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ON SOME AMBIGUITIES IN IBN SĪNĀ'S ANALYSIS OF THE QUANTIFIED HYPOTHETICAL PROPOSITIONS

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Abstract. In his analysis of the hypothetical (*šarṭīyya)* connected (*muttaṣila*) and disjunctive (*munfaṣila*) propositions (*Al-qiyās*, section V), Ibn Sīnā suggests that they can be quantified and presents in section VI a hypothetical system containing the conditional ones, which is exactly parallel to categorical syllogistic and makes use of the same conversion rules and the same proofs. In section VII, he provides four lists of hypothetical quantified propositions whose clauses are themselves quantified and says that the relations of the Aristotelian square of opposition hold for them. In addition, he says that some conditional universal affirmative propositions are equivalent to some universal negative ones with opposed consequents, and to some quantified disjunctive ones. The problem is that these claims are incompatible with each other, since they require two different readings of the universal affirmative conditional proposition, which Ibn Sīnā does not distinguish clearly. In this paper we solve the problem by distinguishing *explicitly* between these two readings and showing that the first one satisfies the conversion rule of the universal affirmative and the relations of the logical square, and validates all the admitted moods, while the second one satisfies the contraposition rule and the equivalences stated by Ibn Sīnā. This accounts for all Ibn Sīnā's claims and makes the system coherent.

Résumé. Dans son analyse des propositions hypothétiques (*šarṭīyya)*, conditionnelles (*muttaṣila*) et disjonctives (*munfaṣila*) (*Al-qiyās*, section V), Ibn Sīnā suggère que ces propositions peuvent être quantifiées et présente dans la section VI de son traité un système hypothétique contenant les propositions conditionnelles, qui est exactement parallèle à la syllogistique des propositions catégoriques et utilise les mêmes règles de conversion et les mêmes démonstrations. Dans la section VII, il présente quatre listes de propositions hypothétiques quantifiées dont les composants sont eux-mêmes quantifiés et affirme que les relations du carré aristotélicien des oppositions valent pour ces propositions. Par ailleurs, il affirme que certaines propositions conditionnelles universelles affirmatives sont équivalentes à certaines universelles négatives dont les conséquents sont opposés, et également à certaines propositions disjonctives. Le problème est que ces thèses sont incompatibles entre elles, car elles requièrent deux lectures différentes de la proposition conditionnelle universelle affirmative, qu'Ibn Sīnā ne distingue pas clairement. Dans le présent article nous résolvons le problème en distinguant explicitement entre ces deux interprétations et en montrant que la première valide la conversion de l'universelle

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affirmative et les relations du carré des oppositions ainsi que tous les modes admis, alors que la seconde valide la règle de contraposition et les équivalences énoncées par Ibn Sīnā, ce qui rend compte des affirmations d'Ibn Sīnā et rend le système cohérent.

1. INTRODUCTION

Ibn Sīnā's hypothetical (*šarṭī*) logic contains a system called *istiṯnāʾī* comparable to the Stoic system and to al-Fārābī's hypothetical (*šarṭī*) system, where disjunctive as well as conditional (called "connected" by Ibn Sīnā) propositions are used, and a system containing quantified hypothetical (*šarṭīyya*)[1] propositions. The latter quantifies over conditional (containing "if… then") as well as disjunctive (containing "either… or") propositions. Ibn Sīnā introduces **A**, **E**, **I** and **O**[2] conditional as well as disjunctive propositions and other complex ones containing **A**, **E**, **I** or **O** predicative clauses. He provides the complete listings of these hypothetical quantified propositions with quantified (categorical) clauses and says that all of them obey the relations of the square of opposition, namely contradiction, contrariety, subcontrariety and subalternation.

Now the relations of the Aristotelian square of opposition hold only under some conditions, among which the most natural one in categorical syllogistic is that **A** and **I** have an import while **E** and **O** do not have an import. In hypothetical logic, the import corresponds to what Wilfrid Hodges calls the "existential augment."[3] However, in his explanations, Ibn Sīnā equates some **A** conditional propositions (which we call **A**c, "c" standing for "connected" (= conditional) to distinguish them from the categorical propositions, called **A**, **E** etc…) with some **E** conditional ones

[1] "Hypothetical" translates *šarṭīyya*, and includes both conditional (or connected = *muttaṣila*) and disjunctive (= *munfaṣila*) propositions. "Conditional" corresponds only to the propositions containing "if… then," whether singular or quantified, although the word used by Ibn Sīnā is rather *muttaṣila*, whose literal translation is "connected."

[2] These letters have been introduced by Medieval logicians to name all quantified categorical propositions. **A** refers to the universal affirmative, **E** refers to the universal negative, **I** refers to the particular affirmative and **O** refers to the particular negative. I use them here for convenience, although Ibn Sīnā did not know them.

[3] See Wilfrid Hodges, *Mathematical background to the logic of Avicenna*, chapter 14, "Propositional logic," p. 255. This book is available at http://wilfridhodges.co.uk/arabic44.pdf. Note that this chapter of the book is devoted to the hypothetical logic of Ibn Sīnā, as one can easily check. So this augment is used in hypothetical logic too, not only in categorical logic. We will argue in the sequel that it is indeed indispensable to validate the conversion rules and the moods in the first hypothetical system containing only conditional propositions.

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(called **E**c, hereafter), without taking into account the existential augment of **A**c. So here, two incompatible opinions about **A**c seem to be held at the same time, namely 1) **A**c contains the augment, 2) **A**c does not contain the augment.

This incompatibility creates a problem, which we raise, but is not raised by Ibn Sīnā himself (who does not seem to see the difficulty), since the equivalences between **A**c and **E**c, and those between **A**c and $\mathbf{A}_D$ (= **A** disjunctive proposition), claimed by Ibn Sīnā, do not hold when **A**c contains the augment, while the relations of the square of opposition, the moods held by Ibn Sīnā in all three figures and the conversions of **A**c and of **I**c, which are both used in the proofs of the hypothetical moods too, are valid only when **A**c does contain the augment.

So the problem is the following: how can we account for these two incompatible claims? How should we formalize the propositions, whether conditional or disjunctive? Can we say that both the relations of the square and the equivalences, the rules and the moods stated by Ibn Sīnā all hold in his frame?

In the sequel, we will show that **A**c can be formalized in two distinct ways and that these two formalizations should be kept but explicitly distinguished in order to account for Ibn Sīnā's claims, or at least for most of them, and for the relations between the conditional and the disjunctive hypothetical propositions. In this analysis, we will be as faithful as possible to the text.

2. PRELIMINARIES

Let us start by clarifying the problem raised and by providing all the definitions needed, which will be used to solve it.

In section V of *Al-qiyās*, Ibn Sīnā develops a new system of hypothetical logic containing quantified hypothetical propositions, both conditional and disjunctive. He starts by defining the operators[4] used in that system, namely the implication (section V, chapter 1) and the disjunction (section V, chapter 2). In what follows, we will consider mainly the conditional ones. The conditional (*muttaṣila*) propositions are of two kinds in Ibn Sīnā's frame: 1) Real implications (= *luzūmīyya*), 2) Not real implications (= *ittifāqīyya*). In the first kind, i. e. the real implication, the consequent is said to really follow from the antecedent, while in the *ittifāqīyya* proposition, no such relation of following is expressed, for

[4] The operators are the logical relations between propositions, such as the implication, the conjunction, the disjunction, the equivalence and so on.

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both the antecedent and the consequent can be semantically or causally independent. The real implication is illustrated by sentences like "If the sun rises, then it is daytime,"[5] or "If this is a human, then it is an animal" where the consequent really follows from the antecedent, while the *ittifāqīyya* proposition is illustrated by examples such as: "If humans exist, then horses exist too"[6] or "If humans exist, then 2 is even (or Void does not exist)."[7] Here there is no dependency between the antecedent and the consequent, since they are independent semantically and causally from each other in both examples. The *ittifāq* is translated by Nabil Shehaby as "chance connection"[8] while it is translated as "agreement" by Wilfrid Hodges. The first translation does not seem to be adequate, since there is no connection at all between "humans exist" and "2 is even," be it chancy or not. The second one seems less problematic, but it must be made more precise, by showing what kind of agreement is expressed by *ittifāq*. As we will see below, the agreement is between the propositions stated and the facts, more precisely between the consequent of the *ittifāqīyya* proposition and the fact corresponding to it.

These two relations are very different from each other, and have distinct truth conditions. Let us first consider the real implication. According to Ibn Sīnā, the relation expressed by the real implication (*luzūm*) can be either causal or semantic or involving relations or other kinds of semantic links. He explains and illustrates the real implication (*luzūm*) by examples illustrating the case where the antecedent may be the cause (*ᶜilla*) of the consequent as in "If the sun rises, then it is daytime,"[9] or the case where the antecedent is the inseparable effect (*maᶜlūl ġayr mufāriq*) of the cause as in "If it is daytime, then the sun rises,"[10] or the case where both are effects of the same cause, as "the thunder and the flash of lightning [are] for the movement of the wind in the clouds,"[11] or the case where they express some relations (*rubbamā kāna muḍāyifan*), as when one says "If $5 > 3$, then $3 < 5$" or "If Paul is the father of John, then John is the son of Paul" (personal examples), or when there are some other semantic links, for instance, when one says "If this is a trian-

[5] Ibn Sīnā, *Al-šifāᵓ*, *Al-mantiq* 4, *Al-qiyās*, ed. S. Zayed, rev. and intr. by I. Madkour (Cairo: Wizārat al-Ṯaqāfa wa-l-Irṣād al-Qawmī, 1964), p. 232.

[6] Ibn Sīnā, *Al-qiyās*, p. 234.

[7] Ibn Sīnā, p. 235.

[8] Nabil Shehaby, *The propositional logic of Avicenna: A translation from Al-shifā Al-qiyās* (Dordrecht: Kluwer, 1973), p. 37.

[9] Ibn Sīnā, *Al-qiyās*, p. 232.

[10] Ibn Sīnā, p. 234.

[11] Ibn Sīnā, p. 234.

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gle, then it is a geometrical figure" (personal example). In all cases, however, there is a strong semantic or causal link between the antecedent and the consequent, which means that the real implication is not a material implication which, as it is defined by modern logicians, is true or false depending only on the truth values of its components, and does not take into account their meanings.

As to *ittifāq*, it is defined as some kind of agreement with the facts. This agreement may concern the two clauses of the proposition as in the following example "If every man is speaking, then every donkey is braying,"[12] or only the consequent, as in the following "If every donkey is talking then every human is talking,"[13] which Ibn Sīnā considers as true on the basis of the truth of its consequent, despite the fact that its antecedent is false.

What are the truth conditions of the real implication [the implicative proposition] and the *ittifāq*? We can say that both are false when the antecedent is true, while the consequent is false. So this case of falsity is common to them both. But the other cases are very different, for the implicative (*luzūmīyya*) proposition can be true when its two clauses are both false, as in the following "If men are stones, men are inert"[14] or when the antecedent is false while the consequent is true as in the sentence "If men are stones, then they are bodies,"[15] or even when the truth values of both clauses are not known as in "If Abdullah is writing, then he is moving his hand"[16] and of course when both clauses are true, as in the first examples given above. But in all these cases, the implicative (*luzūmīyya*) proposition can also be false, if there is no semantic or causal link between the antecedent and the consequent. So the real implication is not truth-functional, for the only settled case is the one where the antecedent is true and the consequent is false, in which case the whole proposition is false.

As to *ittifāq*, it is true when both its clauses are true, and when its consequent alone is true. It is false when its antecedent is true while its consequent is false, and when they are both false.

This being so, we can provide the truth tables of these two relations and compare them with both the material conditional and the conjunction as follows (where "→" stands for the *luzūm*, "If… then" stands for

[12] Ibn Sīnā, *Al-qiyās*, p. 267.
[13] Ibn Sīnā, p. 270.
[14] Ibn Sīnā, p. 261.
[15] Ibn Sīnā, p. 261.
[16] Ibn Sīnā, p. 261.

