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A NOTE ON THE SLOPE OF THE AGGREGATE DEMAND CURVE AT THE ZERO LOWER BOUND

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This paper analyzes a regime-switching New Keynesian model to understand what happens to the aggregate economy when the nominal interest rate hits the zero lower bound (ZLB). Contrary to the literature, our model predicts that the aggregate demand curve is not always upward sloping when the ZLB binds. Instead, it depends on expectations. If the expected duration of the ZLB is short but consistent with expectations surveys, the AD curve can be downward sloping. In that case, the fiscal multiplier is moderate and supply-side reforms are expansionary. These results complement existing findings in the literature.

Keywords: New Keynesian Model, Zero Lower Bound, Fiscal Multipliers, Supply-Side Reforms

1. INTRODUCTION

Since the Great Recession, a strand of literature has looked into the issue of a binding zero nominal interest rate (the zero lower bound, or ZLB) and its impact on the fiscal multiplier. A primary finding is that the government spending multiplier is larger than that in normal times (Eggertsson (2001*a*) and Christiano et al. (2011)). The key mechanism is that when the ZLB is binding, the aggregate demand curve is necessarily upward sloping: without any response in the nominal interest rate, higher inflation stimulates instead of depresses aggregate demand because it lowers the real interest rate. A series of papers, by Christiano et al. (2011), Woodford (2011), Carlstrom et al. (2014), Mertens and Ravn (2014), Boneva et al. (2016), and Eggertsson and Singh (2019), have drawn nearly identical theoretical conclusions with respect to the slope of the AD curve, under various assumptions about the economic environment.

Is this theoretical result supported by empirical evidence? So far the news has not been good. Garín et al. (2019) investigated whether or not the US economy

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operated in the upward-sloping region of the AD schedule between 2008 and 2015, when the nominal interest rate was essentially pegged at zero. Theory suggests that when the AD curve is upward sloping, a positive productivity shock is deflationary, and its effect on output is weak and can even be contractionary.[1] Garín et al. (2019) estimated the responses of output and inflation to a rise in total factor productivity and find an opposite effect on output: while its impact on inflation is negative, its effect on output is strongly positive, higher than in the period 1984–2007. They suggest that there is "some failing of the textbook NK model and its more complicated but closely related medium-scale dynamic stochastic general equilibrium variant." Other studies confirm this result. Wieland (2019) uses VAR techniques to study the impact of oil shocks in and after the Great Recession in the United States and finds that this negative supply shock raised expected inflation but had a contractionary effect on the economy. They also study the impact of the Great East Japan Earthquake in 2011 and find a similar result. These findings are echoed by Cohen-Setton et al. (2017)'s study of the supply-side policies of France in 1936. Their time series and cross-section evidence show that the policies raised prices but lowered output. They present this as a puzzle: there is an apparent "disconnect" between the data and the new-Keynesian model and conclude that the model "is a poor guide to the effects of supply-side shocks in depressed economies."

In this note, we propose a revised version of the New Keynesian model that can potentially reconcile the discrepancy between theory and data. In our model, the slope of the aggregate demand curve is not necessarily upward sloping when the ZLB is binding. Instead, it depends on expectations. When the expected duration of the ZLB is short, the AD curve is downward sloping. Our revision of the New Keynesian model is quite minimal. The basic structure of the model is the same as in all of the aforementioned papers: the economy consists of two states, a normal state and a ZLB state. When in the ZLB state, agents' decisions are complicated by the uncertainty that the future state is either another ZLB state or a normal state. In the referenced papers, the normal state is simplified to be just the steady state of the economy and is not stochastic. This is the assumption that we revise. We assume that both states are stochastic: one state has a binding ZLB, and in the other, the monetary authority implements a Taylor-type interest rate rule, and the evolution of the two states is characterized by a Markov process.

Why does this simple revision lead to a different prediction about the slope of the AD curve? If the normal state is simply the non-stochastic steady state, an implicit assumption is that what happens in the normal state has no impact on the decisions agents make in the ZLB state. This mechanism changes when we assume that in the normal state, the central bank implements a Taylor rule, and the transition probability from a ZLB state to the normal state is positive. Consider a supply-side shock that raises inflation in the ZLB state. If the next state is still a ZLB state, then expected inflation rises, the real rate drops, and the demand goes up. But what if the next state is a normal state? The agents understand that the inflation will be curbed by the Taylor-rule regime in that state.

In this case, expected inflation will decrease, and the real interest rate will go up. Since agents must make this decision under uncertainty, they need to weigh both possibilities. The latter possibility partially offsets the expansionary effect of the former. This effect is magnified when agents must consider the long-run impact of their decisions, especially when on average, the probability of the normal state is higher than that of the ZLB state. As a result, the overall effect on output can be negative, and the AD curve becomes downward sloping.

We find that the AD curve has a negative slope if (1) the transition probability from the normal state to another normal state is high and/or (2) the transition probability from the ZLB state to another ZLB state is relatively low. The magnitude of the former ranges from 0.55 to 1, and of the latter, from 0 to about 0.5. Both sets of values are consistent with findings from empirical research. In particular, survey data show that between 2009 and 2014, the mean expected duration of the ZLB was quite short in the United States, ranging from 4 to 11 quarters (Swanson and Williams (2014)). We also find that the stronger the response of the monetary authority's response to inflation, the more likely that the AD curve is downward sloping.

The policy implications are immediate. If the economy operates in the downward region of the AD curve, the size of the multiplier will be moderate, smaller than under an upward-sloping AD curve. Our computation confirms this result. In our model, the multiplier is higher than 1, but is smaller than those estimated in the aforementioned papers. Two, supply-side reforms are expansionary. In a series of papers, Eggertsson et al. (2014) and others argue that supply-side reforms are contractionary when the ZLB is binding, primarily because the AD curve is upward sloping. Our result confirms that supply shocks have the intended expansionary effect when the AD curve is downward sloping at the ZLB. These results complement existing findings about the ZLB in the literature, as the case of a binding ZLB and a downward-sloping AD curve have not been considered in analyses of both policy issues. Our finding offers additional insights into how macroeconomic policies might play out in the real world.

2. THE MODEL

Our goal is to use a model that is as transparent and tractable as possible. We consider a New Keynesian model that is based on the work of Christiano et al. (2011). We will start with log-linearized system and leave the micro-founded model and the derivations of equilibrium conditions in the appendix.

Economic dynamics are described by the Phillips curve, the IS curve, and a policy equation. The Phillips curve is

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \left[\left(\frac{1}{1-g} + \frac{N}{1-N} \right) \hat{Y}_t - \frac{g}{1-g} \hat{G}_t \right] + u_t, \tag{1}$$

where $\hat{Y}_t$ is the output, π_t is the inflation, $\hat{G}_t$ is a government-spending shock, u_t is a cost-push shock, β is the discount factor, θ is the index of price stickiness,

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$g = G/Y$ is the steady-state spending-output ratio, N is the steady state of hours worked, and $\kappa = (1-\theta)(1-\beta\theta)/\theta$. u_t is typically results from by exogenous variations in mark-ups. This shock is not immediately relevant, but will become useful when we discuss supply-side reforms.

