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WELFARE EFFECTS OF PATENT PROTECTION IN A SEMI-ENDOGENOUS GROWTH MODEL

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This paper examines how strengthening patent protection affects welfare in a nonscale quality-ladder model, which was developed by Segerstrom [*American Economic Review* 88, 1290–1310] and generalized by Li [*American Economic Review* 93, 1009–1017]. In the Segerstrom–Li model, patent protection creates no distortion in static allocation among the production sectors. In order to examine the welfare effects of strengthening patent protection adequately, we incorporate a competitive outside good into the Segerstrom–Li model. In the general model, we derive the welfare-maximizing degree of patent protection analytically by utilizing a linear approximation of the transition path. The result shows that the welfare-maximizing degree of patent protection is weaker when the market share of the outside good is positive than when it is zero. In other words, disregarding the static distortion that patent protection creates leads to excessive patent protection.

Keywords: R&D, Patent Protection, Welfare Analysis, Semi-Endogenous Growth

1. INTRODUCTION

Since the TRIPS (Trade-Related Aspects of Intellectual Property Rights) agreement was signed in 1994, patent protection has been strengthened in many countries. However, strengthening patent protection has opposing welfare effects. On the one hand, it promotes research and development (R&D) activity, and consequently increases welfare. On the other hand, it also raises the price of the good based on the technology protected by patents, and reduces welfare. Therefore, one needs to examine whether strengthening patent protection actually improves welfare and derive the welfare-maximizing level of patent protection. A number of studies have already examined this using an endogenous growth framework [e.g., Iwaisako and Futagami (2003), Kwan and Lai (2003), Futagami and Iwaisako (2007), Lin (2015)]. However, these studies all used growth models including

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a scale effect property, which is largely inconsistent with the existing empirical evidence. Thus, in this paper, we reexamine the welfare effects of strengthening patent protection in a growth model without scale effects. In particular, we employ the nonscale quality-ladder model developed by Segerstrom (1998) and generalized by Li (2003), which has been commonly used in various analyses.[1]

However, we face two challenges in examining the welfare effects of strengthening patent protection in the Segerstrom–Li model. First, in reality, strengthening patent protection promotes innovation by increasing the monopoly power of patentees. This in turn creates static distortions to resource allocation. However, in the Segerstrom–Li model, patent protection creates no distortion in the static allocation among production sectors because the sectors are symmetric. Therefore, in order to examine the welfare effects of patent protection appropriately, we add a good that is not protected by patents and is supplied competitively. We refer to such a good as an "outside good". Note that competitive outside goods are ubiquitous in a real economy, leisure being the standard example. Introducing the outside good into the Segerstrom–Li model leads us to a generalized model where patent protection creates a distortion in the static allocation between the production of the differentiated goods and of the outside good and where a stronger patent protection lowers the production volume of the differentiated goods below its optimal level. Second, in most nonscale growth models, the convergence to a new steady state is not instantaneous. Thus, one needs to take into account the adjustment dynamics to properly assess the welfare effects of a policy change.[2] In this paper, we use Judd's (1982) method as in Helpman (1993) to analytically examine how strengthening patent protection affects welfare. In particular, we derive the welfare-maximizing patent breadth.[3] We find that, in the presence of an outside good, the welfare-maximizing patent breadth is necessarily narrower than in its absence.

Most closely related to our work is O'Donoghue and Zweimüller (2004), who examine the welfare-maximizing patent breadth in a quality ladder model with a competitive outside good.[4] In Appendix 3, they extend the analysis to a nonscale version of their model but, for simplicity's sake, remove the outside good. Furthermore, they focus on the sole steady state. Other studies also examine the welfare effects of patent protection in a nonscale growth model. Futagami and Iwaisako (2007) examine in Section 5 the welfare effects of extending patent length in a nonscale variety expansion model. However, they do not examine the welfare effects during the transition. Lin and Shampine (2018) examine the welfare effects of extending patent length along the transition path in a nonscale variety expansion type growth model. However, they do this only numerically. Therefore, the present paper is a significant contribution to the nonscale growth model literature.[5] In addition, unlike these earlier studies, we base our analysis on a very well recognized nonscale quality-ladder model that has been applied to the analysis of various issues.[6]

The remainder of the paper is structured as follows. Section 2 describes the model and Section 3 derives the equilibrium path. Section 4 considers the effect of stronger patent protection on welfare. Section 5 provides concluding remarks.

2. THE MODEL

As stated in the introduction, we conduct a welfare analysis of strengthening patent protection using a generalized version of the Segerstrom (1998) and Li (2003) model, where patent protection creates static distortions. We employ basically the same notation as Segerstrom (1998) and Li (2003), but add some new variables and parameters.

2.1. Consumers and Workers

As in the Segerstrom–Li model, there is a continuum of differentiated goods, whose qualities are improved by innovations. As detailed below, the firm that invents the state-of-the-art quality of a differentiated good at a point of time produces the good in each future period until the other firms invent the next quality.

The economy has $L(t)$ consumers that inelastically supply one unit of labor at each point of time. $L(t)$ is growing at a constant rate, n, with $L(0) = 1$. Then, the current population is given by $L(t) = e^{nt}$. The intertemporal utility is given by

$$U = \int_0^\infty e^{-(\rho-n)t} \log u(t) dt,$$

where ρ is a subjective discount rate and $u(t)$ is the instantaneous utility at time t.

We introduce a competitive outside good sector where no innovation occurs. This allows us to examine how strengthening patent protection reduces welfare by causing static distortions. We specify the per capita utility as follows[7]:

$$\log u(t) = (1-\gamma) \log D(t) + \gamma \log d_0(t), \qquad 0 \le \gamma \le 1 \tag{1}$$

where $d_0(t)$ denotes the consumption of the outside good, $D(t)$ represents the index of differentiated goods, and $0 \le \gamma \le 1$ is a parameter which, at the consumption optimum, is also the expenditure share of the outside good. $D(t)$ is given by the following CES-type utility function:

$$D(t) = \left\{ \int_0^1 \left[\sum_j \lambda^j d(j, \omega, t) \right]^\alpha d\omega \right\}^{\frac{1}{\alpha}}, \quad 0 \le \alpha < 1, \tag{2}$$

where $d(j, \omega, t)$ denotes the consumption of the good with the jth highest quality in industry ω and $\lambda(> 1)$ denotes the size of the quality improvement obtained from one innovation. Thus, λ^j represents the quality after j times innovations. The elasticity of substitution between any two goods is given by $1/(1-\alpha)$. The utility reduces to that in the Segerstrom–Li model when $\gamma = 0$. A possible interpretation is that the outside good represents all goods other than the differentiated goods, with γ the share of all other goods.

