Macroeconomic Dynamics, **18**, 2014, 65–92. Printed in the United States of America.
doi:10.1017/S1365100512000260

CAN CAPACITY CONSTRAINTS EXPLAIN ASYMMETRIES OF THE BUSINESS CYCLE?

MALTE KNÜPPEL
Deutsche Bundesbank

In this paper, we investigate the ability of a modified real business cycle (RBC) model to reproduce asymmetries observed for macroeconomic variables over the business cycle. To replicate the empirical skewness of major U.S. macroeconomic variables, we introduce a capacity constraint into an otherwise prototypical RBC model. This constraint emerges because of the assumption of kinked marginal costs of utilization, where the kink is located at a utilization rate of 100%. We find that a model with a suitably calibrated cost function reproduces the empirical coefficients of skewness remarkably well.

Keywords: Capacity Utilization, Skewness, RBC Model

1. INTRODUCTION

The notion that macroeconomic variables exhibit asymmetry over the business cycle has a long history in economics. The existence of such asymmetries was claimed early by Mitchell (1927, p. 290), who stated that "Business contractions appear to be a briefer and more violent process than business expansions." Keynes (1936, p. 314), too, observed that "The substitution of a downward for an upward tendency often takes place suddenly and violently, whereas there is, as a rule, no such sharp turning-point when an upward is substituted for a downward tendency." Many results of empirical research point to the importance of asymmetries for macroeconomic variables, as, for instance, do those reported in Goodwin (1993) for output measures and in Neftci (1984) for unemployment, although their importance is not undisputed.[1]

To the best of our knowledge, relatively few attempts have been made to investigate dynamic stochastic general equilibrium models with respect to asymmetries and to compare the resulting asymmetries with those observed in the data. Van Nieuwerburgh and Veldkamp (2006) study a real business cycle (henceforth RBC) model where productivity follows a symmetric Markov-switching process

I would like to thank Beatriz Gaitan, Harald Uhlig, Bernd Lucke, and an anonymous referee for their helpful comments and suggestions. I also wish to thank seminar participants at the 2005 annual conferences of the European Economic Association and the Verein für Socialpolitik for fruitful discussions. This paper represents the author's personal opinions and does not necessarily reflect the views of the Deutsche Bundesbank. Address correspondence to: Malte Knüppel, Deutsche Bundesbank, Wilhelm-Epstein-Straße 14, D-60431 Frankfurt am Main, Germany; e-mail: malte.knueppel@bundesbank.de.

 1365-1005/13

whose state cannot be observed by economic agents. Owing to the additive nature of the productivity shock in their model, the signal-extraction problem the agents face is characterized by a procyclical signal-to-noise ratio, giving rise to several types of asymmetry. Hansen and Prescott (2005) consider an RBC model where there is an upper bound on the number of plants that can be operated. This upper bound is due to a minimum labor requirement per plant and the existence of immobile capital. This capital can be idle in recessions, so that capital's income share is procyclical. Both studies yield satisfactory results concerning the reproduction of the empirical asymmetries investigated. Valderrama (2007) tries to replicate empirical nonlinearities, including asymmetries, using an estimated RBC model with adjustment costs to investment, where the estimation procedure takes nonlinearities into account. However, it turns out that the asymmetries generated by the estimated model do not match those encountered in the data, because the strong correlation of consumption and investment in the model does not allow the reproduction of the distinct empirical asymmetries of these variables.

In this work, we investigate one reason for the existence of asymmetries that is similar to that in Hansen and Prescott (2005), i.e., related to the utilization of capital. We study how the assumption of an upper bound for capital services affects the symmetry properties of the model's variables. However, this upper bound does not emerge from the existence of two types of capital, where the supply of one type is bounded from above, as in Hansen and Prescott (2005). Instead, we investigate an upper bound for capital utilization that is motivated by a kink in the marginal costs of capital utilization. This kink relies on the assumption that the elasticity of marginal cost with respect to utilization jumps to a higher level once capacity utilization reaches 100%. In standard RBC models, such as those described in King and Rebelo (1999), this elasticity is assumed to be constant, implying that no upper bound for utilization exists. We introduce the kinked marginal cost function into an otherwise prototypical RBC model with variable capacity utilization and compare the asymmetries emerging from this model with the asymmetries found in the data. We also study the impact of capacity constraints on second moments by investigating the differences between standard deviations and cross correlations in models with and without capacity constraints.

The paper is set up as follows. In Section 2, we conduct an investigation of the asymmetries of a set of macroeconomic variables, where asymmetries are measured by third standardized moments, i.e., skewness. We also report results for second moments. In Section 3, we present and calibrate the model with capacity constraints. In Section 4, we report simulation results for the model and compare them with the empirical results. We also compare the simulation results with the outcomes of models without capacity constraints. Moreover, we analyze the capacity constraint's impact on stochastic steady-state values, and we investigate the reasons for differing magnitudes of skewness among the model's variables. Finally, we perform an extensive sensitivity analysis in order to study the effects of changes in the utilization cost function and of the additional consideration of investment adjustment costs as in Valderrama (2007). Section 5 concludes.

2. STYLIZED FACTS

Asymmetry in our study will be measured by the coefficient of skewness, i.e., by the standardized third moment.[2] A symmetrically distributed variable always has zero skewness. Thus, if nonzero skewness is present, the variable must have an asymmetric distribution.

If a time series has negative skewness, then there are often fewer observations below the mean than there are observations above the mean, and, on average, those below the mean are larger in absolute value. In a stochastic process with symmetric shocks, negative skewness can emerge if the effects of these shocks are dampened when the realizations of the process lie above its mean, or if the effects of these shocks are amplified when the realizations of the process lie below its mean.

2.1. The Data

Our study is based on postwar U.S. macroeconomic per capita data. The macroeconomic variables investigated here are output, consumption, investment, labor, capital, the real wage, labor productivity, and total factor productivity. The data for each variable, except capital, cover the sample period from the first quarter of 1954, henceforth denoted 1954q1, through 2002q2 and are seasonally adjusted, except for the population series. The series are taken from the National Income and Product Accounts of the Bureau of Economic Analysis if not stated otherwise.

Consumption (henceforth denoted C) is measured as the sum of real personal consumption expenditures for services and nondurable goods and real government consumption. Investment (X) equals the sum of real private consumption of durable goods, real gross private domestic investment, and real government investment. Output (Y) is measured by real gross domestic product (GDP). Labor (H) is the total number of man-hours in nonagricultural establishments. This series is taken from the data set used in Ireland (2004). The real wage (W) is constructed as the ratio of compensation of employees to the product of labor with the consumer price index for all urban consumers. The consumer price index series comes from the Federal Reserve Economic Data database. Data for the capital stock (KA) are available on an annual basis only, and the sample considered ranges from 1954 to 2001.

All series mentioned, except for the real wage, are in per capita terms, which are obtained by dividing the series by the civilian noninstitutional population. Many studies, such as those by Ireland (2004) or King et al. (1988), make direct use of the civilian noninstitutional population series provided by the Bureau of Labor Statistics. This series, however, has been revised several times and contains sharp jumps at the revision dates. For example, owing to revisions, the civilian noninstitutional population increased by more than 0.5% in the first quarters of 1972 and 1990. Although the size of these jumps is small enough to pass unnoticed for volatile series such as investment, they possess a considerable impact on more stable series such as capital and, to a smaller extent, consumption. Moreover, these jumps might have negligible effects on second moments, but could affect third moments to a greater extent.[3] We therefore consider it necessary to smooth the population series before we divide macroeconomic variables by it. To do this, we apply the HP-filter and use the resulting trend as the population series.[4]

We consider two productivity series: labor productivity and total factor productivity. Labor productivity (LP) is defined as the ratio of GDP to labor. The measure that we construct for total factor productivity (TFP) is based on three assumptions: that output is produced by a Cobb–Douglas production function, that quarterly changes in the capital stock are approximately zero, and that the utilization of the capital stock is constant over time.[5] By virtue of these assumptions, TFP can be computed as

$$\ln \mathrm{TFP}_t = \ln Y_t - \alpha \ln \bar{K}_t - (1 - \alpha) \ln H_t, \tag{1}$$

where α is the elasticity of output with respect to capital, and the series $\ln \bar{K}_t$ is simply a linear trend. For the calculation, α is set to $1/3$, which is a value often encountered in the literature.[6]

To induce stationarity and to isolate fluctuations associated with business cycle frequencies, we apply the HP-filter, with the smoothing parameter set to 1,600, to the logarithm of all variables except capital. For the annual capital series, we use the common value of 100. All HP-filtered variables, multiplied by 100, are displayed in Figure 1. The quarterly capital series (K) is constructed simply by inserting the annual value for every quarter of that year.

2.2. Third and Second Moments

The skewness of each variable, as well as the standard deviation, the relative standard deviation with respect to GDP, the correlation with GDP, and the correlation of consumption and investment, is presented in Table 1. No correlation is displayed for capital, because we report results for the annual capital stock KA.

Concerning the coefficients of skewness, we find that capital is the only variable having positive skewness, with a value of about 0.1. The least skewed variables are consumption and the real wage, with coefficients close to −0.1. GDP exhibits a coefficient around −0.4, and both productivity measures have coefficients of about −0.35. The most skewed variables are labor, with a coefficient close to −0.5, and investment, with a coefficient of almost −0.7.

