WORKSHOP

ON THE DEFINITION OF CATASTROPHE CLAIMS AND THE CALCULATION OF THEIR EXPECTED COST FOR THE PURPOSE OF LONG RANGE PLANNING AND PROFIT CENTRE CONTROL

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ABSTRACT

Some reasons are given for paying special attention to the gross cost of catastrophe claims in planning and control. A method is then described of defining catastrophe claims and estimating their expected cost. The various steps in applying the method to real data and its performance for planning and control are discussed and illustrated in conjunction with an investigation carried out on a company portfolio.

KEYWORDS

Large claims; planning and control; fire insurance; machinery breakdown insurance.

1. INTRODUCTION

Reinsurers cannot be expected to pay systematic losses of a direct insurance company. In rating its business the direct company thus has to plan its premium level so that it covers, in addition to "normal" costs, also the expected cost of catastrophe claims and a fair profit to its reinsurers. In planning and long-term forecasting it is therefore natural to consider these two items separately.

In short-term forecasts made during a particular accounting year of the net result of that year, on the other hand, it could be preferable to merge the gross claims cost with the result of reinsurance ceded and predict net cost directly. This is mainly because the reinsurance programme for the year is already agreed so that its actual effect on various levels of the gross claims cost can be judged fairly well. It might, however, still be of some interest to consider the two items separately, especially if gross business and ceded business are separate profit centres.

Another reason for trying to give a precise meaning to the concept of catastrophe claims and for establishing their expected cost is to get a better understanding of the gross results of profit centres. An extraordinary result might often be explained by replacing the observed cost of catastrophe claims by the corresponding expected cost.

A third, technical, reason for treating catastrophe claims separately is that the estimation of their expected cost requires methods different from those appropriate for other claims.

2. THE INFORMATION SYSTEM BACKGROUND

In what follows, a way of handling catastrophe claims will be described. It is used within the framework of an information system showing gross results per profit centre, and within the profit centre per line of business. In this system claims incurred during an accounting year are divided into *small, large* and *catastrophic* claims, as follows.

Small claims are claims with (an estimated) size below a certain limit. Their total cost is equal to their estimated number times a statistical mean-value, which depends on the type of claim, e.g. fire or burglary or water damage.

Remaining claims are large or catastrophic claims. Their sizes are individually estimated by claims adjusters, summing up to a total which, if necessary, is increased by IBNR reserves. Catastrophe claims are known claims with an (individually) estimated amount exceeding a certain high limit. The excess amounts are recorded as cost of catastrophe claims. The remainder of the individually estimated claims amounts, together with the necessary IBNR provisions, is recorded as cost of large claims. Finally, to get the claims incurred figure for the accounting year in question, run-off from previous years of incurrence is added. This is exemplified in Table 1, which comprises all those lines of business for which the concept of catastrophe claims has been defined. These are single or comprehensive lines, where fire claims and/or machinery breakdown claims play a considerable part.

Forecasts on claims cost are made separately for small, large and catastrophic claims. For small claims they are based on predictions of claims numbers, taking observed average claims sizes into account. For large claims, claims cost is predicted directly, taking observed time averages into account. For catastrophic claims forecasts are based on predicted premium volume, taking into account past relations between catastrophic claims costs and premiums according to the method to be described below.

TABLE 1

TOTALS FOR LINES OF BUSINESS SUPPLIED WITH A DEFINITION OF CATASTROPHIC CLAIMS. MSEK (Sw.crowns, millions)

Accounting year	82	83	84	85
Premiums earned	837.4	752.4	719.0	827.3
Small claims cost	62.8	60.3	68.2	73.9
Large claims cost	362.3	370.6	478.6	707.3
Catastrophic claims cost	54.1	0	127.7	56.0
Run-off	26.7	−10.8	−31.1	−42.6
Claims incurred	505.9	420.1	643.4	794.6

3. A METHOD TO DEFINE CATASTROPHE CLAIMS AND TO ESTIMATE THEIR EXPECTED COST: PRINCIPLES

The method we use is very similar to what one would do in establishing an excess-of-loss rate.

We consider certain types of claim, e.g. fire, thought to be potential sources of catastrophe claims. We also consider lines of business covering such types of claim. They may be single lines covering just one type of claim or comprehensive ones covering several types.

For each type of claim considered, j, we define as *possible* catastrophe claims (p.c.c.'s) all claims exceeding a certain limit L_j in constant money-value. The choice of L_j is somewhat arbitrary. It should be sufficiently low to produce a substantial body of p.c.c.'s, so that a theoretical claims size distribution can be fitted. It should not be too low because we want the fitted distribution to have good fit to the (real) catastrophe claims, forming a subset of the p.c.c.'s.

To the observed p.c.c.'s incurred during a certain time period, we fit a theoretical claims size distribution function F_j with $F_j\ (L_j) = 0$.

