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# Numerical simulation of characteristics of propagation of symmetric waves in microwave circular shielded waveguide with a radially inhomogeneous dielectric filling

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## Research Paper

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### Abstract

The paper presents a numerical simulation of the propagation characteristics of symmetric $E$-type and $H$-type waves in microwave circular shielded waveguide with radially inhomogeneous dielectric filling. Using the modified Galerkin method, the calculation of a circular two-layer shielded waveguide was carried out, as a result of which the distribution of the electromagnetic field of the waveguide with linear and parabolic distribution of permeability was determined. The results obtained using the modified Galerkin method were compared with the results obtained using the classical partial domain method, which agree well enough.

## Introduction

Circular non-uniformly filled waveguides, possessing a number of unique features (anomalous dispersion, complex waves, complex resonance [1–7]), are widely used [8–12] in the construction of microwave devices such as attenuators, delay lines, bandpass filters, resonators for radio spectroscopes, etc. Calculation and optimization of the parameters of such devices require the development of numerical and analytical methods for studying waveguides with arbitrary dielectric filling. The possibility of calculating the characteristics of waveguides with filling described by arbitrary analytical functions makes it possible to pose problems of parametric synthesis aimed at the implementation of devices with given characteristics. In addition, algorithms for calculating inhomogeneously filled circular waveguides can be used to study gradient optical fibers [13–21]. This paper proposes a method for calculating the characteristics of wave propagation of a circular shielded waveguide with a radially inhomogeneous dielectric filling, based on a modified Galerkin method as a variant of the spectral method.

## Formulation of the problem

To calculate an inhomogeneously filled circular shielded waveguide, it is proposed to use a modified Galerkin method [22, 23], which is a variant of the general spectral method.

Let us consider the problem of the propagation of symmetric $E$ and $H$-waves in a circular shielded waveguide with partial dielectric filling, the value of the dielectric constant of which arbitrarily depends on the radial coordinates $\varepsilon(r, z, \varphi) = \varepsilon(r)$ (Fig. 1). The value of the magnetic permeability is assumed to be constant.

From Maxwell's equations we get:

$$rotrotE = k_0^2 \varepsilon(r, \varphi)E. \tag{1}$$

Using the following expressions

$$grad(\psi) = r\frac{d\psi}{dr} + \varphi\frac{1}{r}\frac{d\psi}{d\varphi} + z\frac{d\psi}{dz},$$

$$div(E) = \frac{\partial E_r}{\partial r} + \frac{1}{r}\frac{\partial E_\varphi}{\partial \varphi} + \frac{\partial E_z}{\partial z},$$

$$rot(E) = r\left(\frac{1}{r}\frac{\partial E_z}{\partial \varphi} - \frac{\partial E_\varphi}{\partial z}\right) + \varphi\left(\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r}\right) + z\left(\frac{\partial E_\varphi}{\partial r} + \frac{1}{r}E_\varphi - \frac{1}{r}\frac{\partial E_r}{\partial \varphi}\right),$$

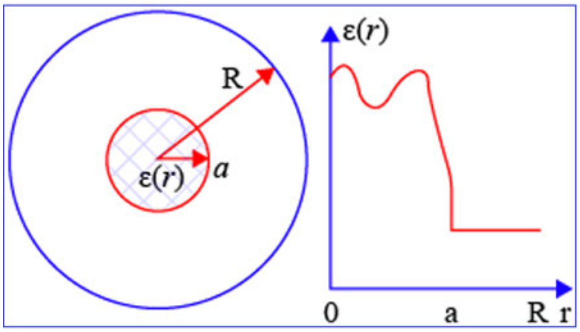


**Fig. 1.** The distribution function of the dielectric constant in the cross-section of the waveguide.

we write equation (1) for the field components in a cylindrical coordinate system:

$$rotrot(E)|_r = \frac{1}{r}\frac{\partial^2 E_\varphi}{\partial r \partial \varphi} + \frac{\partial^2 E_z}{\partial r \partial z} - \frac{1}{r^2}\frac{\partial^2 E_r}{\partial \varphi^2} + \frac{1}{r^2}\frac{\partial E_\varphi}{\partial \varphi} - \frac{\partial^2 E_r}{\partial z^2} = k_0^2 \varepsilon(r, \varphi) E_r, \quad (1a)$$

