MORTALITY DECLINE, RETIREMENT AGE, AND AGGREGATE SAVINGS

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We use an overlapping-generations model with endogenous retirement and saving to study the trade-off between saving and retirement age in response to mortality decline. When life expectancy increases by one year, people delay retirement by about four months. With this magnitude of delay in retirement age, the percentage of lifetime spent in working decreases, and people have to save more for postretirement years. Neither the pure form of sole adjustment through savings nor the proportionality hypothesis is consistent with our results, but the proportionality hypothesis is a better rule of thumb in predicting future behavior. Our choice of the modified Boucekkine et al. (2002) survival function gives a convenient one-to-one correspondence between life expectancy increase and a change in the survival parameter.

Keywords: Mortality Decline, Retirement, Savings

1. INTRODUCTION

Mortality decline has been widespread in many countries in the last century. In the United States, life expectancy at birth increased from 61.0 years in 1933 to 70.9 in 1970, and then further to 78.3 in 2007.[1] Compared with industrialized economies, the speed of mortality decline is even more rapid in newly industrializing economies. The life expectancy in Taiwan increased from 68.7 in 1970 to 78.1 in 2007. According to most official forecasts, the trend of increasing life expectancy is expected to continue in the coming decades.

This paper studies the effects of mortality decline on individuals' retirement age choices and on aggregate savings. From a theoretical point of view, the possible changes of saving behavior during working years and/or retirement age decision in response to mortality decline can be regarded as two sides of the same coin. In principle, an individual may choose to respond by (a) leaving the retirement age unchanged and increasing savings during the working years to provide for postretirement consumption or (b) adjusting mainly along the retirement age dimension by increasing it "proportionally" [Lee and Goldstein (2003)], with the savings

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during working years more or less intact. Less extremely, individuals may respond by adjusting both variables. In this paper we use an overlapping-generations (OLG) model with endogenous retirement and saving to provide quantitative answers regarding the adjustment magnitudes.

Researchers have long been interested in these issues. A standard framework for studying intertemporal consumption in response to demographic or other kinds of shocks is the life-cycle model or its general-equilibrium version (the OLG model with production). Using a continuous-time OLG model (the limiting version of a multiperiod OLG model), researchers such as Lee et al. (2000) and Bloom et al. (2007) examine the effect of mortality decline on individual saving behavior and its aggregate implications.[2] However, their quantitative results differ greatly, ranging from substantial changes in savings according to Lee et al. (2000) to almost zero effect in (the no-pension-system version of) Bloom et al. (2007). Part of the reason for the hugely different magnitudes is that these researchers use different specifications in their papers.[3] This paper tries to reconcile these differences. We first select appropriate ingredients from relevant models and then perform quantitative analysis based on the proposed model.

As indicated in the preceding paragraph, whether retirement age changes substantially or remains unchanged may have an important effect on saving behavior. A by-product of our analysis is that we allow the retirement age to be endogenous, and to determine the magnitude of the effect of mortality decline on retirement age. For this question, the conventional wisdom is that mortality decline leads to a delay in retirement age, because people need to work longer to accumulate enough wealth for the greater expected length of postretirement years. However, some recent papers suggest counterintuitive results that mortality decline can lead to earlier retirement under some conditions [Kalemli-Ozcan and Weil (2010); d'Albis et al. (2012)].[4] Integrating the insights of two strands of research, a partial-equilibrium life-cycle model with endogenous retirement age [such as Kalemli-Ozcan and Weil (2010); d'Albis et al. (2012); or Bloom et al. (2014)] and a general-equilibrium OLG model with a general survival function but exogenous retirement age [such as Lee et al. (2000); d'Albis et al. (2007); or Lau (2009)], we analyze these questions using an OLG model with endogenous retirement and saving.

In the quantitative analysis, a major innovation of this paper is our choice of a specific parametric survival function, which is modified from Boucekkine et al. (2002). The advantages of this specification are that our results are not sensitive to the choice of countries, and there is a one-to-one correspondence between life expectancy and a change of parameter in the survival function. (We elaborate on these characteristics in Section 3.)

Using the OLG model with endogenous retirement and saving and the proposed parametric survival function, we conduct computational analysis. For the type of mortality decline likely to be relevant in the coming decades, it is found that mortality decline leads to a delay in retirement age and an increase in aggregate saving. Neither the pure form of the proportionality hypothesis [Lee and Goldstein

(2003)] nor the sole adjustment through saving is supported. In particular, it is found that an increase of life expectancy of one year leads to a delay of retirement age by about four months. However, the delay in retirement age by this amount is not enough to keep the proportion of lifetime spent on working constant, and it decreases by approximately 0.15 percentage point. Consequently, the aggregate saving rate increases by about 0.034 percentage point. By further comparing with the results of the model with adjustment in savings only and those corresponding to the proportionality hypothesis, it is found that the deviation of the former model is more severe.

The rest of the paper is organized as follows. In Section 2 we describe the OLG model with endogenous retirement and saving, when the individual faces uncertainty in lifetime captured by a general survival function. In Section 3 we discuss a parametric survival function and estimate the maximum age parameter of the survival function. We present our main computational results in Section 4. To provide a clearer perspective on the magnitudes of changes in aggregate saving rate and retirement age, we consider three variants of the model (one with exogenous retirement age, one with retirement age adjusting to maintain a constant proportion of lifetime spent on retirement, and one with the "compression of morbidity" feature) in Section 5. In Section 6, we perform sensitivity analysis using (a) an alternative parametric specification of the survival function and (b) the actual survival data of 13 countries. To illustrate that the model used in this paper can easily be extended to incorporate real-world features, we extend the basic model to include a pay-as-you-go pension system in Section 7. We provide concluding remarks in Section 8.

2. AN OLG MODEL WITH ENDOGENOUS RETIREMENT

We consider a continuous-time OLG model with the retirement age endogenously determined. Following most researchers working with the OLG models, we do not consider the childhood stage and assume that the adult stage starts at age 20. In this paper, "age" may refer to adult age or actual age, depending on the context.

We first consider the decisions of individual consumers/workers. As in Bloom et al. (2007), Kalemli-Ozcan and Weil (2010), and d'Albis et al. (2012), we focus on the extensive margin of labor supply decisions, and also assume that the retirement decision is irreversible. Thus, an individual will supply a constant flow of labor supply, normalized to be 1, before retirement, and zero afterwards. At time t, a cohort-s individual ($s \leq t$) chooses a consumption path (from the current age up to, possibly, maximum age Ω_{ad}) and, if he has not retired yet, a retirement age (R) to maximize[5]

$$\int_{t-s}^{\Omega_{ad}} e^{-\rho(x+s-t)} \frac{l(x)}{l(t-s)} \ln c(s,x)dx - \int_{t-s}^{R} e^{-\rho(x+s-t)} \frac{l(x)}{l(t-s)} \phi(x)\,dx, \quad (1)$$

subject to the flow budget constraint

$$\frac{\partial a(s,x)}{\partial x} = \begin{cases} [r(s+x) + \mu(x)]a(s,x) + w(s+x) - c(s,x) & \text{if } x \leq R \\ [r(s+x) + \mu(x)]a(s,x) - c(s,x) & \text{if } x > R, \end{cases} \tag{2}$$

where ρ is the discount rate, $c(s,x)$ is consumption of a cohort-s individual at age x, $a(s,x)$ is financial assets of a cohort-s individual at age x, $r(t)$ is the (real) interest rate at time t, $w(t)$ is the (real) wage rate at time t, $l(x)$ is the survival probability at age x, $\mu(x) = -\frac{1}{l(x)}\frac{dl(x)}{dx}$ is the mortality rate at age x, and $\phi(x)$ is the disutility of labor at age x. We assume that $\phi(x)$ is non-negative and nondecreasing (i.e., $\phi(x) \geq 0$ and $\phi'(x) \geq 0$), following Sheshinski (2006), Heijdra and Romp (2009), and d'Albis et al. (2012). The specifications of (1) and (2) capture the assumptions that the consumer has no bequest motive and the annuity markets are perfect [as in Yaari (1965); Blanchard (1985); Bloom et al. (2007)].