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ittifāq, "⊃" stands for material implication, "∧" stands for the conjunction and P and Q can be any propositions):

Luzūm			*Ittifāq*			Material implication			Conjunction		
P	$\rightarrow$	Q	If P, then Q			P	$\supset$	Q	P	$\wedge$	Q
1	1/0	1	1	1	1	1	1	1	1	1	1
1	0	0	1	0	0	1	0	0	1	0	0
0	1/0	1	0	1	1	0	1	1	0	0	1
0	1/0	0	0	0	0	0	1	0	0	0	0

As we can see, *ittifāq* is different from both material implication and conjunction. But we can also say that it is indeed truth-functional like these two relations and unlike the real implication (*luzūm*), whose truth conditions are not always settled, since it can be false or true in some cases depending on the absence or the presence of a strong causal or semantic link between its two clauses. But we can also say that *ittifāq*[17] (to which we have not associated any logical symbol) has the truth conditions of its consequent, since it is true when the consequent is true and false when the consequent is false, and for this reason it is useless as a logical relation, since it does not add anything to what the consequent says, its antecedent being just a hypothesis which does not influence the truth value of the whole proposition.

Now what about the conversions? Given these truth tables, we can say that apart from the conjunction, which is commutative, thus convertible, since "P and Q" is equivalent to "Q and P," no other relation provided above is convertible. The real implication is not convertible, since when "$P \rightarrow Q$" is true, "$Q \rightarrow P$" can be false, precisely when Q is true and P is false, for instance, from "If this is a triangle, then this is a geometrical figure," we cannot infer "If this is a geometrical figure then this is a triangle," since a geometrical figure can be a square or a circle, etc. Likewise, *ittifāq* is not convertible too, since when "If P then Q" is true, "If Q then P" is false, when Q is true, but P is false.

So if one adds the quantifiers and wishes to account for the conversion of the two hypothetical conditional affirmative propositions **A**c and **I**c, one has to add something else to these propositions in order to account for these conversions. As to the relations of the square of opposition, they

[17] The truth conditions of *ittifāq* will be modified by some post Avicennian logicians such as al-Ḫūnağī and Ibn ʿArafa, who consider it rather as close to a conjunction (see Afḍal al-Dīn al-Ḫūnağī, *Al-jumal*, available online in the site www.al-mostafa.com, p. 6 and Ibn ʿArafa, *Al-muḫtaṣar fi al-Manṭiq*, manuscript available online in www.al-mostafa.com, p. 26).

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are defined by Ibn Sīnā in *Al-ᶜibāra*,[18] and applied to the hypothetical system too, as we will see below. Their definitions are the following:

1) Contradiction holds when the two propositions are never true and never false together in whatever matter;

2) Contrariety holds when both propositions are never true together in the necessary and the impossible, but can be false together in the possible;

3) Subcontrariety holds when both propositions are never false together in the necessary and the impossible, but can be true together in the possible;

4) Subalternation (called *tadāḫul* by Ibn Sīnā) holds when both propositions are true together (in the necessary for the affirmatives and in the impossible for the negatives) or false together (in the necessary for the negatives and in the impossible for the affirmatives) or when the universal is false while the particular is true (in the possible for affirmatives and negatives).

We will apply those definitions to Ibn Sīnā's analyses of the quantified conditional propositions. Let us now turn to the quantified hypothetical propositions.

3. THE HYPOTHETICAL QUANTIFIED PROPOSITIONS

The quantified conditional and disjunctive propositions are both stated and analysed in section V, chapter 4 of *Al-qiyās*, which is entitled "On the clarifications of the meanings of the universal, the particular, the indefinite and the singular [propositions] for the hypotheticals" (*fī šarḥi maᶜānī al-kullīyya wa al-ğuzᵓīyya wa al-muhmala wa al-šaḫṣīyya fi al-šarṭiyyāti*),[19] where he says "Let us now talk about the hypothetical conditional universal affirmative, so we say: 'Whenever C is B, then H is Z' (*kullamā kana C B fa H Z*)."[20] So the word *kullamā* expresses universality in the affirmative conditional proposition. What the universal conditional affirmative proposition means is expressed as follows: "The hypothetical [conditional] proposition is universal if the consequent follows every positing (*kulla waḍᶜin*) of the antecedent, not only in what is intended (*lā fī al-murādi faqat*), but in the states[21]

[18] Ibn Sīnā, *Al-šifāᵓ*, *Al-manṭiq* 3, *Al-ᶜibāra*, ed. M. El Khodeiri, rev. and intr. by I. Madkour (Cairo: Dar al-Kitāb al-ᶜArabī li-t-Tabᶜ wa-n-Našr, 1970), p. 47.

[19] Ibn Sīnā, *Al-qiyās*, p. 262.

[20] Ibn Sīnā, p. 265.

[21] "State" is a better translation than "situation." This translation is also used in

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(*bal fī al-aḥwāli*)."[22] He adds that these states are "those that require *assuming* the antecedent (*tulzimu faraḍa al-muqaddim*) or can *possibly assume it* (*aw yumkinu an tufraḍa lahu*), and follow it (*wa tatbaᶜahu*) and be joined to it (*wa takūna maᶜahu*)."[23].

This passage is crucial, since it clarifies what Ibn Sīnā means by a universal affirmative conditional proposition, by giving precisions about what entities the quantifier ranges over (the so-called "states [of affairs]") and the nature of the relation between the antecedent and the consequent in such propositions. We can read that the universal conditional proposition requires *assuming* the antecedent to be true. This idea is important to determine the import of such propositions.

As to the particular (*ǧuzᵓī*)[24] conditional affirmative propositions, they are expressed by means of the words *qad yakūn*[25], which I translate by "It happens that." The particular conditional proposition is said to be "here too (*hāhunā ayḍan*)" "like [the one] that you know in the categorical [propositions] (*kamā ᶜalimta fi al-ḥamliyyāti*)."[26] He adds that the particular proposition can follow [in truth] the universal proposition, for just as in categorical logic "the following in the conditional [propositions] [should be such that] if [the following] is true in all the assumptions of the antecedent, then it is true too in some of them, for then, the consequent follows the antecedent in some cases where the antecedent is assumed (*fa-yakūnu ittibāᶜu al-tālī li-baᶜḍ awḍāᶜi al-muqaddim*)."[27] But the particular can also be true when the universal is not true, in which case "the particular is true by itself (*bal huwa al-ḥaqqu nafsuhu*), without being [dependent on the truth] of the universal (*dūna al-kullī*)."[28] This accounts for the subalternation between **A**c and **I**c, for this relation is a valid implication between a universal antecedent and a particular consequent, and it is true when the antecedent is true and also when the antecedent is false, while the consequent is true, as is the case in categorical syllogistic.

Riccardo Strobino's article "Ibn Sīnā's logic," in Edward N. Zalta (ed.), *Stanford encyclopedia of philosophy*, https://plato.stanford.edu/archives/fall2018/entries/ibn-sina-logic (2018), section 3.2. The states should be thought of as "states of affairs."

22 Ibn Sīnā, *Al-qiyās*, p. 272.
23 Ibn Sīnā, p. 272 (my emphasis).
24 Ibn Sīnā, p. 275.
25 Ibn Sīnā, p. 276.
26 Ibn Sīnā, p. 275.
27 Ibn Sīnā, p. 276.
28 Ibn Sīnā, p. 276.

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As to the universal negative, it is analyzed in chapter 5, and expressed by the word "never" (*laysa al-battata*),[29] where *al-battata* expresses the universality in **E**c, while *laysa* expresses the negation. Now, what is negated (or rejected: *marfūᶜ*) is either the real implication (*luzūm*) or the agreement (*ittifāq*),[30] depending on the kind of conditional proposition. Likewise the particular negative contains a negation and is expressed by *qad lā yakūn* ("It happens that not…") or *laysa kullamā* ("Not whenever").[31] In all cases, the universal negative implies the particular negative just as it implies it in categorical logic. So the universal negative rejects the *luzūm* or the *ittifāq* stated in the particular affirmative conditional proposition, while the particular negative rejects the *luzūm* (or the *ittifāq*) stated in the universal affirmative conditional proposition, since **E**c is the contradictory of **I**c and **O**c is the contradictory of **A**c, just as **E** and **I** on the one hand and **A** and **O** on the other hand are contradictories in categorical logic. The conditional propositions with unquantified predicative clauses are stated as follows (C standing for "connected" or "conditional"):

Ac: Whenever (*kullamā*) A is B then H is Z.[32]

Ec: Never (*laysa al-battata*) if A is B then H is Z.[33]

Ic: It happens that (*qad yakūn*) if every A is B then every H is Z.[34]

Oc: Not whenever A is B then C is D.

As to the disjunctive ones, they contain the following quantifying words: *dāʾiman*, *qad yakūn*, *laysa dāʾiman* and *laysa al-battata* in addition to "either… or" (*immā… aw*) expressing the disjunction. These disjunctive propositions are expressed as follows when their clauses are unquantified predicative propositions (D standing for "disjunctive"):

$\mathbf{A}_{\mathrm{D}}$: Always (*dāʾiman*) either A is B or C is D.

$\mathbf{I}_{\mathrm{D}}$: It happens that (*qad yakūn*) either A is B or C is D.

$\mathbf{E}_{\mathrm{D}}$: Never (*laysa al-battata*) either A is B or C is D.

$\mathbf{O}_{\mathrm{D}}$: Not always (*laysa dāʾiman*) either A is B or C is D.[35]

Now these propositions can contain quantified clauses instead of the predicative propositions such as "A is B" or "C is D." In that case their clauses are either **A** or **E** or **I** or **O** categorical propositions. Ibn Sīnā con-

[29] Ibn Sīnā, *Al-qiyās*, p. 280.
[30] Ibn Sīnā, p. 279.
[31] Ibn Sīnā, p. 296.
[32] Ibn Sīnā, p. 265.
[33] Ibn Sīnā, p. 280.
[34] Ibn Sīnā, p. 278.
[35] Ibn Sīnā, p. 373-376.

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siders all possible combinations between these propositions and ends up with 16 combinations for each hypothetical quantified proposition with quantified clauses. He thus provides four lists of 16 propositions, which he states in section VII of *Al-qiyās*.[36] Among these propositions, we will consider only the following, which are involved in the equivalences that we will discuss:

Ac3: Whenever Some A is B then Every C is D
Ac4: Whenever Some A is B then Some C is D
Ac10: Whenever Every A is B then Not every C is D
Ec1: Never when Every A is B, then Every C is D
Ec3: Never when Some A is B, then Every C is D
Ec4: Never when Some A is B, then Some C is D

How can one formalize these propositions, i. e. express them by using the symbolism of first order logic?[37] This will be examined in section 2.

4. THE FORMALIZATION OF THE HYPOTHETICAL PROPOSITIONS

Let us consider first the propositions which contain unquantified predicative clauses. According to Nicholas Rescher, these propositions can be formalized as follows (where "t" stands for time and "A_t" stands for "A is true in t") (Rescher's notation):[38]

$$\begin{array}{ll} \mathbf{A}\ (\text{U. A.}) & \begin{cases} (t)(A_t \supset C_t) \\ (t)\sim(A_t \& \sim C_t) \end{cases} \\ \mathbf{E}\ (\text{U. N.}) & (t)\sim(A_t \& C_t) \\ \mathbf{I}\ (\text{P. A.}) & (\exists t)(A_t \& C_t) \\ \mathbf{O}\ (\text{P. N.}) & (\exists t)(A_t \& \sim C_t) \end{array}$$

Note, here, that despite the fact that Rescher is using the horseshoe in his symbolization of the first proposition, he is *not* talking about material implication in this kind of propositions. Rather he says that the implication used by Ibn Sīnā seems to be close to the "Diodorian implication 'If A, then C,' [which] amounts to 'At each and every time t: If A-at-t, then C-at-t'"[39] as opposed to the Philonian implication, which is indeed

[36] Ibn Sīnā, *Al-qiyās*, p. 361-71.
[37] This symbolism is neutral and can be applied to any logical system, provided some precisions are made with regard to the entities over which the quantifiers range.
[38] Rescher Nicholas, "Avicenna on the logic of 'conditional' propositions," *Notre Dame journal of formal logic*, vol. 4, no. 1 (1963), 48-58, p. 51.
[39] Rescher, p. 50.

