## NOTES

## CAPITAL-LABOR SUBSTITUTION, SECTOR-SPECIFIC EXTERNALITIES, AND INDETERMINACY

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This paper examines the effect of the elasticity of technological substitution on the existence of equilibrium indeterminacy in two-sector economies. Following recent empirical evidence, the elasticity of substitution between capital and labor is below unity and we find that this requires a higher degree of productive externalities in order to still be able to produce indeterminate equilibria. However, empirically realistic rates of substitution do not rule out indeterminacy.

Keywords: Two-Sector Models, Indeterminacy, CES Production Functions, Externalities

## 1. INTRODUCTION

Benhabib and Farmer (1996) and others have shown that belief-driven fluctuations arise in two-sector models at modestly increasing levels of returns to scale. However, almost in its entirety, the existing literature on this subject builds on assuming Cobb–Douglas technologies.<sup>1</sup> This paper will loosen the restriction of Cobb–Douglas form: recent empirical studies suggest that the elasticity of substitution between capital and labor differs from one, thus making Cobb–Douglas a less appropriate choice. For instance, for the aggregate U.S. economy, Klump et al. (2007) and Chirinko (2008) report elasticities of substitution at well below unity.

The current paper investigates how the existence of equilibrium indeterminacy in two-sector competitive economies with sector-specific externalities depends on the elasticity of substitution between capital and labor. We find an inverse relation between the elasticity of substitution and the level of productive externalities that is required to produce indeterminate equilibria. Here our paper relates to Guo and Lansing (2009). Guo and Lansing analyze the indeterminacy

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properties of one-sector economies with non-Cobb-Douglas technologies. Our findings parallel theirs: an elasticity of substitution between capital and labor below one requires a higher levels of productive externalities in order to still be able to produce indeterminate equilibria.<sup>2</sup> Hence, the underlying mechanism seems to be the same for both models. Now, why does a lower elasticity shrink the economy's indeterminacy zone? Belief-driven equilibria in two-sector models come about as follows. Upon, say, optimistic beliefs, people will be willing to substitute current consumption for current investment. The higher output in the investment sector will arise by shifting input factors across sectors and this will create more sectoral externalities. If the technological spillovers are sufficiently strong, the price of investment goods will drop. Along an optimal path, people will increase consumption and reduce investment spending in the future again, which will raise the relative price again, thereby creating self-fulfilling capital gains. The impact of the elasticity of substitution operates through its effect on the mobility of input factors across sectors. Because this mobility is lessened at low levels of substitutability, relative price movements will be smaller and larger externalities will be required.

This being said, our main finding is that indeterminacy cannot be ruled out on purely empirical grounds, in the following sense: based on an elasticity of substitution,  $\sigma$ , of one-half, as recently suggested in Klump et al. (2007), as well as in León-Ledesma et al. (2010), the minimum size of returns to scale for indeterminacy is 1.124 (1.023)-up from 1.077 (1.021) for the Cobb-Douglas case (numbers in parentheses refer to models with variable capital utilization). These values are well placed within the range suggested in empirical studies, for example, Harrison (2003). Yet two caveats apply. First, estimation uncertainty appears to be nonnegligible: Chirinko (2008) surveys existing empirical work on  $\sigma$  and reports values ranging from 0.16 to 3.40. At the lower value, the increasing returns rise to 1.209 (in the constant-capital utilization case), which is on the brink of the plausible range. The issue no longer arises once capital utilization is endogenous: indeterminacy is now consistent with very low values of sigma (even the 0.16 lower bound on Chirinko's range). However, the standard techniques only establish indeterminacy for a small corridor of values for returns to scale. This result mirrors Guo and Harrison (2001), who assume Cobb–Douglas technologies. Further work is needed to determine the properties of equilibrium for higher levels of increasing returns in this case.