For the production side, we assume a standard neoclassical production function with exogenous technological progress,

$$Y(t) = F[K(t), A(t)N(t)], \tag{3}$$

where $Y(t)$, $K(t)$, $N(t)$, and $A(t)$ represent, respectively, output, capital input, labor input, and technological level at time t. Technological progress is represented by

$$A(t) = A(0)e^{gt}, \tag{4}$$

where g is the rate of technological progress. Define the production function in intensive form, which is given by

$$y(t) \equiv \frac{Y(t)}{A(t)N(t)} = F\left[\frac{K(t)}{A(t)N(t)}, 1\right] \equiv f[k(t)]. \tag{5}$$

We consider the steady-state equilibrium of this OLG model. Denote a variable at the steady-state equilibrium with an *. First, it is well known from the production theory that

$$r^*(t) = f'(k^*) - \delta \equiv r^*, \tag{6}$$

where δ is the rate of depreciation of capital, and

$$w^*(t) = A(t)\left[f(k^*) - k^*f'(k^*)\right] \equiv A(t)w^*. \tag{7}$$

Second, conditional on a particular retirement age R, the procedure for obtaining the Keynes–Ramsey rule for the consumer's intertemporal consumption problem is standard. Define the optimal consumption level of a cohort-s individual at age x, conditional on retirement age R, as $c(s,x,R)$. At the steady-state equilibrium, it is well known that

$$\frac{\partial c(s,x,R)}{\partial x} = \left(r^* - \rho\right)c(s,x,R). \tag{8}$$

Third, as in Kalemli-Ozcan and Weil (2010) and d'Albis et al. (2012), it can be shown that the first-order condition for optimal retirement age (R^*) of a cohort-s individual is given by

$$c^*(s, 0, R^*)^{-1} e^{-r^* R^*} w^*(s + R^*) = e^{-\rho R^*} \phi(R^*). \qquad (9)$$

The right-hand side of (9) represents the marginal cost of delaying retirement (at age R^*), discounted back to age 0. The corresponding marginal benefit term is given on the left-hand side of (9). Using the lifetime budget constraint of the individual, (9) can be equivalently represented by

$$\frac{e^{-(r^*-g)R^*} \int_0^{\Omega_{ad}} e^{-\rho x} l(x)\, dx}{\int_0^{R^*} e^{-(r^*-g)x} l(x)\, dx} = e^{-\rho R^*} \phi(R^*). \qquad (10)$$

Finally, following similar analysis in d'Albis (2007) and Lau (2009),[6] it can be shown that the level of capital per unit of effective labor at the steady-state equilibrium, k^*, is defined by

$$k^* = \frac{w^*}{r^* - g - n} \left\{ \left[\frac{\int_0^{R^*} e^{-(r^*-g)x} l(x)\, dx}{\int_0^{\Omega_{ad}} e^{-\rho x} l(x)\, dx} \right] \left[\frac{\int_0^{\Omega_{ad}} e^{-(g+n-r^*+\rho)x} l(x)\, dx}{\int_0^{R^*} e^{-nx} l(x)\, dx} \right] - 1 \right\}, \qquad (11)$$

where r^* and w^* are defined in (6) and (7), and n is the (constant) population growth rate.

The two variables at the steady-state equilibrium (R^* and k^*) are solved for simultaneously from (10) and (11).

3. THE SURVIVAL FUNCTION: A PARAMETRIC SPECIFICATION

The theoretical model of the preceding section allows a general survival function. As a result, it is possible to use actual life tables of individual countries, especially those of industrialized countries for which the assumption of the competitive markets in our theoretical model is more likely to hold, in the computational analysis.

Instead of taking the preceding route, one innovation of this paper is that we adopt a parametric approach to model the survival function. There are two reasons for making this choice. First, although there are broad similarities in the survival functions of industrialized countries in recent years, there are also differences among them, which makes the computational results potentially dependent on the choice of country. To eliminate this possible ambiguity, we choose a parametric survival function so that the computational results do not depend on the country choice, and their dependence on the underlying economic and demographic factors can be seen more transparently. Second, we aim to investigate how mortality decline leads to changes in relevant economic variables, but it is well known that an increase in life expectancy (for example, an increase of one year in life

expectancy at birth) can be caused by different patterns of mortality reductions. Except for researchers familiar with demographic concepts, it is relatively difficult to convey the idea of mortality decline based on, for example, changes in age-specific mortality rates. In communicating our computational results to a wider audience, we consider an increase in life expectancy, which is a more familiar concept. We use a particular parametric survival function to deliver this objective.

Our choice of the survival function is modified from the Boucekkine et al. (2002) specification.

$$l^{\text{BCL}}(x; \beta, \theta) = \frac{e^{-\beta x} - \theta}{1 - \theta}, \tag{12}$$

where β and θ are parameters. As mentioned in Boucekkine et al. (2002, p. 345), the shape of the survival function when $\beta < 0$ and $\theta > 1$ resembles the actual data well, except perhaps toward the end of life.[7]

With the objective of achieving a one-to-one correspondence between life expectancy and changes in the model parameter, we use the equivalent form of (12),

$$l^{\text{BCL}}(x; \beta, \Omega) = \frac{e^{-\beta x} - e^{-\beta \Omega}}{1 - e^{-\beta \Omega}}, \tag{13}$$

where Ω is the maximum age.[8]

There is no difference in (12) and (13) if we interpret Ω as a parameter that may be changing for different survival functions at different times. Our approach in this paper is to treat Ω as a constant even if there is mortality decline (within a relevant range).[9] We call (13) with a fixed value of Ω the "modified Boucekkine et al. (2002) survival function," which is now a one-parameter model. When Ω is unchanged, it can be shown that life expectancy (LE) at birth corresponding to the survival function (13) is[10]

$$\text{LE} = \int_0^\Omega l^{\text{BCL}}(x; \beta, \Omega) \, dx = \frac{1}{\beta} - \frac{\Omega e^{-\beta \Omega}}{1 - e^{-\beta \Omega}}, \tag{14}$$

and there is a one-to-one mapping between life expectancy and the parameter β.

Our next step is to decide on the value of maximum age Ω for computational analysis. Of the 37 countries in the Human Mortality Database, we focus on 25 countries in which the life expectancy for the most recent survival function (men and women combined) is 78.32 or above, where 78.32 is the life expectancy of the United States in 2007. The survival function of the United States (men and women combined) in 2007, which is shown in Figure 1, is taken as the benchmark that future mortality decline is compared with.

