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THE REAL-INTEREST-RATE GAP AS AN INFLATION INDICATOR

KATHARINE S. NEISS
Bank of England

EDWARD NELSON
Bank of England
and
Centre for Economic Policy Research

A long-standing area of research and policy interest is the construction of a measure of monetary policy stance. One measure that has been proposed, as an alternative to indices that employ monetary aggregates or exchange rates, is the spread between the actual real interest rate and its flexible-price, or natural-rate, counterpart. We examine the properties of the natural real interest rate and *real-interest-rate gap* using a dynamic stochastic general equilibrium model. Issues we investigate include the response of the gap and its components to fundamental economic shocks and the indicator and forecasting properties of the real-interest-rate gap for inflation, both in the model and in the data. Our results suggest that the real-interest-rate gap has value as an inflation indicator, supporting a neo-Wicksellian framework.

Keywords: Natural Interest Rate, Real-Interest-Rate Gap, Output Gap, Inflation Indicator

1. INTRODUCTION

In evaluating the consequences of monetary policy actions for the future behavior of inflation, it is often useful to construct a measure of monetary policy stance. Typically, such a measure will use some indicator of monetary or demand conditions, and express that measure relative to a baseline value (such as the value consistent with price stability).

Earlier candidates for monetary stance measures are monetary aggregates or a Monetary Conditions Index (MCI). Monetary aggregates have been criticized for being subject to distortions from financial innovations,[1] and (for broad aggregates)

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for being too far removed from the typical instrument of monetary policy, namely the short-term nominal interest rate.[2] MCIs, which consist of a weighted sum of the interest rate and exchange rate, have the problem that they assign a fixed weight to the exchange rate irrespective of the shock driving the exchange-rate movement. Exchange-rate depreciations are thus interpreted as monetary loosenings, but it is possible for the equilibrium real exchange rate to depreciate under conditions of price stability [King (1997)].[3]

In light of these arguments, one option is instead to construct a measure of monetary policy stance based exclusively on interest rates. In this vein, Woodford (1999, 2000) has advocated the concept of the "natural" or "equilibrium" real rate of interest—the real interest rate associated with flexible prices. Woodford revives the ideas of Wicksell (1898/1936; 1906/1935) within a dynamic stochastic general equilibrium (DSGE) model. He argues that price-level analysis based on the difference between actual and natural real interest rates is preferable to traditional analysis based on the interaction of money demand and supply. In models in which the instrument of monetary policy is a nominal interest rate, this "neo-Wicksellian" analysis of price-level determination avoids the cumbersome procedure of solving for the implied money supply function. In this framework, the key variable for the analysis of "inflationary or deflationary pressures" is "the gap between the current level of the 'natural rate' of interest and the interest rate controlled by the central bank" [Woodford (1999, p. 35)]. In line with this terminology, we define the *real-interest-rate gap* as the spread between the actual and "natural" real interest rate, and this paper studies this gap concept.

The position of real interest rates relative to their natural or "neutral" value has been considered by policymakers too. At the December 9–10, 1998, meeting of the Bank of England's Monetary Policy Committee (MPC), "some members of the Committee found the concept of the neutral rate useful in deciding on interest rate policy, [but] other members found the uncertainty surrounding its level so large that the concept was of little use" [MPC (1999, p. 67)]. Thus, many policymakers have a skeptical attitude toward the value of the real-interest-rate gap concept. The interest-rate gap is also regarded as harder to measure than an output gap. However, we see two key reasons for reconsidering real rate gaps:

(i) Understanding the behavior of the natural real rate appears to be important for understanding the empirical relationship between real interest rates and GDP. The correlations of detrended log GDP with lags of the short-term real rate [$\mathrm{Corr}(y_t, r_{t-k})$] are significantly negative for $k = 0$ to 4 in quarterly U.S. data [see Boldrin et al. (2001)]. However, for U.K. data, it is weakly positive, with, for example, $\mathrm{Corr}(y_t, r_{t-k}) = 0.10$ for $k = 0$ and 0.11 for $k = 4$ on 1980–1999 data.[4] If cyclical variations in the natural real interest rate and potential GDP are negligible, these correlations would simply reflect the relation between real interest rate and output gaps, and would therefore be negative. Evidently, natural real rate and output variations, due to real shocks, are not negligible, and need to be accounted for in monetary policy analysis.

(ii) It is possible that the real-interest-rate gap may actually be measured with *less* uncertainty than the output gap. Although measures of the output gap are common in

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policy analysis, it is worth remembering that many of these measures are based on the assumption that potential GDP evolves according to a deterministic trend (such as a linear, quadratic, or broken-linear trend). Economic theory suggests, instead, that potential GDP, while certainly containing a trend component, also fluctuates over the business cycle in response to all real shocks. The validity of many standard measures of the output gap therefore rests on the hypothesis that the response of potential GDP to these shocks is relatively flat, so that detrended output approximates the output gap well.[5] This is hard to justify in light of results from DSGE models (including ours below) that suggest that potential GDP fluctuates considerably. However, it may be that the response of the *natural real rate* to shocks is relatively flat, in which case the construction of reliable indices of real-rate gap movements is easier, even though data on the natural rate are not available.

These considerations suggest that there are benefits from further study of the real-interest-rate gap concept. To this end, this paper develops a sticky-price DSGE model to examine the behavior of the natural real interest rate and real-interest-rate gap. All DSGE models implicitly provide models of the real-interest-rate gap. In flexible-price models, the behavior of the real-rate gap is trivial—it is zero every period because the real interest rate *is* the natural rate.[6] In sticky-price models,[7] the real-rate gap is zero on average (provided the long-run Phillips curve is vertical), but will not be zero every period (except in the special case where monetary policy eliminates the real effects of price stickiness). In addition to the papers mentioned above, King and Watson (1996), Rotemberg and Woodford (1997), Clarida et al. (1999), and Giannoni (2002) have focused on the behavior of real rates relative to their natural values in sticky-price DSGE models. Of these, only King and Watson have capital formation,[8] and they do not include preference shocks, which could be a key influence on the natural rate. We include both capital formation and preference shocks, as well as other elements absent from the above papers, such as non-time-separable utility. We also consider more than one price-setting specification.

The two main results of this paper are that the natural real rate does not fluctuate greatly over the business cycle, and so, the actual real rate is a reasonable proxy for the real-interest-rate gap; and that, by contrast, potential output variation *is* important at the business-cycle frequency. These results are consistent with points (i) and (ii) above, and have important implications for the modeling of inflation and for policy rules.

Regarding the modeling of inflation, our results are relevant for interpreting recent estimates by Sbordone (2002), Galí and Gertler (1999), and Galí et al. (2002) of the New Keynesian Phillips curve (NKPC). Most sticky-price DSGE models, including the one presented here, imply a close link between the output gap and real marginal cost, so that either series is appropriate to use as the forcing process in estimating the NKPC. However, the above authors find that NKPC's with log real marginal cost[9] as the process driving inflation fit U.S. data with plausible and significant coefficient estimates, whereas specifications with detrended output do not. Galí et al. argue that these findings indicate the presence of labor-market imperfections (such as wage rigidity) that break the relation of output gap to marginal cost. Our results imply that a key factor behind these findings is that detrended output is a poor output gap proxy, because real shocks produce substantial variation in potential GDP. The limitations of detrended output as an output gap proxy have been noted by others, including Galí and Gertler (1999), McCallum and Nelson (1999), and Woodford (2001a). However, quantifying these limitations has been impeded by technical obstacles to computing potential GDP in models that have endogenous state variables, such as capital or lagged consumption. Our Appendix describes an algorithm that overcomes these obstacles, enabling us to compute output and real-interest-rate gaps in models with capital and habit formation.

On policy rules, the behavior of the output gap and natural rate in our model provides mixed support for the interest-rate rule formulated by Taylor (1993). In generating interest-rate prescriptions from a rule, Taylor assumes that cyclical variations in both the natural real rate and potential output can be neglected. Our results support the first of these approximations but not the second.