It is interesting to investigate whether the mentioned coefficients are significantly different from zero in order to evaluate the importance of asymmetries for macroeconomic variables. In order to do this, we use two tests for symmetry of serially correlated data. The first test was proposed by Gasser (1975) and applied by Psaradakis and Sola (2003), among others. Gasser's test assumes a marginal normal distribution of the variable under study and is directly based on the coefficient of skewness. The test statistic has a standard normal distribution under the null of symmetry. The second test, by Bai and Ng (2005), is based

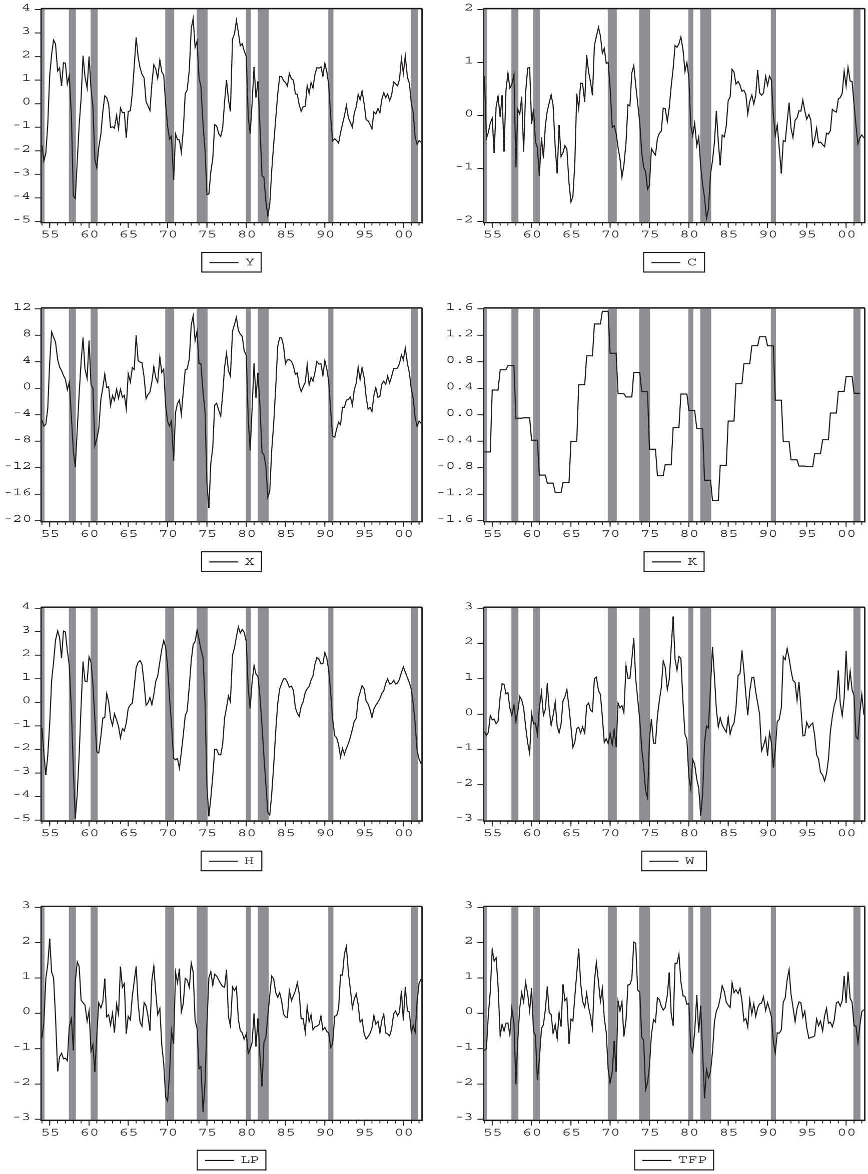

FIGURE 1. HP-filtered time series. Shaded areas denote recessions as dated by the NBER.

on the third and fifth moments of the variable under study and does not require distributional assumptions, but requires the existence of higher-order moments.[7] The test statistic has a χ^2 distribution under the null of symmetry. Psaradakis and Sola (2003) and Bai and Ng (2005) emphasize that the symmetry tests tend to have

https://doi.org/10.1017/S1365100512000260 Published online by Cambridge University Press

TABLE 1. Empirical third and second moments

	Y	C	X	KA	H	W	LP	TFP
Skewness	−0.42	−0.11	−0.69	0.13	−0.49	−0.10	−0.35	−0.34
Gasser's test statistic	−1.74	−0.43	−2.95	0.31	−1.93	−0.42	−1.66	−1.61
p-value	.08	.67	.00	.76	.05	.67	.10	.11
Bai & Ng's test statistic	2.63	1.11	3.63	1.39	3.32	0.81	3.37	3.41
p-value	.10	.29	.06	.24	.07	.37	.07	.06
Std. dev.	1.61	0.71	5.22	0.74	1.78	0.95	0.85	0.80
Rel. std. dev.	1.00	0.44	3.23	0.46	1.11	0.59	0.53	0.50
Corr. with Y	1.00	0.66	0.95		0.88	0.09	0.05	0.71
Corr. with X		0.53						

Note: "Std. dev." denotes "standard deviation"; "rel." stands for "relative."

low power.[8] Thus, the tests often do not reject the null hypothesis of symmetry when the null is incorrect.

The test results are displayed in Table 1. According to both tests, there are no signs of significant asymmetry for capital, consumption, or the real wage. GDP is significantly asymmetric at the 10% significance level according to Gasser's test, and the p-value of Bai and Ng's test only slightly exceeds 10%. At the same level, asymmetry of TFP is significant, according to Bai and Ng's test, and not significant, according to Gasser's test, but the corresponding p-value exceeds 10% by only a small amount. Symmetry of investment can be rejected at the 1% significance level with Gasser's test and at the 10% level with Bai and Ng's test. For labor and labor productivity, there is significant asymmetry at the 10% level with both tests. Thus, the p-values of the tests broadly correspond to the magnitudes of the coefficients of skewness, and variables with absolute values of skewness greater than 0.3 are significantly asymmetric at least at the 10% level according to at least one of the tests.

Concerning second moments, we find the well-known results concerning standard deviations and cross correlations. With respect to volatility, this means that the relative standard deviations of consumption and capital are smallest, investment has the largest relative standard deviation, the volatility of labor is of a magnitude similar to that of the volatility of GDP, and the real wage, LP, and TFP are approximately half as volatile as GDP. The correlation of consumption and investment attains a value of about 0.5, thus being smaller than the respective correlations of both variables with output.

The values of the coefficients of skewness might depend to some degree on the sample chosen. Although this is also true of second moments, this dependence can be expected to be more pronounced for third moments, because the variance of moment estimators increases with the order of the moment considered. As a kind of

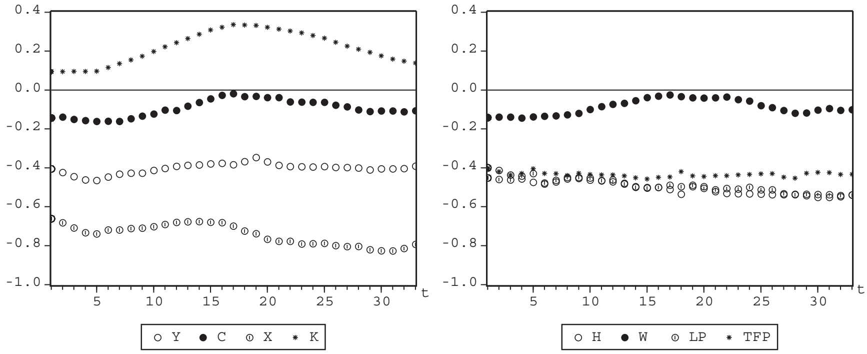

FIGURE 2. Skewness of variables in samples of 162 observations starting $t - 1$ quarters after 1954:1.

robustness check, we vary the sample under study and determine the coefficients of skewness for each sample. We begin with a sample starting in 1954q1 and ending in 1994q2. Then we increase the start and the end of the sample by one quarter. We do so until the resulting sample ends in 2002q2.[9] This implies that the 33 resulting samples are 32 quarters shorter than the original sample.[10] The results of these calculations are presented in Figure 2.

Obviously, most coefficients broadly equal their respective values reported in Table 1 for all samples. Only the positive skewness of capital increases strongly for certain samples, but this might be explained by the small number of distinct values for this series.[11] Therefore, the results concerning capital should be considered with caution. The skewness of both productivity series is moderately more pronounced in all short samples than in the full sample. Interestingly, the coefficients of skewness of consumption and the real wage are very close to each other in each sample.

Summing up, we have found evidence that macroeconomic variables exhibit different magnitudes of skewness. Capital shows weak positive skewness, whereas consumption and the real wage feature weak negative skewness. GDP, LP, TFP, and labor all exhibit at least moderate negative skewness. Finally, investment is strongly negatively skewed.

3. THE MODEL

In this section, we will consider a business cycle model with kinked marginal utilization costs that give rise to a jump in the elasticity of these costs if utilization exceeds a certain threshold. Except for the kinked cost function, the model is a standard RBC model. We will present its setup and its calibration and briefly mention computational details concerning its solution.

3.1. Economic Environment

The economy under consideration is populated by many identical infinitely lived households. Households are assumed to have separable logarithmic preferences over consumption and leisure. We further assume that labor is indivisible and that employment lotteries exist, as suggested by Hansen (1985) and Rogerson (1988). By doing so, we imply that real wages and consumption exhibit the same cyclical behavior. Here this modeling strategy is motivated by the similarity of skewness of these variables found in the previous section.

Because of the assumptions made, the momentary utility function of the stand-in representative household takes the form

$$\mathfrak{U}(\tilde{c}_t, h_t) = \ln \tilde{c}_t - \omega h_t, \tag{2}$$

where $\tilde{c}_t$ denotes consumption and h_t denotes the ratio of time worked to total disposable time in period t. Thus, $1 - h_t$ equals the share of leisure time in period t.

The household ranks alternative streams of consumption and leisure according to the criterion function

$$\sum_{t=s}^{\infty} \beta^{t-s} E_s \left[\mathfrak{U}(\tilde{c}_t, h_t)\right] \quad \text{with } 1 > \beta > 0, \tag{3}$$

where β is the discount rate, and the operator E_s denotes the expectation conditional on information available at time $t = s$.

