WORKSHOP

BONUS MADE EASY[1]

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ABSTRACT

The paper introduces an alternative approach to the traditional experience rating theory in automobile insurance. The approach is based on a simple theory of how high deductibles financed by loans maintain the risk differentiation in an automobile insurance arrangement. Thus the approach differs totally from the usual bonus-malus classes as well as from the credibility based experience rating ideas. The paper is of a theoretical nature and leads up to a mathematical description of how the approach may be optimalized within the framework of a risk model.

KEYWORDS

Bonus-malus systems; optimal deductibles financed by loans.

1. BACKGROUND

From a practical point of view it is well-known that the existing automobile bonus-malus systems possess several considerable disadvantages which are difficult, or even impossible, to handle within the traditional theory of experience rating. The aim of this paper is to introduce an alternative bonus-malus approach which, at least theoretically, eliminates the most important ones of these disadvantages.

2. CRITICISM OF EXISTING BONUS SYSTEMS

To motivate the new bonus-malus (B-M) approach it is appropriate to stress the usual criticism of the existing B-M systems. In particular, the existing systems are, among other things, based on two general characteristics:

(i) The claim amounts are omitted as a posterior tariff criterion.
(ii) At any time the policyholders may leave an insurance company without any further financial commitments to the company.

These characteristics lead to three of the most considerable disadvantages:

(2.1) Regarding an occurred claim, the future loss of bonus will in many cases exceed the claim amount.

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(2.2) The systems create the possibility of malus evasion, that is, the possibility of the policyholders leaving the insurance company to avoid premium increase because of occurred claims.

(2.3) The systems stimulate a slide towards higher average discount rates in the insurance arrangements.

Because only the number of claims (and of course the discount rate) in an insurance period determines the premium in the following period, it follows that (2.1) is an immediate consequence of (i). In many cases (2.1) gives the policyholder a feeling of unfairness, especially if the loss of bonus is much higher than the occurred claim amount. A consequence of this is the well-known bonus hunger behaviour of the policyholders.

Disadvantage (2.2) is of course a consequence of (ii). Malus evaders let the remaining policyholders pay the bill for their (the evaders') claim costs. This has, at least in Norway, been a serious problem in the insurance industry, mainly because of an unsatisfactory exchange of bonus information between the insurance companies.

Because all insurance arrangements attached to existing B-M systems are exposed to bonus hunger as well as malus evasion, it follows that (2.3) is a secondary consequence of (2.1) and (2.2). A higher average rate of discount is contrary to risk differentiation, which is the objective of all B-M systems. In an extreme situation the result might be that the great majority of the policyholders are at, or close to, the maximum rate of discount.

A number of authors have focused on the disadvantages mentioned above, in particular the problem of bonus hunger – see e.g. NORBERG (1975), LEMAIRE (1985) (Chapter 18) and SUNDT (1989). The aim of these authors has not been to solve or eliminate the disadvantages, but rather to take them into the modelling account in connection with the mathematical optimalization of the B-M systems. However, to eliminate the disadvantages one probably has to leave the traditional framework of experience rating, and construct a bonus principle which is basically different. This is precisely the intention of this paper, and in Section 3 we will first introduce the alternative B-M idea, and thereafter place the idea into a mathematical description and notation. The alternative approach may be called a new premium system, and in Section 4 it is shown how the system may be optimalized within the framework of a risk model. In Section 5 some practical deficiencies of the system are discussed, and in Section 6 some concluding remarks are given.

3. AN ALTERNATIVE APPROACH TO EXISTING BONUS SYSTEMS

3.1. Preliminary aspects and assumptions

The fundamental principle of the existing B-M systems simply expresses that the higher the claim frequency of a policyholder, the higher the insurance costs that on average are charged to the policyholder. However, this principle is also valid in an insurance arrangement *consisting of a high maximum deductible which is common to all policyholders*. This follows from the simply fact that good drivers will pay fewer deductibles than bad drivers. Thus we may imagine a premium system where the costs of the incurred deductibles are defined as the malus (the loss of bonus) after a claim occurred. Within this framework it seems natural to assume an individual risk premium above the maximum deductible which is reflected by a priori tariff criteria, but not by a posteriori knowledge about the policyholders. This system defines a malus system rather than a bonus system. However, we may interpret the claim free driving bonus as *avoidance of deductibles.*

Two questions are now appropriate:

(3.1) In what way do we determine the size of the maximum deductible?