2. MODEL

This section describes the DSGE model employed in the paper.

2.1. Households

The economy is inhabited by a large number of households, each of which has preferences defined over its consumption of a composite good (denoted C_t in period t), leisure (which, with time endowment normalized to unity, is denoted $1-N_t$), and real balances (M_t/P_t). A typical household chooses a sequence of consumption, leisure, nominal money and one-period bond holdings (B_{t+1}), and capital (K_{t+1}), to maximize lifetime utility:

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\lambda_{t+j} \frac{\sigma}{\sigma-1} \left(\frac{C_{t+j}}{C_{t+j-1}^h} \right)^{\frac{\sigma-1}{\sigma}} + b(1-N_{t+j}) + \frac{\gamma}{1-\varepsilon} \left(\frac{M_{t+j}}{P_{t+j}} \right)^{1-\varepsilon} \right] \quad (1)$$

subject to a series of real period budget constraints:

$$\begin{aligned} C_{t+j} + \frac{M_{t+j}}{P_{t+j}} + \frac{B_{t+j+1}}{P_{t+j}} + X_{t+j} &= w_{t+j}N_{t+j} + z_{t+j}K_{t+j} + R^G_{t+j-1}\frac{B_{t+j}}{P_{t+j}} \\ &+ \frac{M_{t+j-1}}{P_{t+j}} - \frac{\tau_{t+j}}{P_{t+j}} - \varphi X^{\eta}_{t+j}, \end{aligned} \quad (2)$$

for all $j = 0, 1, \ldots, \infty$, where $b > 0$, $\gamma > 0$, $\varepsilon > 0$, and $0 < \beta < 1$. In (2), X_t is related to K_t by

$$X_{t+j} = K_{t+j+1} - (1-\delta)K_{t+j}, \quad (3)$$

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where $\delta \in [0, 1)$. In equation (2), R_t^G denotes the gross nominal interest rate, N_t denotes labor supplied, τ_t government transfers, and z_t the return on capital. In our numerical work, we consider two settings of the model: a version with no capital variation ($K_t = K^{ss}$ and $X_t = \delta K^{ss}$ for all t), and one with capital accumulation but with adjustment costs operative. With no capital adjustment costs, X_t corresponds to investment expenditure; with capital adjustment costs, we refer to X_t as quasi investment. The size of capital adjustment costs is determined by the parameters $\varphi > 0$ and $\eta (1 < \eta < \infty)$.[10]

The parameter σ indexes the curvature of households' utility function: A larger σ implies greater willingness to shift consumption across time in response to interest-rate changes. Preferences over consumption take on a non-time-separable form to capture the idea that households may exhibit habit formation in their consumption patterns. Parameter $h \in [0, 1)$ indexes the degree of habit formation: If $h = 0$, then households exhibit no habit formation and preferences are time separable. For $0 < h < 1$, utility from current consumption depends on prior consumption. We assume $h > 0$ in light of evidence that this reduces some empirical weaknesses of DSGE models [Fuhrer (2000), Boldrin et al. (2001)].[11] We augment the utility function with a disturbance (λ_t) to consumption preferences, which we interpret as an "IS" or "real demand" shock.

The consumption good C_t is a Dixit–Stiglitz aggregate of a multiplicity of differentiated goods, indexed by $i \in [0, 1]$. Under this scheme, the consumption and price indices are defined as

$$C_t = \left[\int_0^1 C_t(i)^\rho \, di \right]^{\frac{1}{\rho}} \quad \text{and} \quad P_t = \left[\int_0^1 P_t(i)^{\frac{\rho}{1-\rho}} \, di \right]^{\frac{1-\rho}{\rho}}.$$

Substitutability of goods in consumption is governed by $\rho \in (0, 1)$; ρ near 1 implies that goods are fully substitutable and firms are perfectly competitive. The inverse of ρ is therefore the (gross) steady-state markup.

With ψ_t, the Lagrange multiplier on (2), the household's optimality conditions are

$$C_t: \quad \lambda_t \left(\frac{C_t}{C_{t-1}^h} \right)^{\frac{\sigma-1}{\sigma}} \frac{1}{C_t} - \beta h E_t \lambda_{t+1} \left(\frac{C_{t+1}}{C_t^h} \right)^{\frac{\sigma-1}{\sigma}} \frac{1}{C_t} = \psi_t, \tag{4}$$

$$1 - N_t: \quad b = w_t \psi_t, \tag{5}$$

$$M_t: \quad \gamma (M_t/P_t)^{-\varepsilon} = \psi_t - \beta E_t \psi_{t+1} (P_t/P_{t+1}), \tag{6}$$

$$B_{t+1}: \quad 0 = \psi_t - \beta R_t^G E_t \psi_{t+1} (P_t/P_{t+1}), \tag{7}$$

$$K_{t+1}: \quad 0 = \psi_t \left(1 + \varphi \eta X_t^{\eta-1} \right) - \beta E_t \psi_{t+1} \left[(1-\delta) \left(1 + \varphi \eta X_{t+1}^{\eta-1} \right) + z_{t+1} \right], \tag{8}$$

$$\begin{aligned} \psi_t: \quad & C_t + (M_t/P_t) + (B_{t+1}/P_t) + X_t = w_t N_t + z_t K_t + R_{t-1}^G (B_t/P_t) \\ & + [(M_{t-1} - \tau_t)/P_t] - \varphi X_t^\eta. \end{aligned} \tag{9}$$

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2.2. Firms

A continuum of monopolistically competitive firms is indexed by $j \in [0, 1]$. Firm j chooses price (P_{jt}), labor (N_{jt}), and capital (K_{jt}) to maximize profits, given by $P_{jt}(Y_{jt}/P_t) - w_t N_{jt} - z_t K_{jt}$. The demand function it faces is $(P_{jt}/P_t) = (Y_{jt}/Y_t)^{-(1-\rho)}$ and its production function is $Y_{jt} = A_t N_{jt}^{\alpha} K_{jt}^{1-\alpha}$, where Y_t is aggregate GDP, A_t is a technology shock, and $\alpha \in (0, 1)$. In symmetric equilibrium, profit-maximizing conditions are

$$N_t: \quad \alpha(Y_t/N_t) = \mu_t^G w_t, \tag{10}$$

$$K_t: \quad (1-\alpha)(Y_t/K_t) = \mu_t^G z_t, \tag{11}$$

where μ_t^G is the gross markup, which in steady state equals $1/\rho$.

2.3. Market Clearing

Finally, the economy is subject to the following resource constraint:

$$Y_t = C_t + X_t + \varphi X_t^{\eta}, \tag{12}$$

which differs from the usual closed-economy constraint due to capital adjustment costs.