The IS curve is

$$\hat{Y}_t - g\left[\gamma(\sigma-1)+1\right]\hat{G}_t = E_t\left\{\hat{Y}_{t+1} - g\left[\gamma(\sigma-1)+1\right]\hat{G}_{t+1} - (1-g)\left(R_t - r_t^n - \pi_{t+1}\right)\right\}, \qquad (2)$$

where R_t is the nominal interest rate, r_t^n is the natural rate of real interest, $\gamma \in (0, 1)$, and $\sigma > 0$.

The monetary authority's policy rule is

$$R_t = \max\left(0, r_t^n + \phi_1\pi_t + \phi_2\hat{Y}_t\right). \qquad (3)$$

There are two states of the world. In the normal state ($s_t = 1$), the central bank implements a Taylor rule, and there is no active fiscal policy ($\hat{G}_t = G^T = 0$). In a ZLB state ($s_t = 2$), the economy experiences a large adverse shock ($r_t^n = r < 0$), as in Christiano et al. (2011). The shock causes the ZLB to bind ($R_t = 0$), and in the meantime, the fiscal authority will increase government spending ($\hat{G}_t = G^Z > 0$). The evolution of the two states can be characterized by a Markov chain with transition probabilities $p_{ij} = P\left[s_t = j | s_{t-1} = i\right]$, where $i, j = 1, 2$. In each period, there is a probability $p_{11} \geq 0$ that the nominal interest rate responds to inflation actively, $\hat{G}_t = G^T = 0$ and $r_t^n = \bar{r}$, where $\bar{r}$ is the steady state of the nominal interest rate, and a probability $p_{22} \geq 0$ that ($r_t^n = r < 0$), the ZLB binds ($R_t = 0$), and $\hat{G}_t = G^Z > 0$.

In state 1, the Taylor-rule regime ($s_t = 1$), equation (1) becomes

$$\pi_t^T = \beta p_{11} E_t \pi_{t+1}^T + \beta p_{12} E_t \pi_{t+1}^Z + \kappa\left(\frac{1}{1-g} + \frac{N}{1-N}\right)\hat{Y}_t^T, \qquad (4)$$

where $p_{11} + p_{12} = 1$, and T denotes the Taylor-rule regime and Z the ZLB regime.

Combining equations (2) and (3), we have

$$\hat{Y}_t^T = p_{11}E_t\hat{Y}_{t+1}^T + p_{12}E_t\hat{Y}_{t+1}^Z - p_{12}g\left[\gamma(\sigma-1)+1\right]G^Z - (1-g)\left(\phi_1\pi_t^T + \phi_2\hat{Y}_t^T - p_{11}E_t\pi_{t+1}^T - p_{12}E_t\pi_{t+1}^Z\right). \qquad (5)$$

In state 2, the ZLB regime ($s_t = 2$), equation (1) becomes

$$\pi_t^Z = \beta p_{22}E_t\pi_{t+1}^Z + \beta p_{21}E_t\pi_{t+1}^T + \kappa\left[\left(\frac{1}{1-g} + \frac{N}{1-N}\right)\hat{Y}_t^Z - \frac{g}{1-g}G^Z\right], \qquad (6)$$

where $p_{22} + p_{21} = 1$.

The IS curve (2) becomes

$$\hat{Y}_t^Z - g\left[\gamma(\sigma-1)+1\right]G^Z = p_{22}E_t\hat{Y}_{t+1}^Z + p_{21}E_t\hat{Y}_{t+1}^T - p_{22}g\left[\gamma(\sigma-1)+1\right]G^Z + (1-g)\left(r + p_{22}E_t\pi_{t+1}^Z + p_{21}E_t\pi_{t+1}^T\right). \qquad (7)$$

An important feature here is that when the economy is in state 1, economic decisions are affected by expected values of output and inflation in state 2, and vice versa. This feature is absent in the models studied in the literature. In fact, Christiano et al. (2011) and Eggertsson (2010)'s models are special cases of our model. If we let $p_{11} = 1$ and assign the steady state values to output and inflation in state 1, then the model collapses to a version of their models. In that case, (4) and (5) will drop out, and economic dynamics are only determined by two equations

$$\pi_t^Z = \beta p_{22} E_t \pi_{t+1}^Z + \kappa \left[\left(\frac{1}{1-g} + \frac{N}{1-N} \right) \hat{Y}_t^Z - \frac{g}{1-g} G^Z \right], \tag{8}$$

$$\begin{aligned} &\hat{Y}_t^Z - g\left[\gamma(\sigma-1)+1\right] G^Z \\ &\quad = p_{22} E_t \hat{Y}_{t+1}^Z - p_{22} g \left[\gamma(\sigma-1)+1\right] G^Z + (1-g)\left(r + p_{22} E_t \pi_{t+1}^Z\right), \end{aligned} \tag{9}$$

which are simplified versions of (6) and (7).

3. THE SLOPE OF THE AGGREGATE DEMAND CURVE AT THE ZLB

The economic system has a unique equilibrium solution, in which output and inflation are determined by the two fundamental shocks.[2] The slope of the AD and AS curves can be derived as part of the solution process. The appendix describes the solution algorithm, the derivation of the AD and AS relationships, and how to compute their slopes.

Focus on the case that the current state is a ZLB state. As derived in the appendix, the slope of the AD curve is

$$S_{AD} = \frac{1 - p_{22} - p_{21} A(2,2) - (1-g)\, p_{21} A(1,2)}{p_{21} A(2,1) + (1-g)\, p_{21} A(1,1) + (1-g)\, p_{22}}, \tag{10}$$

where $A = \begin{pmatrix} 1 - \beta p_{11} & -\kappa\left(\frac{1}{1-g} + \frac{N}{1-N}\right) \\ (1-g)(\phi_1 - p_{11}) & 1 - p_{11} + (1-g)\,\phi_2 \end{pmatrix}^{-1} \begin{pmatrix} \beta p_{12} & 0 \\ (1-g)\, p_{12} & p_{12} \end{pmatrix}$.

In the special case of $p_{11} = 1$, the slope of the AD curve can be simplified to

$$\frac{1 - p_{22}}{(1-g)\, p_{22}}. \tag{11}$$

Note that this expression is necessarily positive.