We estimate the value of Ω by minimizing the sum of squared residuals (SSR) between a particular survival function (such as that of the United States) and the estimated modified Boucekkine et al. (2002) survival function. Specifically, we restrict Ω to be an integer within the range $\{79, 80, \ldots, 110\}$, and choose Ω to
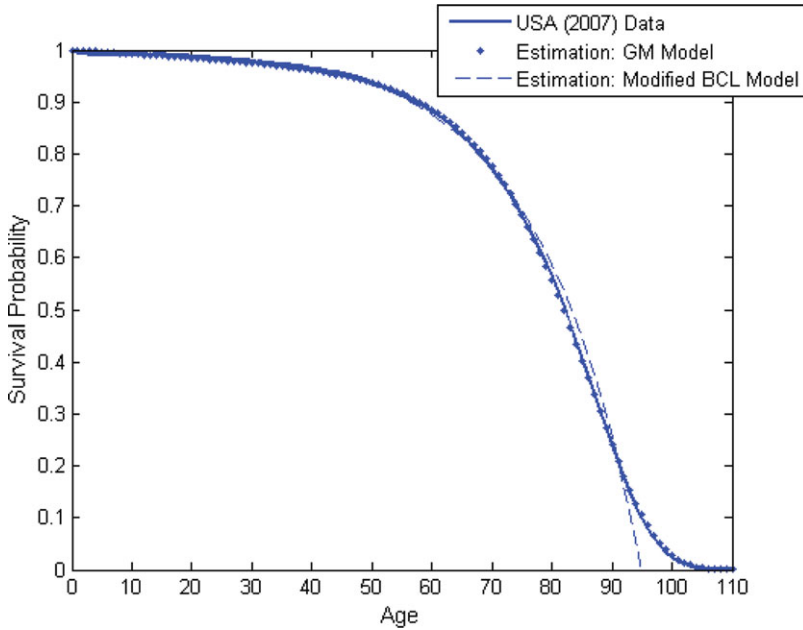
FIGURE 1. Survival functions: data and parametric forms.

minimize[11]

$$\text{SSR}\left(\Omega\right) = \sum_{x=1}^{110} \left[l_x^{\text{data}} - l_x^{\text{BCL}}\left(\Omega\right)\right]^2, \tag{15}$$

where l_x^{data} is the probability of survival to at least age x based on the reference life table data, and $l_x^{\text{BCL}}\left(\Omega\right)$ is generated according to

$$l_x^{\text{BCL}}\left(\Omega\right) = \frac{e^{-\beta(\Omega,78.32)x} - e^{-\beta(\Omega,78.32)\Omega}}{1 - e^{-\beta(\Omega,78.32)\Omega}}, \tag{16}$$

where $\beta\left(\Omega, 78.32\right)$ is solved from the implicit function

$$\psi\left(\beta, \Omega, \text{LE}\right) \equiv \frac{1}{\beta} - \frac{\Omega e^{-\beta\Omega}}{1 - e^{-\beta\Omega}} - \text{LE} = 0, \tag{17}$$

when $\text{LE} = 78.32$ for each value of Ω within the range $\{79, 80, \ldots, 110\}$.

The intuition of this procedure is as follows. For an arbitrary maximum age Ω, we first obtain the corresponding value of the parameter $\beta\left(\Omega, 78.32\right)$ according to (17). That is, we restrict the parameters of the Boucekkine et al. (2002) survival function such that the maximum age is Ω and the life expectancy at birth equals 78.32 (the life expectancy of the United States in 2007). We then calculate the survival probability according to (16). Finally, we compute the SSR according to
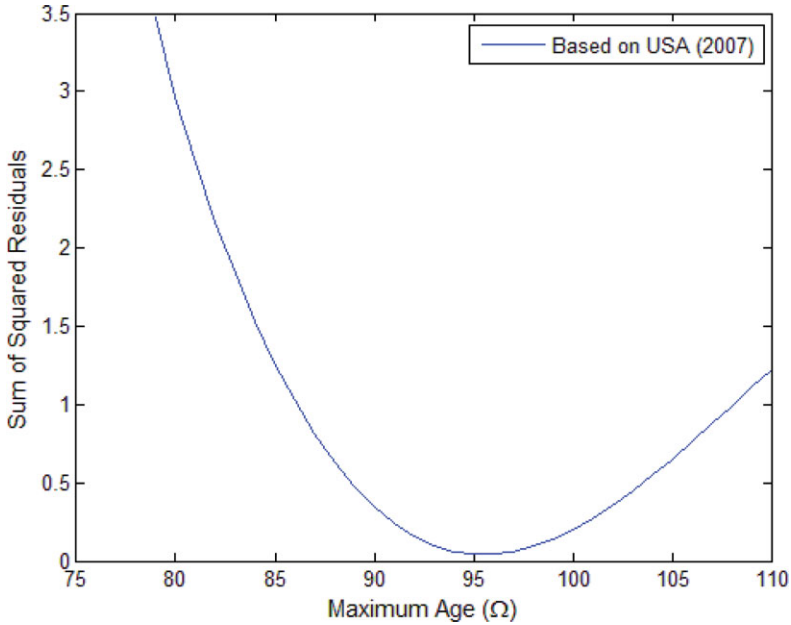
FIGURE 2. Estimation of maximum age parameter.

(15). We repeat the preceding steps for all possible values of Ω and obtain the optimal choice of Ω as the one that minimizes the SSR function.

   The value of SSR versus Ω for the U.S. data is shown in Figure 2, and it can be observed that the least-squares estimate of Ω is 95.[12] A plot of the modified Boucekkine et al. (2002) survival function, together with the U.S. life table data, is presented in Figure 1.

   Applying this procedure to all 25 countries with life expectancy of 78.32 or above, it is found that the optimal choice of Ω is 95 in 13 countries, and is 94 or 96 in 10 countries. A summary of the estimation results is given in Table 1.[13] Based on these results, we use $\Omega = 95$ in the remaining sections.

4. EFFECTS OF MORTALITY DECLINE ON RETIREMENT AND SAVINGS

In the remaining sections of this paper, we conduct a computational analysis of the effects of mortality decline on retirement age and aggregate saving rate.

   For the production side of the model, we assume a Cobb–Douglas production function. It is expressed, in intensive form, as

$$y(t) = k(t)^{\alpha}, \tag{18}$$

where α $(0 < \alpha < 1)$ is the capital share in GDP. With this production function,

TABLE 1. Estimation of maximum age parameter

Least-squares estimate of maximum age (Ω)	Number of countries	Country list
94	7	Denmark (2009), Slovenia (2009), Portugal (2009), Ireland (2009), Germany (2009), Luxembourg (2009), Netherlands (2009)
95	13	U.S.A. (2007), Taiwan (2009), Finland (2009), Belgium (2009), U.K. (2009), New Zealand (2008), Austria (2010), Norway (2009), Israel (2009), Spain (2009), Italy (2008), Sweden (2010), Iceland (2010)
96	3	Canada (2007), Australia (2007), Switzerland (2009)
97	1	France (2009)
98	1	Japan (2009)
	Total: 25	

it can be shown that (11) is simplified as

$$\left(k^*\right)^{1-\alpha} = \left[\frac{(1-\alpha)}{\alpha\,(k^*)^{\alpha-1} - (\delta + g + n)}\right]$$

$$\times \left\{\left[\frac{\int_0^{R^*} e^{-(r^*-g)x} l\,(x)\,dx}{\int_0^{\Omega_{\text{ad}}} e^{-\rho x} l\,(x)\,dx}\right]\left[\frac{\int_0^{\Omega_{\text{ad}}} e^{-(g+n-r^*+\rho)x} l\,(x)\,dx}{\int_0^{R^*} e^{-nx} l\,(x)\,dx}\right] - 1\right\}. \quad \textbf{(19)}$$

For the disutility of labor function, we follow Bloom et al. (2007 2014) to use an exponential form:[14]

$$\phi(x) = m e^{\frac{x}{\Omega_{\text{ad}}}}, \quad \textbf{(20)}$$

where m is a positive parameter. We solve R^* and k^* computationally from (10), (19), and (20).

