traditionally, this is read as a proof that there are 'more' real numbers than there are natural numbers, but this is, in fact, a *bad* argument based on a 'mere' first negation. The positive conclusion one

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actually has is that to every enumeration of the names of real numbers, the name of a quite different number can be built, *etc*. This 'etc.' represents, first, an indefinite way of the higher order process which does not say how many real numbers there are until one, second, positively determines what the real number is, i.e., until one sets the 'second' negation. From the historical point of view, this is exactly what Brouwer, and particularly the constructivist mathematicians like Weyl and Lorenzen, asked for in their attacks on classical mathematics.

Hence, by using the *diagonal argument* as an explanatory device of what the continuity of real numbers consists, we have indicated how Hegel's notions can be fruitfully applied to the subsequent development of mathematics. This is possible on both the conceptual and historical level. Let me briefly indicate how: (1) The fact, e.g., that the elementary concepts of the set-theoretical framework are based on 'bad' decisions fully shows itself not in the notorious phenomena of set-theoretical paradoxes which are, as we know, always reparable by adopting sufficiently drastic measures but in *undecided and undecidable* problems such as *continuum hypothesis*. Their truth is undecided because it was left open—i.e., underdetermined—what the size (cardinality) of the continuum is (sentences like $X = Y$) and how it relates to the cardinality of other sets in general (sentences like $X < Y$). (2) On the other hand, the 'true' reading of the diagonal argument in its suggested role as the explicitation of the continuous aspect of quantity consists in the following observation: In opposition to natural numbers, real numbers cannot be completely specified in advance, but in their constitution they rely internally on the further development of the whole domain of objects.[9] Brouwer's concept of the 'denumerably unfinished' (Brouwer 1907), as applied to real numbers, captures exactly this idea. It is this q*inner* continuity, not their specification as self-standing objects, that makes the real numbers continuous, as opposed to the *external* characterization of their structure by means of topological qualities such as completeness, perfectness, connectedness, etc.

In the next section, I would like to elaborate on these observations by presenting Cantor's approach to mathematics from the dialectical point of view as a kind of paradigm of 'bad' concept making. This is to say that Cantor's concepts are not to be treated negatively in the absolute sense of the word but relatively to their 'negation' (such as that of mathematical finitism) and their subsequent sublation. As such, they are not interesting in themselves, but as an example, quite in accordance with our original plan of demonstrating the radically logical nature of the respective notions.

IV. The mathematical aftermath

The conceptual means of Cantor's set theory, on which his theory of real numbers is based, illustrate the possibilities in which the dialectical process can

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lead us astray, in the direction of taking some differences as existing *in themselves* or introducing them by a *mere* (one) *negation*. This conceptual failure is very general, affecting the thinking in its very core, as known, e.g., from the antinomies of Zeno. Just consider the idea that the in-definite divisibility of the line directly means that the line is composed of infinitely many points which need not be further specified by means of some quantity such as constructability with ruler and compass. Then, however, problems such as *the quadrature of the circle* or *the doubling of the cube* cannot even be phrased: In an indefinite realm of points of the line *there is*, by simple fiat, one corresponding to the circumference of the given circle or the side of the doubled cube no matter whether one can find it or not. In a similar way, Cantor concluded that in talking about sets, numbers, points, etc., one does not need and even cannot have any specific quality delimiting them since every such specification can be overcome in a way indicated by the diagonal argument.

All these are the same 'bad' arguments with which we met in the case of earthly justice or human science. The fact that every system of law will, in the end, meet its limits does not justify one's simply believing in justice behind this world, since it not only doesn't contribute to its development but also hinders the efforts to improve it by earthly means. The illusion of the heavenly argument is thus the very illusion of the first negation as sufficient conceptual means. Such a negation, however, is only the source of the contradiction to be overcome by the second negation as the case of *alogoi logoi* sufficiently manifested and as was even more drastically documented later by the argument of Gödel. According to its 'bad' reading there are some truths of arithmetic which *cannot be proven* by any finite means. What one really has, though, is that in an axiomatic system of arithmetic that is expressively strong enough to capture the usual operations one is always able, in a way similar to Cantor's diagonal means, to construe an arithmetical sentence which is *not provable* from the axioms and is *because* of that true. Since this truth is proven as part of the whole argument, i.e., by finite means, it transcends the original proof-theoretical system simply by definition. The argument is thus not only the instance of setting and overcoming the limit, but also a manifestation of the self-conscious quality of human knowledge to be basically found in the very form of the diagonal construction.

