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SIMULATION-BASED ECONOMETRIC METHODS

by Christian Gouriéroux and Alain Monfort

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1. INTRODUCTION

The accessibility of high-performance computing power has always influenced theoretical and applied econometrics. Gouriéroux and Monfort begin their recent offering, *Simulation-Based Econometric Methods*, with a stylized three-stage classification of the history of statistical econometrics. In the first stage, lasting through the 1960's, models and estimation methods were designed to produce closed-form expressions for the estimators. This spurred thorough investigation of the standard linear model, linear simultaneous equations with the associated instrumental variable techniques, and maximum likelihood estimation within the exponential family. During the 1970's and 1980's the development of powerful numerical optimization routines led to the exploration of procedures without closed-form solutions for the estimators. During this period the general theory of nonlinear statistical inference was developed, and nonlinear micro models such as limited dependent variable models and nonlinear time series models, e.g., ARCH, were explored. The associated estimation principles included maximum likelihood (beyond the exponential family), pseudo-maximum likelihood, nonlinear least squares, and generalized method of moments. Finally, the third stage considers problems without a tractable analytic criterion function. Such problems almost invariably arise from the need to evaluate high-dimensional integrals. The idea is to circumvent the associated numerical problems by a simulation-based approach. The main requirement is therefore that the model may be simulated given the parameters and the exogenous variables. The approach delivers simulated counterparts to standard estimation procedures and has inspired the development of entirely new procedures based on the principle of indirect inference.

Simulation-Based Econometric Methods provides a comprehensive review of this new generation of econometric tools within the classical domain. Although numerous surveys of simulation-based estimation have appeared lately, Gouriéroux and Monfort's treatment is unique in covering both cross-sectional and time-series material in depth. In addition, the emphasis on (purposely) misspecified models within the context of pseudo-maximum likelihood and indirect inference

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procedures provides an intriguing introductory account of these techniques. However, this is not primarily a textbook. The text is based on a set of lectures, which renders the exposition somewhat terse. Moreover, it is purposely narrow in coverage. There is little discussion of the economic issues that motivate the specifications requiring simulation-based estimation, and there is no attempt to provide an overview of the significant empirical results obtained through these techniques. Furthermore, there is no mention of the use of similar computational tools in the areas of Bayesian statistics and bootstrap-based inference procedures. Instead, the book is merely intended as a concise introduction to the main theoretical results, the requisite simulation procedures, and the main implementation issues for simulation-based classical inference techniques. It is written for an audience that has mastered the standard estimation procedures—these methods are reviewed, but the treatment is brief and serves largely to introduce notation before considering simulation-based extensions.

Simulation-Based Econometric Methods is well suited as a source of inspiration and reference for researchers and advanced students alike. It may also serve as required reading for specialized graduate courses but is more likely to be used as supplementary reading for broader courses in which simulation estimation is only briefly discussed. Moreover, Chapter 4 on indirect inference is unique and certainly worth a look for anybody with an interest in the topic. Further, there are good introductory accounts to various technical issues spread throughout the book, e.g., the Metropolis–Hastings algorithm and the infinitesimal generator, that will be of interest to many readers independently of the text's general orientation. Finally, the application chapters bring together a variety of alternative approaches and unifying perspectives that should appeal to both students and researchers who work in related fields. One caveat is that the book can be difficult to read piecemeal. The later chapters, including the applications, build on the earlier material. It may thus require an investment on the part of the reader to become familiar with the notation and organization of the book, even if only a single estimation method or application is of interest. On the other hand, this will be well worth the effort in most instances.

2. CHAPTER ANALYSIS

Gouriéroux and Monfort organize their material in three main parts: an introductory chapter, three chapters on various simulation-based estimation methods, and three chapters dedicated to applications. The emphasis of the latter is, however, squarely on the development of appropriate methodology, rather than on the economics of the applications. Hence, the exposition centers on econometric principles and procedures throughout.