In the general case, however, the sign of the slope is not obvious. We use calibrated simulations to examine its values. Figure 1 shows that the slopes of the AD curve at various values of the transition probabilities. As in Christiano et al. (2011), parameters are calibrated as follows: $\beta = 0.99$, $\phi_1 = 1.5$, $\phi_2 = 0$, $\gamma = 0.29$, $g = 0.2$, $\sigma = 2$, $\theta = 0.85$, and $N = 1/3$. Evidently, the slope can be negative, even when the ZLB binds. This happens when p_{11} is relatively large (between 0.55 and 1), and p_{22} is relatively small (between 0 and 0.5).[3]

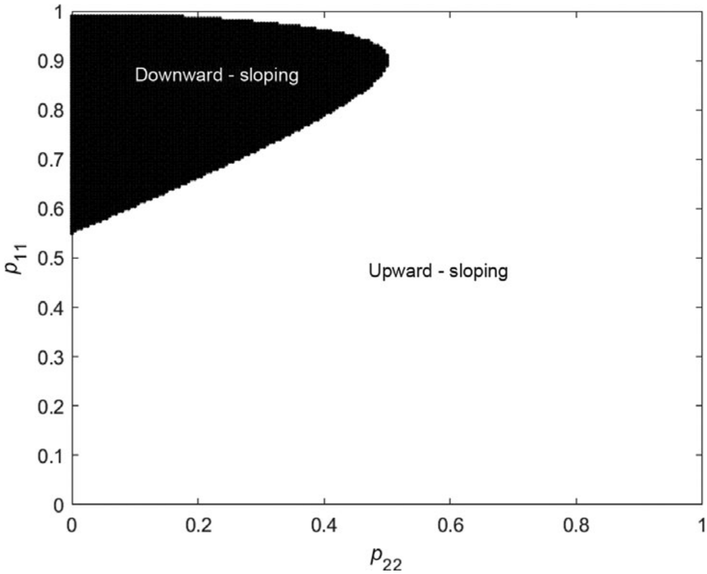

FIGURE 1. Slopes of the AD curve at the ZLB. p_{11}: the transition probability from state 1 to state 1. p_{22}: the transition probability from state 2 to state 2.

The slope of the AD curve is also affected by the central bank's reaction parameter to inflation. In the above analysis, we set this parameter to be 1.5. Next, we let $\phi_1 = 2.5$.[4] Figure 2 reports the result. This change in policy behavior substantially expands the parameter region that allows for a negative slope of the AD curve.

The slope of the AD curve is also affected by three other parameters: the central bank's reaction parameter to output, the degree of price stickiness, and the steady state level of government spending. These are discussed in the "sensitivity analysis" section in the appendix.

Why can the slope of the AD curve be negative when the ZLB binds? To reveal the shape of the AD curve, we need to consider the effect of a supply-side shock. Consider a markup shock that raises inflation. Conventional wisdom suggests that expected inflation rises, the real rate drops, and demand goes up. This mechanism is still there. However, as is evident in equations (6) and (7), today's output is also affected by expected inflation and output when the next state becomes a normal state. In that case, the monetary policy rule kicks in and curbs inflation in the future. Consequently, expected inflation goes down, and current real interest rate actually goes up. This works against the first mechanism. When the probability of a state switch is more likely (p_{22} is small), or/and the monetary policy response to inflation is strong (ϕ_1 is big), this second mechanism is stronger. Its strength is even affected by p_{11}, the probability that a normal state transits to another normal state, because the higher this probability is, the more likely the inflation targeting policy will occur and persist in the long run.

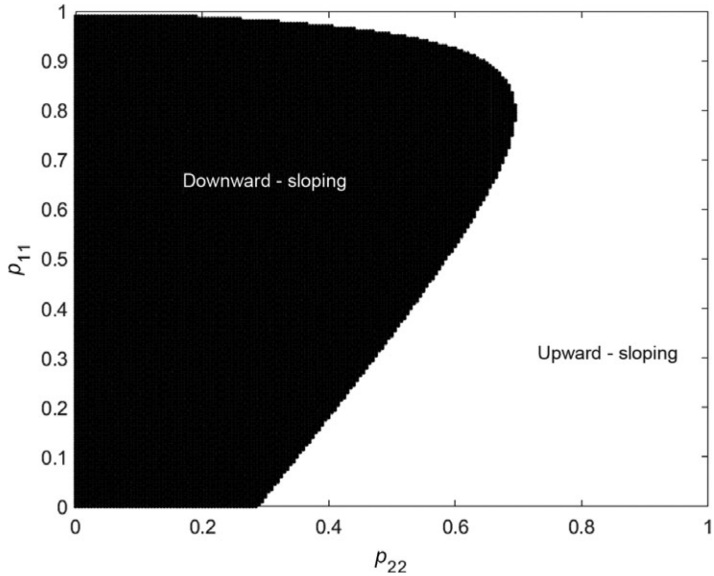

FIGURE 2. Slopes of the AD curve at the ZLB: strict inflation targeting. p_{11}: the transition probability from state 1 to state 1. p_{22}: the transition probability from state 2 to state 2.

The agent must weigh both possibilities when making a decision. If the latter mechanism dominates the first one, aggregate demand goes down, and the slope of the AD curve becomes negative.

In our model setup, we assume that the government spending shock occurs only in the ZLB state. Essentially, it is a proxy for a lean-against-the-wind fiscal stimulus. However, this need not be the case. In reality, the impact of the fiscal stimulus can have a long-lasting effect and extends to the normal state. In that case, what happens to the above mechanism? It should become stronger, and the aggregate demand curve would remain influenced by how agents perceive the possibility of switching back to a normal state. If the government spending shock occurs in a normal state, the Taylor rule regime's response would be to raise the nominal interest rate further. Knowing this, the agents' expectations would be even more influenced by the second mechanism. Consequently, the response in output is more likely to be negative.

4. POLICY IMPLICATIONS

4.1. The Size of the Fiscal Multiplier

Our result immediately suggests that we need to reconsider the policy implications of a binding ZLB combined with a downward-sloping AD curve. At the forefront is the issue of the fiscal multiplier. In the literature, the key finding of

TABLE 1. Government-spending multipliers and the effects of supply-side reforms at the ZLB

Scenario	Downward-sloping AD curves	Upward-sloping AD curves
$\frac{1}{g}\frac{d\hat{Y}_t^Z}{d\hat{G}_t}$	[1.18, 1.29]	$(1.29, +\infty)$
$\frac{d\pi_t^Z}{d\hat{G}_t}$	[0.005, 0.052]	$[0.006, +\infty)$
$-\frac{d\hat{Y}_t^Z}{d\hat{u}_t}$	[0.005, 3.778]	$(-\infty, -0.003]$
$-\frac{d\pi_t^Z}{d\hat{u}_t}$	$[-9.149, -1.006]$	$(-\infty, -1.005]$

a large multiplier rests on the assumption of an upward-sloping AD curve. The case of a negatively sloped AD curve and a zero nominal interest rate has not been considered before. We take up the task here.

Once the equilibrium solution is obtained, the calculation of the fiscal multiplier is quite straightforward. Since equilibrium output and inflation are functions of the government spending shocks $\hat{G}_t$, finding the size of the multiplier amounts to finding $d\hat{Y}_t^Z/d\hat{G}_t/g$, where $1/g$ is a normalizing factor that turns percentage deviations into levels. We leave the lengthy algebra in the appendix, and present the results of our calculation here.