Now, one can gradually see that Cantor's concept formation is in no case dialectically naïve. His definitions of set or cardinal and ordinal number are not delimited in a traditional way of being simply there but are (1) based on stating the in-difference conditions for the given representations and/or are (2) generated by the dialectical process of setting and overcoming the limit, primordially in the process of the transfinite induction. Cantor's set theory, in fact, starts with (3) consciously overcoming one of the most basic contradictions of infinity based on the old observation that, in attempts at measuring the infinite

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totalities such as the set of all natural numbers and the set of all even numbers, one ends up at variance with the seemingly natural and intuitive postulates such as the fifth common notion of Euclid: *the whole is greater than the part*. Thus, one can say that through points (1–3) Cantor meets all the important features of Hegelian dialectics.

The real dialectical failure of Cantor's set theory, as described above, is a serious one, but it is easily overlooked since it is not the objects, sets or numbers that Cantor treats as existing in themselves but the processes that define them. The error of 'bad' concept-making emerges thus on the higher level of being-for-itself: The pure relational structure, including the fact that one is in principle capable of comparing *coherently* the sizes of two infinite totalities, which seemed impossible to Bolzano and his predecessors, does not guarantee that there are infinite quantities to which these abstract relations apply, as the continuum hypothesis and other undecided and/or undecidable results demonstrate. In Hegel's terms, Cantor did not prove himself able to bring the being of sets from its *being-for-itself* to its *being-in-and-for-itself*, leaving us, in the end, with only the empty form of a theory without a clear range of application. The axiomatic theory of Zermelo only manifests this incapability particularly well if compared with the axiomatizations of arithmetic. In spite of their shortcomings, or, better, because of them, these systems were able to extend the 'knowledge' as based on the pre-axiomatic theories significantly, as, e.g., the 'discovery' of the non-standard numbers signifies. The reason is simply that there arose a clear and well-defined difference between the *standard model of arithmetic* and the *axiomatic theory* describing it which, particularly due to Gödel's results, enabled their fruitful dialectical interaction. In set theory the seemingly analogous difference between the *iterative hierarchy* and the axioms describing it turns out to be rather a virtual one, of which conceptual *underdetermination*, as mentioned above, is a significant sign.[10]

From this point of view, the intuitionists' and constructivists' reaction to the set theoretical and logicist attempts at foundations of mathematics strives to secure the original qualitative (immediate) basis (intuition) in the mere quantitative determinations by a careful reconstruction of the original practices (constructions) that led to the concept of quantum. The real numbers are thus, in their being-in-and-for-itself, about to be bound more closely to the computational algorithms and calculations with finite artefacts. The dangerous feature of this opposition, as epitomized not only in Brouwer's *mentalism*, but also in Hilbert's later *finitism*, consists in forcing us to see the original practices, intuitions or signs of language that define the objects of interest as something existing in themselves, without the need of theoretical foundations as captured in the concept of being-for-itself. As such, they are committing a similar mistake—the mistake of 'bad' finitism—as their ideological opponents.

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The truly dialectical way of resolving these mistakes was provided by Weyl, who saw the importance of both Brouwer's recourse to the phenomenal (qualitative) basis of mathematical (quantitative) objects as well as the need for their theoretical and intersubjective foundation as given by the medium of language and its employment within the axiomatic definitions. This means that one has to realize that symbols, in fact, incorporate both the immediacy of the sensuously given and the mediacy of their standing for something else. As Weyl says:

> If Hilbert is not just playing a game of formulae, then he aspires to a theoretical mathematics in contrast to Brouwer's intuitive one. But where is that transcendent world carried by belief, at which its symbols are directed? I do not find it, unless I completely fuse mathematics with physics, and assume that mathematical concepts of number, function, etc. (or Hilbert's symbols) generally partake in the theoretical construction of reality in the same ways as the concept of energy, gravitation, electron, etc. The history of physics shows that intuition and theory must constantly go hand in hand. (Weyl 1998: 140)

What Wittgenstein's sceptical argument and its sceptical solution demonstrated was, in fact, exactly the same point. Mathematical knowledge with its alleged certainty and eternality does not particularly differ from the knowledge of empirical facts. They are both starting in the phenomenal space of various here's and now's and developing it in a way which seemingly points to another world beyond the empirical one but, in fact, only mediates this first world's sensuous immediacy.

In the final section, my goal is to demonstrate how the consequences of bad concept-making and the 'bad' infinity in general can be effectively blocked if one takes into account the social origin of this mediation. In this, the previous comparisons of Hegel's and Wittgenstein's epistemological insights will be used and extended along with a comparison between the master-slave parable, on the one hand, and the problem of a student's mastering some rule, on the other.