To attain a suitable cost differentiation in the risk heterogeneous arrangement, the maximum deductible has to be relatively high, maybe as high as 2000-3000 US dollars (USD). This leads to question number 2:

(3.2) How do we act when knowing that the average policyholder hardly manages (at least in Norway) to cash pay deductibles of more than about 1000 USD?

Let us first look at the latter problem. The new system solves problem (3.2) by giving the policyholders a possibility of financing the incurred deductibles by loans from the insurer. Moreover, this leads to the advantage of smoothing the "loss of bonus" (the deductible) over a period of time, precisely the way that the total loss of bonus is smoothed in the traditional systems.

Before commenting on problem (3.1), we shall illustrate the abovesketched premium system with a simple example: Let us assume that a policyholder has two occurred claims of respectively 5000 USD and 500 USD in periods number 3 and 9 during an insurance period of 15 years. We also assume for simplicity that the deductible loans are ordinary term loans, and that the period of repayments is 5 years. Assume the maximum deductible to be, for instance, 2000 USD, and the premium for large claims above this maximum deductible to be 300 USD during the whole insurance period. Finally, the borrowing rate is assumed to be 10% in arrears. These assumptions lead to a sequence of payments for the policyholder shown in Figure 1. We note that the effect of the alternative system is not essentially different from the effect of a traditional B-M system; the insurance costs increase in the period(s) following an occurred claim. We also note that the loss of bonus is differentiated regarding the size of the claim amounts. Or to be more precise; the loss of bonus will never (except for the interest on the loan) exceed the claim amount, and *hence the bonus hunger effect is eliminated.* In theory the new system will not be exposed to malus evasion either, because the loan is repayed even if the insurance is terminated – see Section 5 for a further discussion on this. Hence, at least theoretically the new system eliminates the disadvantages (2.1), (2.2) and (2.3) in Section 2.

Return to problem (3.1). The solution of this problem ought to be linked to a mathematical optimalization of the system. In addition to problem (3.1), we have to decide a) the amortization form of the deductible loans, b) the length of the repayment period, and c) the rate of interest. The conditions a), b) and c) are in practice given by the money market. Thus it may seem meaningless to find mathematical "optimal" lending conditions. However, these conditions will never be absolute, therefore it may be after all interesting to find optimal values at least for some of the conditions.

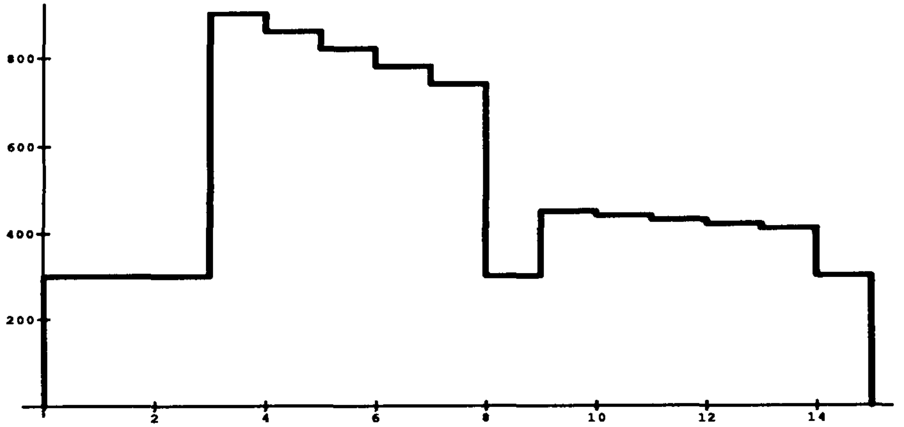

FIGURE 1. The payments for the policyholder over a period of 15 years.

