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Spin effects in nonlinear Compton scattering in ultrashort linearly-polarized laser pulses

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Abstract

The nonlinear Compton scattering by a linearly polarized laser pulse of finite duration is analyzed, with a focus on the spin effects of target electrons. We show that, although the Compton scattering accompanied by the electron no-spin flip is dominant, for some energy regions of Compton photons their emission is dominated by the process leading to the electron spin flip. This feature is observed for different pulse durations, and can be treated as a signature of quantum behavior. Similar conclusions are reached when analyzing the scattered electron energy spectra. This time, the sensitivity of spin effects to the carrier-envelope phase of the driving pulse is demonstrated. The possibility of electron acceleration by means of Compton scattering is also discussed.

Keywords: Compton scattering; Short pulse; Spin effects; Ultra-intense laser pulse

1. INTRODUCTION

Compton scattering of a laser beam with relativistic electrons has become an efficient source of highly polarized, nearly monoenergetic, and tunable high-energy photons. These Compton photon beams have various industrial, medical, and scientific applications. Therefore, it is of great importance to theoretically predict spectral and spatial distributions of Compton photon beams, and to control their properties using the incident beam parameters. While the theory of a Compton process induced by a monochromatic plane wave laser field is well documented in the literature (for recent reviews, see Di Piazza *et al.*, 2012; Ehlotzky *et al.*, 2009; Rashchupkin *et al.*, 2012), there is a need to fully understand the situation when a finite laser pulse collides with the target particles.

In the first analysis of a Compton process beyond the monochromatic plane wave approximation, the scattering by a Klein-Gordon particle was considered (Neville & Rohrlich, 1971). The Compton scattering by a Dirac particle for long laser pulses, in which case the slowly-varying envelope approximation is justified, was considered (Narozhny & Fofanov, 1996). For intense, few-cycle pulses, which are used in modern experimental setups, a more adequate treatment of the temporal structure of an incident laser pulse is necessary. Such a treatment has been used in very recent papers (Boca *et al.*, 2012*a*, 2012*b*; Boca & Florescu, 2009; 2011; Heinzl *et al.*, 2010; Krajewska & Kamiński, 2012*a*; Mackenroth & Di Piazza, 2011; Mackenroth *et al.*, 2010; Seipt & Kämpfer, 2011). In these papers, the laser field has been modeled by a plane-wave-fronted pulse (PWFP), which has finite extension in the propagation direction but has infinite extension in the transverse direction. As argued (Bulanov *et al.*, 2011), this approximation is justified for highly energetic electrons moving in a laser pulse, since the action of the laser ponderomotive force pushing these electrons aside with respect to the pulse propagation direction can be neglected. For completeness, let us also mention that such a description has been proposed by Neville and Rohrlich (1971).

Various aspects of Compton scattering; including the carrier-envelope phase effects, the shape effects, the sensitivity of the Compton radiation to the incident pulse duration, were analyzed (Boca *et al.*, 2012*a*, 2012*b*; Boca & Florescu, 2009; 2011; Heinzl *et al.*, 2010; Krajewska & Kamiński, 2012*a*; Mackenroth & Di Piazza, 2011; Mackenroth *et al.*, 2010; Seipt & Kämpfer, 2011). However, in only one of these papers were the electron spin effects in Compton scattering by a pulsed laser field explicitly studied (Boca *et al.*, 2012*b*). Prior investigations of spin effects in Compton scattering concerned a monochromatic plane wave field (Bolshedvorsky & Polityko, 2000; Ivanov *et al.*, 2004; Panek *et al.*, 2002). It followed from these investigations

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that the processes which are not accompanied by the electron spin flip dominate above those which are. It is the aim of the present paper to check whether this is also the case for the Compton process induced by a finite pulse. More importantly, we will investigate whether the spin effects vary with the incident pulse duration or with its carrier-envelope phase. For this purpose, we will consider the linearly polarized laser field impinging on the target electron. This is in contrast to the situation analyzed in (Boca *et al.*, 2012*b*), where the circularly polarized laser pulse was considered. Let us also note that a lot of effort has been made in the literature to describe the classical counterpart of Compton scattering, which is Thomson scattering (Lau *et al.*, 2003; Popa, 2011; 2012; Umstadter, 2003). It appears that for some parameters of the colliding beams, the Compton scattering spectra averaged over the electron spin give exactly the same results as Thomson scattering (Boca & Florescu, 2011; Heinzl *et al.*, 2010; Seipt & Kämpfer, 2011), i.e., for those parameters the spin effects do not play a significant role.

2. THEORY

In this paper, the formulas are given in units such that $\hbar = 1$, whereas the numerical results are presented in relativistic units in which $\hbar = m_e = c = 1$, where m_e is the electron rest mass. We use the light-cone coordinates (Krajewska & Kamiński, 2012*a*). Namely, for a given space direction determined by a unit vector $\boldsymbol{n}$ and for an arbitrary four-vector a we keep the following notations: $a^{\|} = \boldsymbol{n}\cdot\boldsymbol{a}$, $a^{-} = a^0 - a^{\|}$, $a^{+} = (a^0 + a^{\|})/2$, and $\boldsymbol{a}^{\perp} = \boldsymbol{a} - a^{\|}\boldsymbol{n}$. For the four-vectors, we use both the contravariant (a^0, a^1, a^2, a^3) and the standard (a_0, a_x, a_y, a_z) notations.

Using the S-matrix formalism of quantum electrodynamics (QED), we derive that the probability amplitude for the Compton process, $e^-_{\boldsymbol{p}_i\lambda_i} \to e^-_{\boldsymbol{p}_f\lambda_f} + \gamma_{\boldsymbol{K}\sigma}$, with the initial and final electron momenta and spin polarizations $\boldsymbol{p}_i\lambda_i$ and $\boldsymbol{p}_f\lambda_f$, respectively, equals

$$\mathscr{A}(e^-_{\boldsymbol{p}_i\lambda_i} \to e^-_{\boldsymbol{p}_f\lambda_f} + \gamma_{\boldsymbol{K}\sigma}) = -ie\int d^4x\, j^{(++)}_{\boldsymbol{p}_f\lambda_f,\boldsymbol{p}_i\lambda_i}(x)\cdot A^{(-)}_{\boldsymbol{K}\sigma}(x), \qquad (1)$$

where $\boldsymbol{K}\sigma$ denotes the Compton photon momentum and polarization. Here, we consider the case when both the laser pulse and the Compton photon are linearly polarized. In Eq. (1),

$$A^{(-)}_{\boldsymbol{K}\sigma}(x) = \sqrt{\frac{1}{2\varepsilon_0\omega_{\boldsymbol{K}}V}}\,\varepsilon_{\boldsymbol{K}\sigma}e^{iK\cdot x}, \qquad (2)$$

where V is the quantization volume, ε_0 is the vacuum electric permittivity, $\omega_{\boldsymbol{K}} = cK^0 = c|\boldsymbol{K}|$ $(K\cdot K = 0)$, and $\varepsilon_{\boldsymbol{K}\sigma} = (0, \boldsymbol{\varepsilon}_{\boldsymbol{K}\sigma})$ is the polarization four-vector satisfying the conditions,

$$K\cdot\varepsilon_{\boldsymbol{K}\sigma} = 0, \quad \varepsilon_{\boldsymbol{K}\sigma}\cdot\varepsilon_{\boldsymbol{K}\sigma'} = -\delta_{\sigma\sigma'}, \qquad (3)$$

for $\sigma, \sigma' = 1, 2$. Moreover, $j^{(++)}_{\boldsymbol{p}_f\lambda_f,\boldsymbol{p}_i\lambda_i}(x)$ is the matrix element of the electron current operator with its ν-component equal to

$$[j^{(++)}_{\boldsymbol{p}_f\lambda_f,\boldsymbol{p}_i\lambda_i}(x)]^{\nu} = \bar{\psi}^{(+)}_{\boldsymbol{p}_f\lambda_f}(x)\gamma^{\nu}\psi^{(+)}_{\boldsymbol{p}_i\lambda_i}(x). \qquad (4)$$