We also calculate the equilibrium values of two variables: (a) proportion of (adult) lifetime spent on working,[15]

$$\frac{\int_0^{R^*} l\,(x)\,dx}{\int_0^{\Omega_{\text{ad}}} l\,(x)\,dx}, \quad \textbf{(21)}$$

and (b) aggregate saving rate, which is given by

$$1 - (1-\alpha)\left[\frac{\int_0^{R^*} e^{-(r^*-g)x} l\,(x)\,dx}{\int_0^{\Omega_{\text{ad}}} e^{-\rho x} l\,(x)\,dx}\right]\left[\frac{\int_0^{\Omega_{\text{ad}}} e^{-(g+n-r^*+\rho)x} l\,(x)\,dx}{\int_0^{R^*} e^{-nx} l\,(x)\,dx}\right]. \quad \textbf{(22)}$$

In the computational analysis, we follow the literature [such as Barro et al. (1995)] to assume that $\alpha = 0.3$, $\delta = 0.05$, $\rho = 0.02$, $n = 0.01$, and $g = 0.02$.[16] Based on the estimation results in Section 3, we use $\Omega_{\text{ad}} = \Omega - 20 = 75$.

TABLE 2. Mortality decline, retirement age, and aggregate saving rate

	The main model		With exogenous retirement age 65 (i.e., $R^* = 45$)		With retirement age adjusted according to the proportionality hypothesis	
Life expectancy	78.32	85	78.32	85	78.32	85
Retirement age	65	67.2	65 (Exogenous)	65 (Exogenous)	65	67.9
Proportion of adult lifetime spent working	72.68%	71.70%	72.68%	68.47%	72.68% (Exogenous)	72.68% (Exogenous)
Aggregate saving rate	28.59%	28.82%	28.59%	29.75%	28.59%	28.55%

To study the effects of mortality decline, we consider the range of life expectancy from 78 to 85 in the computational analysis, with the lowest value (78) slightly less than 78.32, the life expectancy for the United States in 2007. We consider an increase in life expectancy up to 85 years old, which is about a 10% increase from its current value, and which is higher than the life expectancy of any other current population (except for Japanese women). For each life expectancy (LE) in this range, we set $\Omega = 95$ and calculate β (95, LE) according to the implicit function ψ $(\beta, \Omega, \text{LE}) = 0$ in (17), and then obtain the survival function based on adult age,

$$l_{\text{ad}}^{\text{BCL}}(x) = \frac{l^{\text{BCL}}(x + 20)}{l^{\text{BCL}}(20)} = \frac{e^{-(x+20)\beta(95,\text{LE})} - e^{-95\beta(95,\text{LE})}}{e^{-20\beta(95,\text{LE})} - e^{-95\beta(95,\text{LE})}}. \tag{23}$$

Substituting this survival function into (10), (19), and (20), we obtain the steady-state equilibrium values of R^* and k^*.[17]

The computational results are presented in the upper panel of Figure 3 and in column (1) of Table 2. When life expectancy increases from 78.32 to 85, optimal retirement age increases from 65 to 67.2, implying that the proportion of adult lifetime spent on working decreases 0.98 percentage points (from 72.68% to 71.70%).[18] At the same time, aggregate saving rate increases by 0.23 percentage point (from 28.59% to 28.82%). Intuitively, when life expectancy increases, optimal retirement age increases as well.[19] However, it increases less than proportionally, resulting in a decrease in the proportion of adult life spent on working. Because individuals have a relatively smaller percentage of working life, they adjust to the rise in life expectancy by increasing saving during the working years. Aggregating the level of savings of individuals across different cohorts, it is found that there is a mild increase in the aggregate saving rate.

It can be observed from the upper panel of Figure 3 that the relationships are quite linear in this range. We conclude that when life expectancy increases by one year (in this range), retirement age increases by about four months, and aggregate saving rate increases by about 0.034 percentage points.
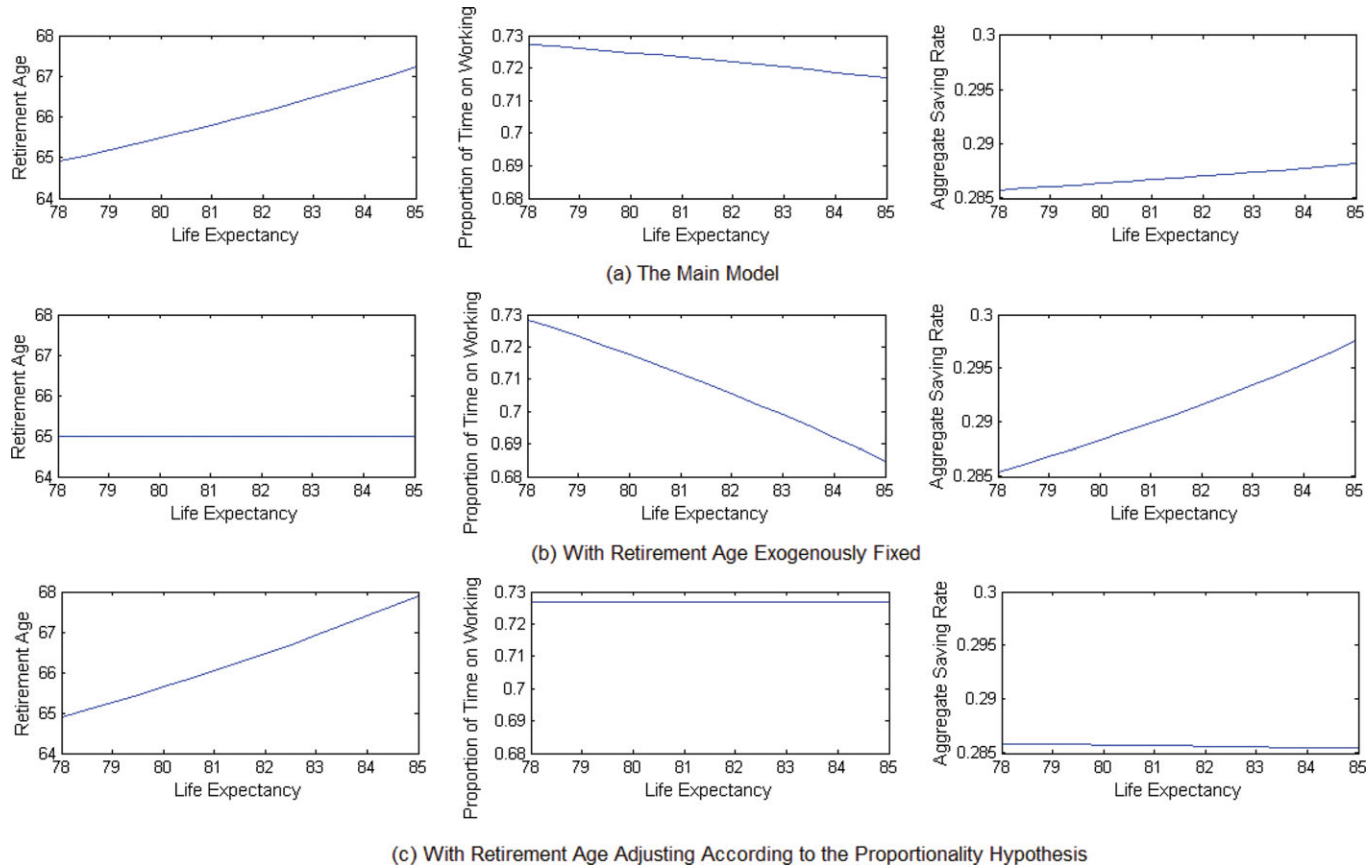
(a) The Main Model

(b) With Retirement Age Exogenously Fixed

(c) With Retirement Age Adjusting According to the Proportionality Hypothesis

FIGURE 3. Mortality decline, retirement age, and aggregate saving rate: main model and two variants.