Now, stress item a), the amortization form of the loans. In principle we ought to choose an amortization form which imitates the *traditional* influence of the premiums in the time periods following a claim. More precisely, an amortization form where the repayments are high during the first periods following a claim and then gradually fall. Moreover, this satisfies the desire of the policyholders to repay most of the claim costs shortly after the claim has occurred. Within *annuity loans* the repayments are exactly the same in the repayment period, while the repayments are not decreasing enough within ordinary *term loans*. Hence, these alternatives of the amortization form are ignored. However, there exists an alternative fulfilling all the mentioned properties, that is, the *exponential amortization form*. This form is also relatively handy in the mathematical computations.

Before touching the mathematical description of the alternative system, one last assumption concerning the financing of the deductibles has to be made. In a practical application of the new system it is of course the policyholders who decide how much to pay cash, and how much to borrow. Hence, a deductible is partially financed by a cash payment greater than or equal to zero, and partially by a sum borrowed from the insurer. However, to simplify the mathematical analysis we assume the *entire* deductible of an occurred claim to be financed by a loan. This is an advantage because the costs are then smoothed over a period of time. In addition, a full-financing by loans is computationally easier to analyse.

3.2. Mathematical description

Assume the following mathematical description of the alternative system: Let Y_i; $i = 1, 2, \ldots$ be the values at time zero of the claim amounts of a policyholder that occurred at the time points T_i; $i = 1, 2, \ldots$, respectively. Let Z_i be the value at time zero of the amount payed by the policyholder of claim number i, and assume Z_i on the ordinary excess-of-loss form

$$Z_i = \min(Y_i, b), \tag{1}$$

where b is interpreted as the value at time zero of the common *maximum deductible* of all policyholders at time T_i.

Let π be the inflation discount intensity related to the values at time zero of the claim amounts. Hence it follows that the *future nominal value* of Z_i at time T_i is $Z_i \exp(\pi T_i)$. Note besides that the deductible (b at time zero) is thought of as following the inflation intensity π.

Let $Z_i \exp(\pi T_i)$ be fully financed by a loan from the insurer. The loan is charged a rate of interest δ and continuously amortized by a stream of payment $\{r_i(s); s \geq 0\}$, where $s = 0$ refers to the time T_i of the claim occurrence.

The payment stream of loan number i has to satisfy (see e.g. GERBER (1990), Chapter 1)

$$Z_i \exp(\pi T_i) = \int_0^\infty v^s r_i(s)\, ds, \tag{2}$$

where $v^s = \exp(-\delta s)$ = the interest discount factor at time s.

Let $N(t)$ be the number of claims occurred in the time interval $(0, t]$. Then

$$r(t) = \sum_{i=1}^{N(t)} r_i(t - T_i) \tag{3}$$

is the *amortization rate* of the policyholder at time t.

Assume an exponential form of amortization, that is,

$$r_i(s) = B_i \exp(-\rho s). \tag{4}$$

B_i is here called "the initial amortization level", and may be interpreted as *interest + repayments* in the first repayment year. When the rate of interest δ is known, ρ expresses the *amortization profile* of the sums borrowed, that is, the obliquity of the repayments, or to which extent the repayments should be high in the beginning and then gradually decreasing.

From (2) and (4) we obtain

$$Z_i \exp(\pi T_i) = \int_0^\infty \exp(-\delta s)\, B_i \exp(-\rho s)\, ds$$

$$= \frac{B_i}{\delta + \rho},$$

or

$$B_i = Z_i \exp(\pi T_i)(\delta + \rho). \tag{5}$$

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Formula (5) gives the relationship between ρ and "the initial amortization level" B_i when the rate of interest δ and the sum borrowed $Z_i \exp(\pi T_i)$ are known. In particular, we see that $\rho = 0$ (constant amortization) implies $B_i = \delta Z_i \exp(\pi T_i)$, which means solely repaying interest to infinity. Henceforth, we will assume $\rho \geq 0$.