Solving the maximization problem of the household, we now derive the demand functions for the differentiated good in industry ω and the outside good. First, in

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each differentiated goods industry, the households consume only the good with the lowest quality-adjusted price, which is provided by the firm having the highest quality in equilibrium. Letting $j(\omega, t)$ denote the highest quality in industry ω, the demand for the good with the highest quality in industry ω is given by

$$d(\omega, t) = \frac{(\lambda^{j(\omega,t)})^{\varepsilon}\ p(\omega, t)^{-(\varepsilon+1)}}{\int_0^1 \left[(\lambda^{j(\omega',t)})^{\varepsilon}\ p(\omega', t)^{-\varepsilon}\right] d\omega'}(1-\gamma)c(t), \tag{3}$$

where $p(\omega, t)$ denotes the price of the good with the highest quality in industry ω, $c(t)$ denotes the per capita consumption, and where $\varepsilon \equiv \alpha/(1-\alpha)$. $1/(1-\alpha)$ denotes the elasticity of substitution, and thus ε denotes the elasticity of substitution minus one, and $\varepsilon \geq -1$. Second, we take the wage rate as a numeraire, that is, $w(t) = 1$. We assume that one unit of the outside good can be produced using one unit of labor. Because the market for the good is perfectly competitive, the price of the good must be equal to $w(t) = 1$. Therefore, the demand for the outside good is given by

$$d_0(t) = \gamma c(t). \tag{4}$$

Using (3) and (4), we derive the Euler equation for dynamic maximization as follows:

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho. \tag{5}$$

2.2. Product Markets

We now consider how strengthening patent protection affects the behavior of firms. Generally, two policy instruments influence the degree of patent protection. One is the patent length, which determines how long the patentee can produce and sell the product exclusively. The other is the patent breadth, which determines the scope of the products that the patentee can prevent other firms from producing and selling.[8] In the quality-ladder model, products of different qualities within the same product line are perfect substitutes. Therefore, the patent breadth represents the range of quality that the patent authority forbids other producers to produce.

In practice, the patent authority controls both patent length and patent breadth. However, for simplicity, we assume that the patent length is fixed and infinite and that the patent authority controls only the patent breadth.[9] Following Li (2001), we incorporate patent breadth as follows.[10] When the state-of-the-art quality in industry ω is given by $\lambda^{j(\omega)}$, firms other than the patentee of the state-of-the-art-quality product cannot legally produce products with a higher quality than $\lambda^{j(\omega)}/\beta$, where $\beta \in [1, \lambda]$.[11] Then, β can be interpreted as representing the patent breadth. In this setting, a higher β implies a broader patent breadth. The patent protection is maximal when $\beta = \lambda$. When $\beta = 1$, patent protection is inexistent.

The pricing strategy of a firm depends on the patent breadth. The optimal price level for the firm holding the patent for a state-of-the-art good is such that the other firms cannot earn positive profits by entering the market for that good. More precisely, the patentee of the state-of-the-art good, the quality of which is λ^j, chooses a price such that the quality-adjusted price of the good is no higher than the quality-adjusted price charged by the other producers. The other producers can legally produce a product of quality of no more than λ^j/β. Therefore, when the price chosen by the other producers is p', the patentee can exclude them from the market by charging a price p that satisfies $p/\lambda^j \leq p'/(\lambda^j/\beta)$. The lowest price that the other producers can charge is given by their marginal cost, that is, by $w(t) = 1$. Thus, the limit price of the patentee is given by

$$p(\omega, t) = \beta(\leq \lambda).$$

When $\lambda > 1/\alpha$, the patentee charges a price equal to $1/\alpha$ even if $\beta = \lambda$. For simplicity, we limit the range of λ to $1 < \lambda < 1/\alpha$. Then, the patentee charges a price equal to β ($\leq \lambda$), which is determined by the patent breadth.

Substituting this into the demand function for the good in industry ω, (3), yields

$$d(\omega, t) = \frac{1}{\beta} \frac{(\lambda^{j(\omega,t)})^{\epsilon}}{Q(t)} (1-\gamma)c(t).$$

Therefore, the profit earned by the patentee in industry ω is given by

$$\pi(\omega, t) = \frac{\beta - 1}{\beta} \frac{(\lambda^{j(\omega,t)})^{\epsilon}}{Q(t)} (1-\gamma)c(t)L(t), \tag{6}$$

where

$$Q(t) \equiv \int_0^1 \left(\lambda^{j(\omega',t)}\right)^{\epsilon} d\omega'$$

is the average quality level across all industries.

2.3. R&D Races

Next, we consider the behavior of R&D firms. By devoting ℓ_i units of labor to R&D activities in industry ω, the R&D firms will invent the next highest quality, the $(j(\omega, t) + 1)$-th quality, with instantaneous probability

$$I(t)^i = \frac{\tilde{A}(\omega, t)}{X(\omega, t)} \ell_i \quad \text{where} \quad \tilde{A}(\omega, t) = \frac{AQ(t)^{\phi}}{\lambda^{\varepsilon[j(\omega,t)+1]}}, \tag{7}$$

where $X(\omega, t)$ is the R&D difficulty index in industry ω. The index $X(\omega, t)$ evolves according to

$$\frac{\partial X(\omega, t)/(\partial t)}{X(\omega, t)} = \mu I(\omega, t),$$

where $I(\omega, t)$ denotes the aggregate R&D investment in industry ω. Thus, the difficulty increases with the investment in R&D. This makes sustainable growth without population growth impossible and plays a key role in eliminating scale

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effects. Moreover, equation (7) implies that R&D generates other positive and negative externalities. First, $\lambda^{\varepsilon[j(\omega,t)+1]}$ in (7) represents a negative spillover. $\lambda^{j(\omega,t)+1}$ is the quality that is obtained by the next innovation in industry ω. Therefore, (7) shows that the higher the quality in an industry, the lower the probability of a successful innovation. This is a negative intraindustry spillover, and ε represents the strength of the spillover. Second, $Q(t)^{\phi}$ in (7) represents a positive interindustry spillover, and ϕ represents the strength of the spillover. The higher the average quality $Q(t)$, the higher the probability of a successful innovation.

Letting $v(\omega,t)$ denote the value of the patent of the state-of-the-art quality in industry ω, the profit of a R&D firm is given by $v(\omega,t)[\tilde{A}(\omega,t)/X(\omega,t)]\ell_i - \ell_i$ under the R&D technology detailed above. For a finite size of R&D activities in equilibrium, we have

$$v(\omega,t)\frac{\tilde{A}(\omega,t)}{X(\omega,t)} = 1. \tag{8}$$

Next, consider the no-arbitrage condition. The holders of a patent earns dividends equal to $\pi(\omega,t)dt$ and capital gains equal to $\partial v(\omega,t)/(\partial t)dt$ over a time interval of length dt. Moreover, during this time interval, the firm producing the state-of-the-art product is exposed to the risk of being leapfrogged by another firm with probability $I(\omega,t)dt$, where $I(\omega,t)$ is the innovation rate. Thus, the shareholders make a capital loss of $v(\omega,t)$ with probability $I(\omega,t)dt$. Therefore, we obtain the following no-arbitrage condition between the stock of the patentee of a state-of-the-art product and a riskless asset:

$$r(t)v(\omega,t) = \pi(\omega,t) + \frac{\partial v(\omega,t)}{\partial t} - I(\omega,t)v(\omega,t).$$

Substituting (6) and (8) into this no-arbitrage condition yields

$$\begin{aligned} r(t) + I(\omega,t) &= \frac{\pi(\omega,t)}{v(\omega,t)} + \frac{\partial v(\omega,t)/(\partial t)}{v(\omega,t)} \\ &= \frac{\beta-1}{\beta}\frac{A(1-\gamma)c(t)L(t)}{X(\omega,t)Q(t)^{1-\phi}} + \mu I(\omega,t) - \phi\frac{\dot{Q}(t)}{Q(t)}. \end{aligned} \tag{9}$$

Suppose that the R&D difficulty indices are symmetric in all industries, that is, that $X(\omega,0) = X_0$. Then, (9) guarantees the existence of the symmetric equilibrium path, where $I(\omega,t) = I(t)$ and $X(\omega,t) = X(t)$ $\forall\ t = [0,\infty]$. We focus our analysis on the symmetric equilibrium path.

