Macroeconomic Dynamics, **22**, 2018, 279–306. Printed in the United States of America.
doi:10.1017/S1365100516000171

ASYMMETRY AND THE AMPLITUDE OF BUSINESS CYCLE FLUCTUATIONS: A QUANTITATIVE INVESTIGATION OF THE ROLE OF FINANCIAL FRICTIONS

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We examine the quantitative significance of financial frictions that reduce firms' access to credit in explaining asymmetric business cycles characterized by disproportionately severe downturns. Using rate spread data to calibrate the severity of these frictions, we successfully match several key features of U.S. data. Specifically, although output and consumption are relatively symmetric (with output being slightly more asymmetric), investment and hours worked display significant asymmetry over the business cycle. We also demonstrate that our financial frictions are capable of significantly amplifying adverse shocks during severe downturns. Although the data suggest that these frictions are only active occasionally, our results indicate that they are still a significant source of macroeconomic volatility over the business cycle.

Keywords: Business Cycle Asymmetry, Moral Hazard, Agency Costs, Liquidity Shocks, Occasionally Binding Constraints

1. INTRODUCTION

Empirical evidence suggests that U.S. business cycles are asymmetric, and that this asymmetry can be subdivided into two broad categories of steepness and deepness. Steepness captures the fact that sharp contractions are often followed by long protracted recoveries, whereas deepness captures the fact that business cycle troughs are often deeper than peaks are tall. Early works by Neftci (1984), Hamilton (1989), Sichel (1993), and Acemoglu and Scott (1997) clearly identify the presence of these forms of asymmetry in many macroeconomic aggregates,

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https://doi.org/10.1017/S1365100516000171 Published online by Cambridge University Press

such as real output, hours worked, unemployment, and investment. Furthermore, key stylized facts regarding U.S. business cycle asymmetry, such as investment, employment, and hours worked being more asymmetric than output, have been widely documented in the literature [see Sichel (1993), Hansen and Prescott (2005), and McKay and Reis (2008)]. Although the empirical evidence related to asymmetric business cycles has been observed for many years, the possible mechanisms within a dynamic general equilibrium model that can generate the degree of asymmetry observed in the data are still being explored.

Acemoglu and Scott (1997) use intertemporal increasing returns arising from endogenous variations in the profitability of firms' investment choices to generate asymmetric business cycles. Caballero and Hammour (1996) and McKay and Reis (2008) introduce similar mechanisms, but focus on the adoption of new technology and the optimal timing of creative destruction. Hansen and Prescott (2005) manipulate occasionally binding capacity constraints to generate sufficient degrees of deepness[1] in output and hours worked to match the data, whereas Knuppel (2014) generates realistic levels of skewness in aggregate variables by incorporating a kink into the marginal costs of capital utilization. Van Nieuwerburgh and Veldkamp (2006) incorporate asymmetric learning over the business cycle to capture the degree of steepness observed in the data. Our paper examines the quantitative significance of financial frictions that amplify adverse productivity shocks in matching the asymmetry observed in U.S. output, consumption, investment, and hours worked data.[2]

Financial frictions enter our model through entrepreneur-run projects that take two periods to complete, face both moral hazard and idiosyncratic liquidity shocks, and require outside financing.[3] Liquidity shocks represent a sudden need to raise additional funds after the installation of inputs in order to bring a project to completion. Moral hazard takes the form of private benefits that an entrepreneur could receive from shirking, in which case his project is less likely to successfully produce output. The incentive constraints arising from moral hazard considerations bind only when the economy is in a sufficiently adverse state; however, equity contracts are structured so that entrepreneurs never find it optimal to shirk. An adverse productivity shock has three distinct effects on output. First, it reduces the expected output of projects, causing a reduction in initial investments. Second, it has the potential to exacerbate the moral hazard problem, leading to a further reduction in initial investment to satisfy the incentive constraints. Third, as project size falls because of the first two effects, investors become less willing to provide additional funds in response to the project-specific liquidity shocks, causing fewer projects to run to completion. Given that the incentive constraints bind only during severe economic downturns, they cause firms additional losses in both current investment funding and future liquidity provisions, thereby exacerbating the severity of the downturn and creating business cycle asymmetry.

A common measure of the importance of tight credit conditions is the risk spread between the rates on 3-month nonfinancial commercial paper and 3-month T-bills. The asymmetry in this measure is evident in the spikes that tend to occur during

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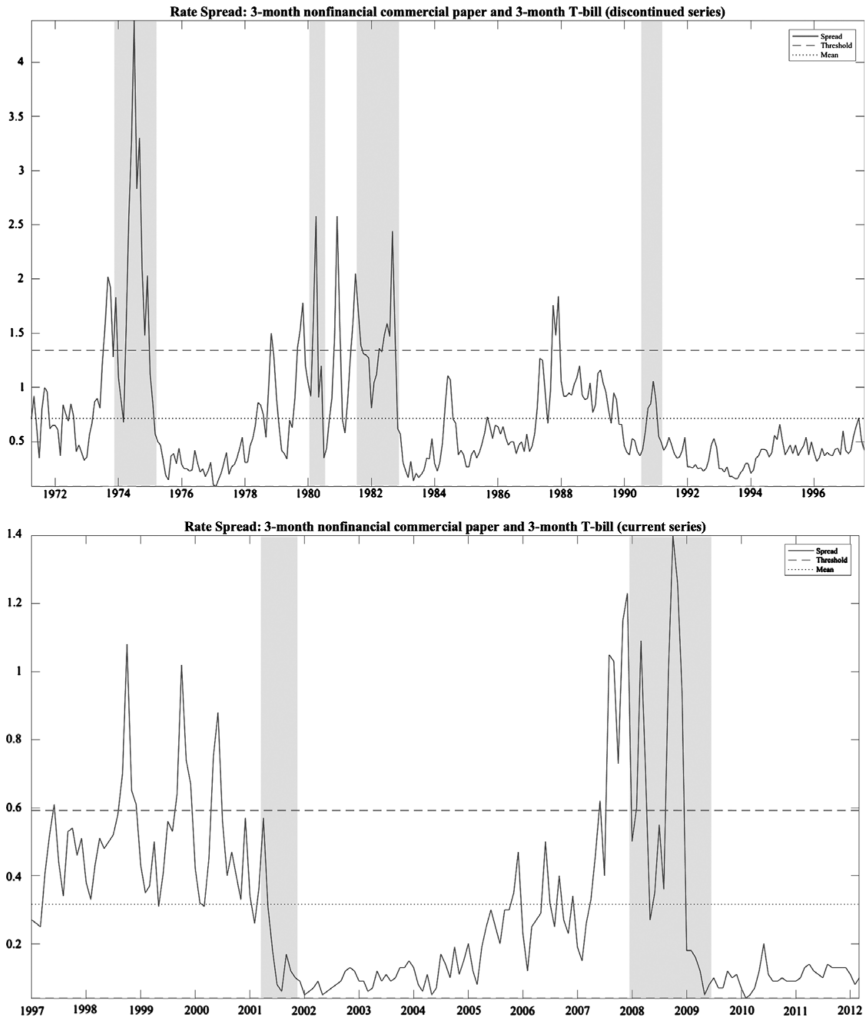

FIGURE 1. Lending over the business cycle.

economic downturns, as illustrated in Figure 1, which also shows the narrowest symmetric band around the mean of the series that includes its minimum. This asymmetric increase in the wedge between investors' and firms' valuations of funds provides a target for calibrating the severity of the agency problem presented in this paper. In particular, the size of the agency rent (described in the following) is set so that the computed time path for the spread in the firms' shadow price of funds spikes outside of its symmetric band with a frequency equal to that observed in the rate spread data.

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Other papers have addressed the degree to which financial constraints generate asymmetries in business cycles. Mendoza (2010) demonstrates that an occasionally binding leverage constraint is capable of drastically amplifying small shocks in a thoughtfully calibrated DSGE (dynamic stochastic general equilibrium) model, giving rise to sudden stops.[4] Li and Dressler (2011) include an occasionally binding international borrowing constraint in a small open economy model and demonstrate that the degree of steepness asymmetry generated by the model depends on the initial debt level of the country. The primary difference between the model presented in this paper and these works is that we focus on the extent to which our calibrated model can replicate the degree of deepness asymmetry observed in the data while simultaneously retaining a strong fit to standard business cycle facts. Mendoza (2010), although successful in generating significant amplification of adverse shocks, focuses on matching the properties of sudden stops, not long-run asymmetric behavior. Li and Dressler (2011) focus on steepness rather than deepness, and find that they must use unrealistically high levels of international debt to generate statistically significant asymmetry.

Besides asymmetry, we also consider our mechanism's ability to amplify business cycles. Ultimately, we find that although our financial frictions are only operational occasionally, they significantly contribute to our model's volatility over the business cycle.

We summarize the findings of this paper: (i) Our financial frictions generate quantitatively significant levels of asymmetry in several key variables. In particular, our model predicts the skewness of output, consumption, investment, and hours worked to be −0.22, −0.09, −0.86, and −0.76 compared with the values of −0.36, −0.16, −0.91, and −0.34 in the data. (ii) Our model replicates the fact that investment and hours worked both display more asymmetry than output. In terms of deepness, our model implies values of 0.88, 0.88, and 0.95 for investment, hours worked, and output, respectively, compared with the values of 0.80, 0.89, and 0.98 in the data. In addition, investment is more asymmetric than hours worked in terms of skewness (−0.86 vs. −0.76 in the model compared with −0.91 vs. −0.34 in the data). (iii) The model also implies that consumption is less asymmetric than output, as in the data. Specifically, our model generates deepness measures of 0.98 and 0.95 for consumption and output, respectively, compared with 1.02 and 0.98 in the data. (iv) Restricting attention to a downturn, we find that our financial frictions amplify the percentage decline in output, investment, and hours worked, at the trough, by 33.0%, 47.3%, and 120.7%, respectively. (v) Although financial frictions are only active occasionally, their presence significantly amplifies business cycle volatility, with the standard deviation of output rising by 11.6% and of hours worked by 59.3%.