Our main goal is to understand how variations in the transition probabilities affect the multiplier. So we construct a fine grid of values for both p_{11} and p_{22}, between 0 and 1, and compute the size of the multiplier for each combination of the two probabilities. All other parameters are still calibrated as before. Then, we group these multipliers according to the corresponding slopes of the AD curve and examine their sizes.

The first row of Table 1 shows that when the AD curve is downward sloping, the government spending multiplier ranges from 1.18 to 1.29, but when the AD curve is upward sloping, the multiplier is always higher than 1.29. These values are certainly calibration-dependent, but the division between the two parameter regions is quite clear-cut: the multiplier is larger when the AD curve is upward sloping. Row two of Table 1 shows that a government spending shock is inflationary in both cases, which is an expected result.

It is known that the fiscal multiplier is small when the ZLB does not bind. For example, Christiano et al. (2011) used the exact same calibrated parameters in a standard New Keynesian model without the ZLB and found that the multiplier is equal to 1.05. What we find here is something new but perhaps not surprising: when the ZLB binds but the AD curve still has the normal slope, the multiplier is larger than in a standard model, but is smaller than in the case of an upward-sloping AD curve.

4.2. The Effect of Supply-Side Reforms

Recent research suggests that if supply-side reforms take place when the ZLB binds, the effect can be contractionary (Eggertsson (2012), Eggertsson et al. (2014), and Fernández-Villaverde (2014)). The economic mechanism is similar to the logic behind a large fiscal multiplier: supply-side reforms improve efficiency and reduce price and wage markups, and thus reduce marginal costs for firms. This in turn lowers expected inflation. If the ZLB binds, lower *ex ante* inflation raises the real interest rate and causes aggregate demand to fall. Eggertsson et al. (2014) go so far as to make a prediction that European structural reforms would harm the recovery, since the European nominal interest rate had been kept at near-zero levels for a prolonged period of time in the wake of the 2008 financial crisis.

We note that this theory again hinges on the assumption of an upward-sloping AD curve: when both curves slope up and the AD curve is steeper (a necessary condition for equilibrium uniqueness), a rightward shift of the AS curve reduces rather than increases output.[5] When the AD curve is downward sloping, what is the effect of supply-side reforms? A natural conclusion is that it would be expansionary, as a rightward shift of the AS curve raises output. Let us examine the model's predictions.

Recall that there is a cost-push shock in the Phillips curve (equation (1))

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \left[\left(\frac{1}{1-g} + \frac{N}{1-N} \right) \hat{Y}_t - \frac{g}{1-g} \hat{G}_t \right] + \hat{u}_t. \tag{12}$$

The cost-push shock is the easiest way to proxy supply-side reforms: a negative shock represents a reduction in marginal cost caused by policies that improve economic efficiency. The effect of the reform is thus represented by the impact of the shock on output and inflation. We run simulations and compute the derivatives of output and inflation with respect to the cost-push shock. The derivatives are calculated in the appendix. As before, a fine grid of values for the transition probabilities are used to compute a large number of derivatives, which are then grouped according to the slopes of the AD curve. The last two rows of Table 1 report the results. Note that there are negative signs in front of the two derivatives, because supply-side reforms are represented by a negative rather than a positive shock.

The result does confirm our conjecture. When the AD curve is upward sloping, the reform causes both output and inflation to go down, as predicted by Eggertsson et al. (2014) and others. When AD is downward sloping, the reform becomes expansionary: output's response is positive, while inflation's response remains negative. Theoretically, this is hardly remarkable. Yet its practical implications are substantial: the European reforms could be a success after all.

5. DISCUSSION

The above analysis suggests that the slope of the aggregate demand curve depends crucially on expectations. A logical question to ask is, how did market expectations react to the ZLB policy in the United States between 2009 and 2014?

A number of empirical studies suggest that the expected duration of the ZLB regime was quite short. Swanson and Williams (2014) and Kulish et al. (2017) examine the Federal Reserve Bank of New York's Survey of Primary Dealers and the Blue Chip Financial Forecasts and find that prior to the shift to calendar-based forward guidance policies in August 2011, median expected duration of the ZLB was invariably below 5 quarters. After the Fed announced in August 2011 that it expected to keep the funds rate near zero "at least through mid-2013," there was a noticeable rise in median expectations, to about 8–11 quarters. Swanson and Williams (2014) find that intermediate and long-term bond yields were very responsive to macroeconomic announcements prior to August 2011, indicating that they were not constrained by the ZLB policy. This, they believe, suggests that "the financial markets did not expect the zero bound to constrain the funds rate for more than a few quarters." These result corroborate an earlier study by Bauer and Rudebusch (2013), who use a nonnegative dynamic term structure model to study the signaling effect of the Fed's bond purchase program. They draw a nearly identical conclusion that prior to August 2011, financial markets expected the zero bound to constrain US short-term rates for only a few quarters, and after that date, the funds rate was expected to lift off much later. These findings, combined with our theoretical result, suggest that the US aggregate demand could well be in the downward-sloping regime during the period after the Great Recession.

What about a country like Japan, where the ZLB prevailed for a prolonged period of time? Our conjecture is that Japan's case is not covered by our theory. Instead, it is related to a different strand of literature. In solving the model, we have restricted our solution to a unique, determinate equilibrium. There is another strand of literature that studies expectations-driven liquidity traps, following the work by Benhabib et al. (2001; 2002) and Schmitt-Grohé and Uribe (2001). For expectation-driven dynamics to prevail, the equilibrium solutions need to be indeterminate. For example, in Benhabib et al. (2001), there exist two steady states – an intended steady state where monetary policy is active and an unintended steady state where the central bank is assumed to respond passively to inflation. Mertens and Ravn (2014), Boneva et al. (2016), and Schmitt-Grohé and Uribe (2017) study economic dynamics in this type of equilibria when the ZLB binds. The general consensus is that the fiscal multiplier is quite small. In the context of our model, an indeterminate equilibrium will occur when the expected duration of the ZLB is very long (p_{22} is large). Since our focus is on understanding the economic mechanism relevant to a downward-sloping aggregate demand curve, we do not delve into that issue here.

In our model, the transition probabilities are exogenously given. While this is standard practice in the literature, it is no doubt a convenient abstract from reality. Economies that are in ZLB regimes may well have different dynamic paths if their policies differ significantly. For example, an economy with a more active fiscal authority may be able to get out of the low state faster. In that case, the transition probabilities become endogenous. Our hunch is that in that case, expectations would play an even bigger role, and the response of market expectations to policies would continue to have a substantial effect on the transition trajectory of the economy.

6. CONCLUSIONS

In this paper, we demonstrate that when it comes to the issue of the zero lower bound, expectations matter. When there are two states of the world, and agents must consider the consequences of policies in both states, the predictions of the model depart from the conventional wisdom: the AD curve can be downward sloping even when the ZLB binds, the fiscal multiplier is not necessarily large, and supply-side reforms can be expansionary. All depend on expectations. The result of the paper highlights the difficulty in predicting the impact of macroeconomic policies in a liquidity trap using dynamic general equilibrium models.