2.4. Equilibrium

To investigate the dynamics of the model, we loglinearize the above optimality conditions and technological constraints around the steady state. The resulting equations are as follows (with the superscript *ss* denoting steady-state values):

Consumption

$$\frac{\beta h(\sigma-1)}{\sigma(1-\beta h)} E_t c_{t+1} = \frac{\beta h^2 \sigma - \beta h^2 + \beta h \sigma - 1}{\sigma(1-\beta h)} c_t - \frac{h(\sigma-1)}{\sigma(1-\beta h)} c_{t-1} - \psi_t + \frac{1-\beta h \rho_\lambda}{1-\beta h} \lambda_t, \tag{13}$$

Labor-market equilibrium

$$0 = y_t - n_t + \psi_t - \mu_t, \tag{14}$$

Money demand

$$0 = -(1/\varepsilon)\psi_t - rm_t - [1/(\varepsilon R^{ss})]R_t, \tag{15}$$

Euler equation

$$E_t \psi_{t+1} = \psi_t - r_t, \tag{16}$$

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Fisher equation

$$E_t\pi_{t+1} = R_t - r_t, \tag{17}$$

Quasi investment

$$(1-\delta)E_t x_{t+1} + \frac{(1-\alpha)\rho Y^{ss}/K^{ss}}{(\eta-1)\varphi\eta(X^{ss})^{\eta-1}} E_t(y_{t+1} - k_{t+1} - \mu_{t+1})$$
$$= x_t + \frac{1}{(\eta-1)\varphi\eta(X^{ss})^{\eta-1}} r_t, \tag{18}$$

Law of motion for capital

$$k_{t+1} = \delta x_t + (1-\delta)k_t, \tag{19}$$

Resource constraint

$$y_t = (C^{ss}/Y^{ss})c_t + \left[\left(X^{ss} + \varphi\eta X^{ss^{\eta}}\right)/Y^{ss}\right]x_t, \tag{20}$$

Production function

$$y_t = a_t + \alpha n_t + (1-\alpha)k_t, \tag{21}$$

where y_t, c_t, k_t, n_t, x_t, rm_t, μ_t, and a_t are the log deviations of Y_t, C_t, K_t, N_t, X_t, (M_t/P_t), μ_t^G, and A_t, respectively, from their steady-state values. Similarly, the Lagrange multiplier and demand shock should now be interpreted as log deviations from the steady state of the corresponding variables in the original nonlinear model. The demeaned quarterly net inflation rate is denoted as π_t, and the demeaned net nominal and real interest rate are denoted as R_t and r_t, respectively. To complete the model, we need a price-setting equation and a policy rule.

2.5. Price Setting

The natural real interest rate is the real rate that prevails in the case of fully flexible prices, whereas the actual real interest rate is the real rate that prevails under sticky prices.[12] Our procedure for obtaining a sticky-price equilibrium consists of two steps. First, we solve for the natural real interest rate (r_t^*) and natural output (in logs, y_t^*) by obtaining the flexible-price equilibrium of the loglinear model above.[13] This flexible-price equilibrium is characterized by a constant markup [$\mu_t = 0$; see, e.g., Ireland (1997)], and so, it can be obtained by imposing this condition. Second, we specify a model of gradual price adjustment based on the Calvo–Rotemberg model. Following Roberts (1995), we write the New Keynesian Phillips curve as

$$\beta E_t\pi_{t+1} = \pi_t + \alpha_\mu \mu_t, \tag{22}$$

where $\alpha_\mu > 0$. As a robustness check, we also report results for the Fuhrer and Moore (1995) pricing specification:

$$0.5E_t\pi_{t+1} = \pi_t - 0.5\pi_{t-1} + \alpha_\mu \mu_t. \tag{23}$$

In both models, the size of α_μ governs the degree to which prices are sticky: The larger is α_μ, the more flexible are goods prices.[14]

2.6. Shocks

There are two real shocks in this model: a technology and a real demand (IS) shock. These shocks are assumed to follow stationary AR(1) processes. We denote the AR parameter for a_t by ρ_a and its innovation process by e_{at}, while for the demand shock λ_t, the AR(1) parameter is ρ_λ and the innovation process is $e_{\lambda t}$.

2.7. Policy Rule

Under flexible prices, actual and natural real interest rates coincide irrespective of the monetary policy rule. But monetary policy has real effects when prices are sticky, so the policy rule specification will have implications for the real interest rate gap. We have examined the properties of the real interest rate gap under a variety of different rules, and in this paper focus on a policy rule estimated on U.K. data and described in Section 3.

2.8. Open-Economy Considerations

Our model has no explicit open-economy elements. Notwithstanding this, we believe that it has value as a model of interest-rate behavior for a small open economy. The conditions under which a small economy's real interest rate is dictated solely by global factors are highly stringent. Consequently, domestic rather than purely global factors may be important in the determination of real interest rates.

This point is particularly relevant for a model intended for monetary policy analysis. Inflation-targeting central banks typically use a short-term nominal-interest-rate instrument, which, combined with some inflation inertia, implies short-run policy influence over the short-term real interest rate, even though these economies are highly open and part of a global capital market. Indeed, many would consider the following essential for realistic policy analysis: (i) central bank control of nominal rates and short-run influence over real rates, (ii) inflation persistence, and (iii) investment in physical capital being important for the business cycle and being a channel through which monetary policy affects aggregate demand. Our closed-economy model can (under certain settings) satisfy (i)–(iii); yet very few existing open-economy DSGE models can. In the Obstfeld and Rogoff (1995) model, for example, real interest rates are invariant to domestic monetary policy and equal to the foreign rate every period. The failure of standard open-economy models to satisfy (ii) is shown in McCallum and Nelson (2000); and allowing for capital formation is often problematic in open-economy models.

We therefore proceed with our closed-economy analysis, but take the openness of the UK economy into account in our calibration. To this end, we note that openness affects goods market behavior by adding net export demand to total domestic aggregate demand. Consider the Euler equation for consumption, for simplicity abstracting from habit formation ($h = 0$). Then, $c_t = E_t c_{t+1} - \sigma r_t + \sigma(1 - \rho_\lambda)\lambda_t$, and iterations produce

$$c_t = -\sigma E_t \Sigma_{j=0}^{\infty} r_{t+j} + \sigma \lambda_t, \tag{24}$$

and so, consumption depends on current and expected future real short rates and the IS shock. Now, suppose that net foreign demand for domestic output, in logs nx_t, is given by

$$nx_t = b_1 q_t + \kappa_t, \tag{25}$$

where q_t is the log real exchange rate (an increase in q_t being a depreciation), κ_t a shock to foreign demand, and $b_1 > 0$. Combining this with the real interest parity condition $q_t = E_t q_{t+1} - r_t + u_t$, where the shock u_t includes both the foreign real interest rate and the exchange-rate risk premium, we can write (25) as

$$nx_t = -b_1 E_t \Sigma_{j=0}^{\infty} r_{t+j} + b_1 E_t \Sigma_{j=0}^{\infty} u_{t+j} + \kappa_t. \tag{26}$$

Thus, aggregate non investment demand is given by

$$s_c c_t + s_{nx} nx_t = -(s_c \sigma + s_{nx} b_1) E_t \Sigma_{j=0}^{\infty} r_{t+j} + \text{exogenous shocks}, \tag{27}$$

where s_c and s_{nx} are the steady-state shares of consumption and net exports in GDP. Open-economy influences raise the interest elasticity of (non investment) aggregate demand from $s_c\sigma$ to $(s_c\sigma + s_{nx} b_1)$ since the real interest rate affects the real exchange rate and hence foreign demand. We take this into account by calibrating σ to a higher value than suggested by the interest elasticity of consumption alone.

Finally, we discuss the relevance for our analysis of two other aspects of open-economy analysis. First, Euler equation (16) still holds under openness. However, some open-economy models assume finite horizons or an endogenous discount factor, making the external asset position relevant for consumption [e.g., Smets and Wouters (2002)]. The evidence suggests, however, that the business-cycle frequency dynamics of endogenous variables—the frequency with which the present paper is concerned—are little changed by the introduction of these features [Kollmann (1991), Kim and Kose (2000)].

Second, openness puts imports into the consumer price index, in principle creating an extra channel through which shocks affect inflation. However, there is little empirical support for exchange-rate terms in the Phillips curve [Stock and Watson (2001)], possibly because of slow or incomplete pass-through of exchange-rate movements to import prices. This suggests that our use of a Phillips curve with no explicit open-economy terms is a reasonable approximation.

3. MODEL CALIBRATION AND PROPERTIES

In this section we describe the responses of the natural real rate and the real-interest-rate gap in a calibrated version of our model. We first turn to our calibration.

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3.1. Calibration

The parameter values assigned to our model are similar to those found in earlier work on sticky-price DSGE models, including King and Watson (1996) and McCallum and Nelson (1999). These are presented in Table 1, and we now discuss the key choices.

