

the absolute convergence of its Fourier series, and chapter 11 proves the identity of the Carleson and Helson classes of sets.

In the last chapter properties of rare and lacunary series are considered. The same class of perfect sets of constant ratio of dissection considered in chapter 6 reappears here to play an important role in this apparently unrelated subject.

Most of these results are appearing in book form for the first time. Much of the material is the original work of the authors and some of the results are in fact new. Although most of the problems are very special, the crucial counterexamples and restrictions of more general theories are often explained by just such situations as are discussed here. The authors' reasons for the study of these sets are ably put forward in a very short preface.

The book is one of a newer series by this press and is well bound and printed. It contains a useful index of definitions and notations and a full bibliography.

P. S. Bullen, Paris

The application of continued fractions and their generalizations to problems in approximation theory, by Alexey Nikolaevitch Khovanskii. Translated by Peter Wynn. P. Noordhoff N. V., Groningen, 1963. xii + 212 pages. Dfl. 28. -

This book on continued fractions is devoted to certain selected topics in the analytic theory, with particular emphasis on those aspects that deal with rational approximations to functions and with numerical applications and computations. It is a translation into English of the Russian work written in 1956 by A. N. Khovanskii.

The first chapter is concerned with an exposition of important recurrence relations and analytic theory of continued fractions, in particular, with transformations of continued fractions, transformations of series into equivalent and corresponding continued fractions, and considerations of convergence theory. Chapter II is devoted to continued fraction expansions of certain functions. Here is derived a solution of a certain Riccati equation with the help of continued fractions, from which are found continued fraction expansions of binomial functions,  $\sqrt[x]{x}$ ,  $\ln x$ ,  $e^x$ , trigonometric, inverse trigonometric, and hyperbolic functions, and  $\int_0^x dx/(1+x^k)$ . Also derived are continued fraction expansions for the ratios of Bessel functions, the ratios of hypergeometric functions, Prym's function, and the incomplete gamma

function. In chapter III are studied further methods for the development of rational function approximations to functions. Examples are given of the numerical computation. In chapter IV are considered the generalized continued fractions proposed by Euler. Particular attention is given to methods associated with matrices, and examples demonstrate the computation of roots and the solution of polynomial equations. The book concludes with a list of 17 books in the Russian language on the general theory of continued fractions, a list of 109 books and articles used in this text, and a list of 10 supplementary references (added by the translator).

In the translation of this book, the translator states that he has allowed himself a certain degree of freedom.

This text contains a tremendous mass of valuable formulas in continued fraction theory. Due to this fact, it can be considered as a useful reference manual for such formulas as well as a text on methods for research in analysis and in computational work.

E. Frank, Chicago, Illinois

Calculus of Variations, by I. M. Gelfand and S. V. Fomin.  
Revised English edition, translated and edited by R. I. Silverman.  
Prentice-Hall, Inc., Englewood Cliffs, N. J., 1963. 232 + vii pages.  
\$7.95.

Of three recent volumes bearing the same title, all translated from the Russian [Elsgolc (1962), Akhiezer (1962)], the present text is the most modern and sophisticated.

After a brief introduction to the theory of functionals, necessary conditions for extreme values of simple integrals are derived from the first variation with more than the usual rigour. The first three chapters of the book are devoted to this problem and its generalizations: the fixed end problem for  $n$  unknown functions, functionals depending on higher order derivatives, variational problems with subsidiary conditions, discontinuous solutions. The sections dealing with subsidiary conditions are somewhat disappointing as they make no mention at all of the deep results of Carathéodory (*Acta Math.* 47(1926), 199-236; *Comment. Math. Helv.* 5(1933), 1-19) which must nowadays be regarded as classical. It must also be pointed out that the authors' definition of "non-holonomic" constraints (p. 48) is incomplete and therefore misleading in that they do not distinguish at all between integrable and non-integrable conditions.

Chapter 4 introduces the canonical equations of motion and furnishes an introduction to the Hamilton-Jacobi theory and its