Chapter 1 serves to establish notation, review classical estimation procedures, and motivate the use of simulation-based methods. The general properties of estimators obtained by maximizing a smooth, parametric criterion function are presented, generic first-order conditions are displayed, and derivative-

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based optimization algorithms are discussed. Illustrations cover maximum likelihood (ML), pseudo-maximum likelihood (PML), nonlinear least squares (NLLS), and generalized method of moments (GMM). Cases yielding intractable criterion functions as a result of the presence of high-dimensional integrals are provided, including dynamic models with latent endogenous variables, limited dependent variable models, models with unobserved aggregation or heterogeneity features, nonlinear dynamic models with unobserved factors, and dynamic programming models. These examples are merely outlined, with more thorough analysis being postponed for later. The chapter concludes by categorizing two distinct simulation procedures for dynamic models: path versus conditional simulations. The former generates the endogenous variables conditional on prior observed exogenous and simulated endogenous variables, whereas the latter generates the endogenous variables conditional on prior observed exogenous and observed endogenous variables. The latter approach utilizes more information and is thus generally preferable, but the requisite distribution for conditional simulation is often unknown, leaving path simulations as the only feasible strategy.

Chapter 2 begins with a discussion of the principles behind simulation-based estimation. The basic idea is to calibrate the parameter vector so that the properties of the simulated series resemble those of the observed data. But which features should serve as the calibration criterion? Gouriéroux and Monfort stress that the naive approach of matching simulated series to the observed data point by point (pathwise calibration) is misguided and will not provide a consistent estimator. Instead, standard ergodicity and stationarity conditions imply convergence of cross-sectional or time-series averages, suggesting calibration based on sample moments and corresponding simulated moments. This also renders the usual first-order (moment) conditions arising from classical procedures, or even economic theory, relevant as estimation criteria for simulation-based methods. Consequently, the chapter proceeds to the method of simulated moments (MSM). First, the properties of standard GMM procedures are reviewed. If the moments are not available in closed form, MSM estimators are obtained by replacing the analytic moments in the GMM procedure by simulated counterparts. If the underlying model defines a fully specified probabilistic structure, a natural simulator exploits the associated data-generating process for simulation, the so-called frequency simulator. Unfortunately, the resulting estimator is not always satisfactory—it may not be differentiable in the parameters (invalidating derivative-based optimization algorithms), or it may imply a large amount of variability over (simulated) outcomes—motivating the discussion of importance sampling, designed to endow the simulators with more desirable attributes. The sequel reviews the similarities between the MSM and the GMM estimators. Most importantly, the MSM estimator is consistent for a fixed number, S, of simulated series used in the approximation of the moment conditions. Moreover, the asymptotic covariance matrix of the MSM estimator exceeds that of the GMM estimator by a factor of $1/S$, when the optimal weighting matrix is employed. In effect, the simulation error vanishes for S large, as the approximation to the population moments improves. Moreover, the simulated paths are mutually independent, so convergence to the GMM case occurs at the rate of root-S.

Chapter 3 concentrates on variants of simulated maximum likelihood (SML). If the likelihood is intractable, there are two potential scenarios with different implications for the implementation of SML. The simpler case arises when an unbiased simulator for the conditional distribution given past observed exogenous and endogenous variables is available. In this case, conditional simulations are feasible, and SML proceeds by averaging the simulated likelihood contributions observation by observation. Formally, this produces an inconsistent estimator (for a given number, S, of simulated draws per observation) because an unbiased conditional density, by Jensen's inequality, does not imply an unbiased log-density. Nonetheless, if S diverges to infinity at an appropriate rate, consistency is retained, and the number of simulations required to obtain an approximately unbiased estimator is usually not prohibitive. Moreover, it is often possible to provide first-order corrections for the asymptotic bias. Hence, if the SML estimator is feasible, it is likely preferable to the typically inefficient, albeit consistent, MSM estimators. In dynamic models with latent endogenous variables, the conditional distribution given past observables is usually not tractable, and alternative SML strategies must be employed. The book considers two avenues: accelerated versions of importance sampling based on path simulations and simulated expectation maximization (EM) methods utilizing the Metropolis–Hastings algorithm. The implementation of these methods must be designed on a case-by-case basis but has provided impressive results in a number of important applications.

The chapter then moves on to PML methods. Following a review stressing the class of distributions that results in consistent and asymptotically normal PML estimators for the conditional mean parameters (PML1) or mean and variance parameters (PML2), the simulated PML is introduced. Again, the estimator is inconsistent for a fixed S, but preserves the asymptotic properties of PML, if S grows appropriately with sample size. Finally, bias correction methods are illustrated for a static model estimated by simulated NLLS. An appendix provides details on the Metropolis–Hastings algorithm.

