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Laser wakefield acceleration in narrow plasma-filled channels

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Abstract

The wakefield acceleration driven by a single laser pulse in a narrow plasma-filled solid waveguide is considered. Parameters of the waveguide which provide the maximum energy gain of accelerated particles at a given length and peak power of the driver are found in the approximation of a relativistically weak driver. The obtained formulas are illustrated by calculating the electron energy gain achievable with the terawatt laser of Utsunomiya University.

Keywords: Channeling; Laser wakefield acceleration; Waveguide

1. INTRODUCTION

Laser-driven wakefield acceleration of charged particles in a plasma has been a field of active research since 1979 (Tajima & Dawson, 1979). This activity is encouraged by very high accelerating gradients possible inside the plasma. Several reviews of the subject are available now (Esarey *et al.*, 1996; Leemans *et al.*, 1998; Ogata & Nakajima, 1998).

The main obstacle on the way from high accelerating gradients to high energy gains (of gigaelectronvolt range) is a short acceleration distance, typically limited by the diffraction of the driving laser pulse (the driver). To overcome the diffraction, it was proposed to use plasma channels, that is, nonuniform plasmas with a local density minimum on the axis (Esarey *et al.*, 1996).

In this article, we consider the channeling inside a narrow solid-wall waveguide (Lotov, 1998). The idea of this method is the following. The laser beam is focused onto the entrance to a metal or dielectric tube (waveguide). The tube radius R is of the order of the laser beam radius σ_r in the focus, so the beam excites a fundamental waveguide mode. The waveguide is filled with a cold gas (hydrogen) with density n. While moving through the waveguide, the beam instantly ionizes both the hydrogen and the inner parts of the waveguide walls. In the plasma channel formed in this way, the driver propagates with no diffraction and excites the wakefield usable for particle acceleration. The advantage of this channeling method is that it offers the possibility of precise driver trajectory control that is necessary for most conceivable applications of the laser wakefield acceleration.

There are at least two reasons for filling the waveguide with hydrogen. First, the hydrogen can be most easily fully ionized. A fully ionized inner plasma is necessary for stable propagation of the driver. Second, in hydrogen the number of nucleons per electron is the smallest. A low nucleon density of the accelerating medium is required to preserve the small emittance of the accelerated beam (Skrinsky, 1997).

In Section 2, we derive an expression for the maximum possible energy gain of accelerated particles which can be achieved at a given duration and peak power of the driver. We restrict our consideration to the case of relativistically weak laser beams and narrow (as compared to the plasma skin depth) channels. In Section 3, we illustrate the resultant formulas by calculating the energy gain achievable with the terawatt laser of Utsunomiya University. In Section 4, we summarize the main findings.

2. OPTIMIZATION FOR THE MAXIMUM ENERGY GAIN

We use the cylindrical coordinates (r, φ, z) with the z-axis as the direction of driver propagation. Assume that the driver is circularly polarized and the amplitude of its electric field, near the entrance to the channel, is

$$E_d = E_{dm} \exp\left(-\frac{r^2}{2\sigma_r^2} - \frac{(z-ct)^2}{2\sigma_z^2}\right), \tag{1}$$

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where c is the light velocity in vacuum, t is the time, and we use the subscript m to designate the maximum amplitude of a quantity. The maximum field amplitude can be expressed in terms of the peak beam power P_m:

$$E_{dm} = \sqrt{\frac{4P_m}{c\sigma_r^2}}. \tag{2}$$

For efficient excitation of the wakefield (both in uniform plasmas and in plasma-filled channels), the driver length σ_z needs to be of the order of the plasma skin depth c/ω_p, where $\omega_p = \sqrt{4\pi ne^2/m}$ is the plasma electron frequency, n is the plasma density (of the inner plasma in the case of waveguide channeling), and the other notations are common. Throughout the article, we assume the laser beam is relativistically weak, that is, it has the maximum normalized vector potential

$$a_{dm} = \frac{eE_{dm}}{mc\omega} \ll 1, \tag{3}$$

where $\omega = 2\pi c/\lambda$ is the beam frequency, and the beam wavelength in vacuum is $\lambda \ll c/\omega_p$.

