J. Plasma Physics (2012), *vol.* 78, *part* 2, *pp.* 203–205. © Cambridge University Press 2012
doi:10.1017/S0022377811000584

Study on stability and growth rate of the dust acoustic waves in vortex-like ion distribution

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(Received 3 August 2011; revised 23 November 2011; accepted 5 December 2011; first published online 6 January 2012)

Abstract. For vortex-like ion distribution dusty plasmas, the modified KP equation is obtained by using the traditional perturbation method. The growth rate of a solitary wave solution for a higher order disturbance propagating in an arbitrary direction is obtained. We find that the solitary wave is stable under higher order disturbance in this system. However, the growth rate is different with different propagating direction.

1. Introduction

Dusty plasma has been an interesting research topic in the past decade [1–3]. Dusty plasma is an ionized gas containing small particles of solid matter that acquires a large electric charge by collecting electrons and ions from the plasma. The dusty plasma has been observed widely in planetary rings, asteroid zones, cometary's tails, magnetosphere, as well as the lower part of the earth's atmosphere [4]. In the laboratory, the dust grains exist as impurities and influence significantly the behavior of the surrounding plasma [4]. Since the masses of the dust grains are much greater than that of both electrons and the ions in the dusty plasma, the dust grains nearly have no consequence on high frequent oscillations except for the damping factor. Rao has predicted the existence of dust acoustic (DA) waves in the unmagnetized dusty plasma, which has also been observed in the experiments [5–9]. The dust grains can significantly change the characteristic of the dusty plasmas in the laboratory. Dusty plasmas have also been widely applied in industry.

There are many research works on dusty plasmas [10], such as: the solitary waves [11], instability [12], and the effect of the dust size distribution [13–15]. It is well known that the Korteweg-de Vries (KdV) equation can describe the small but finite amplitude nonlinear waves in one-dimensional dusty plasma. If there are transverse disturbances, this kind of waves can be described by the Kadomtsev-Petviashvili (KP) equation or Zakharov-Ktsov equation [12, 16–19].

When there are solitary waves in higher dimensional dusty plasmas, some higher order disturbances may exist. The propagation direction of this kind of disturbances may be in the same direction of the solitary waves, or in any directions that are relative to the propagation direction of the solitary waves. This kind of small disturbances may either have no effect on the solitary waves (i.e. the solitary wave is stable) or destroy the solitary waves as the time increases (i.e. the solitary wave is unstable). Such kind of work has been done if the ion distribution of the dusty plasma is in the Boltzmann distribution [16]. However, research work for vortex-like ion distribution remains unsolved. This paper will focus on this kind of work. We assume that the direction of the solitary wave is in the positive x direction. The angle of the disturbances to the x axis is θ. For vortex-like ion distribution dusty plasmas, the modified KP equation is obtained by using the traditional perturbation method. The growth rate of a solitary wave solution for a higher-order disturbance propagating in an arbitrary direction is obtained. We find that the solitary wave is stable under higher-order disturbance in this system; however, the growth rate is different with the different propagating direction.

2. Derivation of the modified KP equation

As discussed below, we studied the 2D nonlinear waves in a vortex-like ion distribution dusty plasma. For simplicity and generality, we assume that the propagation direction of the waves is in the positive x direction and there are no external magnetic fields. Meanwhile, the higher-order disturbance is propagating in an arbitrary direction. The dusty plasma contains three kinds of species of dust grains with heavy mass and negative charges, ions, and electrons. The equation of motion of this 2D dusty plasma can be written as follows:

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_x) + \frac{\partial}{\partial y}(n_d u_y) = 0 \tag{1}$$

$$\frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} = Z_d \frac{\partial \phi}{\partial x} \tag{2}$$

$$\frac{\partial u_y}{\partial t} + u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} = Z_d \frac{\partial \phi}{\partial y} \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = n_d - n_i \tag{4}$$

where n_d is the density of the dust grains, u_x, u_y are the velocities of the dust fluid in the x, y direction respectively, n_i is the density of ions, and ϕ is the electrical potential. All the variables are in dimensionless form. n_d is normalized by n_{d0}; Z_d by Z_{d0}; (x, y), spatial coordinate (x, y), time, velocity, and ϕ are normalized by $\lambda_{Dd} = (T_{eff}/4\pi Z_{d0} n_{d0} e)^{1/2}$, $\omega_{pd}^{-1} = (m_d/4\pi n_{d0} Z_{d0}^2 e^2)^{1/2}$, $c_d = (Z_{d0} T_{eff}/m_d)^{1/2}$, and T_{eff}/e respectively.

In general case, the ion satisfies Boltzmann distribution. However, in some cases, it may satisfy the vortex-like ion distribution. It has been realized in experiments and studied by many researchers [20]. It can be written in the following form for small amplitude approximation [20]:

$$n_i = 1 - \phi - \frac{4}{3}a(-\phi)^{3/2} \tag{5}$$

where $a > 0$. It is noted that this ion density is due to free and trapped ion distribution [20–26].

For this case, we use traditional perturbation method as follows:

$$\xi = \varepsilon^{1/4}(x - ct) \tag{6}$$

$$\eta = \varepsilon^{1/2} y \tag{7}$$

$$\tau = \varepsilon^{3/4} t \tag{8}$$

where c is the propagation velocity of the solitary waves and $\varepsilon \ll 1$ is a small parameter. All the quantities are expanded in the following from:

$$n_d = 1 + \varepsilon n_1 + \varepsilon^{3/2} n_2 + \cdots \tag{9}$$

$$u_x = \varepsilon u_1 + \varepsilon^{3/2} u_2 + \cdots \tag{10}$$

$$u_y = \varepsilon^{1/2}(\varepsilon v_1 + \varepsilon^{3/2} v_2 + \cdots) \tag{11}$$

$$\phi = \varepsilon \phi_1 + \varepsilon^{3/2}\phi_2 + \cdots . \tag{12}$$

Equation (9) shows that the density of the dust grain is 1 (dimensionless) in the thermal equilibrium state. Equations (10) and (11) show that the velocity of the dust grain is zero when the system is in the thermal equilibrium state. Equation (12) shows that the electrical potential is zero when the system is in the thermal equilibrium state.