2.4. The Labor Market and the Dynamics of $Q(t)$

First, the total labor demand for R&D is given by $\int_0^1 L_I(\omega)d\omega$, where $L_I(\omega)$ denotes the aggregate volume of labor engaged in R&D activities. At the symmetric equilibrium, the probability of success in innovation is symmetric across industries. We thus obtain $L_I(\omega) = \frac{X(t)I(t)\lambda^{\varepsilon[j(\omega,t)+1]}}{AQ(t)^{\phi}}$. Therefore, the total labor demand

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for R&D is $\frac{X(t)I(t)\lambda^{\varepsilon}Q(t)^{1-\phi}}{A}$. Second, the total labor demand for the production of differentiated goods is given by $\int_0^1 d(\omega,t)L(t)d\omega = \frac{(1-\gamma)c(t)L(t)}{\beta}$ and the labor demand for production of the outside good is given by $d_0(t)L(t) = \gamma c(t)L(t)$. Hence, the labor market-clearing condition is

$$1 = \left[(1-\gamma)\frac{1}{\beta} + \gamma\right]c(t) + \frac{X(t)I(t)\lambda^{\varepsilon}Q(t)^{1-\phi}}{AL(t)}. \tag{10}$$

This equation shows that an increase in β reduces the labor demand for the production of differentiated goods and consequently distorts the resource allocation between the differentiated goods and the outside good. As shown later, this implies that a stronger patent protection (a higher β) reduces welfare.

Next, we consider the dynamics of the average quality across industries. We derive the increase of $Q(t)$ in an infinitesimal time interval dt. In each industry, innovation occurs with probability $I(t)dt$. However, by the law of large numbers, the measure of industries that successfully invent a higher quality good is $I(t)dt$ with certainty. Therefore, the increase of $Q(t)$ in time interval dt is given by $\dot{Q}(t)dt = [\lambda^{\varepsilon}Q(t) - Q(t)](I(t)dt)$. Thus,

$$\frac{\dot{Q}(t)}{Q(t)} = (\lambda^{\varepsilon} - 1)\, I(t). \tag{11}$$

3. BALANCED GROWTH EQUILIBRIUM

To describe the equilibrium path, we define $z(t) \equiv AL(t)/(X(t)Q(t)^{1-\phi})$ and $y(t) \equiv AL(t)c(t)/(X(t)Q(t)^{1-\phi})$. These two variables are constant on the balanced growth path (BGP). From (7), $AQ(t)^{\phi}/(X(t)Q(t))$ represents the R&D productivity. Thus, we can roughly interpret $z(t)$, which is the product of the R&D productivity and the labor population, as "R&D productivity." From the labor market equilibrium condition, (10), the consumption expenditure, $c(t)L(t)$ is proportional to the labor devoted to production $[(1-\gamma)(1/\beta)+\gamma]c(t)L(t)$. Since $y(t)$ is the product of $c(t)L(t)$ with the R&D productivity $AQ(t)^{\phi}/(X(t)Q(t))$, we can roughly interpret $y(t)$ as "production labor" (per unit of R&D cost). Note that $z(t)$ is a state variable, whereas $y(t)$ is a jump variable. Rewriting the labor market equilibrium condition, (10), using $z(t)$ and $y(t)$, we obtain the equilibrium R&D intensity

$$I(t) = \lambda^{-\varepsilon}\left\{z(t) - \left[(1-\gamma)\frac{1}{\beta} + \gamma\right]y(t)\right\}. \tag{12}$$

This equation shows that higher R&D productivity $z(t)$ and smaller production labor per unit of R&D cost $y(t)$ increase the equilibrium R&D intensity. Substituting (11) into the no-arbitrage condition, (9), and rewriting this using $z(t)$ and

$y(t)$, we obtain the equilibrium interest rate

$$r(t) = \frac{\beta - 1}{\beta}(1-\gamma)y(t) + (\mu - 1)I(t) - \phi\frac{\dot{Q}(t)}{Q(t)}.$$

Using this equation, (5), and (12), the market equilibrium path is characterized by the dynamic system:

$$\frac{\dot{z}(t)}{z(t)} = n - \left[\mu + (1-\phi)(\lambda^{\varepsilon} - 1)\right]\lambda^{-\varepsilon}\left\{z(t) - \left[(1-\gamma)\frac{1}{\beta} + \gamma\right]y(t)\right\}, \quad \mathbf{(13)}$$

$$\frac{\dot{y}(t)}{y(t)} = y(t) - z(t) - (\rho - n). \quad \mathbf{(14)}$$

The other endogenous variables are determined by $z(t)$ and $y(t)$. On the BGP, $z(t)$ is constant. From (13), we obtain

$$I = n/\left[\mu + (1-\phi)(\lambda^{\varepsilon} - 1)\right]. \quad \mathbf{(15)}$$

Further, from (13) and (14), $z(t)$ and $y(t)$ satisfy on the BGP

$$z = \frac{\lambda^{\varepsilon} I + \left[(1-\gamma)\frac{1}{\beta} + \gamma\right](\rho - n)}{(1-\gamma)(1-\frac{1}{\beta})} \quad \text{and} \quad y = \frac{\lambda^{\varepsilon} I + (\rho - n)}{(1-\gamma)(1-\frac{1}{\beta})}. \quad \mathbf{(16)}$$

Here and in the following, terms without the time index "(t)" are used to represent values on the BGP.

Using comparative statics, we derive the long-run effects on $I(t)$, $z(t)$, and $y(t)$ of an increase of β. From (15), the innovation rate is independent of β, and thus strengthening patent protection does not affect the long-run innovation rate. Differentiating y with respect to β yields

$$y_{\beta}\left(\equiv \frac{\partial y}{\partial \beta}\right) = -\frac{y}{\left(1 - \frac{1}{\beta}\right)\beta^2} < 0. \quad \mathbf{(17)}$$

From (14), $z = y - (\rho - n)$ on the BGP. Therefore,

$$z_{\beta}\left(\equiv \frac{\partial z}{\partial \beta}\right) = y_{\beta} < 0. \quad \mathbf{(18)}$$

Because $z(t)$ is a state variable, the model exhibits transitional dynamics that must be taken into consideration when evaluating a policy. To do so, we derive the linearized system of $z(t)$ and $y(t)$ in the neighborhood of the BGP and compute the transition path of $I(t)$ and $c(t)$. The linearized system of (13) and (14) is given by

$$\begin{pmatrix} \dot{z}(t) \\ \dot{y}(t) \end{pmatrix} = \begin{pmatrix} -\frac{n}{\lambda^{\varepsilon} I}z & \left[(1-\gamma)\frac{1}{\beta} + \gamma\right]\frac{n}{\lambda^{\varepsilon} I}z \\ -y & y \end{pmatrix}\begin{pmatrix} z(t) - z \\ y(t) - y \end{pmatrix}. \quad \mathbf{(19)}$$