The remainder of the paper is organized as follows. Section 2 presents the structure of the model, which is solved in Section 3. Section 4 addresses the calibration of the model. Section 5 discusses the model's results, and Section 6 concludes with a brief discussion of potential extensions of the paper.

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2. THE MODEL

We consider an infinite-horizon growth model where the economy is populated by a continuum of households of measure one and the members of each household pool and share risk perfectly. All households are identical and a representative household consists of an investor, a continuum of entrepreneurs, and a continuum of workers, each of measure one. At the beginning of each period, every entrepreneur is endowed with a plan for a project that requires outside funding to rent capital and hire labor from other households. The workers of the household all supply labor to the entrepreneurs of other households in exchange for the market clearing wage, while the investor manages the household's portfolio. This portfolio consists of the household's equity holdings in outside projects, its capital position, and its holdings of a real liquid asset, which finance outside projects' future cost overruns [see Holmstrom and Tirole (1997)].[5]

2.1. Household Sector: The Entrepreneurs' Problems

Each entrepreneur of the representative household starts a new project indexed by $i \in [0, 1]$ every period. These projects take two periods to complete. During the first period, time t, capital and labor must be acquired for use in the project. The inclusion of capital as a factor of production represents a departure from the model presented in Atolia et al. (2011), who abstract from capital accumulation. As we show later, the addition of capital allows the current model to match the standard business cycle facts more closely and makes further quantitative exercises possible. To finance his resource costs, an entrepreneur sells shares, s_t^i, in his project at price p_t^i.[6] Therefore, the first-period resource financing constraint faced by entrepreneur i at time t is given by

$$w_t n_{1,t}^i + r_t k_{t+1}^i = p_t^i s_t^i, \tag{1}$$

where $n_{1,t}^i$ and k_{t+1}^i denotes the labor and capital inputs of the project and w_t and r_t denote their respective factor prices. The reader may also note the difference in timing between the capital rental rate, r_t, and the capital stock, k_{t+1}^i. In our model, production does not occur until the second period, time $t+1$, but all input costs are paid up front at time t. Therefore, the difference in timing was chosen to retain the convention of dating the capital stock by the period when it is used in production. This timing change will alter the form of the capital Euler equation slightly (see Section 3.1).

At the start of the second period, time $t+1$, the aggregate productivity of the economy, $\theta_{t+1} > 0$, is realized. This value of productivity, along with the previously installed quantities of capital and labor, determines the potential output of project i at time $t+1$,

$$y_{t+1}^i = \theta_{t+1}(k_{t+1}^i)^\alpha (n_{1,t}^i)^{1-\alpha}, \tag{2}$$

where $0 < \alpha < 1$.

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Each project also experiences an idiosyncratic cost overrun, ρ^i_{t+1}, at the start of the second period, $t+1$, that requires the entrepreneur to employ an additional $n^i_{2,t+1}$ hours of labor immediately or forgo the output of the project. That is, the cost overrun is given by

$$\rho^i_{t+1} = n^i_{2,t+1}, \tag{3}$$

when measured in terms of units of labor. The total cost/wage bill, $w_{t+1}n^i_{2,t+1}$, for these additional labor hours must be paid using the real liquid asset. This financing requirement is what facilitates the interpretation of this cost overrun as a liquidity shock. [See Holmstrom and Tirole (1997).]

Entrepreneurs also lack the resources required to fund the second-period labor need internally, so they return to their first-period investors seeking additional funds. Investors were aware of this potential need when they made their first-period investments. Thus, they planned for it by allocating some of their household's resources at time t to building up a balance of the real liquid asset, M_{t+1}, that can be used to meet the liquidity need at the start of time $t+1$. After observing both the aggregate and idiosyncratic shocks, the investors form a rule to determine how they will finance the liquidity need.

This rule is characterized in Lemma 1 and Corollary 1.[7]

LEMMA 1. *If, in period $t+1$, investors finance the liquidity need for project i with liquidity shock ρ^i_{t+1}, they also finance the liquidity need for any project h if $\rho^h_{t+1} \leq \rho^i_{t+1}$.*

COROLLARY 1. *Let the distribution $F(\cdot)$ of ρ have support $[0, \bar{\rho}]$, where $0 < \bar{\rho} \leq \infty$. Then there exists a unique $\rho^\star_{t+1} \in (0, \bar{\rho}]$ such that all projects with $\rho^i_{t+1} \leq \rho^\star_{t+1}$ will have their liquidity needs financed.*

For projects that have their liquidity needs financed, the per-share contribution, $m^i_{t+1}(\rho^i_{t+1})$, of the investors is such that the investors finance the total cost of the liquidity shock:

$$m^i_{t+1}(\rho^i_{t+1})s^i_t = \rho^i_{t+1}w_{t+1}. \tag{4}$$

The success of a project that has its liquidity need met is still uncertain. Entrepreneurs possess a hidden action, their choice of effort, which affects their projects' probability of success. If an entrepreneur is diligent, his project will succeed with high probability p_{H}. If he chooses to shirk his responsibilities and engage in a privately beneficial activity, then his project's probability of success will fall to p_{L}.

Investors are aware of this agency problem. The cost of shirking is assumed to be sufficiently high, as determined by a large value for $\Delta p = p_{\mathrm{H}} - p_{\mathrm{L}}$, so that all projects with diligent entrepreneurs have a positive expected net present value, whereas all projects with nondiligent (shirking) entrepreneurs have a negative expected net present value. Therefore, it is never advantageous for the investor to allow the entrepreneur to shirk [see Tirole (2006)]. Thus, investors structure equity contracts to guarantee effort by the entrepreneur. Specifically, incentive

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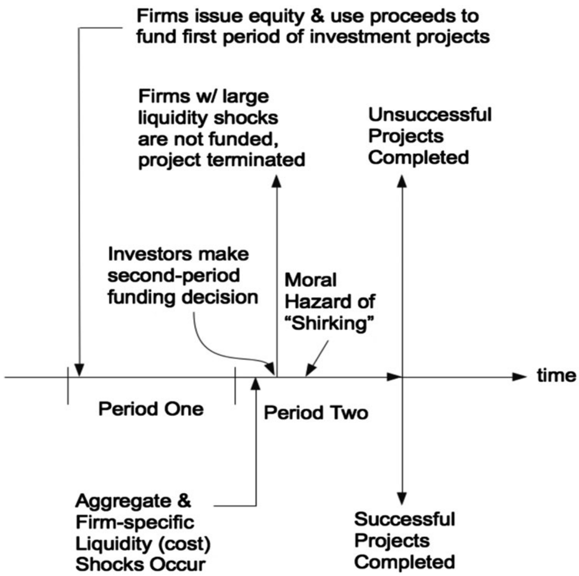

FIGURE 2. Timing of risky investment projects.

compatibility (IC) constraints (to be described later) are respected to ensure that entrepreneurs are always diligent.

The timing of the projects described is summarized in the timeline found in Figure 2.

Because all of a project's inputs are purchased in advance, any output, y^i_{t+1}, generated by a project must be divided between its shareholders. Outside investors are entitled to s^i_t of this output, leaving $1 - s^i_t$ for the entrepreneurs. Given the assumption that it is always optimal to induce the entrepreneur to be diligent, and that there exists a liquidity financing threshold, the expected output from project i at time $t+1$ is $p_{\mathrm{H}} y^i_{t+1} F(\rho^{\star}_{t+1})$, where p_{H} denotes the project's probability of success and $F(\rho^{\star}_{t+1})$ denotes the likelihood the project's second-period liquidity need will be met.

The following family of incentive compatibility (IC) constraints, one for each realization of θ_{t+1} (as y^i_{t+1} depends on θ_{t+1}),

$$p_{\mathrm{H}}(1 - s^i_t) y^i_{t+1} \geq p_{\mathrm{L}}(1 - s^i_t) y^i_{t+1} + J s^i_t, \tag{5}$$

guarantees that the entrepreneur will always prefer diligence over shirking. In particular, the IC constraints in (5) ensure that for any realization of θ_{t+1}, the entrepreneur's share of expected output when diligent (left-hand side) is at least as large as the sum of his share of expected output when shirking and his private benefit from shirking (right-hand side). The entrepreneur's private benefit, $J s^i_t$, is assumed to depend on both a scale parameter, J, and the share of the project sold

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to outside investors, s_t^i. As project size is increasing in s_t^i, its presence in this term captures the fact that the entrepreneur's private benefit from shirking increases as the project becomes larger. [See Atolia et al. (2011) for more details on this point.]

The IC constraints (5) can be written more compactly as

$$(1 - s_t^i) y_{t+1}^i \geq A s_t^i, \tag{6}$$

where $A = J/\Delta p$ denotes the entrepreneur's agency rent and the right-hand side of equation (6) represents the minimum payment to the entrepreneur that would preserve the entrepreneur's incentive not to shirk [see Holmstrom and Tirole (1997) and Tirole (2006)].

The entrepreneur issues shares, rents capital, and hires labor at time t in order to maximize the value of his share, $(1 - s_t^i)$, of the project's expected future output,

$$\Pi_t^i = \max_{s_t^i, k_{t+1}^i, n_{1,t}^i} E_t \left\{ \beta \frac{U_{C,t+1}}{U_{C,t}} (1 - s_t^i) p_{\mathrm{H}} y_{t+1}^i F(\rho_{t+1}^\star) \right\}, \tag{7}$$

discounted using the household's stochastic discount factor $\beta U_{C,t+1}/U_{C,t}$, where β is the household's discount factor and $U_{C,t+1}/U_{C,t}$ is its intertemporal marginal rate of substitution (MRS) in consumption (where $U_{C,t}$ denotes the partial derivative of the household's utility function with respect to C at date t). The maximization of (7) is subject to the entrepreneur's first-period resource financing constraint (1) and his incentive compatibility (IC) constraints in (6), taking w_t, r_t, p_t^i, and intertemporal MRS in consumption as given.