The habit formation parameter is set to $h = 0.8$, in line with Fuhrer's (2000) estimate. Parameter σ indexes utility curvature and determines the interest elasticity of non investment expenditure. Because of habit formation, our calibrated value for σ must lie in (0, 1). Many studies set σ close to unity, but estimates of Euler and optimizing IS equations on U.S. data in Hall (1988), McCallum and Nelson (1999), Fuhrer (2000), and Ireland (2001) suggest a lower σ value—around 0.2.[15] As discussed in Section 2.8, the openness of the U.K. economy justifies a higher value. We therefore choose $\sigma = 0.6$.[16]

Capital adjustment cost parameters are calibrated so that (quasi) investment is considerably more interest elastic than consumption, but not implausibly so. The capital adjustment cost settings in Table 1 imply a semi-elasticity of investment with respect to the short-term real interest rate of about 3.2%.[17] In our presentation of model properties below, we indicate the effect on the results of our capital adjustment cost assumptions.

Before turning to the properties of the natural rate in our model, we summarize the evidence presented in Neiss and Nelson (2001) on the match of our model with U.K. data [when the model is solved using rule (28), below]. Our model essentially matches the autocorrelations of annual inflation, nominal interest rates, detrended output, and actual real rates and, to a lesser extent, several other dynamic

TABLE 1. Model calibration

Parameter	Description	Quarterly value
α	Labor share	0.64
β	Discount factor	0.99
σ	Parameter indexing the curvature of the utility function	0.6
h	Habit formation parameter	0.8
δ	Rate of depreciation	0.025
$1/\sigma\varepsilon$	Scale elasticity of money demand	1
φ	Capital adjustment cost parameter	0.75
η	Capital adjustment cost parameter	2
$1/\rho, \mu$	Steady-state gross markup	1.25
ρ_λ	AR(1) parameter, IS shock	0.33
ρ_a	AR(1) parameter, technology shock	0.95
σ^2_{ea}	Variance of technology innovations	$(0.007)^2$
$\sigma^2_{e\lambda}$	Variance of IS innovations	$(0.01)^2$
N^{ss}	Steady-state fraction of time in employment	0.33
α_μ	Degree of nominal rigidity under sticky prices. Calibrated value corresponds to a 75% probability that firm is unable to change price	0.086

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relationships, including the real/nominal interest-rate relation. The correlation between GDP and lags of the real rate [Corr(y_t, r_{t-k}) for $k > 0$] is generally positive in the model, which agrees qualitatively with U.K. data. The main exception is for $k = 0$, which is slightly negative (-0.03) in the model. The model's most serious drawback is that it generates negative values of Corr(y_t, r_{t-k}) for $k < 0$, which in the data are mostly positive. These comparisons suggest future work could focus on modifying the aggregate demand specification.

3.2. Response of the Natural Real Rate to Shocks

Figures 1A and 1B plot the model responses of the natural real interest rate (r_t^*) to technology and real demand shocks.[18] Three cases are considered: a setting of the model with no capital, capital formation with adjustment costs, and costless capital adjustment. We present the first and third cases to indicate the effect our specification of capital formation has on the behavior of the natural real rate.

Figure 1A indicates that a temporary 1% shock to technology raises r_t^* by about 5 basis points when capital adjustment is costless, but that it reduces it otherwise. In the no-capital case, a technology shock raises output and consumption today by more than in future periods. The current period's aggregate supply constraint on the community's ability to consume has been relaxed, and the constraint in future periods relaxed by a smaller and diminishing amount. Households would like to smooth their consumption of the higher output (especially given the habit formation in their preferences), and attempt to postpone much of their higher

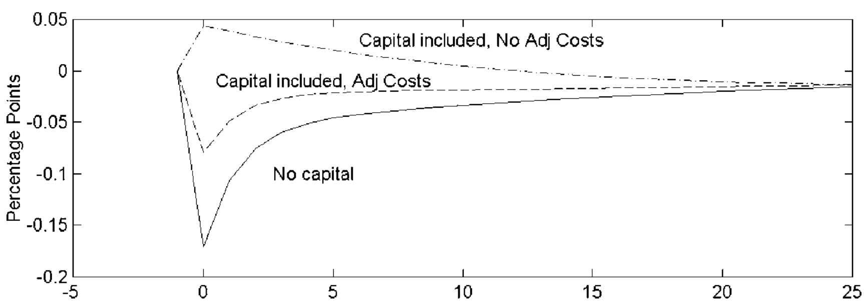

Figure 1A. Natural real rate response to technology shock.

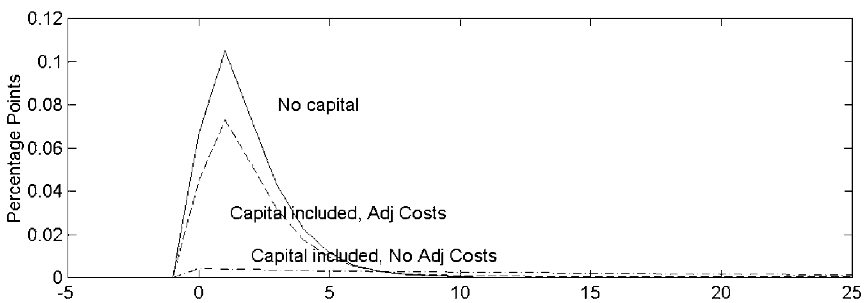

Figure 1B. Natural real rate response to real demand shock.

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consumption. But, in equilibrium, all output must be consumed today. The natural real interest rate declines to ensure that this occurs.[19]

When capital can vary, investment rises because the technology shock boosts profitability of production. This tends to raise r_t^*. Offsetting pressure comes from households' wish to save part of the extra income, and the disincentive to rapid investment from adjustment costs. The net effect is a fall in r_t^*, though less than in the no-capital case.

Figure 1B gives responses to a temporary 1% shock to real demand. Unlike a monetary policy shock, this affects values of real variables under price flexibility. The shock raises consumption demand. With fully flexible capital, only a small change in the real rate is needed to facilitate a decline in investment to make room for higher consumption; hence, the limited response of r_t^*. If there is no capital or if firms face large adjustment costs, then the real rate must rise by more to dampen the rise in consumption.

A common feature of Figures 1A and 1B is that the natural real interest rate responds more when capital cannot adjust costlessly.[20] A flexible-price, flexible-capital model implies almost no variation in r_t^*, since quantities bear the bulk of the adjustment.

Responses of potential GDP (Figures 2A and 2B) are the mirror image of the real rate responses. The model setting without capital is associated with the *smallest*

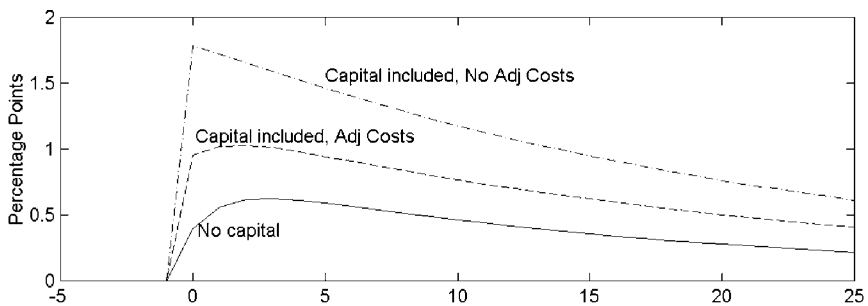

FIGURE 2A. Potential output response to technology shock.