5. VARIANTS OF THE MODEL

We now consider three variants of the model: (a) when retirement age is assumed to be unchanged, (b) when retirement age is assumed to adjust to maintain a constant proportion of lifetime spent on working, and (c) when compression of morbidity is present [Fries (1980); Bloom et al. (2007)]. We consider the first two variants in order to see whether the computational results in the preceding section are closer to the proportionality hypothesis or to the hypothesis of adjustment solely through savings. We are also interested to see whether our computational results are robust or not to the compression of morbidity specification.

5.1. When Retirement Age Is Fixed

In many papers adopting the OLG framework, the retirement age is conveniently assumed to be exogenously fixed. The exogenous retirement age model can be interpreted as a special case of the model in Section 2, when the disutility of labor function $\phi(x)$ increases extremely sharply when $x = R^*$ (i.e., $\lim_{x \to R^*} \phi'(x) = \infty$).

For easy comparison with the results in Section 4, we now assume that people always retire at age 65 ($R^* = 45$) irrespective of life expectancy. Using $R^* = 45$, we can obtain the proportion of lifetime spent on working according to (21). We can also solve for the remaining variable, k^*, from (19). Once k^* is obtained, aggregate saving rate is calculated according to (22).

The computational results for different values of life expectancy are presented in the middle panel of Figure 3 and in column (2) of Table 2. When life expectancy increases from 78.32 to 85, the proportion of adult lifetime spent on working (until the exogenous retirement age of 65) decreases by a much larger amount of 4.21 percentage points (from 72.68% to 68.47%). Because a significantly larger proportion of (adult) lifetime is spent on retirement, the saving levels during working years have to increase more substantially. After aggregating across people at different ages, it is found that the aggregate saving rate increases by 1.16 percentage points (from 28.59% to 29.75%). Compared with the results of the model with endogenous retirement age (presented in the upper panel of Figure 3), the adjustment of these two variables in the exogenous retirement age model is in the same direction, but the magnitudes are much more substantial.

5.2. When Retirement Age Adjusts According to the Proportionality Hypothesis

We now consider the opposite extreme case: when retirement rate is adjusted so that the resulting percentage of time spent on working is constant irrespective of the level of life expectancy.

To facilitate comparison with the results in Section 4, we now assume that the individual's retirement age is chosen so that the proportion of lifetime spent on working is always the same (at 72.68%). Computationally, we use this condition

to calculate R^* from (21) for a given modified Boucekkine et al. (2002) survival function with life expectancy in the relevant range. We then calculate k^* according to (19) and aggregate saving rate according to (22).

The results are presented in the lower panel of Figure 3 and in column (3) of Table 2. When life expectancy increases from 78.32 to 85, R^* increases from 65 to 67.9 in order to keep the proportion of lifetime spent on working at 72.68%. Moreover, the aggregate saving rate decreases slightly from 28.59% to 28.55%. The last result is not surprising, because if the percentage of time spent on working is unchanged, the aggregate saving rate is unlikely to change drastically.

With these additional computational results from Sections 5.1 and 5.2, we can provide a more comprehensive interpretation of the results for the endogenous retirement age model in Section 4, in regard to whether it is more inconsistent with the proportionality hypothesis or the hypothesis of adjustment in savings only.[20] For the model corresponding to the proportionality hypothesis, when life expectancy increases from 78.32 to 85, the proportion of lifetime spent on working is unchanged, and the saving rate of the economy, obtained by aggregating the saving levels of different cohorts, is effectively constant (a drop of 0.04 percentage point from the original 28.59%). On the other hand, aggregate saving rate increases by 1.16 percentage points if the consumers change their saving behavior but not retirement age (i.e., the OLG model with exogenous retirement age). Relative to these two extreme cases, the increase in aggregate saving rate is 0.23 percentage point when the consumers are able to adjust both retirement and saving behavior. We conclude that between these two pure forms of adjustment, the deviation of the computational results from the proportionality hypothesis is less severe than that from the hypothesis of sole adjustment in saving. Between the two extreme hypotheses in response to mortality changes, the proportionality hypothesis is a better rule of thumb to predict saving behavior.

5.3. When Compression of Morbidity Is Present

It is commonly believed that people nowadays not only live longer, but also remain relatively healthy for a longer period of time, when compared with people a few generations ago. This is consistent with the hypothesis of compression of morbidity [Fries (1980)] that with mortality decline, there is a decrease in the proportion of lifetime spent in poor health. Based on this idea, Bloom et al. (2007) assume that health status, and thus disutility of labor, depend on age relative to life expectancy. We examine whether the computational results in Section 4 are affected or not by incorporating this hypothesis.

We now modify the model in Section 2 by replacing the disutility of labor function $\phi(x)$ with $\phi[x/\varepsilon(x)]$, where

$$\varepsilon(x) = \frac{\int_x^{\Omega_{ad}} l(q)\,dq}{l(x)} \tag{24}$$

is life expectancy at age x. In the computational analysis, we also adopt the exponential form [as in Bloom et al. (2007)]

$$\phi \left[\frac{x}{\varepsilon (x)} \right] = h e^{\frac{x}{\varepsilon(x)}}, \tag{25}$$

where h is a positive parameter.[21] We then solve for the equilibrium values R^* and k^* from (19) and

$$\frac{e^{-(r^*-g)R^*} \int_0^{\Omega_{\text{ad}}} e^{-\rho x} l(x)\, dx}{\int_0^{R^*} e^{-(r^*-g)x} l(x)\, dx} = h e^{\frac{R^*}{\varepsilon(R^*)} - \rho R^*}. \tag{26}$$

The computational results are presented in the upper panel of Figure 4. When life expectancy increases from 78.32 to 85, optimal retirement age increases from 65 to 67.1, the proportion of lifetime spent on working decreases by 1.23 percentage points (from 72.68% to 71.45%), and the aggregate saving rate increases by 0.29 percentage point (from 28.59% to 28.88%). Compared with the results in Section 4 in which disutility of labor depends on age only, the presence of the compression of morbidity phenomenon does not cause any major change in the magnitudes of the adjustment of retirement age and savings.

6. COMPARING THE MODIFIED BOUCEKKINE ET AL. (2002) SPECIFICATION AND OTHER APPROACHES

In preceding sections, we use a parametric survival function with one parameter: the modified Boucekkine et al. (2002) form. Although this survival function provides a convenient specification (especially regarding the one-to-one correspondence between life expectancy and the mortality parameter β) and resembles recent life table data fairly well (see, for example, Figure 1), one may be interested to know whether the quantitative results reported in this paper would remain similar or not when (a) other parametric specifications or (b) the actual data are used. We study this question in Sections 6.1 and 6.2, respectively.