From (4) and (5) we have

$$r_i(s) = Z_i(\delta+\rho) \exp(\pi T_i) \exp(-\rho s). \tag{6}$$

Therefore, from (3) we finally obtain the expression

$$r(t) = \sum_{i=1}^{N(t)} Z_i(\delta+\rho) \exp(\pi T_i - \rho(t-T_i)). \tag{7}$$

To obtain an impression of the effect of ρ, it may be suitable to take a closer look at the function (6). Under assumptions of $\delta = 10\%$ and $Z_i \exp(\pi T_i) = 1$, Figure 2 shows the stream of payments $r_i(s)$ for some specified values of ρ. Note that the higher ρ is, the higher the payments are during the first repayment period(s). In the case of $\rho = 0$, we see that only 10% interest of $Z_i \exp(\pi T_i) = 1$ is continuously payed.

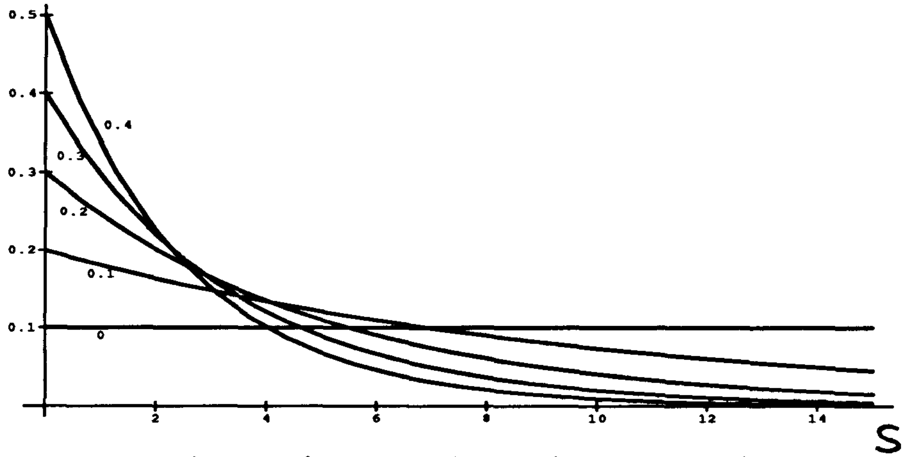

FIGURE 2. The stream of payments $\{r_i(s);\, s \geq 0\}$ when $\rho = \{0, 0.1, 0.2, 0.3, 0.4\}$.

4. A MATHEMATICAL OPTIMALIZATION DESIGN

4.1. Model assumptions

To carry through an optimalization of the new system, a claim risk model has to be built. In this paper we assume the widely accepted negative binomial model, see e.g. LEMAIRE (1991):

> The claim number process $\{N(t);\, t \geq 0\}$ of a policyholder is a homogeneous Poisson process given the claim intensity Θ. Let Θ follow a gamma distribution Gamma (α, β). Assume also the values at time zero $Y_1, Y_2, \ldots$ of the claim amounts to be independent and identically distributed (i.i.d.), and independent of $\{N(t);\, t \geq 0\}$ and of Θ.

Under these assumptions we also easily establish the values at time zero of the sums borrowed, $\{Z_i = \min(Y_i, b);\ i = 1, 2, \ldots\}$, to be i.i.d. and independent of $\{N(t);\ t \geq 0\}$ and of Θ.

4.2. Choice of loss function

Within the risk model in subsection 4.1 and the mathematical description in subsection 3.2, we want to minimize an *expected loss function* to find some optimal parameter values of the system.

The theoretical individual risk intensity of the policyholder at time t is easily evaluated as $Q(t) = \exp(\pi t)\Theta EY$. Now, the point is to estimate $Q(t)$ using a loss function which includes the amortization rate $r(t)$. In a real application of the system we have already indicated the suitability of a constant *individual* premium for all risks above the maximum deductible. For simplicity, we henceforth disregard this individual differentiation, and instead we assume a constant *collective* premium. Hence, let $p(t)$ be this premium of large claims at time t:

$$p(t) = \exp(\pi t)p = \exp(\pi t)\ E\Theta E(Y-Z), \tag{8}$$

where Y and Z are the values at time $t = 0$ of the random claim amount and the random sum borrowed, respectively. Now, write