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Let J denote the Jacobian matrix of the dynamic system on the right-hand side (RHS) of (19). The determinant of J is negative, $\det J = -(1-\gamma)(1-\frac{1}{\beta})\frac{n}{\lambda^{\varepsilon} I} zy < 0$. Therefore, one characteristic root is negative and the other is positive, and thus the BGP is a saddle point. Because $y(t)$ is a jump variable, whereas $z(t)$ is a state variable, the market equilibrium path is uniquely determined. Moreover, one can show that the negative root of the characteristic equation is smaller than $-n$.[12] Let ν denote the negative characteristic root and $h = [1, \Lambda]^T$ denote the associated characteristic vector. Solving $Jh = \nu h$ for Λ yields

$$\Lambda = \left[(1-\gamma)\frac{1}{\beta} + \gamma\right]^{-1} \left[1 - \frac{\lambda^{\varepsilon} I}{z} \frac{(-\nu)}{n}\right]. \tag{20}$$

Using the characteristic root and vector, one obtains for the market equilibrium path $z(t) = z + [z(0) - z]\, e^{\nu t}$ and $y(t) = y + [z(0) - z]\, \Lambda e^{\nu t}$. Differentiating these expressions with respect to β, the responses of $z(t)$ and $y(t)$ to a marginal increase in β are obtained as

$$\frac{\partial z(t)}{\partial \beta} = z_{\beta}\left(1 - e^{\nu t}\right),$$

$$\frac{\partial y(t)}{\partial \beta} = y_{\beta} - z_{\beta} \Lambda e^{\nu t}.$$

Next, let's derive the effects of a change in β on the path of $I(t)$. Suppose that the economy is on the BGP until patent protection is changed at time 0. Using (12), one obtains

$$\frac{\partial I(t)}{\partial \beta} = \lambda^{-\varepsilon} \left[\frac{\partial z(t)}{\partial \beta} - \left(\frac{1-\gamma}{\beta} + \gamma\right)\frac{\partial y(t)}{\partial \beta} + \frac{1-\gamma}{\beta^2} y\right].$$

Substituting (17), (18), and (20) into this equation leads to

$$\frac{\partial I(t)}{\partial \beta} = I \frac{-z_{\beta}}{z} \frac{(-\nu)}{n} e^{\nu t} > 0. \tag{21}$$

This shows that an increase in β enhances innovation in the short run, although this effect disappears in the long run because $\nu < 0$. Summarizing we have following:

PROPOSITION 1. *A strengthening of patent protection (an increase in patent breadth β) enhances innovation initially. The size of the enhancement is given by (21). In the long run, this enhancement disappears.*

Because of the initial decrease in production labor, the innovation $I(t)$ initially jumps to a higher level as shown in Figure 1. Then, the increase in the innovation

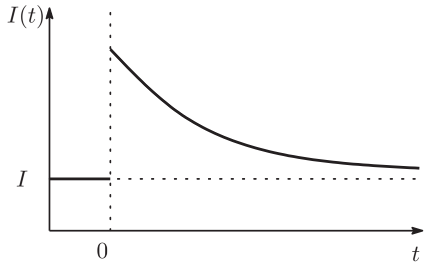

FIGURE 1. Phase diagram and the transition after an increase in β.

raises the difficulty of R&D $X(t)$. Thus, $I(t)$ returns over time to the original steady-state level.

4. WELFARE ANALYSIS

We now examine analytically how a marginal increase in patent breadth, starting from the BGP, affects welfare.

First, the per capita utility from the consumption of differentiated goods can be expressed as

$$D(t) = \frac{1}{\beta} Q(t)^{\frac{1}{\varepsilon}} (1-\gamma) c(t).$$

Substituting this into (1), the instantaneous per capita utility can be written as

$$\log u(t) = \log c(t) - (1-\gamma)\log\beta + (1-\gamma)\frac{1}{\varepsilon}\log Q(t) + \log\left[(1-\gamma)^{1-\gamma}\gamma^{\gamma}\right].$$

Solving the differential equation for $Q(t)$, (11), we have

$$\log Q(t) = (\lambda^{\varepsilon} - 1)\int_0^t I(s)ds + \log Q(0),$$

which means that the average quality across industry represents the accumulation of past innovation successes. These equations imply that the utility depends on the total consumption, $c(t)$, on the prices of differentiated goods as a function of the patent breadth, β, and on the accumulated volume of past innovations, $\int_0^t I(s)ds$. Therefore, the effect on the instantaneous utility of a strengthening of patent protection consists of three parts:

$$\frac{\partial \log u(t)}{\partial \beta} = \frac{\partial \log c(t)}{\partial \beta} - (1-\gamma)\frac{1}{\beta} + (1-\gamma)\frac{\lambda^{\varepsilon}-1}{\varepsilon}\frac{\partial}{\partial\beta}\left(\int_0^t I(\tau)d\tau\right).$$

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The welfare of a household is $U = \int_0^\infty e^{-(\rho-n)t} \log u(t) dt$. Thus, the effect of a marginal increase in patent breadth on welfare is given by

$$\frac{\partial U}{\partial \beta} = \int_0^\infty e^{-(\rho-n)t} \frac{\partial \log u(t)}{\partial \beta} dt = \underbrace{\int_0^\infty e^{-(\rho-n)t} \frac{\partial \log c(t)}{\partial \beta} dt}_{\substack{\text{consumption expenditure effect} \\ (+)}}$$

$$+ \underbrace{-\frac{1}{\rho-n}(1-\gamma)\frac{1}{\beta}}_{\substack{\text{competition-reducing effect} \\ (-)}}$$

$$+ \underbrace{(1-\gamma)\frac{\lambda^\varepsilon - 1}{\varepsilon} \int_0^\infty e^{-(\rho-n)t} \frac{\partial}{\partial \beta}\left(\int_0^t I(\tau) d\tau\right) dt}_{\substack{\text{innovation-enhancing effect} \\ (+)}} . \quad \textbf{(22)}$$

A strengthening of patent protection affects welfare through the following three channels. First, it enhances innovation in the short run, as shown in Proposition 1, and raises welfare. This is the *innovation-enhancing effect*, captured by the third term on the RHS of (22). Second, it allows the patentee to charge a higher price and thus reduces welfare. This is the *competition-reducing effect*, captured by the second term on the RHS of (22). Finally, it affects the production volume through enhancing innovation and thus affects the consumption expenditure $c(t)$. This is the *consumption expenditure effect*, captured by the first term on the RHS of (22).

Let's evaluate these different effects. First, derive the magnitude of the innovation-enhancing effect from (21) as follows:

$$(1-\gamma)\frac{\lambda^\varepsilon - 1}{\varepsilon} \int_0^\infty e^{-(\rho-n)t} \frac{\partial}{\partial \beta}\left(\int_0^t I(\tau) d\tau\right) dt$$
$$= (1-\gamma)\frac{\lambda^\varepsilon - 1}{\varepsilon} \frac{1}{(\rho-n)(\rho-n-\nu)} I\left(\frac{-\nu}{n}\right)\frac{-z_\beta}{z} > 0. \quad \textbf{(23)}$$

An increase in β enhances innovation in the short run. Thus, the innovation-enhancing effect is positive.

Second, we can evaluate the consumption expenditure effect as[13]:

$$\int_0^\infty e^{-(\rho-n)t} \frac{\partial \log c(t)}{\partial \beta} dt$$
$$= \frac{-y_\beta}{y}\left\{\frac{1}{\rho-n}\left(\frac{y}{z}-1\right) - \frac{1}{\rho-n-\nu}\left(\frac{1-\gamma}{\beta}+\gamma\right)^{-1} \frac{\lambda^\varepsilon I}{z}\frac{-(\nu+n)}{n}\right\}. \quad \textbf{(24)}$$

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From (16), $y/z > 1$ and $\nu + n < 0$. Therefore, the consumer expenditure effect is ambiguous. However, a numerical analysis shows that it is usually positive.