$$rotrot(E)|_\varphi = \frac{1}{r^2}E_\varphi + \frac{1}{r}\frac{\partial^2 E_r}{\partial r \partial \varphi} - \frac{1}{r^2}\frac{\partial E_r}{\partial \varphi} + \frac{1}{r}\frac{\partial^2 E_z}{\partial \varphi \partial z} - \frac{\partial^2 E_\varphi}{\partial r^2} - \frac{1}{r}\frac{\partial E_\varphi}{\partial r} - \frac{\partial^2 E_\varphi}{\partial z^2} = k_0^2 \varepsilon(r, \varphi) E_\varphi, \quad (1b)$$

$$rotrot(E)|_z = \frac{\partial^2 E_r}{\partial r \partial z} + \frac{1}{r}\frac{\partial E_r}{\partial z} + \frac{1}{r}\frac{\partial^2 E_\varphi}{\partial \varphi \partial z} - \frac{\partial^2 E_z}{\partial r^2} - \frac{1}{r}\frac{\partial E_z}{\partial r} - \frac{1}{r^2}\frac{\partial^2 E_z}{\partial \varphi^2} = k_0^2 \varepsilon(r, \varphi) E_z. \quad (1c)$$

We represent the wave fields of the guiding structure in the form of expansions in terms of eigenfunctions of the Dirichlet and Neumann boundary value problems for a uniformly filled circular waveguide. The connection between the components of the electric field, in accordance with the spectral method, is established through the coefficients of the series of expansions substituted in (1).

## Symmetrical *H*-waves

In the absence of the angular dependence of the field, we assume $\partial/\partial\varphi = 0$, $E_r = 0$, $E_z = 0$. In this case, equation (1) will be reduced to a single equation for the $\varphi$ component of the electric field

$$\frac{1}{r^2}E_\varphi - \frac{\partial^2 E_\varphi}{\partial r^2} - \frac{1}{r}\frac{\partial E_\varphi}{\partial r} - \frac{\partial^2 E_\varphi}{\partial z^2} = k_0^2 \varepsilon(r) E_\varphi.$$

Writing $E_\varphi(r, \varphi, z) = E_\varphi(r, \varphi)e^{-i\beta z}$ we obtain an equation for the transverse coordinate function

$$\frac{\partial^2 E_\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial E_\varphi}{\partial r} - \frac{1}{r^2}E_\varphi + (k_0^2\varepsilon(r) - \beta^2)E_\varphi = 0, \quad (2)$$

where $\varepsilon(r) = \begin{cases} \varepsilon_1 - \frac{\varepsilon_1 - \varepsilon_2}{a^2}r^2, r \le a \\ 1, a \le r \le R. \end{cases}$,

$\varepsilon -$ relative dielectric constant.

Assuming the dependence of the field on the longitudinal coordinate and time, we obtain equations for the components of the electric field:

$$\begin{cases} \frac{\partial^2 E_\varphi}{\partial a^2} + (k_0^2\varepsilon(r, a) - \beta^2)E_\varphi - \frac{\partial^2 E_a}{\partial r \partial a} + i\beta\frac{\partial E_\theta}{\partial r} = 0, \\ \frac{\partial^2 E_a}{\partial r^2} + (k_0^2\varepsilon(r, a) - \beta^2)E_a - \frac{\partial^2 E_r}{\partial r \partial a} + i\beta\frac{\partial E_\theta}{\partial a} = 0, \\ \frac{\partial^2 E_\theta}{\partial r^2} + \frac{\partial^2 E_\theta}{\partial a^2} + k_0^2\varepsilon(r, a)E_\theta + i\beta\frac{\partial E_r}{\partial r} + i\beta\frac{\partial E_a}{\partial a} = 0. \end{cases} \quad (2a)$$

The solution to equation (2a) will be sought [23] in the form:

$$E_\varphi(r) = \sum_{n=0}^{N} b_n J_1(\alpha_n r), \quad (3)$$

where $J_1(\alpha_n r)$ is the Bessel function of the 1st order, the coefficients $\alpha_n$ are determined taking into account the boundary condition $E_\varphi(r = R)$ from equation $J_1(\alpha_n R) = 0$.