Here, $\psi^{(+)}_{\boldsymbol{p}\lambda}(x)$ is the Volkov solution of the Dirac equation coupled to the electromagnetic field (Krajewska & Kamiński, 2010; Volkov, 1935):

$$\psi^{(+)}_{\boldsymbol{p}\lambda}(x) = \sqrt{\frac{m_e c^2}{VE_{\boldsymbol{p}}}}\left(1 - \frac{e}{2k\cdot p}\not{A}\not{k}\right)u^{(+)}_{\boldsymbol{p}\lambda}e^{-iS^{(+)}_p(x)}, \qquad (5)$$

with

$$S^{(+)}_p(x) = p\cdot x + \int^{k\cdot x}\left[\frac{eA(\phi)\cdot p}{k\cdot p} - \frac{e^2A^2(\phi)}{2k\cdot p}\right]d\phi. \qquad (6)$$

Moreover, $E_{\boldsymbol{p}} = cp^0$, $p = (p^0, \boldsymbol{p})$, $p\cdot p = m_e^2c^2$, and $u^{(+)}_{\boldsymbol{p}\lambda}$ are the free-electron bispinor normalized such that

$$\bar{u}^{(+)}_{\boldsymbol{p}\lambda}u^{(+)}_{\boldsymbol{p}\lambda'} = \delta_{\lambda\lambda'}. \qquad (7)$$

The four-vector potential $A(k\cdot x)$ in Eq. (5) represents an external electromagnetic radiation generated by lasers, in the case when a transverse variation of the laser field in a focus is negligible (Bulanov *et al.*, 2011). In other words, $A(k\cdot x)$ represents the plane-wave-fronted pulse. In this case, $k\cdot A(k\cdot x) = 0$ and $k\cdot k = 0$, which allows one to exactly solve the Dirac equation in the electromagnetic field.

From now on, we use the Coulomb gauge for the radiation field, in which case the electric and magnetic field components are equal to

$$\boldsymbol{\mathscr{E}}(k\cdot x) = -\partial_t\boldsymbol{A}(k\cdot x) = -ck^0\boldsymbol{A}'(k\cdot x), \qquad (8)$$

$$\boldsymbol{\mathscr{B}}(k\cdot x) = \nabla\times\boldsymbol{A}(k\cdot x) = -\boldsymbol{k}\times\boldsymbol{A}'(k\cdot x), \qquad (9)$$

where "prime" means the derivative with respect to $k\cdot x$. Because the electric field generated by lasers has to fulfill the following condition,

$$\int_{-\infty}^{\infty}\boldsymbol{\mathscr{E}}(ck^0t - \boldsymbol{k}\cdot\boldsymbol{r})dt = 0, \qquad (10)$$

we have also that

$$\lim_{t\to-\infty}\boldsymbol{A}(ck^0t - \boldsymbol{k}\cdot\boldsymbol{r}) = \lim_{t\to\infty}\boldsymbol{A}(ck^0t - \boldsymbol{k}\cdot\boldsymbol{r}). \qquad (11)$$

If a pulse lasts for a period T_p, its fundamental frequency is defined as $\omega = 2\pi/T_p$ and the laser field four-vector is $k = k^0(1, \boldsymbol{n})$. Here, $\omega = ck^0$ whereas $\boldsymbol{n}$ is a unit vector defining a direction of the laser pulse propagation. In order to interpret the momentum p in the Volkov solution (5) as the

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asymptotic momentum of the free electron, we take

$$A(k\cdot x)=A_0\varepsilon f(k\cdot x), \tag{12}$$

with the shape function $f(k\cdot x)$ such that $f(k\cdot x)=0$ for $k\cdot x<0$ and for $k\cdot x>2\pi$. In addition, ε is the linear polarization vector of the laser field such that $\varepsilon^2=-1$ and $k\cdot\varepsilon=0$. Note that the last conditions do not uniquely determine ε. It can be shown that the transformation:

$$\varepsilon\to\varepsilon+a(k\cdot x)k, \tag{13}$$

with an arbitrary differentiable function $a(k\cdot x)$, does not violate these conditions provided that $k\cdot k=0$, which is indeed the case. Note that Eq. (13) is a special case of the gauge transformation. The observable quantities, like the probability distributions that we derive below, should be invariant with respect to such a transformation. The invariance with respect to the gauge transformation, Eq. (13), permits us to choose ε as the spacelike vector, since its zeroth-component can be canceled by the gauge transformation,

$$\varepsilon\to\varepsilon-\frac{\varepsilon^0}{k^0}k. \tag{14}$$

The same concerns $\varepsilon_{K\sigma}$. For this reason, we keep ε and $\varepsilon_{K\sigma}$ as the spacelike four-vectors.

The probability amplitude for the Compton process (1) becomes

$$\mathscr{A}(e^-_{p_i\lambda_i}\to e^-_{p_f\lambda_f}+\gamma_{K\sigma})=i\sqrt{\frac{2\pi\alpha c(m_ec^2)^2}{E_{p_f}E_{p_i}\omega_K V^3}}\mathscr{A}, \tag{15}$$

where α is the fine-structure constant, $\alpha=e^2/(4\pi\varepsilon_0 c)$, and

$$\begin{aligned}\mathscr{A}=&\int d^4x\, e^{-i(S^{(+)}_{p_i}(x)-S^{(+)}_{p_f}(x)-K\cdot x)}\\&\times\bar{u}^{(+)}_{p_f\lambda_f}\left(1-\mu\frac{m_ec}{2p_f\cdot k}f(k\cdot x)\not{\varepsilon}\not{k}\right)\not{\varepsilon}_{K\sigma}\\&\times\left(1+\mu\frac{m_ec}{2p_i\cdot k}f(k\cdot x)\not{\varepsilon}\not{k}\right)u^{(+)}_{p_i\lambda_i}.\end{aligned} \tag{16}$$

Here, we have introduced the scaled amplitude of the vector potential,

$$\mu=\frac{|eA_0|}{m_ec}, \tag{17}$$

which measures the peak intensity of the laser field. After some algebraic manipulations, we find that a phase present in Eq. (16) equals

$$S^{(+)}_{p_i}(x)-S^{(+)}_{p_f}(x)-K\cdot x=(\bar{p}_i-\bar{p}_f-K)\cdot x+G(k\cdot x), \tag{18}$$

where the laser-dressed momenta have been introduced,

$$\bar{p}=p-\mu m_ec\frac{p\cdot\varepsilon}{p\cdot k}\langle f\rangle k+\frac{1}{2}(\mu m_ec)^2\frac{\langle f^2\rangle}{p\cdot k}k. \tag{19}$$

Since $\bar{p}$ is polarization-dependent one observes asymmetries in angular distributions of the Compton photons (Krajewska & Kamiński, 2012*a*). Here, we understand that

$$\langle f^j\rangle=\frac{1}{T_p}\int_{\boldsymbol{k}\cdot\boldsymbol{r}/ck^0}^{T_p+\boldsymbol{k}\cdot\boldsymbol{r}/ck^0}dt\,[f(ck^0t-\boldsymbol{k}\cdot\boldsymbol{r})]^j, \tag{20}$$

where $j=1,2$. In Eq. (18), we have also introduced

$$\begin{aligned}G(k\cdot x)=\int_0^{k\cdot x}d\phi\Big[&-\mu m_ec\left(\frac{p_i\cdot\varepsilon}{p_i\cdot k}-\frac{p_f\cdot\varepsilon}{p_f\cdot k}\right)(f(\phi)-\langle f\rangle)\\&+\frac{1}{2}(\mu m_ec)^2\left(\frac{1}{p_i\cdot k}-\frac{1}{p_f\cdot k}\right)(f^2(\phi)-\langle f^2\rangle)\Big].\end{aligned} \tag{21}$$