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what modern logicians call "material implication."[40] The Diodorian implication, as Łukasiewicz stresses in his article, is very different from the Philonian one, which takes into account only the truth values of the antecedent and the consequent. For he says that according to Diodorus of Cronos "an implication is true if and only if *it has not been the case, and it is not possible* that it leads to a falsity starting from a truth."[41] So this Diodorian implication looks very much like the strict implication championed by C. I. Lewis, as Łukasiewicz himself says in his text and in note 18.[42]

In all cases, the two pairs of contradictories are **A**c / **O**c and **E**c / **I**c, as in categorical logic. So the formalizations should account for these contradictions and also for the other relations between the quantified propositions, namely, contrariety, subcontrariety and subalternation. But as we can see, the logical structures of the hypothetical conditional quantified propositions are parallel to the modern formalizations of the usual quantified categorical propositions. And as is well known, the modern formalizations of the quantified categorical propositions validate only the contradictions **A** / **O** and **E** / **I**; they do not validate contrariety, subcontrariety and subalternation, given that **A** has no import in its modern interpretation. Consequently it is not contrary to **E**, since both **A** and **E** can be true together when their antecedent is false, nor does it entail **I**, since a conditional does not imply a conjunction.

So these formalizations of the quantified conditional propositions should not validate the last three relations, although they do validate the contradictions. This raises a problem, since Ibn Sīnā says explicitly that all the relations of the square are valid in his hypothetical logic as well as in his categorical logic. Therefore one must search for other formalizations of these hypothetical quantified conditional propositions in order to account for Ibn Sīnā's claims about the relations of the square.

On the other hand, two further problems could be raised with regard to the formalizations above. The first has to do with the formalization of **I**c and **O**c as conjunctions, which some people reject, for according to

[40] Jan Łukasiewicz, "Contribution à l'histoire de la logique des propositions," transl. in French in Jean Largeault (ed.) *Logique mathématique: Textes* (Paris: Armand Colin, 1972), p. 9-25, p. 15.

[41] My translation. In French, what is said is the following: "Diodore de Cronos soutenait au contraire qu'une implication est vraie si et seulement *s'il n'a pas été ni n'est possible* qu'elle parte du vrai et aboutisse au faux" (Łukasiewicz, p. 15, emphasis in the original text).

[42] Łukasiewicz, p. 15.

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them, **I**c and **O**c should not contain a conjunction, since Ibn Sīnā expresses them by using "if… then." The second is that the formalization of **A**c given above does not validate **A**c conversion, nor Darapti and Felapton, which are both held by Ibn Sīnā in his hypothetical logic too.

These two moods and all other ones are stated and proved in *Al-qiyās*, section VI, chapter 1, where Ibn Sīnā establishes a parallel between this particular hypothetical system and categorical syllogistic, for he says in the very beginning of the chapter what follows:

> The moods containing conditionals are those composed of two conditional [propositions] which share a term, I mean [here] an antecedent and a consequent. And this is (*wa yakūnu ḏālika*) in the fashion (*ᶜalā hayʾati*) of the three *categorical* figures (*al-aškāl al-ḥamlīyya*).[43]

Thus the parallel is clearly assumed and will appear throughout the whole chapter in the moods and the proofs provided. Let us give the proofs of Darapti and Felapton, together with that of Datisi, where **A**c and **I**c "conversions" are used without any explanation.

The hypothetical Darapti is stated as follows:

> Whenever C is D then H is Z (**A**c; minor premise)
> And Whenever C is D then A is B (**A**c; major premise)
> It follows that: It happens that if H is Z, then A is B[44]

Its proof is said to be made "by the *conversion* of the minor, which reduces it to the first figure"[45] (my emphasis). This "conversion" puts on an **A**c proposition and is supposed to lead to an **I**c proposition, so that the mood is reduced to Darii in the first figure, just as happens in categorical syllogistic, which is also evoked in the same page (line 8). Unfortunately, Ibn Sīnā does not say what this "conversion" looks like and how one can convert an implicative **A** proposition, since all implications (not only material implication) are asymmetric, i. e. not convertible, as we saw above.

We can add that in the very short chapter (section VII, chapter 3) devoted to conversion in the context of hypothetical logic, Ibn Sīnā evokes only **E**-conversion[46], i. e. the conversion of the universal negative conditional proposition (which is uncontroversial), together with contraposition (*ᶜaks al-naqīḍ*). It seems then that Ibn Sīnā deliberately omits to define **A**c and **I**c conversions, which raises many questions with regard to their validity. This being so, the proof of Darapti is very doubtful and

[43] Ibn Sīnā, *Al-qiyās*, p. 295 (my emphasis).
[44] Ibn Sīnā, p. 302.
[45] Ibn Sīnā, p. 302.
[46] Ibn Sīnā, p. 385.

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very unconvincing unless one provides a logically acceptable account of **A**c-conversion.

As to the hypothetical Felapton, it is stated as follows:

> Whenever C is D, then H is Z (**A**c; minor premise)
> And Never if C is D, then A is B (**E**c; major premise)
> It follows that: Not whenever H is Z, then A is B[47]

Here too, the proof makes use of **A**c-"conversion," for Ibn Sīnā says explicitly "It is proved by the conversion of the minor (*yubayyanu bi-ᶜaksi al-ṣuġrā*),"[48] without any clear account of this "conversion."

In another mood, namely, Datisi, he also uses **I**c-"conversion," without any explanation of that particular "conversion." For Datisi is the following mood:

> It happens that if C is D then H is Z (**I**c; minor premise)
> And Whenever C is D, then A is B (**A**c; major premise)
> It follows that It happens that if H is Z then A is B[49]

Here it is said "to be proved by the conversion of the minor (*yubayyanu bi-ᶜaksi al-ṣuġrā*)" too. The minor being a particular proposition **I**c, he seems to use an **I**c "conversion," which he had not defined nor shown to be valid previously.

These passages show clearly the weaknesses in the proofs of the hypothetical moods and the absence of any definition or logical account of one of the major rules used in syllogistic, namely, conversion. I could say that, because of these weaknesses, one should just reject Ibn Sīnā's hypothetical moods as being invalid, because they are not properly proved, and consequently this whole hypothetical system. But I won't choose this option, because it would be too easy and would be devastating for the whole hypothetical logic, which contains several systems, mixing in an interesting way between conditional, disjunctive and predicative propositions, and giving rise to many new moods as shown, for instance, in *Arabic logic, from al-Fārābī to Averroes*[50] and in many of Wilfrid Hodges' writings. So the problem is now the following: how can one account for these two conversions and also for **E**c-conversion, which is also used in the proofs of the second figure moods?

As we saw above, Rescher and many scholars after him, for instance,

[47] Ibn Sīnā, *Al-qiyās*, p. 302.
[48] Ibn Sīnā, p. 303.
[49] Ibn Sīnā, p. 303.
[50] Saloua Chatti, *Arabic logic, from al-Fārābī to Averroes* (Birkhäuser, 2019).

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Wilfrid Hodges (in several writings) and Zia Movahed[51] just formalize the **I**c proposition as a conjunction, and by doing so, justify at least **I**c-conversion, since the conjunction is commutative. But as we just noted, this formalization by a conjunction has been criticized by Riccardo Strobino who, it must be said, does not provide any alternative formalization in his article on Ibn Sīnā's logic (2018), nor does he even question Ibn Sīnā's use of **A**c and **I**c "conversions" in the context of hypothetical logic.[52]

Now against this first objection – that the **I**c proposition should not be rendered by a conjunction – Wilfrid Hodges suggests in his article "Identifying Ibn Sīnā's hypothetical logic I: Sentences' forms"[53] that the idea that both **A**c and **I**c contain a conditional is strongly encouraged by Rescher's "translations" of these propositions, which are expressed as follows in Rescher's article:

(i) Always [i. e. "at all times" or "in all cases"]: when A, then (also) C;
(ii) Sometimes: when A, then (also) C.[54]

As we can see, and as Wilfrid Hodges rightly stresses, the propositions stated after the words "always" and "sometimes" are exactly the same, which strongly suggests that if this **A**c proposition contains an implication, the same implication should also be present in **I**c. According to Wilfrid Hodges, these "translations" are misleading because **A**c and **I**c do not have the same logical behaviour: **I**c is simply convertible, while **A**c is only partially convertible, since its conversion leads to a particular proposition. But we can note first that Rescher himself, despite his "translations" formalizes the **I**c proposition as a conjunction. So his translations are incompatible with his formalizations. Secondly, in hypothetical logic, both **A**c and **I**c are said by Ibn Sīnā to express an implication, not only a conjunction, especially those called *luzūmīyya*.

For this reason, Riccardo Strobino says that according to Ibn Sīnā "all *muttaṣil* statements express relations of dependency, not just co-

[51] Zia Movahed, "A critical examination of Ibn Sīnā's theory of the conditional syllogism," *Sophia perennis*, vol. 1, no. 1, 2009.

[52] See in particular Strobino, "Ibn Sīnā's logic," section 4.4, where he just states the "conversions," without making any remark about their alleged "validity," while paradoxically saying at the same time that conversion "is an *essential* method of proof for certain hypothetical syllogisms" (my emphasis, section 4.4). My question is then: if it is so essential in hypothetical logic too, why is it not defined by Ibn Sīnā and how can one account for it with the modern tools that Ibn Sīnā himself did not possess?

[53] Wilfrid Hodges, "Identifying Ibn Sīnā's hypothetical logic I: Sentences' forms", draft, November 2017, p. 6.

[54] Rescher, "Avicenna on the logic of 'conditional' propositions," p. 50.

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instantiation at all or at some times (*Qiyās* V.1)."[55] He adds that in his [presumably informal] explanations, Ibn Sīnā "seems to treat them [i. e. the **I**c propositions] as genuine conditionals… (*Qiyās* V.4)"[56] His third argument is that if **I**c were a conjunction, this "would be incompatible with two other views" that Ibn Sīnā defends, namely 1) "there are *muttaṣil* **A**-propositions in the *luzūmī* sense involving impossible antecedents and impossible consequents (e. g. *Qiyās*, V.4, 273)" and 2) "from a *muttaṣil* **A**-conditional, the corresponding *muttaṣil* **I**-conditional always follows (*Qiyās* V.4, 276)."[57]

Let us consider these points one by one. Point 1 says that all **A**c and **I**c propositions express authentic implications, since the consequent depends on the antecedent and follows it, so that the whole implicative proposition would be true if there is such a following and false if the consequent does not follow from the antecedent. So this proposition is not simply a conjunction, which does not involve any dependency between its components. Thus Rescher's, Movahed's and Hodges' formalizations of **I**c do not seem in accordance with Ibn Sīnā's explanations about the particular (= **I**c) *luzūmīyya* proposition.

This point, however, is partly wrong because of the use of "all," since the *ittifāqīyya* particular propositions do not express any kind of dependency as claimed by Ibn Sīnā himself. So even if we consider that it applies to the *luzūmīyya* propositions, it does not apply anyway to all conditional (*muttaṣila*) propositions, as claimed by Strobino.

Now although Ibn Sīnā does indeed say that the *luzūmīyya* propositions are implications, he also says that both **A**c and **I**c should be convertible and he does use these two conversions in his hypothetical logic, as we saw above. So the problem is: how can one convert an implication? One cannot change the logical facts, and it is a fact that no implication whatsoever is convertible in any logical system, unless it is a double implication, i. e. an equivalence (= "if P then Q and if Q then P").[58] To solve

[55] Riccardo Strobino, "Ibn Sīnā's logic," note 5.

[56] Strobino, note 5.

[57] Strobino, note 5.

[58] We could alternatively consider the option of formalizing the conditional propositions as *double* implications, since this would account for all the conversions in a logically satisfying way and consequently for all the proofs provided by Ibn Sīnā. However, Ibn Sīnā does not admit this option, for according to him, this would introduce serious changes in the formulations of the conversion rules and ultimately of the moods too (Darapti would become something like Darapta, for instance) which he explicitly rejects (see *Al-qiyās*, p. 391-392). For then, **A**c conversion would lead from a universal proposition to a universal proposition too, not to a particular one, as is the case in categorical syllogistic. So it is the parallelism between categorical syllogistic and this

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this particular problem, one can copy the solution for the convertibility of the categorical **A** already provided in categorical syllogistic, since Ibn Sīnā explicitly establishes a parallel between categorical syllogistic with all its rules and this specific hypothetical system.

In categorical syllogistic **A**-conversion holds because **A** is given an existential import. The import of a categorical proposition is defined as follows:

Definition. A proposition has an import if and only if it cannot be true when its subject does not exist.

Corollary. If a proposition can be true when its subject does not exist, then it does not have an import.

