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Squishy oscillations

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The dynamics of soft porous media involves complex interactions between fluid flow and elasticity. The recent paper by Fiori *et al.* (*J. Fluid Mech.*, vol. 974, 2023, A2) highlights phenomena relating to the periodic loading of such poro-elastic media, including hysteresis and the localisation of deformation at high frequencies. These effects could result in rectification and steady streaming in many important applications.

Key words: porous media, flow–structure interactions

1. Overview

Poro-elasticity characterises a wide range of materials including soils and rocks, the tissues of plants and animals, and various gels involved in manufactured pharmaceuticals. These materials fundamentally consist of at least two phases: an elastic scaffold and a permeating fluid. In many cases, such as a bathroom sponge, the scaffold is intrinsically elastic, and flows are driven by external mechanical forces. In others, such as hydrophilic gels (hydrogels), the scaffold can derive its elasticity in part from interactions with the interstitial fluid, and flows can be driven by internal osmotic gradients. There is a vast literature discussing the physico-chemical characterisations of different systems. However, from a continuum, fluid-mechanical perspective, the macroscopic characteristics of flows through all these types of poro-elastic media are similar, describable by the same equations. There is relatively little written from the continuum perspective, and the paper by Fiori, Pramanik & MacMinn (2023) provides new insight into the fundamental character of deformation and flow in poro-elastic media.

The particular focus of the paper by Fiori *et al.* is the response of a poro-elastic medium to periodic forcing. Such forcing might be found in the perivascular system surrounding arterial blood vessels in the brain (e.g. Kelley 2021), the forcing by ocean tides of melt-water flows in sub-glacial till (Warburton, Hewitt & Neufeld 2023), and the response of ground water to solid-Earth tides (Allègre *et al.* 2016), to give just a few examples in very different contexts and at vastly different scales. Two principles are highlighted by this study. The first is that, although the mechanical description is of fluid flow, the response of

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the system is diffusional in character, giving rise to the sorts of evanescent diffusion waves seen in the thermal response of the ground and of thick-walled buildings to seasonal cycles and typified by Stokes's second problem of a flat plate oscillating in its own plane adjacent to a viscous fluid with inertia. The second is that nonlinearities in the material properties of a poro-elastic medium and in the nature of the forcing can lead to pronounced hysteresis.

2. Fundamentals

As described above, poro-elastic media exist at widely different scales and in different contexts, studied by scientists from different disciplines. When that happens, terminology can sometimes limit the cross-fertilisation of ideas. For example, a physical chemist or colloid scientist might normally describe the flux of one component of a mixture relative to another as driven by gradients in chemical potential or concentration, whereas an earth scientist might rather describe groundwater flow as driven by gradients in pore pressure. Additionally, the latter might describe the stresses within the porous scaffold as the effective stress in the manner of Terzaghi (1943), while the former might describe the same pressures as osmotic. At the macroscopic, continuum level, these physical descriptions are equivalent. So, while Fiori *et al.* use the language of soil scientists, their results are equally applicable to colloidal gels and biological tissue.

A macroscopic starting point is to consider the pressure P measured by a transducer large enough to sample both phases of the mixture, be it a solution (mixed at the molecular scale), a colloidal suspension, a gel, a sponge or a rock. Additionally, consider the pressure p measured in a chamber separated from the mixture by a semi-permeable membrane that allows just one component (the fluid) to pass freely. The difference $P - p$ is the osmotic pressure $\varPi$. This description is familiar in the context of salt solutions, for example, but it is applicable and useful in all the contexts mentioned above. For example, osmotic pressure thus defined is used as an important descriptor for particle suspensions (Deboeuf *et al.* 2009). Whereas the total pressure P and the solvent pressure p are dependent variables of the macroscopic system, the osmotic pressure $\varPi(\phi)$ is a material property dependent on the concentration (volume fraction) of the solute in the mixture ϕ.

Armed with these definitions, it can be shown (Peppin, Elliot & Worster 2005) that Darcy's law for flow through a porous medium and Fick's law for the diffusion of a solute are equivalent, with the permeability k and the bulk diffusivity D related by

$$D = \frac{kM}{\mu}, \tag{2.1a}$$

where

$$M = \phi \frac{\mathrm{d}\varPi}{\mathrm{d}\phi} \tag{2.1b}$$

is the osmotic modulus and μ is the dynamic viscosity of the solvent. Note that the bulk diffusivity of a medium (tendency towards uniform concentration) is distinct from the self-diffusivity of components of the medium. They are equal for solutions but the self-diffusivity is essentially zero for concentrated suspensions and gels while the bulk diffusivity is substantial. It is unusual to think of the permeability of a salt solution or the diffusivity of a porous medium but this universal equivalence comes into its own when discussing colloidal suspensions, gels and poro-elastic media. The diffusivity given by (2.1) is that derived in Fiori *et al.* once one identifies the osmotic pressure $\varPi$ with $-\sigma'$, where σ' is the effective stress of the elastic scaffold, given that $\phi = 1 - \phi_f$, where ϕ_f is the porosity of the medium, the volume fraction of solvent (interstitial fluid).

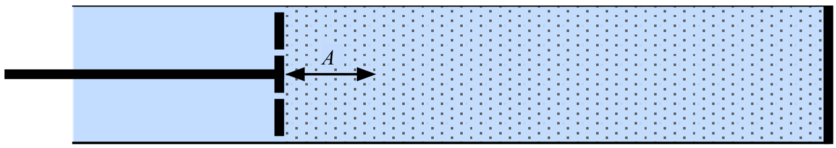

Figure 1. Schematic diagram of a saturated poro-elastic medium between a porous piston to the left and an impermeable wall to the right. Oscillatory compression is begun from time $t = 0$. After Fiori *et al.* (2023).