2.2. Household Sector: Workers' and Investor's Choices

The representative household's period utility function U has standard properties and is given by

$$U(C_t, L_t) = \log C_t + \eta \log L_t, \tag{8}$$

where $\eta > 0$ is a parameter, C_t is consumption, and L_t is leisure. Thus, the household derives utility from consumption and leisure. The household's discount factor is $\beta \in (0, 1)$.

All of the household's agents engage in separate income-generating activities during the time period. Based on the household's consumption–leisure decision, the workers provide labor, n_t, which is one source of income, $w_t n_t$. The entrepreneurs start new projects in each period (which are indexed by $i \in [0, 1]$) and retain shares $(1 - s_t^i)$ in those projects. The shares retained in projects started in period $t - 1$ (indexed by $l \in [0, 1]$) mature in period t and yield profits in the amount Π_t^l, thus providing another source of income for the household.

The final source of income is from the household's assets, which are managed by the investor. He determines and implements the household's optimal consumption–saving and portfolio allocation decisions. The investor accumulates k_{t+1} units of capital to be carried into the next period, which depreciates at the rate δ per period.

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He rents this capital out to the entrepreneurs of other households, for which he receives an advance payment of $r_t k_{t+1}$ in the current period. In addition, the investor buys s_t^j shares of projects externally operated by other households, where $j \in [0, 1]$. As the number of shares of each project is normalized to 1, s_t^j shares entitle the household to a corresponding fraction of the project's output in period $t+1$, provided the project is eventually successful. A necessary condition for the project to produce output is that its random liquidity need at the beginning of period $t+1$ is financed. This liquidity need arises from the fact that the entrepreneur needs to pay for unanticipated extra costs of operations in period $t+1$ *before* the project's output becomes available. The provision of this liquidity is the third investment option for the household. In particular, the household carries or costlessly stores M_{t+1} units of the aggregate good, which yield zero net return but are available to finance the liquidity needs at the beginning of period $t+1$.

In addition to making the investment decisions for the next period, the household's investor also determines which of the ongoing projects (of other households in which he invested in period $t-1$) will have their liquidity needs financed in period t. This decision is made after observing the current-period aggregate shock (θ_t) and the individual realization of ρ_t^j. As discussed earlier, this latter decision would take the form of a cutoff value for the liquidity shock, $\rho_t^\star$.

Because only projects that have their liquidity need financed will produce (with probability p_{H}), the household's total income, Z_t, is

$$Z_t = w_t n_t + r_t k_{t+1} + \int_0^1 \Pi_t^l dl + \int_0^1 p_{\mathrm{H}} y_t^j s_{t-1}^j I(\rho_t^j \leq \rho_t^\star) dj, \qquad \mathbf{(9)}$$

where I denotes an indicator function, that is, 1 when $\rho_t^j \leq \rho_t^\star$ and zero otherwise, and the last two terms on the right-hand side are, respectively, the profits from the maturing projects started by entrepreneurs of the household and the return from the investment in the maturing projects of the other households.

Furthermore, as the liquidity needs must be financed out of the liquid asset, M_t, carried into period t, we have the following constraint on liquidity financing:

$$\int_0^1 m_t^j(\rho_t^j) s_{t-1}^j I(\rho_t^j \leq \rho_t^\star) dj \leq M_t. \qquad \mathbf{(10)}$$

Finally, the household's (consolidated) budget constraint is given by

$$C_t + \int_0^1 p_t^j s_t^j dj + \int_0^1 m_t^j(\rho_t^j) s_{t-1}^j I(\rho_t^j \leq \rho_t^\star) dj + M_{t+1} + k_{t+1} \leq M_t + (1-\delta) k_t + Z_t, \qquad \mathbf{(11)}$$

where the right-hand side is the total resources available to the household: the liquidity carried from the last period, the undepreciated capital stock, and the income described in (9). The left-hand side is the use of those funds: consumption, the purchase of shares in new projects, the meeting of the liquidity needs of the

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existing projects, the provision for the liquidity need for the next period, and the accumulation of capital for the next period.

The household also faces a time constraint that states that all time (which is normalized to 1 each period) is spent either working or taking leisure:

$$n_t + L_t \leq 1. \tag{12}$$

The household solves

$$\max_{\left\{C_t, n_t, L_t, k_{t+1}, \rho_t^*, M_{t+1}, \left\{s_t^j\right\}_{j\in[0,1]}\right\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, L_t\right), \tag{13}$$

subject to (10)–(12), taking w_t, r_t, and p_t^j, $j \in [0, 1]$, as given.

3. SOLVING THE MODEL

In this section, we first solve the optimization of the representative household, which is followed by solving the problem of the representative entrepreneur.

3.1. Solution to the Household's Problem

The trade-off between working and taking leisure for the household yields the following familiar Euler equation:[8]

$$w_t U_{C,t} = U_{L,t}. \tag{14}$$

The household's investment decision is more complicated. It must allocate its resources between current consumption and the four other competing uses. The optimality condition for the accumulation of capital (k_{t+1}) is

$$1 - r_t = \beta E_t \left\{ \frac{U_{C,t+1}}{U_{C,t}} (1 - \delta) \right\}, \tag{15}$$

where the left-hand side is the net period-t cost of acquiring one unit of capital, which is less than 1, as the rent (r_t) on a unit of the acquired capital is received in period t itself. In period $t + 1$, the household receives back $(1 - \delta)$ units of undepreciated capital, which has a present discounted value given by the right-hand side.

The optimality conditions for the decision to finance the liquidity needs of maturing projects ($\rho_t^\star$), the choice of liquidity (M_{t+1}), and investment in shares

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(s_t^j) yield the following equations:

$$U_{C,t} + \lambda_t = U_{C,t} \frac{p_{\mathrm{H}} y_t^j}{m_t(\rho_t^\star)}, \tag{16}$$

$$U_{C,t} = \beta E_t \left\{ U_{C,t+1} \left[\frac{p_{\mathrm{H}} y_{t+1}}{m_{t+1}(\rho_{t+1}^\star)} \right] \right\}, \tag{17}$$

$$U_{C,t} = \beta E_t \left\{ U_{C,t+1} \left[\frac{p_{\mathrm{H}} y_{t+1} F(\rho_{t+1}^\star)}{p_t^j} \right] \left[\frac{y_{t+1}^j}{y_{t+1}} - \frac{\bar{m}_{t+1}(\rho_{t+1}^\star)}{m_{t+1}(\rho_{t+1}^\star)} \right] \right\}, \tag{18}$$

where y_t is the period-t output from a typical project that was started in period $t-1$, λ_t is the Lagrange multiplier on the liquidity financing constraint (10), and

$$\bar{m}_{t+1}(\rho_{t+1}^\star) = \int_0^{\rho_{t+1}^\star} m_{t+1}(\rho_{t+1}) \frac{f(\rho)}{F(\rho_{t+1}^\star)} d\rho \tag{19}$$

denotes the average liquidity need, conditional on the need being financed.

To understand the intuition behind (16), it is useful to use (4) in (16) to obtain

$$\rho_t^* = \frac{1}{1 + \frac{\lambda_t}{U_{C,t}}} \frac{p_H s_{t-1}^j y_t^j}{w_t}. \tag{20}$$

This expression for financing the liquidity need is fairly intuitive. For example, when liquidity is in abundant supply, λ_t is zero and we have

$$\rho_t^* w_t = p_H s_{t-1}^j y_t^j, \tag{21}$$

where the left-hand side is the liquidity need of the marginal firm and the right-hand side is the expected output accruing to the investor, conditional on the liquidity need being financed. The liquidity need of a project will be financed up to this amount because the past investment decision is not relevant for liquidity financing. In addition, because the investor is diversified over a large number of identical projects, he is risk-neutral with respect to any single project. When liquidity is limited, λ_t is positive, and (16) says that the amount of liquidity supplied to firms is accordingly reduced—-a fact brought out more clearly by (20).

In equations (17) and (18), the left-hand side is the (current marginal utility) cost of the choice and the right-hand side is its (expected discounted future) marginal benefit. In equation (17), the term in parentheses is the gross one-period (marginal) return to liquidity because the numerator ($p_{\mathrm{H}} y_{t+1}$) is the (per-share marginal) output from financing the liquidity need and the denominator [$m_{t+1}(\rho_{t+1}^\star)$] is the cost. Hence, (17) equates the expected discounted (future) marginal benefit on the right-hand side to the marginal cost on the left-hand side.

Equation (18), after imposition of symmetry across projects, simplifies to

$$U_{C,t} = \beta E_t \left\{ U_{C,t+1} \left[\frac{p_H y_{t+1} F\left(\rho_{t+1}^*\right)}{p_t} \right] \left[1 - \frac{\bar{m}_{t+1}\left(\rho_{t+1}^*\right)}{m_{t+1}\left(\rho_{t+1}^*\right)} \right] \right\}. \tag{22}$$

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The term in the first parentheses is the gross return on shares in the absence of a liquidity shock in the second period. The term in the second parentheses captures the reduction in gross return caused by the need for second-period liquidity financing. This term is also intuitive. For example, consider the case where the average liquidity need, $\bar{m}_{t+1}\left(\rho^*_{t+1}\right)$, is zero. In that case, the gross return from shares is unaffected. As the average liquidity financing $[\bar{m}_{t+1}\left(\rho^*_{t+1}\right)]$ goes up, the return on investment in shares falls. Overall, (18) determines the price of shares of the project based on the household's preferences and the project's characteristics.

3.2. Solution to the Entrepreneur's Problem

Before we can solve the entrepreneur's problem, we must first consider a few details regarding the IC constraint and the distribution of both the aggregate and idiosyncratic shocks.

Recall that, as it is never optimal for the investor to allow the entrepreneur to shirk, the IC constraint must be satisfied for all possible future productivity levels.