For a certain line of business, i, the number of p.c.c.'s per year from claim type j is assumed to follow a Poisson distribution with parameter N_{ij} which is estimated from the observed number of p.c.c.'s per year. We have $N_{ij} = 0$ for claim types not covered by the line. For single lines only one N_{ij} can be different from zero.

Under the usual assumption of independence between claims numbers and amounts, the expected yearly cost of p.c.c.'s for line i then is

$$\sum_j N_{ij} \int_0^\infty (1 - F_j(x))\ dx. \tag{1}$$

Putting

$$N_i = \sum_j N_{ij}, \qquad G_i(x) = \sum_j N_{ij} F_j(x) / N_i \tag{2}$$

this may be written

$$N_i \int_0^\infty (1 - G_i(x))\ dx. \tag{3}$$

In the following we drop the index i as we are considering a fixed line of business.

A catastrophe claim for the line of business considered we define as a claim exceeding a limit x_0, to be defined below. We further establish the highest EML (estimated maximum loss per event and insured risk) in the line of business at the time when the investigation is done, and transform it into the same money-value as all other amounts. Denote this by $x_1 (x_1 > x_0)$.

We consider x_1 as an upper bound for catastrophe claims. In case one risk has a much higher EML than remaining risks within the line of business, it might have been realistic to use a somewhat lower upper limit in the integral (4), below.

Also, in fitting theoretical claims size distributions to the p.c.c.'s, EML-constraints should properly have been taken into consideration. As most p.c.c.'s are small compared to the EML's this would, hopefully, not have affected the fitting procedure very much. For the time being, these ideas have not been pursued any further.

The expected catastrophe claims amount per p.c.c., i.e. the expected amount in excess of x_0 per p.c.c., for claim type j then is

$$\int_{x_0}^{x_1} (1 - F_j(x))dx \tag{4}$$

and the expected cost per year of catastrophe claims for the line of business thus is, using (2) and dropping the index i

$$N \int_{x_0}^{x_1} (1 - G(x))\, dx. \tag{5}$$

The lower limit x_0 for catastrophe claims is chosen as ten per cent of earned premiums gross, transformed into constant money-value, within the line of business during the accounting year preceding the year of the investigation. For the time being, this choice is rather pragmatic. It stems from management's subjective opinion on what constitutes a serious impact by a single event on the loss ratio of a line of business. Further considerations should entail comparisons with the normal loss ratio variations.

If necessary, however, the limit is raised to give an average time between two consecutive catastrophe claims of at least two years ("you can't have a catastrophe every year").

The expected number of catastrophe claims per year is

$$N(1 - G(x_0)) \tag{6}$$

and the supplementary rule is interpreted so as to require this quantity to be less then one half. This gives a lower bound for x_0.

TABLE 2

PLANNING PERIOD 1983–85. MSEK

Planning year	83	84	85
Premiums earned	233.9	269.3	307.0
Comm. & expenses	−79.8	−90.5	−102.6
Small claims cost	−39.7	−45.8	−52.9
Large claims cost	−88.8	−102.1	−117.4
Res. before cat. claims	25.6	30.9	34.1
Cat. claims cost	−11.6	−13.3	−15.3
Underwriting res. gross	14.0	17.6	18.8
Cost of reinsurance	−2.3	−2.7	−3.1
Underwriting res. net	11.7	14.9	15.7

In order not to burden the total business with catastrophes that are too small, even if they might be catastrophic according to the above rules for the specific line considered, we also prescribe, as a second supplementary rule, that x_0 must not be less than a chosen fixed amount, common to all lines.

Finally the yearly expected catastrophe claims cost according to formula (5) is expressed as a percentage of earned premiums gross during the accounting year preceding the year of investigation.

One investigation of this kind was done in 1981, the observed time period being the years 1971–80. In 1982 the results were used for the planning period 1983–85, applying the expected catastrophe claims cost percentages to predicted premiums earned, gross. This is exemplified in Table 2 for one of the lines concerned. In this case the expected percentage was equal to 5.

4. SOME DETAILS ON THE PRACTICAL APPLICATION OF THE METHOD

In the investigation of 1981 we included four single lines of business. These were Fire/property and loss-of-profits, and Machinery breakdown/property and loss-of-profits. In addition three comprehensive lines were considered with respect to their claims of the four types mentioned.

For these lines all p.c.c.'s incurred during the years 1971–80 were recorded by type of claim, the p.c.c. limit L being 1 MSEK for fire types and 0.4 MSEK for machinery breakdown types in the money-value of 1971. This corresponds to 2.3 and 0.9 MSEK in the money-value of 1980, according to the index chosen.

The material was inspected with respect to trends, but no obvious ones seemed to be present. This confirmed the simple model assumption of constant yearly p.c.c. rates N.

To each type of claims three families of claims size distribution functions were fitted, and the goodness-of-fit was judged by the chi-square criterion. The three families were: the Gamma family, the Lognormal family and the Pareto family. In all cases the Pareto family gave the best fit.

For the Pareto family we have

$$1 - F(x) = (x/L)^{-a} \qquad x > L.$$

The maximum-likelihood estimates of the Pareto parameter were as follows, for the four different types of claims.