Let us find the distribution of driver intensity inside a circular channel of radius R. For the most efficient excitation of the fundamental waveguide mode, we need

$$\sigma_r \approx 0.5R \tag{4}$$

(Lotov, 1998). Then 85% of the beam energy falls into TE_{11} mode, the peak driver electric field on the waveguide axis is

$$E_{wm} \approx 0.76E_{dm}, \tag{5}$$

and the amplitude of the electric field in the channel is

$$E_w = E_{wm}\sqrt{J_0^2\left(\frac{\mu r}{R}\right) + J_2^2\left(\frac{\mu r}{R}\right)}\exp\left(-\frac{(z-ct)^2}{2\sigma_z^2}\right). \tag{6}$$

Here J_0 and J_2 are Bessel functions, and $\mu \approx 1.84$ is the first zero of the derivative of J_1 with respect to its argument; in derivation of Eq. (6), we have neglected the influence of the inner plasma on the driver motion and used the assumption $R \gg \lambda$.

In the linear approximation (when $a_{wm} \ll 1$), the small perturbation of the plasma electron density δn is related to the normalized vector potential of the driver a_w as

$$\frac{\partial^2 \delta n}{\partial t^2} + \omega_p^2 \delta n = \frac{nc^2}{2}\Delta a_w^2, \quad a_w = \frac{eE_w}{mc\omega} \tag{7}$$

(Esarey *et al.*, 1996), where Δ is the Laplacian. In the case of narrow channels (when $R \ll \sigma_z$), the dominant contribution to the Laplacian is made by its radial part, so the right-hand side of Eq. (7) reads as

$$\frac{ne^2E_{wm}^2}{2m^2\omega^2}\left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}(J_0^2+J_2^2)\right)\exp\left(-\frac{(z-ct)^2}{\sigma_z^2}\right), \tag{8}$$

whence the amplitude of the density perturbation behind the driver is

$$\delta n_m = \frac{ne^2E_{wm}^2}{2m^2\omega^2}\left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}(J_0^2+J_2^2)\right)\cdot\frac{\sqrt{\pi}\sigma_z}{c\omega_p}\exp\left(-\frac{\sigma_z^2\omega_p^2}{4c^2}\right). \tag{9}$$

Note that we formally obtain $\delta n_m < 0$ near the axis and $\delta n_m > 0$ near the wall, which means that the plasma density oscillates in opposite phases in these two regions.

With the density perturbation in hand, we find the wakefield potential Φ from the equation

$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}\Phi - \frac{\omega_p^2}{c^2}\Phi = 4\pi e^2\delta n, \tag{10}$$

with the boundary condition

$$\Phi|_{r=B} = 0 \tag{11}$$

(Lotov, 1998). For narrow channels (with $R \ll c/\omega_p$), the solution to Eq. (10) is particularly simple. To obtain the amplitude $\Phi_0(r)$ of the wakefield potential oscillations, we neglect the second term in the left-hand side of Eq. (10) (since it is roughly $c^2/(\omega_p R)^2$ times smaller than the first term) and insert the amplitude of density oscillations (9) into the right-hand side:

$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}\Phi_0 = \frac{2\pi\sqrt{\pi}ne^4\sigma_z E_{wm}^2}{m^2c\omega^2\omega_p}\left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}(J_0^2+J_2^2)\right) \times \exp\left(-\frac{\sigma_z^2\omega_p^2}{4c^2}\right).$$

Then we twice integrate in r and find

$$\Phi_0(r) = \frac{2\pi\sqrt{\pi}ne^4\sigma_z E_{wm}^2}{m^2c\omega^2\omega_p}\left(J_0^2\left(\frac{\mu r}{R}\right) + J_2^2\left(\frac{\mu r}{R}\right) - C\right) \times \exp\left(-\frac{\sigma_z^2\omega_p^2}{4c^2}\right) \tag{12}$$

with the constant

$$C = J_0^2(\mu) + J_2^2(\mu) \approx 0.2. \tag{13}$$

Taking into account both terms on the left-hand side of Eq. (10) would yield a somewhat smaller value of Φ_0. The electromagnetic force exerted by the wakefield on an axi-