The rest of this paper is organized as follows. Section 2 describes the details of the model. The calibration and the steady state are outlined in Section 3. Section 4 discusses the existence of equilibrium indeterminacy. Variable capital utilization is introduced in Section 5, and Section 6 concludes.

#### 2. THE MODEL

The artificial economy consists of consumers and firms. Firms are arranged in two production sectors that produce either the consumption good or the investment good by using capital and labor. The two input factors can move freely between sectors in each production period. Firms have access to a CES technology that features sector-specific externalities. The external effect comes from the aggregate output in the respective sector. All markets are fully competitive in every other respect.

### 2.1. Firms

In the consumption sector, a large number of measure-one firms produce output using the production technology

$$c_{t} = M \left[ \alpha k_{ct}^{\rho} + (1 - \alpha) h_{ct}^{\rho} \right]^{\frac{1}{\rho}} C_{t}^{\frac{\mu_{c}}{1 + \mu_{c}}}, \qquad M > 0, \quad 0 < \alpha < 1, \quad \rho = \frac{\sigma - 1}{\sigma}.$$

Here  $c_t$ ,  $h_{ct}$ ,  $k_{ct}$ , and  $\alpha$  are the output of the (final) consumption good, the firm-level labor used in the consumption sector, the firm-level capital used in the consumption sector, and the distribution parameter of capital in production.  $\sigma$  stands for the elasticity of substitution between capital and labor in production. At  $\sigma = 1$ , the production function is of Cobb–Douglas form. For  $\sigma \to 0$  (or  $\sigma \to \infty$ ), capital and labor become perfect complements (or perfect substitutes).  $C_t$  denotes the level of average production in the consumption sector and  $\mu_c$  measures the degree of sector-specific externalities. Thus,  $\mu_c > 0$  implies positive externalities and the expression  $C_t^{\mu_c/(1+\mu_c)}$  represents the productive externality in the consumption sector. M is an efficiency term, which will be recomputed whenever  $\sigma$  is varied. In the Cobb–Douglas case, M is equal to one.

Analogously, a typical firm in the investment sector has access to technology

$$x_{t} = N \left[ \alpha k_{xt}^{\rho} + (1 - \alpha) h_{xt}^{\rho} \right]^{\frac{1}{\rho}} X_{t}^{\frac{\mu_{x}}{1 + \mu_{x}}}, \qquad N > 0,$$

where  $x_t$  and  $h_{x,t}$  ( $k_{x,t}$ ) stand for the output of the investment goods and firm-level labor (capital) used in the investment goods sector. The term  $X_t^{\mu_x/(1+\mu_x)}$  represents the productive externality in the investment sector. Expressions for the constants M and N will be derived in Section 3.

In line with Benhabib and Farmer (1996), we assume that the elasticities of substitution are the same in both sectors. The reasoning for this is threefold. First, this assumption allows us to isolate the effect of different degrees of elasticity on indeterminacy. Second, we are able to compare our results to existing work on indeterminacy in two-sector models. Last, we are not aware of any systematic empirical differences between consumption and investment sectors' elasticities of substitution [the above-mentioned recent studies report estimates for the aggregate economy; Young (2010) is one of the few exceptions, but he does not report any systematic differences across sectors].<sup>3</sup>

Given the assumptions that factor markets are perfectly competitive and that inputs are perfectly mobile across the two sectors, the firms' profit maximization implies

$$\frac{c_t}{h_{ct}} \frac{(1-\alpha)h_{ct}^{\rho}}{\left[\alpha k_{ct}^{\rho} + (1-\alpha)h_{ct}^{\rho}\right]} = w_t = p_t \frac{x_t}{h_{xt}} \frac{(1-\alpha)h_{xt}^{\rho}}{\left[\alpha k_{xt}^{\rho} + (1-\alpha)h_{xt}^{\rho}\right]}$$
(1)

and

$$\frac{c_t}{k_{ct}} \frac{\alpha k_{ct}^{\rho}}{\left[\alpha k_{ct}^{\rho} + (1-\alpha)h_{ct}^{\rho}\right]} = r_t = p_t \frac{x_t}{k_{xt}} \frac{\alpha k_{xt}^{\rho}}{\left[\alpha k_{xt}^{\rho} + (1-\alpha)h_{xt}^{\rho}\right]}.$$
(2)