Note that the space-time integral (16) can be expressed in term of integrals

$$C^{(n)}=\int d^4x[f(k\cdot x)]^n e^{-i(\bar{p}_i-\bar{p}_f-K)\cdot x-iG(k\cdot x)}, \tag{22}$$

where $n=0,1,2$. Passing to the light-cone variables (Krajewska & Kamiński, 2012*a*) we see that the integral over x^- is limited to the finite region, $0\leqslant x^-\leqslant 2\pi/k^0$, provided that n is not equal to 0. For $n=0$, we need to transform the respective integral to a more suitable form, which is done by applying the Boca-Florescu transformation (Krajewska & Kamiński, 2012*a*). This leads to

$$\begin{aligned}C^{(0)}=&\int d^4x\left(\tilde{a}f(k\cdot x)+\tilde{b}f^2(k\cdot x)\right)\\&\times e^{-i(\bar{p}_i-\bar{p}_f-K)\cdot x-iG(k\cdot x)},\end{aligned} \tag{23}$$

where

$$\tilde{a}=\frac{k^0}{Q^0}\mu m_ec\left(\frac{p_i\cdot\varepsilon}{p_i\cdot k}-\frac{p_f\cdot\varepsilon}{p_f\cdot k}\right), \tag{24}$$

$$\tilde{b}=-\frac{k^0}{2Q^0}(\mu m_ec)^2\left(\frac{1}{p_i\cdot k}-\frac{1}{p_f\cdot k}\right), \tag{25}$$

with the four-vector Q defined as

$$Q=p_i-p_f-K. \tag{26}$$

The above equations define parameters $\tilde{a}$ and $\tilde{b}$, provided that $Q^0\neq 0$. We will show below that this condition is always satisfied (see, Eq. (35)).

Note that the laser-dressed momenta (19) are gauge-dependent, as they change their values if the gauge

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transformation, Eq. (13), with a constant a is applied. For the first time, such a definition of the laser-dressed momenta has been introduced in the context of Compton scattering (Krajewska & Kamiński, 2012a). At the same time, we would like to note that in the formulation presented above only the difference $\bar{p}_\mathrm{i} - \bar{p}_\mathrm{f}$ appears. The point being that this difference is gauge-invariant. Therefore, we can redefine the laser-dressed momenta such that

$$\bar{p} \to \bar{p} - s_0 k - s_1 \varepsilon, \tag{27}$$

with, in principle, arbitrary real constants s_0 and s_1. These constants can still depend on time-averaged shape functions (namely, $\langle f \rangle$ or $\langle f^2 \rangle$), however they should vanish for a vanishing laser field in order to keep the correspondence to the free particle case. Note that the new definition (27) preserves the difference $\bar{p}_\mathrm{i} - \bar{p}_\mathrm{f}$. Therefore, one can use this fact to define the gauge-invariant laser-dressed momenta. By making the substitution $\varepsilon \to \varepsilon + ak$ (where a is constant), we find that the dressed momenta do not depend on a provided that $s_1 = -\mu m_\mathrm{e} c \langle f \rangle$, while retaining an arbitrary s_0. Let us stress, however, that such a modification of the momentum dressing does not affect any observable quantity, and should be considered only as a mathematical operation.

$G(k \cdot x)$, given by Eq. (21), is a periodic function of $k \cdot x$, meaning that $G(0) = G(2\pi)$. Hence, we can make the following Fourier expansion,

$$[f(k \cdot x)]^n e^{-iG(k \cdot x)} = \sum_{N=-\infty}^{\infty} G_N^{(n)} e^{-iNk \cdot x}. \tag{28}$$

Using this series expansion we can rewrite Eq. (16) such that

$$\mathscr{A} = \sum_N D_N \int d^4 x e^{-i(\bar{p}_\mathrm{i} - \bar{p}_\mathrm{f} - K + Nk) \cdot x}, \tag{29}$$

where

$$\begin{aligned} D_N = & \bar{u}^{(+)}_{\boldsymbol{p}_\mathrm{f}\lambda_\mathrm{f}} \not{\varepsilon}_{K\sigma} u^{(+)}_{\boldsymbol{p}_\mathrm{i}\lambda_\mathrm{i}} G_N^{(0)} \\ & + \frac{1}{2}\mu m_\mathrm{e} c \left(\frac{1}{p_\mathrm{i} \cdot k} \bar{u}^{(+)}_{\boldsymbol{p}_\mathrm{f}\lambda_\mathrm{f}} \not{\varepsilon}_{K\sigma} \not{\varepsilon} \not{k} u^{(+)}_{\boldsymbol{p}_\mathrm{i}\lambda_\mathrm{i}} \right. \\ & \left. - \frac{1}{p_\mathrm{f} \cdot k} \bar{u}^{(+)}_{\boldsymbol{p}_\mathrm{f}\lambda_\mathrm{f}} \not{\varepsilon} \not{k} \not{\varepsilon}_{K\sigma} u^{(+)}_{\boldsymbol{p}_\mathrm{i}\lambda_\mathrm{i}} \right) G_N^{(1)} \\ & - \frac{(\mu m_\mathrm{e} c)^2}{4(p_\mathrm{i} \cdot k)(p_\mathrm{f} \cdot k)} \bar{u}^{(+)}_{\boldsymbol{p}_\mathrm{f}\lambda_\mathrm{f}} \not{\varepsilon} \not{k} \not{\varepsilon}_{K\sigma} \not{\varepsilon} \not{k} u^{(+)}_{\boldsymbol{p}_\mathrm{i}\lambda_\mathrm{i}} G_N^{(2)}, \end{aligned} \tag{30}$$

and where $G_N^{(0)}$ must be replaced by $\tilde{a} G_N^{(1)} + \tilde{b} G_N^{(2)}$, as it follows from the Boca-Florescu transformation (23). Now, performing the space-time integration in Eq. (29) and keeping in mind that $0 \leqslant x^- \leqslant 2\pi/k^0$, we arrive at

$$\mathscr{A} = \sum_N (2\pi)^3 \delta^{(1)}(P_N^-) \delta^{(2)}(\boldsymbol{P}_N^\perp) D_N \frac{1 - e^{-2\pi i P_N^+/k^0}}{i P_N^+}, \tag{31}$$

where

$$P_N = \bar{p}_\mathrm{i} - \bar{p}_\mathrm{f} - K + Nk. \tag{32}$$

Note that P_N^- and $\boldsymbol{P}_N^\perp$ are independent of N.

2.1. Compton-Photon Energy Distribution

In order to solve the momentum conservation conditions imposed by the delta functions in Eq. (31), let us introduce the four-vector $w = p_\mathrm{i} - K$, so that

$$p_\mathrm{f}^0 = p_\mathrm{f}^\parallel + w^-, \quad \boldsymbol{p}_\mathrm{f}^\perp = \boldsymbol{w}^\perp. \tag{33}$$

Since the electron mass is different from zero, it follows from the first equation that $w^- > 0$, and

$$p_\mathrm{f}^\parallel = \frac{(m_\mathrm{e} c)^2 - (w^-)^2 + \boldsymbol{w}_\perp^2}{2w^-} = \frac{K \cdot p_\mathrm{i}}{w^-} + w^\parallel, \tag{34}$$

which means that

$$Q^0 = p_\mathrm{i}^0 - p_\mathrm{f}^0 - K^0 = p_\mathrm{i}^\parallel - p_\mathrm{f}^\parallel - K^\parallel = -\frac{p_\mathrm{i} \cdot K}{w^-} < 0. \tag{35}$$

This is exactly the applicability condition for the Boca-Florescu transformation.