The production function is defined by

$$\tilde{y}_t = z_t (u_t \tilde{k}_t)^{\alpha} (\nu^t h_t)^{1-\alpha} \quad \text{with } 0 < \alpha < 1, \tag{4}$$

where z_t is a stationary random variable that permits temporary changes in TFP (henceforth simply productivity), $\tilde{y}_t$ denotes output, $\tilde{k}_t$ is the stock of capital, $\ln(\nu)$ is the constant growth rate of labor-augmenting technical progress, and u_t is the utilization rate of capital. Obviously, without further restrictions, the household would choose a utilization rate that was as high as possible. The standard method of ruling out this possibility is the assumption of utilization costs, which are commonly modeled as a convex increasing function of the utilization rate. Many authors, such as King and Rebelo (1999), choose the depreciation rate to be determined by this cost function, so that the depreciation rate increases with higher utilization. An alternative approach is used by Christiano et al. (2001) and Smets and Wouters (2003), who model utilization costs in terms of foregone output. We decide to pursue the former approach here, so that the resource constraint of the economy is given simply by

$$\tilde{y}_t = \tilde{c}_t + \tilde{x}_t, \tag{5}$$

where $\tilde{x}_t$ is gross investment. The model is transformed to a stationary one by writing it in terms of the stationary variables $c_t = \tilde{c}_t/\nu^t$, $y_t = \tilde{y}_t/\nu^t$, $k_t = \tilde{k}_t/\nu^t$, and $x_t = \tilde{x}_t/\nu^t$.

The capital stock evolves according to

$$\nu k_{t+1} = \{1 - [\delta + g\,(u_t)]\}\, k_t + x_t, \tag{6}$$

where $\delta + g(u_t)$ is the depreciation rate of capital. Thus, $g(u_t)$ is the stochastic part of the depreciation rate, and the function $g(u_t)k_t$ denotes the costs of utilization. These costs are assumed to be convex and increasing in u_t.

Concerning the exogenous process for z_t, we consider the first-order autoregressive process

$$\ln z_t = (1-\rho)\ln \bar{z} + \rho \ln z_{t-1} + \varepsilon_t \quad \varepsilon_t \sim \text{i.i.d. } \mathcal{N}(0, \sigma^2), \tag{7}$$

where $\bar{z}$ is constant and greater than zero, and ε_t is Gaussian white noise.

The share of time spent working cannot exceed total time, so the household fulfills the time-endowment constraint $h_t \leq 1$. In addition, none of the variables mentioned can become negative except for investment, and the transversality condition given by

$$\lim_{t\to\infty} \beta^t \frac{1}{c_t} k_{t+1} = 0$$

must be fulfilled.

Maximizing the criterion function subject to the constraints presented leads to the first-order conditions

$$\omega = \frac{1}{c_t}\,(1-\alpha)\,\frac{y_t}{h_t},$$

$$\frac{1}{c_t} = \frac{\beta}{\nu} E_t\left(\frac{1}{c_{t+1}}\left\{\alpha\frac{y_{t+1}}{k_{t+1}} + 1 - [\delta + g\,(u_{t+1})]\right\}\right),$$

and

$$\frac{\partial g\,(u_t)}{\partial u_t} k_t = \alpha \frac{y_t}{u_t}. \tag{8}$$

The last condition states that the marginal costs of utilization must equal the marginal returns.

3.2. The Nonstochastic Steady State

The nonstochastic steady state of the economy is given by the values that the variables adopt if the logarithm of productivity equals its unconditional mean $\ln \bar{z}$ for all t with certainty. For utilization and its cost, we impose the nonstochastic steady-state values

$$\bar{u} = 1$$

and

$$g\,(\bar{u}) = 0,$$

thereby implying that the nonstochastic steady state of the model is identical to the nonstochastic steady state of an equivalent model with constant capital utilization.

3.3. Capacity Constraints and the Function of Utilization Costs

For an investigation of this economy, additional assumptions about the cost function $g(u_t)k_t$ are necessary. If the model is log-linearized in order to solve it, one does not have to specify a functional form for $g(u_t)$, but only needs to determine the elasticity of marginal costs with respect to utilization, given by

$$\eta = \frac{u_t \left.\dfrac{\partial^2 [g(u_t)\, k_t]}{\partial u_t^2}\right|_{u_t=\bar{u}}}{\left.\dfrac{\partial [g(u_t)\, k_t]}{\partial u_t}\right|_{u_t=\bar{u}}}.$$

However, because we aim at a numerical solution, we have to rely on a specific functional form of $g(u_t)$, which we choose to be

$$g(u_t) = \alpha \frac{\bar{y}}{\bar{k}} \frac{1}{1+\eta}\left(u_t^{1+\eta} - 1\right) \quad \text{with } \eta > 0,$$

where η must be positive to guarantee convexity of $g(u_t)$. Note that this function fulfills the requirement on $g(u_t)$ with respect to its value at the nonstochastic steady state. In addition, at the nonstochastic steady state with $\bar{u} = 1$, the first derivative of $g(u_t)$ is always equal to $\alpha\frac{\bar{y}}{\bar{k}}$, independent of the value of η. Therefore, we have that the first-order condition with respect to utilization, (8), which here becomes

$$\alpha \frac{\bar{y}}{\bar{k}} u_t^{\eta} k_t = \alpha \frac{y_t}{u_t}, \tag{9}$$

is satisfied for all possible values of η in the nonstochastic steady state.

The first-order condition (9) implies that when η goes to infinity, the marginal costs of utilization also go to infinity if u_t exceeds one, and are zero if u_t is lower than one. Thus, in this case, the household always sets u_t equal to one and the model is identical to a model with fixed utilization. If, in contrast, η is close to zero, marginal costs hardly vary with utilization, and u_t exhibits high volatility.

The assumption of a constant elasticity of marginal costs with respect to utilization implies that the utilization rate can become infinitely large, so that even in the short run, there is no upper bound on the supply of capital services. Thus, if a positive shock to productivity occurs in a certain period, utilization can always increase, independent of the size of the shock and the size of the capital stock. This assumption seems problematic, because there are physical limits to many kinds of capital services that cannot be exceeded. For example, machines employed in production cannot be used for more than 24 hours per day. Upper bounds on the services they can provide are obviously present. The same is true of most other

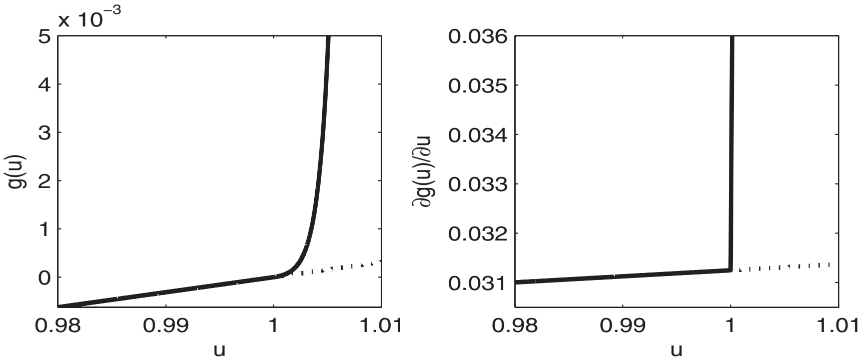

FIGURE 3. Costs of utilization (left panel) and marginal costs of utilization (right panel) with $\eta_2 = 0.39$ and $\eta_1 = 1{,}000$ (solid lines), and with $\eta_1 = \eta_2 = 0.39$ (dotted lines).

kinds of capital, such as land for agricultural production, where the amount of crop per unit of land cannot be increased to infinity.

These reasons lead us to impose a nonconstant elasticity of marginal costs with respect to utilization. We assume that there are two possible values for this elasticity, and that they differ depending on the magnitude of the utilization rate with respect to its nonstochastic steady-state value. That is, we assume that $g(u_t)$ is described by the function

$$g(u_t) = \begin{cases} \alpha \frac{\bar{y}}{\bar{k}} \dfrac{1}{1+\eta_1}\left(u_t^{1+\eta_1} - 1\right) & \text{if } u_t > 1 \\ \alpha \dfrac{\bar{y}}{\bar{k}} \dfrac{1}{1+\eta_2}\left(u_t^{1+\eta_2} - 1\right) & \text{if } u_t \leq 1. \end{cases} \qquad \mathbf{(10)}$$

This function is related to the capacity constraint mentioned earlier by the value of η_1. If η_1 goes to infinity, u_t will never exceed one. Because, as stated earlier, the first derivative of $g(u_t)$ at the steady state is independent of η, the function described by (10) is differentiable for all u_t.[12] In order to illustrate the possible behavior of the costs of utilization, we plot two functions $g(u_t)$ and two functions $\partial g(u_t)/\partial u_t$ in Figure 3. When k_t equals 1, these functions correspond to the costs and marginal costs of utilization, respectively. In each panel, one function is characterized by a large increase in the elasticity with respect to utilization if the utilization rate exceeds 100%, while the elasticity used for the other function is constant.[13]

It should be noted that capacity constraints could also emerge for other reasons than those mentioned.[14] Moreover, although a relation between utilization and depreciation is often assumed, the functional form of this relation given earlier is only one among many potential candidates.[15] Unfortunately, concerning the functional form, there appears to be no empirical guidance in the literature. Therefore,

https://doi.org/10.1017/S1365100512000260 Published online by Cambridge University Press

TABLE 2. Calibrated parameters and nonstochastic steady-state values

α	β	δ	ν	ω	η_1	η_2	ρ	σ	$\bar{h}$
0.333	0.988	0.015	1.004	4.181	1000	0.39	0.985	0.00354	0.2

the only possibility for judging whether the assumptions made here with regard to utilization costs are not unreasonable is a comparison of the model's implications with the empirical data. However, even if satisfactory results are obtained using the functional form given by (10), this does, of course, not rule out the possibility that other functional forms could generate similar outcomes. In the sensitivity analysis conducted in Section 4.3, the function (10) will be modified in order to gain further insights.