Equation (1) denotes the demand for labor in the consumption and in the investment sector, respectively. Because both sectors hire from the same pool of homogenous workers, the real wage paid,  $w_t$ , is the same in both sectors. Analogously, equation (2) states that the marginal products of capital in the two sectors must equal the rental rate,  $r_t$ . Resources in this economy are fully allocated to the two production sectors. We will only consider symmetric competitive equilibria. Thus, it is easy to show that factor intensities are identical across both sectors; the shares of total labor,  $h_t$ , and of total capital,  $k_t$ , used in the consumption good sector are identical, denoted by  $\Omega_t$ :

$$\Omega_t \equiv \frac{h_{ct}}{h_t} = \frac{k_{ct}}{k_t}$$
 and  $1 - \Omega_t \equiv \frac{h_{xt}}{h_t} = \frac{k_{xt}}{k_t}$ .

Furthermore, in symmetric equilibrium  $c_t = C_t$  and  $x_t = X_t$ , allowing us to rewrite the production technologies as

$$c_t = M \left[ \alpha k_t^{\rho} + (1 - \alpha) h_t^{\rho} \right]^{\frac{1 + \mu_c}{\rho}} \Omega_t^{1 + \mu_c}$$
(3)

and

$$x_t = N \Big[ \alpha k_t^{\rho} + (1 - \alpha) h_t^{\rho} \Big]^{\frac{1 + \mu_x}{\rho}} (1 - \Omega_t)^{1 + \mu_x}.$$
(4)

The relative price of the investment good in terms of the consumption good is then given by

$$p_{t} = \frac{M^{1+\mu_{c}}}{N^{1+\mu_{x}}} \Big[ \alpha k_{t}^{\rho} + (1-\alpha) h_{t}^{\rho} \Big]^{\frac{\mu_{c}-\mu_{x}}{\rho}} \frac{\Omega_{t}^{\mu_{c}}}{(1-\Omega_{t})^{\mu_{x}}}.$$
(5)

#### 2.2. People

People are represented by a stand-in agent. This representative's preferences depend only on consumption and labor and they are characterized by the lifetime utility function

$$\sum_{t=0}^{\infty} \beta^t u(c_t, h_t), \qquad 0 < \beta < 1.$$

Here  $0 < \beta < 1$  denotes the subjective discount factor and the periodic utility u(., .) takes on the functional form

$$u(c_t, h_t) = \ln c_t - \phi h_t, \qquad \phi > 0.$$

The linearity of the periodic utility in labor corresponds to the indivisible labor concept formulated by Hansen (1985) and Rogerson (1988). Physical capital is subject to evaporative decay at the constant rate  $0 < \delta < 1$ . Capital accumulates according to

$$k_{t+1} = (1 - \delta)k_t + x_t.$$
 (6)

Finally, the agent faces the period budget constraint

$$c_t + p_t x_t = w_t h_t + r_t k_t. ag{7}$$

In the perfect-foresight competitive equilibrium, the consumer's first-order conditions are

$$\frac{w_t}{c_t} = \phi, \tag{8}$$

$$\frac{p_t}{c_t} = \beta \frac{1}{c_{t+1}} \left[ r_{t+1} + p_{t+1}(1-\delta) \right]$$
(9)

plus the usual transversality condition

$$\lim_{t \to \infty} \beta^t \frac{p_t k_{t+1}}{c_t} = 0.$$

Equation (8) denotes the consumption–leisure tradeoff and (9) is the consumption Euler equation, which describes optimal intertemporal savings.

