B.A.J. **9**, V, 1227-1231 (2003)

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Insurance: Mathematics & Economics

Volume **33**(1), 2003

Ballotta, L. & Haberman, S. *Valuation of guaranteed annuity conversion options.* 87-108. In this note we introduce a theoretical model for the pricing and valuation of guaranteed annuity conversion options associated with certain deferred annuity pension-type contracts in the U.K. The valuation approach is based on the similarity between the payoff structure of the contract and a call option written on a coupon-bearing bond. The model makes use of a one-factor Heath–Jarrow–Morton framework for the term structure of interest rates. Numerical results are investigated and the sensitivity of the price of the option to changes in the key parameters is also analyzed.

Becherer, D. *Rational hedging and valuation of integrated risks under constant absolute risk aversion.* 1-28. We study a rational valuation and hedging principle for contingent claims which integrate tradable and non-tradable sources of risk. The principle is based on the preferences of a rational investor with constant absolute risk aversion, and uses exponential utility-indifference arguments. Properties of this valuation and of a corresponding hedging strategy are analyzed in a general semi-martingale market framework. To obtain further constructive results and properties, a more specific class of semi-complete product models is studied in detail. This yields a computation scheme, simple valuation bounds, and results on diversification and information effects.

Blake, D., Cairns, A. J. G. & Dowd, K. *Pensionmetrics 2: stochastic pension plan design during the distribution phase.* 29-47. We consider the choices available to a defined contribution (DC) pension plan member at the time of retirement for conversion of his pension fund into a stream of retirement income. In particular, we compare the purchase at retirement age of a conventional life annuity (i.e., a bond-based investment) with distribution programmes involving differing exposures to equities during retirement. The residual fund at the time of the plan member's death can either be bequested to his estate or revert to the life office in exchange for the payment of survival credits while alive. The most important decision, in terms of cost to the plan member, is the level of equity investment. We also find that the optimal age to annuitise depends on the bequest utility and the investment performance of the fund during retirement.

Chiu, S. N. & Yin, C. C. *The time of ruin, the surplus prior to ruin and the deficit at ruin for the classical risk process perturbed by diffusion.* 59-66. The paper studies the joint distribution of the time of ruin, the surplus prior to ruin and the deficit at ruin for the classical risk process that is perturbed by diffusion. We prove that the expected discounted penalty satisfies an integro-differential equation of renewal type, the solution of which can be expressed as a convolution formula. The asymptotic behaviour of the expected discounted penalty as the initial capital tends to infinity is discussed.

Deelstra, G., Grasselli, M. & Koehl, P.-F. *Optimal investment strategies in the presence of a minimum guarantee.* 189-207. In a continuous-time framework, we consider the problem of a Defined Contribution Pension Fund in the presence of a minimum guarantee. The problem

of the fund manager is to invest the initial wealth and the (stochastic) contribution flow into the financial market, in order to maximize the expected utility function of the terminal wealth under the constraint that the terminal wealth must exceed the minimum guarantee. We assume that the stochastic interest rates follow the affine dynamics, including the Cox–Ingersoll–Ross (CIR) model [Econometrica 53 (1985) 385] and the Vasiek model. The optimal investment strategies are obtained by assuming the completeness of financial markets and a CRRA utility function. Explicit formulae for the optimal investment strategies are included for different examples of guarantees and contributions.

FRAGNELLI, V. & MARINA, M. E. *A fair procedure in insurance.* 75-85. We consider a random variable R, representing a risk that has to be insured by n companies and which a premium ϕ is assigned to. How should they share the risk and the premium in order to be better off? Under suitable hypotheses, there exists an optimal decomposition of the risk. According to this division, we develop a procedure that results in an allocation of $(\phi. R)$ that is efficient, individually rational and envy-free.

FROSTIG, E. *Properties of the power family of fractional age approximations.* 163-171. We derive some properties of the power family of fractional age approximations introduced by Jones and Mereu [IME 27; IME 30]. The different approximations are compared with respect to stochastic ordering. We also compare the power approximations with the unknown life survival function.

HEILPERN, S. *A rank-dependent generalization of zero utility principle.* 67-73. This paper is devoted to a zero utility principle under the rank-dependent utility theory. The theoretical background is presented and the basic properties of such premium principles are studied.