In categorical syllogistic, the affirmatives have an import while the negatives do not have an import. This opinion is said by Terence Parsons (2006)[59] to be held by Aristotle, and it is indeed held by both al-Fārābī and Ibn Sīnā as shown in several writings.[60] If we consider Ibn Sīnā's opinion about the import, we can say that he holds explicitly the thesis that affirmative propositions have an import while negative propositions do not in *Al-maqūlāt*,[61] where he says that "Zayd who does not exist [*al-maᶜdūm*] is seeing" is false, while "Zayd who does not exist [*al-maᶜdūm*] is not seeing" is true because it is negative, for it means something like "it is not the case that Zayd who does not exist [*al-maᶜdūm*] is seeing," and this is true, precisely because Zayd does not exist. He repeats the same idea and applies it to quantified propositions in *Al-ᶜibāra*, where he says:

particular hypothetical system that prevents him from using the double implication in that system.

59 Terence Parsons, "The traditional square of opposition," in Edward N. Zalta, (ed.), *Stanford encyclopedia of philosophy*, http://plato.stanford.edu/entries/square/index.html (2006).

60 See, for instance, Saloua Chatti, "Logical oppositions in Arabic logic, Avicenna and Averroes," in *Around and beyond the square of opposition* (Basel: Springer, 2012), p. 21-40; Saloua Chatti, "Existential import in Avicenna's modal logic," *Arabic sciences and philosophy*, vol. 26 (2016), p. 45-71; Saloua Chatti, *Arabic logic, from al-Fārābī to Averroes* (Birkhaüser, 2019), in particular p. 34ff, for al-Fārābī, who says that the affirmative sentences "every man is white," "some men are black" are "all *false* if their subjects *do not exist*" (*Al-maqūlāt*, p. 124.13-14), while the negative sentences, e. g. "not every man is white" *are true*, when their *subjects do not exist.* For Ibn Sīnā, see p. 39ff in Chatti, *Arabic logic*; and Wilfrid Hodges, "Affirmative and negative in Ibn Sīnā," in Catarina Dutilh Novaes & Ole Hjortland Thomassen (ed.), *Insolubles and consequences: Essays in honour of Stephen Read* (Lightning Source, 2012).

61 Ibn Sīnā, *Al-šifā*, *Al-manṭiq* 2, *Al-maqūlāt*, ed. G. Anawati, M. El Khodeiri, A. F. El Ehwani, S. Zayed, rev. and intr. by I. Madkour (Cairo, 1959), p. 259.

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> We say: The real nature of an affirmative [proposition] (*inna ḥaqīqata al-īǧāb*) is to [state] the judgement that the predicate is true of an existent subject (*al-hukmu bi-wuǧūd al-maḥmūl li-al-mawḍūᶜ*), and it is *impossible to state the judgement* that something *is true of a non-existent* thing (*wa mustaḥīlun an yuḥkama ᶜalā ġair al-mawǧūdi bi-anna šayʾan mawǧūdun lahu*). Every subject of an affirmative proposition is satisfied either in the world or in the mind. If one says: Every icosahedron is so-and-so, what is meant by this is that every icosahedron, regardless of where it is found (*yūǧadu kayfa kāna*), is so-and-so. This does not mean that every non-existent icosahedron is a non-existent so-and-so, for if it is non-existent, its properties are non-existent too, since it is not possible for it to be non-existent while its properties are existent…[62]

He adds that these mathematical entities should be considered as existent:

> If the icosahedron doesn't exist then it doesn't satisfy any description, because being non-existent, it can't satisfy any description. If it doesn't exist, how could it be the case that it satisfies something?[63]

The same opinion is expressed in *Al-naǧāt*, where he says what follows:

> … For the negation is true of a non-existent subject, while the affirmation, whether indefinite or definite, is true *only of an existent subject*, so it is true to say that "The phoenix is not seeing" [= "It is not the case that the phoenix is seeing"], while it is not true to say that "The phoenix is not-seeing."[64]

Note that, here, he is talking about the metathetic (*maᶜdūla*) propositions, i. e. the propositions containing a metathetic predicate (a predicate preceded by "not"), such as "Zayd is not-seeing." But according to him, these metathetic propositions are affirmative, and they behave just as the simple affirmative propositions, as he says earlier in the same paragraph.[65]

So we can say that the opinion according to which the affirmative propositions have an import while the negative ones do not is indeed Ibn Sīnā's genuine opinion about the import of all categorical singular, unquantified and quantified propositions.

Now given the parallelism between categorical syllogistic and this first hypothetical system, a parallel solution should hold in this first hy-

[62] Ibn Sīnā, *Al-ᶜibāra*, p. 79-80 (my emphasis).

[63] Ibn Sīnā, *Al-ᶜibāra*, p. 80.

[64] Ibn Sīnā, *Al-naǧāt*, ed. Muḥyī al-Dīn Sabrī al-Kurdī, 2nd ed. (Cairo, 1938), p. 16 (my translation; my emphasis).

[65] Ibn Sīnā, *Al-naǧāt*, p. 15.

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pothetical system too. We will see below that there is some evidence in the text in favour of that opinion even in hypothetical logic.

As to point 2, according to which the **I**c propositions are "genuine conditionals," we can first say that, according to Ibn Sīnā, this is true only of the *luzūmīyya* proposition; it is not true of the *ittifāqīyya* proposition. Ibn Sīnā's evidence for this interpretation of the **I**c *luzūmīyya* proposition can be found in section V, chapter 4 of *Al-qiyās* where he says that the proposition "It happens that if something is an animal then it is a human, that is, if he is talking, and this is so by necessity."[66] By this example, he wants to show that being a human follows (necessarily) from being an animal, at least for some individuals, namely, those who can talk. This is why despite the particular character of this proposition, it is said to involve a necessary implication, provided some conditions are met. The presence of "it happens that" indicates that the *luzūm* occurs only in some cases, not in all cases. These cases involve some presupposed conditions, which account for the necessity of the implication.

He also provides other examples, which, he says, can also be interpreted as involving a *luzūm* between the antecedent and the consequent, despite the fact that their components do not seem at first sight to be necessarily related. The first example is the following: "It happens that if this is a human, then it is a writer."[67] One could think that being a human does not imply being a writer even in some cases. But Ibn Sīnā justifies the presence of a *luzūm* in this example by saying that being a human implies in some cases being a writer, i. e. precisely when we have some evidence that this human is able of inscribing or writing (*yarqumu*[68]) something.

Other examples are provided at page 278, where Ibn Sīnā says: "… As an example, [we can have] 'It happens that if every human is moving his hand, then every human is writing,' and this occurs when each of them moves his hand only when he starts writing; and *this is not impossible*."[69] We can note here that adding "this is not impossible" could show that he is not himself convinced by the presence of a necessary *luzūm* in this proposition. For the example given seems unconvincing.

Elsewhere, he says that "'It happens that if every human is writing,

[66] Ibn Sīnā, *Al-qiyās*, p. 276.
[67] Ibn Sīnā, *Al-qiyās*, p. 276.
[68] The verb *raqama* in classical Arabic means *kataba* (= to write) even if nowadays, it means rather "to number." In both cases, however, the person shows some ability in writing.
[69] Ibn Sīnā, *Al-qiyās*, p. 278 (my emphasis).

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then no human is throwing [something] (or 'is able of throwing [something]'), so that every human does not know how to throw [something] (*fa-kull insān ǧāhilun bi-al-rimāya*),' and this happens if we suppose that every human is weak and does not devote himself to anything else except learning how to write."[70]

Now we can note that Ibn Sīnā's examples are rather strange and justify the relation of following by means of some presuppositions which ought to be present and thought of, in order for the following to hold. But if one supposes that in some past state of affairs, for instance, in prehistoric times, there were no humans at all, then the consequent "it is a human" would not follow necessarily from the antecedent "it is an animal" in the proposition "It happens that if something is an animal then it is a human." So the *luzūm* in this kind of sentences is not obvious at all. Likewise, if one supposes that no human in some particular place or at some particular time is able to write, then the consequent "it is a writer" could not follow from the antecedent "it is a human." So the *luzūm* is not obvious in the particular propositions, even if it could seem plausible at first sight. The two last examples are even stranger and not plausible since why should a human move his hand only when he is writing? Anyone can move his hand in any other circumstance and even without being aware of that moving. Likewise, why should a writer be unable to throw something? This seems very strange too and does not account for any kind of following, since "writing" does not in any sense imply "not being able to throw [something]." This shows that Ibn Sīnā's arguments for the presence of a real implication in **I**c are not at all convincing and just seem contrived. We will return to this point with more precisions below.

The third point stressed by R. Strobino is that interpreting **I**c as a conjunction would be incompatible with two ideas endorsed by Ibn Sīnā which are 1) Some **A** propositions in the *luzūmī* sense have impossible antecedents and impossible consequents and 2) **A** propositions always entail **I** propositions in hypothetical logic.

This means that since the *luzūm* can hold even when the antecedent and the consequent of the proposition are impossible, that is, always false, these *luzūmīyya* propositions cannot be considered as conjunctions, since a conjunction can never be true when its components are false. Now it is true that a *luzūmīyya* proposition can be true when its antecedent and its consequent are false, as we saw in section 1. But this truth holds especially when the proposition is not quantified as when

[70] Ibn Sīnā, *Al-qiyās*, p. 278.

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one says “if men are stones then men are inert” or when it is universal as when one says “Whenever men are stones, men are inert.” But as far as I can see, there is no example of *luzūmīyya* particular propositions (i. e. starting by “it happens that”) where the antecedent and the consequent are impossible. For Ibn Sīnā never says, for instance, “It happens that if 5 is even, then 5 is divisible by 2” or anything like that. So if these propositions involving impossible clauses are true, they would be universal, not particular. In this kind of propositions, the particular character would be very strange.

However, it remains true that the particular propositions follow from the universal ones by subalternation and that the *luzūmīyya* universal propositions can be true even with impossible antecedents and consequents. But the universal propositions which entail the particular ones are those which have an import, not those which do not have an import. And the universal proposition whose antecedent and whose consequent are impossible is true only when it does not have an import, i. e. only when it is formalized as $(\forall s)(Ps \rightarrow Qs)$, in which case it does not entail the particular proposition, since the particular follows only from the **A**c which has an import, i. e. the one which is symbolized as follows: $(\exists s)Ps \wedge (\forall s)(Ps \rightarrow Qs)$. So, even if it is true that the **A**c propositions entail by subalternation the **I**c propositions, this entailment occurs only under some conditions, which are required in order for the subalternation to be valid. When **A**c has no import, it does not entail **I**c, as we noted earlier.

This problem of the entailment of **I** by **A** is also raised in categorical syllogistic and in that system, it is solved by attributing an existential import to all affirmative propositions and denying it from the negative ones. Likewise, a parallel solution should be provided in Ibn Sīnā’s hypothetical logic. Ibn Sīnā seems to hold such a solution when he says:

> Likewise for the following in the conditional [propositions], if it is true in all cases where the antecedent is assumed (*iḏā ṣadaqa fī kulli waḍᶜin li-l-muqaddim*), it is true of some of them (*ṣadaqa ᶜalā al-baᶜḍ*), so that the consequent follows the antecedent in some of these situations (*li-baᶜdi awḍāᶜi al-muqaddim*).[71]

In this passage, Ibn Sīnā considers that both **A**c and **I**c have an import. So if they are formalized as implications, they should contain an augment. Thus they correspond to the categorical **A** and **I** with existential import. The import of Ac hypothetical propositions is also clearly

[71] Ibn Sīnā, *Al-qiyās*, p. 276.

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presupposed in the proof by *ekthesis* of the hypothetical Bocardo, where Ibn Sīnā says what follows:

> From a minor universal affirmative and a major particular negative, Whenever C is D then H is Z and not whenever C is D then A is B; therefore Not whenever H is Z then A is B. This can be proved only by *reductio ad absurdum* or by *ekthesis* by saying: *Let the state where "C is D" and "not (A is B)," be the state where "K is T,"* then "Never if K is T, then A is B;" thus we say: "whenever C is D, then H is Z" and "it happens that when C is D then K is T," it follows that if H is Z then K is T and "never if K is T then A is B;" from which it follows that "not whenever H is Z, then A is B."[72]

In this proof, which is parallel to the proof of the categorical Bocardo, he presupposes that the state "K is T" is an actual illustration of a state where "C is D" and "not (A is B)." So there is a state where C is D is the case while "A is B" is not, and that particular state is the one where "K is T." This is possible because even if the **O**c proposition "not whenever C is D then A is B" does not have an import, i. e. does not require the actuality of the state "C is D," to be true, the **A**c proposition "whenever C is D then H is Z" presupposes that "C is D" is the case and requires that to be itself true.