6.1. The Gompertz–Makeham Specification

Although the Boucekkine et al. (2002) specification fits recent life table data reasonably well for most ages, its fits at very old ages are not good. In the demographic literature, it is well known that the Gompertz–Makeham specification, which involves three parameters, provide a better fit, especially at very old ages.[22] The age-specific mortality rate of this specification is given by

$$\mu^{\text{GM}}(x) = \mu_0 + \mu_1 e^{\mu_2 x}. \tag{27}$$

From (27), we can obtain the survival function as

$$l^{\text{GM}}(x) = e^{-\int_0^x \mu^{\text{GM}}(q)\,dq} = e^{-\left[\mu_0 x + \frac{\mu_1}{\mu_2}(e^{\mu_2 x} - 1)\right]}. \tag{28}$$

(a) With Compression of Morbidity

(b) With Actual Life Table Data

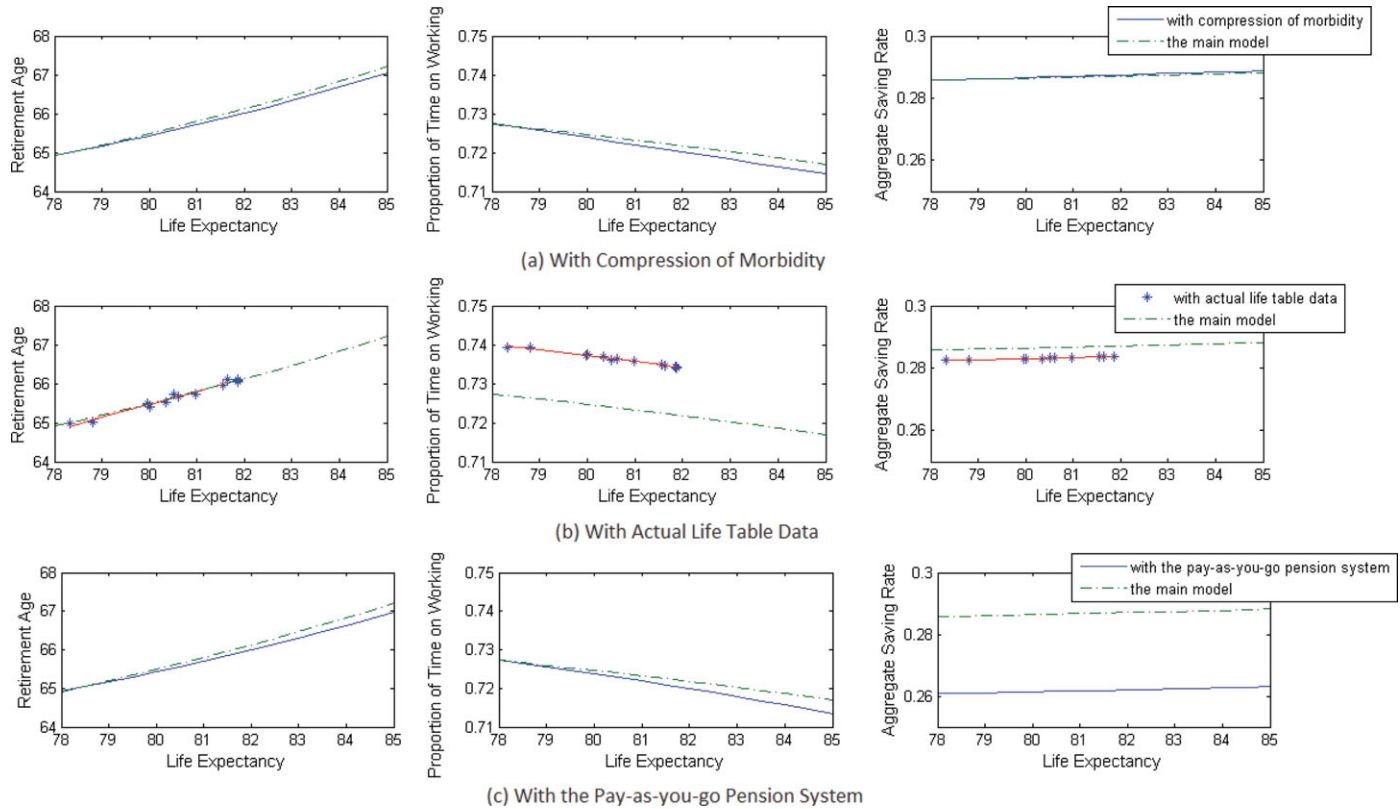(c) With the Pay-as-you-go Pension System

FIGURE 4. Mortality decline, retirement age, and aggregate saving rate: robustness analysis.

The Gompertz–Makeham specification has been applied in, for example, Heijdra and Romp (2009).

Compared with the Gompertz–Makeham survival function, the modified Boucekkine et al. (2002) specification (with a fixed maximum age) provides the advantage that the future survival function can be uniquely obtained by the life expectancy information, but the disadvantage is the poorer data fit. We would like to see whether this poorer fit, particularly at very old ages, causes huge or small discrepancies for economic variables that we are interested in. Because this task cannot be done for future survival function without imposing more restrictions on how the parameters change, we consider in this section only the "current" survival function (2007 U.S. data, as presented in Figure 1) with a particular value of life expectancy, not survival functions with higher life expectancy.

Using the nonlinear least-squares procedure,[23] the Gompertz–Makeham parameters are estimated to be $\mu_0 = 6.94 \times 10^{-4}$, $\mu_1 = 2.94 \times 10^{-5}$, and $\mu_2 = 0.0944$. The resulting Gompertz–Makeham survival function is also given in Figure 1. It can be observed that the fit is better than that of the modified Boucekkine et al. (2002) survival function.

We now conduct a computational analysis similar to that in Section 4, except that we use a survival function based on

$$l_{\text{ad}}^{\text{GM}}(x) = \frac{l^{\text{GM}}(x+20)}{l^{\text{GM}}(20)}, \tag{29}$$

where $l^{\text{GM}}(.)$ is defined in (28). To facilitate comparison with the results in Section 4, we use the same value (0.7603) of the parameter m in the disutility function of labor (20). The computational results of the OLG model using the Gompertz–Makeham specification are as follows: optimal retirement age is 65.04 ($R^* = 45.04$), proportion of lifetime spent working is 72.63%, and aggregate saving rate is 28.61%. These results are very similar to those in Section 4. It is concluded that the values of the key economic variables of the OLG model based on the modified Boucekkine et al. (2002) and Gompertz-Makeham specifications applied to the 2007 U.S. data are very similar.

6.2. Using Actual Life Table Data

We now perform the computational analysis based on actual data. To enhance compatibility between the modified Boucekkine et al. (2002) survival function (with a particular value of maximum age) and the actual life table data, we only analyze those countries in which the maximum age parameter (Ω) of this survival function is estimated to be 95. As seen in Table 1, there are altogether 13 countries in this list, with life expectancy varying from 78.32 (United States) to 81.87 (Iceland).

We conduct computational analyses for the 13 countries using the model in which the disutility of labor function is given by (20), and then compare the results with those in Section 4. As in that section, we use the U.S. data to calibrate

the coefficient in (20). Our method is exactly the same as before, except that the modified Boucekkine et al. (2002) survival function is replaced by the actual data. After transforming the actual life table data as a function of adult age, we obtain R^* and k^* by solving (10), (19), and (20).[24]

The computational results are presented in the middle panel of Figure 4, where each country is represented by a point with "∗." We further apply the ordinary-least-squares regression method to obtain the best-fit linear relations based on these points. The lines are quite similar to those based on the modified Boucekkine et al. (2002) survival function (also shown in the middle panel of Figure 4).[25] When life expectancy varies within the range of 78.32 to 81.87, an increase in life expectancy by one year leads to a delay of retirement age by 4 months, a decrease in the proportion of lifetime spent on working by 0.15 percentage point, and an increase in aggregate saving rate by 0.034 percentage point. These magnitudes are almost identical to those in Section 4. Our computational results using the OLG model suggest that the modified Boucekkine et al. (2002) survival function is a reasonable alternative to using the actual life table data.