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ally moving (in the direction $\mathbf{e}_z$) ultrarelativistic electron is just the gradient of Φ:

$$\mathbf{F} = -e(\mathbf{E} + [\mathbf{e}_z \times \mathbf{B}]) = \nabla\Phi, \tag{14}$$

and the spatial period of the wakefield is $2\pi c/\omega_p$, so the maximum accelerating force (on the axis) is

$$F_{zm} = \frac{\omega_p}{c}\Phi_0(0) = \frac{\sqrt{\pi}e^2\sigma_z E_{wm}^2\omega_p^2}{2mc^2\omega^2}(1-C)\exp\left(-\frac{\sigma_z^2\omega_p^2}{4c^2}\right). \tag{15}$$

Let us find the acceleration length for the case of a narrow waveguide. Assuming the diffraction is completely suppressed by the waveguide, we have three effects that can limit the acceleration length: driver depletion, detuning of the driver and accelerated particles, and the dispersive broadening of the driver. The latter two effects are determined by the dispersion relation for electromagnetic waves in the plasma-filled waveguide:

$$\omega^2 = k_z^2c^2 + \alpha^2, \quad \alpha^2 = \omega_p^2 + \frac{\mu^2c^2}{R^2}, \tag{16}$$

where $k_z \approx 2\pi/\lambda$ is the z-component of the wave-vector. The group velocity of the driver is

$$v_g = \frac{d\omega}{dk_z} = \frac{k_z c^2}{\omega}, \tag{17}$$

and its relativistic factor is

$$\gamma_b = \left(1 - \frac{v_g^2}{c^2}\right)^{-1/2} = \frac{\omega}{\alpha}, \tag{18}$$

whence we obtain the detuning length

$$L_d = 4\gamma_b^2\frac{c}{\omega_p} = \frac{4\omega^2c}{\alpha^2\omega_p} \tag{19}$$

(Esarey *et al.*, 1996). The length of dispersive broadening L_b can be estimated as the distance traveled by the driver until its length increases $\sqrt{2}$ times (see, e.g., Jackson, 1962):

$$L_b = c\sigma_z^2\left(\frac{d^2\omega}{dk_z^2}\right)^{-1} \sim \frac{c\omega^3}{\alpha^2\omega_p^2}. \tag{20}$$

The ratio of the two distances is

$$\frac{L_b}{L_d} \sim \frac{\omega}{4\omega_p} \gg 1, \tag{21}$$

so the pulse broadening can be safely ignored.

The depletion length for relativistically weak drivers in the unbounded plasma is typically $|a_{dm}|^{-2}$ times greater than the detuning length (Esarey *et al.*, 1996). Thus, we can neglect the energy lost by the driver in the inner plasma as compared to the initial driver energy. The driver damping on the waveguide walls is somewhat difficult to estimate. The calculation of this damping rate will be the subject of a separate study. However, it was experimentally measured to be quite low at high laser intensities (Jackel *et al.*, 1995; Borghesi *et al.*, 1998; Dorchies *et al.*, 1999), so we assume the pulse depletion length L_p to be longer than L_d.