This import of the **A** propositions, which is shown in the proof by *ekthesis* is also stressed by Wilfrid Hodges who says, talking about al-Fārābī (who also uses *ekthesis* to prove the categorical Bocardo):

> We turn to Bocardo. Here al-Fārābī uses ecthesis, in line with Galen *Institutio logica* [83] 10.8, 24, 1-9 [...]. By those conventions the premise "Every B is a C" has existential import, so it implies that B is nonempty. Hence al-Fārābī's reading of "Some B is not an A" yields that there is an individual that is a B and not an A, and so D is nonempty.[73]

Likewise, in hypothetical logic, "Whenever C is D, then H is Z" presupposes and implies that "C is D" is the case, just like B is presupposed to be nonempty in "Every B is C."

But we will see in the last section that **A**c can also be without import in some cases, and even that this absence of import is required sometimes to validate some rules (for instance, contraposition).

Now in the passage named by Riccardo Strobino "*Qiyās*, V.4, 273" which, according to him, shows that some true conditionals have impossible antecedents, Ibn Sīnā uses the particle *law* to express the conditional. But this particle, in Arabic, expresses counterfactual condition-

[72] Ibn Sīnā, *Al-qiyās*, p. 303 (emphasis added).

[73] See Wilfrid Hodges, Introduction, in Saloua Chatti and Wilfrid Hodges, *Al-Fārābī, Syllogism, an abridgement of Aristotle's Prior analytics*, (Bloomsbury Edition, 2020), p. 53.

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als, not indicative ones. And this is corroborated by the following passage, where Ibn Sīnā says what follows: "For 'If this were two (*law kāna hāḏā ṯinwatan*) and were not divisible by two equals, then this two would be one (*fardan*),' this would be true even if the antecedent is impossible (*muḥāl*)."[74]

But one has to note that although Ibn Sīnā evokes sometimes counterfactual conditionals with impossible antecedents, he does not use them in his hypothetical quantified propositions and moods, for neither the **A**c propositions nor the **I**c ones contain the particle *law* (= "if it were…"). Rather he explicitly says that these counterfactual propositions with impossible antecedents are used in some sciences, where the scientist relies on proofs by *reductio ad absurdum*.[75] So it is in the context of such proofs that they are useful. While the clauses of the **A**c propositions that he uses in his hypothetical logic are expressed by using variables, since these clauses are expressed as "*A* is *B*," or "*H* is *Z*," and when they are quantified as "Every *A* is *B*" or "Some *C* is *D*," so that the meanings of their antecedents and their consequents are not known. This being so, one cannot know if "*A* is *B*" or "Some *C* is *D*" express impossibilities or not.

Now in view of the example considered by Ibn Sīnā above, the counterfactual conditional could be seen as close to the *luzūm*. But this is not always true, because the counterfactual conditional is entirely different from the *luzūm* for at least two reasons:

1) The antecedent of a counterfactual conditional is already known to be false, and the person who uses this counterfactual conditional makes the supposition that it is true in order to see what would ensue from it if it were true. He or she asks: what would happen if the antecedent were supposed true?

2) The relation between the antecedent and the consequent of the counterfactual conditional is not always necessary, and it does not warrant the truth of the consequent even when the antecedent is supposed true.

Take, for instance, the following sentence, which is a classical example of such counterfactual conditionals:

(1) "If John F. Kennedy had not been killed, he would have been elected President of the United States once again."

In such a sentence, we know that the antecedent is false since, unfortunately, J. F. Kennedy has indeed been killed. But the sentence is not

74 Ibn Sīnā, *Al-qiyās*, p. 273.
75 Ibn Sīnā, p. 273.

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necessarily true since nothing warrants that J. F. Kennedy would have been elected once again even if he had still been alive at that time. For one could also say:

(2) "If John F. Kennedy had not been killed, he would not have been elected President of the United States once again."

In all cases, neither (1) nor (2) can be known to be true with certainty. This is so because in such sentences, there is no necessary link between the antecedent and the consequent, unlike what happens with the *luzūm*.

While things are different if we consider the *luzūmīyya* sentences. For the antecedent of a *luzūmīyya* sentence could be either true or false. So we cannot say that its falsity is known in advance. Furthermore, we can determine the truth of a *luzūmīyya* sentence quite clearly, if we take into account the meanings of its antecedent and its consequent. This makes this kind of implicative propositions very different from the counterfactual conditionals.

Now if we follow Ibn Sīnā's explanations and consider that both **A**c and **I**c propositions should be formalized as conditionals, how could we account for the conversions and for all the moods stated and proved by Ibn Sīnā?

For as is well known, $(\forall s)(Ps \to Qs)$ does not entail $(\exists s)(Qs \to Ps)$, which means that **A**c-conversion does not hold if **A**c and **I**c are simple conditionals. But Ibn Sīnā admits **A**c-conversion in his hypothetical logic too. So we must formalize **I**c and **A**c accordingly. Likewise, $(\exists s)(Ps \to Qs)$ does not entail $(\exists s)(Qs \to Ps)$, which means that **I**c-conversion does not hold if **I**c is formalized as a simple conditional. So we must find the adequate formalization which validates this conversion too. And as I stressed above, this can be said not only of material implication, but also of all kinds of implications, since all of them are asymmetric.

Furthermore if both **A**c and **I**c are formalized as simple conditionals, then not only Darapti and Felapton, but also Disamis, Datisi, Ferison and Bocardo do not hold, which is just unacceptable!

To show this, let us take Disamis as an example. If **A**c and **I**c are interpreted as simple conditionals, this mood would be formalized as follows (where P is the middle, Q is the major and R is the minor):

$$[(\exists s)(Ps \to Qs) \wedge (\forall s)(Ps \to Rs)] \overset{?}{\vdash} (\exists s)(Rs \to Qs).$$

When we consider two states s_1 and s_2, and the modern definitions of the quantifiers, which render the universal quantifier by a series of

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conjunctions and the existential one by a series of (inclusive) disjunctions[76] we would formalize the mood as follows [where "→" stands for the conditional, and "∧" stands for the conjunction]:

$$\{[(Ps_1 \to Qs_1) \vee (Ps_2 \to Qs_2)] \wedge [(Ps_1 \to Rs_1) \wedge (Ps_2 \to Rs_2)]\} \overset{?}{\vdash} [(Rs_1 \to Qs_1) \vee (Rs_2 \to Qs_2)].$$

Here we can easily show that the two premises do not entail the conclusion since we can find a case of falsity under the conclusion when the two premises are true. This case of falsity occurs when Ps_1, Ps_2, Qs_1 and Qs_2 are all false while Rs_1 and Rs_2 are both true. In that specific case, the main conjunction in the left, which relates the two premises, is true, while the disjunction in the right, which expresses the conclusion, is false, because its elements are both false. Since the premises are both true, while the conclusion is false, this means that the premises *do not entail* the conclusion. The same can be said about all third figure moods, which are no more valid (i. e. productive) when both **A**c and **I**c are formalized as simple conditionals.

But Ibn Sīnā does hold all third figure moods in his hypothetical logic too, so **I**c must be formalized accordingly i. e. in a way that validates all of these moods. The formalization of the particular proposition by a conjunction does validate these moods in categorical syllogistic, and it would validate them in hypothetical logic too, if **I**c were formalized as a conjunction. Now this formalization of **I**c by a conjunction seems to some people inadequate, because it does not warrant the presence of a strong link between the antecedent and the consequent. But as we just saw, Ibn Sīnā's own explanations of the presence of this "strong link" are not convincing at all. Besides that, is there any other formalization that would account for all the rules and the moods held by Ibn Sīnā in this specific hypothetical system? We will discuss this point in the next section.

As to **E**c, it is in all cases the contradictory of **I**c. It is usually formalized as a simple conditional expressed as "$(\forall s)(Ps \to \sim Qs)$." Thus formalized, **E**c does indeed contradict **I**c, which is usually formalized as "$(\exists s)(Ps \wedge Qs)$," since $(\forall s)(Ps \to \sim Qs) \equiv \sim(\exists s)(Ps \wedge Qs)$.

[76] See, for instance, the definition provided by W. V. O. Quine in *Methods of logic* (1950), French transl. *Méthodes de logique* (Paris: Armand Colin, 1973), which is the following: "'$(\exists x)Fx$' and '$(x)Fx$' become respectively: $Fa \vee Fb \vee \ldots Fh$; $Fa.Fb.\ldots.Fh$" (p. 128, French translation; Quine uses "(x)" instead of $(\forall x)$ for the universal quantifier, and "." (a dot) to express the conjunction instead of the more frequently used symbol "∧").

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Besides that, Ibn Sīnā himself does hold in *Al-qiyās* the following equivalence: "$(P \to Q) \equiv \sim(P \wedge \sim Q)$"[77] and also the following: "$(P \to Q) \equiv (\sim P \vee Q)$."[78] So these formalizations are not foreign to him. The latter accounts for the translatability between conditional and disjunctive propositions which will also be considered below.

This formalization of **I**c by a conjunction, however, does not account for the *luzūmī* character of this particular proposition claimed by Ibn Sīnā. Nevertheless, despite the fact that Ibn Sīnā's arguments are not very convincing, can we find another formalization which would account for this alleged *luzūmī* character? If we consider that this formalization should also preserve the conversions and the relations of the square of oppositions, which are also claimed by Ibn Sīnā, we would choose the following formalization for **I**c and its contradictory **E**c:

$$\mathbf{I}\text{c}: (\exists s)Ps \wedge (\exists s)(Ps \to Qs)$$
$$\mathbf{E}\text{c}: \sim[(\exists s)Ps \wedge (\exists s)(Ps \to Qs)]$$

This accounts for the *luzūmī* character of **I**c. It also accounts for Ibn Sīnā's claim that the negation denies "the implication itself," not only the consequent, and for the conversions as well as the relations of the square of opposition. For with these formalizations, **A**c does entail **I**c and **E**c does entail **O**c, since the following implications are logically (i. e. formally) valid:

$$\mathbf{A}\text{c}: (\exists s)Ps \wedge (\forall s)(Ps \to Qs) \text{ implies } \mathbf{I}\text{c}: (\exists s)Ps \wedge (\exists s)(Ps \to Qs)$$
$$\mathbf{E}\text{c}: \sim[(\exists s)Ps \wedge (\exists s)(Ps \to Qs)] \text{ implies } \mathbf{O}\text{c}: \sim[(\exists s)Ps \wedge (\forall s)(Ps \to Qs)]$$

We can show this validity for the first couple of propositions by considering two states s_1 and s_2 and translating accordingly the two formulas:

$$\{(Ps_1 \vee Ps_2) \wedge [(Ps_1 \to Qs_1) \wedge (Ps_2 \to Qs_2)]\}$$
$$\vdash \{(Ps_1 \vee Ps_2) \wedge [(Ps_1 \to Qs_1) \vee (Ps_2 \to Qs_2)]\}$$

This entailment, which accounts for subalternation, obviously holds since a conjunction entails an inclusive disjunction, given that when the conjunction is true, its two conjuncts are true, which means that they remain true in the proposition **I**c. So when **A**c is true, **I**c is true too. They are thus related by subalternation.

[77] Ibn Sīnā, *Al-qiyās*, p. 280.
[78] Ibn Sīnā, p. 251.

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Likewise, all the other relations of the square are valid. As to **A**c-conversion and **I**c-conversion, they are both valid, for the following entailments hold, as one can easily show:

Ac-conversion: $(\exists s)Ps \wedge (\forall s)(Ps \to Qs) \vdash (\exists s)Qs \wedge (\exists s)(Qs \to Ps)$

Ic-conversion: $(\exists s)Ps \wedge (\exists s)(Ps \to Qs) \vdash (\exists s)Qs \wedge (\exists s)(Qs \to Ps)$

The presence of the augment in **A**c can be justified by the following passage of *Al-qiyās*:

> When we say: "If A is B, then H is Z," we assume from this (*nūǧibu min hāḏa*) that at any time where "A is B" *is the case* and when A is B then H is Z, as if the fact that H is Z follows the fact that A is B, *in so far as in effect* A is B (*min ḥayṯu huwa kāʾinun A [huwa] B*)[79]

In this passage, Ibn Sīnā stresses the idea that the antecedent of **A**c must be true in order for **A**c itself to be true. Naturally **O**c should be formalized accordingly, being the contradictory of **A**c.