3.4. Calibration

Several parameters are calibrated according to the values reported in King and Rebelo (1999). These include the share of capital income, α, which is set to $1/3$, the growth rate of the economy, $\ln(\nu)$, which is set to 0.004, the share of time devoted to work in the nonstochastic steady state, $\bar{h}$, which is set to 20%, and the discount factor, β, which is determined by $\beta = \nu/(1 + r)$, where r is the average quarterly real interest rate, $r = 0.065/4$. The quarterly depreciation rate in the nonstochastic steady state δ is set to 0.015, which is in line with the result reported in Stokey and Rebelo (1995). As mentioned in Section 3.3, utilization in the nonstochastic steady state $\bar{u}$ is set to 1. With these values, the capital-to-output ratio in the nonstochastic steady state equals 10.67,[16] and the consumption-to-output ratio attains a value of 0.80. Finally, because the value of $\bar{z}$ affects neither the dynamics of the model nor the great ratios ($\bar{c}/\bar{y}$, $\bar{k}/\bar{y}$, $\bar{x}/\bar{y}$), but only the scale of the economy, we normalize it to 1.

The choice of the parameter values of the process for productivity (7) depends on the costs of utilization. We decide to set the persistence parameter ρ equal to 0.985. This choice is based on Table 5 in King and Rebelo (1999), according to which a value of 0.978 corresponds to the case of constant capacity utilization, and a value of 0.989 corresponds to an almost costless variation of capacity utilization. So we have chosen an intermediate value for ρ that appears appropriate in the case of moderate marginal costs with respect to utilization. For η_1, we choose a value of 1,000, because this value turns out to be large enough to be considered equivalent to a constraint that sets an upper limit to utilization. The values of η_2 and σ are then determined by the requirement that the model should replicate the standard deviation and skewness of output as measured by GDP. We find that a value of η_2 equal to 0.39 and a value of σ equal to 0.00354 fulfill these requirements.

Table 2 summarizes the calibrated parameter and steady-state values.

3.5. Solution

Because the model under study can be expected to exhibit pronounced asymmetries as a result of the constraint on capacity utilization, a solution by log-linearization would not be appropriate. In fact, any solution method imposing a smooth functional form on the decision rules of the household could be problematic, because the decision rules can be expected to be kinked at the point where the capacity constraint starts to bind. Therefore, we solve the model by value-function iteration, as was also done by Hansen and Prescott (2005).

This approach requires k_t and z_t to be discrete-valued variables. In the setting of the maximization problem here, both variables are continuous. This problem is addressed by the common approach of transforming k_t and z_t into discrete-valued variables, i.e., by choosing grids that these variables lie on. The choice of the number of grid points for k_t and z_t is subject to the trade-off between accuracy and computing time. Although the choice of the number of grid points for k_t is to some extent arbitrary, the range of this grid must be chosen in such a way that it contains the complete ergodic set, i.e., the set that k_t does not leave once it has entered it. For such a set to exist, z_t has to be bounded. Concerning the AR(1)-process for z_t, Tauchen (1986) proposes a discrete-valued approximation by an m-state Markov chain, which evidently leads to boundedness of z_t. Following Hansen and Prescott (2005), we set m to 15 and choose the values attained by the Markov chain in such a way that they cover $b = \pm 2$ standard deviations of the process for productivity from $\ln(\bar{z})$. The grid for k_t consists of 1,200 evenly spaced grid points. The values $\ln z_t$ can adopt, the corresponding states, and the transition matrix of the Markov chain are given in the Appendix.

4. RESULTS

4.1. Summary Statistics

To find the moments implied by the model economy, we ran 50,000 simulations, each one yielding 2,194 observations. We disregard the first 2,000 observations, so that 194 observations remain. For every variable except the depreciation rate, utilization, and utilization costs, we take logarithms and apply the HP-filter. In addition to the variables contained in the model, we also report results for TFP measured as if utilization and capital were constant and as if utilization were equal to one, thus as

$$\mathrm{tfp}_t = \frac{y_t}{\bar{k}^\alpha h_t^{1-\alpha}}.$$

This variable corresponds to the empirical measure of total factor productivity given by (1). Henceforth, in the context of the model economy, we will refer to tfp_t as total factor productivity.[17] The variable labor productivity is constructed as $\mathrm{lp}_t = y_t/h_t$. This definition corresponds to the definition used for the calculation of empirical labor productivity. The quantity $2/3 \cdot \mathrm{lp}_t$, of course, simply equals the real wage w_t. Concerning capital, we construct an annual variable ka_t with

TABLE 3. Third and second moments

	y	c	x	ka	h	w	lp	tfp
Skewness								
Model	−0.42	−0.12	−0.68	0.07	−0.50	−0.12	−0.12	−0.31
Data	*−0.42*	*−0.11*	*−0.69*	*0.13*	*−0.49*	*−0.10*	*−0.35*	*−0.34*
Std. dev.								
Model	1.61	0.42	6.95	0.70	1.25	0.42	0.42	0.80
Data	*1.61*	*0.71*	*5.22*	*0.74*	*1.78*	*0.95*	*0.85*	*0.80*
Rel. std. dev.								
Model	1.00	0.26	4.31	0.43	0.77	0.26	0.26	0.49
Data	*1.00*	*0.44*	*3.23*	*0.46*	*1.11*	*0.59*	*0.53*	*0.50*
Corr. with y								
Model	1.00	0.90	0.98		0.98	0.90	0.90	0.98
Data	*1.00*	*0.66*	*0.95*		*0.88*	*0.09*	*0.05*	*0.71*
Corr. with x								
Model		0.84						
Data		*0.53*						

Note: "Std. dev." denotes "standard deviation"; "rel." stands for "relative."

$t = 4, 8, \ldots$ by considering only every fourth value of the quarterly capital series k_t in order to make the results from the model comparable to the empirical results. The annual capital series is calculated prior to the application of the HP-filter.

Coefficients of skewness, standard deviations, relative standard deviations, and correlations generated by the model economy are displayed in Table 3. For convenience, we again show the respective values found in the empirical data. Standard deviations are multiplied by 100.

Concerning the coefficients of skewness, the results of the model correspond remarkably well to the empirical results for most variables. The differences between the coefficients of skewness from the model and those from the empirical data do not exceed 0.03 for the variables consumption, investment, real wage, labor, and TFP. Moreover, it turns out that capital is the only positively skewed variable in the model, attaining a coefficient of skewness only 0.06 smaller than its empirical counterpart. The skewness of LP in the model is considerably less pronounced than that in the data. The skewness of output in the model matches the empirical skewness by construction, i.e., because of the choice of η_2.

Concerning second moments, many variables from the model are less volatile than their empirical counterparts, above all consumption, and the associated variables real wage and LP. In contrast to this, the empirical standard deviations of investment, capital, and TFP are fairly well reproduced by the model. The strongest deviations of simulated cross correlations from their empirical counterparts are observed for the real wage and LP. The correlation of consumption and investment equals 0.84, thereby exceeding its empirical counterpart by about 0.3. In light of the results reported in Valderrama (2007), it is noteworthy that the model

TABLE 4. Second and third moments of all models

	y	c, w, lp	x	ka	h	tfp	u	$g(u)$	k	z
Skewness										
Cap. constr.	−0.42	−0.12	−0.68	0.07	−0.50	−0.31	−1.22	−1.21	0.04	0.00
Const. util.	−0.01	0.01	−0.18	−0.01	−0.02	0.00	−0.02	0.05	−0.01	0.00
Sym. util.	−0.01	0.00	−0.21	−0.01	−0.02	−0.01	0.03	0.05	−0.01	0.00
Std. dev.										
Cap. constr.	1.61	0.42	6.95	0.70	1.25	0.80	1.95	0.06	0.30	0.49
Const. util.	1.61	0.55	6.12	0.92	1.13	0.88	0.00	0.00	0.40	0.87
Sym. util.	1.61	0.41	6.59	0.69	1.24	0.80	2.64	0.08	0.30	0.41
Rel. std. dev.										
Cap. constr.	1.00	0.26	4.31	0.43	0.77	0.49	1.21	0.04	0.19	0.31
Const. util.	1.00	0.34	3.80	0.57	0.70	0.55	0.00	0.00	0.25	0.54
Sym. util.	1.00	0.25	4.09	0.43	0.77	0.50	1.64	0.05	0.18	0.26
Corr. with y										
Cap. constr.	1.00	0.90	0.98		0.98	0.98	0.61	0.61	−0.04	0.93
Const. util.	1.00	0.91	0.98		0.98	0.98	0.62	0.62	0.05	0.99
Sym. util.	1.00	0.93	0.99		0.99	0.99	0.56	0.56	−0.06	0.98
Corr. with x										
Cap. constr.		0.84								
Const. util.		0.85								
Sym. util.		0.89								

Note: "Std. dev." denotes standard deviation, "rel." stands for relative, "cap. constr." stands for the model with capacity constraint, "const. util." for the model with constant utilization, and "sym. util." for the model with symmetric utilization.

reproduces the different magnitudes of the empirical skewness of consumption and investment, although it overstates the correlation of these variables.

To investigate to what extent the coefficients of skewness of the model are caused by the capacity constraint, we simulate two models without such a constraint. In one model, we set η_2 to 1,000, so that utilization becomes virtually constant. The parameter ρ is set to 0.98 and the parameter σ to 0.00634.[18] All remaining parameters are unchanged. We will refer to this model as the model with constant utilization. In the other model, we set η_1 to 0.39, so that utilization can be varied, but the marginal costs of utilization are not kinked at $u_t = 1$, which means that utilization can be expected to be approximately symmetric. In this model, the parameter ρ is set to 0.988 and the parameter σ to 0.00292.[19] This model will be referred to as the model with symmetric utilization. Results of the simulations with these two models and with the model with capacity constraint are presented in Table 4. Obviously, in the models without capacity constraint, the skewness of all variables except for investment is close to zero. The skewness of investment equals about −0.18 in the model with constant utilization and about −0.21 in the model with symmetric utilization. To the best of our knowledge, the skewness of investment in prototypical RBC models has not been documented in the literature yet and seems at least noteworthy.

Hence, for the model with capacity constraint, we can conclude that it is indeed only the capacity constraint that causes the skewness of all variables except investment and that strongly amplifies the negative skewness of investment. The most pronounced negative skewness of the model with capacity constraint is observed for the variables utilization and depreciation rate,[20] with coefficients of about -1.2. This result is not surprising, because, in contrast to the other variables, these two variables virtually cannot exceed their nonstochastic steady-state values.