#### 3. CALIBRATION AND UNIQUE STEADY STATE

Before proceeding to analysis of the existence of equilibrium indeterminacy, we will describe the parametric specification of the model and assign parameter values. Using a CES technology is not standard, and comparing distinct values of the elasticity of substitution entails potential pitfalls. That is, by varying the elasticity of substitution, the inputs' efficiency is affected and different factor shares and different levels of output per worker can apply at any given capital–labor ratio. To avoid this bias, Klump and Saam (2008) propose the use of a normalized CES function. The normalized CES function is defined to maintain all stationary state quantities of the benchmark case constant. Our benchmark is Cobb–Douglas or  $\sigma = 1$  and parameters that are dependent on  $\sigma$  will be recomputed whenever  $\sigma$  is varied, so that the associated stationary quantities remain unchanged.

Parameters denoted by a bar represent the benchmark Cobb–Douglas case. These parameter values are calibrated to match U.S. postwar aggregate data (Table 1). The capital share of income,  $\overline{\alpha}$ , is equal to 30% in order to match the average capital share in GNP [Gollin (2002)]. Households' time spent on working,  $\overline{h}$ , is equal to 30% of the total time endowment. This value is chosen to replicate the long-run fraction of noncivilian working-age employment, 75%, and the steady-state fraction of time spent in market activity of 40% [Kydland (1995)]. The discount factor is set so that the steady state net return to capital is 4%. The

$\overline{\alpha}$	0.3
$\overline{\beta}$	0.99
$\overline{\delta}$	0.025
$\overline{h}$	0.3
η	1

TABLE 1 Calibration

# quarterly depreciation rate of capital, $\overline{\delta}$ , is equal to 2.5% in order to match the steady-state ratio of investment to capital.

The benchmark stationary state is obtained by solving the steady-state versions of equations (2) to (9) at  $\sigma = 1$ :

$$\overline{\Omega} = 1 - \frac{\overline{\alpha}\overline{\delta}}{\frac{1}{\overline{\beta}} - 1 + \overline{\delta}},$$

$$\overline{k} = \left\{ \frac{\left[ (1 - \overline{\Omega}) \overline{h}^{1 - \overline{\alpha}} \right]^{1 + \mu_x}}{\overline{\delta}} \right\}^{\frac{1}{1 - \overline{\alpha}(1 + \mu_x)}}$$
$$\frac{\overline{\beta} \overline{r}}{\overline{p}} = 1 - \overline{\beta}(1 - \overline{\delta}).$$

,

We are now able to determine *M* and *N*:

$$M = \frac{\overline{c}^{\frac{1}{1+\mu_c}}}{\overline{\Omega} \left[\alpha \overline{k}^{\rho} + (1-\alpha)\overline{h}^{\rho}\right]^{\frac{1}{\rho}}} \quad \text{and} \quad N = \frac{\overline{x}^{\frac{1}{1+\mu_x}}}{(1-\overline{\Omega}) \left[\alpha \overline{k}^{\rho} + (1-\alpha)\overline{h}^{\rho}\right]^{\frac{1}{\rho}}}.$$

The last parameter to be determined is the distribution parameter  $\alpha$ . We adopt Klump and Saam's (2008) mechanism, which is recomputing  $\alpha$  whenever  $\sigma$  is varied to maintain the capital share at the benchmark level. Therefore, the calibration of  $\alpha$  is described by

$$\alpha = \frac{\overline{\alpha}}{\overline{\alpha} + (1 - \overline{\alpha}) \left(\frac{\overline{k}}{\overline{h}}\right)^{\rho}}.$$

#### 4. EXISTENCE OF EQUILIBRIUM INDETERMINACY

We will now investigate the relationship between the sector-specific externalities and the elasticity of capital-labor substitution in producing equilibrium indeterminacy. Because the plausibility of indeterminacy is an empirical issue, all results will be presented numerically.