LEMMA 2. *Given a particular period-t allocation, let $\theta_{L,t+1}$ denote the lowest possible productivity level that could be realized in period $t+1$. Then, if the IC constraint is satisfied for $\theta_{L,t+1}$, it will be satisfied for all realizations of θ_{t+1}.*

Proof. Inspection of equation (6) [after making use of (2)] indicates that the IC constraint for a particular realization of θ_{t+1} is satisfied as long as

$$\theta_{t+1} \geq \tilde{\theta} \equiv \frac{As^i_t}{(1-s^i_t)(k^i_{t+1})^{\alpha}(n^i_{1,t})^{1-\alpha}}. \tag{23}$$

Thus, if the IC constraint is satisfied for $\theta_{L,t+1} \geq \tilde{\theta}$, then (23) is satisfied for all possible realizations of θ_{t+1} and the result follows. ■

By virtue of Lemma 2, the family of IC constraints in (6) is reduced to a single IC constraint,

$$(1-s^i_t)y^i_{L,t+1} \geq As^i_t, \tag{24}$$

where

$$y^i_{L,t+1} \equiv \theta_{L,t+1}(k^i_{t+1})^{\alpha}(n^i_{1,t})^{1-\alpha}. \tag{25}$$

For this simplification to work, it is necessary that there be indeed a well-defined value of $\theta_{L,t+1}$. To this end, we assume that θ_{t+1} follows an AR(1) process in logs,

$$\log(\theta_{t+1}) = \phi\log(\theta_t) + \epsilon_{t+1}, \tag{26}$$

where $0 < \phi < 1$ and ϵ is drawn from a symmetrically truncated ($\pm$ 2.5 std. dev.) normal distribution with mean 0 and variance σ^2_{ϵ}. For this process, note that there is a well-defined minimum for θ_{t+1}, given the current value of θ_t. In particular, truncation at the lower end implies that

$$\theta_{L,t+1} = \theta^{\phi}_t \exp(\epsilon_{\mathrm{L}}), \tag{27}$$

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where ϵ_L is the lowest realization of the shock.[9]

The entrepreneur is aware that his current actions will effect his likelihood of receiving liquidity financing next period. Thus, how his choices of s_t^i, k_{t+1}^i, and $n_{1,t}^i$ impact $F(\rho_{t+1}^\star)$ is taken into account when the maximization is performed. Using equation (2) to remove y_t^i, the expression for $\rho_t^\star$ in (20) can be written from the perspective of the entrepreneur as

$$\rho_t^\star = \left(\frac{1}{1+\frac{\lambda_t}{U_{C,t}}}\right)\frac{p_H s_{t-1}^i \theta_t (k_t^i)^\alpha (n_{1,t-1}^i)^{1-\alpha}}{w_t}. \quad (28)$$

Updating this expression for $\rho_t^\star$ by one period and substituting it into equation (7) yields the following objective function for the entrepreneur:

$$\max_{s_t^i, k_{t+1}^i, n_{1,t}^i} E_t \left\{ \beta \frac{U_{C,t+1}}{U_{C,t}} (1-s_t^i) p_H \theta_{t+1} (k_{t+1}^i)^\alpha (n_{1,t}^i)^{1-\alpha} F\left(\frac{p_H s_t^i \theta_{t+1} (k_{t+1}^i)^\alpha (n_{1,t}^i)^{1-\alpha}}{\left(1+\frac{\lambda_{t+1}}{U_{C,t+1}}\right) w_{t+1}} \right) \right\}, \quad (29)$$

where the maximization is subject to the resource financing constraint (1) and the IC constraint (24).

Note that in order to solve the entrepreneur's problem, we must specify a functional form for $F(\cdot)$, the distribution of the second-period idiosyncratic liquidity shock. We assume that $F(\cdot)$ belongs to the family of truncated power-law distributions. In particular,

$$F(\rho) = \left(\frac{\rho}{\bar{\rho}}\right)^e, \quad (30)$$

where $\bar{\rho}$ is the upper limit of the support of the truncated distribution, zero being the lower limit. Parameter $e \in (0, 1]$ controls the shape of the distribution, with smaller values resulting in higher probabilities of smaller shocks. This generalizes the distribution used by Atolia et al. (2011), which is a special case of (30) with $e = 1$. This change allows the model to be calibrated to a specific value of $n_{1,ss}/n_{ss}$, which is fixed at 0.5 in their paper.

There are two possible solutions to the entrepreneur's problem: one where the IC constraint binds and one where the IC constraint is naturally satisfied (nonbinding).[10] In the binding IC constraint case, the entrepreneur chooses s_t^i,

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k^i_{t+1}, and $n^i_{1,t}$ to maximize (29) subject to (1) and (24). Solving this problem yields

$$s^i_t = \frac{y^i_{L,t+1}}{A + y^i_{L,t+1}}, \tag{31}$$

$$\mu^I_t = [(1+2e) - 2s^i_t(1+e)]\left[\frac{(1+e)p^i_t}{s^i_t y^i_{L,t+1}}\right], \tag{32}$$

$$\mu^R_t = \frac{s^i_t(1+e)^2 - e(1+e)}{s^i_t} + \mu^I_t\left[\frac{y^i_{L,t+1}}{p^i_t s^i_t}\right], \tag{33}$$

$$(1-\alpha)r_t k^i_{t+1} = \alpha w_t n^i_{1,t}, \tag{34}$$

where μ^R_t and μ^I_t are, respectively, the Lagrange multipliers on the resource financing constraint (1) and the IC constraint (24).

In the nonbinding case, the entrepreneur solves the same problem as before, but ignores the IC constraint. Solving this problem yields

$$s^i_t = \bar{s} \equiv \frac{1+2e}{2(1+e)}, \tag{35}$$

$$\mu^I_t = 0, \tag{36}$$

$$\mu^R_t = \bar{\mu}^R \equiv \frac{\bar{s}(1+e)^2 - e(1+e)}{\bar{s}}, \tag{37}$$

along with (34), which continues to hold in the nonbinding case.

Comparing the solutions for the two cases in (31)–(34) and (34)–(37) provides very useful insights into the mechanism through which moral hazard affects the macrodynamics in the model. To see this mechanism, note that when moral hazard is operating in the model and the IC constraint binds, $\mu^I_t > 0$. Equation (31) then implies $s^i_t < \bar{s}$. Thus, investors incentivize the entrepreneurs by leaving them with a greater stake in the project. However, this reduces the resources that can be committed by the investors, and hence the shadow price of resources (μ^R_t) goes up. In fact, starting with (33), some simple algebra using other equations shows that

$$\mu^R_t - \bar{\mu}^R = \left(\frac{\bar{s}}{s^i_t} - 1\right)\left[\frac{2(1+e)^2}{s^i_t} - \frac{e(1+e)}{\bar{s}}\right] > 0, \tag{38}$$

because $s^i_t < \bar{s}$, and as $e \in (0, 1]$, the terms in both the parentheses and square brackets in (38) are positive when s^i_t is below $\bar{s}$. Moreover, as moral hazard bites more severely, s^i_t falls and μ^R_t rises. In summary, in the model, financial frictions arising from moral hazard operate through the amount of equity that can be credibly committed to outside investors in the first period without jeopardizing incentives. Financial frictions reduce outside equity and the resultant financing, which, in turn, reduces the size of projects and the quantity of factors employed by them.

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The procedure for checking whether the IC constraint binds is as follows. We solve the model assuming that the IC constraint is nonbinding and find the value for $n^i_{1,t}$. Let $n^\star_{1,t}$ be the value of $n^i_{1,t}$ found from (24) assuming $s^i_t = \bar{s}$, which is given by

$$n^\star_{1,t} = \left[\frac{A\bar{s}}{(1-\bar{s})\theta_{L,t+1}(k^i_{t+1})^\alpha} \right]^{\frac{1}{1-\alpha}}. \quad \textbf{(39)}$$

We compare $n^i_{1,t}$ with the threshold value $n^\star_{1,t}$ derived from the IC constraint. If the value of $n^i_{1,t} > n^\star_{1,t}$, then the IC constraint is satisfied and the nonbinding solution is the correct solution. However, if the value for $n^i_{1,t}$ is less than $n^\star_{1,t}$, then the binding solution must be used. The two solutions coincide when $n^i_{1,t} = n^\star_{1,t}$.

3.3. Competitive Equilibrium

This section describes the competitive equilibrium for this economy.

DEFINITION. *Given the initial stock of capital, k_0, and its distribution, $k^i_0, \forall i \in [0, 1]$ over various projects, the amount of labor committed to initial projects, n_{-1}, and its distribution, $n^i_{-1}, \forall i \in [0, 1]$ over various projects, the initial stock of liquidity, M_0, the initial equity holdings, $s^j_{-1}, \forall j \in [0, 1]$, and the stochastic process of productivity (26), the competitive equilibrium for this economy is the set of sequences of prices $\{r_t\}^\infty_{t=0}$, $\{w_t\}^\infty_{t=0}$, and $\{p^j_t, \forall j \in [0, 1]\}^\infty_{t=0}$ and allocations $\{C_t\}^\infty_{t=0}$, $\{n_t\}^\infty_{t=0}$, $\{L_t\}^\infty_{t=0}$, $\{M_{t+1}\}^\infty_{t=0}$, $\{k_{t+1}\}^\infty_{t=0}$, $\{\rho^\star_t\}^\infty_{t=0}$, $\{s^j_t, \forall j \in [0, 1]\}^\infty_{t=0}$, $\{s^i_t, \forall i \in [0, 1]\}^\infty_{t=0}$, $\{k^i_{t+1}, \forall i \in [0, 1]\}^\infty_{t=0}$, $\{n^i_{1,t}, \forall i \in [0, 1]\}^\infty_{t=0}$, and $\{n^i_{2,t}, \forall i \in [0, 1]\}^\infty_{t=0}$ such that*