As shown in Appendix E on https://sites.google.com/site/tatsuroiwaisako/, one can decompose the welfare effect into a term that depends on γ, $F(\beta, \gamma)$, and a term that is independent of γ, $G(\beta)$,

$$\frac{\partial U}{\partial \beta} = \frac{-y_\beta}{y} \frac{1}{(\rho - n)(\rho - n - \nu)} \left(\frac{-\nu}{n}\right) \left(\frac{1-\gamma}{\beta} + \gamma\right)^{-1} \times \frac{\lambda^\varepsilon I}{z} \left[F(\beta, \gamma) + G(\beta)\right], \tag{25}$$

where

$$\begin{aligned} F(\beta, \gamma) &= -\left(\frac{\rho - n - \nu}{-\nu}\right) n \left(\frac{1-\gamma}{\beta} + \gamma\right) \frac{z}{\lambda^\varepsilon I} \\ &\quad \times \left\{[(1-\gamma)\beta + \gamma] - \left(\frac{1-\gamma}{\beta} + \gamma\right)^{-1}\right\} \\ &\quad + \frac{1-\lambda^{-\varepsilon}}{\varepsilon}(\lambda^\varepsilon I + \rho - n)\gamma, \\ G(\beta) &= \frac{1-\lambda^{-\varepsilon}}{\varepsilon}(\lambda^\varepsilon I + \rho - n)\frac{1}{\beta - 1} - \rho. \end{aligned}$$

We now show that $F(\beta, \gamma)$ is a decreasing function of β. Using (16), one can rewrite $F(\beta, \gamma)$ as

$$\begin{aligned} F(\beta, \gamma) &= -\left(\frac{\rho - n - \nu}{-\nu}\right) n \frac{\left(\frac{1-\gamma}{\beta} + \gamma\right)}{(1-\gamma)(1-\frac{1}{\beta})} \left[1 + \left(\frac{1-\gamma}{\beta} + \gamma\right) \frac{\rho - n}{\lambda^\varepsilon I}\right] \\ &\quad \times \frac{\gamma(1-\gamma)(\beta-1)^2}{\beta\left(\frac{1-\gamma}{\beta} + \gamma\right)} + \frac{1-\lambda^{-\varepsilon}}{\varepsilon}(\lambda^\varepsilon I + \rho - n)\gamma, \\ &= -\left(\frac{\rho - n - \nu}{-\nu}\right) n \left[\beta + ((1-\gamma) + \gamma\beta) \frac{\rho - n}{\lambda^\varepsilon I}\right] \\ &\quad \times \frac{\gamma(\beta - 1)}{\beta} + \frac{1-\lambda^{-\varepsilon}}{\varepsilon}(\lambda^\varepsilon I + \rho - n)\gamma. \end{aligned}$$

Because $(-\nu)$ is a decreasing function of β, see Appendix F of the unpublished appendices, it can be shown that $\partial F(\beta, \gamma)/(\partial \beta) < 0$. Moreover, because $G(\beta)$ is a decreasing function of β, $F(\beta, \gamma) + G(\beta)$ is a decreasing function of β. Also, $\lim_{\beta \searrow 1} G(\beta) = +\infty$ and thus $\lim_{\beta \searrow 1}[F(\beta, \gamma) + G(\beta)] = +\infty$. From (25), one obtains $\lim_{\beta \searrow 1} \partial U/(\partial \beta) > 0$. Therefore, if $\partial U/(\partial \beta)|_{\beta = \lambda} < 0$, a unique level β^* satisfies $\partial U/(\partial \beta) = 0$ in $(1, \lambda)$ and $\partial U/(\partial \beta) < 0 (> 0)$, if and only if β is larger

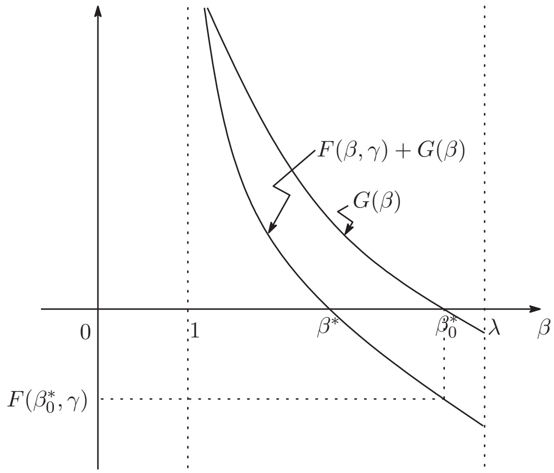

FIGURE 2. The welfare-maximizing patent breadth.

(smaller) than β^*, as in Figure 2. The results are summarized in the following proposition.

PROPOSITION 2. *If* $\partial U/(\partial\beta)|_{\beta=\lambda} < 0$, *a unique level of patent breadth* $\beta^* \in (1, \lambda)$ *satisfies* $\partial U/(\partial\beta) = 0$. *An increase in patent breadth* β *raises welfare if the initial patent breadth is narrower than* β^*, *and lowers it otherwise.*

We refer to β^* as *the welfare-maximizing patent breadth.*

Before examining the welfare-maximizing patent breadth in the case with an outside good, let us consider the simpler case without an outside good (i.e., the Segerstrom–Li case). In that case, $\gamma = 0$ and $F(\beta, \gamma) = 0$. From (25), the welfare-maximizing is given the unique value β_0^* of β such that $G(\beta_0^*) = 0$. We have

$$(\beta_0^* - 1)\rho = \frac{1-\lambda^{-\varepsilon}}{\varepsilon}(\lambda^{\varepsilon} I + \rho - n). \tag{26}$$

Interestingly, one can show that $\beta^* < \beta_0^*$, see Appendix A. That is, the presence of an outside good reduces the welfare-maximizing patent breadth.

Let now assume that $\beta_0^* < \lambda$. Using (26), one recognizes that $\beta_0^* < \lambda$ is equivalent to

$$(\lambda - 1)\rho > \frac{1-\lambda^{-\varepsilon}}{\varepsilon}(\lambda^{\varepsilon} I + \rho - n).$$

This last inequality also guarantees that $\beta^* < \lambda$ since $\beta^* < \beta_0^*$. We thus have the following:

PROPOSITION 3. *Suppose that* $(\lambda - 1)\rho > \frac{1-\lambda^{-\varepsilon}}{\varepsilon}(\lambda^{\varepsilon} I + \rho - n)$. *Then,* $\beta^* \in (1, \lambda)$, *that is, the welfare-maximizing patent breadth is interior. The presence of an outside good* ($\gamma > 0$) *always reduces the welfare-maximizing patent breadth* β^*.

Table 1. Parameter values for our calculations

ρ	λ	n	ε	ϕ
0.07	1.4	0.01	0.1	0.1

To intuitively understand this result, consider how the presence of the outside good affects the positive and negative welfare effects of a broadening of the patent breadth. First, a positive share of the outside good, which means a lower share for the innovative goods, weakens the negative welfare effect of an increased patent protection through the competition-reducing effect, (22). The positive share of the outside good also weakens the positive welfare effect through innovation-enhancing, (22). One cannot sign the impact on the positive welfare effect through consumption expenditure, (22). However, the presence of the outside good necessarily reduces the welfare-maximizing patent breadth. From this, we can infer that the weakening of the positive welfare effects overwhelm the reduction of the negative effects.