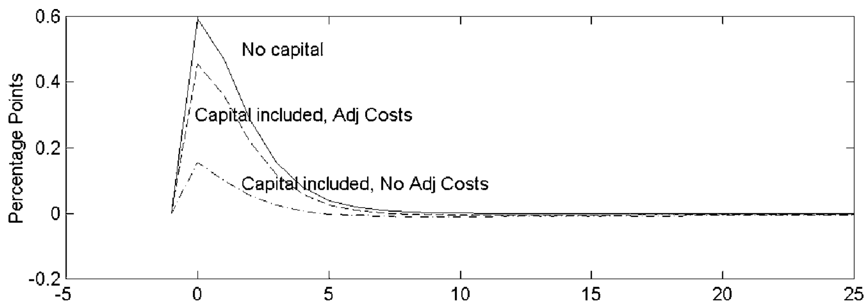

FIGURE 2B. Potential output response to real demand shock.

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increase in potential GDP in response to the technology shock and the *largest* in response to a real demand shock. Impediments to adjusting capital restrain the rise in investment, and thus output, that a technology shock would otherwise induce. And when the capital stock cannot be varied at all, it is not possible for higher consumption demand to be satisfied partly by a rise in consumption's share of income (which a rise in real rates tends to induce because investment is more interest-elastic than consumption). So, the real demand shock leads to sharp increases in both real interest rates and output.

As noted in Section 2.8, our choice of $\sigma = 0.6$ serves to approximate the effects of openness on real interest rate dynamics. We checked the accuracy of this approximation by examining the r_t^* response in the McCallum and Nelson (2000) open-economy model to a domestic IS shock, with utility given by equation (1) but with $\sigma = 0.2$. Because the MN model has no capital, this experiment is an open-economy analogue of the no-capital case in Figure 1A. The size and shape of the open-economy r^* response closely resemble those in Figure 1A [see Neiss and Nelson (2001)]. Thus, our approximation of open-economy effects appears to be a good one.

3.3. Response of the Real-Interest-Rate Gap to Shocks

We now turn to the sticky-price case, and examine the response of the real interest rate gap—the spread between actual and natural real interest rates—to shocks. This requires that we choose a price-adjustment specification and policy rule to close the model. We use Calvo pricing here. For the monetary policy rule, an estimated rule on 1980–1999 U.K. data would be desirable because our empirical results will examine the behavior of inflation in that sample. But changes in the U.K. monetary policy regime over 1980–1999 mean full-sample estimates are not very reliable. Instead, we use the following U.K. policy rule estimated by Nelson (2001) for 1992.4–1997.1:

$$4^* R_t = \rho_R 4^* R_{t-1} + (1-\rho_R) 1.267 E_t \Delta_4 p_{t+1} + (1-\rho_R) 0.47 y_t + 4^* e_{Rt}, \quad \textbf{(28)}$$

where $\rho_R = 0.29$, $\Delta_4 p_{t+1} = \Sigma_{j=0}^{3} \pi_{t+1-j}$ is one-period-ahead annual inflation, and the variance of the policy shock e_{Rt} is $(0.001)^2$. Rule (28) is similar to the empirical Taylor (1993) rules of Clarida et al. (1999). Under (28), the nominal interest rate responds to expected future inflation (by more than one-for-one in the long run), and detrended output (y_t). Policymakers respond to detrended output presumably because they regard it as a good proxy for the output gap—an assumption common among empirical researchers, though not one for which we will find support.

Figures 3A to 3C display the response of the real-interest-rate gap, detrended output, and the output gap to technology, demand, and policy shocks, respectively, with policy rule (28), and capital formation subject to adjustment costs.

Figure 3A depicts the effect of a technology shock. The shock generates an increase in the real-interest-rate gap—an effective policy tightening—for two reasons. First, the natural rate falls, and so, for a given actual real interest rate, monetary policy is tighter. Second, the policy rule responds to the level of output, and so, a productivity shock induces a rise in the nominal, and hence real, interest rate.

An IS shock, however, raises both the actual and natural rate (Figure 3B). Thus, in contrast to the technology-shock case, the policy response—a tightening in response to higher output—does tend to contain the opening-up of a real rate gap. However, the rise in the actual rate is less than the rise in the natural rate, and so, the overall policy stance is looser.

Figure 3C simply illustrates that a monetary tightening affects only the actual real interest rate. The real rate gap therefore rises one-for-one with the actual rate. The output and output-gap responses are identical because monetary policy cannot affect potential GDP.

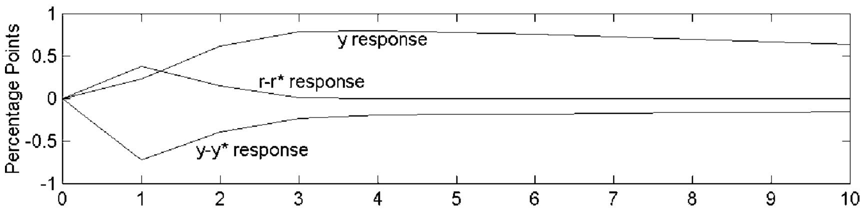

FIGURE 3A. Responses to technology shock.

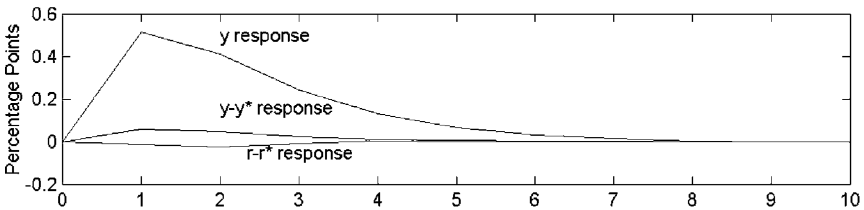

FIGURE 3B. Responses to real demand shock.

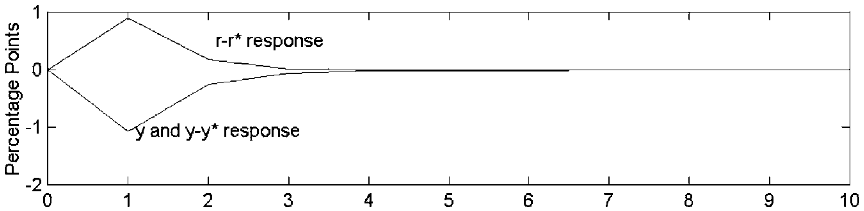

FIGURE 3C. Responses to monetary policy shock.

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The response of the real-interest-rate gap to different shocks illustrates how a policy rule like (28), which responds to the level of output, can yield perverse results. Effectively, the rule responds symmetrically to positive supply-and-demand shocks. This is because it does not take into account that the natural values of both the real interest rate and output have been affected by the real shocks. The natural real interest rate falls in response to the technology shock, and so, the monetary policy response should be to lower, not raise, interest rates. In the case of a real demand shock, the natural rate increases, and a policy aimed at minimizing output gap and inflation variations would allow the real rate to rise in line with the natural rate. However, rule (28) does not raise the real rate enough in response to the shock to maintain a zero gap. As discussed below, a policy rule that responds to output leads, especially in the technology-shock case, to counterproductive results because detrended output is a poor indicator of the output gap in our model. Figures 3A–3C demonstrate the mirror-image relationship between the output gap and the real-interest-rate gap, which we also discuss below.