Substituting (3) into (2), we obtain

$$-\sum_{n=0}^{N} b_n\left[\frac{\partial^2 J_1(\alpha_n r)}{\partial r^2} + \frac{1}{r}\frac{\partial J_1(\alpha_n r)}{\partial r} - \frac{1}{r^2}J_1(\alpha_n r)\right] = \sum_{n=0}^{N} b_n k_0^2 \varepsilon(r) J_1(\alpha_n r) - \sum_{n=0}^{N} b_n \beta^2 J_1(\alpha_n r).$$

Considering that

$$\frac{\partial^2 J_1(\alpha_n r)}{\partial r^2} + \frac{1}{r}\frac{\partial J_1(\alpha_n r)}{\partial r} - \frac{1}{r^2}J_1(\alpha_n r) = -\alpha_n^2 J_1(\alpha_n r),$$

we get

$$\sum_{n=0}^{N} b_n(\alpha_n^2 + \beta^2)J_1(\alpha_n r) = \sum_{n=0}^{N} b_n k_0^2 \varepsilon(r) J_1(\alpha_n r). \quad (4)$$

Multiplying both sides of equation (4) by $rJ_1(\alpha_q r)$ and integrating within $r \in [0;R]$, we obtain the equation

$$(\alpha_q^2 + \beta^2)Q_q b_q = \sum_{n=0}^{N} b_n k_0^2 \int_0^R \varepsilon(r) r J_1(\alpha_n r) J_1(\alpha_q r) dr. \quad (5)$$

Here we used the orthogonality condition for the Bessel functions:

$$\int_0^R r J_1(\alpha_n r) J_1(\alpha_q r) dr = \begin{cases} Q_q, q = n \\ 0, q \ne n \end{cases},$$

where $Q_q = 0.5R^2 J_0^2(\alpha_n R)$, which takes place, since in this case the Bessel functions are a solution to the homogeneous boundary value problem on the Bessel equation.

Equation (5) can be represented in matrix form:

$$M \cdot b = T \cdot b, \quad (6)$$

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where

$$M_{q,n} = \begin{cases} (\alpha_q^2 + \beta^2)Q_q,\ q = n, \\ 0,\ q \neq n, \end{cases}$$

$$T_{q,n} = k_0^2 \int_0^R \varepsilon(r) r J_1(\alpha_n r) J_1(\alpha_q r) dr.$$

Writing equation (6) in the form $(M - T) \cdot b = 0$ and equating the determinant of matrix $(M - T)$ to zero, we obtain the dispersion equation for symmetric $H$-waves propagating in a circular waveguide with an arbitrary dependence of $\varepsilon$ on $r$:

$$Det(\beta) = M - T = 0. \quad (7)$$

Note that the matrix $\boldsymbol{T}$ does not depend on $\beta$, therefore, when solving the dispersion equation (7), it is calculated only once, which significantly reduces the search time for the roots of the dispersion equation. Note that, when deriving equations (6) and (7), no restrictions were imposed on the form of dependence $\varepsilon(r)$, i.e. this method allows one to calculate symmetric $H$-waves with a completely arbitrary nature of the change in the dielectric constant along the transverse coordinate, while $\varepsilon$ can also be a complex quantity, which allows, for example, calculating waveguides with a complex absorption distribution in the cross-section, that is, to solve non-self-adjoint boundary value problems, in which the identity of the differential operators of the direct and adjoint boundary value problems is not satisfied.

## Symmetrical *E*-waves

For symmetric $E$-waves, we put

$$\frac{\partial}{\partial \varphi} = 0,\ E_\varphi = 0,\ H_r = H_z = 0.$$

In this case, equation (1) transforms into a system of two equations:

$$i\beta \frac{\partial E_z}{\partial r} + (k_0^2 \varepsilon(r) - \beta^2) E_r = 0,$$
$$i\beta \frac{\partial E_r}{\partial r} + i\beta \frac{1}{r} E_r + \frac{\partial^2 E_\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial E_z}{\partial r} + k_0^2 \varepsilon(r) E_z = 0.$$

Introducing variable $\widetilde{E_z} = i\beta \cdot E_z$, we arrive at the equations:

$$\begin{aligned} &\frac{\partial \widetilde{E_z}}{\partial r} + (k_0^2 \varepsilon(r) - \beta^2) E_r = 0, \\ &\frac{\partial^2 \widetilde{E_z}}{\partial r^2} + \frac{1}{r}\frac{\partial \widetilde{E_z}}{\partial_r} + k_0^2 \varepsilon(r) \widetilde{E_z} - \beta^2 \left( \frac{\partial E_r}{\partial_r} + \frac{1}{r} E_r \right) = 0. \end{aligned} \quad (8)$$

The boundary conditions on an ideally conducting surface for the tangential and normal components of the electric field $E_\tau\Big|_s = 0,\ \frac{\partial E_n}{\partial_n}\Big|_n = 0$ [18], in this case lead to the equation

$$E_z|_{r=R} = 0. \quad (9)$$

The components of the electric field in accordance with the spectral method will be sought in the form of autonomous expansions:

$$\widetilde{E_z} = \sum_{n=0}^{N} A_n J_0(\alpha_n r),\ E_r = \sum_{m=0}^{N} B_m J_1(\alpha_m r). \quad (10)$$

Taking into account the first boundary condition (9), the wave numbers $\alpha_n$ are determined from equation $J_0(\alpha_n R) = 0$.