Since $\boldsymbol{P}_N^\perp$ and P_N^- do not depend explicitly on N, and

$$\int d^3 p_\mathrm{f}\, \delta^{(1)}(P_N^-) \delta^{(2)}(\boldsymbol{P}_N^\perp) = \frac{k^0 p_\mathrm{f}^0}{k \cdot p_\mathrm{f}}, \tag{36}$$

we derive the differential distribution of energy emitted as Compton photons with polarization σ in the space direction $\boldsymbol{n}_{\boldsymbol{K}}$, provided that the initial and final electron momentum and spin polarization are $\boldsymbol{p}_\mathrm{i}\lambda_\mathrm{i}$ and $\boldsymbol{p}_\mathrm{f}\lambda_\mathrm{f}$, respectively,

$$\begin{aligned} \frac{d^3 E_C(\boldsymbol{n}_{\boldsymbol{K}}\sigma; \boldsymbol{p}_\mathrm{i}\lambda_\mathrm{i}; \boldsymbol{p}_\mathrm{f}\lambda_\mathrm{f})}{d\omega_{\boldsymbol{K}} d^2\Omega_{\boldsymbol{K}}} = & \frac{\alpha (m_\mathrm{e} c)^2 k^0 (K^0)^2}{(2\pi)^2 p_\mathrm{i}^0 (k \cdot p_\mathrm{f})} \\ & \times \left| \sum_N D_N \frac{1 - e^{-2\pi i P_N^0/k^0}}{P_N^0} \right|^2. \end{aligned} \tag{37}$$

Here, one has to remember that the electron final four-momentum p_f is such that

$$\boldsymbol{p}_\mathrm{f}^\perp = \boldsymbol{w}^\perp, \tag{38}$$

$$p_\mathrm{f}^\parallel = \frac{(m_\mathrm{e} c)^2 + (\boldsymbol{w}^\perp)^2 - (w^-)^2}{2w^-}, \tag{39}$$

$$p_\mathrm{f}^0 = \frac{(m_\mathrm{e} c)^2 + (\boldsymbol{w}^\perp)^2 + (w^-)^2}{2w^-}, \tag{40}$$

which follows from the four-momentum conservation conditions. One can also check that the above solution of the momentum conservation conditions, $P_N^- = 0$ and $\boldsymbol{P}_N^\perp = \boldsymbol{0}$, satisfies the on-mass shell relation, $p_f \cdot p_f = (m_e c)^2$.

2.2. Electron Probability Distribution

In order to obtain the energy-angular distribution of the final electrons scattered in the Compton process we have to solve the above-mentioned momentum conservation conditions for the given initial and final electron momenta. Elementary algebra shows that the Compton photon momentum K is

$$K^0 = \frac{(q^-)^2 + (\boldsymbol{q}^\perp)^2}{2q^-}, \; K^\parallel = -\frac{(q^-)^2 - (\boldsymbol{q}^\perp)^2}{2q^-}, \; \boldsymbol{K}^\perp = \boldsymbol{q}^\perp, \tag{41}$$

where $q = p_i - p_f$. Since the propagation of the Compton photon cannot be parallel to the propagation of the laser beam (hence $K^- > 0$), we get the condition $q^- > 0$, which puts restrictions on the final electron momentum. Indeed, it leads to the inequality

$$x^2 \sin^2\theta_f - 2xp_i^- \cos\theta_f + (m_e c)^2 - (p_i^-)^2 < 0, \tag{42}$$

in which $x = |\boldsymbol{p}_f| > 0$, and θ_f is the angle between the momentum $\boldsymbol{p}_f$ and the direction of propagation of the laser pulse. In order to have real solutions of this inequality, we get the restrictions for θ_f,

$$(p_i^-)^2 - (m_e c)^2 \sin^2\theta_f > 0, \tag{43}$$

as well as for x,

$$\max(0, x_-) < x = |\boldsymbol{p}_f| < \max(0, x_+), \tag{44}$$

where

$$x_\pm = \frac{p_i^- \cos\theta_f \pm \sqrt{(p_i^-)^2 - (m_e c)^2 \sin^2\theta_f}}{\sin^2\theta_f}. \tag{45}$$

Using standard methods based on the scattering matrix element, we can derive the expression for the energy-angular probability distribution of final electrons. Since

$$\int d^3K \delta^{(1)}(P_N^-)\delta^{(2)}(\boldsymbol{P}_N^\perp) = \frac{k^0 K^0}{k \cdot K}, \tag{46}$$

we obtain that this distribution is given as

$$\frac{d^3P(p_f; \lambda_i, \lambda_f, \sigma)}{dE_{\boldsymbol{p}_f} d^2\Omega_f} = \frac{\alpha(m_e c)^2 k^0 |\boldsymbol{p}_f|}{(2\pi)^2 E_{\boldsymbol{p}_i}(k \cdot K)} \times \left| \sum_{N=-\infty}^{\infty} D_N \frac{1 - e^{-2\pi i P_N^+/k^0}}{P_N^+} \right|^2, \tag{47}$$

where λ_i, λ_f, and σ label the spins of the initial and final electrons, and the polarization of the Compton photon, respectively.

2.3. Shape Function

For a particular realization of the theory derived above, we choose the laser pulse determined by the four-vector potential,

$$A(k \cdot x) = A_0 B \varepsilon f(k \cdot x). \tag{48}$$

Here, A_0 is related to the parameter μ (Eq. (17)), whereas the constant B is adjusted such that the energy contained in the laser pulse is constant (irrespective of its duration). For the shape function, we choose

$$\begin{aligned} f'(k \cdot \boldsymbol{x}) &= N_A \sin^2\left(\frac{k_L \cdot x}{2N_{osc}}\right) \sin(k_L \cdot x + \chi) \\ &= N_A \sin^2\left(\frac{k \cdot x}{2}\right) \sin(N_{osc} k \cdot x + \chi), \end{aligned} \tag{49}$$

for $0 \leqslant k_L \cdot x \leqslant 2\pi N_{osc}$, and 0 otherwise. The pulse duration equals $T_p = 2\pi N_{osc}/\omega_L$, hence $k_L = (\omega_L/c)(1, \boldsymbol{n}) = N_{osc} k$. Moreover, ω_L is the carrier frequency of the laser pulse whereas N_{osc} (where $N_{osc} = 1, 2, \ldots$) defines the number of field oscillations contained in the pulse. Here, χ is the carrier-envelope phase. Note that the shape function (49) determines the electric and magnetic fields of the laser pulse, Eqs. (8) and (9). Thus, the shape function for the four-vector potential equals

$$f(k \cdot x) = \int_0^{k \cdot x} d\phi f'(\phi), \tag{50}$$

and vanishes for $k \cdot x < 0$ and $k \cdot x > 2\pi$. In Eq. (49), the normalization constant N_A is defined such that

$$\frac{1}{T_p} \int_{\boldsymbol{k}\cdot\boldsymbol{r}/ck^0}^{T_p + \boldsymbol{k}\cdot\boldsymbol{r}/ck^0} dt [f'(ck^0 t - \boldsymbol{k} \cdot \boldsymbol{r})]^2 = \frac{1}{2}, \tag{51}$$

where "prime" still means the derivative with respect to the argument $k \cdot x$. Therefore, if we keep the energy within the pulse fixed, we need to scale the maximum value of the vector potential by $\sqrt{N_{osc}}$ when changing the number of laser field oscillations within the pulse, N_{osc}. In other words, $B = \sqrt{N_{osc}}$ has to be chosen in Eq. (48).