The same idea can also be found in the following passage:

> When we say "Whenever C is B, then H is Z" we don't only mean by "whenever" the generalizing of what is intended (*taʿmīm al-murād*), so that what is expressed is like saying "Every time where C is B, then H is Z;" rather it involves generalizing every state (*ḥāl*) connected (*yaqtarinu*) to the sentence "Every C is B" so that *any state* or condition related to [that sentence], *which makes "C is B" true* (*mawǧūdan*) cannot do so without also making "H is Z" true.[80]

Here too, he says explicitly that the implication is true when it makes the consequent true whenever the antecedent is itself true.

As to **E**c-conversion, it is valid too and would be expressed as follows:

Ec-conversion: $\sim\{(\exists s)Ps \wedge (\exists s)(Ps \to Qs)\} \vdash \sim\{(\exists s)Qs \wedge (\exists s)(Qs \to Ps)\}$

Unfortunately, the above formalization of **E**c does not account for some very important moods held by Ibn Sīnā, namely, Celarent from the first figure and Cesare from the second figure. This is a real problem, especially for Celarent, since as all moods of the first figure, Celarent is said to be "perfect (*qiyāsun kāmil*)"[81] by Ibn Sīnā. This mood is stated as follows in Ibn Sīnā's hypothetical system:

[79] Ibn Sīnā, *Al-qiyās*, p. 263 (my emphasis).
[80] Ibn Sīnā, p. 265 (my emphasis).
[81] Ibn Sīnā, p. 296.

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Whenever A is B, then C is D
Never if C is D then H is Z
It follows that Never if A is B, then H is Z[82]

The premises of this mood and its conclusion would be expressed as follows, if **E**c were formalized as above (where "A is B" is rendered by P, "C is D" by Q, and "H is Z" by R):

$$\text{Premises: } \{[(\exists s)Ps \wedge (\forall s)(Ps \rightarrow Qs)] \wedge \sim[(\exists s)Qs \wedge (\exists s)(Qs \rightarrow Rs)]\}$$
$$\text{Conclusion: } \sim[(\exists s)Ps \wedge (\exists s)(Ps \rightarrow Rs)]$$

With two states s_1 and s_2, we would have the following:

$$\text{Premises: } \{(Ps_1 \vee Ps_2) \wedge [(Ps_1 \rightarrow Qs_1) \wedge (Ps_2 \rightarrow Qs_2)]\}$$
$$\wedge \sim\{(Qs_1 \vee Qs_2) \wedge [(Qs_1 \rightarrow Rs_1) \vee (Qs_2 \rightarrow Rs_2)]\}$$
$$\text{Conclusion: } \sim\{(Ps_1 \vee Ps_2) \wedge [(Ps_1 \rightarrow Rs_1) \vee (Ps_2 \rightarrow Rs_2)]\}$$

Unfortunately, the premises do not entail the conclusion with this formalization of **E**c, for we could have a case of falsity under the conclusion, when both premises are true, precisely, when Ps_1, Qs_1, and Qs_2 are true, while Rs_1, Rs_2, and Ps_2 are false. In this case, the two implications in **E**c (the major premise) are false, which makes the external negation of **E**c true, the whole **A**c (the minor premise) is true, since its disjunction is true and its two implications are true too, but the conclusion is false, since its external negation is false, given that its disjunction is true ("Ps_1" being true), while its second implication, namely, "$Ps_2 \rightarrow Rs_2$" is true (Ps_2 and Rs_2 being false), even if its first implication [= "$Ps_1 \rightarrow Rs_1$"] is false (Ps_1 being true, while Rs_1 is false).

This being so, the mood becomes invalid! This result is just unacceptable, especially if we consider that Celarent is a first figure mood, i. e. a perfect mood, as Ibn Sīnā rightly stresses. So there is something wrong in this formalization of **E**c, and consequently that of **I**c, since they are both related, so that if one of them is rejected, the other one should be rejected too.

One has thus to reconsider the previous formalizations of **I**c as a conjunction and **E**c as its negation, which is also the one held by all logicians in categorical logic. In favour of these formalizations, we can first say that Ibn Sīnā's account of the *luzūmi* character of **I**c is not convincing at all. And he seems himself not very convinced by this *luzūm* in **I**c since, when evoking his second example ("It happens that if this is human then this is a writer"), he says that "There is no harm (*lā baʾsa*) in

[82] Ibn Sīnā, *Al-qiyās*, p. 296.

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considering this proposition as true by agreement or by necessity, just as the particular is true both absolutely and by necessity."[83] Likewise, when stating one of his last examples, he says that the situations described "are not impossible" as if he was apologizing for the strangeness of his example. Secondly and more importantly, the last formalizations validate all the conversions, all the moods of all figures and all the relations of the square of opposition. They also clearly exhibit the parallelism stressed by Ibn Sīnā between categorical syllogistic and this specific hypothetical system. So they are logically much more adequate than the formalization involving a real implication in **I**c.

We can add that in categorical logic, modern logicians all agree about the fact that "$(\exists x)(Fx \rightarrow Gx)$" does not render adequately the particular affirmative proposition, either because it is "trivial"[84] as Quine says in *Methods of logic* or because "An **I** proposition of the form 'Some Φ's are Ψ's' is symbolized as '$(\exists x)(\Phi x . \Psi x)$,' which asserts that there is at least one thing having both the property Φ and the property Ψ. But the proposition '$(\exists x)(\Phi x \supset \Psi x)$' asserts only that there is at least one object which either has the property Ψ or does not have the property Φ, which is a very different and much weaker assertion"[85] as Irving Copi says in *Symbolic logic*.

Likewise, Ibn Sīnā's **I**c proposition would assert something like "There is at least one state where the antecedent and the consequent are both true." This is not very different from what **I**c actually says in all the examples provided by Ibn Sīnā, for we can say that there is at least one situation where "this is an animal" and "this is a human" are both true, or where "this is a human" and "this is writing" are both true, etc. since nothing warrants that being an animal really implies being a human, or that being a human really implies being a writer.

In addition, Ibn Sīnā does indeed hold the equivalence between "$P \rightarrow Q$" and "$\sim P \vee Q$" in his hypothetical logic, which Irving Copi evokes in his quotation, even if Ibn Sīnā does not endorse material implication.

Last but not least, **I**c could not anyway be formalized as "$(\exists s)(Ps \rightarrow Qs)$," since it should be convertible, and this implication is not convertible.[86] This is why we have added the augment in its formalization. But

[83] Ibn Sīnā, *Al-qiyās*, p. 277.

[84] Quine, *Méthodes de logique*, p. 128.

[85] Irving M. Copi, *Symbolic logic*, 3rd ed. (London: Macmillan, 1967), p. 78.

[86] The problem of **A**'s and **I**'s conversions is not raised by modern logicians in their own systems, because these traditional conversions are not among the rules that they use in their systems, even if **I** is naturally convertible since it is rendered as a conjunction, which is commutative. On the contrary, all conversions are crucial rules

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with the augment, it is very close to a conjunction, since its truth conditions are almost the same, when we consider one state, as we can see in the following tables:.

P	$\wedge$	$(P$	$\to$	$Q)$	/	P	$\wedge$	Q
1	**1/0**	1	1/0	1		1	**1**	1
1	**0**	1	0	0		1	**0**	0
0	**0**	0	1/0	1		0	**0**	1
0	**0**	0	1/0	0		0	**0**	1

And they are exactly the same when there is a real entailment between P and Q, since then, we would have the following truth conditions (with one state):

P	$\wedge$	$(P$	$\to$	$Q)$
1	**1**	1	1	1
1	**0**	1	0	0
0	**0**	0	1/0	1
0	**0**	0	1/0	0

So we can say that there is no harm (*lā baʾsa*, as Ibn Sīnā says!) in considering the **Ic** propositions as conjunctions instead of real implications.

Given these explanations, the four kinds of conditional propositions should be formalized as follows:

Ac: $(\exists s)Ps \wedge (\forall s)(Ps \to Qs)$

Ic: $(\exists s)(Ps \wedge Qs)$

Ec $= (\forall s)(Ps \to {\sim}Qs)$ $[= {\sim}$**Ic** $= {\sim}(\exists s)(Ps \wedge Qs)]$

Oc $= {\sim}$**Ac** $= {\sim}[(\exists s)Ps \wedge (\forall s)(Ps \to Qs)]$

Consequently, the **Ac** and **Ec** propositions that we will discuss below should be formalized as follows:

Ac3: $(\exists s)I_1s \wedge (\forall s)(I_1s \to A_2s)$

Ac4: $(\exists s)I_1s \wedge (\forall s)(I_1s \to I_2s)$

Ac10: $(\exists s)A_1s \wedge (\forall s)(A_1s \to O_2s)$

Ec1: $(\forall s)(A_1s \to {\sim}A_2s)$ $[= (\forall s)(A_1s \to O_2s)]$

Ec3: $(\forall s)(I_1s \to {\sim}A_2s)$ $[= (\forall s)(I_1s \to O_2s)]$

Ec4: $(\forall s)(I_1s \to {\sim}I_2s)$ $[= (\forall s)(I_1s \to E_2s)]$

in the syllogistic. This is why they must be given much importance in this context.

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However, although these formalizations validate all the relations of the square and give rise to some very interesting geometrical figures, they do not seem to account for some equivalences held by Ibn Sīnā. In the next section, we will analyse in detail these equivalences and the problems that they raise.

5. THE INCOMPATIBILITY BETWEEN THESE FORMALIZATIONS AND THE EQUIVALENCES CLAIMED BY IBN SĪNĀ

As we said above, these formalizations validate all the relations of the square, which are admitted by Ibn Sīnā in his hypothetical logic too, as appears in the following quotation (and elsewhere in the same section):

> … and you know [what] contradiction, contrariety, subcontrariety and subalternation [signify] so we don't need to tell you again about them, for they are defined as they are in the case of the predicative propositions.[87]

Besides that, these propositions give rise to many logical figures, from squares to figures of 16 vertices and beyond, through hexagons and various kinds of octagons. For this reason, they are very interesting for the theory of oppositions.

However, in his informal explanations, Ibn Sīnā does not seem to interpret **A**c and **O**c as we did above, i. e. by adding the augments, although he does indeed say, in several parts of his text, that the relations of the square hold in the hypothetical logic too.

This raises a problem for in his informal explanations, Ibn Sīnā equates some **A**c propositions with some **E**c ones. For instance, he says in the passage below that the following **E**c proposition equals an **A**c one:

> The sentence "Never when every A is B then every C is D" (*laysa al-battata iḏā kāna kull A B fa-kull C D*) in its general sense, is equivalent in force to (*fī quwwati*) the sentence "Whenever every A is B, then not every C is D" (*kullamā kāna kull A B, fa-laysa kull C D*) and in the sense of the connection and the real implication, it equals "whenever every A is B, then it does not follow (*laysa yalzamu*) that every C is D."[88]

So, according to what he is saying in this quotation, the following **E**c: "Never when A_1 then A_2" [= **E**c1 above = $(\forall s)(A_1 s \to \sim A_2 s)$] is equivalent to the following **A**c: "Whenever A_1 then not A_2" [= **A**c10 above] (where A_1 = Every A is B and A_2 = Every C is D).

But such an equivalence holds only if this **A**c, i. e. **A**c10 above, is formalized as follows: $(\forall s)(A_1 s \to \sim A_2 s)$, for only in that case, we would have

[87] Ibn Sīnā, *Al-qiyās,* p. 362.
[88] Ibn Sīnā, p. 366.5-7 (my translation).

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the following equivalence: (**E**c1) $(\forall s)(A_1 s \rightarrow {\sim}A_2 s) \equiv (\forall s)(A_1 s \rightarrow {\sim}A_2 s)$ (**A**c10).