Substituting (10) into (8), we obtain a system of two functional equations:

$$\begin{aligned} &-\sum_{n=0}^{N} A_n \alpha_n J_1(\alpha_n r) + \sum_{m=0}^{N} B_m (k_0^2 \varepsilon(r) - \beta^2) J_1(\alpha_m r) = 0, \\ &\sum_{n=0}^{N} A_n \left( \frac{\partial^2 J_0(\alpha_n r)}{\partial r^2} + \frac{1}{r}\frac{\partial J_0(\alpha_n r)}{\partial r} + k_0^2 \varepsilon(r) J_0(\alpha_n r) \right) - \\ &-\beta^2 \sum_{m=0}^{N} B_m \left( \frac{\partial J_1(\alpha_m r)}{\partial r} + \frac{1}{r} J_1(\alpha_m r) \right) = 0. \end{aligned} \quad (11)$$

Taking into account the equalities

$$\frac{\partial^2 J_0(\alpha_n r)}{\partial r^2} + \frac{1}{r}\frac{\partial J_0(\alpha_n r)}{\partial r} = -\alpha_0^2 J(r),$$

$$\frac{\partial J_1(\alpha_m r)}{\partial r} = \alpha_m J_0(\alpha_m r) - \frac{1}{r} J_1(\alpha_m r),$$

system (11) can be rewritten as

$$-\sum_{n=0}^{N} A_n \alpha_n J_1(\alpha_n r) + \sum_{m=0}^{N} B_m (k_0^2 \varepsilon(r) - \beta^2) J_1(\alpha_m r) = 0, \quad (12a)$$

$$-\sum_{n=0}^{N} A_n (k_0^2 \varepsilon(r) - \alpha^2) J_0(\alpha_n r) - \beta^2 \sum_{m=0}^{N} B_m \alpha_m J_0(\alpha_m r) = 0. \quad (12b)$$

Multiplying equation (12a) by $r J_1(\alpha_q r) = 0$, equation (12b) by $r J_0(\alpha_q r) = 0$ and integrating within $r \in [0;R]$, we obtain the system of equations:

$$\begin{aligned} &-A_q \alpha_q Q_q + k_0^2 \sum_{m=0}^{N} B_m \int_0^R r\varepsilon(r) J_1(\alpha_m r) J_1(\alpha_q r) dr - \\ &- B_q \beta^2 Q_q = 0, \\ &k_0^2 \sum_{n=0}^{N} A_n \int_0^R r\varepsilon(r) J_0(\alpha_n r) J_0(\alpha_q r) dr - \\ &- A_q \alpha_q^2 Q_q - B_q \beta^2 \alpha_q Q_q = 0. \end{aligned} \quad (13)$$

Here we used the orthogonality conditions for the Bessel functions

$$\int_0^R r J_0(\alpha_n r) J_0(\alpha_q r) dr = \int_0^R r J_1(\alpha_n r) J_1(\alpha_q r) dr = $$
$$= \begin{cases} Q_q,\ Q = n \\ 0,\ q \neq n \end{cases},$$

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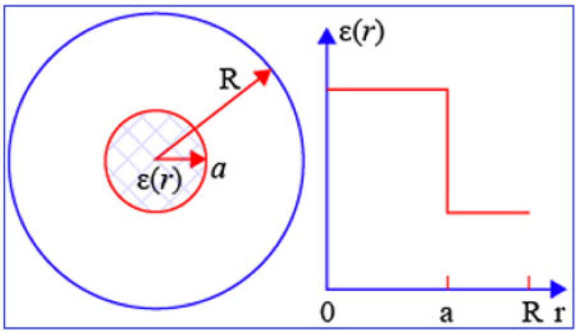


**Fig. 2.** Circular waveguide with a dielectric rod.

where $Q_q = (R^2/2)J_1^2(\alpha_q R)$, since in this case the Bessel functions are a solution to a homogeneous boundary value problem.