Fire: number of p.c.c. = 190, $a = 1.26$ (chi-square 6.57, 9 classes)
Fire loss-of-profits: number of p.c.c. = 131, $a = 1.52$ (13.65, 9)
Machinery breakdown: number of p.c.c. = 70, $a = 1.40$ (3.83, 7)
Machinery loss-of-profits: number of p.c.c. = 57, $a = 1.15$ (7.75, 7)

The maximum-likelihood estimates were computed from the individual observations divided by L, and not from the grouped sample, as the inverted value of the arithmetic mean of the natural logarithms. The number of classes for the chi-

TABLE 3

Line of business	N	x_0	Cat. claims No.	Cat. claims cost
Fire	17	33.2	0.60	11%
Fire loss-of-profits	12	19.0	0.50	13%
Machinery breakdown	12	10.0	0.25	7%
Machinery loss-of-profit	6	10.0	0.40	28%
Comprehensive no. 1	5	15.3	0.36	5%
Comprehensive no. 2	2	10.0	0.26	9%
Comprehensive no. 3	2	10.0	0.18	8%
			2.55	

square test is mentioned to give the reader the possibility of his own personal choice of degrees of freedom in this situation. Obviously, the fit is fairly good.

The fixed amount according to the second supplementary rule for the catastrophe limit was, pragmatically, chosen as 10 MSEK 1980, corresponding to 4.3 MSEK in the money-value of 1971. This had to be used for the Machinery breakdown/property and loss-of-profits lines and for comprehensive lines no. 2 and 3.

The first supplementary rule had to be used for the Fire/loss-of-profits line, which exhibits this line as a little "dangerous". Strictly speaking, it should have been used also for Fire. However, as the number of catastrophe claims according to the main rule is close to 0.5, cf. Table 5, no correction was made.

Expected number of p.c.c.'s per year (N), limits x_0 (MSEK, money-value of 1980) and expected yearly catastrophe claims numbers and costs (% of premiums earned) are shown in Table 3 for each of the seven lines considered.

5. COMPARISON BETWEEN ESTIMATED AND OBSERVED NUMBERS AND AMOUNTS OF CATASTROPHE CLAIMS DURING THE PERIOD 1982–85

The expected number of catastrophe claims, computed according to formula (6) for each line of business and totalled over the seven different lines, amounts to 2.55 per year. The actual numbers have varied between zero and four.

In Table 4, the first line, showing actual catastrophe claims costs, is reproduced from Table 1. The second line shows the corresponding expected costs. In

TABLE 4

ACTUAL VS EXPECTED CATASTROPHE CLAIMS COST 1982–85. TOTALS FOR ALL LINES FOR WHICH CATASTROPHIC CLAIMS HAVE BEEN DEFINED. MSEK, gross

Accounting year	82	83	84	85	82–85
Actual cat. claims cost	54.1	0	127.7	56.0	237.8
Expected claims cost	82.4	69.1	64.5	75.2	291.2
Actual/expected %	66	0	198	74	82

TABLE 5

LOSS RATIOS BASED ON ACTUAL VS EXPECTED CATASTROPHE CLAIMS COST. TOTALS FOR ALL LINES FOR WHICH CATASTROPHIC CLAIMS ARE DEFINED. GROSS

Accounting year	82	83	84	85
Actual loss-ratio, %	60.4	55.8	89.5	96.0
Expected loss-ratio, %	63.8	65.0	80.7	98.4

establishing actual catastrophe claims, the catastrophe limit x_0 is chosen as 10% of actual premiums instead of predicted ones, for lines following the main rule. For remaining lines it is indexed according to the same index as used in the investigation.

The expected costs of catastrophe claims have accordingly been computed by applying the percentages stated above to actual instead of predicted premiums.

In Table 5, Table 4 is combined with Table 1 to produce loss ratios, inclusive of run-off, based on actual and expected catastrophe claims cost respectively.

During the period shown we have had a strong increase of medium-sized large claims, of the order of magnitude some millions Sw. crowns per claim. This trend is a little more clearly spelt out by the second row than by the first one. The variance about the fitted regression line is 25.3 for the second series compared to 81.8 for the first one. Of course, the smoothing effect is larger for individual lines.

6. CONCLUSIONS

The method has worked fairly well during the years it has been in operation. There has been a positive response by management to the extra information on catastrophic claims that is supplied. For long-range planning, graduation of catastrophe claims costs is indispensable, be it done by the method described or otherwise.

The parameters of the method, i.e. the p.c.c. claims size distribution, expected yearly numbers of p.c.c.'s per line of business and catastrophe claims limits, should be updated every fifth year or so. In doing this more care should be taken, if possible, to state the desired degree of graduation. The possibility of trends, in frequencies and/or amounts, remaining after transformation of data into constant money-value, should not be neglected.

It may also be mentioned that until now we have treated property and loss-of-profits claims separately. In a new investigation, which has just started, we will try to merge such claims originating from the same event.

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