7. WITH THE PAY-AS-YOU-GO PENSION SYSTEM

As mentioned in the Introduction, we keep the OLG model simple to highlight the changes of saving and retirement age in response to mortality decline, so that the computational results are not affected by other specifications. As a result, the model does not capture many real-world features. One may wonder whether this simple model can handle these features or not. In this section, we briefly address this question by incorporating one such important feature: a social security system. A by-product of this exercise is that we can examine whether our results are robust or not in the presence of a pay-as-you-go system.

Consider a pure pay-as-you-go retirement system with pure transfer such that the social security payments by workers in any period are entirely transferred to the surviving retirees. In each period, each worker pays a fraction (τ) of his wage income. On the other hand, each retiree obtains a fraction b, the replacement ratio, of the market wage (in real terms) when he retires.[26] In this environment, individuals' budget constraints are affected, because of the changes in wage income (when working) and retirement income (after retiring). The new budget constraint of a cohort-s individual is given by

$$\frac{\partial a\,(s,x)}{\partial x} = \begin{cases} [r(s+x)+\mu(x)]\,a(s,x)+(1-\tau)\,w\,(s+x)-c\,(s,x) & \text{if } x \leq R \\ [r(s+x)+\mu(x)]\,a(s,x)+bw\,(s+R)-c\,(s,x) & \text{if } x > R. \end{cases}$$
$$(30)$$

For convenience, we assume in the following analysis that the replacement ratio (b) is exogenously fixed. Following steps similar to those in Section 2, it can be shown that at the steady-state equilibrium, τ^* (the social security tax rate), k^*, and

R^* are determined according to

$$\tau^* = \frac{be^{gR^*} \int_{R^*}^{\Omega_{ad}} e^{-(n+g)x} l(x)\, dx}{\int_0^{R^*} e^{-nx} l(x)\, dx},$$

(31)

$$k^* = \frac{w^*}{r^* - g - n} \left\{ \left[\frac{(1-\tau) \int_0^{R^*} e^{-(r^*-g)x} l(x)\, dx + be^{gR^*} \int_{R^*}^{\Omega_{ad}} e^{-r^*x} l(x)\, dx}{\int_0^{\Omega_{ad}} e^{-\rho x} l(x)\, dx} \right] \right.$$

$$\left. \times \left[\frac{\int_0^{\Omega_{ad}} e^{-(g+n-r^*+\rho)x} l(x)\, dx}{\int_0^{R^*} e^{-nx} l(x)\, dx} \right] - 1 \right\},$$

(32)

and

$$\frac{e^{-(r^*-g)R^*} \left[(1-\tau-b) + bg \frac{\int_{R^*}^{\Omega_{ad}} e^{-r^*x} l(x) dx}{e^{-r^*R^*} l(R^*)} \right] \int_0^{\Omega_{ad}} e^{-\rho x} l(x)\, dx}{(1-\tau) \int_0^{R^*} e^{-(r^*-g)x} l(x)\, dx + be^{gR^*} \int_{R^*}^{\Omega_{ad}} e^{-r^*x} l(x)\, dx} = e^{-\rho R^*} \phi(R^*).$$

(33)

For the computational analysis, we assume that $b = 0.4$, which is close to the average figure for the United States [Sheshinski (2008, p. 2)]. Together with the modified Boucekkine et al. (2002) survival function (23), we obtain the steady-state equilibrium values of τ^*, k^*, and R^* from (31) to (33).

The computational results are presented in the lower panel of Figure 4. When life expectancy increases from 78.32 to 85, optimal retirement age increases from 65 to 67.0, leading to a decrease of 1.38 percentage points (from 72.68% to 71.30%) in the proportion of adult lifetime spent on working. At the same time, aggregate saving rate increases by 0.21 percentage point (from 26.09% to 26.30%). The major conclusions based on the basic model remain robust when a pay-as-you-go system is introduced.[27]

8. CONCLUSION

Mortality decline has been observed almost everywhere in recent years, and is expected to continue in the future. This paper studies the response of retirement age and saving to mortality decline. Two extreme forms of response have been mentioned in the literature. Researchers using the model with exogenous retirement age [such as Lee et al. (2000); Lau (2009)] implicitly assume that the adjustment is solely to saving but not to retirement age. On the other hand, the proportionality hypothesis [Lee and Goldstein (2003)] suggests that the adjustment falls mainly on retirement age, with saving level more or less unchanged. Our computational results based on an OLG model with endogenous retirement and saving suggest a simple rule of thumb that may be useful for policy analysts: when life expectancy increases by one year, people respond by delaying retirement age by about four months. The endogenous increase in retirement age of this magnitude in response

to mortality decline is not enough to attain the proportionality benchmark, resulting in a decrease in the proportion of (adult) lifetime spent working. Consequently, people need to save more for postretirement years, and aggregate saving rate increases mildly.

In comparison with the proportionality hypothesis and the model with sole adjustment through saving, it is found that the deviation of the model with sole adjustment by saving is more severe. We find that the conclusions based on models with exogenous retirement age significantly overstate the effects on aggregate saving: once the retirement age is allowed to be determined endogenously, the percentage point increase of the aggregate saving rate is dropped dramatically to about one-fifth of the original change.

Besides the substantive results, our paper also makes a methodological contribution by using a particular survival function: the modified Boucekkine et al. (2002) form. This survival function delivers the advantages that the results are not sensitive to country choice, and it gives a convenient one-to-one correspondence between a change in life expectancy and a change in the survival parameter. Moreover, the computational results are robust when we use the Gompertz–Makeham specification or actual life table data instead of the modified Boucekkine et al. (2002) specification. We believe that the modified Boucekkine et al. (2002) survival function will be useful in future studies.

In order to understand the trade-off between saving and retirement age transparently, we use an OLG model with only these essential features in this paper. There are at least two possible extensions to incorporate other relevant features. First, it has been argued [in, for example, Costa (1998); Bloom et al. (2014)] that wealth effect is important in affecting the changes of retirement age over time. Using a partial equilibrium framework, the results in Bloom et al. (2014) suggest that both mortality decline and increasing wealth are important in explaining changes in retirement age. However, they do not consider the general equilibrium effects due to changes in interest rate and wage rate. It is interesting to incorporate wealth effects into a general equilibrium OLG model with endogenous retirement and saving. Second, the importance of annuity market imperfection on saving and economic growth has been emphasized in a recent study using an OLG model with endogenous growth [Heijdra and Mierau (2012)]. Another possible research topic is whether and how changes in the degree of annuity market imperfection [Hansen and Imrohoroğlu (2008); Heijdra and Mierau (2012)] affect the trade-off between saving and retirement age adjustment in response to mortality decline.

NOTES

1. Our data source is the Human Mortality Database (www.mortality.org), and we downloaded the data in early 2012. Note that although in principle it is more appropriate to use cohort life tables, we use period life table information because cohort life tables for people born recently are not available.