Substituting these expansions into (1)–(4), we obtain:

$$\frac{\partial}{\partial \xi}\left(\frac{\partial \phi_1}{\partial \tau} + a\sqrt{-\phi_1}\frac{\partial \phi_1}{\partial \xi} + \frac{1}{2}\frac{\partial^3 \phi_1}{\partial \xi^3}\right) + \frac{1}{2}\frac{\partial^2 \phi_1}{\partial \eta^2} = 0 \tag{13}$$

where $u_1 = -c\phi_1$, $n_1 = -\phi_1$, and $c^2 = 1$. Letting: $\zeta = \xi - 8\tau$, $\psi = \phi/225$, and $\tau' = 8\tau$, (13) can be rewritten as:

$$16\frac{\partial \psi}{\partial \tau} - 16\frac{\partial \psi}{\partial \zeta} + 30\psi^{1/2}\frac{\partial \psi}{\partial \zeta} + \frac{\partial^3 \psi}{\partial \zeta^3} + \frac{\partial^2 \psi}{\partial \eta^2} = 0 \tag{14}$$

$$\frac{\partial \varphi}{\partial \zeta} = \psi. \tag{15}$$

We assume that there are following higher-order disturbances:

$$\psi = \psi_0(\zeta) + \delta\psi(\zeta)\exp(ik_1\zeta + ik_2\eta + i\gamma\tau) + c.c.. \tag{16}$$

$$\varphi = \varphi_0(\zeta) + \delta\varphi(\zeta)\exp(ik_1\zeta + ik_2\eta + i\gamma\tau) + c.c.. \tag{17}$$

where $\psi_0(\zeta) = \sec h^4\zeta$ is one of a solitary wave solution of the MKP equation. $\delta\psi, \delta\varphi$ is a complex function of ζ. $(k_1, k_2) = k(\cos\theta, \sin\theta)$ are real values, θ is the angle between the positive x axis and the direction of the wave vector, and γ is the growth rate. $c.c.$ is complex conjugate. Substituting (16) and (17) into (14) and (15), we obtain the linear equation of:

$$L(\delta\psi(\zeta))_\zeta = f \tag{18}$$

$$\delta\varphi_\zeta = \delta\psi - ik_1\delta\varphi \tag{19}$$

where $L = \frac{d^2}{d\zeta^2} + 30\psi_0^{1/2} - 16c$ and $f = i[16(\gamma + ck_1) - 30k_1\psi_0^{1/2} + k_1^3]\delta\psi + 3k_1^2\delta\psi_\zeta - 3ik_1\delta\psi_{\zeta\zeta} + k_2^2\delta\varphi$.

For long wave approximation, we expand it in this way:

$$\delta\psi = \delta\psi_0 + k\delta\psi_1 + \cdots, \quad \delta\varphi = \delta\varphi_0 + k\delta\varphi_1 + \cdots,$$

$$\gamma = k\gamma_1 + k^2\gamma_2 + \cdots, \quad f = kf_1 + k^2 f_2 + \cdots.$$

For simplicity, we let $\beta_n = \left\langle \frac{\psi_0^n}{\psi_{0\zeta}^2} \right\rangle$ and $\alpha_n = \langle \psi_0^n \rangle$. And for arbitrary function's average operator, we define $\langle h(\zeta)\rangle = \frac{1}{\lambda}\int_0^\lambda h(\zeta)d\zeta$, and thus obtain:

$$\delta\psi_0 = \frac{\partial \psi_0}{\partial \zeta}, \delta\varphi_0 = \psi_0$$

By a tedious calculation, we finally obtain the dispersive relation as follows:

$$\left(\gamma_1 + c\cos\theta - \frac{\sin^2\theta}{16\cos\theta}\right)\left(\gamma_1^2 - \frac{c\sin^2\theta}{28}\right) = 0 \tag{20}$$

There are three solutions of the growth rate.

$$\gamma_1 = -c\cos\theta + \frac{\sin^2\theta}{16\cos\theta} \tag{21}$$

$$\gamma_1 = \sqrt{\frac{c}{28}}\sin\theta \tag{22}$$

$$\gamma_1 = -\sqrt{\frac{c}{28}}\sin\theta \tag{23}$$

If γ is real, the solitary wave is stable. However, the signs of γ determine the propagation direction of the solitary waves. If there are imaginary frequency of the γ, for example, $\gamma = i\gamma_r$, where γ_r is real, we then conclude that if γ_r is negative, the solitary wave is unstable because the disturbances exponentially increases with the time.

All the solutions of the growth rate of (21)–(23) are real, i.e. the disturbances are stable. For special case, $\theta = 0$, $\gamma_1 = -c$, namely, there is only one solution of the growth rate if the disturbance is in the x direction. However, there are two solutions if the disturbances is in the y direction, i.e. if $\theta = \frac{\pi}{2}$, $\gamma_1 = -\sqrt{\frac{c}{28}}$, and $\gamma_1 = \sqrt{\frac{c}{28}}$.

In this case, the magnitude of the growth rate is smaller than that in the x direction.

From above analysis, we conclude that the solitary waves are stable when there are higher-order disturbances in vortex-like ion distribution dusty plasmas. However, the growth rate depends on the propagation direction of the disturbances.

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