$$\Theta EY = \Theta EZ + \Theta E(Y-Z).$$

Then one can interpret $p(t)$ as an estimator of $\exp(\pi t)\ \Theta E(Y-Z)$. If we now just let $r(t)$ be an estimator of $\exp(\pi t)\Theta EZ$ and use the *traditional* expected quadratic loss function

$$E[p(t)+r(t)-Q(t)]^2,$$

we will in the first place obtain a loss expression dependent on the time t, which is not a desirable situation. In the second place $r(t)$ would not alone be a sufficiently good estimator of $\exp(\pi t)\Theta EZ$. Owing to the fact that the loss of bonus (the sums borrowed) is payed in arrears, the amortization rate $r(t)$ is too small during the first periods according to the true intensity $\exp(\pi t)\Theta EZ$.

However, to solve these problems we may construct a loss function which integrates the total cash flow of the policyholder over a period of time. The actual loss function ought to reflect the total financing of a) the large claim risks and of b) all deductibles occurred in the actual optimalization period.

The following expected quadratic loss function takes care of the mentioned objections in a reasonable way:

$$E\left[\int_0^M v^t(p(t)+r(t))\,dt + v^M S(M) - \int_0^M v^t Q(t)\,dt\right]^2, \tag{9}$$

where

M = a restricted time horizon.

v^t = $\exp(-(\pi+\omega)t)$ = total discount factor at time t, with the *inflation* discount intensity π and a *mathematical weight* discount intensity ω. $\exp(-\omega t)$ is hereby interpreted as a weight function; we see e.g. that $\omega = 0$ implies a uniform weight function over the time period $(0, M]$.

$p(t)$ = $\exp(\pi t)\, E\Theta E(Y-Z)$
= the large claim premium at time t.

$r(t)$ = $\sum_{i=1}^{N(t)} Z_i(\delta+\rho)\exp(\pi T_i - \rho(t-T_i))$
= the amortization rate of the policyholder at time t.

$Q(t)$ = $\exp(\pi t)\,\Theta EY$
= the theoretical risk intensity at time t.

$S(M)$ = $\sum_{i=1}^{N(M)} \int_M^\infty \exp(-\pi(t-M))\, r_i(t-T_i)\, dt$
= the value at time M of all future repayments caused by claims occurred in $(0, M]$.

Summary:

Loss function (9) may be interpreted as the expected quadratic deviation between a mathematical value at time zero of the actual cash flow of the policyholder and the corresponding mathematical value at time zero of the theoretical risk intensity of the policyholder over the time period $(0, M]$. Note that all raised loans during $(0, M]$ have to be repayed, and hence one has to include $v^M S(M)$ in the loss function.

4.3. Computation of the expected loss function

To minimize (9) analytically or numerically with respect to e.g. the system parameters δ, ρ and b, the function has to be of algebraic nature. To obtain an algebraic form of (9) some statistical computations have to be made.

Let

$$Z(t) = \sum_{i=1}^{N(t)} Z_i \exp((\pi+\rho)T_i). \tag{10}$$

Then by (7)

$$r(t) = (\delta+\rho)\exp(-\rho t)\, Z(t), \tag{11}$$

and by simple algebra we obtain

$$v^M S(M) = \left(\frac{\delta+\rho}{\pi+\rho}\right)\exp(-(\pi+\omega+\rho)M)\, Z(M). \tag{12}$$

Introduce the annuity

$$\bar{a}_{\overline{M}|} = \int_0^M \exp(-\omega t)\, dt = \omega^{-1}(1-\exp(-\omega M)),$$

and the expression

$$\Psi = \int_0^M v^t r(t)\, dt = (\delta+\rho)\int_0^M \exp(-(\pi+\omega+\rho)t)\, Z(t)\, dt.$$

Then function (9) may be written as

$$\begin{aligned}(13) \qquad & E\Psi^2 + 2E[\Psi v^M S(M)] + E[v^M S(M)]^2 + \\ & + 2\bar{a}_{\overline{M}|} E[(\Psi + v^M S(M))(p - \Theta EY)] + \\ & + \bar{a}^2_{\overline{M}|}[p^2 - 2p(E\Theta)(EY) + E\Theta^2 (EY)^2].\end{aligned}$$