3. SPIN EFFECTS

3.1. Compton-Photon Energy Distribution

The relativistic quantum theory formulated in Section 2 is relativistically invariant, and so we can arbitrarily choose the reference system in which we perform our calculations. For numerical illustration, we consider the reference system such that the laser pulse propagates along the z-axis

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($\boldsymbol{n} = \boldsymbol{e}_z$) and the electron has the initial momentum $\boldsymbol{p}_\mathrm{i} = -10^{-4} m_\mathrm{e} c \boldsymbol{e}_z$, i.e., we consider the head-on configuration. More importantly, since we choose $\boldsymbol{p}_\mathrm{i} \neq \boldsymbol{0}$, it is meaningful to define the electron helicity. The carrier frequency of the incident laser pulse in the chosen reference frame is $\omega_\mathrm{L} = 0.1 m_e c^2$, and so the process must be treated on quantum-mechanical grounds. In addition, we consider a laser pulse characterized by the parameters $\mu = 10$ and $\chi = 0$. We assume that the Compton photon is scattered in the direction $\boldsymbol{n}_{\boldsymbol{K}}$ specified by the polar and azimuthal angles $\theta_{\boldsymbol{K}} = 0.2\pi$ and $\varphi_{\boldsymbol{K}} = 0$ (in radians) which defines unambiguously the (xz)-plane, that we call the scattering plane.

We consider the linearly polarized vectors of the laser pulse $\boldsymbol{\varepsilon}$ and the Compton photon $\boldsymbol{\varepsilon}_{\boldsymbol{K}}$ in four different configurations, each of them either lying in the (xz)-plane or being perpendicular to this plane. From now on, λ_i and λ_f will denote the initial and final helicities of the electron. They can be $+1$ if the electron spin projection on its propagation direction is $+\frac{1}{2}$, and -1 otherwise. In the presented figures, we will show the results for two cases: when the electron helicity does not change during the process ($\lambda_\mathrm{i}\lambda_\mathrm{f} = +1$) and when it does change ($\lambda_\mathrm{i}\lambda_\mathrm{f} = -1$) due to the Compton photon emission. The results for different pulse durations will be shown.

In Figure 1, we show the energy spectra of Compton radiation which is emitted when the laser pulse polarization vector lies in the scattering plane, and the laser pulse contains four oscillations ($N_\mathrm{osc} = 4$). Comparing all panels, we observe the dominant contribution for those Compton photons which are also polarized in the scattering plane (see, the upper panels). In this case, the process which conserves helicity of the colliding electron dominates for small energies of emitted photons, when $\omega_{\boldsymbol{K}} < 2 m_\mathrm{e} c^2$. For larger Compton photon energies, $\omega_{\boldsymbol{K}} > 2 m_e c^2$, both processes (with and without the electron spin flip) seem to occur with comparable probabilities. To see a more detailed structure of these distributions we present them in Figure 2 over a very narrow range of $\omega_{\boldsymbol{K}}$. These distributions exhibit very rapid oscillations, which is a typical feature of the Compton radiation spectra (Boca & Florescu, 2009; Krajewska & Kamiński, 2012*a*; Panek *et al.*, 2002; Seipt & Kämpfer, 2011). Interestingly, there are some energies $\omega_{\boldsymbol{K}}$ for which the Compton radiation is predominantly emitted in the process that does not conserve the colliding electron helicity. This is in contrast to the case when Compton scattering is induced by a monochromatic plane wave laser field (Panek *et al.*, 2002) or even by a finite laser pulse of circular polarization (Boca *et al.*, 2012*b*). We anticipate therefore that this effect is most pronounced for a linearly polarized laser pulse colliding with a target electron.

As one can see in Figure 1, the emission of photons polarized perpendicularly to the scattering plane is suppressed as compared to the emission of photons polarized in that plane. On contrary, in Figure 3 we demonstrate that photon emission in both directions is comparable. We would like to stress that this is a purely quantum effect and so it cannot be observed in Thomson scattering. Here, similar to Figure 1,

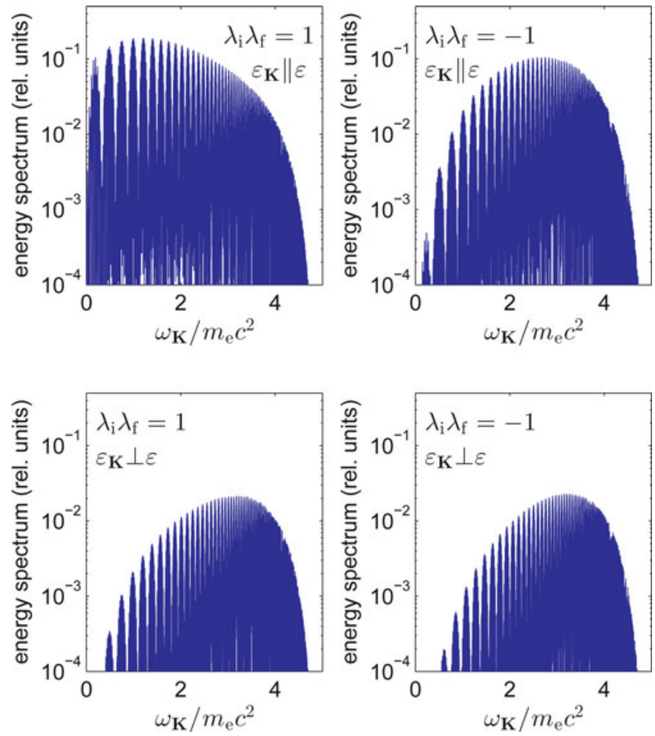

Fig. 1. (Color online) The energy spectra of Compton photons (37) emitted in the head-on collision of a short laser pulse and the relativistic electron. The laser pulse propagates in the z-direction whereas its polarization vector $\boldsymbol{\varepsilon}$ is in the (xz)-plane. In addition, the central frequency of the laser field is $\omega_\mathrm{L} = 0.1 m_e c^2$, its strength is determined by the parameter $\mu = 10$, and the carrier-envelope phase is $\chi = 0$. The results are for the four-cycle laser pulse ($N_\mathrm{osc} = 4$). The electron initial momentum is $\boldsymbol{p}_\mathrm{i} = -10^{-4} m_e c \boldsymbol{e}_z$. The Compton photon is detected in the direction specified by the polar and azimuthal angles $\theta_{\boldsymbol{K}} = 0.2\pi$ and $\varphi_{\boldsymbol{K}} = 0$ (in radians). While the upper panels are for the case when both the laser pulse and the Compton photon are polarized in the same direction, the lower panels are for the case when both vectors are perpendicular to each other. At the same time, the left column represents the results for the case when the electron helicity is conserved during the process ($\lambda_\mathrm{i}\lambda_\mathrm{f} = +1$), while the right column represents the results for the case when the electron helicity flips due to the Compton photon emission ($\lambda_\mathrm{i}\lambda_\mathrm{f} = -1$).

the no-spin-flipping process is by far more efficient over the shown range of Compton photon energies, i.e., when $\omega_{\boldsymbol{K}} < 2 m_\mathrm{e} c^2$.