1. *Given prices $\{r_t\}^\infty_{t=0}$, $\{w_t\}^\infty_{t=0}$, and $\{p^j_t, \forall j \in [0, 1]\}^\infty_{t=0}$, the allocations $\{C_t\}^\infty_{t=0}$, $\{n_t\}^\infty_{t=0}$, $\{L_t\}^\infty_{t=0}$, $\{M_{t+1}\}^\infty_{t=0}$, $\{k_{t+1}\}^\infty_{t=0}$, $\{\rho^\star_t\}^\infty_{t=0}$, and $\{s^j_t, \forall j \in [0, 1]\}^\infty_{t=0}$ solve the representative household's problem (13) subject to (10)–(12).*
2. *Given prices $\{r_t\}^\infty_{t=0}$, $\{w_t\}^\infty_{t=0}$, and $\{p^j_t, \forall j \in [0, 1]\}^\infty_{t=0}$ and the household's stochastic discount factor, the allocations $\{s^i_t, \forall i \in [0, 1]\}^\infty_{t=0}$, $\{k^i_{t+1}, \forall i \in [0, 1]\}^\infty_{t=0}$, and $\{n^i_{1,t}, \forall i \in [0, 1]\}^\infty_{t=0}$ solve entrepreneur i's problem (29) subject to (1) and (24). In addition, for every t, if $\rho^i_t \leq \rho^\star_t$, in accordance with equation (3), the entrepreneur hires $n^i_{2,t}$ additional units of labor to complete the project started in period t-1.*
3. *For every t, markets for goods, labor, capital, and equities clear.*

Recall that all projects are ex ante identical. To simplify the market clearing conditions, we make use of this feature/symmetry of the environment. In particular, the market clearing conditions for capital and equity are given by

$$s_t \equiv s^i_t = s^j_t, \quad \textbf{(40)}$$

$$k_t = k^i_t. \quad \textbf{(41)}$$

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In addition, the symmetry across projects also implies that

$$p_t \equiv p_t^i, \tag{42}$$

$$n_{1,t} \equiv n^i. \tag{43}$$

Furthermore, the constraint on the provision of liquidity can now be written as[11]

$$\int_0^{\rho_t^\star} s_{t-1} m_t(\rho_t) f(\rho_t) d\rho \leq M_t, \tag{44}$$

which on application of the assumption of the functional form of $F(\rho)$ in (30) and evaluation of the integral reduces to

$$w_t \left(\frac{e}{1+e}\right) \frac{\rho_t^{\star 1+e}}{\bar{\rho}^e} \leq M_t. \tag{45}$$

The labor market clearing condition is given by

$$n_{1,t} + \bar{n}_{2,t}(\rho_t^\star) F(\rho_t^\star) = n_t, \tag{46}$$

where

$$\bar{n}_{2,t}(\rho_t^\star) = \int_0^{\rho_t^\star} \rho \frac{f(\rho_t)}{F(\rho_t^\star)} d\rho \tag{47}$$

denotes the average additional labor requirement, conditional on receiving liquidity financing. Under the distributional assumptions for ρ, the labor market clearing condition reduces to

$$n_{1,t} + \left(\frac{e}{1+e}\right) \frac{\rho_t^{\star 1+e}}{\bar{\rho}^e} = n_t. \tag{48}$$

All goods produced at equilibrium are from projects that have liquidity needs less than or equal to $\rho_t^\star$. Thus for all projects/goods for which $\rho_t^i \leq \rho_t^\star$, we have

$$y_t^i = y_t = \theta_t (k_t^i)^\alpha (n_{1,t-1}^i)^{1-\alpha}, \tag{49}$$

and the goods-market-clearing condition is

$$C_t + M_{t+1} + k_{t+1} = Y_t + (1-\delta) k_t + M_t, \tag{50}$$

where

$$Y_t = p_{\mathrm{H}} y_t F(\rho_t^\star) \tag{51}$$

denotes the aggregate output of the economy at time t.

The model can be summarized by the following equations, (1), (12), (14)–(18), (25)–(27), (31)–(34), (45), (48)–(51). These 19 equations contain the 19 distinct variables s, p, Y, y, y_{L}, θ, θ_{L}, n_1, $\rho^\star$, w, L, n, C, M, r, k, λ, μ^I, and μ^R. When the IC constraint is slack, we set $\mu_t^I = 0$ and drop (31) from the system [or equivalently we replace (31) with (35)]. If the liquidity financing constraint is slack, we set $\lambda_t = 0$ and drop (45) from the system.

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4. CALIBRATION

In this section, we provide an overview of the data targets used to bring the model in line with features of the aggregate U.S. economy.

4.1. Preference/Production Parameters and Liquidity Shocks

The model is calibrated to a quarterly frequency with the discount rate, β, set to 0.99, implying an annual interest rate of approximately 4%. The rate of capital depreciation, δ, is set to 0.02, resulting in 8% annual depreciation, and we follow convention by setting the steady-state level of hours worked equal to its long-run average in the data, 0.36. This restriction on hours allows us to back out the utility parameter on leisure, η. Last, as is standard, we target the capital share of output $\frac{1}{3}$, which gives $\alpha = 0.36$. In our model, unlike standard growth models, α differs from capital's share of output. The reason is that labor hours used in production are not the only source of income for labor. Workers also receive labor income as part of the cost overrun, and to be consistent with proprietor's income, some of the profits accruing to entrepreneurs must be attributed to labor.

The persistence in the productivity shock process, ϕ, and the standard deviation of its innovations, σ_ϵ, are set to 0.933 and 0.0085, respectively. These values were chosen so that the log of the Solow residual derived from our model, using the standard Cobb–Douglas production function ($y = \hat{\theta} k^\alpha n^{1-\alpha}$, $\alpha = 1/3$), has a first-order autocorrelation of 0.95 and a percent volatility of 2.45.[12]

The two remaining parameters, $\bar{\rho}$ and e, govern the distribution of the liquidity shock process. To determine the values of these parameters, we target the fraction of firms that have their liquidity shocks financed in the steady state and the fraction of total hours worked that are devoted to (first-period) production. For the first target, we follow Atolia et al. (2011) and set $F(\rho^\star_{ss}) = 0.85$, so that 85 percent of projects receive their second-period liquidity funding in the steady state. For the second target, we depart from Atolia et al. (2011). We set e to target $n_{1,ss}/n_{ss} = 0.9$, so that 90 percent of steady-state hours worked come from production workers and only 10 percent of steady-state hours arise due to from cost overruns.[13] Although the specific values for these data targets are plausible, they are not based on any specific facts. However, we conducted a sensitivity analysis on these values and found that our model's second and third moments are not significantly affected by changes in these targets (results available from the authors upon request). A full description of the model's parameters can be found in Table 1.

4.2. Model Volatility and Severity of the Agency Problem

In order to provide a measure of the quantitative impact of moral hazard on the economy, we calibrate the severity of the agency problem in the model, using data on the spread between the rate paid on three-month nonfinancial commercial paper and three-month U.S. Treasury bills.[14] Both panels of Figure 1 present this spread, along with the narrowest symmetric band around the series' mean that

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TABLE 1. Parameters and steady state values

		Parameters		
$\alpha = 0.36$	$\beta = 0.99$	$\eta = 0.71$	$\delta = 0.02$	$\phi = 0.933$
$\sigma_\epsilon = 0.0085$	$p_{\mathrm{H}} = 0.9$	$p_{\mathrm{L}} = 0.4$	$\bar{\rho} = 6.1817$	$e = 0.0714$
		Steady state		
$s = 0.5333$	$p = 0.5294$	$n = 0.36$	$n_1 = 0.3240$	$\frac{n_1}{n} = 0.9$
$w = 0.5602$	$r = 0.0298$	$k = 3.3835$	$\rho^\star = 0.6353$	$F(\rho^\star) = 0.85$
$\frac{c}{Y} = 0.8819$	$\frac{k}{Y} = 5.9060$	$\frac{M}{Y} = 0.0352$	$Y = 0.5729$	$y = 0.7489$

includes its minimum. This spread has asymmetric fluctuations with large positive spikes outside of the symmetric band during economic downturns. We interpret these extreme values as being indicative of moral hazard that exposes investors to disproportionately higher risk during downturns. The divergence in the valuation of funds by the "inside" entrepreneurs and the "outside" investors represented by the rate spread in the data is captured in the model by $\mu_t^R - \bar{\mu}^R$, the spread between the entrepreneurs' shadow prices of funds with and without financial friction.

Given this interpretation, we simultaneously set the entrepreneur's agency rent, $A = \frac{J}{\Delta p}$, and the volatility of the shock process, ϵ, so that the shadow price spread, $\mu_t^R - \bar{\mu}^R$, mimics the asymmetry of the rate spread data mentioned in the preceding and the volatility of the model-implied Solow residuals matches that found in the literature. In particular, we set $A = \frac{J}{\Delta p}$ so that $\mu_t^R - \bar{\mu}^R$ spikes outside its symmetric band about 14 percent of the time, matching the frequency found in the data. Figure 3 shows how similar the spikes in the shadow price spread are to those found in the rate spread data. Having calibrated $\frac{J}{\Delta p}$ to match the asymmetry of the rate spread data, we simply set $p_{\mathrm{H}} = 0.9$, so that the entrepreneur's project succeeds with a high probability when the entrepreneur is diligent.[15]

In models with occasionally binding constraints, the degree of asymmetry generated depends on the frequency with which the constraints bind. Hansen and Prescott (2005) calibrate the frequency of binding of their capacity constraint to target the level of deepness asymmetry of U.S. output. They show that their model is capable of generating time series for hours worked and investment that are more asymmetric than output. Our calibration strategy is more general. We do not target the asymmetry of output. Instead, we target features of financial data and then evaluate our model based on its ability both to replicate the *level* of asymmetry observed in output, consumption, investment, and hours worked and to match the *relative ordering* of the asymmetries present in these variables.