Proposition 3 analytically shows that the presence of an outside good reduces the welfare-maximizing patent breadth. However, we cannot analytically examine whether an increase in the share of the outside good necessarily reduces the welfare-maximizing patent breadth. Thus, we examine it using a numerical example using the plausible parameter values in Table 1. Mehra and Prescott (2003) estimated the average real return on the stock market for the past century at 0.07. At the steady state, ρ is equal to the interest rate r. Thus, we set $\rho = 0.07$. We set n to 0.01 to match the world population growth rate. Basu (1996) estimated that the mark-up belongs in the interval [1.1,1.4], we choose $\lambda = 1.4$. Further, we set $\varepsilon = 0.1$. Consequently, the elasticity of substitution among industries, $\varepsilon + 1$, is 1.1. Finally, we set the R&D spillover parameters. The positive interindustry spillover has only a small impact and thus we set $\phi = 0.1$. We use the following three different values for the R&D difficulty parameter: $\mu = 1.0$, 1.5, and 1.9.

This numerical example shows that a larger share of the outside good decreases the welfare-maximizing patent breadth, at least when γ is small. When μ is high, an increase in the share of the outside good decreases the welfare-maximizing patent breadth, even if γ is quite large, as in the right panel of Figure 3.[14]

Finally let examine how the presence of the outside good changes the effects of the other parameters on the welfare-maximizing patent breadth. Assume first that there is no outside good, that is equivalently, that $\gamma = 0$. Rewriting (26) and using (15), we obtain

$$\beta_0^* = \frac{1-\lambda^{-\varepsilon}}{\varepsilon}\left[\frac{\phi(\lambda^{\varepsilon}-1)+1-\mu}{(1-\phi)(\lambda^{\varepsilon}-1)+\mu}\frac{n}{\rho}+1\right]+1.$$

Differentiating the RHS of the equation with respect to each parameter, we find that

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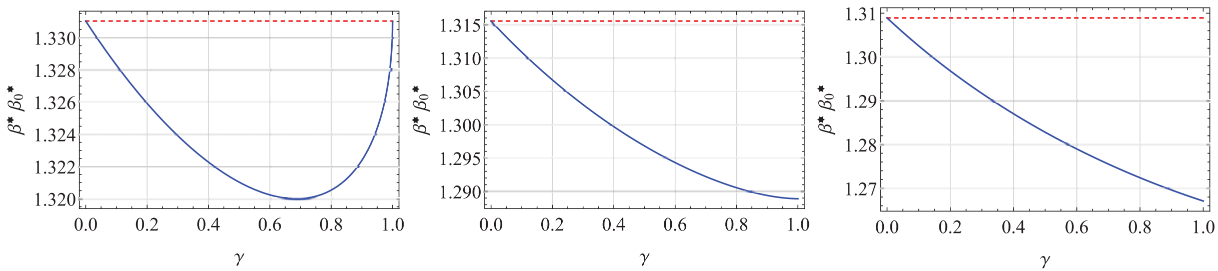

$\mu = 1.0$ $\mu = 1.5$ $\mu = 1.9$

FIGURE 3. Relation between β^*/β_0^* and γ. In each panel, the solid line shows the value of β^* and the dotted line the value of β_0^*.

- $\lambda \uparrow$ and/or $\phi \uparrow$ and/or $\mu \downarrow \implies \beta_0^* \uparrow$.
- $n \uparrow$ and/or $\rho \downarrow \implies \beta_0^* \uparrow (\downarrow)$, if $\mu < (>) 1 + \phi(\lambda^\varepsilon - 1)$.

Let's now assume that there is an outside good ($\gamma > 0$), the effects are far more complicated and we must rely on numerical examples to get a grasp of the situation. Appendix B shows the numerical results. They show that the signs of the effects on the welfare-maximizing patent breadth are not affected by the presence of an outside good.

5. CONCLUSION

To evaluate the welfare effect of strengthening patent protection adequately, we generalized the Segerstrom–Li model by introducing an outside good that is supplied competitively. In the generalized model, patent protection creates static distortions. We showed that the presence of the outside good necessarily reduces the welfare-maximizing patent breadth. In other words, ignoring outside goods leads to an excessive protection of patents. This is an important finding since, with the exception of O'Donoghue and Zweimüller (2004), most related studies based on nonscale quality-ladder models have overlooked the static distortion effect arising from patent protection.

Furthermore, this paper makes a contribution in terms of welfare analyses in a nonscale growth model. In typical nonscale growth models, a policy change induces a transition to a new steady state that must be accounted for in order to properly evaluate the welfare impacts.[15] However, most of the existing studies based on nonscale growth models concentrate on the steady state and neglect the transition. This paper demonstrates how the linear approximation of the transition path suggested in Judd (1982) can be used for an analytical investigation of welfare impacts in nonscale growth models.

NOTES

1. The model is a semi-endogenous growth model, in the sense that the innovation rate is endogenous although the long-run growth rate does not depend on preference parameters, such as a subjective discount rate.
2. The exceptions are Peretto (2007, 2011), which analytically examine the cumulated welfare change in a nonscale growth model with simple transition paths.
3. In the Segerstrom–Li model, where there is no outside good, one can derive the welfare-maximizing patent breadth by controlling the patent breadth in such a way that the market equilibrium path coincides with the socially optimal path. However, in the generalized model with an outside good, this is impossible and one must use Judd's method.
4. In their model, they use the term "noninnovative good" instead of "outside good."
5. Kwan and Lai (2003) and Cysne and Turchick (2012) examined the welfare effects of strengthening intellectual property rights (IPR) protection analytically in a growth model with transitions. However, their model is a simple AK model based on Rivera-Batiz and Romer (1991) and therefore exhibits a scale effect.
6. Among others, Sener (2001) uses this model to investigate the impact of trade liberalization on wage inequality in the presence of Schumpeterian unemployment. Dinopoulos and Segerstrom (1999)

incorporate an endogenous supply of skills into the Segerstrom (1998) model and examine the effects of trade liberalization. Cozzi and Impullitti (2010) extend Dinopoulos and Segerstrom's (1999) to a model with heterogeneous industries and examine how the shift in the composition of public demand toward high-tech goods affects R&D, the relative wages of skilled labor, and the supply of skills. Using Dinopoulos and Segerstrom (1999), Impullitti (2010) examines numerically how foreign competition affects the optimal R&D subsidy in the United States. Moreover, Dinopoulos and Segerstrom (2007, 2010), Parello (2008), and Iwaisako et al. (2011) extend the model to a North–South framework and examine the effects of strengthening IPR protection in the South.

7. Assuming Cobb–Douglas utility, the production of the outside good and that of the patented goods are both necessarily positive, as shown later in (3) and (4). That is, the economy does not specialize in either good.

8. Other patent instruments include intellectual appropriability and the division rule governing profits between basic and applied researchers. Cozzi and Spinesi (2006), Chu et al. (2012), Chu and Furukawa (2011, 2013), Chu and Pan (2013), and Cozzi and Galli (2013) have examined their effects.

9. Judd (1985), Iwaisako and Futagami (2003), Futagami and Iwaisako (2007), Lin (2015), and Zeng et al. (2014) examine how patent length affects social welfare. As shown by Futagami and Iwaisako (2007), under the assumption of finite patent length the equilibrium paths are complicated even if the production function is AK. To avoid this difficulty, other studies have incorporated a time-invariant probability of imitation into their models, interpreting a decrease in probability as strengthening patent protection. See Grinols and Lin (2006), Furukawa (2007), Horii and Iwaisako (2007), and Palokangas (2011).