4. INDICATOR PROPERTIES OF THE REAL-INTEREST-RATE GAP

We now examine some dynamic properties of our model, focusing on statistics that describe the relationship between the real-interest-rate gap, aggregate demand measures, and inflation. Table 2 gives selected correlations and standard deviations for four model settings (constant or fluctuating capital, with Calvo or

Table 2. Model statistics

	Model setting			
Statistic[a]	No capital, Calvo pricing	Capital with adjustment costs, Calvo pricing	No capital, FM pricing	Capital with adjustment costs, FM pricing
$\mathrm{SD}(y_t)$	1.59	2.22	1.76	2.70
$\mathrm{SD}(y_t^*)$	1.81	2.82	1.81	2.82
$\mathrm{SD}(y_t - y_t^*)$	0.30	0.80	0.19	0.69
$\mathrm{SD}(r_t)$	1.05	1.06	1.25	1.32
$\mathrm{SD}(r_t^*)$	0.96	0.56	0.96	0.57
$\mathrm{SD}(r_t - r_t^*)$	1.41	1.21	1.68	1.53
$\mathrm{Corr}(y_t, y_t^*)$	0.99	0.98	0.99	0.97
$\mathrm{Corr}(y_t, y_t - y_t^*)$	−0.68	−0.68	−0.20	−0.06
$\mathrm{Corr}(r_t, r_t^*)$	0.02	−0.03	−0.14	−0.17
$\mathrm{Corr}(r_t, r_t - r_t^*)$	0.73	0.88	0.82	0.93
$\mathrm{Corr}(y_t, r_t)$	−0.02	−0.048	−0.04	−0.03
$\mathrm{Corr}(y_t, r_{t-1})$	−0.02	0.06	−0.004	0.09
$\mathrm{Corr}(y_t - y_t^*, r_t - r_t^*)$	−0.58	−0.74	−0.93	−0.97

[a] SD(•) is the standard deviation; Corr(•, •) is the simple correlation coefficient. Standard deviations of interest-rate variables are annualized. Statistics reported are computed from analytical formulas for the model moments.

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Fuhrer–Moore price setting). The variables focused upon are log output (y_t), the real interest rate (r_t), log natural output (y_t^*), the natural real rate (r_t^*), the output gap ($y_t - y_t^*$), and the real rate gap ($r_t - r_t^*$).

Consistent with Figures 3A–3C, Table 2 indicates a negative relation between the output and real rate gaps for all model settings (correlations from -0.58 to -0.97).[21] If our model had white-noise shocks and no capital or habit formation, there would be a perfect inverse relation between the two gaps—see equation (24). The more general production, preference, and shock specifications break the exact relation, but it remains a close one.

On the other hand, differences between the two series emerge when we analyze the behavior of their individual components. In particular, the behavior of the real interest rate (r_t) is a reasonable approximation of real-rate gap behavior; the correlation between the two series is high. Correspondingly, r_t^* varies much less than r_t. This contrasts with the finding by Rotemberg and Woodford (1997), who report a standard deviation of r_t^* of 3.7%, higher than the actual rate, but is consistent with King and Watson's (1996) finding of low real rate variability under price flexibility. Rotemberg and Woodford's estimate may be attributable to larger shock variances than are typical in other studies.

By contrast, the level of y_t is not a good index of the output gap $y_t - y_t^*$. In fact, the two series are *inversely* related, with correlations from -0.06 to -0.68, and the output gap's standard deviation is well below that of output. Because of the absence of steady-state growth in our model, y_t in our model corresponds to detrended output in the data. These results call into question the widespread practice of measuring the output gap by detrended (actual) GDP. Indeed, variations in detrended GDP are dominated by variations in potential GDP—the two series' correlation ranges from 0.97 to 0.99—and so, it is invalid to treat potential GDP as constant or growing steadily.

This finding has implications for the interpretation of NKPC estimates of Sbordone (2002) and Galí and Gertler (1999). Most sticky-price DSGE models imply a close relation of the output gap to real marginal cost, so that either series is appropriate as the forcing process in estimation of the NKPC. However, Galí and Gertler, for example, find that estimates of the NKPC on U.S. data are plausible and correctly signed if marginal cost (equivalently, the markup) is used, but incorrectly signed if detrended output is the driving process. They recognize that this could arise from detrended output being a poor output gap proxy, but Galí et al. (2002) argue instead for models with labor-market imperfections that break the relation of the output gap to marginal cost. Our model instead suggests that a key factor behind the NKPC estimates is that detrended GDP is a weak output gap proxy, and that the failure of NKPC estimates with detrended GDP is not compelling evidence of the importance of labor rigidities.

To examine the forecasting properties of the real-interest-rate gap for inflation, we now report averages across model simulations of estimates of the regression,

$$\Delta_4 p_t = b_{10} + b_{11}(r_{t-1} - r_{t-1}^*) + b_{12}\Delta_4 p_{t-1},$$

which can be thought of as the inflation equation in a one-lag VAR consisting of annual inflation, $\Delta_4 p_t$, and the real rate gap. We compare these to results with the output gap:

$$\Delta_4 p_t = b_{20} + b_{21}(y_{t-1} - y^*_{t-1}) + b_{22}\Delta_4 p_{t-1}.$$

We supplement these results by reporting regressions that replace the gap variables with their observable components: r_{t-1} instead of $(r_{t-1} - r^*_{t-1})$, and y_{t-1} instead of $(y_{t-1} - y^*_{t-1})$. These last two regressions can be thought of as what a researcher might estimate if they approximated the natural real rate by a constant in constructing a real-interest-rate gap series, or potential output by a trend in constructing an output gap series.

Regressions on the artificial datasets are summarized in Table 3,[22] and show that both the lagged real rate gap and lagged output gap are highly significant when each is added to an autoregression for inflation. Moreover, the signs of the estimated coefficients have economic interpretations: The positive-output-gap coefficient reflects an "excess demand" influence on inflation, and the real-rate-gap coefficient reflects the inverse relationship between the real rate gap and excess demand.[23]

Table 3. Regressions on simulated data: Dependent variable $\Delta_4 p_t$ (annual inflation rate)

	Coefficient on			
Model setting	$r_{t-1} - r^*_{t-1}$	$\Delta_4 p_{t-1}$	SEE	SD($\Delta_4 p_t$)
No capital, Calvo	−0.90 (0.08)	0.96 (0.01)	0.40	2.44
Capital with adj. costs, Calvo	−1.16 (0.11)	0.97 (0.01)	0.47	3.03
	Coefficient on			
	$y_{t-1} - y^*_{t-1}$	$\Delta_4 p_{t-1}$	SEE	SD($\Delta_4 p_t$)
No capital, Calvo	2.80 (0.19)	0.72 (0.02)	0.35	2.44
Capital with adj. costs, Calvo	0.70 (0.06)	0.87 (0.01)	0.45	3.03
	Coefficient on			
	r_{t-1}	$\Delta_4 p_{t-1}$	SEE	SD($\Delta_4 p_t$)
No capital, Calvo	−1.00 (0.13)	1.01 (0.01)	0.44	2.44
Capital with adj. costs, Calvo	−1.20 (0.14)	0.99 (0.01)	0.49	3.03
	Coefficient on			
	y_{t-1}	$\Delta_4 p_{t-1}$	SEE	SD($\Delta_4 p_t$)
No capital, Calvo	0.00 (0.04)	0.98 (0.02)	0.40	2.44
Capital with adj. costs, Calvo	−0.05 (0.05)	0.95 (0.03)	0.47	3.03

Note: Numbers reported in cols. 2 and 3 are the means across simulations of parameter estimates and their corresponding standard errors.

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A key question is how much of the fit of these regressions is due to the explanatory power provided by the "unobservable" components of the gaps. The answer is provided by the regressions in Table 3 that use actual values of r_t and y_t as regressors rather than the corresponding gaps. For the regressions with r_t, the omission of r_t^* results in a loss of some explanatory power (confirming that $r_t - r_t^*$ is the appropriate inflation indicator) but—because the low r_t^* variability—the loss is minor. So, the real interest rate again seems a reasonable proxy for the real rate gap for the purpose of forecasting inflation.

By contrast, replacing $y_{t-1} - y_{t-1}^*$ by y_{t-1} produces loss of significance and a "wrongly" signed coefficient. Thus, both the real rate gap and the output gap can serve as indicators of inflation—but leading-indicator properties of the rate gap largely come from the observable component, whereas those of the output gap are due to the unobservable potential GDP component. Our results suggest that cyclical fluctuations in potential GDP are great enough to make detrended output an unreliable output gap proxy, but that the accompanying fluctuations in the natural real rate are quite small. Consequently, the real interest rate has value as an indicator of demand pressure and inflation.