Chapter 4 concerns indirect inference. This chapter highlights the versatility of simulation-based estimation: a structural (maintained) model may be consistently estimated by indirect means via an instrumental or auxiliary model that need have nothing to do with the structural model. In other words, estimation may be performed via a purposely misspecified auxiliary model. This is obviously useful when the structural model is intractable or computationally taxing. Even under a misspecified model, the auxiliary parameter vector will typically converge to a pseudo-true value. If the auxiliary model is estimated on a simulated series generated by the structural model also, compatibility between the actual and simulated data may be gauged by the closeness of the corresponding auxil-

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iary parameter estimates. Of course, each choice of the structural parameter vector generates a distinct simulated series. Indirect inference estimates the structural model by minimizing the distance between the auxiliary parameter obtained from the data and the simulated series over the structural parameter vector in a suitable quadratic form. Consequently, the structural parameter is calibrated by matching the properties of the simulated series to those of the actual data along the dimensions captured by the auxiliary model. Alternatively, if the auxiliary model is estimated by (quasi-)maximum likelihood, one may estimate the structural parameters directly by minimizing a quadratic form in the simulated auxiliary score evaluated at the estimated auxiliary parameter vector—an approach advocated by Gallant and Tauchen (1996) as a part of their efficient method of moments (EMM) procedure. The chapter next reviews the properties of indirect inference estimators. Subject to regularity, consistency and asymptotic normality applies, with the asymptotic covariance matrix resembling that of standard GMM or MSM. In fact, indirect inference contains both the EMM and MSM procedures as special cases.

The chapter concludes with some revealing illustrations of the flexibility of indirect inference. It has several potential uses beyond providing a consistent estimation procedure when a direct approach is infeasible. First, it may be utilized as a pure computational shortcut, estimating a simple auxiliary likelihood rather than dealing directly with a more complex structural likelihood and then obtaining consistent structural parameter estimates in the second simulation-based stage, often without significant loss of efficiency. Second, it may be beneficial to use the structural model as the auxiliary model also. One such application invokes an inconsistent estimator of the structural parameter vector, say, a Kalman filter in a nonlinear state space model, and then corrects the asymptotic bias in the second stage. Alternatively, if the first-stage estimator is consistent, the second-stage simulation-based estimator may serve to remove an asymptotic second-order bias. Third, the refinement of the indirect inference principle embodied in the EMM approach allows for powerful and constructive model diagnostics based on tests applied to the scores of a well-designed auxiliary model. The chapter concludes with a useful appendix outlining the derivation of the asymptotic properties for the indirect inference estimator.

Chapter 5 discusses applications to limited dependent variable models. This type of model provided the original impetus for the development of simulation methods, and there is a substantial literature surveying the area (for a recent example and extensive references, see, e.g., Stern, 1997). The chapter provides an in-depth review of discrete choice models, stressing the form of the likelihood and the associated likelihood equation (score vector). Frequently, both MSM and SML—or the method of simulated scores (MSS), if based on direct simulations of the score vector—are feasible, but care must be taken to ensure consistency of the SML or MSS estimator such as, e.g., combining two different simulators in the approximation of the scores. It is again noted that the frequency simulator tends to have undesirable properties, and a variety of alternatives is explored,

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including the so-called Stern and Geweke, Hajivassiliou, and Keane (GHK) estimators. Both assure smoothness of the criterion function, improve precision, and carry relatively low computational costs. The remainder of the chapter deals with indirect inference in qualitative models and the choice of simulators in specific limited dependent variable models. Implementation of the Stern and GHK simulators is illustrated, and techniques such as acceptance-rejectance and Gibbs sampling are discussed. Finally, the chapter provides a brief summary of some empirical studies. However, the perspective remains theoretical, with an emphasis on the implementation of simulation estimators in the particular settings explored.

Chapter 6 concerns applications to financial series. The first part addresses the issue of estimating continuous-time models from discretely observed data. If the model is specified in terms of a stochastic differential equation (SDE), the distribution of the discretely observed data is generally unknown, but realizations from the system are readily simulated with a high degree of precision. This renders indirect inference a natural candidate. The discrete Euler approximation to the SDE, often invoked in practical applications, may serve as an auxiliary model, with the asymptotic bias being corrected in the second indirect inference step. Similarly, if the SDE contains unobserved factors, a popular discrete-time approximation is given by the state space form, and this may be estimated via the Kalman filter. Although the procedure is inconsistent, the second indirect inference step provides the appropriate bias correction. A few Monte Carlo experiments for models allowing for direct ML estimation document satisfactory performance of the indirect inference procedure, whereas the popular (naive) approach of estimating directly from the discrete Euler approximation is entirely inadequate. Further illustrations show how indirect inference may utilize implied (Black–Scholes) volatilities as the basis for an auxiliary model for the purpose of estimating an underlying stochastic volatility (SV) model. Such examples underscore the great flexibility of indirect inference in designing economically relevant auxiliary models for estimation of structural models with latent variables.