V. The philosophical aftermath

Wittgenstein's solution to his paradox—circumscribed as the transfer from the 'bad' (in)finity of some instances (such as specific cats) to the true infinity of the developed concept (cat)—lies in the very observation that there is no third object ('third cat') to be used as a *tertium comparationis* of the wrong and right uses. There is only the ability of two (ideal) speakers to achieve some stable use of our concepts. As a result, there is no beyond of externally or internally given objects, no cat to be

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known in itself, but only our capability of theoretically charged knowledge about this 'cat', i.e., our practices of recognizing the 'cat' *for us*, mediated by the phenomenon of the intersubjectively rooted rule-following. Accordingly, the rules and their following are not primarily something explicitly given—explained, e.g., by some words that have to be understood and thus depending on some other rules—but *customs* or *institutions* (Wittgenstein 1954: §199).

The reason that I, to be able to follow some rule, need there to have somebody else stems from the normative nature of rule-following. As Wittgenstein famously argued, purely private control of the application of some rule does not allow me to differentiate between the case when I am following a rule and when I only think that I am following one (Wittgenstein 1954: §202). As a result, the difference between right and wrong collapses into the mere expression that somebody finds something right or wrong, which lacks any objective validity. This, in fact, is exactly the situation preceding the 'life and death fight' of Hegel's parable, in which normativity does not arise until one's biological life is risked. Read epistemologically, by risking the certainty of the Cartesian first-person view, the fight of mere 'private' opinions leads to the emergence of intersubjective and thus fallible knowledge.[11] In his reflections on rule-following, Wittgenstein develops exactly these ideas that discuss the conditions under which one is mastering some rule. These turn out to consist in the mutual conditioning of the pupil (servant) and his teacher (master) in its necessary evolution into the dialogue of two equal partners within the institution of rule-following.

Let us look at this as a variation on the inside vs. outside problem and the respective kinds of knowledge. The idea that there is something like immediate and thus absolutely certain knowledge is the idea of the 'bad' finiteness, stemming from the opinion that because of its certainty the claim 'something looks like X' is epistemologically prior to and thus more certain than the claim 'something is X'.[12] Both Hegel in his *Phenomenology of Spirit* and Wittgenstein in his *Philosophical Investigations* are systematically criticizing this by pointing out that there cannot be—for simple conceptual reasons—any unmediated, i.e., infallible 'knowledge', and that its mediation and the source of fallibility consists in its social nature. The dialectic of mediation originates from the fact that a mere and thus 'bad' negation of the privateness of knowledge, of its total dependency on the cognizing subject, leads to the 'bad' infinity of the opposite idea of the objective world lying beyond our limited cognitive capacities.

These extremes are sublated in the concept of intersubjectively grounded knowledge being dependent on society but not on its individual members. Here, society cannot be conceived as a mere totality of social atoms, i.e., as a kind of bigger individual differing from the single ones only by a greater complexity, but as a normatively stratified whole held together by mutual acknowledgment of its elements, i.e., on internal, not external grounds. To recognize and to articulate

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this insight, the propaedeutic case of real numbers and their internally conceived continuity can be mentioned so as to proceed to the more complicated phenomena of the social contract and its interiorization through the adoption of a certain attitude. As in the case of numbers, one does not approach the social laws and rules in a Hobbesian external way. Society is both finite in that it depends on the opinions and particular deeds of individuals, and infinite in that it is rooted in the attitudes of individuals to each other and to institutions such as justice that transcend individual needs.

The social duality of knowledge, as grounded in the concept of mutual acknowledgment of two (ideal) speakers, might be further explained as being responsible for the above mentioned separation of the institution of knowledge and of what it is knowledge about. In general, one gets the separation of what is differentiated from what it is to be independently applied to, as mirrored in the elementary sentence 'S is P' in its opposition to the proto-judgement 'this (here and now) is A'. In this way, one can replace the 'bad' concept-making involved in the belief in two kinds of entities (such as, e.g., the intuition and concept)—to be cognized by different cognitive capacities (such as, e.g., senses and mind) and all the traditional dualities—by a theory that gives these dualities the true internal meaning of socially stratified knowledge. This is thoroughly justified and elaborated in Brandom's *Making It Explicit* (Brandom 1994), particularly in his characterization of knowledge as a *hybrid deontic status*.