10. Spinesi (2011) also analyzed the effect of patent breadth on innovation in a quality-ladder model.

11. In this paper, we implicitly assume that the product with the quality level that lies between the state-of-the-art quality and the second-highest quality can be produced and consumed.

12. Proof: see Appendix C of unpublished appendices on https://sites.google.com/site/tatsuroiwaisako/

13. For the derivation, see Appendix D of the unpublished appendices on https://sites.google.com/site/tatsuroiwaisako/.

14. For the other values of μ, a larger share of the outside good decreases the welfare-maximizing patent breadth when γ is smaller than a certain value. See Appendix G on https://sites.google.com/site/tatsuroiwaisako/.

15. In the Segerstrom (1998) model, the speed of convergence is low and the economy is in transition for a long time, as Steger (2003) demonstrated using an empirically plausible baseline set of parameters. Thus, the welfare change associated with the transition is quite important and needs to be examined analytically.

REFERENCES

Basu, Susanto (1996) Procyclical productivity: Increasing returns or cyclical utilization. *Quarterly Journal of Economics* 111, 709–751.

Chu, Angus, Guido Cozzi, and Silvia Galli (2012) Does intellectual monopoly stimulate or stifle innovation? *European Economic Review* 56(4), 727–746.

Chu, Angus and Yuichi Furukawa (2011) On the optimal mix of patent instruments. *Journal of Economic Dynamics and Control* 35, 1964–1975.

Chu, Angus and Yuichi Furukawa (2013) Patentability and knowledge spillovers of basic R&D. *Southern Economic Journal* 79, 928–945.

Chu, Angus and Shiyuan Pan (2013) The escape-infringement effect of blocking patents on innovation and economic growth, *Macroeconomic Dynamics* 17(04), 955–969.

Cozzi, Guido and Silvia Galli (2014) Sequential R&D and blocking patents in the dynamics of growth, *Journal of Economic Growth* 19(2), 183–219.

Cozzi, Guido and Giammario Impullitti (2010) Government spending composition, technical change, and wage inequality. *Journal of the European Economic Association* 8(6), 1325–1358.

Cozzi, Guido and Luca Spinesi (2006) Intellectual appropriability, product differentiation, and growth. *Macroeconomic Dynamics* 10(01), 39–55.

Cysne, Rubens P. and David Turchick (2012) Intellectual property rights protection and endogenous economic growth revisited. *Journal of Economic Dynamics and Control* 36(6), 851–861.

Dinopoulos, Elias and Paul S. Segerstrom (1999) A Schumpeterian model of protection and relative wages. *American Economic Review* 89, 450–472.

Dinopoulos, Elias and Paul S. Segerstrom (2007) North–South Trade and Economic Growth. CEPR discussion papers no. 5887.

Dinopoulos, Elias and Paul S. Segerstrom (2010) Intellectual property rights, multinational firms and economic growth. *Journal of Development Economics* 92, 13–27.

Furukawa, Yuichi (2007) The protection of intellectual property rights and endogenous growth: Is stronger always better? *Journal of Economic Dynamics and Control* 31, 3644–3670.

Futagami, Koichi and Tatsuro Iwaisako (2007) Dynamic analysis of patent policy in an endogenous growth model. *Journal of Economic Theory* 132, 306–334.

Grinols, Earl and Hwan C. Lin (2006) Global patent protection: Channels of north and south welfare gain. *Journal of Economic Dynamics and Control* 30, 205–227.

Helpman, Elhanan (1993) Innovation, imitation, and intellectual property rights. *Econometrica* 61, 1247–1280.

Horii, Ryo and Tatsuro Iwaisako (2007) Economic growth with imperfect protection of intellectual property rights. *Journal of Economics (Zeitschrift für Nationalökonomie)* 90, 45–85.

Impullitti, Giammario (2010) International competition and U.S. R&D subsidies: A quantitative welfare analysis. *International Economic Review* 51(4), 1127–1158.

Iwaisako, Tatsuro and Koichi Futagami (2003) Patent policy in an endogenous growth model. *Journal of Economics (Zeitschrift für Nationalökonomie)* 78, 239–258.

Iwaisako, Tatsuro, Hitoshi Tanaka, and Koichi Futagami (2011) A welfare analysis of global patent protection in a model with endogenous innovation and foreign direct investment. *European Economic Review* 55, 1137–1151.

Judd, Kenneth L. (1982) An alternative to steady-state comparisons in perfect foresight models. *Economics Letters* 10, 55–59.

Judd, Kenneth L. (1985) On the performance of patents. *Econometrica* 53, 567–585.

Kwan, Yum K. and Edwin L.-C. Lai (2003) Intellectual property rights protection and endogenous economic growth. *Journal of Economic Dynamics and Control* 27, 853–873.

Li, Chol-Won (2001) On the policy implications of endogenous technological progress. *Economic Journal* 111, 164–179.

Li, Chol-Won (2003) Endogenous growth without scale effects: Comment. *American Economic Review* 93, 1009–1017.

Lin, Hwan C. (2015) Creative destruction and optimal patent life in a variety-expansion growth model. *Southern Economic Journal* 81(3), 803–828.

Lin, Hwan and L. F. Shampine (2018) R&D-based calibrated growth models with finite-length patents: A novel relaxation algorithm for solving an autonomous FDE system of mixed type. *Computational Economics* 51(1), 123–158.

Mehra, Rajnish and Esward C. Prescott (2003) The equity premium in retrospect. In G. M. Constantinides, M. Harris, and R. M. Stulz (eds.), *Handbook of the Economics of Finance*, pp. 889–938. Amsterdam, NH: Elsevier.

O'Donoghue, Ted and Josef Zweimüller (2004) Patents in a model of endogenous growth. *Journal of Economic Growth* 9, 81–123.

Parello, Carmelo P. (2008) A north–south model of intellectual property rights protection and skill accumulation. *Journal of Development Economics* 85, 253–281.

Palokangas, Tapio (2011) Optimal patent length and breadth in an economy with creative destruction and non-diversifiable risk. *Journal of Economics (Zeitschrift für Nationalökonomie)* 102, 1–27.

Peretto, Pietro F. (2007) Corporate taxes, growth and welfare in a Schumpeterian economy. *Journal of Economic Theory* 137(1), 353–382.

Peretto, Pietro F. (2011) The growth and welfare effects of deficit–financed dividend tax cuts. *Journal of Money, Credit and Banking* 43(5), 835–869.

Rivera-Batiz, Luis and Paul M. Romer (1991) Economic integration and endogenous growth. *Quarterly Journal of Economics* 106, 531–555.

Segerstrom, Paul S. (1998) Endogenous growth without scale effects. *American Economic Review* 88, 1290–1310.

Sener, Fuat (2001) Schumpeterian unemployment, trade and wages. *Journal of International Economics* 54, 119–148.

Spinesi, Luca (2011) Probabilistic heterogeneous patent protection and innovation incentives. *B.E. Journal of Economic Analysis and Policy (Contributions)* 11(1), 1–44.

Steger, Thomas M. (2003) The Segerstrom model: Stability, speed of convergence and policy implications. *Economics Bulletin* 15, 1–8.

Zeng, Jinli, Jie Zhang, and Michael K. Fung (2014) Patents and price regulation in an R&D growth model. *Macroeconomic Dynamics* 18, 1–22.