NOTES

1. A more accurate description of the theory's prediction is as follows. A positive TFP shock raises the natural rate of real interest. When the ZLB is binding, the nominal interest rate cannot be reduced to accommodate this change. Inflation and expected inflation decrease, which raises the real interest rate and decreases demand. This works against the expansionary effect of higher productivity. As a result, the increase in output is always lower than that in an inflation targeting regime. If the expected duration of the ZLB is long, the effect of rising TFP can even be contractionary.
2. For our purpose, we only consider the case of a unique, determinate equilibrium solution. When the expected duration of the ZLB is very long, the economic system will become indeterminate, and there can be multiple equilibrium solutions that are driven by nonfundamental variables (Eggertsson (2011*b*), Carlstrom et al. (2014), Eggertsson and Singh (2019)). Our model can produce this result. We do not include it in the analysis as it is not directly relevant to the issue considered. See our discussion in Section 5.
3. Although not shown, the AS curve is found to be always upward sloping at the ZLB.
4. High values for ϕ_1 are often used to represent a hawkish central bank that reacts strongly to inflation. For example, Eggertsson et al. (2014) set ϕ_1 to 10 to represent a harsh inflation targeting regime in Europe.
5. What if the AD curve is flatter than the AS curve? In that case, indeterminacy occurs, and there can be multiple equilibria, and the result of the reform can be unpredictable. See Eggertsson (2011), Carlstrom et al. (2014), Mertens and Ravn (2014), Boneva et al. (2016), and Eggertsson and Singh (2019). Our model can reproduce this result. As pointed out earlier, we do not present that analysis here, as it is not directly relevant.

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A: APPENDIX

A.1. THE MICRO-FOUNDED MODEL

The model is based on Christiano et al. (2011).

The representative household's utility function is

$$U = E_0 \sum_{t=0}^{\infty} d_t \left\{ \frac{\left[C_t^{\gamma} (1 - N_t)^{1-\gamma}\right]^{1-\sigma} - 1}{1 - \sigma} + \upsilon(G_t) \right\}, \qquad \textbf{(A1)}$$

where E_0 is the conditional expectations operator, C_t consumption, N_t hours worked, G_t (exogenous) government spending. $\upsilon(\cdot)$ is concave. Parameter restrictions are $\sigma > 0$ and

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$\gamma \in (0, 1)$. d_t is a stochastic cumulative discount factor, given by

$$d_t = \begin{cases} \frac{1}{1+r_1^n} \times \frac{1}{1+r_2^n} \times \cdots \times \frac{1}{1+r_t^n} & t \geq 1 \\ 1 & t = 0 \end{cases}. \quad \textbf{(A2)}$$

Assume that the natural rate of interest r_t^n follows a two-state Markov process

$$\Pr\left[r_{t+1}^n = r | r_t^n = r\right] = p_{22}, \quad \textbf{(A3)}$$

$$\Pr\left[r_{t+1}^n = \bar{r} | r_t^n = r\right] = 1 - p_{22} = p_{21}, \quad \textbf{(A4)}$$

$$\Pr\left[r_{t+1}^n = \bar{r} | r_t^n = \bar{r}\right] = p_{11}, \quad \textbf{(A5)}$$

$$\Pr\left[r_{t+1}^n = r | r_t^n = \bar{r}\right] = 1 - p_{11} = p_{12}, \quad \textbf{(A6)}$$

where $\bar{r} = \frac{1}{\beta} - 1$ is the steady state of r_t^n, and $r < 0$ represents an adverse shock that is large enough to force the ZLB to bind. In Christiano et al. (2011), the assumption is essentially $p_{11} = 1$.

The agent's budget constraint is

$$P_t C_t + B_{t+1} = B_t(1 + R_t) + W_t N_t + T_t, \quad \textbf{(A7)}$$

where P_t denotes prices, B_t the quantity of one-period bonds, R_t the nominal interest rate, W_t wages, and T_t firms' profits net of lump-sum taxes.

The final goods market is competitive, while the intermediate goods market is monopolistically competitive. The production function for the representative final goods producer is

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad \textbf{(A8)}$$

where Y_t is the output, $Y_t(i)$ is the intermediate good i, and $\varepsilon > 1$ is the elasticity of substitution between differentiated goods.

Solving the final goods producer's profit maximization problem gives the demand function for intermediate good i

$$P_t(i) = P_t\left[\frac{Y_t}{Y_t(i)}\right]^{\frac{1}{\varepsilon}}, \quad \textbf{(A9)}$$

where $P_t(i)$ is the price of intermediate good i.

The production function for intermediate good producer i is

$$Y_t(i) = N_t(i). \quad \textbf{(A10)}$$

Assume a Calvo-style price setting. At time t, there is only fraction θ of intermediate good firms that can set prices while the rest $1 - \theta$ still use prices set in the last period

$$P_t(i) = P_{t-1}(i). \quad \textbf{(A11)}$$

Suppose that intermediate good producer i can set its price at time t. She maximizes the following discounted profit function subject to the Calvo-style price-setting friction, the production function (A10), and the demand function for its good (A9)

$$E_t \sum_{j=0}^{\infty} \beta^j \upsilon_{t+j}\left[P_{t+j}(i)\, Y_{t+j}(i) - (1 - \upsilon)\, W_{t+j} N_{t+j}(i)\right], \quad \textbf{(A12)}$$

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where υ_{t+j} is the multiplier on the household budget constraint in the Lagrangian representation of the household problem and $\upsilon = \frac{1}{\varepsilon}$ a subsidy that corrects the steady-state inefficiency due to monopoly power.

The monetary authority's policy reaction function is

$$R_t = \max\left[0, (1+\pi_t)^{\phi_1}\left(\frac{Y_t}{Y}\right)^{\phi_2} - 1\right], \tag{A13}$$

where π_t denotes inflation, Y_t output, Y the steady state of output, ϕ_1 the policy response to inflation, and ϕ_2 the policy response to the output gap.

The exogenous government spending follows the same two-state Markov process. In state 1, it is assumed to remain at its steady state, while government spending increases by a fixed value whenever the ZLB binds.

The economy's resource constraint is

$$C_t + G_t = Y_t. \tag{A14}$$

A.2. DERIVING THE SLOPE OF THE AD CURVE

The economic system consists of equations (4)–(7). For completeness, the cost-push shock is added to the system, representing supply-side reforms. Thus, there are three exogenous shocks: a government spending shock, a real interest rate shock, and a cost-push shock. As in Christiano et al. (2011), we simplify the monetary policy rule by setting the reaction parameter to output ϕ_2 to 0. This does not affect the conclusion of the paper.