Second moments are hardly affected by the introduction of a capacity constraint. Rather, they depend on the possibility of varying capital utilization. Therefore, one often finds noticeable differences between the model with constant utilization and models with varying utilization, whereas, in many cases, the model with capacity constraint produces results that are similar to those for the model with symmetric utilization. For most variables, the correlations with output are rather similar across models. The correlation of consumption and investment also reaches similar values for the models with capacity constraint and with constant utilization, whereas the correlation is slightly greater in the model with symmetric utilization.

An interesting feature of the economy with capacity constraint is given by its behavior in the stochastic steady state. In contrast to the nonstochastic steady state, the stochastic steady state is characterized by uncertainty about future values of productivity. That is, the model attains its stochastic steady state when productivity is always at its steady-state level, but the household believes that productivity is uncertain and evolves according to (7). Owing to uncertainty about future income, precautionary saving in our models leads to a value of the capital stock that is higher than its nonstochastic steady-state value. This increase is unrelated to the existence of a capacity constraint. However, owing to the capacity constraint, the stochastic steady-state value of capital increases further. This effect occurs because the household loses the possibility of adjusting its production to new levels via the variation of utilization as soon the capacity constraint is reached. Therefore, the household seeks to prevent the constraint from binding, and it does so by accumulating a larger capital stock. To assess the quantitative importance of the additional capital accumulation, we report the stochastic steady-state values and the mean values over all simulations for all models, as well as the nonstochastic steady-state values of the models' variables, in Table 5.

The values in the nonstochastic steady state are identical for all models. The values in the stochastic steady state can be regarded as a kind of median, because, in 50% of all cases, productivity is higher than its stochastic steady-state value.[21] As a consequence, the other variables can be expected to lie above or below their stochastic steady-state values in about 50% of all cases.

In the models without capacity constraint, the nonstochastic steady-state values of all variables are very close to their counterparts in the stochastic steady state. The increase in the capital stock due to precautionary saving is below 0.05% in both cases. The existence of a capacity constraint, however, causes an increase

TABLE 5. Stochastic steady-state and mean values

		Deviations from nsss in %					
		Cap. constr.		Const. util.		Sym. util.	
	nsss level	sss	Mean	sss	Mean	sss	Mean
y	0.65	0.1	−0.2	0.0	0.1	0.0	0.1
c	0.52	0.2	0.1	0.0	0.1	0.0	0.1
x	0.13	−0.1	−1.3	0.0	0.2	0.0	0.0
k	6.97	1.5	1.6	0.0	0.2	0.0	0.3
h	0.20	−0.1	−0.3	0.0	0.0	0.0	0.0
w, 2/3lp	2.18	0.2	0.1	0.0	0.1	0.0	0.1
tfp	1.00	0.2	0.0	0.0	0.1	0.0	0.1
u	1.00	−1.0	−1.7	0.0	0.0	0.0	−0.1
$100(\delta + g(u))$	1.50	−2.0	−3.6	0.0	0.0	−0.1	−0.1

Note: "Cap. constr." stands for the model with capacity constraint, "const. util." for the model with constant utilization, and "sym. util." for the model with symmetric utilization. "nsss" denotes the nonstochastic steady state and "sss" the stochastic steady state.

in the capital stock of 1.5%. Thus, the additional capital accumulation due to the capacity constraint is much greater than the additional capital accumulation due to the existence of uncertainty.

In contrast to the stochastic steady-state values, the means of the capital stock in the models without capacity constraint moderately exceed the nonstochastic steady-state value, namely by 0.2 to 0.3%. Given that stochastic steady-state values can be thought of as median values and given that the skewness of capital is close to zero, according to the results in Table 4, this result might appear puzzling at first sight. However, the reason is simply that, in Table 5, we consider levels, although in Table 4 we consider log-levels of all variables except for the depreciation rate and utilization. Because the logarithm is a concave function, variables in levels tend to have higher values of skewness than variables in log-levels. Yet the effect of the log function on skewness appears to be rather small for the variables considered here. In the model with capacity constraint, the mean values of all variables that exhibit negative skewness according to the results in Table 4 are lower than their respective values in the stochastic steady state.

Apart from capital, strong deviations of the stochastic steady state from the nonstochastic steady state in the model with capacity constraint are found only for utilization and, consequently, for the depreciation rate. To prevent the capacity constraint from binding too often, in the stochastic steady state the household chooses a utilization rate that is 1% lower than in the nonstochastic steady state, leading to a decrease in the depreciation rate by 2%. On average, utilization is 1.7% lower and the depreciation rate is 3.6% lower than in the nonstochastic steady state, so that the average depreciation rate equals about 1.45% instead of 1.5%.

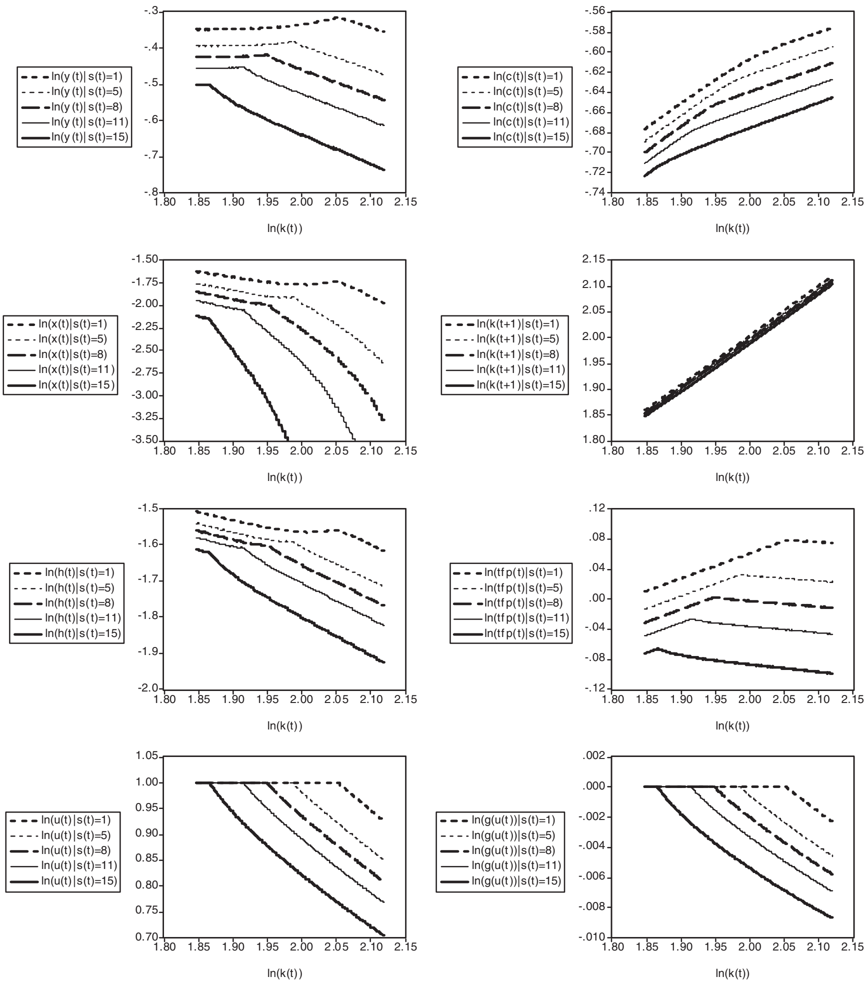

FIGURE 4. Decision rules.

4.2. Reasons for Skewness

Further insight into the behavior of the variables in the model with capacity constraints can be gained by investigating the corresponding decision rules. These rules are displayed in Figure 4. We present decision rules for all endogenous variables given the capital stock in period t and a level of productivity corresponding to productivity in the states 1, 5, 8, 11, and 15 of the underlying Markov chain.[22] As can be seen for all variables except capital, the decision rules are kinked. These kinks are located at those values of the state variables k_t and s_t where the kink in the marginal utilization costs occurs. For capital, kinks exist as well, but they are not visible because the slopes of the decision rules change only marginally and

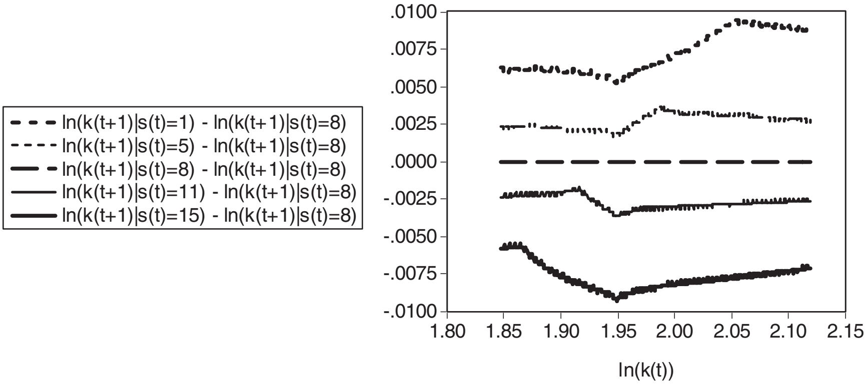

FIGURE 5. Differences between decision rules for $\ln(k_{t+1})$.

because the decision rules for different states are very close to each other. Yet the kinks become visible looking at the differences between the decision rules for the five states mentioned and the decision rule for state 8, as displayed in Figure 5.