The system of equation (13) can be written in matrix form:

$$\begin{bmatrix} T^{(0,0)}\, T^{(0,1)} \\ T^{(1,0)}\, T^{(1,1)} \end{bmatrix} \cdot \begin{bmatrix} A \\ B \end{bmatrix} = 0, \quad (14)$$

where

$$T^{(0,0)}_{q,m} = -\alpha_q Q_q \delta_{q,m},$$
$$T^{(0,1)}_{q,m} = k_0^2 \int_0^R r\varepsilon(r) J_1(\alpha_m r) J_1(\alpha_q r)dr - \beta^2 Q_q \delta_{q,m},$$
$$T^{(1,0)}_{q,m} = k_0^2 \int_0^R r\varepsilon(r) J_0(\alpha_n r) J_0(\alpha_q r)dr - \alpha_q^2 Q_q \delta_{q,m}, \quad (15)$$
$$T^{(1,1)}_{q,m} = -\beta^2 \alpha_q Q_q \delta_{q,m},$$

$\delta_{q,n}$ −Kronecker symbol.

Equating the determinant of matrix equation (14) to zero, we obtain a dispersion equation describing the symmetric $E$-waves of a circular waveguide with an arbitrary radial dielectric filling.

## Numerical implementation of algorithms

Two-layer shielded waveguide. As an example, we use equations (7) and (14) to calculate the simplest test structure – a circular waveguide with a homogeneous dielectric rod (i.e. $\varepsilon(r) = \varepsilon = const$, Fig. 2) and compare the results with the exact ones obtained by the classical method of partial regions.

The calculations were carried out for a waveguide with parameters: $R = 20\text{mm}$, $a = 10\text{mm}$, $\varepsilon = 3$, at a frequency of $f = 10\text{GHz}$.

The classical calculation method gives the following results: for symmetric $H$-waves $\beta_H = 2376891/\text{m}$, for symmetric $E$-waves $\beta_E = 22755000/\text{m}$.

The calculation of test structures using the proposed technique was carried out by substituting the function $\varepsilon(r) = \begin{cases} 3, r \le a \\ 1, a \le r \le R \end{cases}$ into equations (6) and (14).

The convergence of solutions obtained by the modified Galerkin method for symmetric $E$ and $H$-waves is shown in Table 1 and in Fig. 3.

From Table 1 and Fig. 3 it follows that the convergence of the modified Galerkin method is monotonic and occurs rather quickly (already at $N = 5$ the difference between longitudinal wave numbers does not exceed 1.5%).

**Table 1.** Calculation by the modified Galerkin method

| No | Symmetric $E$-waves ($\beta_E$ = 22755000/m) | Symmetric $H$-waves ($\beta_H$ = 2376891/m) |
|---|---|---|
| 1 | 2 331 366 | 2 347 509 |
| 2 | 2 343 096 | 2 365 369 |
| 3 | 2 323 624 | 2 372 738 |
| 4 | 2 318 978 | 2 374 027 |
| 5 | 2 309 273 | 2 375 506 |
| 6 | 2 307 095 | 2 375 777 |
| 7 | 2 301 335 | 2 376 259 |
| 8 | 2 300 207 | 2 376 347 |
| 9 | 2 296 397 | 237 655 |
| 10 | 2 295 748 | 2 376 586 |
| 11 | 229 304 | 2 376 686 |
| 12 | 2 292 637 | 2 376 704 |
| 13 | 2 290 613 | 2 376 758 |
| 14 | 2 290 347 | 2 376 768 |
| 15 | 2 288 776 | 2 376 800 |

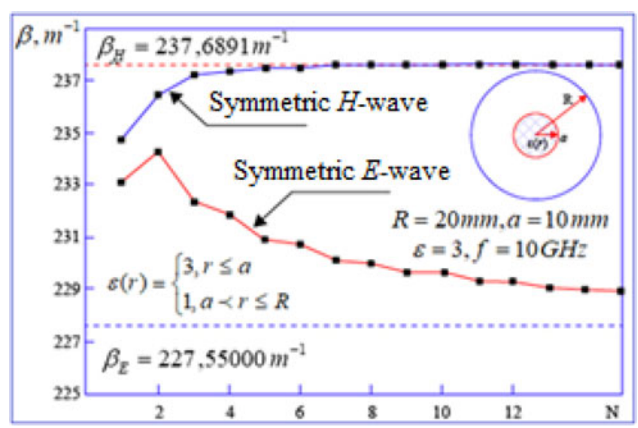


**Fig. 3.** Convergence in integral characteristics.

Figure 3 also shows that in the case of symmetric $H$-waves, convergence occurs faster, which, apparently, is associated with the difference in the number of equations to be solved (one equation (2) for symmetric $H$-waves and two equation (8) for symmetric $E$-waves).

In Fig. 4, the dotted line shows the dependences of the field components $H_z$ and $E_\varphi$ on the coordinate $r$, calculated for the symmetric $H$-wave at $N = 5$.

From the graphs shown in Fig. 4 that the field distributions calculated by two different methods practically coincide.