By (13) we have to find the 1.- and 2.-order moments of the $Z(t)$-process, that is $EZ(t)$ and $E[Z(s)\, Z(t)]$. However, the stochastic process $Z(t)$ does not have independent waiting times between steps, and hence the calculations become somewhat complex. We may however show that $Z(t)$ has the same distribution as

$$(14) \qquad Z^* = \sum_{i=1}^{N^*} Z_i \exp((\pi+\rho)\, U_i),$$

where

$$\begin{gathered}\text{given } \Theta = \theta,\ N^* \sim \text{Poisson}(\theta t), \\ U_1, \ldots, U_{N^*} \text{ are i.i.d.} \sim \text{Uniform}\,[0, t], \\ Z_1, \ldots, Z_{N^*} \text{ are i.i.d.},\end{gathered}$$

and where N^*, the U_i's and the Z_i's are stochastically independent. This result was in general discovered by JUNG (1963); see also BÜHLMANN (1970), pp. 57-60. By standard statistical calculations we then obtain

$$(15) \qquad EZ(t) = E\Theta \frac{EZ}{(\pi+\rho)}[\exp((\pi+\rho)t) - 1],$$

and for $0 \le s \le t$

$$\begin{aligned}(16) \quad E[Z(s)\, Z(t)] = {} & E\Theta \frac{EZ^2}{2(\pi+\rho)}\text{k}\,[\exp(2(\pi+\rho)s) - 1] + \\ & + E\Theta^2 \frac{(EZ)^2}{(\pi+\rho)^2}[\exp((\pi+\rho)s) - 1]\,[\exp((\pi+\rho)t) - 1].\end{aligned}$$

To obtain an algebraic form of the expected loss function (13), one has to complete seven isolated computations. Below, these computations are noted as

$\psi_1, \ldots, \psi_7$ (remember the integral definition of Ψ):

(17) $\psi_1 = E\Psi^2$

(18) $\psi_2 = E[\Psi v^M S(M)]$

(19) $\psi_3 = E[v^M S(M)]^2$

(20) $\psi_4 = E(\Theta\Psi)$

(21) $\psi_5 = E[\Theta v^M S(M)]$

(22) $\psi_6 = E\Psi$

(23) $\psi_7 = E[v^M S(M)]$.

In this paper we restrict ourselves to indicate that (17)-(23) are easily calculated by use of standard statistical methods. The clue is here to use the expressions (15) and (16). Thus, for instance, we have

$$\psi_1 = E\Psi^2 = (\delta+\rho)^2 E\left[\int_0^M Z(t)\exp\left(-(\pi+\omega+\rho)\,t\right)dt\right]^2$$

$$= (\delta+\rho)^2 \int_0^M ds \int_s^M E[Z(s)\,Z(t)]\exp\left(-(\pi+\omega+\rho)\,(s+t)\right)dt.$$

Finally, we establish the expected loss function

$$E\left[\int_0^M v^t(p(t)+r(t))\,dt + v^M S(M) - \int_0^M v^t Q(t)\,dt\right]^2 \tag{24}$$

$$= \psi_1 + 2\psi_2 + \psi_3 + 2\bar{a}_{\overline{M}|}[p\psi_6 - EY\psi_4 + p\psi_7 - EY\psi_5] +$$
$$+ \bar{a}^2_{\overline{M}|}[p^2 - 2pE\Theta EY + E\Theta^2(EY)^2].$$

4.4. Comments on the loss function

Under the model assumptions of subsection 4.1 we have

$$E\Theta = \alpha/\beta, \qquad E\Theta^2 = \alpha(\alpha+1)/\beta^2.$$

If the claim amount distribution is assumed known, the function (24) depends on eight unknown parameters. Two of them, α and β, can e.g. be estimated by the maximum likelihood estimators described by LEMAIRE (1985), Chapter 12. Further, it seems natural to keep the inflation intensity π, the mathematical weight intensity ω and the time horizon M constant (they might also be considered as random variables). Thus the actual optimalization (varying) parameters are the remaining *system parameters* δ, ρ and b.