APPENDIX A: THE PROOF OF PROPOSITION 3

In this appendix, we prove Proposition 3.

We show that $F(\beta_0^*, \gamma) < 0$. Using (26), we can rewrite $F(\beta_0^*, \gamma)$ as follows:

$$F(\beta_0^*, \gamma) = \gamma(\beta_0^* - 1)H(\beta_0^*, \gamma),$$

where

$$H(\beta_0^*, \gamma) = -\left(\frac{\rho - n - \nu}{-\nu}\right) n \left[1 + \left(\frac{1-\gamma}{\beta_0^*} + \gamma\right)\frac{\rho - n}{\lambda^\varepsilon I}\right] + \rho. \tag{27}$$

Rewriting the RHS of (27) yields

$$\begin{aligned} H(\beta_0^*, \gamma) &= (\rho - n)\frac{n}{\nu}\left[1 + \left(\frac{1-\gamma}{\beta_0^*} + \gamma\right)\frac{\rho - n}{\lambda^\varepsilon I}\right] - n\left[1 + \left(\frac{1-\gamma}{\beta_0^*} + \gamma\right)\frac{\rho - n}{\lambda^\varepsilon I}\right] + \rho \\ &= (\rho - n)\left\{\left[1 + \left(\frac{1-\gamma}{\beta_0^*} + \gamma\right)\frac{\rho - n}{\lambda^\varepsilon I}\right]\frac{n}{\nu} + \left[1 - \left(\frac{1-\gamma}{\beta_0^*} + \gamma\right)\frac{n}{\lambda^\varepsilon I}\right]\right\}. \end{aligned}$$

Furthermore, using z, we rewrite this as

$$H(\beta_0^*, \gamma) = (\rho - n)\left\{(1-\gamma)\left(1 - \frac{1}{\beta_0^*}\right)\frac{zn}{\lambda^\varepsilon I \nu} + \left[1 - \left(\frac{1-\gamma}{\beta_0^*} + \gamma\right)\frac{n}{\lambda^\varepsilon I}\right]\right\}.$$

We let $f(\nu)$ denote the characteristic function of the Jacobian matrix of the linearized system, J, that is, $f(\nu) \equiv \nu^2 - \mathrm{tr} J \nu + \det J = 0$. Rewriting yields $\nu - \mathrm{tr} J = -\det J \frac{1}{\nu}$. Expressing $\mathrm{tr} J$ and $\det J$ by making use of z and y and substituting them into this equation, we obtain

$$\frac{\nu}{y} + \frac{n}{\lambda^\varepsilon I}\frac{z}{y} - 1 = (1-\gamma)\left(1 - \frac{1}{\beta_0^*}\right)\frac{zn}{\lambda^\varepsilon I \nu}.$$

Substituting this into $H(\beta_0^*, \gamma)$ and rewriting it, we obtain

$$H(\beta_0^*, \gamma) = (\rho - n)\left\{\frac{\nu}{y} + \left[\frac{z}{y} - \left(\frac{1-\gamma}{\beta_0^*} + \gamma\right)\right]\frac{n}{\lambda^{\varepsilon} I}\right\} = (\rho - n)\frac{1}{y}(\nu + n),$$

where we use $[z - (\frac{1-\gamma}{\beta_0^*} + \gamma)y] = \lambda^{\varepsilon} I$ from (12). Furthermore, using $\nu < -n$, we show that $H(\beta_0^*, \gamma) < 0$.

Hence, we obtain $F(\beta_0^*, \gamma) + G(\beta_0^*) = F(\beta_0^*, \gamma) < 0$. As shown above, $F(\beta, \gamma) + G(\beta)$ is a decreasing function of β, and thus we can show that $\beta^* < \beta_0^*$.

We focus our analysis on the case where the welfare-maximizing patent breadth is interior, that is, $\beta^* < \lambda$. For this purpose, we assume $\beta_0^* < \lambda$. From (26), we rewrite the condition as follows:

$$(\lambda - 1)\rho > \frac{1 - \lambda^{-\varepsilon}}{\varepsilon}(\lambda^{\varepsilon} I + \rho - n).$$

This guarantees that $\beta^* < \lambda$ because $\beta_0^* < \lambda$.

APPENDIX B: THE EFFECTS OF THE PARAMETERS ON THE WELFARE-MAXIMIZING PATENT BREADTH (NUMERICAL EXAMPLES)

In this appendix, by using numerical examples, we examine the relations between the welfare-maximizing patent breadth and industry spillovers, the quality increment, the population growth rate, and the discount rate.

Basically, we set the values of the parameters to those in the numerical example in Section 4. From Figure 4, we can see that the signs of the effects of ϕ, λ, n, ρ, and μ on the welfare-maximizing patent breadth in the presence of an outside good are the same as those in the no-outside good case $\gamma = 0$.

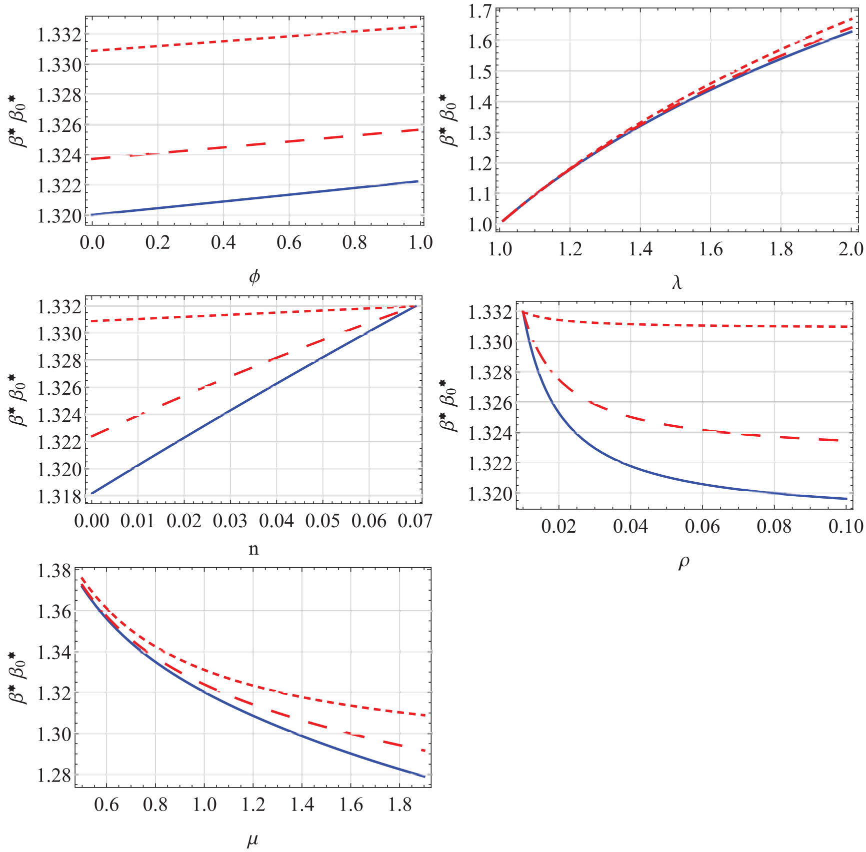

FIGURE 4. Relation between β^*/β_0^* and ϕ, λ, n, ρ, μ. In each panels, the dotted, broken, and solid lines show the value of β_0^*, the value of β^* when $\gamma = 0.3$ and $\gamma = 0.6$, respectively.

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