Thus, using the example of a test problem with an exact solution, a high accuracy, efficiency of the method, and fast convergence of the solution obtained using the modified Galerkin method are shown.

## Calculation of a waveguide with a rod, the dielectric constant of which changes according to the parabolic law

Based on equation (15), the dispersion characteristics of symmetric $E$-waves propagating in a circular waveguide with partial

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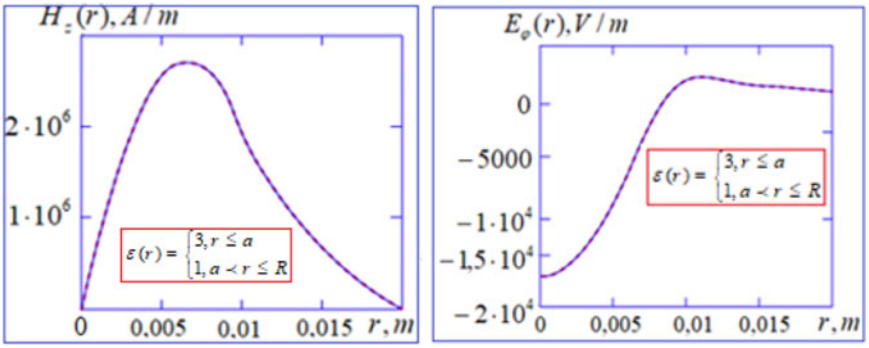


**Fig. 4.** Field distribution of the first symmetric *H*-wave: dotted line – partial domain method, solid line – modified Galerkin method.

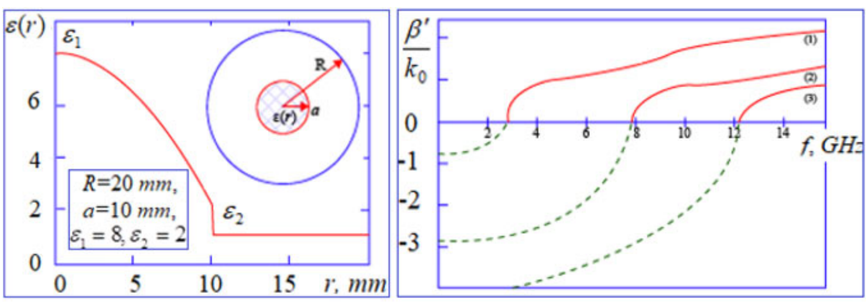


**Fig. 5.** Dispersion characteristics of symmetric *E*-waves of a circular waveguide with a parabolic profile of dielectric filling.

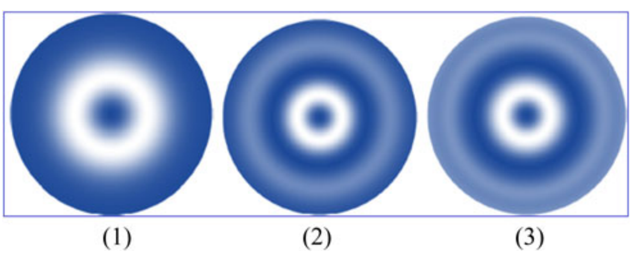


**Fig. 6.** Distribution of power flux density symmetric *E*-waves at frequency $f = 14$ GHz.

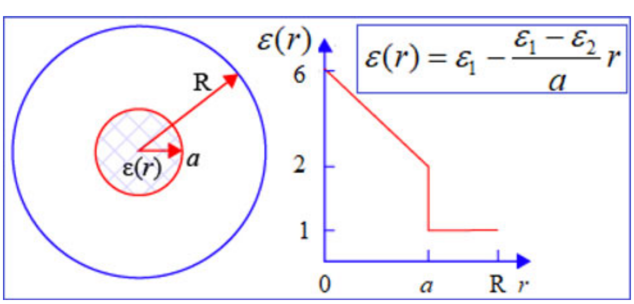


**Fig. 7.** Dielectric constant function.

dielectric filling, the permeability of which changes according to the parabolic law, are described by the equation:

$$\varepsilon(r) = \begin{cases} \varepsilon_1 - \dfrac{\varepsilon_1 - \varepsilon_2}{a^2} r^2, r \le a \\ 1, a \le r \le R. \end{cases}$$

Substituting this expression in (15) and calculating the integrals (numerically or analytically), we obtain a solution to the dispersion problem. Note that for any calculation of the integrals from (15) is carried out only once, since they do not depend on either the frequency or the longitudinal wavenumber, and are determined only by the filling parameters. This is an unconditional advantage of this method, which makes it possible to significantly reduce the time for calculating the characteristics of the structure.