The next set of applications considers the extraction of exact moment conditions from SDE's. The Hansen and Scheinkman (1995) procedure based on the infinitesimal generator is reviewed. Unfortunately, this procedure generally fails to deliver a full set of identifying conditions for models involving latent factors. MSM may provide a feasible alternative in this setting, but it is unlikely to match the efficiency of well-designed indirect inference or EMM procedures. Finally, it is noted that indirect inference provides a useful approach to a number of discrete-time estimation problems in the area. The featured application involves a Factor ARCH model using a discretized state space form as the auxiliary model and exploiting the Kitagawa filtering algorithm for first-step estimation. Finally, it is noted that direct implementation of SML is feasible for some discrete-time SV models via an accelerated Gaussian importance sampler. In general, this chapter appears richer in terms of empirically relevant economic illustrations than the other application-oriented chapters.

Chapter 7 concludes the book with a review of regime switching models. The section on endogenously switching regimes considers disequilibrium models. Static models with multiple micro markets or nonlinear demand and supply schedules generally imply intractable likelihoods, and PML or SPML methods are necessary. Some experiments indicate that properly designed SPML methods perform well. Dynamic disequilibrium models tend to render PML infeasible, but SPML is typically viable. Furthermore, case-specific SML techniques have been designed for some important applications.

The exogenous regime switching models are classified as Markovian or non-Markovian in the observables (conditional on the regime). For Markovian models, the Kitagawa algorithm may be tractable for a low number of (unobserved) regimes, leading to the approach popularized by Hamilton (1989). For non-Markovian linear models with latent endogenous variables, the system is conveniently cast in a switching state space form. Conditional on a history of the regime vector, the standard state space form applies, leading to the so-called partial Kalman filter. Combining the output of the filter for the expected mean and covariance matrix with simulation techniques allows for evaluation of the likelihood function. A number of interesting models arise as special cases: switching ARMA models, switching factor models, dynamic switching regressions, time deformation models, and models with endogenously missing data.

3. CONCLUDING REMARKS

Simulation-based econometric techniques are here to stay. It is safe to conjecture that the continuing improvements to low-cost computational power will render the distinction between estimation based on numerically tractable criterion functions and estimation based on simulation techniques arcane within a decade. Software will invoke simulation tools in the same routine fashion that we today extract or compute p-values for test statistics in standard probability distributions that are not available in exact closed form, e.g., the normal, F-, or t-distributions. Analytic solutions will simply serve as benchmarks for illustration in the manner that exact distributional results for, e.g., ordinary least squares (OLS) may be used today. This inevitable trend toward reliance on simulation tools is driven by the fact that they render numerous new models tractable; they enhance the number of inference techniques available in complex settings, thus allowing for new trade-offs between the stringency of the maintained assumptions and the associated statistical efficiency; they enable entirely new diagnostic tools to be applied in a variety of scenarios; and they offer the possibility of using economic criteria for formal statistical calibration or estimation. Specifically, applied work across all fields should receive a boost from the enhanced capability of handling more empirically realistic specifications of structural models. Currently, this style of work is severely limited because of computational problems arising from the incorporation of factors such as unobserved heterogeneity and aggregation. Likewise, more appropriate time-series specifications for macroeconomic and financial applications will be explored once the analysis of nonlinear dynamic latent variable specifications becomes practical. Moreover, as the computational obstacles disappear theorists will be further inspired to pursue the different inference principles and computational approaches to their ultimate and logical endpoint within increasingly general settings. The recent explosion of research in indirect inference principles, Markov chain Monte Carlo techniques, Bayesian analysis, and bootstrap methods is surely a mere precursor of things to come.

In spite of its limited size and scope, *Simulation-Based Econometric Methods* probably provides the most comprehensive introduction to the classical-based inference techniques within this area. As such, it is strongly recommended to all with an interest in simulation-based estimation. The strength is the unifying approach, detailing implementation in both cross-section and time-series applications. Moreover, the integration of PML and indirect inference methods in the review is unrivaled and sets the book apart from alternative offerings. Nonetheless, a reader interested in specific topics may also want to consult the specialized exposition in, e.g., Stern (1997) for cross-section applications or Tauchen (1997) for time-series applications.

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