Now with this formalization, **A**c10, which is considered as equivalent to **E**c1, does not contain the augment. If it did contain the augment, it would be formalized as follows: $(\exists s)A_1 s \wedge (\forall s)(A_1 s \rightarrow {\sim}A_2 s) \equiv (\exists s)A_1 s \wedge (\forall s)(A_1 s \rightarrow O_2 s)$ (= **A**c10) and would not be equivalent to **E**c1, which does not contain any augment and is formalized as "$(\forall s)(A_1 s \rightarrow {\sim}A_2 s)$." In that case the only valid implication is the one that leads from **A**c10 to **E**c1, since a conjunction implies its elements. Unfortunately, there is no implication which would lead from **E**c1 to **A**c10, as we will show below, unlike what Ibn Sīnā seems to think.

But Ibn Sīnā defends the idea that both implications hold and he provides two proofs to show their validity. The first one is supposed to show that **E**c1 implies **A**c10 and the second one shows that **A**c10 implies **E**c1.

The first proof runs as follows:

> If "Never when every A is B then every C is D" [= ${\sim}(\exists s)(A_1 s \wedge A_2 s)$] is true then "whenever every A is B, then not every C is D" is true too; otherwise its contradictory (*naqīḍuhā*), i. e. the following: "Not whenever every A is B, then not every C is D" is true… in which case (*fa-yakūnu ḥīnaʾidhin*) "it happens that (*qad kāna*) every A is B and also (*wa maʿahu*) every C is D" [= $(\exists s)(A_1 s \wedge A_2 s)$ = **I**c1 above]; but we said "Never when every A is B then every C is D," which is absurd.[89]

If we summarize this proof, we can say that, according to him:

– If (1) "${\sim}(\exists s)(A_1 s \wedge A_2 s)$" does not imply "whenever A_1 then ${\sim}A_2$";

– Then (2) "${\sim}(\exists s)(A_1 s \wedge A_2 s)$" would be true and "whenever A_1 then ${\sim}A_2$" would be false;

– In which case (3) "Not whenever A_1 then ${\sim}A_2$" would be true [= ${\sim}(\forall s)(A_1 s \rightarrow {\sim}A_2 s)$];

– Consequently (4) "$(\exists s)(A_1 s \wedge A_2 s)$" (= **I**c) would also be true;

– But this contradicts "${\sim}(\exists s)(A_1 s \wedge A_2 s)$" (the initial **E**c1 proposition) and leads to an absurdity.

In this proof, we can see that Ibn Sīnā is assuming that (3) entails (4). But (3) entails (4) only if **A**c10 does not contain the augment. If **A**c did contain the augment, it would be formalized in another way, and in that case **E**c1 would not imply **A**c10, as one can easily show. So **A**c is presupposed to be merely a conditional in this proof. It is not assumed to contain an augment related by a conjunction to the conditional.

As to the second proof [that **A**c10 implies **E**c1], it runs as follows:

[89] Ibn Sīnā, *Al-qiyās*, p. 366.

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> If the sentence "Whenever every A is B, then every C is D" is true and the sentence "Never when every A is B, then not every C is D" is not true, then its contradictory would be true, that is, "It happens that when every A is B then not every C is D." The sentence "Every A is B" would then be true while its consequent would not be "every C is D," since it [would be] "not every C is D;" but we said that "whenever every A is B," then the consequent must be "every C is D," and this is absurd (*ḫalf*).[90]

We can express this proof as follows:

– If "whenever A_1 then A_2" does not imply "never when A_1 then $\sim A_2$," then the first would be true and the second [proposition], namely "$(\forall s)(A_1 s \to \sim\sim A_2 s)$," which is equivalent to "$\sim(\exists s)(A_1 s \wedge \sim A_2 s)$" would be false;

– Consequently "$(\exists s)(A_1 s \wedge \sim A_2 s)$" would be true, being the contradictory of "$\sim(\exists s)(A_1 s \wedge \sim A_2 s)$;" therefore both "$A_1 s$" and "$\sim A_2 s$" would be true too (since their conjunction is true);

– But what was said in the beginning is that "whenever A_1 then A_2" is true; so A_2 (the consequent of that proposition) should be true too, since A_1, its antecedent, is assumed to be true;

– However "A_2" and "$\sim A_2$" cannot be true together, being contradictory. This is absurd.

This proof presupposes first that **A**c's consequent should be true [since A_1 is true], second it assumes that the **E**c proposition is false, and consequently its contradictory, namely **I**c, i. e. "$(\exists s)(A_1 s \wedge \sim A_2 s)$" is true, but **I**c is true only when both "$A_1 s$" and "$\sim A_2 s$" are true. The result is that **A**c and this **I**c, i. e. "$(\exists s)(A_1 s \wedge \sim A_2 s)$" cannot be true together.

Consequently, **A**c can be formalized as "$(\forall s)(A_1 s \to A_2 s)$" or as "$(\exists s)A_1 s \wedge (\forall s)(A_1 s \to A_2 s)$," since both are incompatible with "$(\exists s)(A_1 s \wedge \sim A_2 s)$," the first proposition being its contradictory, while the second proposition is its contrary.

So this proof does not presuppose that **A**c should necessarily be formalized without the augment; it can be formalized with or without the augment. It thus admits both interpretations. In other words, this **A**c always implies that **E**c, with or without the augment, which means that we do have the following entailments:

$$(\forall s)(A_1 s \to A_2 s) \vdash (\forall s)(A_1 s \to \sim\sim A_2 s)$$

[= **A**c without the augment entails **E**c];

[90] Ibn Sīnā, *Al-qiyās*, p. 367.

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$$(\exists s)A_1s \wedge (\forall s)(A_1s \to A_2s) \vdash (\forall s)(A_1s \to \sim\sim A_2s)$$

[= **A**c with the augment entails **E**c too].

Nevertheless, despite the adequacy of that particular proof, Ibn Sīnā's theory as a whole raises a problem, for if **A**c does not contain the augment, the equivalences hold but the relations of the square do not hold, unlike what Ibn Sīnā says.

Let us explain this more clearly. If **A**c does not contain the augment, it is formalized as $(\forall s)(Ps \to Qs)$. In this case, it is not the contrary of **E**c, which is formalized as $(\forall s)(Ps \to \sim Qs)$, because both can be true together, precisely when their [common] antecedent is false. As a consequence, subcontrariety and subalternation do not hold too. So almost all the relations of the square do not hold since in that case the formalizations of the hypothetical quantified propositions make them parallel to the modern formalizations of the categorical propositions. And as is well known, in modern logic only contradiction holds. Likewise, here too, all the relations of the square, except contradiction, do not hold if **A**c does not contain the augment, they hold only when it contains the augment. As a consequence, Ibn Sīnā 's opinion on the alleged equivalence between some **A**c propositions and some **E**c ones is not compatible with his claim that the relations of the square hold in this system too.

Furthermore, he applies the following claim to other **A**c and **E**c propositions:

> The way these are reduced (*waǧhu al-ruǧūʿi*) is to keep the quantity of the proposition as it was and to change the quality, the antecedent remaining as it was and being followed by a contradictory consequent.[91]

So according to him, the following are also equivalent:

– "Never when some A is B then every C is D" (**E**c3) and "whenever some A is B then not every C is D" (**A**c3);[92]

– "Never when some A is B, then some C is D" (**E**c4) and "whenever some A is B, then no C is D" (**A**c4).[93]

But these equivalences presuppose, like the first one, that **A**c does not contain the augment. For all these, as for the first one, he proves that the **E**c proposition implies the **A**c one, and that the **A**c proposition implies the **E**c one. However, as we saw above, **A**c does imply **E**c, in both formalizations of **A**c, but **E**c implies **A**c only when **A**c does not contain the augment, i. e. is formalized as "$(\forall s)(A_1s \to A_2s)$." So if **A**c contains

[91] Ibn Sīnā, *Al-qiyās*, p. 366.
[92] Ibn Sīnā, p. 366.
[93] Ibn Sīnā, p. 366.

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the augment, it cannot be entailed by its corresponding **E**c, although it does entail it. Therefore the only **A**c that is entailed by **E**c is the one that does not contain the augment.

On the other hand, however, he says what follows:

> The two universal affirmatives which have two contradictory consequents are *contrary* (*fī quwwati al-mutaḍāddatayni*), so they are both false and they are not contradictory. This is so because one of these affirmatives has the power of the universal negative, which is opposed to the other one by *contrariety*.[94]

So according to him, both **A**c with "A_2" as consequent and **A**c with "$\sim A_2$" as consequent are contrary, because the second **A**c is equivalent to an **E**c. But if the first **A**c is formalized as "$(\forall s)(A_1 s \to A_2 s)$" and the second **A**c is formalized as "$(\forall s)(A_1 s \to \sim A_2 s)$," then they are not contrary, unlike what Ibn Sīnā says, because thus formalized, they can be true together, precisely when $A_1 s$ is false.

This is so because these two universal propositions are contrary only when: (1) the first **A**c is formalized as "$(\exists s)A_1 s \wedge (\forall s)(A_1 s \to A_2 s)$" and the second **A**c is formalized as "$(\forall s)(A_1 s \to \sim A_2 s)$;" or (2) the first **A**c is formalized as "$(\exists s)A_1 s \wedge (\forall s)(A_1 s \to A_2 s)$" and the second **A**c is formalized as "$(\exists s)A_1 s \wedge (\forall s)(A_1 s \to \sim A_2 s)$;" or (3) the first **A**c is formalized as "$(\forall s)(A_1 s \to A_2 s)$" and the second **A**c is formalized as "$(\exists s)A_1 s \wedge (\forall s)(A_1 s \to \sim A_2 s)$."

For only in these three cases, the two **A**c's are contrary, as we can show by a simple calculus and as has been shown in the case of categorical propositions by Chatti & Schang in "The cube, the square and the problem of existential import."[95] But given that according to Ibn Sīnā, only the affirmative (categorical) propositions have an import, as we have shown above and given the parallelism between the categorical propositions and the hypothetical conditional ones, (2) and (3) can hardly be admitted in his theory.

On the other hand, he also says that the following **O**c: "Not whenever A_1 then A_2" implies the following **I**c: "It happens that when A_1 then $\sim A_2$."

His proof is the following:

> If there is no implication, then "Not whenever A_1 then A_2" would be true and "It happens that when A_1 then $\sim A_2$" would be false.

[94] Ibn Sīnā, *Al-qiyās*, p. 368 (my emphasis).

[95] Saloua Chatti and Fabien Schang, "The cube, the square and the problem of existential import," *History and philosophy of logic*, vol. 34, no. 2 (2013): 101-32, esp. p. 114.

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> In that case, (1) "Never when A_1 then $\sim A_2$" would be true.
> And (1) implies (2) "Whenever A_1 then A_2"
> But we said: "Not whenever A_1 then A_2," which is absurd.[96]

Here too, he presupposes that **E**c implies **A**c with a contradictory consequent (since he considers that (1) implies (2)), which, as we showed above, is valid only when **A**c has no augment.

According to him, this means that **O**c and **I**c are subcontrary since he claims:

> And these two particulars can be true together (*qad taṣduqāni maᶜan*).[97]

Unfortunately, if **O**c and **I**c are formalized as "$(\exists s)(A_1 s \wedge \sim A_2 s)$" and "$(\exists s)(A_1 s \wedge A_2 s)$" respectively, they cannot be true together, because when "$A_2 s$" is true, "$\sim A_2 s$" is false, which means that if the first conjunction is true, the second one is false. So both particulars cannot be true together when formalized in that way. They cannot therefore be subcontrary as Ibn Sīnā claims.

In another part of the text, he also says what follows:

> If the universal is true, then the particular, that is, its *subaltern*, which is implied by it, is also true, and if the particular is false then the universal is false, but not conversely (in both cases).[98]

This means that he explicitly holds subalternation between **A**c and **I**c and between **E**c and **O**c.

But then, he should admit the case where **A**c contains the augment, while **E**c does not contain the augment, given that the subalternation between the two affirmative propositions is valid only when **A**c contains the augment, while the one between the two negatives is valid only when **O**c contains the augment. Otherwise, both subalternations are not valid.

However, this alternative, even if it validates the contrarieties and all the relations of the square, is not in accordance with the equivalences held, as we have just shown. It does not validate either the principle of contraposition, i. e. the following equivalence: "$(\forall s)(Ps \rightarrow Qs) \equiv (\forall s)(\sim Qs \rightarrow \sim Ps)$," for this principle is valid only when **A**c is expressed *without* the augment.