The unique equilibrium solution of the system has four equations, each expresses an endogenous variable as a function of the fundamental shocks. Since there are only two state values for each shock, and there are no endogenous state variables, the endogenous variables are invariant functions of the state values of shocks, and so are their expected values. Consequently, we can write $\pi_t^T = E_t\pi_{t+1}^T = \pi^T$, $\pi_t^Z = E_t\pi_{t+1}^Z = \pi^Z$, $\hat{Y}_t^T = E_t\hat{Y}_{t+1}^T = \hat{Y}^T$, and $\hat{Y}_t^Z = E_t\hat{Y}_{t+1}^Z = \hat{Y}^Z$.

Equations (4) and (5) can be described by the following dynamic system

$$\begin{pmatrix} \pi^T \\ \hat{Y}^T \end{pmatrix} = A\begin{pmatrix} \pi^Z \\ \hat{Y}^Z \end{pmatrix} + BG^Z, \tag{A15}$$

or

$$\pi^T = A(1,1)\,\pi^Z + A(1,2)\,\hat{Y}^Z + B(1,1)\,G^Z, \tag{A16}$$

$$\hat{Y}^T = A(2,1)\,\pi^Z + A(2,2)\,\hat{Y}^Z + B(2,1)\,G^Z, \tag{A17}$$

where $A = \begin{pmatrix} 1-\beta p_{11} & -\kappa\left(\frac{1}{1-g} + \frac{N}{1-N}\right) \\ (1-g)(\phi_1 - p_{11}) & 1 - p_{11} + (1-g)\,\phi_2 \end{pmatrix}^{-1} \begin{pmatrix} \beta p_{12} & 0 \\ (1-g)\,p_{12} & p_{12} \end{pmatrix}$ and $B = \begin{pmatrix} 1-\beta p_{11} & -\kappa\left(\frac{1}{1-g} + \frac{N}{1-N}\right) \\ (1-g)(\phi_1 - p_{11}) & 1 - p_{11} + (1-g)\,\phi_2 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ -p_{12}g\left[\gamma(\sigma-1)+1\right] \end{pmatrix}$. Note that in state 1, the real interest rate shock and the cost-push shock do not appear directly in the system. They only happen in state 2.

Plugging (A16) and (A17) into (6) yields the AS curve at the ZLB

$$\pi^Z = \frac{\beta p_{21} A(1,2) + \kappa\left(\frac{1}{1-g} + \frac{N}{1-N}\right)}{1 - \beta p_{22} - \beta p_{21} A(1,1)} \hat{Y}^Z + \frac{\beta p_{21} B(1,1) - \frac{g\kappa}{1-g}}{1 - \beta p_{22} - \beta p_{21} A(1,1)} G^Z + \frac{1}{1 - \beta p_{22} - \beta p_{21} A(1,1)} u. \quad \textbf{(A18)}$$

Therefore, the slope of the AS curve is

$$S_{AS} = \frac{\beta p_{21} A(1,2) + \kappa\left(\frac{1}{1-g} + \frac{N}{1-N}\right)}{1 - \beta p_{22} - \beta p_{21} A(1,1)}. \quad \textbf{(A19)}$$

Plugging (A16) and (A17) into (7) yields the AD curve at the ZLB

$$\pi^Z = \frac{1 - p_{22} - p_{21} A(2,2) - (1-g)\, p_{21} A(1,2)}{p_{21} A(2,1) + (1-g)\, p_{21} A(1,1) + (1-g)\, p_{22}} \hat{Y}^Z - \frac{p_{21} B(2,1) + (1-g)\, p_{21} B(1,1) + (1-p_{22})\, g\left[\gamma(\sigma-1)+1\right]}{p_{21} A(2,1) + (1-g)\, p_{21} A(1,1) + (1-g)\, p_{22}} G^Z - \frac{1-g}{p_{21} A(2,1) + (1-g)\, p_{21} A(1,1) + (1-g)\, p_{22}} r, \quad \textbf{(A20)}$$

where the slope of the AD curve is

$$S_{AD} = \frac{1 - p_{22} - p_{21} A(2,2) - (1-g)\, p_{21} A(1,2)}{p_{21} A(2,1) + (1-g)\, p_{21} A(1,1) + (1-g)\, p_{22}}. \quad \textbf{(A21)}$$

A.3. DERIVING THE FISCAL MULTIPLIER AND THE EFFECT OF SUPPLY-SIDE REFORMS

Continue from the derivations of the previous section.

Define the relative slope of AD and AS curves as

$$S = S_{AD} - S_{AS}. \quad \textbf{(A22)}$$

where $S_{AS} = \frac{\beta p_{21} A(1,2) + \kappa\left(\frac{1}{1-g} + \frac{N}{1-N}\right)}{1 - \beta p_{22} - \beta p_{21} A(1,1)}$.

The equilibrium solution is obtained by equating the AS curve (A18) with the AD curve (A20)

$$\pi^Z = c_1 u + c_2 G^Z + c_3 r, \quad \textbf{(A23)}$$

$$\pi^Z = d_1 u + d_2 G^Z + d_3 r, \quad \textbf{(A24)}$$

where $c_1 = \frac{1}{S\left[1 - \beta p_{22} - \beta p_{21} A(1,1)\right]}$,

$c_2 = \frac{1}{S}\left\{\frac{p_{21} B(2,1) + (1-g) p_{21} B(1,1) + (1-p_{22}) g\left[\gamma(\sigma-1)+1\right]}{p_{21} A(2,1) + (1-g) p_{21} A(1,1) + (1-g) p_{22}} + \frac{\beta p_{21} B(1,1) - \frac{g\kappa}{1-g}}{1 - \beta p_{22} - \beta p_{21} A(1,1)}\right\}$,

$c_3 = \frac{1-g}{S\left[p_{21} A(2,1) + (1-g) p_{21} A(1,1) + (1-g) p_{22}\right]}$,

$d_1 = S_{AD} \times \frac{1}{S\left[1 - \beta p_{22} - \beta p_{21} A(1,1)\right]}$,

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$d_2 = S_{AD} \times \frac{1}{S}\left\{\frac{p_{21}B(2,1)+(1-g)p_{21}B(1,1)+(1-p_{22})g\left[\gamma(\sigma-1)+1\right]}{p_{21}A(2,1)+(1-g)p_{21}A(1,1)+(1-g)p_{22}} + \frac{\beta p_{21}B(1,1)-\frac{g\kappa}{1-g}}{1-\beta p_{22}-\beta p_{21}A(1,1)}\right\}$
$- \frac{p_{21}B(2,1)+(1-g)p_{21}B(1,1)+(1-p_{22})g\left[\gamma(\sigma-1)+1\right]}{p_{21}A(2,1)+(1-g)p_{21}A(1,1)+(1-g)p_{22}}$,
and $d_3 = S_{AD} \times \frac{1-g}{S\left[p_{21}A(2,1)+(1-g)p_{21}A(1,1)+(1-g)p_{22}\right]} - \frac{1-g}{p_{21}A(2,1)+(1-g)p_{21}A(1,1)+(1-g)p_{22}}$.