**FIGURE 1.** Parameter constellation for indeterminacy when  $\mu_X = \mu_C > 0$ .

We begin by restricting  $\mu_x = \mu_c > 0$ . Figure 1 displays the parameter constellation for equilibrium indeterminacy. The downward-sloped curve separates determinacy (below curve) and indeterminacy zones (above it). For indeterminacy to arise, the level of externalities must be inversely related to the elasticity of substitution. This general picture is similar to Guo and Lansing's (2009) exploration into one-sector models. It can be understood as follows. Sunspot equilibria imply that reallocations of factor inputs are profitable for people, and this arises from realizing the scale economies. However, the lower the elasticity of substitution, the more costly the reallocation and hence larger returns to scale are required.

At  $\sigma = 1$ , the production technology boils down to the Cobb–Douglas case and we find the minimum  $\mu_x = 0.0774$ , which is, of course, exactly the same number that Harrison (2001) reports. As noted earlier, recent empirical work by Klump et al. (2007) and Chirinko (2008) suggests  $\sigma$  between 0.4 and 0.6 for the U.S. economy. This raises the minimum increasing returns to no more than 1.1103 (at  $\sigma = 0.6$ ) to 1.1405 (at  $\sigma = 0.4$ ). However, if the technology approaches the Leontief case (say,  $\sigma = 0.01$ ), then indeterminacy becomes very unlikely ( $\mu_x = 0.7140$ ).<sup>4</sup>

It is also of interest to explore  $\mu_x > 0$  and  $\mu_c = 0$ , which draws appeal from empirical results; namely, Harrison (2003) finds increasing returns only in the investment sector. Figure 2 plots this case. Again, the equilibrium regions of determinacy and indeterminacy are separated by a downward-sloping curve. Moreover, this curve is identical to that in Figure 1. That is, indeterminacy arises regardless of the value of  $\mu_c$ . This is reminiscent of Harrison (2001) and Weder (2000): indeterminacy in the two-sector model is determined by the externalities in the investment sector.



**FIGURE 2.** Parameter constellation for indeterminacy when  $\mu_X > 0$ ,  $\mu_C = 0$ .

To sum up, indeterminacy remains an empirically plausible phenomenon. Klump et al. (2007) and León-Ledesma et al. (2010) find a  $\sigma$  around 0.5, for which the minimum size of returns to scale for indeterminacy is 1.1236—up from 1.077 for the Cobb–Douglas case. This value of increasing returns still lies within the range suggested in empirical studies [for example, Harrison (2003)].

#### 5. INDETERMINACY AND VARIABLE CAPITAL UTILIZATION

Next, we will incorporate variable capital utilization. Capital utilization displays a strong procyclical pattern in data [see King and Rebelo (1999)]. Accordingly, we now assume that the capital utilization rate,  $u_t$ , can be endogenously set by the agents and that the sectors' technologies become

$$c_{t} = M \left[ \alpha (u_{t}k_{ct})^{\rho} + (1-\alpha)h_{ct}^{\rho} \right]^{\frac{1}{\rho}} C_{t}^{\frac{\mu_{c}}{1+\mu_{c}}}$$

and

$$x_{t} = N \left[ \alpha (u_{t} k_{xt})^{\rho} + (1 - \alpha) h_{xt}^{\rho} \right]^{\frac{1}{\rho}} X_{t}^{\frac{\mu_{x}}{1 + \mu_{x}}}$$

Furthermore, as in most studies with variable capital utilization, the rate of physical depreciation is an increasing function of the utilization rate

$$\delta_t = \frac{1}{\theta} u_t^{\theta}, \qquad \theta > 1.$$

1

Working with the same calibration and normalization of technologies as before, Figure 3 shows how varying the degree of capital–labor substitution affects this economy.<sup>5</sup> Again, indeterminacy becomes less likely for low levels of elasticity of substitution between capital and labor. This parallels our findings above. However, the presence of variable capital utilization significantly lowers the returns to scale that are needed for self-fulfilling beliefs. For example, in the benchmark Cobb– Douglas case, externalities as low as 1.0206 to 1.0262 generate indeterminacy;