So Ibn Sīnā's claims are incompatible with each other, which raises a problem: how should we formalize **A**c and **O**c, if we wish to account for all of Ibn Sīnā's claims? For if we add the augments, the relations of the

96 Ibn Sīnā, *Al-qiyās*, p. 371.
97 Ibn Sīnā, p. 371.
98 Ibn Sīnā, p. 372 (my emphasis).

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square hold but not the equivalences stated by Ibn Sīnā nor the principle of contraposition, and if we remove the augments, the equivalences stated and the principle of contraposition hold but not the relations of the square.

We will try to solve this problem in the next section.

6. TWO POSSIBLE SOLUTIONS TO THE PROBLEM

To answer this question, we can choose between two possible solutions:

1) We can weaken the equivalences held by Ibn Sīnā by considering them as implications instead. In that case **A**c would contain the augment, and no other interpretation of **A**c would be admitted;

2) We can keep the equivalences and the relations of the square, but distinguish between two kinds of **A**c and accordingly two kinds of **O**c, which are their respective contradictories.

In the first case, the **A**c propositions imply the **E**c ones, but not conversely. As a consequence all the equivalences become implications where the antecedents would be the **A**c propositions and the consequents would be the **E**c ones, and at the same time, all the relations of the square hold. For instance, we would have the following implications, involving **A**c and **E**c propositions and **I**c and **O**c ones:

– "Whenever some A is B then not every C is D" (**A**c3) implies "Never when some A is B then every C is D" (**E**c3), but not conversely;

– "It happens that when every A is B then not every C is D" implies "Not whenever every A is B then every C is D," but not conversely.

This would make the text coherent. But this solution does not conform what Ibn Sīnā actually says, since he explicitly claims and proves that some **E**c propositions imply some **A**c ones, which would not be acceptable if we choose this solution.

In addition, it has another inconvenience, which is that it does not conform also the equivalences or the implications stated between some conditional propositions and some disjunctive ones. For according to Ibn Sīnā, "$(P \to Q) \equiv (\sim P \vee Q)$," as he says several times in *Al-qiyās*,[99] from which it follows that if **A**c is interpreted as a simple conditional, then the following equivalence holds: **A**c $= (\forall s)(Ps \to Qs) \equiv (\forall s)(\sim Ps \vee Qs) =$ $\mathbf{E}_D$.

However, if **A**c contains the augment, this equivalence would not hold,

[99] Ibn Sīnā, *Al-qiyās*, for instance p. 251.

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since "$(\exists s)Ps \wedge (\forall s)(Ps \rightarrow Qs)$" is not equivalent to "$(\forall s)({\sim}Ps \vee Qs)$," although it implies it.

In addition, he says what follows:

> The disjunctive proposition where the disjunction is real and whose elements are affirmative, implies the conditional proposition whose antecedent is the contradictory of one of the elements of the disjunctive, while its consequent is the other element [of the disjunctive]..., if these elements have the same quantity and the same quality.[100]

For instance: "Always either every A is B or every C is D" implies "Whenever not (every A is B) then every C is D," and also "Whenever not (every C is D) then every A is B."[101]

If we formalize these propositions, we get the following entailments: (1) $(\forall s)(A_1 s \veebar A_2 s) \vdash (\forall s)({\sim}A_1 s \rightarrow A_2 s)$ and (2) $(\forall s)(A_1 s \veebar A_2 s) \vdash (\forall s)({\sim}A_2 s \rightarrow A_1 s)$ [where "$\veebar$" stands for the exclusive disjunction].

Unfortunately, these entailments hold only when **A**c does not contain the augment. So, here, what is presupposed is that the disjunctive propositions and the conditional ones are mutually translatable only when **A**c does not contain the augment. For if we add the augment to **A**c, we would have the following:

$$(\forall s)(A_1 s \veebar A_2 s) \overset{?}{\vdash} (\exists s){\sim}A_1 s \wedge (\forall s)({\sim}A_1 s \rightarrow A_2 s)$$

$$(\forall s)(A_1 s \veebar A_2 s) \overset{?}{\vdash} (\exists s){\sim}A_2 s \wedge (\forall s)({\sim}A_2 s \rightarrow A_1 s)$$

Unfortunately, these implications do not hold, as suggested by the question marks, since the second proposition may be false when the first one is true.

This means that the entailments between disjunctive and conditional propositions, held by Ibn Sīnā, are not valid when **A**c contains the augment. They are valid only when **A**c does not contain the augment.

On the other hand, he also holds the following, where the disjunctive is not real (i. e. not exclusive) and has negative elements: "Always either no A is B or no C is D" entails "Whenever some A is B, then no C is D."[102]

When formalized, this can be expressed as follows:

$$(\forall s)(E_1 s \vee E_2 s) \vdash (\forall s)(I_1 s \rightarrow E_2 s) \text{ (}\mathbf{A}\text{c without the augment)} \quad (1)$$

$$(\forall s)(E_1 s \vee E_2 s) \overset{?}{\vdash} (\exists s)I_1 s \wedge (\forall s)(I_1 s \rightarrow E_2 s) \text{ (}\mathbf{A}\text{c with the augment)} \quad (2)$$

[100] Ibn Sīnā, *Al-qiyās*, p. 376.
[101] Ibn Sīnā, p. 376.
[102] Ibn Sīnā, p. 378.

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Unfortunately, (1) is valid but (2) is not valid, as suggested by the question mark and as can be verified by a simple calculus. This too shows that **A**c should not contain the augment if one wants to account for the mutual translatability of conditional and disjunctive propositions in Ibn Sīnā's frame.

On the other hand, the **A**c proposition is also said to entail $\mathbf{A}_\mathrm{D}$. In this case, both entailments hold, for the **A**c propositions "$(\exists s)I_1 s \wedge (\forall s)(I_1 s \rightarrow E_2 s)$" and "$(\forall s)(I_1 s \rightarrow E_2 s)$," i. e. with or without the augment, both entail the $\mathbf{A}_\mathrm{D}$ disjunctive proposition "$(\forall s)(E_1 s \vee E_2 s)$." This shows that the only entailment that holds is the one where the first proposition is **A**c and the second proposition is $\mathbf{A}_\mathrm{D}$, unlike what Ibn Sīnā says.

So this first solution is not very satisfying because it does not account adequately for the real relations between the conditional propositions and the disjunctive ones, and for what is claimed in the text.

This is why one has to search for another solution. This other solution could be the following second solution: **A**c (and consequently **O**c) can be expressed in two ways: (1) without the augment, (2) with the augment, and these two formalizations must be explicitly distinguished from each other in order for the whole system to be coherent.

In addition, one must say exactly which interpretation validates which rule or mood, in order to clarify all the claims endorsed by Ibn Sīnā.

This clarification can be made as follows, for as has been previously shown, the first interpretation of **A**c (the one where **A**c does *not have* an import) validates the following:

– The equivalences between some **A**c propositions and some **E**c ones;
– The equivalences between some **A**c propositions and some $\mathbf{A}_\mathrm{D}$ ones;
– The principle of contraposition.

While the second interpretation of **A**c (the one where **A**c *has* an import) validates the following:

– All the relations of the square;
– Some entailments between some **A**c propositions and some $\mathbf{E}_\mathrm{C}$ ones but not conversely;
– Some entailments between some **A**c propositions and some $\mathbf{A}_\mathrm{D}$ ones, but not conversely;
– **A**c and **I**c conversions;
– All the valid moods of all figures, including Darapti and Felapton.

Since all these relations, entailments, moods, rules and principles are indeed held by Ibn Sīnā, one has to take these admissions into account and to formalize the propositions accordingly in order for the whole sys-

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tem to be admissible and free of confusion and/or contradictions.

This solution accounts for all these formulas, moods, rules and principles, although it has the inconvenience of interpreting some propositions in two possible ways. But this inconvenience is not really a problem, once the two interpretations are clearly distinguished from each other and stated in a clear way. For we can find two interpretations of the same operator even in modern logic, where the disjunction, for instance, can be interpreted either as an inclusive disjunction, when it means: either P or Q or both or as an exclusive disjunction, when it means: either P or Q but not both.

However, in Ibn Sīnā's text, we don't find a clear distinction between two possible interpretations, although he does distinguish between two kinds of disjunctive propositions in his second hypothetical system. This is so because he uses a natural language (i. e. Arabic) and does not use a specific symbolism (apart from the term variables), for these distinctions appear clearly only when one formalizes Ibn Sīnā's propositions. The fact that he did not provide these distinctions shows that something is not clear in the text, especially with regard to the augments.

Nevertheless, if one wishes to make the system consistent, one has to credit him with these findings, even if he did not explicitly exhibit them. One reason for this is that he did provide the equivalences between the unquantified conditional propositions and the unquantified disjunctive ones. So we cannot say that they are foreign to him. Another reason is that he did talk about the augments of **A**c and **O**c in both his categorical logic and his hypothetical one. So here too, the idea is not foreign to him.

For instance, the equivalence "$(P \to Q) \equiv (\sim P \vee Q)$" can be found in the following quotation:

> When they say: "Not A is B or C is D" [...] it is undoubtedly (*lā maḥāla*) a hypothetical (*šartiyya*), [...], it thus resembles the following conditional: "If A is B, then C is D" [...][103]

He also holds the following equivalence: "$(P \vee Q) \equiv (\sim P \to Q)$," for he says:

> The word *immā* (or) does not only mean an explicit conflict (*ᶜinād*), but also that the second [is true] (*kāʾinun*) when the first is not.[104]

We can also credit him with the following equivalence, which he uses in several places: $(P \to Q) \equiv \sim(P \wedge \sim Q)$ and with the principle of contraposition, namely the following equivalence: $(P \to Q) \equiv (\sim Q \to \sim P)$, which

[103] Ibn Sīnā, *Al-qiyās*, p. 251.
[104] Ibn Sīnā, p. 244.

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he states explicitly in *Al-qiyās*. In all these, the conditionals should not contain the augments. Otherwise, the equivalences would not be valid.

However, on the other hand, he also endorses the following claims:

1) **A**c and **E**c are contrary;
2) **I**c and **O**c are subcontrary;
3) **A**c implies **I**c and **E**c implies **O**c;
4) **I**c-conversion is valid;
5) **A**c-conversion is valid;
6) Darapti and Felapton are productive moods.

All these are said to hold in both categorical and hypothetical logics. But these theses all require the presence of the augment in both **A**c (and **A**) and **O**c (and **O**).

This is why one has to admit the two interpretations of **A**c in order to account for all of Ibn Sīnā's claims. One has also and above all to separate between these interpretations in order to warrant the consistency of the whole system.

Ibn Sīnā did indeed provide these interpretations, but he did not provide a precise account of which rules, principles, moods and implications are validated by each of these two **A**c's. But when one applies the modern symbolism to Ibn Sīnā's propositions and accounts for each one in a precise way, one can complement his analysis and make it both clear and consistent. This means that his system is perfectly able to validate all what he considered as valid, provided some precisions are made, and even if he himself did not provide explicitly these precisions. Thus complemented, the system appears to be very nicely consistent and very rich.

7. CONCLUSION

Ibn Sīnā's hypothetical logic is very much influenced by his categorical syllogistic, since according to him, the categorical syllogistic is the ultimate reference and the basis of all his subsequent systems, including the hypothetical logic.

But his hypothetical logic also introduces some new developments that Ibn Sīnā expresses in several ways. Among these developments, we find those where the hypothetical propositions contain quantified elements. In his analysis of these complex propositions, Ibn Sīnā holds several equivalences between **A**c and **E**c hypothetical propositions on the one hand and between conditional and disjunctive propositions on the other hand and claims that the relations of the square of opposition hold in hypothetical logic too.

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Unfortunately, these claims cannot hold together, being incompatible. In what precedes, we have shown that one can validate all of them by separating two kinds of **A**c conditional propositions (and their respective **O**c contradictories), the first kind being complemented by an existential augment while the second kind would not contain it.

However, although Ibn Sīnā does indeed talk in one way or another about these two interpretations of **A**c and their respective contradictories, he does not really separate them in a clear way. This separation is, however, indispensable to validate all the rules, moods, equivalences, logical relations and principles that he holds in his system, and above all to make the system coherent, clear and free of confusions. This is so because **A**c without the augment validates some equivalences and rules, while **A**c with the augment validates the relations of the square, the conversions and all the syllogistic moods. Separating the two interpretations of these propositions is thus the only way to make the system coherent and to account at the same time for its richness, which is indubitable.

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