MORILLO, I. & BERMÚDEZ, L. *Bonus-malus system using an exponential loss function with an inverse Gaussian distribution.* 49-57. In most companies, the importance of auto-insurance has prompted their actuaries to look for different tariff systems that distribute the exact weight of each risk among portfolios. The use of quadratic loss functions in most of classical bonus–malus systems leads to very high maluses. To avoid this problem, we use an exponential loss function and subsequently obtain the result for the Poisson–Inverse Gaussian model. We show, with a numerical example, that this new model fits better than the classical one for our data. Furthermore, it provides maluses that are not so high, as well as other advantages for actuaries.

RAMSAY, C. M. *A solution to the ruin problem for Pareto distributions.* 109-116. An expression is derived for $\Psi(u)$, the probability of ultimate ruin given an initial reserve of u in the case of a Pareto distribution. Laplace transforms and exponential integrals are used to derive this expression, which involves a single integral of real valued functions along the positive real line. Most importantly, the integrand is not of an oscillating kind. This expression for $\Psi(u)$ is new, and may be used to form the basis of a more refined set of asymptotic approximations to $\Psi(u)$. Finally, it is shown that $\Psi(u)$ can be expressed as the expected value of a function of a two parameter gamma random variable.

SCHOUTENS, W. & STUDER, M. *Short-term risk management using stochastic Taylor expansions under Levy models.* 173-188. The Taylor expansion is a powerful tool in the analysis of deterministic functions. A stochastic Taylor expansion together with some general existence results have been developed for diffusion processes and some other classes of processes. We explicitly calculate a stochastic Taylor expansion for multivariate Poisson processes. An extension to diffusion processes with Poisson jumps is straightforward. The expansion is used for two financial applications in the context of risk management.

WU, R., WANG, G. & WEI, L. *Joint distributions of some actuarial random vectors containing the time of ruin.* 147-161. In this paper, we introduce the renewal measure of the defective renewal sequence constituted by the zero points of the classical risk model $U(t), t \geq 0$. The density function of this renewal measure is derived. By this density function together with the strong Markov property of the surplus process, we obtain the explicit expressions for the ruin probability and the joint distributions of actuarial random vectors $(T, U(T^-), |U(T)|)$ and $(T, U(T^-), |U(T)|, \sup_{0\leq t<T} U(t), \sup_{T\leq t<L} U(t), \inf_{0\leq t<L} U(t))$, where T represents the time of ruin and L the time of the surplus process leaving zero ultimately. Finally, a special case with the claim amount being exponentially distributed is considered.

WU, X. & YUEN, K. C. *A discrete-time risk model with interaction between classes of business.* 117-133. In this paper, a discrete-time risk model with dependent classes of business called the interaction (IR) model is proposed. The model assumes that the number of claims in one class is governed not only by its underlying risk but also by the risks in other classes. For a family of claim-number distributions, the IR model is examined. Numerical studies are carried out to compare the finite-time ruin probabilities of the model to those of other correlated aggregate claims models in the literature. For the infinite-time ruin probabilities, comparisons between these models in terms of their adjustment coefficients are also made.

YANG, H. *Ruin theory in a financial corporation model with credit risk.* 135-145. This paper builds a new risk model for a firm which is sensitive to its credit quality. A modified Jarrow, Lando and Turnbull model (Markov chain model) is used to model the credit rating. Recursive equations for finite time ruin probability and distribution of ruin time are derived. Coupled Volterra type integral equation systems for ultimate ruin probability, severity of ruin and joint distribution of surplus before and after ruin are also obtained. Some numerical results are included.

SCANDINAVIAN ACTUARIAL JOURNAL

No. 1, 2003

MACDONALD, A. S., WATERS, H. R. & WEKWETE, C. T. *The genetics of breast and ovarian cancer I: A model of family history.* 1-27. We present a Markov model of breast cancer (BC) and ovarian cancer (OC) and estimate its transition intensities, mainly using United Kingdom population data. In the case of BC and OC, we estimate intensities according to BRCA1 and BRCA2 genotype. We use this to estimate the probabilities that an applicant for insurance has a BRCA1 or BRCA2 mutation, given complete or incomplete knowledge of her family history of BC and OC. Life (and other) insurance underwriters typically have incomplete knowledge of family history, for example no information on the number of healthy relatives. We show how these probabilities depend strongly on estimates of the mutation frequencies and penetrances, and conclude that it may not be appropriate to apply risk estimates based on studies of high-risk families to other groups.