So, the multiplier is

$$\frac{1}{g}\frac{d\hat{Y}^Z}{dG^Z} = \frac{1}{Sg}\left\{\frac{p_{21}B(2,1)+(1-g)\,p_{21}B(1,1)+(1-p_{22})\,g\left[\gamma(\sigma-1)+1\right]}{p_{21}A(2,1)+(1-g)\,p_{21}A(1,1)+(1-g)\,p_{22}} + \frac{\beta p_{21}B(1,1)-\frac{g\kappa}{1-g}}{1-\beta p_{22}-\beta p_{21}A(1,1)}\right\}, \quad \textbf{(A25)}$$

and the effect of government spending on inflation can be represented by

$$\frac{d\pi^Z}{dG^Z} = S_{AD}\frac{d\hat{Y}^Z}{dG^Z} - \frac{p_{21}B(2,1)+(1-g)\,p_{21}B(1,1)+(1-p_{22})\,g\left[\gamma(\sigma-1)+1\right]}{p_{21}A(2,1)+(1-g)\,p_{21}A(1,1)+(1-g)\,p_{22}}. \quad \textbf{(A26)}$$

The effects of supply-side reforms are

$$-\frac{d\hat{Y}^Z}{du} = -\frac{1}{S\left[1-\beta p_{22}-\beta p_{21}A(1,1)\right]}, \quad \textbf{(A27)}$$

$$-\frac{d\pi^Z}{du} = -\frac{S_{AD}}{S\left[1-\beta p_{22}-\beta p_{21}A(1,1)\right]}. \quad \textbf{(A28)}$$

In the special case of $p_{11}=1$, $p_{12}=1-p_{11}=0$, and matrices A and B reduce to

$$\begin{aligned} A &= \begin{pmatrix} 1-\beta p_{11} & -\kappa\left(\frac{1}{1-g}+\frac{N}{1-N}\right) \\ (1-g)(\phi_1-p_{11}) & 1-p_{11}+(1-g)\,\phi_2 \end{pmatrix}^{-1}\begin{pmatrix} \beta p_{12} & 0 \\ (1-g)\,p_{12} & p_{12} \end{pmatrix} \\ &= \begin{pmatrix} 1-\beta & -\kappa\left(\frac{1}{1-g}+\frac{N}{1-N}\right) \\ (1-g)(\phi_1-1) & (1-g)\,\phi_2 \end{pmatrix}^{-1}\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \end{aligned} \quad \textbf{(A29)}$$

$$\begin{aligned} B &= \begin{pmatrix} 1-\beta p_{11} & -\kappa\left(\frac{1}{1-g}+\frac{N}{1-N}\right) \\ (1-g)(\phi_1-p_{11}) & 1-p_{11}+(1-g)\,\phi_2 \end{pmatrix}^{-1}\begin{pmatrix} 0 \\ -p_{12}g\left[\gamma(\sigma-1)+1\right] \end{pmatrix} \\ &= \begin{pmatrix} 1-\beta & -\kappa\left(\frac{1}{1-g}+\frac{N}{1-N}\right) \\ (1-g)(\phi_1-1) & (1-g)\,\phi_2 \end{pmatrix}^{-1}\begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \end{aligned} \quad \textbf{(A30)}$$

In this case, the slope of the AS curve (A19), the slope of the AD curve (A21), and the relative slope (A22) are simplified to

$$S_{AS} = \frac{\beta p_{21}A(1,2)+\kappa\left(\frac{1}{1-g}+\frac{N}{1-N}\right)}{1-\beta p_{22}-\beta p_{21}A(1,1)}$$

$$= \frac{\kappa\left(\frac{1}{1-g} + \frac{N}{1-N}\right)}{1 - \beta p_{22}}$$

$$= \frac{\kappa\left[1 + \frac{N}{1-N}(1-g)\right]}{(1 - \beta p_{22})(1-g)}, \quad \textbf{(A31)}$$

$$S_{AD} = \frac{1 - p_{22} - p_{21}A(2,2) - (1-g)\, p_{21}A(1,2)}{p_{21}A(2,1) + (1-g)\, p_{21}A(1,1) + (1-g)\, p_{22}}$$

$$= \frac{1 - p_{22}}{(1-g)\, p_{22}}, \quad \textbf{(A32)}$$

$$S = S_{AD} - S_{AS}$$

$$= \frac{1 - p_{22}}{(1-g)\, p_{22}} - \frac{\kappa\left[1 + \frac{N}{1-N}(1-g)\right]}{(1 - \beta p_{22})(1-g)}$$

$$= \frac{(1 - \beta p_{22})(1 - p_{22}) - p_{22}\kappa\left[1 + \frac{N}{1-N}(1-g)\right]}{(1 - \beta p_{22})(1-g)\, p_{22}}$$

$$= \frac{\triangle}{(1 - \beta p_{22})(1-g)\, p_{22}} \quad \textbf{(A33)}$$

where we use the fact that $A(1,2) = A(1,1) = A(2,2) = A(2,1) = A(1,1) = 0$ and define $\triangle = (1 - \beta p_{22})(1 - p_{22}) - p_{22}\kappa\left[1 + \frac{N}{1-N}(1-g)\right]$.

So, the multiplier (A25) is reduced to

$$\frac{1}{g}\frac{d\hat{Y}^Z}{dG^Z} = \frac{1}{Sg}\left\{\frac{p_{21}B(2,1) + (1-g)\, p_{21}B(1,1) + (1 - p_{22})\, g\left[\gamma(\sigma - 1) + 1\right]}{p_{21}A(2,1) + (1-g)\, p_{21}A(1,1) + (1-g)\, p_{22}} + \frac{\beta p_{21}B(1,1) - \frac{g\kappa}{1-g}}{1 - \beta p_{22} - \beta p_{21}A(1,1)}\right\}$$

$$= \frac{(1 - \beta p_{22})(1-g)\, p_{22}}{\triangle g}\left\{\frac{(1 - p_{22})\, g\left[\gamma(\sigma - 1) + 1\right]}{(1-g)\, p_{22}} - \frac{g\kappa}{(1 - \beta p_{22})(1-g)}\right\}$$

$$= \frac{(1 - \beta p_{22})(1 - p_{22})\left[\gamma(\sigma - 1) + 1\right] - p_{22}\kappa}{\triangle}, \quad \textbf{(A34)}$$

where we use the fact that $B(2,1) = B(1,1) = 0$, and $A(2,1) = A(1,1) = 0$. The simplified version of the multiplier (A34) is exactly the same as in Christiano et al. (2011).

A.4. SENSITIVITY ANALYSIS

In Section 3, we examine how the slope of the demand curve varies in response to the transition probabilities and the inflation coefficient in the Taylor rule. In this section, we conduct sensitivity analyses with respect to more parameters.

The parameters that affect the slope of the AD curve are ϕ_1 (policy reaction to inflation), ϕ_2 (policy reaction to output), g (steady state level of government spending), and θ (degree of price stickiness). ϕ_1 has been analyzed in Figure 2. We consider how the slope of the AD curve is affected by the other parameters here.