5. EMPIRICAL PROPERTIES OF THE REAL-INTEREST-RATE GAP

We now construct, for the United Kingdom, empirical counterparts to our model's natural real interest rate and real-interest-rate gap series, and investigate the relation of the resulting gap series to inflation. In our model, r_t^* is a combination of technology shocks (a_t) and IS shocks (λ_t). To measure a_t, we use Solow residuals constructed from U.K. data.[24] To measure λ_t, we use equations (13), (16), and (17), which imply that λ_t is a linear combination of $E_t \Delta c_{t+2}$, $E_t \Delta c_{t+1}$, Δc_t, $E_t \pi_{t+1}$, and R_t. We measure R_t by the Treasury bill rate, c_t by the log of private nondurables consumption, and π_t by the quarterly RPIX inflation. We measure expected values by forecasts from an eight-lag VAR estimated over 1976.1–1999.4, consisting of R_t, Δc_t, and π_t, plus two dummy variables, DERM_t and $\mathrm{D924}_t$. These dummies capture UK monetary policy regime changes, and are nonzero during the UK's membership of the Exchange Rate Mechanism (ERM) (1990.4–1992.3) and the inflation targeting period (from 1992.4), respectively. We then use our baseline model with capital formation to generate r_t^* and $(r_t - r_t^*)$ series for the United Kingdom.

Table 4 gives simple correlations between annual UK inflation ($\Delta_4 p_t$) and lags of our real-rate gap series. Correlations between inflation and the actual real rate are also given. The inflation/real-rate gap correlation is negative, and tends to be more negative than the correlations between inflation and the real rate, in keeping with our model's predictions.

Our model describes fluctuations for a specific steady state and monetary policy rule. However, over 1980–1999, there were changes in the UK's monetary policy regime—notably the shifts to ERM membership and inflation targeting—and different unconditional mean real interest rates across regimes; the mean was higher before inflation targeting. In evaluating our real rate-gap series, it is desirable to account for these shifts. Therefore, Table 4 also gives *partial* correlations, controlling

TABLE 4. Correlations between inflation and real-interest-rate gap, UK data for sample period 1980.1–1999.4

Statistic	$k=0$	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$	$k=7$	$k=8$
Simple correlations									
Corr $(\Delta_4 p_t, r_{t-k} - r^*_{t-k})$	−0.13	−0.21	−0.26	−0.40	−0.47	−0.40	−0.42	−0.46	−0.48
Corr $(\Delta_4 p_t, r_{t-k})$	0.01	−0.10	−0.18	−0.35	−0.44	−0.38	−0.42	−0.47	−0.52
Partial correlations, conditional on policy regime breaks									
Corr $(\Delta_4 p_t, r_{t-k} - r^*_{t-k})$	−0.50	−0.54	−0.58	−0.66	−0.73	−0.62	−0.60	−0.60	−0.59
Corr $(\Delta_4 p_t, r_{t-k})$	−0.37	−0.45	−0.52	−0.61	−0.70	−0.61	−0.59	−0.61	−0.61

for the two shifts. Essentially, these are correlations between the real rate, the real-rate gap, and inflation, once all have been purged of their correlation with the dummy variables DERM_t and D924_t. The partial correlations are more negative than the simple correlations, confirming that allowing for regime changes is important. Also, both the simple and the partial inflation/real-rate-gap correlations tend to be more negative for low k than those of inflation and the real rate.

Finally, we estimate on 1979.1–1999.4 U.K. data the equation estimated on artificial data in Table 3: a regression of $\Delta_4 p_t$ on $\Delta_4 p_{t-1}$ and $(r_{t-1} - r^*_{t-1})$. A constant and the policy-shift dummies are also included. The coefficient estimate on $(r_{t-1} - r^*_{t-1})$ is negative, and a test for excluding it rejects with p-value higher than 0.001. Thus, U.K. data suggest that the real-interest-rate gap is a valuable indicator of inflation.

6. CONCLUSIONS

This paper examined what a DSGE model has to say about the real-interest-rate gap as an indicator of inflation. A shortcoming of the concept is that it requires the construction of an "unobservable" natural real rate series. However, the same is true of the output-gap concept, and our results suggest that output-gap series as often constructed can be misleading. Under these circumstances, a real-interest-rate gap series is useful in evaluating monetary policy stance and inflationary pressure, in keeping with the neo-Wicksellian framework of Woodford (1999, 2000). Our results suggest that there are benefits to allowing for natural rate movements when modeling inflation, and basing the construction of natural rate series on economic theory. The quantitative benefits may be modest, however, because of the low degree of cyclical variation in the natural rate.

NOTES

1. See Svensson (1999) for a recent criticism of monetary aggregates.
2. As shown in Poole (1970), an interest-rate instrument rule typically implies that output, the interest rate, and prices are insulated from the effect of money demand shocks.
3. See Eika et al. (1996) for a discussion of MCI weights.

4. The UK real rate data used are those constructed in Section 5.

5. Similarly, production-function-based approaches to measuring potential GDP typically do not keep track of the distinction between actual and natural values of productive inputs. For example, measuring flexible-price labor supply by the total labor force involves assuming inelastic labor supply. A DSGE model such as ours provides a way of keeping track of flexible-price values of variables and of their response to real shocks.

6. This is so even for models, such as those in Beaudry and Guay (1996), that enrich the dynamics of real-business-cycle models, but maintain the assumption of price flexibility.

7. Here, we presume that the stickiness of prices lasts more than one period.

8. Woodford (2000) adds endogenous capital formation to his model. One difference between his model and ours is that our calibration of preference parameters, which strongly affects the interest elasticity of aggregate demand, is more in line with empirical studies such as Fuhrer (2000) and Ireland (2000). Another difference is our method of measuring the natural rate when there are endogenous state variables (see the Appendix).

9. Equivalently, with the log markup, as in equation (22).

10. Our specification of capital adjustment costs follows Casares and McCallum (2000).

11. An antecedent of the recent use of habit formation in DSGE models is Duesenberry's (1949, pp. 24–25) argument for a "consumption habit formation process" in the modeling of U.S. consumption behavior.

12. As in other sticky-price DSGE models, the assumption of monopolistic competition among firms provides groundwork for the assumption of gradual price setting.

13. A monetary policy rule needs to be appended to this system to complete the model, but the solution for potential output will be invariant to the rule selected.

14. For the Calvo model, the flexible-price equilibrium is reached as $\alpha \to \infty$.

15. Higher σ values are estimated by Rotemberg and Woodford (1997) and Amato and Laubach (2002), but these are not based on conventional econometric estimation procedures such as instrumental variables or maximum likelihood.

16. Boldrin et al. (2001) take logarithmic preferences (the limit as $\sigma \to 1$) and set $h = 0.9$. Beaudry and Guay (1996) assume log utility and estimate h to be 0.3 to 0.5. Woodford (2000) sets $\sigma \to 1$ and $h = 0$. All these parameter settings seem to make consumption too interest elastic, compared to the estimates cited above.

17. Capital adjustment costs are also important for generating realistic output behavior under sticky prices. Without capital adjustment costs, output exhibits an extremely elastic response to monetary policy shocks [see Casares and McCallum (2000)].

18. Details on how we computed these impulse responses are presented in the Appendix.

19. Figure 1A matches Woodford's (2001b) result that temporary a_t shocks (i.e., $0 \le \rho_a < 1$) induce a countercyclical natural rate response. If a_t movements were permanent (i.e., $\rho_a = 1.0$), and there were no habit formation in preferences, the technology shock would leave the equilibrium real rate unchanged, as in Clarida et al. (1999).

20. This is true even if preferences do not exhibit habit formation, although habit formation magnifies the real rate responses to both technology and real demand shocks.