Note that Figure 2 unveils regimes in which the Compton radiation is mostly emitted via a spin-flipping process. The question arises whether the demonstrated frequency differences could be resolved experimentally. To answer this question, let us keep in mind that the presented spectra relate to a particular choice of the reference frame. The parameters used in this section, when transformed to the laboratory frame (LAB), correspond to the head-on collision of a Ti:Sapphire laser beam ($\omega_\mathrm{L}^\mathrm{LAB} = 1.548$ eV) with a highly relativistic electron such that $|\boldsymbol{p}_\mathrm{i}^\mathrm{LAB}| = 7.9$ GeV$/c$. Due to their interaction, a highly energetic photon is produced; for instance, with energy $\omega_{\boldsymbol{K}}^\mathrm{LAB} = 3$ GeV and at the angle $\theta_{\boldsymbol{K}}^\mathrm{LAB} = \pi - 6 \times 10^{-5}\pi$ if $\omega_{\boldsymbol{K}} = 2 m_\mathrm{e} c^2$ and $\theta_{\boldsymbol{K}} = 0.2\pi$. What is however more important (see, Figure 2), the spin-flipping configuration is preferable over the energy interval of

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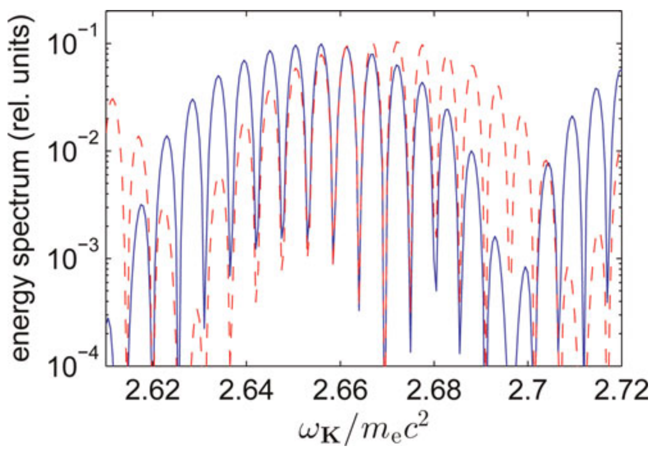

Fig. 2. (Color online) The same as in the top panels of Figure 1 but over a small region of Compton photon energies. The detailed comparison of energy spectra is shown for the Compton process which conserves helicity of the colliding electron (solid line) and which does not (dashed line). For some energies, the Compton radiation is predominantly emitted in the process which does not conserve the target electron helicity.

Compton photons roughly $0.03 m_{\mathrm{e}}c^2$ (in the chosen reference frame) or, equivalently, 45 MeV (in the laboratory frame). We believe therefore that even in view of finite energy spread and emittance of available electron beams, it is possible to resolve such differences experimentally.

As we have mentioned above, the characteristic property of Compton radiation spectra are very rapid oscillations. These oscillations are due to multiphoton interferences which are characteristic for other strong-field QED processes as well. For the Compton scattering by a long laser pulse, we also observe an additional structure, as shown in Figure 4. The results presented in Figure 4 are for the 32-cycle pulse polarized in the scattering plane. In this case, there appears an additional structure in the region of low-energy Compton photons. The same is present in Figure 5 where the 32-cycle pulse is polarized in the direction perpendicular to the scattering plane. We suspect that this structure results from a destructive interference between the Compton radiation emitted during the raise of the pulse and its fall off, which does not occur for short pulses (see, Figs. 1 and 2). Moreover, in Figure 5 we observe that the envelope of oscillations exhibits additional modulations. Other than that, the other features discussed above for Compton scattering by a four-cycle pulse stay the same, i.e., they do not depend on the pulse duration. Note that the aforementioned modulations (Fig. 5) are typical multiphoton modulations observed for processes induced by long laser pulses. Such multiphoton peaks in Compton scattering has been observed (Boca & Florescu, 2009, 2011; Seipt & Kämpfer, 2011; Krajewska & Kamiński, 2012*a*).

3.2. Electron Probability Distribution

Next, we would like to discuss the electron probability distributions, as they allow one to establish a closer connection to an experiment measuring only the properties of scattered

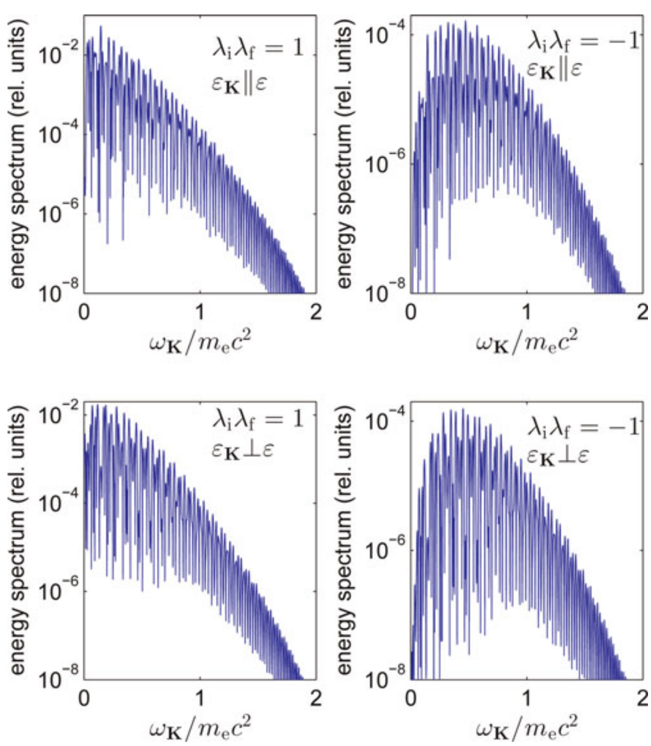

Fig. 3. (Color online) The same as in Figure 1 but the polarization vector of the incident laser pulse $\boldsymbol{\varepsilon}$ is perpendicular to the (*xz*)-plane.

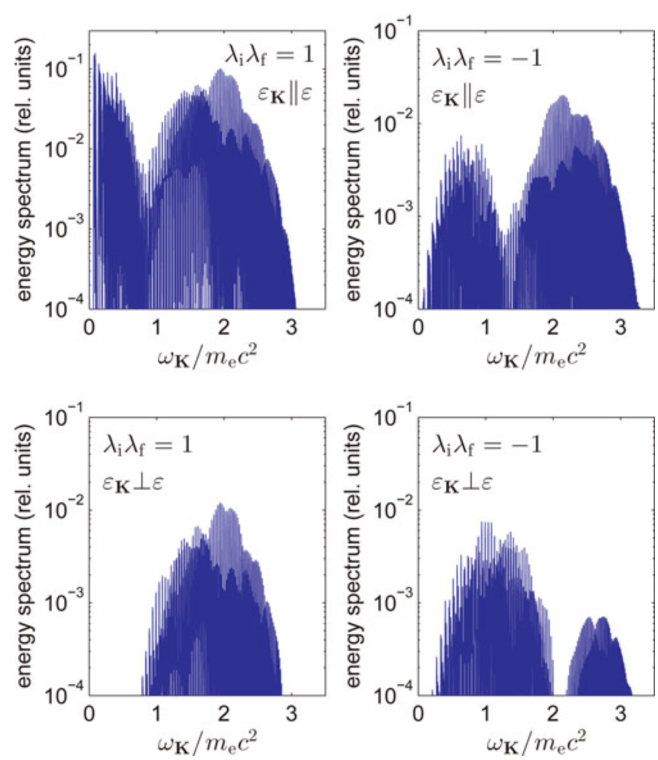

Fig. 4. (Color online) The same as in Figure 1 but for the 32-cycle incident laser pulse ($N_{\mathrm{osc}} = 32$).

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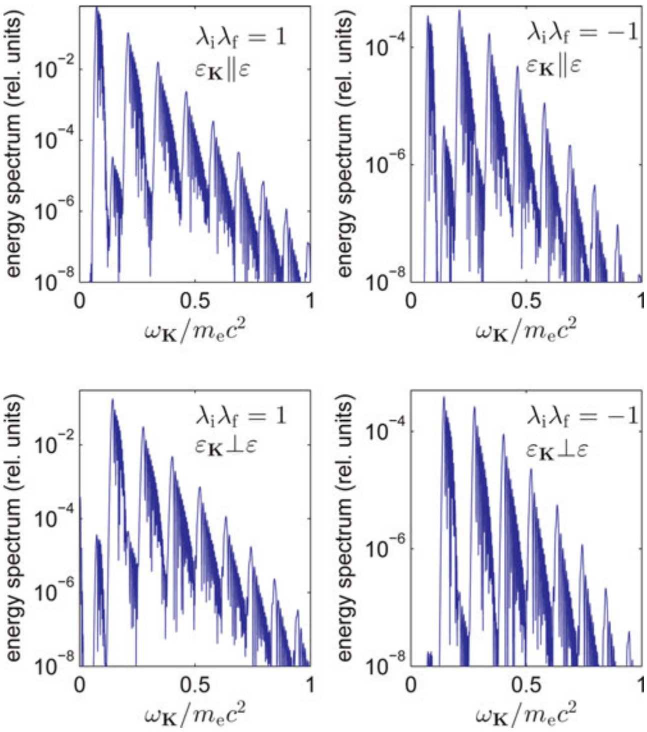

Fig. 5. (Color online) The same as in Figure 4 but the polarization vector of the laser pulse **ε** is perpendicular to the (*xz*)-plane.