5. RESULTS

We are now ready to investigate the effect of the financial frictions on the performance of our benchmark model. As our focus is on assessing the role of

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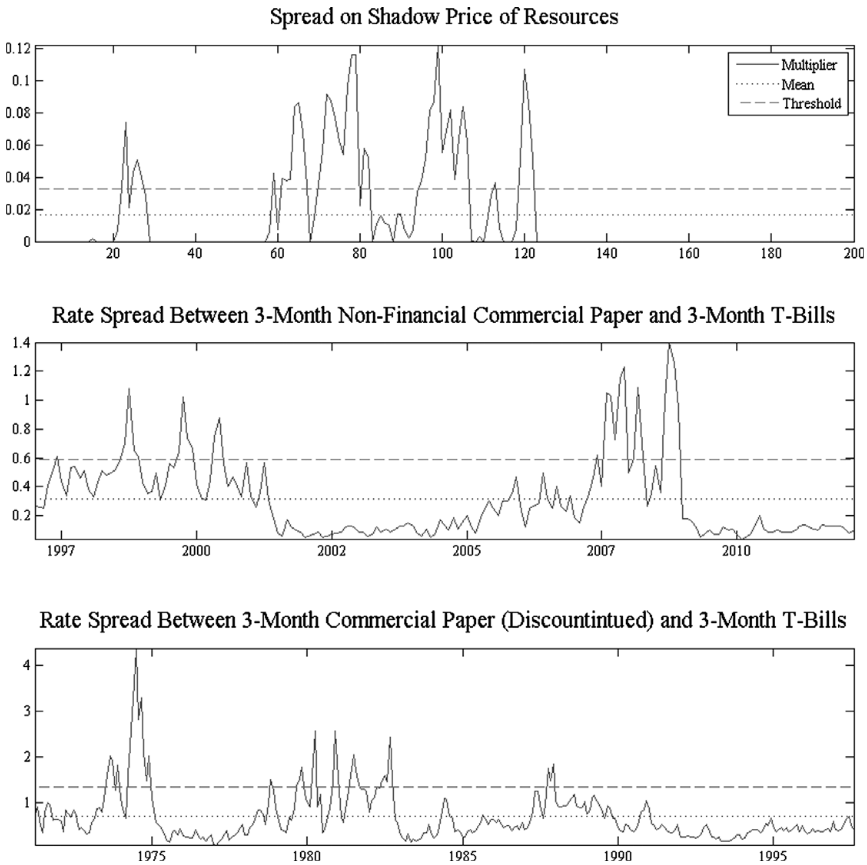

FIGURE 3. Shadow price of funds and rate spread data.

financial frictions, we will, when necessary, compare the results of our benchmark model with a ‘no-frictions" version of our model where moral hazard has been shut down by setting the entrepreneur’s agency rent, $\frac{J}{\Delta p}$, to a very low value (close to zero). In both cases, the deterministic extended path (DEP) method is used to compute an initial solution. This solution is then used to estimate initial values for the parameters of the model’s conditional expectation functions so that the generalized stochastic simulation algorithm (GSSA) can be used to improve the accuracy of the approximation.[16] One of the primary benefits of GSSA relative to other stochastic simulation methods, such as the parameterized expectations approach (PEA), is that one can achieve a much higher degree of accuracy with a shorter stochastic simulation. We use a 100,000-period simulation path to approximate a solution to our model, and all second and third moments are derived using this 100,000-period path.

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TABLE 2. Second moments

	Data[a]	Benchmark	No-frictions
	Volatility (%)		
σ_Y	1.53	1.57	1.41
σ_c/σ_Y	0.81	0.42	0.44
σ_{inv}/σ_Y	4.44	6.26	5.54
σ_n/σ_Y	1.07	0.53	0.37
	Correlation with output		
Y	1.00	1.00	1.00
c	0.87	0.92	0.95
inv	0.91	0.95	0.97
n	0.87	0.87	0.95
	Autocorrelation		
Y	0.87	0.82	0.82
c	0.88	0.80	0.79
inv	0.84	0.78	0.83
n	0.93	0.73	0.74

[a] Data are taken from FRED and range from 1964Q1 to 2014Q2. Our measure of output (Y) comes from the GNPC96 series for real gross national product, whereas our measures of consumption (c) and investment (inv) are taken from the PCECC96 series for real personal consumption expenditures and the GPDIC1 series for real gross private domestic investment, respectively. Total hours (n) are computed as the product of the PAYEMS series of total nonfarm employment and the AWHNONAG series, which measures average hours worked per week.

5.1. Basic Characteristics of Business Cycle Fluctuations

Our benchmark model with financial frictions provides a reasonable match to the data in terms of percent volatility and correlation with output. A brief summary of these results is presented in Table 2.[17] The model is seen to successfully match the relative ordering of the volatility of output, consumption, and investment found in the data, indicating that it is consistent with consumption-smoothing behavior—a feature not captured by Atolia et al. (2011). Also, our benchmark model successfully matches the strong procyclicality of hours worked found in the data, which is in sharp contrast to Atolia et al. (2011), where hours worked appear countercyclical. In addition, the presence of the binding IC constraint in the benchmark version of the model is shown to add volatility not present in the no-frictions variant, bringing the model closer to the data in terms of output volatility and the volatility of hours worked relative to output.

5.2. Financial Frictions and the Severity of Downturns

In this subsection, we establish—in steps—that financial frictions can lead to qualitatively significant business cycle asymmetry by amplifying adverse productivity shocks.

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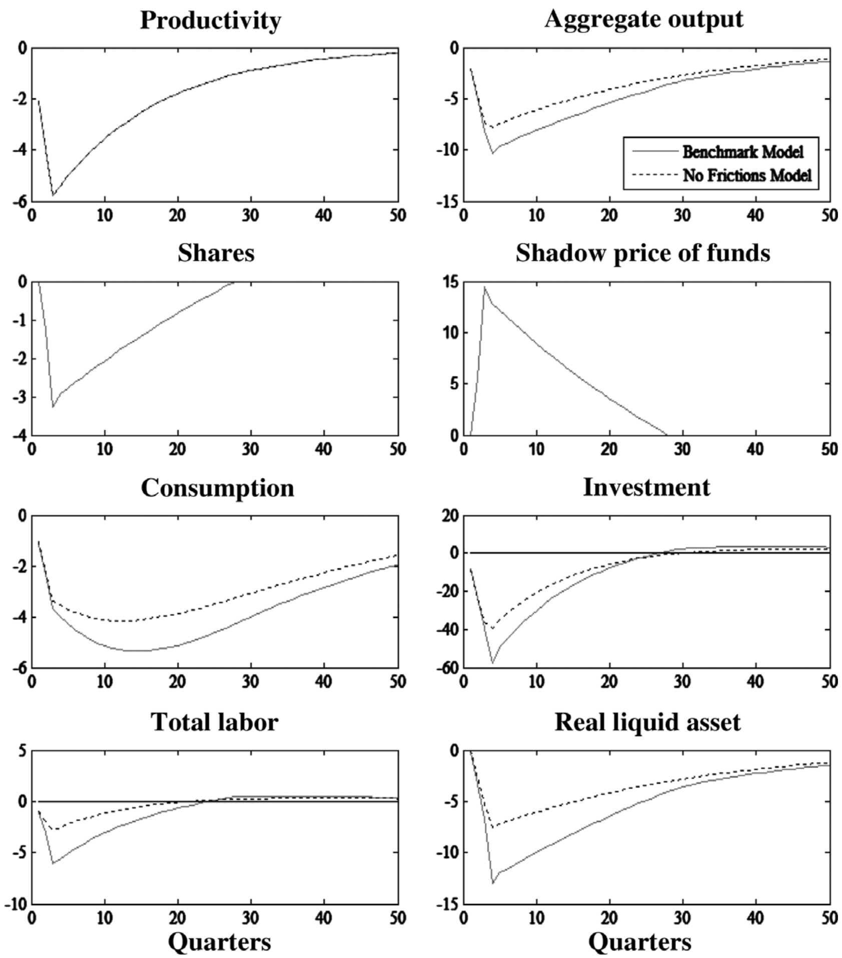

FIGURE 4. Impulse response functions.

We begin by presenting the impulse response functions of the key variables. Specifically, the innovations in the economy's productivity shock process are set to ϵ_L for the first three periods, and to zero (neutral shock) thereafter. This three-step shock process is chosen given that starting from the steady state, the first two shocks are needed to bring the IC constraint just past the point of binding and the third shock is used to cause the IC constraint to bind more severely. Figure 4 presents plots of these impulse response functions for several key variables of the benchmark model, where the y-axis measures the percent deviation from the steady state. To highlight the role played by the financial frictions, the figures also show (in dashed lines) the response of the no-frictions variant, where the effect of moral hazard has been shut down by setting the entrepreneur's agency rent, $\frac{J}{\Delta p}$, to a very low value. As expected, the benchmark model's impulse responses fall farther from their steady-state level, indicating an exacerbation in the intensity of downturns.

These impulse response functions also allow us to measure the quantitative significance of our financial frictions' amplification mechanism. Following Gertler and Kiyotaki (2011), we compare our benchmark model with financial frictions to the no-frictions variant using differences in the percent deviation from steady state at the trough, as well as differences in accumulated losses during a downturn (crisis), to gauge the magnitude of amplification. We find that when financial frictions are present, the trough in the responses of output, investment and hours worked falls from −7.78%, −39.34%, and −2.77% to −10.34%, −57.94%, and −6.10%, respectively. Furthermore, the presence of financial frictions amplifies the cumulative losses of output, investment, and hours worked by 25.59%, 28.11%, and 375.98%, respectively. These results indicate that our model's financial frictions are capable of significantly amplifying adverse productivity shocks. This result differs from Cordoba and Ripoll (2004), who find that financial frictions arising from collateral constraints [in the spirit of Kiyotaki and Moore (1997)] lead to very little amplification of adverse shocks for standard preferences and technology and typical parameter values.