The results of calculating the dispersion characteristics of symmetric *E*-waves of a circular waveguide with a parabolic profile of the dielectric filling are shown in Fig. 5. Figure 6 shows the distribution of the Umov-Poynting vector over the cross-section of the waveguide, calculated for three modes at frequency $f = 14$ GHz (points 1, 2, 3 in Fig. 5).

## Calculation of a waveguide with a rod, the dielectric constant of which varies linearly

Based on equation (7), the structure is calculated in the form of a circular waveguide with partial dielectric filling, the permeability of which varies linearly (Fig. 7) within $r \in [0/a]$. The calculations were carried out for a waveguide with parameters $R = 20$ mm, $a = 10$ mm, $\varepsilon(r) = \varepsilon_1 - ((\varepsilon_1 - \varepsilon_2)/a)r$, $\varepsilon_1 = 6$, $\varepsilon_2 = 2$ frequency $f = 10$ GHz.

For comparison, the calculation of the same structure was performed with the representation of the linear profile of the dielectric constant in the form of a step approximation (Fig. 7) with the number of steps equal to 20. The results of the calculation of the field distribution obtained by solving the dispersion equation (7) are shown in Fig. 8. The results of calculating the field distribution, performed according to the proposed technique and using the partial domain method, coincide with the graphic accuracy.

## Conclusions and recommendation

On the basis of the method developed in this work, algorithms have been developed for calculating the characteristics of symmetric waves of a cylindrical waveguide with an axisymmetric dielectric filling, which has a radial dependence of the dielectric

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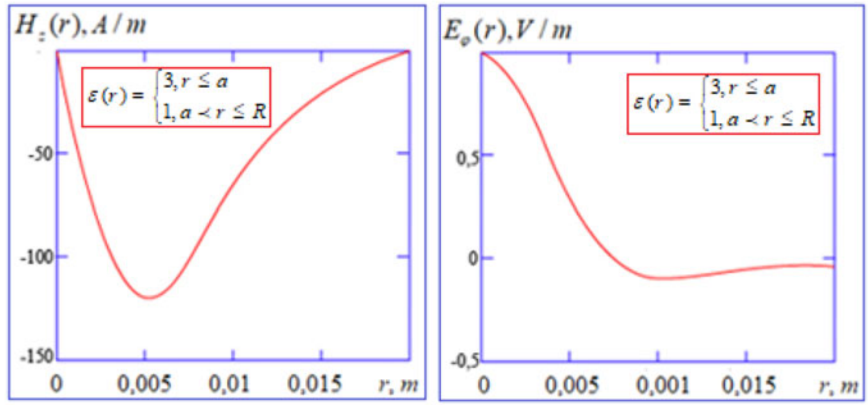


**Fig. 8.** Results of calculating the distribution of the wave field $H_{01}$ of a waveguide with a linear distribution of permeability.

constant The procedure for composing algorithms is a modified Galerkin method, in which a variational procedure is applied to functional relations following directly from Maxwell's equations, and corresponds to the canons of the spectral method.

On the example of three boundary value problems, the correctness and efficiency of the modified Galerkin method as a variant of the spectral method are confirmed.

The method is an alternative partial domain method in cases where the latter requires a multilayer approximation of the dielectric filling function, and can be extended to all waveguides with coordinate screening surfaces that provide complete sets of eigenfunctions of boundary value problems for comparison waveguides.