Looking at the kinked decision rules, it becomes rather obvious that they give rise to asymmetric behavior of the simulated variables. Suppose that the log of the capital stock equals $\ln(\check{k}_t) = 1.95$, which is a value slightly below the nonstochastic steady state, and that state 8 of the Markov chain is present, i.e., productivity is at its steady-state value of 1, so that the economy is close to a kink in the decision rules. If, in the next period, productivity increases because, say, state 5 occurs, any control variable q grows by approximately $\ln(q_{t+1}|s_{t+1} = 5, \check{k}_t) - \ln(q_t|s_t = 8, \check{k}_t)$, because the capital stock changes only by a small amount; i.e., $\ln k_{t+1} \approx \ln \check{k}_t$. In an analogous manner, if productivity decreases in the next period because state 11 occurs, q falls by approximately $\ln(q_{t+1}|s_{t+1} = 11, \check{k}_t) - \ln(q_t|s_t = 8, \check{k}_t)$. Both possible changes of productivity have the same probability, and both possible new values of log productivity have the same distance from zero, which means that the shocks under consideration are symmetric. The reaction of the control variables, however, is asymmetric, because the value of $\ln(q_{t+1}|s_{t+1} = 5, \check{k}_t) - \ln(q_t|s_t = 8, \check{k}_t)$ exceeds the absolute value of $\ln(q_{t+1}|s_{t+1} = 11, \check{k}_t) - \ln(q_t|s_t = 8, \check{k}_t)$, as can be seen in the decision rules. Thus, $E[q_{t+1}|s_t = 8, \check{k}_t]$ is skewed to the left. Similar reasoning applies for almost all values of the set (k_t, s_t). Exceptions are given by very small and large values of k_t, where the utilization constraint either always or never binds, independent of the value of s_t.[23] It is also obvious that some variables, such as investment and utilization, are more skewed than others, such as consumption and TFP. The ratio of, for example, $\ln(q_{t+1}|s_{t+1} = 11, \check{k}_t) - \ln(q_t|s_t = 8, \check{k}_t)$ to the absolute value of $\ln(q_{t+1}|s_{t+1} = 5, \check{k}_t) - \ln(q_t|s_t = 8, \check{k}_t)$ is much higher for investment and utilization than for consumption and TFP.

The decision rules also give an intuition for why cross correlations are hardly affected by the introduction of the capacity constraint. First, note that the cross

https://doi.org/10.1017/S1365100512000260 Published online by Cambridge University Press

TABLE 6. Skewness with modified utility functions

Utility	y	c	x	ka	h	w, lp	tfp	u	$g(u)$	k
Original	−0.42	−0.12	−0.68	0.07	−0.50	−0.12	−0.31	−1.22	−1.21	0.04
$\Phi = 1$	−0.42	−0.12	−0.68	0.07	−0.50	−0.12	−0.31	−1.21	−1.20	0.04
$\Phi = 0$	−0.43	−0.14	−0.69	0.06	−0.52	−0.11	−0.32	−1.21	−1.20	0.04
$\Phi = -1$	−0.44	−0.16	−0.71	0.05	−0.53	−0.10	−0.32	−1.20	−1.19	0.03

correlations of the models with constant and with symmetric utilization are very similar. Second, note that, in the economy with capacity constraint, the decision rules are all kinked in the same region of the set (k_t, s_t), i.e., in the region where the economy is on the verge of changing from one regime to the other regime. One regime here is characterized by constant utilization, the other regime by variable utilization. Because cross correlations are similar in both regimes, and because all variables undergo a possible regime switch simultaneously, the resulting cross correlations are approximately equal to their averages over both regimes.

According to the results in Table 3, some variables are more skewed than others. Based on Figure 4, it is straightforward to give an explanation of why, for example, the depreciation rate (i.e., the utilization costs) and utilization are strongly negatively skewed. They are virtually bounded above, in contrast to the other variables of the model. However, it is not self-evident why, for example, output exhibits stronger skewness than consumption or why capital has positive skewness.

One possible reason that consumption is less skewed than most other variables could be given by the preferences of the household. Because its utility is logarithmic in consumption, its expected utility from consumption can be approximated by

$$E\left[\ln c_t\right] \approx \ln \bar{c} + \frac{E\left[c_t - \bar{c}\right]}{\bar{c}} - \frac{E\left[(c_t - \bar{c})^2\right]}{2\bar{c}^2} + \frac{E\left[(c_t - \bar{c})^3\right]}{3\bar{c}^3}.$$

As this approximation shows, the household has a preference for positively skewed consumption, because $E[\ln c_t]$ is increasing in $E[(c_t - \bar{c})^3]$.

To investigate whether preferences can explain why consumption is less negatively skewed than most other variables, we simulate the model with capacity constraint using a modified momentary utility function. Instead of (2), we employ the approximation

$$\mathfrak{U}(c_t, h_t) = \ln \bar{c} + \frac{(c_t - \bar{c})}{\bar{c}} - \frac{(c_t - \bar{c})^2}{2\bar{c}^2} + \Phi \frac{(c_t - \bar{c})^3}{3\bar{c}^3} - \frac{(c_t - \bar{c})^4}{4\bar{c}^4} - \omega h_t.$$

For the parameter Φ, we consider the values 1, 0, and −1. The value of 1 is used to check the validity of the approximation. The coefficients of skewness emerging from these utility functions are presented in Table 6. We also show the coefficients of skewness obtained with the original utility function (2).

The approximation with $\Phi = 1$ appears to be sufficiently exact. Differences with respect to the results with the original utility function are hardly observable. When the value of is Φ lowered to zero, i.e., when the household is indifferent to skewness of consumption instead of preferring positive skewness, the skewness of consumption indeed attains a lower value. However, the decrease is fairly small. Instead of -0.12, the coefficient equals -0.14. When Φ is set to -1, so that the household has preferences for negatively skewed consumption, the coefficient of skewness decreases to -0.16. Thus, it can be concluded that preferences play, at best, a minor role in the explanation of the skewness of consumption. Moreover, they cannot explain differences among the magnitudes of skewness of different variables. When Φ is set to values lower than 1, not only the skewness of consumption, but also the skewness of output, investment, capital, and TFP, become marginally smaller.

So, although the different magnitudes of skewness are obvious from looking at the decision rules, we cannot offer satisfactory economic explanations for these differences yet. The strong skewness of utilization and utilization costs due to their quasi-boundedness may be regarded as the only exception. The topic of possible reasons for the differing magnitudes of skewness is again addressed, but not completely resolved, in the following sensitivity analysis. It therefore remains to be scrutinized in future research.

4.3. Sensitivity Analysis

In the following, we will investigate how the reproduction of empirical third moments by the model depends on certain modeling decisions or parameter values. We focus on the function chosen for the utilization costs and on the additional consideration of investment adjustment costs.[24]

Concerning the utilization cost function, we relax the capacity constraint and set η_1 to values ranging from 1 to 10, so that exceeding a utilization rate of 100% becomes less costly. Moreover, the threshold of the utilization cost function (10) is modified so that the elasticity of marginal costs with respect to utilization is higher if $u_t > \tau$, where τ can now deviate from the nonstochastic steady-state value $\tau = \bar{u} = 1$ considered previously. We choose the values $\tau = 1.01$ and $\tau = 0.99$, which correspond to deviations from $\tau = 1$ of about half a standard deviation of u_t in the original model. Finally, we also allow for the presence of symmetric investment adjustment costs, following the specification used by Valderrama (2007), so that the new budget constraint is given by

$$y_t = c_t + x_t + \frac{\phi}{2} k_t \left(\frac{\bar{x}}{\bar{k}} - \frac{x_t}{k_t} \right)^2 .$$

Thus, adjustment costs increase with deviations of the investment–capital ratio from its value in the nonstochastic steady state.

Table 7. Sensitivity analysis of skewness with respect to parameters of adjustment- and utilization-cost functions

	y	c, w, lp	x	ka	h	tfp	u	$g(u)$	k	z
					Capacity constraint					
Original	−0.42	−0.12	−0.68	0.07	−0.50	−0.31	−1.22	−1.21	0.04	0.00
$\eta_1 = 10$	−0.40	−0.11	−0.66	0.07	−0.48	−0.29	−1.17	−1.16	0.04	0.00
$\eta_1 = 4$	−0.36	−0.10	−0.63	0.06	−0.45	−0.26	−1.07	−1.05	0.04	0.00
$\eta_1 = 2$	−0.31	−0.08	−0.59	0.06	−0.39	−0.22	−0.89	−0.87	0.04	0.00
$\eta_1 = 1$	−0.22	−0.05	−0.49	0.03	−0.28	−0.15	−0.56	−0.53	0.02	0.00
$\tau = 1.01$	−0.26	0.09	−0.47	0.07	−0.34	−0.14	−1.28	−1.26	0.04	0.00
$\tau = 0.99$	−1.26	−0.72	−1.58	0.07	−1.34	−1.09	−2.73	−2.72	0.04	0.00
$\tau = 0.99$, $\eta_1 = 4$	−0.42	−0.13	−0.68	0.06	−0.50	−0.31	−1.31	−1.29	0.04	0.00
$\phi = 3$	−0.23	−0.18	−0.33	0.00	−0.27	−0.20	−1.11	−1.10	0.00	0.00
$\phi = 10$	−0.16	−0.16	−0.20	−0.02	−0.16	−0.16	−1.13	−1.12	−0.01	0.00
					Constant utilization					
Original	−0.01	0.01	−0.17	0.00	−0.02	0.00	−0.02	0.05	0.00	0.00
$\phi = 3$	−0.01	0.01	−0.12	0.00	−0.03	0.00	−0.02	0.05	0.00	0.00
$\phi = 10$	0.00	0.01	−0.07	0.00	−0.02	0.00	−0.01	0.05	0.00	0.00
					Symmetric utilization					
Original	−0.01	0.01	−0.21	−0.01	−0.02	0.00	0.03	0.05	−0.01	0.00
$\phi = 3$	−0.01	0.00	−0.10	0.00	−0.03	0.00	0.02	0.03	0.00	0.00
$\phi = 10$	0.00	0.00	−0.05	0.00	−0.02	0.00	0.02	0.03	0.00	0.00

Note: The original parameterization refers to η_1 =1,000, $\tau = 1$, $\phi = 0$ in the case of the model with capacity constraints, and to $\phi = 0$ in the models with constant and symmetric utilization, respectively.