**FIGURE 3.** Parameter constellation for indeterminacy with variable utilization when  $\mu_X > 0$ ,  $\mu_C = 0$ .

this replicates Guo and Harrison (2001). Based on an elasticity of substitution of one-half, the minimum size of returns to scale for indeterminacy is 1.0229 to 1.0301. These values of increasing returns lie within the range suggested in empirical studies [Basu and Fernald (1997)]. This being said, it is easy to see that indeterminacy arises only in a small parametric corridor.<sup>6</sup> This is a clear conundrum: the smallness of the parametric range decreases the plausibility of indeterminacy.

Finally, we will shed light on how the dynamics of the artificial economy are affected by capital–labor substitution. Figure 4 plots the impulse responses of key macro variables to a one-time sunspot shock for the model with variable capital utilization (the equivalent qualitative findings can be shown to apply in a fixed-utilization model). The variables are output, consumption, the labor share, and employment. To be more precise, we let  $\sigma = 0.5$  and  $\sigma = 1$  and trace out the interplay of capital–labor substitution and sector-specific externalities with regards to dynamics. Externalities in the investment sector are set at 1.024.

Figure 4 displays five findings. First, reducing  $\sigma$  decreases the volatility of output, consumption, and employment. Second, reducing  $\sigma$  lowers the persistence of these variables. The reasoning is as follows: the smaller substitutability lessens the mobility of input factors across sectors. This dampens the mechanism that drives indeterminacy, and accordingly the impact of sunspots shocks is smaller. Third, consumption (employment) is less (about the same as) volatile than output. Fourth, the labor share is countercyclical for  $\sigma < 1$  and obviously remains constant for the Cobb–Douglas case, and it is procyclical for  $\sigma > 1$  [this is similar to Guo and Lansing (2009)]. Last, there is one fact that the sunspot-driven model cannot replicate: consumption is countercyclical, as in Benhabib and Farmer (1996).



**FIGURE 4.** Impulse responses of key macro variables to a one-time sunspot shock for the model with variable capital utilization. Elasticities of substitution are  $\sigma = 0.5$  and  $\sigma = 1$  (solid line).

#### 6. CONCLUDING REMARKS

This paper has examined the effect of the elasticity of technological substitution on the existence of equilibrium indeterminacy in two-sector economies. Following recent empirical evidence, the elasticity of substitution between capital and labor is below unity, and we showed that this requires a higher degree of productive externalities to still be able to produce indeterminate equilibria. This parallels Guo and Lansing's (2009) analysis of one-sector economies.

Do empirically realistic rates of substitution rule out indeterminacy? Klump et al. (2007) suggest values for  $\sigma$  of about one half. Given this value, the minimum size of returns to scale for indeterminacy is 1.1236 for the model with constant capital utilization and 1.0229 for the variable–capital utilization version. Hence, it appears that the minimum requirement for indeterminacy is not outside the usually considered acceptable range of scale economies.

#### NOTES

1. Nishimura and Venditti (2006) is the only exception that we are aware of.

2. Pintus (2006) also examines one-sector models with non-Cobb-Douglas technologies.

3. However, our analysis below suggests that the investment sector's technology generates indeterminacy; hence, we suspect that if  $\sigma s$  differed across sectors, the investment sector would still drive the results.

4. For values smaller than  $\sigma \approx 0.01$ , the Jacobian matrix becomes singular, and accordingly the figure is truncated at that point.

5. The calibration implies that  $\theta = 1.404$ .

6. To be more precise, for parameter constellations above the upper curve, the steady state is a source and linear methods do not allow predictions regarding possible endogenous cycles to be made. Hence, we cannot rule out nonuniqueness.

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