Although the preceding results highlight the fact that downturns are exacerbated by the financial frictions, they are silent about the effect of the frictions during upturns. Figure 5 presents plots of the variables' simulated time paths in levels for the first 200 periods for both the benchmark and no-frictions models. During times of neutral or high productivity, the variable time paths of the two models lie on top of each other, but during periods of sufficiently low aggregate productivity, they diverge, with the time paths of the benchmark model with financial frictions falling below their no-frictions counterparts. Together, the plots of the models' impulse response functions and time paths clearly indicate that financial frictions arising from moral hazard exacerbate the intensity of downturns, but leave upturns unaffected.

The fact that financial frictions exacerbate downturns implies that they must also amplify the volatility of the business cycle. However, because our calibration strategy indicates that financial frictions are only active occasionally, their effect on mean volatility over the business cycle could be relatively small. Table 2 presents results suggesting that this is not the case; i.e., inclusion of financial frictions significantly amplifies volatility. For example, when measured as percent standard deviation, the volatility of output rises from 1.41% for the no-frictions model to 1.57% for the benchmark model with financial frictions, an increase of approximately 11.6%. The effect for labor is much larger, at about 59.3%. These results indicate that even though financial frictions only impact the economy occasionally, they still contribute significantly to the volatility observed over the business cycle.

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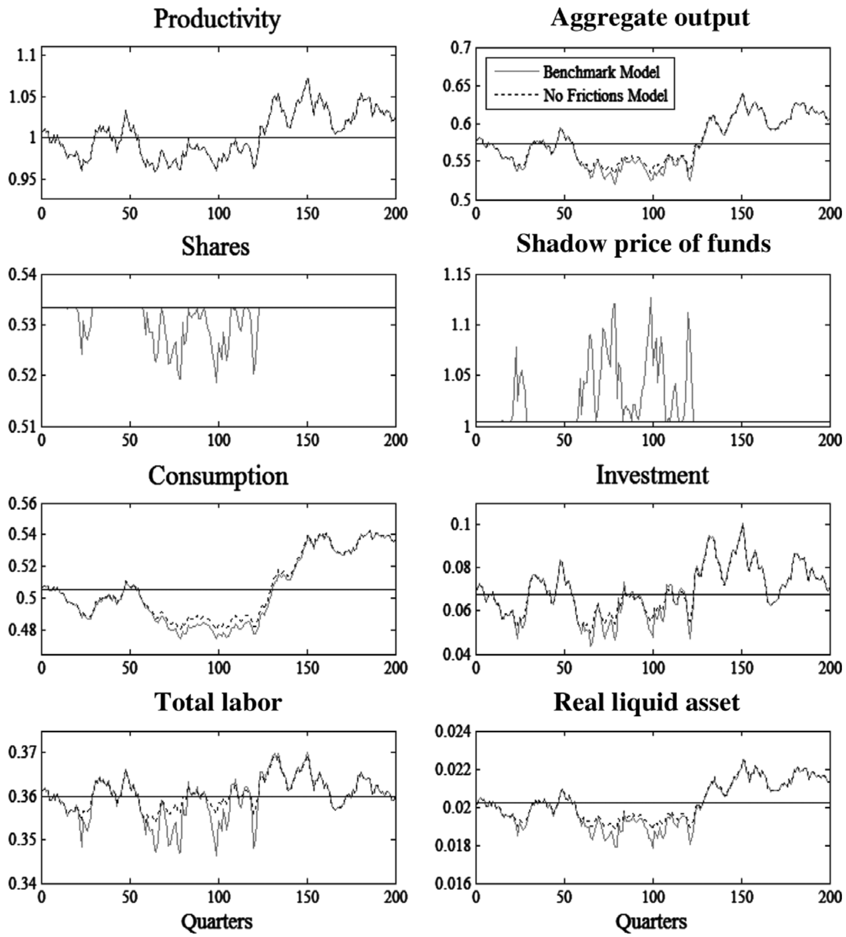

FIGURE 5. Simulated time paths.

5.3. Asymmetry of Business Cycle Fluctuations

Although the results of the preceding subsection demonstrate that financial frictions exacerbate downturns relative to the no-frictions counterpart, they do not establish that the resulting asymmetry shows up in our benchmark model. It is conceivable, albeit unlikely, that the no-frictions model is, in fact, asymmetric, with disproportionately large upturns relative to downturns. In this case, financial frictions that exacerbate downturns will actually work to remove or mitigate asymmetry, rather than induce it. To conclusively make the case that financial frictions generate asymmetric fluctuations, we subject both the benchmark and no-frictions models to a pair of equal but opposite shocks. The *difference* in

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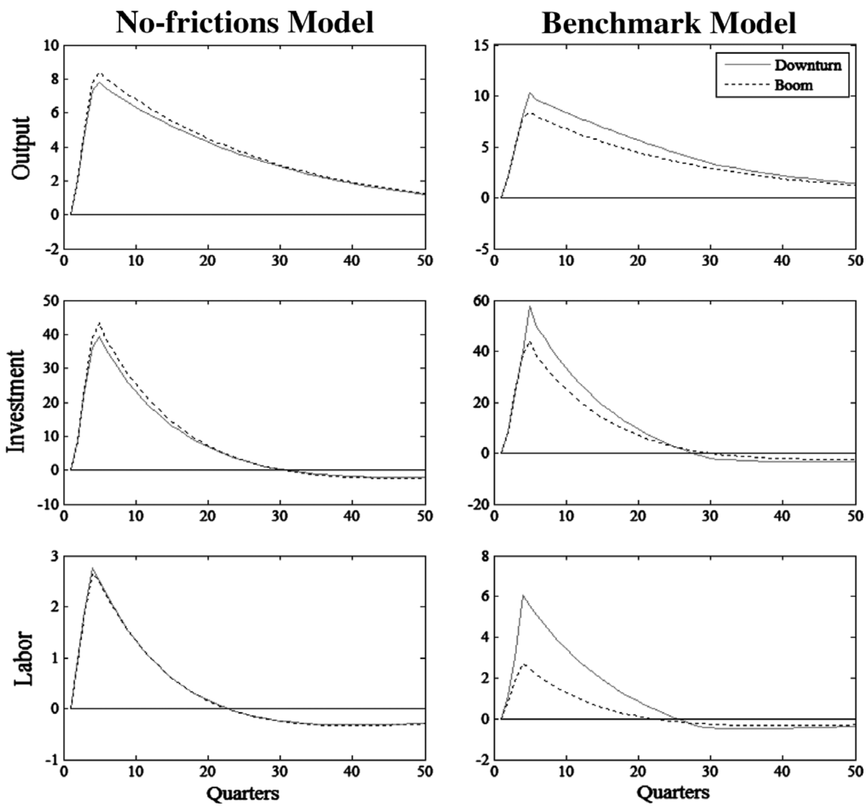

FIGURE 6. Asymmetry plots.

response of each model to this pair of shocks provides an assessment of the level of asymmetry present in the model.

To be precise, both specifications are subjected to a short downturn and a short expansion composed of shocks of magnitude ϵ_L in the first three periods. The profiles for aggregate output, investment, and aggregate labor found from the downturn and expansion are converted to percent deviations from steady state. The downturn profiles are scaled by -1 so that they can be plotted on top of the expansions (see Figure 6). The first column of plots in Figure 6 confirms that little to no asymmetry is present in the no-frictions model, whereas the second column confirms its presence in the benchmark model with financial frictions. More specifically, the benchmark model's paths clearly show that downturns are more severe than expansions. Taken together, these results demonstrate that the presence of moral hazard, and the resulting financial frictions, lead to asymmetric business cycles by exacerbating economic downturns.

Having established that financial frictions are the source of asymmetry in the model, we now turn to quantifying the degree of this asymmetry. Table 3 presents

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TABLE 3. Third moments

	Skewness			Deepness[a]		
	Data[a]	Benchmark	No-frictions	Data[b]	Benchmark	No-frictions
Y	−0.36	−0.22	−0.02	0.98	0.95	0.99
c	−0.16	−0.09	−0.02	1.02	0.98	0.99
inv	−0.91	−0.86	−0.24	0.80	0.88	0.94
n	−0.34	−0.76	−0.04	0.89	0.88	0.99

[a] Deepness(X) = Average % deviation above trend/Average % deviation below trend.
[b] Data are taken from FRED and range from 1964Q1 to 2014Q2. Our measure of output (Y) comes from the GNPC96 series for real gross national product, whereas our measures of consumption (c) and investment (inv) are taken from the PCECC96 series for real personal consumption expenditures and the GPDIC1 series for real gross private domestic investment, respectively. Total hours (n) are computed as the product of the PAYEMS series of total nonfarm employment and the AWHNONAG series, which measures average hours worked per week.

both the skewness and the deepness of the model's key variables. Three main results stand out from Table 3. First, practically no asymmetry is generated by our model when financial frictions are not operational. This is evidenced by the skewness values near zero and the deepness values near one found in the third and fourth columns of Table 3. Second, when financial frictions are operational, our model generates quantitatively significant levels of asymmetry, as in the data. Specifically, our model predicts the skewness of output, consumption, investment, and hours worked to be −0.22, −0.09, −0.86, and −0.76, respectively, compared to the values of −0.36, −0.16, −0.91, and −0.34 found in the data. Similar conclusions emerge from looking at the deepness statistics in Table 3. Third, our benchmark model also captures the relative ordering of asymmetry statistics across our key variables, with consumption displaying less asymmetry than output and investment and hours worked displaying more. This is clear from the deepness of output, consumption, investment, and hours worked of 0.95, 0.98, 0.88, and 0.88, respectively, generated by our model. Moreover, the model also reproduces greater asymmetry of investment relative to hours worked as measured by skewness (−0.86 vs. −0.76) as in the data (−0.91 vs. −0.34). Therefore, our benchmark model with financial friction can *simultaneously* capture the standard business cycle facts mentioned earlier and replicate *both* the level and relative ordering of asymmetry statistics found in the U.S. data.