MACDONALD, A. S., WATERS, H. R. & WEKWETE, C. T. *The genetics of breast and ovarian cancer II: A model of critical illness insurance.* 28-50. We present a model of breast cancer (BC) and ovarian cancer (OC) and other events that would lead to a claim under a Critical Illness (CI) insurance policy, and estimate its transition intensities, mainly using United Kingdom population data. We use this to estimate the costs of CI insurance in the presence of a family history of BC or OC, using the probabilities from Part I of carrying a BRCA1 or BRCA2

mutation, given the family history. In practice, the family history may not include all relevant facts; we look at the range of costs depending on what is known. We show the effect of lower penetrance than is observed in high-risk families. Finally, we consider what the cost of adverse selection might be, were insurers unable to use genetic test or family history information.

NIELSEN, J. P. *Smoothing and prediction with a view to actuarial science, biostatistics and finance.* 51-74. In this paper we review twenty-two papers on smoothing and prediction that were submitted for the Danish doctorate thesis (which is more comprehensive than a PhD) in the spring of 2002. Most of the papers deal with theoretical considerations; however, applications to actuarial science, biostatistics and finance are also discussed. The review mentions some of the most important background literature connected to the thesis and it includes a few remarks on the ten-year working process.

POLITIS, K. *Semiparametric estimation for non-ruin probabilities.* 75-96. We consider the classical risk model with unknown claim size distribution F and unknown Poisson arrival rate λ. Given a sample of claims from F and a sample of interarrival times for these claims, we construct an estimator for the function $Z(u)$, which gives the probability of non-ruin in that model for initial surplus u. We obtain strong consistency and asymptotic normality for that estimator for a large class of claim distributions F. Confidence bounds for $Z(u)$ based on the bootstrap are also given and illustrated by some numerical examples.

SCANDINAVIAN ACTUARIAL JOURNAL

No. 2, 2003

FROSTIG, E., HABERMAN, S. & LEVIKSON, B. *Generalized life insurance: ruin probabilities.* 136-152. We study ruin probabilities for generalized life insurance programs. These programs include among others whole life and long term care contracts. Clearly, in such programs the claims in successive years are dependent, hence the structure of our problem is different from that of ruin probabilities in general insurance where claims over time are independent. First, we develop algorithms calculating the ruin probabilities for life and LTC insurance programs. Further, upper and lower bounds for these probabilities are derived. These new bounds take into account the joint distribution of claims over time.

IRBÄCK, J. *Asymptotic theory for a risk process with a high dividend barrier.* 97-118. The only way to avoid ruin in the classical model of the collective risk theory is that the surplus increases to infinity. We consider a modified model with a dividend barrier that prevents this behaviour. It is shown that there is a simple approximation formula for the time of ruin when the level of the dividend barrier is high and the Cramér-Lundberg condition is satisfied. A numerical example is presented in the case when the claims are exponentially distributed. The relation to queuing theory is used to derive the proportion of time the surplus is below some given level.

LANDSMAN, Z. M. & MAKOV, U. E. *Sequential quasi-credibility for scale dispersion models.* 119-135. The sequential approach to credibility, developed by Landsman and Makov is extended to the scale dispersion family, which contains distributions often used in actuarial science: log-normal, Weibull, Half normal, Stable, Pareto, to mention only a few. For members of this family a sequential quasi-credibility formula is devised, which can also be used for heavy tailed claims. The results are illustrated by a study of log-normal claims.

LINNEMANN, P. *An actuarial analysis of participating life insurance.* 153-176. An actuarial model is developed to reveal the intrinsic nature of participating life insurance. The basic safe-side

criterion is examined. It is established how the first-order prospective net premium reserve includes safety margins or bonus loadings, and it is demonstrated how the bonus loadings are currently released. It is demonstrated how surplus may be distributed and accumulated as a terminal bonus in an equitable way. The level premium is divided into a variable recurrent single premium and a variable natural premium, and an alternative to the prospective net premium reserve is examined. A capitalization of future safety margins or bonus loadings, which are related to past premiums and the paid-up benefit, may allow the insurance company a considerable increase in investment freedom. The theory is illustrated by numerical results.

https://doi.org/10.1017/S1357321700004517 Published online by Cambridge University Press