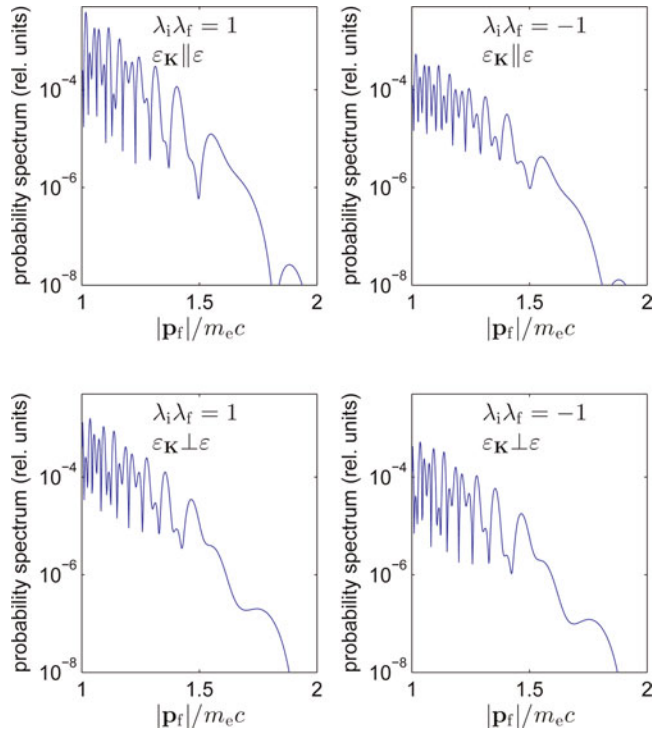

Fig. 6. (Color online) The probability distributions of final electron after it scatters off a short laser pulse, Eq. (47). The presented results are in the reference frame in which the laser pulse propagates in the *z*-direction, $\mu = 10$, $\omega_L = 0.1m_ec^2$, $N_{osc} = 4$, $\chi = 0$, and the polarization vector points in the *y*-direction. The electron initial momentum is $\boldsymbol{p}_i = -10^{-4}m_ec\boldsymbol{e}_z$, and the polar and azimuthal angles of the final electron are $\theta_f = 0.2\pi$ and $\varphi_f = 0$ (in radians).

electrons and letting the scattered photons to pass unobserved. The presented electron distributions were calculated based on theory introduced in Section 2.2. As specified in the previous section, λ_i and λ_f denote here the initial and final electron helicities; in particular, $\lambda_i\lambda_f = 1$ relates to the process which conserves the electron spin whereas $\lambda_i\lambda_f = -1$ is for otherwise.

In Figure 6, we show the probability distributions of final electron which is scattered off a short laser pulse in the counterpropagating setup; namely, we choose the reference frame in which the pulse propagates in the *z*-direction ($\boldsymbol{n} = \boldsymbol{e}_z$) whereas the initial electron moves with the opposite momentum $\boldsymbol{p}_i = -10^{-4}m_ec\boldsymbol{e}_z$. In the chosen reference frame, the carrier frequency of the laser pulse is $\omega_L = 0.1m_ec^2$. The maximum strength of the field is characterized by the parameter $\mu = 10$, and the carrier-envelope phase is $\chi = 0$. The Compton photon is scattered in the *y*-direction, while the final electron direction is described by the angles $\theta_f = 0.2\pi$ and $\varphi_f = 0$ (in radians). This time, we define the scattering plane with respect to the initial and final electron momenta, which is still the (*xz*)-plane. As one can see in Figure 6, the electron scattering occurs predominantly through the no-spin-flipping process, which happens for both considered configurations. If we change the carrier-envelope phase to $\chi = \pi/2$, as presented in Figure 7, the spin-flipping process becomes dominant if $\boldsymbol{\varepsilon}_K \| \boldsymbol{\varepsilon}$. Therefore, one can control the properties of the process by changing the phase χ.

In closing this section, we would like to point out that usually the process with $\lambda_i\lambda_f = -1$ is suppressed with respect to the no-spin-flipping process. However, for some particular geometries, in particular, when the laser-field polarization is perpendicular to the scattering plane, we have found the opposite situation. This has been illustrated in Figure 7 (upper row) but it is even more clear in Figure 8. In the latter case, the results are shown in a different reference frame, in which the initial momentum of the electron is $\boldsymbol{p}_i = -m_ec\boldsymbol{e}_z$. In particular, when comparing the panels in the upper row of Figure 8, one observes roughly an order of magnitude enhancement of the probability distribution for the spin-flipping process. Even though less pronounced, such an enhancement is also observed in the lower panel of this figure over the whole range of energies.

4. REMARKS ON THE ELECTRON ACCELERATION

A lot of attention in literature has been devoted to electron acceleration by lasers, which includes wakefield acceleration in plasma (Esarey *et al.*, 2009; Malka, 2012; Tajima & Dawson, 1979; Umstadter, 2003), electron acceleration by a standing wave (Galkin *et al.*, 2012; Korobkin *et al.*, 2013), or Thomson scattering in an all-optical setup (Kulagin *et al.*, 2008). Here, we would like to comment on the possibility of electron acceleration via the Compton process.

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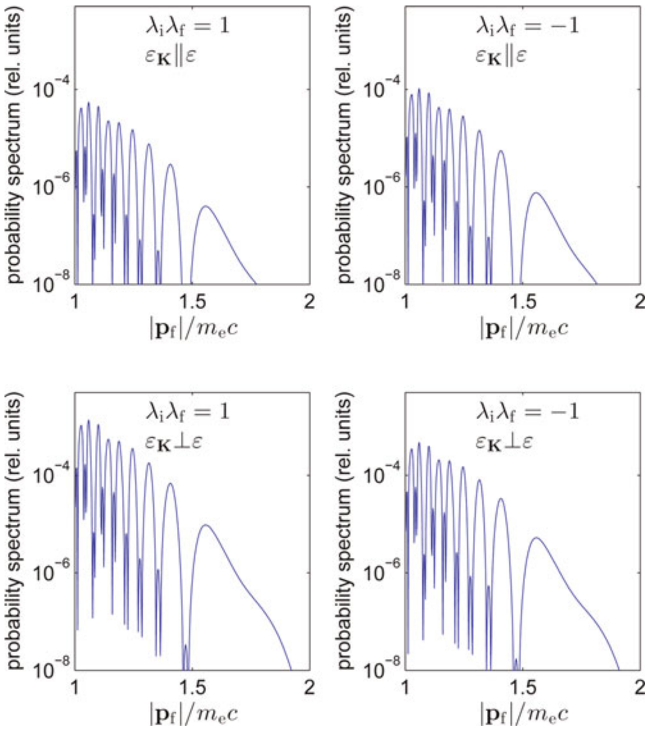

Fig. 7. (Color online) The same as in Figure 6 but for $\chi = \pi/2$.