As for the relation of mathematics, with its alleged eternal truths, to the sociality of knowledge, what particularly comes to mind are our previous remarks concerning the context-dependence of the constitution of real numbers on those previously constituted ones, as captured in the 'true' reading of the diagonal argument. The argument's epitomization of the self-conscious and self-reflective dimension of knowledge provides another important example that is also to be found in other self-referential results of modern logic and mathematics, such as Gödel's theorems. The most adequate mathematical expression of the social duality might be found in the axiomatic (and thus inferentialist) design in which the axioms, theorems or mathematical sentences are treated not as dead artefacts, but as something one must state and defend against somebody else. Technically, such an idea was elaborated by Weyl's and Hilbert's pupil Paul Lorenzen within the apparatus of dialogical logic and its game-theoretical semantics (Lorenzen 1962).

VI. Conclusion

The paper dealt with the concept of 'bad' infinity from several points of view of different generality: (1) The most general point was that of a logical or

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epistemological nature concerning the very talk about our world and its development from purely qualitative, proto-linguistic differentiations to standard talk about quantitatively different objects and their contingent properties. (2) This was introduced within the narrower mathematical context, particularly that of establishing the concept of quantum, to be generalized into the philosophical problem of 'bad' concept-making and its comparison with the problem of rule-following. (3) The goal of these comparisons was, first, to show that 'bad' infinity does not primarily mirror the problem of the difference and superiority of actual over potential infinity, as it goes deeper than that. The second goal was to demonstrate that, because of this deeper significance, the 'bad' infinity can be applied indiscernibly to mathematical development after Hegel, particularly to the antithetical approaches of set theory (actualist concept of infinity) and constructivist tradition (potentialist concept of infinity). Both of these points are at variance with the still prevalent evaluation of Hegel's logical doctrine and its applicability to mathematics, particularly within the Anglo-American philosophical tradition.

Against this background, the role of mathematics and its philosophy in Hegel's (as well as Wittgenstein's) system is clear: It does not represent the subject 'in itself' but is rather a means of helping philosophy to express knowledge *about* knowledge, via the dialectical way of being both, its expression and—due to its own self-conscious and social nature—instantiation.

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Notes

[1] Bolzano's opinion can be cited as rather characteristic: '[…] some philosophers, particularly of more recent times, like Hegel and his followers, are not satisfied with this infinity so well known to mathematicians. They call it contemptuously "the bad infinity" and claim to know a much higher one, the true, the *qualitative infinity* which they find especially in *God* and generally only in the *absolute*. If they, like Hegel, Erdmann and others, imagine the mathematical infinity

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only as a quantity which is *variable* and has no limit to its growth [...], then I would agree with them in their criticism of this concept of a quantity itself never *reaching* but only *growing* into infinity. A *truly infinite* quantity, e.g. the length of the whole straight line unbounded in both directions [...] needs precisely not to be variable, as it is in fact not in the example mentioned' (Bolzano 2004: §11). For similar remarks see also Cantor (1932: 391).

[2] See, e.g., Žižek's list of what Hegel 'cannot think' (Žižek 2012: 455, 457–58) and his remarks about Hegel's 'inadequate understanding of mathematics, his reduction of mathematics to the very model of the abstract "spurious infinity"'.

[3] See, particularly, Stekeler-Weithofer (1992) on Hegel's logic and the chapter on the philosophy of mathematics in Stekeler-Weithofer (2005).

[4] *WL* = Hegel, *The Science of Logic*, ed. G. di Giovanni (Cambridge: Cambridge University Press, 2010)/Hegel, *Wissenschaft der Logik I* (Frankfurt: Suhrkamp, 1986c).

[5] See *WL*: 226–27/310, which does not refer to Berkeley, but to Carnot.

[6] *JS* = Hegel, *Jenaer Schriften 1801–1807* (Frankfurt: Suhrkamp, 1986b).

[7] See Stekeler-Weithofer (1992) for the exegesis of Hegel's logical terminology along analytical lines.

[8] *EPW* = Hegel, *The Logic of Hegel*, trans. W. Wallace (Oxford: Clarendon Press, 1874)/Hegel, *Enzyklopädie der philosophischen Wissenschaften I* (Frankfurt: Suhrkamp 1986a).

[9] In this form, this observation is due to Wittgenstein (1956: 131).

[10] I do not have enough space here to elaborate on this remark. For some broader context of where this argument is going see, e.g., Feferman (1998: 7).

[11] The epistemic interpretation of the master-slave dialectic can be found in Stekeler-Weithofer (2014).

[12] For arguments against this opinion in this form see Sellars (1956).

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