In this connection, *analytical* optimal parameter solutions are in general difficult to find. However, *numerical* solutions are easily computed by a computer system, for example the mathematical software system Mathematica. Note that the maximum deductible b enters into the function (24) via the moments EZ and EZ^2. Thus, an approximating optimalization of b demands a statistical analysis of the claim amounts in a representative claim portfolio. Also the premium of large claims, $p(t)$, has to be estimated in association with a real claim portfolio.

Note finally that the alternative premium system may be mathematically compared with traditional B-M systems via the expected loss function (9). Or to be more precise; within each of the traditional B-M systems one may construct an estimator to the estimand $\int_0^M v^t Q(t)\,dt$. By using these estimators in loss function (9), we are able to compare the expected losses of the traditional B-M systems with the expected loss of the alternative system, and hence find the best mathematically fitted system.

4.5. The loss function for the special case M = ∞

To give some more information on the structure of the loss function, one may exhibit the function for the special case when the time horizon M tends to infinity. Assume in this case that $\omega > 0$, which is in accordance with economic theory. When $M = \infty$, we see from (12), (16) and (19) that ψ_3 tends to zero. By (18), (21) and (23) then also ψ_2, ψ_5 and ψ_7 tend to zero. In formula (24) thus only ψ_1, ψ_4 and ψ_6 remain different from zero. Straightforward calculation gives

$$\psi_1 = \frac{1}{2\omega}\left(\frac{\delta+\rho}{\pi+\omega+\rho}\right)^2\left[E\Theta EZ^2 + \frac{2}{\omega}E\Theta^2(EZ)^2\right],$$

$$\psi_4 = \frac{1}{\omega}\left(\frac{\delta+\rho}{\pi+\omega+\rho}\right)E\Theta^2 EZ,$$

$$\psi_6 = \frac{1}{\omega}\left(\frac{\delta+\rho}{\pi+\omega+\rho}\right)E\Theta EZ.$$

Inserting $p = E\Theta(EY-EZ)$ the loss function may then be put into the following form

$$x^2 A_1(b) - 2xA_2(b) + A_3(b),$$

with

$$x = \frac{\delta+\rho}{\pi+\omega+\rho} \tag{25}$$

$$A_1(b) = \frac{1}{2\omega}\left[E\Theta EZ^2 + \frac{2}{\omega}E\Theta^2(EZ)^2\right] \tag{26}$$

$$A_2(b) = \left(\frac{1}{\omega}\right)^2 EZ[(E\Theta)^2 EZ + \mathrm{Var}\,\Theta EY] \tag{27}$$

$$A_3(b) = \left(\frac{1}{\omega}\right)^2 [(E\Theta)^2(EZ)^2 + \mathrm{Var}\,\Theta\,(EY)^2]. \tag{28}$$

The influence of the system parameters δ and ρ is contained in x, and thus is separated from that of the system parameter b.

For fixed b the loss function attains its minimum for

$$x = x(b) = A_2(b)/A_1(b), \tag{29}$$

and the minimum is

$$\min(b) = A_3(b) - A_2(b^2)/A_1(b). \tag{30}$$

Denoting the claim amount c.d.f. by F, we have

$$EZ = \int_0^b [1 - F(y)]\, dy \tag{31}$$

$$EZ^2 = \int_0^{b^2} [1 - F(\sqrt{y})]\, dy. \tag{32}$$

Thus EZ and EZ^2 are continuous functions of b. If F is continuous, they are also differentiable. The same is then also true for min (b). Thus, for special choices of F it should not be difficult to minimize min (b) with respect to b, and thereby obtain a global minimum.

For the moment we content ourselves with the following remarks:
By (25) optimal values of δ and ρ for fixed b are related by

$$\delta(b) = [x(b) - 1]\,\rho(b) + (\pi + \omega)\, x(b).$$

Thus the interest intensity $\delta(b)$ is greater than, equal to or less than the market interest intensity $\pi + \omega$ according as $x(b)$ is greater than, equal to or less than one.