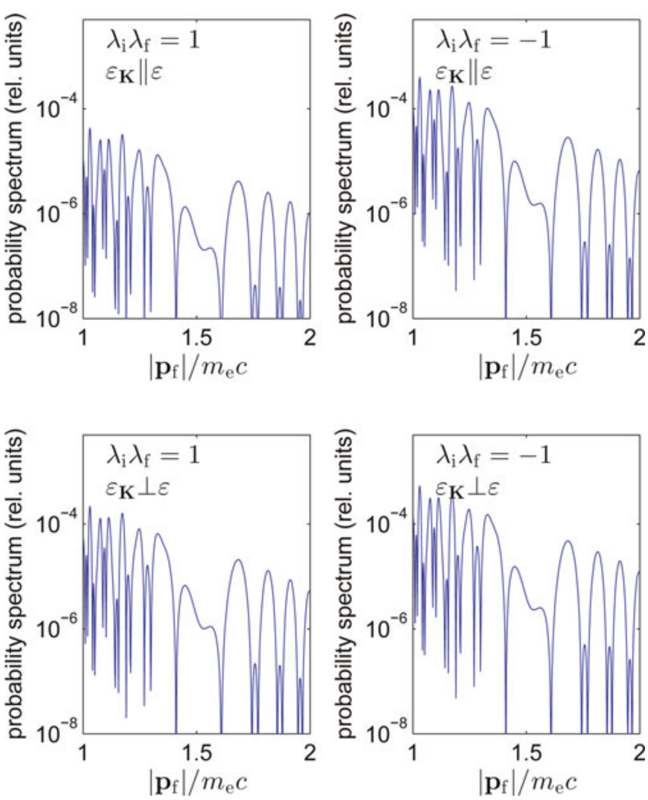

Fig. 8. (Color online) The same as in Figure 7 but in the reference frame where $\boldsymbol{p}_{\rm i} = -m_e c\boldsymbol{e}_z$. For the considered parameters, the spin-flipping process dominates.

As is known from classical electrodynamics, electrons interacting with a plane-wave-fronted pulse cannot be accelerated or decelerated. We meet a completely different situation in QED, where during the Compton process energies of the initial and final electrons differ from each other. Usually, when studying the Compton effect, one is mostly interested in efficient creation of photons as energetic as possible. This goal is achieved by decelerating the high-energy electron. The natural question arises: Is it possible to accelerate already high-energy electron beams by means of the Compton process in short laser pulses? The answer to this question (within the PWFP approximation) is positive, as follows from our discussion of the conservation conditions. Indeed, let us choose the reference frame, *the computational reference frame*, in which electrons are initially at rest, hence $p_{\rm i}^- = m_{\rm e}c$. In this frame, there are restrictions imposed on possible momenta of final electrons

$$0 < |\boldsymbol{p}_{\rm f}| < p_{\max} = 2m_{\rm e}c\frac{\cos\theta_{\rm f}}{\sin^2\theta_{\rm f}}, \tag{52}$$

where $\cos\theta_f > 0$ and θ_f is the angle between the laser pulse and final electron propagation directions. Therefore, the electron energy cannot exceed its maximum value equal to

$$E_{\max} = m_{\rm e}c^2(1 + 2\cot^2\theta_{\rm f}), \tag{53}$$

which is larger than the initial electron energy $m_e c^2$. By making the Lorentz boost along the laser field propagation such that its central frequency $\omega_{\rm L}$ scales to the laboratory one as

$$\omega_{\rm L}^{\rm LAB} = \gamma(1-\beta)\omega_{\rm L} = \sqrt{\frac{1-\beta}{1+\beta}}\,\omega_{\rm L}, \tag{54}$$

the above maximum energy transforms into

$$E_{\max}^{\rm LAB} = m_{\rm e}c^2\gamma[1 + 2(1-\beta)\cot^2\theta_{\rm f}]. \tag{55}$$

Hence, the maximum relative gain of energy by electrons in the Compton scattering is

$$\delta_{\max}^{\rm LAB} = \frac{E_{\max}^{\rm LAB}}{m_{\rm e}c^2\gamma} - 1 = 2(1-\beta)\cot^2\theta_{\rm f} > 0, \tag{56}$$

and can be arbitrary large for sufficiently small $\theta_{\rm f}$.

In Figure 9 we present the relative gain of the electron energy in the laboratory frame as a function of the momentum of a scattered electron in the computational frame. We observe that for sufficiently small scattering angles $\theta_{\rm f}$ the electron can significantly be accelerated, but this process is accompanied by the enormous increase of the Compton photon energy, even larger than the initial electron energy. Due to this fact, probabilities for the acceleration process are many orders of magnitude smaller than the probabilities in which electrons are decelerated. In this sense, although possible, the Compton scattering in short laser pulses cannot be considered as an efficient mechanism for the acceleration of electrons.

5. CONCLUSIONS

We investigated the Compton scattering by short laser pulses, with the emphasis on spin effects. The head-on configuration of the colliding electron and the laser pulse was considered. The laser pulse was treated as a plane-wave-fronted field, which is along the lines developed in our recent papers (Krajewska & Kamiński, 2012*a*; 2012*b*). Using this model to describe the incident laser field, we demonstrated that the Compton photon energy spectra are very rapidly oscillating functions which are modulated if the incident laser pulse is long enough. Our numerical results showed that, in general, the Compton photon emission is most pronounced if the polarization vector of the emitted radiation is in the scattering plane and there is no spin-flip of the colliding electron. There exists, however, some energy intervals where the Compton photon emission with the electron spin-flip is dominant; in contrast to Compton scattering by a monochromatic plane wave laser field (Panek *et al.*, 2002). As we demonstrated, the same is true for the electron energy spectra. When analyzing the latter, we pointed out that there is a possibility to manipulate with spin effects by changing the carrier-envelope phase of the incident laser pulse. Finally, we discussed the possibility of electron acceleration by means of Compton scattering by short laser pulses.

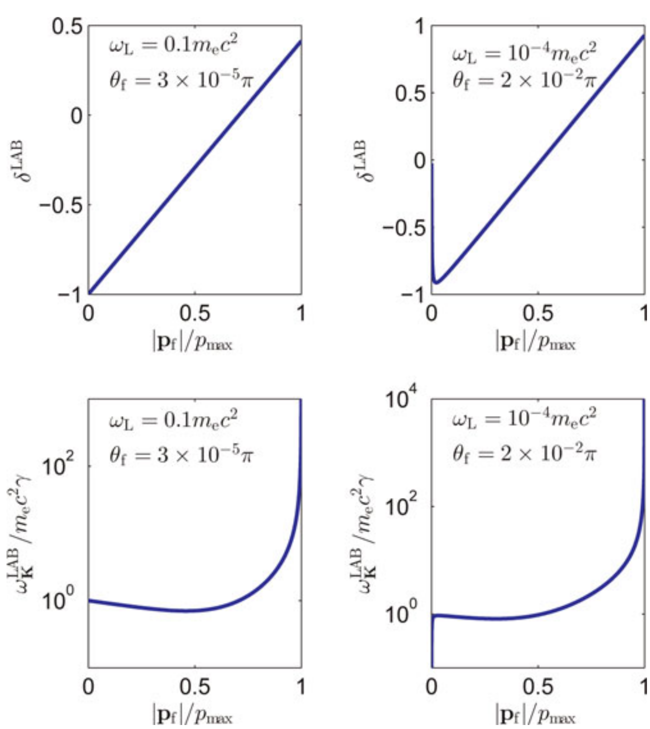

Fig. 9. (Color online) The upper row presents the relative energy gain of electrons in the laboratory reference frame, $\delta^{LAB} = E_{p_f}^{LAB}/E_{p_i}^{LAB} - 1$, as functions of the electron final momentum in the computational reference frame, for two central laser frequencies and scattering angles (also in the computational reference frame). The lower row presents the ratio of emitted Compton photon energy to the initial electron energy in the laboratory reference frame. In both cases the central laser frequency in the laboratory frame is $\omega_L^{LAB} = 1.548$eV, which uniquely determines the γ and β parameters for the Lorentz transformation between these two reference frames, Eq. (54).

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