21. Our findings of a negative detrended output/output-gap relationship, a strong real rate/real-rate gap relationship, and a close inverse relationship between the real-rate gap and output gaps are robust to assuming different values of σ and much lower capital adjustment costs. For example, setting $\varphi = 0.10$ (which delivers an investment standard deviation twice that of our baseline calibration), we have $\mathrm{Corr}(y_t, y_t - y_t^*) = -0.38$, $\mathrm{Corr}(r_t, r_t - r_t^*) = 0.96$, and $\mathrm{Corr}(y_t - y_t^*, r_t - r_t^*) = -0.85$. Keeping $\varphi = 0.75$ but using $\sigma = 0.9$ instead of our baseline $\sigma = 0.6$, we have $\mathrm{Corr}(y_t, y_t - y_t^*) = -0.38$, $\mathrm{Corr}(r_t, r_t - r_t^*) = 0.49$, and $\mathrm{Corr}(y_t - y_t^*, r_t - r_t^*) = -0.80$.

22. The reported regressions are for Calvo price setting; results with FM price setting are similar [Neiss and Nelson (2001)].

23. It would not be sensible to interpret the estimated coefficients further beyond this sign interpretation because they are reduced form. In particular, if interpreted as structural equations, the estimated

inflation regressions appear to suggest that long-run nonzero output and real-interest-rate gaps can be obtained by altering the long-run inflation rate. However, the structural model that generated the regression data does not have this property.

24. See the working paper version of this article [Neiss and Nelson (2001)] for data sources. We quadratically detrend our empirical Solow residual series to construct an empirical counterpart to the a_t series.

25. No e_{Rt} term appears because monetary neutrality holds under flexible prices.

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APPENDIX: CALCULATION OF OUTPUT AND INTEREST-RATE GAPS

Here we describe our procedure for calculating (log) potential output (y_t^*) and the natural real interest rate (r_t^*) in our model. This permits our computation of impulse responses for the natural real rate in Section 3, and analysis of the output gap ($y_t - y_t^*$) and real interest rate gap ($r_t - r_t^*$) in Section 4.

Because of capital and habit formation in our model, both the period t capital stock and the prior period's consumption level are state variables, in addition to the exogenous technology (a_t), real demand (λ_t), and monetary policy (e_{Rt}) shocks. The model's solutions for y_t^* and r_t^* cannot be written as simple functions of the exogenous shocks alone. To see this, consider output under sticky prices. From the production function,

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$$y_t = a_t + (1-\alpha)k_t + \alpha n_t. \qquad \textbf{(A.1)}$$

Similarly, output under flexible prices, or potential output, may be written as

$$y_t^* = a_t + (1-\alpha)k_t^* + \alpha n_t^*, \qquad \textbf{(A.2)}$$

where asterisks denote flexible-price values. Had we assumed inelastic labor supply and an exogenous capital stock, then y_t^* would simply be a linear combination of two current-dated exogenous variables, a_t and k_t^*, and calculation of $y_t - y_t^*$ would be trivial. However, for the more general case the solution to k_t^* and n_t^* are functions of the whole state vector:

$$k_t^* = \phi_1 \boldsymbol{s}_{t-1}^*, \qquad \textbf{(A.3)}$$

$$n_t^* = \phi_2 \boldsymbol{s}_{t-1}^*, \qquad \textbf{(A.4)}$$

where the ϕ_i are 1×4 coefficient vectors, and $\boldsymbol{s}_t^* = [k_t^*\, c_{t-1}^*\, a_t \lambda_t]'$ is the state vector under flexible prices.[25]

To calculate y_t^*, one procedure is to condition on *actual* capital and consumption k_t and c_{t-1}—in effect replacing the unobserved flexible-price variables in $\boldsymbol{s}_t^*$ with their sticky-price counterparts, and using equations (30)–(32) to compute y_t^*. This procedure seems illegitimate because the behavior of the two endogenous state variables, capital and consumption, will be a function of monetary shocks and the monetary policy rule under sticky prices. However, Woodford (2000) has made a case for this method of defining the natural output and interest rate, and we discuss it further below.

The following is our method for calculating y_t^* and r_t^*. In essence, we want to express y_t^* and r_t^* as functions of the exogenous variables only—effectively "substituting out" k_t^* and c_{t-1}^* of the solution equations. To this end, we note that since we are assuming (vector) autoregressive processes for the exogenous elements of $\boldsymbol{s}_t^*$, $\boldsymbol{s}_t^*$ follows the law of motion $\boldsymbol{s}_t^* = \boldsymbol{R}\boldsymbol{s}_{t-1}^* + \boldsymbol{\Psi}_\varepsilon \varepsilon_t$, where ε_t is a vector of innovations. $\boldsymbol{s}_t^*$ thus has a vector moving-average representation giving each element of $\boldsymbol{s}_t^*$ as a (possibly infinite) distributed lag of the ε_t. Specifically, k_t^* and c_t^* may be written as $k_t^* = f_1(\varepsilon_{t-1}, \varepsilon_{t-2}, \varepsilon_{t-3}, \ldots)$ and $c_t^* = f_2(\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots)$, where the $f_i(\bullet)$ are linear functions. It follows that

$$y_t^* = a_t + f_3(\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots). \qquad \textbf{(A.5)}$$

Since the shocks a_t and λ_t each are infinite moving averages of the ε_t, it follows that

$$y_t^* = f_4(a_t, a_{t-1}, \ldots, \lambda_t, \lambda_{t-1}, \ldots). \qquad \textbf{(A.6)}$$

By the same argument, r_t^* may be written as

$$r_t^* = f_5(a_t, a_{t-1}, \ldots, \lambda_t, \lambda_{t-1}, \ldots). \qquad \textbf{(A.7)}$$

To obtain the impulse responses and simulation results reported in the text, we approximate the right-hand sides of (A.6) and (A.7) by a long but finite distributed lag of a_t and λ_t. Our complete procedure for calculating the gaps is as follows:

(i) Solve the model under flexible prices.
(ii) Simulate the model; using data generated from these simulations, run regressions of the form $y_t^* = c_1 a_t + \cdots + c_j a_{t-j} + d_1 \lambda_t + \cdots + d_j \lambda_{t-j}$, and $r_t^* = g_1 a_t + \cdots + g_j a_{t-j} + h_1 \lambda_t + \cdots + h_j \lambda_{t-j}$ for a finite j that is high enough to generate a good fit.

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(iii) Store the c_i, d_i, g_i, and h_i coefficients (actually, averages of each coefficient across simulations).

(iv) Solve the model incorporating sticky prices. Augment the exogenous variable vector with y_t^* and r_t^*, where these are defined as the indicated linear combinations of current and lagged a_t and λ_t.

(v) Define the output gap as $y_t - [y_t^*$ as defined in step (iv)], and the real interest rate gap as $r_t - [r_t^*$ as defined in step (iv)]. We found that this procedure generates measures of r_t^* and y_t^* that are near-perfect approximations for the correct measures given by equations (A.6) and (A.7).

Woodford (2000) has argued instead for defining the natural rate conditional on the realized (sticky-price) values of endogenous state variables such as the capital stock. He argues that this is preferable to our procedure because "[i]t seems odd to define the economy's "natural" level of activity . . . in a way that makes irrelevant the capital stock that actually exists and the effects of this upon the economy's productive capacity" (Woodford, 2000, p. 67). However, our procedure does not make the actual capital stock irrelevant: while the flexible-price capital stock concept appears in our definitions of natural levels of variables, it is the actual, existing capital stock that appears in the production function, capital law of motion, capital adjustment cost function, and resource constraint of the sticky-price economy.

We discuss some of the differences between our method and Woodford's in more detail in Neiss and Nelson (2001). To us, the major two advantages of our procedure are: (a) Woodford's method implies that a surprise loosening of monetary policy can (permanently) raise potential output, whereas our method implies that it cannot; (b) our natural rate definition readily extends to the case of endogenous state variables beside the capital stock. The physical constraint evoked by Woodford is applicable only to state variables that enter the production function, whereas our model contains an endogenous state variable—lagged consumption—that does not enter the production function.