As b tends to infinity, EZ and EZ^2 tend to EY and EY^2 respectively. From (26)-(28) we see that

$$A_2(\infty) = A_3(\infty) = A_1(\infty) - \frac{1}{2\omega} E\Theta EY^2 = \left(\frac{1}{\omega}\right)^2 E\Theta^2 (EY)^2.$$

Thus by (29), $x(\infty) < 1$.

For b tending to zero, $A_1(b)$ will be of the order of magnitude b^2. $A_2(b)$ will be of the order of magnitude b, because of the second term within the paranthesis. Thus by (29), $x(0+) = \infty$. This means that there is (at least) one b with $x(b) = 1$. From (26)-(32) it can be shown that for such a b we will have $x'(b) < 0$ and $\min'(b) > 0$, if $F(y) > 0$ for $y > 0$. This proves that there is exactly one value of b with $x(b) = 1$ and that $x(b) > 1$ to the left of this point and $x(b) < 1$ to the right of it. Furthermore, min (b) has, at least locally, a minimum to the left of the point. This indicates that the optimal δ-value is greater than $\pi + \omega$, or, in other words, the interest intensity for the loan should be greater than the market interest intensity.

5. PRACTICAL SYSTEM DEFICIENCIES

In general it is often difficult, or even impossible, to eliminate deficiencies of an existing financial market system without generating other system deficiencies. The automobile insurance B-M principle seems typically to be characterized by this two-sided effect, and hence it is not difficult to point out some general practical deficiencies of the alternative B-M approach. An obvious one is that a high common deductible necessarily involves a *lower total premium income* compared with traditional bonus systems, and thereby generates a lower insurance profit to the insurer. Another deficiency is the *credit risk* of the policyholders, or, more precisely, it is not certain that the policyholders are able to repay their deductible loans. Hence, the insurer has to, in one way or another, make conditions linked to the individual solvence security in order to meet possible losses. One way of doing this is e.g. that the insurer demands the policyholders to save an amount of money in each insurance period to build up an individual risk reserve to cover (parts of) future incurred deductibles. A "claim risk account" with the insurer should, in regard to reduce the credit risk and to maximize the rate of interest on deposits, be closed for withdrawals during the insurance periods, except for financing incurred deductibles. Thus, the premium and claim costs of the policyholders will also have a more uniform dispersion during the insurance periods.

6. CONCLUDING REMARKS

In theory the alternative B-M approach eliminates the most important disadvantages of the existing B-M systems. A policyholder will for instance within the existing systems, unlike the alternative approach, often make a profit by asking a bank for a credit to cover an occurred claim cost, instead of reporting the claim to the insurer. This seems obvious, but can also under some specified conditions be explicitly shown by comparing the effective rate of interest on a banking credit with the "effective rate of interest" on the loss of insurance bonus. By constructing a B-M approach which eliminates bonus hunger, one also avoids mathematical risk modelling which includes assumptions about bonus hunger, as e.g. NORBERG (1975), LEMAIRE (1985) (Chapter 18) and SUNDT (1989) have built into their models.

On the other hand the alternative B-M approach contains, as pointed out in Section 5, some practical deficiencies like credit risk and lower premium income. The point is however that these deficiencies *are just relevant for the (existing) insurers, and not for the policyholders*. In other words; the alternative approach is less favourable to the existing insurers than to their customers. Thus, it seems conceivable that the traditional insurance industry at once will be rather sceptical about introducing the alternative B-M approach to the insurance market. It seems, however, more probable that the possible initiators in this connection will be the (future) financial institutions—or cooperations between institutions—which consist of a *superior* banking service and a *minor* (automobile) insurance service. In the first place these institutions are generally interested in introducing customer-friendly products to increase their market share and market profit in the insurance market. In the second place, and under these circumstances, they probably interpret the problem of lower premium income as of secondary importance, while they obviously have the best qualifications to handle the problem of credit risk. Finally, and in the third place, these institutions already have the general administrative device which the alternative B-M approach demands, or stated in its extreme form, an optimal combination of actuarial and banking knowledge and culture.

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