2. Other papers such as Boucekkine et al. (2002), Heijdra and Romp (2009), and Zhang and Zhang (2009) also address this question, but they focus on other mechanisms as well. For example, Boucekkine et al. (2002) and Zhang and Zhang (2009) model human capital investment together with

retirement decisions, and Heijdra and Romp (2009) consider retirement decision in the presence of a pension system. Although these extra factors are important in the respective circumstances, we find it easier to understand the intuition of the saving versus retirement age trade-off by focusing on a model with only life-cycle consumption and retirement decisions. After the results of the main model are presented and the intuition made clear, we extend the model to incorporate a pay-as-you-go pension system in Section 7.

3. There are at least two differences relevant to the issues discussed in this paper. For the specification of the human survival function, Lee et al. (2000) use a general survival function, but Bloom et al. (2007) assume an exponential function. For retirement age, Lee et al. (2000) assume that it is exogenous, whereas Bloom et al. (2007) assume that it is determined endogenously.

4. The intuition of the counterintuitive results of Kalemli-Ozcan and Weil (2010) and d'Albis et al. (2012), as well as a discussion of the similarities and differences of these two papers, can be found in Section 3.3 of d'Albis et al. (2012).

5. Because the childhood stage is not considered in this model, a cohort-s individual corresponds to one whose adult age is zero (i.e., one who starts to work) at time s.

6. Specifically, it can be shown, using procedures similar to those leading to (21) of Lau (2009), that consumption per unit of effective labor at the steady-state equilibrium (c^*) can be expressed as

$$c^* = w^* \left[\frac{\int_0^{R^*} e^{-(r^*-g)x} l(x)\, dx}{\int_0^{\Omega_{\mathrm{ad}}} e^{-\rho x} l(x)\, dx} \right] \left[\frac{\int_0^{\Omega_{\mathrm{ad}}} e^{-(g+n-r^*+\rho)x} l(x)\, dx}{\int_0^{R^*} e^{-nx} l(x)\, dx} \right].$$

Next, using procedures similar to those leading to (18) of d'Albis (2007), we obtain

$$a^* = \frac{c^* - w^*}{r^* - g - n},$$

where a^* is the level of financial assets per unit of effective labor at the steady-state equilibrium. Equation (11) is obtained after using the equilibrium condition of the financial market, $k^* = a^*$, where k^* can be interpreted as the demand for financial assets and a^* is the supply of financial assets.

7. The discrepancy may be important for statistical analysis regarding survival probabilities at very old ages, but it is less severe in terms of saving and retirement behavior, which are the focus of this paper, because the probabilities of survival are already quite low for very old ages. The analysis in Section 6 deals with these issues.

8. Note that Ω refers to actual age and it is related to Ω_{ad} (the maximum age expressed in adult age) in Section 2 according to $\Omega_{\mathrm{ad}} = \Omega - 20$.

9. This specification is consistent with the observation that "(t)he average length of life has risen from 47 to 73 years in this century, but the maximum life span has not increased" [Fries (1980, p. 130)].

10. Moreover, LE $< \Omega/2$ when $\beta > 0$, and LE $> \Omega/2$ when $\beta < 0$. We focus on the $\beta < 0$ segment for empirical relevance.

11. Note that because actual life table data are available only in discrete years, the objective function in (15) is expressed in discrete years also. Note also that the maximum age in the data set is 110, and that the lowest value of Ω in the estimation is 79, because it cannot be lower than LE $= 78.32$ in our reference life table.

12. Besides using the maximum age in our data set (110), we have also used the maximum age of the modified Boucekkine et al. (2002) survival function (Ω) as the upper limit of summation in the SSR function (15), and have found that the optimal choice of Ω is still 95.

13. Following the suggestion of a referee, we have also considered the minimization of

$$\mathrm{SSR}\,(\Omega) = \sum_{x=21}^{110} \left[\frac{l_x^{\mathrm{data}}}{l_{20}^{\mathrm{data}}} - \frac{l_x^{\mathrm{BCL}}(\Omega)}{l_{20}^{\mathrm{BCL}}(\Omega)} \right]^2.$$

The optimal choice of Ω is always the same, whether it is based on this criterion or (15), for each of the 25 countries we examine.

14. The functional forms used in Bloom et al. (2007) and Bloom et al. (2014) are slightly different. In Bloom et al. (2007), the disutility of labor depends on age relative to life expectancy, to capture the compression of morbidity hypothesis; in Bloom et al. (2014), it depends linearly on age-specific mortality rates and thus varies exponentially with age. We choose (25) in Section 5.3, which is the same as in Bloom et al. (2007). To maintain a similar functional form, we specify age relative to maximum age in (20) in this section.

15. Lee (2001) studies how mortality decline affects the changes in expected length of retirement of U.S. males from 1850 to 1990. The measure of expected length of retirement years, $\int_{R^*}^{\Omega_{ad}} l\,(x)\,dx$, used in Lee (2001) is simply the difference of the numerator and denominator terms of (21) in this paper.

16. To maintain a constant population growth rate ($n = 0.01$), we assume implicitly that the increase in life expectancy is accompanied by a decrease in fertility. Note that the fertility decision is not modeled explicitly in most OLG models, including the model used in this paper.

17. We use the restriction that the retirement age is 65 (i.e., $R^* = 45$ in adult age) for the modified Boucekkine et al. (2002) survival function with LE = 78.32 to obtain the value of the parameter m in (20). We then let the parameter m be fixed at this value (0.7603), and obtain other computational results.

18. To facilitate comparison across different cases, the initial life expectancy for all three cases in Table 2 is fixed at 78.32 (instead of 78).

19. This is not inconsistent with the results in d'Albis et al. (2012), because mortality reductions in recent years have tended to concentrate more on older ages.

20. Because the model with endogenous retirement and savings is more general than either of these two pure forms of adjustment, it is not surprising that these restricted versions are not consistent with the computational results in Section 4. The major motivation of the analyses in Sections 5.1 and 5.2 is to determine, based on the quantitative results, which restricted version may be used by policy makers as a better rule of thumb.

21. As in Section 4, we calibrate the value of h by assuming $R^* = 45$ when LE = 78.32.

22. Other parametric methods for modeling the human survival function include the hyperbolic tangent function in Faruqee (2003) and the de Moivre specification in Bruce and Turnovsky (2013).

23. The objective function of the least-squares problem is

$$SSR\left(\mu_0, \mu_1, \mu_2\right) = \sum_{x=1}^{110}\left[l_x^{\text{data}} - l_x^{\text{GM}}\left(\mu_0, \mu_1, \mu_2\right)\right]^2,$$

where $l_x^{\text{GM}}\left(\mu_0, \mu_1, \mu_2\right)$ is generated according to (28).

24. Because the actual life table data are expressed in discrete years only, we have to adjust our computational method slightly by replacing integration with summation. Moreover, to allow R^* to be a noninteger, we use linear approximation between two adjacent years. For example, we approximate $\int_0^{45.3} e^{-nx} l\,(x)\,dx$ by $(1 - 0.3)\sum_{x=0}^{45} e^{-nx} l_x + 0.3\sum_{x=0}^{46} e^{-nx} l_x$.

25. The lines are particularly similar for retirement age. The deviation is most severe for the proportion of lifetime spent on working, but the maximum deviation is still less than 2%.

26. This assumption is used to capture the fact that the social security benefit in the United States, for example, is adjusted for inflation but not for change due to technological progress. Note that the idealized version of a pure pay-as-you-go system specified in this section is unlikely to capture all aspects of real-world systems perfectly.

27. The only major change is that the aggregate saving rate is always lower when the social security system is present, irrespective of the value of life expectancy. This result is also consistent with the conventional wisdom about the effect of social security on saving behavior.

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