3. Hysteresis

Hysteresis in periodically forced flows through poro-elastic media has been observed experimentally (MacMinn, Dufresne & Wettlaufer 2015; Hewitt *et al.* 2016). As an idealisation to reveal important physical principles analytically, Fiori *et al.* consider the one-dimensional system illustrated in figure 1, described by the equations

$$\frac{\partial \phi}{\partial t} = \frac{\partial}{\partial x}\left(D(\phi)\frac{\partial \phi}{\partial x}\right), \tag{3.1}$$

$$\phi\frac{\mathrm{d}a}{\mathrm{d}t} = -D(\phi)\left.\frac{\partial \phi}{\partial x}\right|_{x=a(t)}, \quad \left.\frac{\partial \phi}{\partial x}\right|_{x=L} = 0, \tag{3.2a,b}$$

with the boundary location $a(t) = (A/2)(1 - \cos \omega t)$ a prescribed sinusoidal compression. The physical description of the system combines mass conservation of both scaffold and fluid with Darcy's law for flow of the fluid through the scaffold and a mechanical stress balance relating the fluid pressure to the effective stress (osmotic pressure), together with a constitutive relationship between the effective stress and the porosity. These all combine to form the diffusion equation (3.1), with the boundary conditions (3.2*a*,*b*) representing mass conservation at the porous piston and the impermeable wall, respectively.

This system is constitutively nonlinear given that, in general, k, M and, therefore, D are functions of ϕ. It is also kinematically nonlinear given that the first boundary condition is applied at the moving piston. However, it can be linearised for sufficiently small displacements such that $|\phi - \phi_0| \ll \phi_0$ and $A \ll \sqrt{D/\omega}$, where ϕ_0 is the solid fraction of the relaxed state, by taking the diffusivity D to be constant and applying the first boundary condition at $x = 0$. In common with other sinusoidally forced diffusion equations, a diffusion wave is generated along the medium, with displacements localised near the piston at high frequencies and varying linearly with distance from the piston at low frequencies. There is a phase shift between the stress and strain (displacement) at the piston, as illustrated for small-amplitude oscillations in figure 2(*a*). The downwards trend of the major axis of the phase portrait shows the general increase in compressive stress $-\sigma'$ with strain a. Note that $\sigma' > 0$ indicates that, in this scenario, the porous medium is adhered to the piston and must be retracted by external forces rather than just relaxing elastically.

The symmetric, almost linear response for small-amplitude deformations, illustrated by figure 2(*a*), contrasts significantly with the nonlinear response resulting from a maximum strain of 20 %, shown in figure 2(*b*). The hysteresis shown in figure 2(*a*) results simply from the phase lag between stress and strain associated with a diffusion wave. The much stronger hysteresis shown in figure 2(*b*) is a consequence both of the fact that the permeability decreases with decreasing porosity and that, given the constitutive model for the elasticity of the scaffold that these authors employ (Hencky elasticity), the osmotic modulus increases with decreasing porosity, which represents a form of strain

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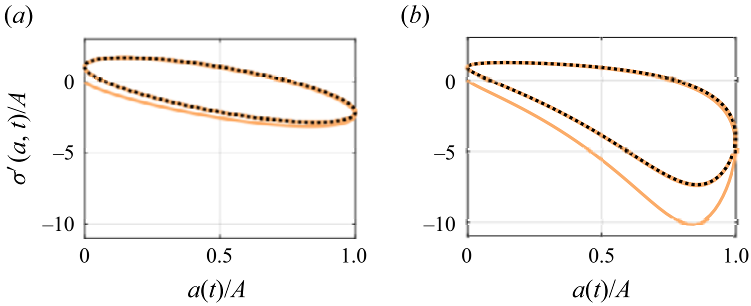

Figure 2. Stress vs strain at the porous piston for rapid, small-amplitude (*a*) and large-amplitude (*b*) oscillatory compressions of a poro-elastic medium. Reproduced from Fiori *et al.* (2023).

hardening under compression. Note, by scaling (2.1) and (3.2*a*), that, for rapid oscillations with $\sqrt{D/\omega} \ll L$, the strain $-\sigma' = \Pi \propto \sqrt{\mu\omega M/k}A$, so strain hardening and decreasing permeability act similarly in modifying the stress–strain relationship at the porous piston, illustrated in figure 2.

The sort of set-up and analysis presented by Fiori *et al.* could potentially be exploited to determine the material properties of poro-elastic media. By varying the amplitude and frequency of the forcing and measuring the amplitude and phase of the response, one can, in principle, determine both the permeability and elastic modulus of the medium, and the frequency domain can provide a robust framework within which to analyse such data (Géraud *et al.* 2020). In terms of modelling, there are very many problems of practical interest involving flows in periodically forced poro-elastic media. In addition to the examples mentioned above, the sorts of nonlinearities highlighted here could perhaps lead to flow rectification and be exploited in microfluidic diodes. Other applications, such as microfluidic actuators (D'Eramo *et al.* 2018), require an understanding of the elastic, morphological response of gels to various stimuli promoting fluid flow and differential swelling. Such responses are beginning to be explored from the perspective of fluid mechanics (Butler & Montenegro-Johnson 2022). The associated continuum modelling can be challenging but recent developments by Webber, Etzold & Worster (2023) suggest a tractable way of linearising the governing equations of poro-elasticity for small deviatoric strains while allowing large, locally isotropic strains associated with swelling and drying. We can look forward to many more fluid-mechanical studies of gels and other poro-elastic materials in the future.

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