To obtain the energy gain of accelerated particles W, we multiply the detuning length Eq. (19) by the accelerating force Eq. (15), and substitute Eqs. (2), (4), (5), (13), and (16) into the product:

$$W \approx \frac{7.7P_m e^2\sigma_z\omega_p}{(c^2 + \omega_p^2R^2/\mu^2)mc^2}\cdot\exp\left(-\frac{\sigma_z^2\omega_p^2}{4c^2}\right). \tag{22}$$

We can see from this expression that, for $R \ll c/\omega_p$, the energy gain does not depend on the waveguide radius if the beam is initially properly focused. The energy gain has its maximum at the plasma density such that

$$\frac{c}{\omega_p} = \frac{\sigma_z}{\sqrt{2}}, \quad \text{or} \quad n = \frac{mc^2}{2\pi e^2\sigma_z^2}. \tag{23}$$

It approximately equals

$$W \approx \frac{6.6P_m e^2}{mc^3}. \tag{24}$$

To provide this energy gain, the driver should propagate in the waveguide over

$$\frac{L_d c}{\omega\sigma_r^2} = \frac{16\omega}{\omega_p(\mu^2 + \omega_p^2R^2/c^2)} \approx \frac{5\omega}{\omega_p} \approx \frac{20\sigma_z}{\lambda} \tag{25}$$

Rayleigh lengths.

3. NUMERICAL EXAMPLE

Here we calculate the electron energy gain achievable with the terawatt laser of Utsunomiya University. The expected parameters of the laser, the required parameters of the channel and plasma, and several calculated parameters of the accelerator are given in Table 1. In the table we analyze two different cases:

$$\text{(A) } R = 0.6\ c/\omega_p; \qquad \text{(B) } R = 0.8\ c/\omega_p. \tag{26}$$

For smaller values of R, the linear theory [formulas (7) and (10)] is not valid, and the energy gain possibly gets smaller because of saturation of the wakefield ($\delta n_m/n$ cannot be greater than unity). For greater values of R, the waveguide cannot be considered as narrow, and the exact solution to

Table 1. *Parameters of the waveguide wakefield accelerator*

Parameter	Value (A)	Value (B)	Formula
Beam peak power, P_m	0.5 TW	0.5 TW	
Beam half-length, σ_z	30 μm	30 μm	
Beam wavelength, λ	800 nm	800 nm	
Plasma (gas) density, n	$6.3 \cdot 10^{16}$ cm^{-3}	$6.3 \cdot 10^{16}$ cm^{-3}	(23)
Waveguide radius, R	12.7 μm	17 μm	(26)
Beam rms radius in focus, σ_r	6.4 μm	8.5 μm	(4)
Channel length, L_d	23 cm	37 cm	(19)
Plasma wavelength, $2\pi c/\omega_p$	130 μm	130 μm	(23)
Maximum accelerating gradient, F_{zm}	8 MeV/cm	4.4 MeV/cm	(15)
Channel length (in Rayleigh lengths)	710	660	(25)
Relative density perturbation, $\delta n_m/n$	0.75	0.25	(9)
Laser strength parameter, a_{dm}	0.3	0.23	(3)
Error of narrow waveguide approximation	10%	20%	(27)
Energy gain, W	175 MeV	160 MeV	(22)

Eqs. (7) and (10) is required. Assuming the waveguide to be narrow gives an error of the order of

$$\frac{\omega_p^2 R^2}{\mu^2 c^2}, \tag{27}$$

which is acceptably low for $R < 0.8\ c/\omega_p$.

We see that the maximum energy gain can be as high as 175 MeV. Note that, even though the laser is not very intense, the plasma wave is close to nonlinear. However, since the excited wakefield is mainly transversal, the strong perturbation of the electron density does not result in accelerating gradients of the order of the wavebreaking limit $E_{wb} = \sqrt{4\pi nmc^2}$.

4. CONCLUSION

We have analyzed the possibility of laser wakefield acceleration in gas-filled solid waveguides. Unlike the acceleration in an unbounded plasma, in the waveguides the most efficient acceleration occurs (in the case of relativistically weak drivers) at driver and waveguide radii somewhat smaller than the plasma skin-depth. The excited wakefield is mainly transversal, that is, plasma electrons oscillate mainly across the channel, and the achievable accelerating field is much smaller than the wavebreaking limit even for strong perturbations of the electron density. The maximum energy gain of the accelerated particles is fully determined by the peak laser power unless the driver is focused so tightly that the plasma response becomes nonlinear.

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