6. CONCLUSION

This paper addresses two important related questions regarding the ability of financial frictions to generate quantitatively significant asymmetry and amplification of business cycle fluctuations. We introduce financial frictions into our model through entrepreneur-run firms that face both moral hazard and idiosyncratic cost overruns. The basic structure of our model follows Atolia et al. (2011), but overcomes many salient shortcomings of previous work. Most notably, we provide a

https://doi.org/10.1017/S1365100516000171 Published online by Cambridge University Press

strategy for connecting the level of moral hazard in the model with characteristics found in U.S. data. With the severity of the agency problem calibrated to a realistic level, we are able to address the question regarding the quantitative impact of financial frictions in a model that is able to replicate stylized business cycle facts.

Using our calibrated model, we examine the role played by financial frictions in generating asymmetries and exacerbating the fluctuations found in the business cycle. For our benchmark calibrated model, not only do we find quantitatively significant levels of asymmetry in key variables, but also this asymmetry replicates stylized facts found in the data. Specifically, consumption is found to be less asymmetric than output, whereas investment and hours worked are found to be more asymmetric than output. Furthermore, when measured in terms of skewness, we find that our model is also consistent with the empirical observation that investment is more asymmetric than hours worked. Although our financial frictions are only operational occasionally, they are found to amplify business cycle volatility significantly. For example, the presence of financial frictions was shown to increase the volatility of output and hours worked by about 11.6% and 59.3%, respectively. Taken together, these results indicate that the presence of financial frictions can lead to asymmetric business cycles by exacerbating downturns while leaving upturns unaffected.

The basic framework of this paper can be extended in many directions. One interesting extension would be the inclusion of labor market search, which would allow examination of the effect of fluctuations in credit access on the behavior of labor market variables. Another natural extension is the inclusion of long-lived firms and firm-level heterogeneity.

NOTES

1. Deepness is measured as the mean percentage deviation above trend relative to the mean percentage deviation below trend. A symmetric series has a deepness measure of approximately 1.00, whereas series with larger downturns have a deepness measure less than 1.00.

2. Kocherlakota (2000) demonstrates that financial frictions arising from endogenous borrowing constraints have the potential to significantly amplify and propagate large adverse income shocks. He reports that the degree of this amplification depends crucially on the parameters of the model. Given that a rigorous calibration was outside the scope of his paper, Kocherlakota (2000) leaves questions regarding the quantitative significance of financial frictions in generating business cycle asymmetry for future research.

3. This model is an elaboration of Atolia et al. (2011), which employs the modeling strategy of Holmstrom and Tirole (1997) and Tirole (2006) to capture the importance of liquidity constraints in the presence of moral hazard.

4. In a related paper, Dagher (2014) generates sudden stops through the combination of trend shocks and endogenous borrowing constraints.

5. The real liquid asset can be viewed as an investment in a storage technology, as in Kiyotaki and Moore (2005). The relevant characteristic of the storage technology for our purposes is that its output is available for use at the *beginning* of the next period to meet the needs of the production technology for additional resources. Kiyotaki and Moore (2012) (in a separate paper) and Cui and Radde (2013) focus on aspects of the exogenous liquidity shocks to financial assets as determined by their resalability in order to examine issues related to monetary policy and the cyclical holdings of liquid assets.

https://doi.org/10.1017/S1365100516000171 Published online by Cambridge University Press

6. After the total shares for a project are normalized to 1, s_t^i denotes the fraction of the project sold to outside investors.

7. Their proofs are fairly intuitive, and hence have been skipped.

8. Detailed derivations of the first-order conditions are available from the authors upon request.

9. The truncation at the upper end is imposed to maintain the symmetry of the shock process, as we are specifically interested in asymmetry generated endogenously by the financial frictions.

10. Detailed derivations of both the binding and nonbinding solutions are available from the authors upon request.

11. This equilibrium condition is written as an inequality constraint because it may be optimal for households to withhold liquidity in severely depressed times, so that they do not exhaust their current stock of M every period. However, we track the Lagrange multipliers on this constraint and they remain positive over our entire simulation.

12. It is common in the RBC literature to set $\phi = 0.95$ and $\sigma_\epsilon = 0.008$ when calibrating a quarterly RBC model with Cobb–Douglas technology. The outcome of this process is a TFP series whose log has an autocorrelation of 0.95 and a percent volatility of approximately 2.45. We thank an anonymous referee for suggesting this strategy for calibrating ϕ and σ_ϵ, given our nonstandard production function.

13. They assume a uniform distribution for liquidity shocks, which corresponds to $e = 1$ in our case. As a result, about half of hours worked in the steady state are due to cost overruns in their model, which are very high.

14. This is in sharp contrast to Atolia et al. (2011), who provide no data target for the severity of the agency problem and simply choose a level that allows their incentive constraint to bind occasionally.

15. As p_{H} only enters the model as a scale term, its level will not influence the volatility or asymmetry generated by the model. However, together the levels of p_{H} and p_{L} will influence the severity of the agency problem, which will influence volatility and asymmetry. We deal with this issue by embedding $\Delta p = p_{\mathrm{H}} - p_{\mathrm{L}}$ into the agency rent, A, and calibrating this value to rate spreads as described earlier.

16. See Heer and Maussner (2008, 2009) for an overview of DEP and how it compares, in terms of both accuracy and computation time, to other alternative methods such as log-linearization, value function iteration, and the parameterized expectations approach (PEA). For a description of GSSA, see Judd et al. (2011) and Maliar and Maliar (2014). Also, a brief Computational Appendix to this paper is available from the authors upon request.

17. All summary statistics are computed after HP-filtering the model's results.

REFERENCES

Acemoglu, Daron and Andrew Scott (1997) Asymmetric business cycles: Theory and time-series evidence. *Journal of Monetary Economics* 40(3), 501–533.

Atolia, Manoj, Tor Einarsson, and Milton Marquis (2011) Understanding liquidity shortages during severe economic downturns. *Journal of Economic Dynamics and Control* 35(3), 330–343.

Caballero, Richard J. and Mohamad Hammour (1996) On the timing and efficiency of creative destruction. *Quarterly Journal of Economics* 111(3), 805–852.

Cordoba, Juan-Carlos and Marla Ripoll (2004) Credit cycles redux. *International Economic Review* 45(4), 1011–1046.

Cui, Wei and Soren Radde (2013) Search-Based Endogenous Illiquidity. Manuscript, University College London.

Dagher, Jihad C. (2014) Sudden stops, output drops, and credit collapses. *Macroeconomic Dynamics* 18(4), 812–837.

Gertler, Mark and Nobuhiro Kiyotaki (2011) Financial intermediation and credit policy in business cycle analysis. In Benjamin M. Friedman and Michael Woodford (eds.), *Handbook of Monetary Economics*, pp. 547–599. San Diego, CA: Elsevier.

Hamilton, James D. (1989) A new approach to the economic analysis of non-stationary time series and the business cycle. *Econometrica* 57(2), 357–384.

https://doi.org/10.1017/S1365100516000171 Published online by Cambridge University Press

Hansen, Gary D. and Edward C. Prescott (2005) Capacity constraints, asymmetries, and the business cycle. *Review of Economic Dynamics* 8(4), 850–865.

Heer, Burkhard and Alfred Maussner (2008) Computation of business cycle models: A comparision of numerical methods. *Macroeconomic Dynamics* 12(5), 641–663.

Heer, Burkhard and Alfred Maussner (2009) *Dynamic General Equilibrium Modeling: Computational Methods and Applications,* 2nd ed. Heidelberg, Germany: Springer Verlag.

Holmstrom, Bengt and Jean Tirole (1997) Financial intermediation, loanable funds, and the real sector. *Quarterly Journal of Economics* 112(3), 663–691.

Judd, Kenneth L., Lilia Maliar, and Serguei Maliar (2011) Numerically stable and accurate stochastic simulation approaches for solving dynamic economic models. *Quantitative Economics* 2(2), 173–210.

Kiyotaki, Nobuhiro and John Moore (1997) Credit cycles. *Journal of Political Economy* 105(2), 211–241.

Kiyotaki, Nobuhiro and John Moore (2005) Liquidity and asset prices. *International Economic Review* 46(2), 317–349.

Kiyotaki, Nobuhiro and John Moore (2012) Liquidity, Business Cycles, and Monetary Policy. NBER working paper 17934.

Knuppel, Malte (2014) Can capacity constraints explain asymmetries of the business cycle? *Macroeconomic Dynamics* 18(1), 65–92.

Kocherlakota, Narayana R. (2000) Creating business cycles through credit constraints. *Federal Reserve Bank of Minneapolis Quarterly Review* 24(3), 2–10.

Li, Shuyun M. and Scott Dressler (2011) Business cycle asymmetry via occasionally binding international borrowing constraints. *Journal of Macroeconomics* 33(1), 33–41.

Maliar, Lilia and Serguei Maliar (2014) Numerical methods for large-scale dynamic economic models. In Karl Schmedders and Kenneth L. Judd (eds.), *Handbook of Computational Economics*, pp. 325–477. Amsterdam: Elsevier.

McKay, Alisdair and Ricardo Reis (2008) The brevity and violence of contractions and expansions. *Journal of Monetary Economics* 55(4), 738–751.

Mendoza, Enrique G. (2010) Sudden stops, financial crises, and leverage. *American Economic Review* 100(5), 1941–1966.

Neftci, Salih N. (1984) Are economic time series asymmetric over the business cycle? *Journal of Political Economy* 92(2), 307–328.

Sichel, Daniel E. (1993) Business cycle asymmetry: A deeper look. *Economic Inquiry* 31(2), 224–236.

Tirole, Jean (2006) *The Theory of Corporate Finance.* Princeton, NJ: Princeton University Press.

Van Nieuwerburgh, Stijn and Laura Veldkamp (2006) Learning asymmetries in real business cycles. *Journal of Monetary Economics* 53(4), 753–772.

https://doi.org/10.1017/S1365100516000171 Published online by Cambridge University Press