The results of the sensitivity analyses can be found in Tables 7 and 8. As expected, the asymmetries of all variables decrease if η_1 decreases. With a value of $\eta_1 = 10$, however, the coefficients of skewness are almost identical to those obtained with η_1 =1,000. This result illustrates that it is not necessary to impose an upper bound for utilization, but it is enough to render high utilization sufficiently costly, to replicate the empirical skewness coefficients. If η_1 equals 1, most skewness coefficients attain about half of the magnitudes observed with the original model. So there is still a considerable amount of asymmetry generated by a relatively moderate kink in the marginal costs of utilization.

For most variables, the results obtained with $\eta_1 = 1$ are comparable in magnitude to those obtained with the kink being located at $\tau = 1.01$ and η_1 =1,000. The major differences are given by the facts that, in the latter case, the utilization rate is more strongly skewed and consumption is weakly positively skewed. If the kink is located at $\tau = 0.99$, the model generates pronounced asymmetries for all variables except the capital stock. For example, the skewness of output roughly triples compared to that in the original model.[25]

https://doi.org/10.1017/S1365100512000260 Published online by Cambridge University Press

TABLE 8. Sensitivity analysis of second moments with respect to parameters of of adjustment- and utilization-cost functions

	y	c, w, lp	x	ka	h	tfp	u	$g(u)$	k	z	r_{cx}
					Capacity constraint						
Original	1.61	0.42	6.95	0.70	1.25	0.80	1.95	0.06	0.30	0.49	0.84
$\eta_1 = 10$	1.63	0.42	6.99	0.71	1.26	0.80	1.99	0.06	0.31	0.49	0.85
$\eta_1 = 4$	1.65	0.42	7.07	0.72	1.27	0.82	2.06	0.06	0.31	0.49	0.85
$\eta_1 = 2$	1.68	0.43	7.18	0.73	1.30	0.83	2.19	0.07	0.32	0.49	0.86
$\eta_1 = 1$	1.76	0.44	7.49	0.77	1.36	0.87	2.46	0.08	0.33	0.49	0.87
$\tau = 1.01$	2.28	0.62	9.40	0.70	1.72	1.15	2.10	0.06	0.31	0.49	0.88
$\tau = 0.99$	1.64	0.44	6.96	0.70	1.26	0.82	1.53	0.05	0.31	0.49	0.80
$\tau = 0.99$, $\eta_1 = 4$	1.71	0.44	7.36	0.72	1.33	0.84	2.03	0.06	0.31	0.49	0.84
$\phi = 3$	1.04	0.50	3.32	0.38	0.56	0.68	1.38	0.04	0.15	0.49	0.96
$\phi = 10$	0.79	0.54	1.81	0.20	0.25	0.62	1.07	0.03	0.08	0.49	0.98
					Constant utilization						
Original	1.61	0.55	6.12	0.92	1.13	0.88	0.00	0.00	0.40	0.87	0.85
$\phi = 3$	1.34	0.64	4.21	0.67	0.73	0.87	0.00	0.00	0.28	0.87	0.94
$\phi = 10$	1.11	0.75	2.58	0.43	0.37	0.87	0.00	0.00	0.17	0.87	0.97
					Symmetric utilization						
Original	1.61	0.41	6.60	0.69	1.24	0.80	2.65	0.08	0.30	0.41	0.89
$\phi = 3$	0.98	0.48	2.99	0.31	0.51	0.65	1.86	0.06	0.12	0.41	0.97
$\phi = 10$	0.74	0.52	1.65	0.14	0.23	0.59	1.49	0.05	0.05	0.42	0.98

Note: The original parameterization refers to η_1 =1,000, $\tau = 1$, $\phi = 0$ in the case of the model with capacity constraints, and to $\phi = 0$ in the models with constant and symmetric utilization, respectively. r_{cx} denotes the correlation of consumption and investment.

The foregoing results actually suggest that there might be other combinations of τ and η_1 giving rise to results similar to those obtained with the original model. Indeed, choosing $\tau = 0.99$ and $\eta_1 = 4$ yields asymmetries comparable to those with $\tau = 1$ and η_1 =1,000. So a utilization rate that is virtually bounded at its nonstochastic steady-state value of 1 and a less pronounced kink at a lower utilization rate are both specifications yielding results that are in line with empirical asymmetries. Therefore, one should not overemphasize the importance of the functional form and parameterization of the utilization costs in the original model. Rather, the more general conclusion to be drawn from the sensitivity analyses is that an appropriately calibrated increasing elasticity of the marginal utilization costs is a good candidate for the explanation of business cycle asymmetries. However, because the underlying idea of an upper limit to capital services is relatively easy to motivate and to understand, and because various types and effects of capacity constraints have been studied in the economic literature, this calibration might be considered the most interesting and important one.

Concerning the investment adjustment costs, it turns out that their presence does not cause additional asymmetries per se, as indicated by the results obtained with constant and with symmetric utilization. Rather, the adjustment costs lower the asymmetries observed for investment. This result is likely to be due to the decrease in the volatility of investment. Because we are actually considering the (HP-filtered) logarithm of investment, strongly negative values are less probable if the volatility is small.[26]

If investment adjustment costs are introduced into the model with capacity constraint, this also leads to a decrease in volatility and, therefore, in the asymmetry of investment. Although the asymmetry of utilization and utilization costs is hardly affected, the asymmetry of most other variables considered also decreases considerably. Since the volatility of these variables also decreases, and utilization and utilization costs are the only variables considered that are not subject to the logarithmic transformation, the reason for the decrease in the asymmetries of the other variables is presumably the same as described earlier for investment. Consumption, however, becomes moderately more skewed and volatile than in the original model. This result is caused by the strong correlation of investment and consumption in the presence of adjustment costs. For example, with $\phi = 10$, this correlation attains a value of 0.98. If two variables are almost perfectly correlated, their skewness, obviously, also has to be very similar. The reason for the higher correlation can be understood when the dynamics close to the kink of utilization costs is considered. Whereas in the absence of adjustment costs, investment can easily be adapted to smooth consumption, a greater burden of adjustment falls on consumption if ϕ is large. To be more precise, with $\phi = 0$ and a utilization rate close to 100%, a positive productivity shock can be absorbed by strongly and immediately increasing investment, and by increasing consumption smoothly by smaller amounts in the current and subsequent periods. A negative productivity shock of the same size can be absorbed by a decrease of investment larger in size than the increase considered previously. Of course, the larger size is caused by the comparatively stronger drop in output due to the asymmetric utilization costs. Such reactions of investment become more costly if ϕ is larger than zero. Thus, consumption then has to react more strongly to the productivity shocks and the corresponding output changes on impact, leading to a stronger synchronization of consumption and investment dynamics, and, consequently, to more skewed and volatile consumption. Because the empirical asymmetries of consumption and investment are rather distinct, the presence of investment adjustment costs as considered by Valderrama (2007) thus tends to hamper the model's ability to reproduce these asymmetries.

5. CONCLUSION

In this work, we have analyzed the consequences of the existence of capacity constraints for the asymmetries emerging from an otherwise prototypical RBC model. The capacity constraint originates from the assumption of an upper bound on the utilization of capital, caused by kinked marginal utilization costs. We have compared the asymmetries caused by a model with such a constraint to the asymmetries present in the data, and we have found that the model is able to replicate the asymmetries of most variables, i.e., of output, consumption, investment, capital, labor, the real wage, and (measured) TFP very well. Only the skewness of LP is more pronounced in the data than in the model.

To verify that it is the capacity constraint that causes the model's asymmetries, we have simulated two alternative models without constraint and found that only investment exhibits noteworthy skewness. Comparing the model with capacity constraint with the alternative models, we find that the existence of the capacity constraint leads to increased capital accumulation and lower utilization. Comparing the models with each other, we also note that the introduction of a capacity constraint has negligible effects on standard deviations and cross correlations.

The differing magnitudes of skewness of the model's variables can be inferred from the corresponding decision rules, but it is difficult to find explanations for these differences. Only the reason for the strong negative skewness of utilization and utilization costs is obvious. The other differences in magnitudes of skewness remain largely unexplained. Interestingly, the weak skewness of consumption is found to be basically unrelated to the preference for positively skewed consumption implied by the utility function.

Although capacity constraints for capital services can be motivated easily and are studied frequently in the economic literature, it should be noted that other utilization cost functions that also imply an increasing elasticity of marginal costs with respect to utilization could reproduce empirical business cycle asymmetries sufficiently well, too. The presence of investment adjustment costs as considered by Valderrama (2007), however, tends to hamper the model's ability to reproduce empirical asymmetries, because these costs lead to a strong synchronization of consumption and investment dynamics, whereas the empirical asymmetries of these variables differ considerably.

Future research could address the question of the reasons for the differing magnitudes of skewness in greater depth. In view of the results in Canova (1998), it would, of course, also be interesting to investigate whether the skewness found in the data and that produced by the model still coincide well if other detrending procedures are used.

NOTES

1. See, for example, DeLong and Summers (1986) or Bai and Ng (2005).
2. It is understood that there are several other types of asymmetry that can be of interest, including steepness and sharpness as considered, for instance, by Clements and Krolzig (2003). The type of asymmetry that is associated with skewness is sometimes labeled *deepness* in the literature, a term coined by Sichel (1993).
3. When we speak of third moments, we always refer to third *standardized* moments, i.e., skewness.
4. We set the smoothing parameter to the standard value of 14,400 for the monthly population series. Then we construct quarterly values by taking averages of the resulting trend.

5. In making the second assumption, we follow Cooley and Prescott (1995). The assumption of constant utilization is not in line with the model presented later on. However, the aim here is not to measure true TFP, but to use a simple empirical measure of TFP. This measure can then be compared with a model-based measure that is constructed in the same way.

6. A brief survey of the different possibilities of calculating α may be found in Christiano (1988). The value $1/3$ is employed by King and Rebelo (1999), among others.

7. For the test, we use the Bartlett kernel as discussed by Newey and West (1987) and no prewhitening.

8. Bai and Ng (2005) stress that low power occurs in small samples with strong serial correlation. Psaradakis and Sola (2003) state that low power is observed with detrended data.

9. For the quarterly capital series, the last three samples considered all end in 2001q4.

10. The length of 32 quarters was chosen because, according to Burns and Mitchell (1946), this value corresponds to the maximum length of one business cycle. We can therefore be confident that we have excluded at least one cycle for the first and the last sample.