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## References


1. **Lina D, Xueming L, Dong M, Leiran W and GuoxI W** (2012) Experimental observation of dissipative soliton resonance in an anomalous-dispersion fiber laser. *Optics Express* **20**, 265–270.
2. **Jun L, Yu C, Pinghua T, Changwen X, Chujun Z, Han Z and Shuangchun W** (2015) Generation and evolution of mode-locked noise-like square-wave pulses in a large-anomalous-dispersion Er-doped ring fiber laser. *Optics Express* **23**, 6418–6427.
3. **Shestopalov YV** (2018) Complex waves in a dielectric waveguide. *Wave Motion (North-Holland Publishing Company)* **82**, 16–19.
4. **Shestopalov YV and Kuzmina EA** (2018) On a rigorous proof of the existence of complex waves in a dielectric waveguide of circular cross section. *Progress in Electromagnetics Research* **82**, 137–164.
5. **Mazur M and Mazur J** (2011) Operation of the phase shifter using complex waves of the circular waveguide with periodical ferrite-dielectric filling. *Journal of Electromagnetic Waves and Applications* **25**, 935–947.
6. **Smirnov YG and Valovik DV** (2015) Guided electromagnetic waves propagating in a plane dielectric waveguide with nonlinear permittivity. *Physical Review A* **91**, 345–354.
7. **Kusaykin OP, Melezhik PN, Poyedinchuk AE, Provalov SA and Seleznyov DG** (2016) Surface and leaky waves of a planar dielectric waveguide with a diffraction grating. *IET Microwaves, Antennas & Propagation* **10**, 61–67.
8. **Calignanoa F, Peverini OA, Addamo G, Paonessa F, Manfredi D, Galati M, Salmi A, Atzeni E, Minetola P and Iuliano L** (2019) High-performance microwave waveguide devices produced by laser powder bed fusion process. *Procedia CIRP* **79**, 85–88.
9. **Caballero ED, Esteban H, Belenguer A and Boria V** (2012) Efficient analysis of substrate integrated waveguide devices using hybrid mode matching between cylindrical and guided modes. *IEEE Transactions on Microwave Theory and Techniques* **60**, 232–243.
10. **Nair D and Webb JP** (2003) Optimization of microwave devices using 3-D finite elements and the design sensitivity of the frequency response. *IEEE Transactions on Magnetics* **39**, 1325–1328.
11. **Belenguer A, Esteban H and Boria VE** (2014) Novel empty substrate integrated waveguide for high-performance microwave integrated circuits. *IEEE Transactions on Microwave Theory and Techniques* **62**, 832–839.
12. **Fang K, Matheny MH, Luan X and Painter O** (2016) Optical transduction and routing of microwave phonons in cavity-optomechanical circuits. *Nature Photonics* **10**, 489–496.
13. **Liu W, Zhang Y, Triki H, Mirzazadeh M, Ekici M, Zhou Q, Biswas A and Belic M** (2019) Interaction properties of solitonics in inhomogeneous optical fibers. *Nonlinear Dynamics* **95**, 557–563.
14. **Silveira M, Frizera A, Leal-Junior A, Ribeiro D, Marques C, Blanc W and Díaz CAR** (2020) Transmission–reflection analysis in high scattering optical fibers: a comparison with single-mode optical fiber. *Optical Fiber Technology* **58**, 123–131.
15. **Islamov IJ, Ismibayli EG, Gaziyev YG, Ahmadova SR and Abdullayev RS** (2019) Modeling of the electromagnetic field of a rectangular waveguide with side holes. *Progress In Electromagnetics Research* **81**, 127–132.
16. **Islamov IJ, Shukurov NM, Abdullayev RS, Hashimov KK and Khalilov AI** Diffraction of electromagnetic waves of rectangular waveguides with a longitudinal. *IEEE Conferences 2020 Wave Electronics and its Application in Information and Telecommunication Systems (WECONF)*, INSPEC Accession Number: 19806145.
17. **Khalilov AI, Islamov IJ, Hunbataliyev EZ, Shukurov NM and Abdullayev RS** Modeling microwave signals transmitted through a rectangular waveguide. *IEEE Conferences 2020 Wave Electronics and its Application in Information and Telecommunication Systems (WECONF)*, INSPEC Accession Number: 19806152.
18. **Islamov IJ and Ismibayli EG** (2018) Experimental study of characteristics of microwave devices transition from rectangular waveguide to the megaphone. *IFAC-PapersOnLine* **51**, 477–479.
19. **Ismibayli EG and Islamov IJ** (2018) New approach to definition of potential of the electric field created by set distribution in space of electric charges. *IFAC-PapersOnLine* **51**, 410–414.
20. **Islamov IJ, Ismibayli EG, Hasanov MH, Gaziyev YG and Abdullayev RS** (2018) Electrodynamics characteristics of the no resonant system of transverse slits located in the wide wall of a rectangular waveguide. *Progress in Electromagnetics Research Letters* **8**, 23–29.
21. **Islamov IJ, Ismibayli EG, Hasanov MH, Gaziyev YG, Ahmadova SR and Abdullayev RS** (2019) Calculation of the electromagnetic field of a rectangular waveguide with chiral medium. *Progress in Electromagnetics Research* **84**, 97–114.
22. **Alsuyuti MM, Doha EH, Ezz-Eldien SS, Bayoumi BI and Baleanu D** (2019) Modified Galerkin algorithm for solving multitype fractional differential equations. *Mathematical Methods in the Applied Sciences* **42**, 1389–1412.
23. **Duvigneau R** (2020) CAD-consistent adaptive refinement using a NURBS-based discontinuous Galerkin method. *International Journal for Numerical Methods in Fluids* **92**, 1096–1117.

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