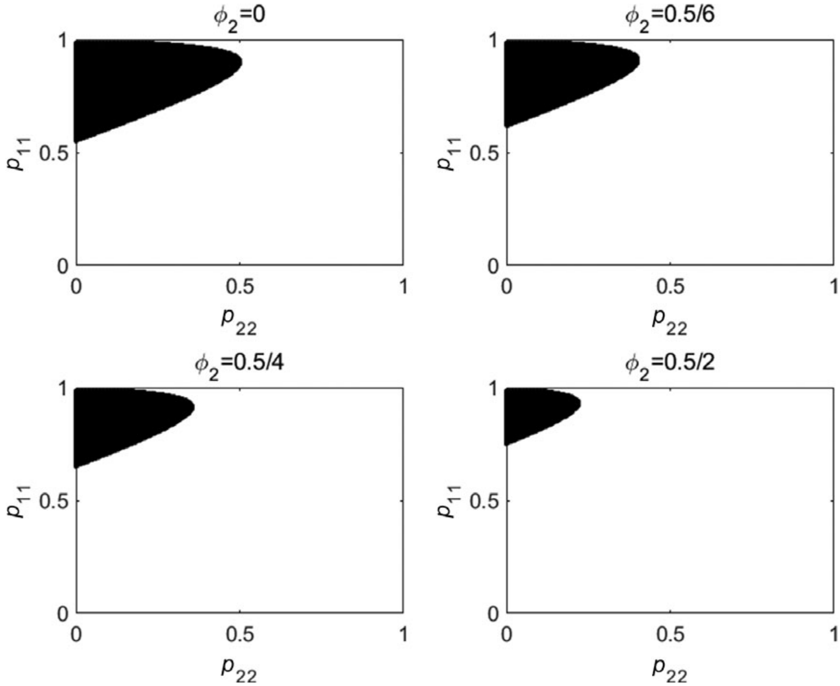

FIGURE A.1. Sensitivity of slopes of the AD curve at the ZLB to the monetary policy's response to output (ϕ_2). p_{11}: the transition probability from state 1 to state 1. p_{22}: the transition probability from state 2 to state 2.

Figure A.1 shows the effect of ϕ_2 on the slope of the AD curve. When plotting the figure, ϕ_1 is set at the benchmark value of 1.5. Evidently, a higher policy response to output tends to reduce the region of the negative slope. Next, we consider how the degree of nominal rigidity affects the slope of the AD curve at the ZLB, holding ϕ_1 and ϕ_2 at their baseline values. As Figure A.2 shows, stickier prices (higher values of θ) make it more likely for the AD curve to be downward sloping. In both figures, it remains true that combinations of high p_{11} and low p_{22} are more likely to result in a downward-sloping AD curve. Finally, we vary the steady-state level of government spending and examine its impact in Figure A.3. When g increases, there is a very slight increase in the parameter regions leading to negative slopes, but the magnitude is small.

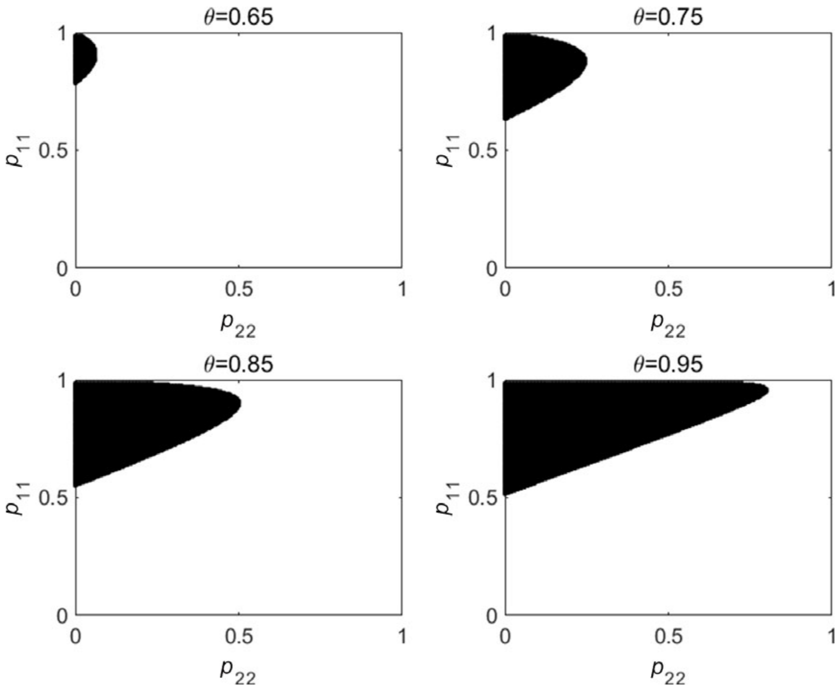

FIGURE A.2. Sensitivity of slopes of the AD curve at the ZLB to price stickiness (θ). p_{11}: the transition probability from state 1 to state 1. p_{22}: the transition probability from state 2 to state 2.

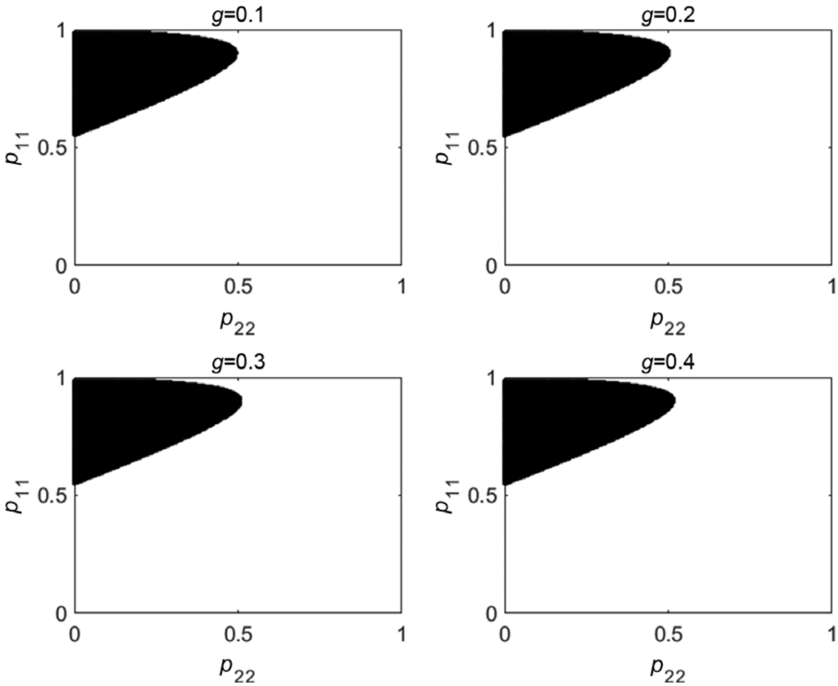

FIGURE A.3. Sensitivity of slopes of the AD curve at the ZLB to the share of government spending in the steady state. p_{11}: the transition probability from state 1 to state 1. p_{22}: the transition probability from state 2 to state 2.