11. The capital series has only 48 distinct values.

12. The conditions for the maximization by value-function iteration that will be employed to solve the model do not require differentiability of the constraints. We could therefore also have directly constrained u_t to be less than or equal to one. However, we prefer the specification presented here, because it is more flexible and nests the case of u_t never being larger than one if η_1 approaches infinity. Moreover, we can consider a lower value of η_1 in the sensitivity analysis.

13. The parameter values and the value for the ratio $\bar{y}/\bar{k}$ chosen here correspond to those used later.

14. For example, in Gilchrist and Williams (2000), capacity constraints are the result of a putty-clay technology.

15. Concerning the assumption of varying utilization rates and depreciation rates, see, for example, Taubman and Wilkinson (1970).

16. This translates into a ratio of about 2.67 for annual values.

17. Remember that z_t is simply labeled productivity.

18. As mentioned previously, a value for ρ close to 0.98 is suggested in King and Rebelo (1999) for an economy with constant utilization. The value for σ was chosen to replicate the standard deviation of GDP.

19. Again, a value for ρ equal to about 0.988 is suggested in King and Rebelo (1999) for an economy with highly variable utilization, and the value for σ was chosen to replicate the standard deviation of GDP.

20. The depreciation rate is given by $\delta + g(u_t)$. Because δ is constant, the central moments of the depreciation rate such as skewness, standard deviation, and cross correlation are identical to the central moments of $g(u_t)$.

21. For productivity, the stochastic steady state is identical to the nonstochastic steady state.

22. The respective values of productivity are reported in the Appendix.

23. Of course, the reasoning presented cannot be applied to the decision rules with $s_t = 1$ and $s_t = 15$, either.

24. Variations of the other model parameters, as well as of parameters related to the numerical solution method, hardly affect the asymmetries produced by the model. The corresponding simulation results are available upon request.

25. Although the changes with respect to skewness might seem large with respect to the apparently small changes in τ, one should bear in mind that the standard deviations of u_t, depending on the calibration considered, only range from 0.015 to 0.021, so that the changes with respect to τ are actually large as well.

26. This supposition is supported by the results of simulations with a higher value of σ not reported here. In that case, the volatility and the asymmetry of investment both increase. Such a relation between the variance and the skewness of a random variable being subject to a monotonic concave transformation can be illustrated using the lognormal distribution. Suppose that the variable N has a normal distribution with variance σ^2, so that e^N is lognormally distributed. Then $-e^N$, a monotonic concave transformation

https://doi.org/10.1017/S1365100512000260 Published online by Cambridge University Press

of N, has a negative skewness equal to $-(e^{\sigma^2}+2)\sqrt{e^{\sigma^2}-1}$, which is decreasing in the variance σ^2. Of course, the variance of $-e^N$ is increasing in σ^2. The logarithm is a monotonic concave transformation as well, and the statistical relations described here are likely to be the reason that the skewness of the logarithm of investment is less pronounced when investment becomes less volatile.

REFERENCES

Bai, J. and S. Ng (2005) Tests of skewness, kurtosis, and normality for time series data. *Journal of Business and Economic Statistics* 23(1), 49–60.

Burns, A.M. and W.C. Mitchell (1946) *Measuring Business Cycles.* New York: National Bureau of Economic Research.

Canova, F. (1998) Detrending and business cycle facts. *Journal of Monetary Economics* 41, 475–512.

Christiano, L.J. (1988) Why does inventory investment fluctuate so much? *Journal of Monetary Economics Volume* 21, 247–280.

Christiano, L., M. Eichenbaum, and C. Evans (2001) Nominal rigidities and the dynamic effects of a shock to monetary policy. Unpublished manuscript.

Clements, M.P. and H.-M. Krolzig (2003) Business cycle asymmetries: Characterisation and testing based on Markov-switching autoregressions. *Journal of Business and Economic Statistics* 21, 196 – 211.

Cooley, T.F. and E.C. Prescott (1995) Economic growth and business cycles. In T.F. Cooley (ed.), *Frontiers of Business Cycle Research,* chap. 1, pp. 1–38. Princeton, NJ: Princeton University Press.

DeLong, J.B. and L.H. Summers (1986) Are business cycles symmetrical? In R.J. Gordon (ed.), *The American Business Cycle: Continuity and Change,* pp. 166–178. University of Chicago Press for the National Bureau of Economic Research.

Gasser, T. (1975) Goodness-of-fit-tests for correlated data. *Biometrika* 62(3), 563–570.

Gilchrist, S. and J.C. Williams (2000) Putty-clay and investment: A business cycle analysis. *Journal of Political Economy* 108(5), 928–960.

Goodwin, T.H. (1993) Business-cycle analysis with a Markov-switching model. *Journal of Business and Economic Statistics* 11(3), 331–339.

Hansen, G.D. (1985) Indivisible labor and the business cycle. *Journal of Monetary Economics* 16, 309–327.

Hansen, G.D. and E.C. Prescott (2005) Capacity constraints, asymmetries, and the business cycle. *Review of Economic Dynamics* 8(4), 850–865.

Ireland, P.N. (2004) A method for taking models to the data. *Journal of Economic Dynamics and Control* 28, 1205–1226.

Keynes, J.M. (1936) *The General Theory of Employment, Interest and Money.* Macmillan.

King, R.G., C.I. Plosser, and S.T. Rebelo (1988) Production, growth and business cycles: I. The basic neoclassical model. *Journal of Monetary Economics* 21(2/3), 195–232.

King, R.G. and S.T. Rebelo (1999) Resuscitating real business cycles. In J.B. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics,* vol. 1B, chap. 14. Amsterdam: North-Holland.

Mitchell, W.C. (1927) *Business Cycles: The Problem and Its Setting.* New York: National Bureau of Economic Research.

Neftci, S.N. (1984) Are economic time series asymmetric over the business cycle? *Journal of Political Economy* 92(2), 307–328.

Newey, W.K. and K.D. West (1987) A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica* 55(3), 703–708.

Psaradakis, Z. and M. Sola (2003) On detrending and cyclical asymmetry. *Journal of Applied Econometrics* 18(3), 271–289.

Rogerson, R. (1988) Indivisible labor, lotteries and equilibrium. *Journal of Monetary Economics* 21(1), 3–16.

Sichel, D.E. (1993) Business cycle asymmetry: A deeper look. *Economic Inquiry* 31, 224–236.

Smets, F. and R. Wouters (2003) An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association* 1(5), 1123–1175.
Stokey, N. and S.T. Rebelo (1995) Growth effects of flat-rate taxes. *Journal of Political Economy* 103, 519–550.
Taubman, P. and M. Wilkinson (1970) User cost, capital utilization and investment theory. *International Economic Review* 11(2), 209–15.
Tauchen, G. (1986) Finite state Markov-chain approximations to univariate and vector autoregressions. *Economics Letters* 20, 177–81.
Valderrama, D. (2007) Statistical nonlinearities in the business cycle: A challenge for the canonical RBC model. *Journal of Economic Dynamics and Control* 31(9), 2957–2983.
Van Nieuwerburgh, S. and L. Veldkamp (2006) Learning asymmetries in real business cycles. *Journal of Monetary Economics* 53(4), 753–772.

APPENDIX: PARAMETERS OF DISCRETE PROCESS FOR PRODUCTIVITY

The values $\ln z_t$ can attain are given by

$$100 \ln z_t = \begin{cases} 4.103 & \text{if } s_t = 1 \\ 3.517 & \text{if } s_t = 2 \\ 2.931 & \text{if } s_t = 3 \\ 2.345 & \text{if } s_t = 4 \\ 1.758 & \text{if } s_t = 5 \\ 1.172 & \text{if } s_t = 6 \\ 0.586 & \text{if } s_t = 7 \\ 0.000 & \text{if } s_t = 8 \end{cases}, \quad 100 \ln z_t = \begin{cases} -0.586 & \text{if } s_t = 9 \\ -1.172 & \text{if } s_t = 10 \\ -1.758 & \text{if } s_t = 11 \\ -2.345 & \text{if } s_t = 12 \\ -2.931 & \text{if } s_t = 13 \\ -3.517 & \text{if } s_t = 14 \\ -4.103 & \text{if } s_t = 15 \end{cases},$$

with s_t being the state of the Markov chain with transition matrix $\mathbf{P}$ given by

$$\mathbf{P} = \begin{bmatrix}
.74 & .16 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 \\
.25 & .59 & .17 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 \\
.01 & .24 & .59 & .17 & .01 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 \\
.00 & .01 & .23 & .59 & .18 & .01 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 \\
.00 & .00 & .01 & .22 & .59 & .18 & .01 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 \\
.00 & .00 & .00 & .01 & .22 & .59 & .19 & .01 & .00 & .00 & .00 & .00 & .00 & .00 & .00 \\
.00 & .00 & .00 & .00 & .01 & .21 & .59 & .20 & .01 & .00 & .00 & .00 & .00 & .00 & .00 \\
.00 & .00 & .00 & .00 & .00 & .01 & .20 & .59 & .20 & .01 & .00 & .00 & .00 & .00 & .00 \\
.00 & .00 & .00 & .00 & .00 & .00 & .01 & .20 & .59 & .21 & .01 & .00 & .00 & .00 & .00 \\
.00 & .00 & .00 & .00 & .00 & .00 & .00 & .01 & .19 & .59 & .22 & .01 & .00 & .00 & .00 \\
.00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .01 & .18 & .59 & .22 & .01 & .00 & .00 \\
.00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .01 & .18 & .59 & .23 & .01 & .00 \\
.00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .01 & .17 & .59 & .24 & .01 \\
.00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .17 & .59 & .25 \\
.00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .16 & .74
\end{bmatrix}.$$

Element $\mathbf{P}_{i,j}$ of this matrix contains the transition probability $\Pr(s_{t+1} = j \,|s_t = i)$.

https://doi.org/10.1017/S1365100512000260 Published online by Cambridge University Press