A.A.S. **4**, II, 287-301 (2009)

CALENDAR YEAR EFFECTS, CLAIMS INFLATION AND THE CHAIN-LADDER TECHNIQUE

By D. Brydon and R. J. Verrall

ABSTRACT

This paper examines the chain-ladder technique using the recently developed theory for age-period-cohort models. The theory was set out by Kuang *et al.* (2008a), and we believe that it has some significant implications for claims reserving and the chain-ladder technique. This paper applies the age-period-cohort model using the over-dispersed Poisson framework, and examines a number of experiments in order to understand better how the chain-ladder technique deals with calendar year effects. The conclusions from these investigations are that the basic chain-ladder technique may have some fundamental difficulties in many circumstances. We would therefore recommend that it should be used with caution, and that the data are examined in detail before any projections are made. This has particular importance in the context of solvency calculations since the chain-ladder technique can impose some specific patterns into the projections.

KEYWORDS

Chain-Ladder Technique; Claims Reserving; Solvency

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1. Introduction

The chain-ladder technique is one of the most widely-used methods in claims reserving in general insurance. It has become so commonly accepted as a reserving method that it is very often the first, and even sometimes, the only method applied. The parameters of the chain-ladder technique are often used to describe the run-off pattern, and it is often stated that the development factors are a useful way to communicate about the development of reserves to non-specialists. When a method becomes as familiar as this, it has disadvantages as well as advantages. The disadvantages include that it may be applied with too little thought; that it may be applied to the wrong data; that it becomes so accepted that it is regarded as "correct"; and that too little thought is given to the underlying assumptions in its use. In this

https://doi.org/10.1017/S1748499500000749 Published online by Cambridge University Press

paper, we consider the implications of some recently published papers on the use of the chain-ladder technique in the context of claims inflation and calendar year effects. Specifically, the aim is to understand the way that the chain-ladder technique projects forward any past inflation in the data, by carrying out a number of simplified experiments and examining the properties of the chain-ladder projections. It is often stated that using data which has not been adjusted for past inflation will result in this being projected forward in a sensible way (which is assumed to be conservative). For example, Corollary 3.5 of Taylor (2000) states that "The unadjusted chain ladder will provide an estimate of outstanding loss liability about as reasonable as from the inflation adjusted chain ladder if future (total) inflation is expected to be about the same on average as in the past." Thus, we examine in this paper whether the statement that is often made about the chain-ladder technique, that it "projects a weighted average of past claims inflation into the future" is true. Also, if it is true in some way, we consider the way in which past claims inflation may be projected forward. Since the chain-ladder technique does not include any parameters that relate to the calendar years, it is not possible to use analytical methods in an investigation of this kind. This is the reason why the approach taken is to use a set of experiments designed to show how the chain-ladder technique deals with different situations regarding the calendar year effects.

The title of this paper contains the phrases "claims inflation" and "calendar year effects". Since inflation operates in calendar time, it is sometimes assumed that any effects that manifest themselves along the diagonals of a claims run-off triangle will be associated with claims inflation. This is not necessarily true, and we prefer to use the term "calendar year effects". For a discussion of reasons why this is the case, see, for example, Section 1.2.2 of Taylor (2000). For example, calendar year effects could result from changes in the tightness of underwriting, changes in the prudence of case reserve estimates or inflation in claim numbers. In other words, they are not due to inflation in average claim sizes alone.

Jones *et al.* (2006), which is the report of the General Insurance Reserving Issues Taskforce (GRIT) state very strongly that "the development of more sophisticated mathematical reserving techniques need not be a priority at this stage". The reason given for this view is that "the key requirement is currently to help actuaries to understand better the business which they are trying to reserve. Without this enhanced understanding more sophisticated methods are likely to add no value, and indeed could be misleading in their apparent spurious accuracy". We find this an unfortunate view, with which we cannot agree. Firstly, it is through mathematical techniques that it is possible to get a better understanding of even apparently simple methods, such as the chain-ladder technique. Secondly, if mathematical techniques are not developed, they will not be available when the actuarial profession finds that they are needed after all! We believe that if actuaries are to be confident in the use of unsophisticated methods, then it is necessary to use the more sophisticated mathematics and statistics that will enable the actuary to "understand how a well understood method reacts to the changes".

The actuarial literature contains many papers which consider stochastic models for claims reserving. In the context of the chain-ladder technique, Mack (1993) and Renshaw & Verrall (1998) present alternative approaches, and England & Verrall (2002) provides a useful overview. Also, the books by Wuthrich & Merz (2008) and Taylor (2000) provide good background reading material. Taylor (2000) considers the calendar year effects in more detail than Wuthrich & Merz (2008). In the context of the specific subject material in this paper, a significant paper is Barnett & Zehnwirth (2000) which also considers the calendar year effects in claims data. Barnett & Zehnwirth (2000) also makes the point that the blind application of the chain-ladder technique can be misleading. We believe that this can now be considered more carefully, using the advances made in the recent papers by Kuang *et al.* (2008a and 2008b). It is for this reason that this paper considers the chain-ladder technique and looks at the implicit assumptions it makes about the calendar year effects using the age-period-cohort models defined by Kuang *et al.* (2008a).

Another method for claims reserving which has some relevance to the discussion in this paper is the separation technique. This was first defined by Verbeek (1972), with a convenient calculation method suggested by Taylor (1977). The aim of the separation technique is to estimate the calendar year effects, but it assumes that the accident year effects are already known, or can be controlled out. This illustrates an underlying difficulty with the format of the data in a claims triangle, which is shared by data in the age-period-cohort model in other contexts. This is that it is not at all straightforward to estimate the underlying components of the data in all three directions at the same time. There are significant identification difficulties, which usually result in the use of models that just consider two out of the three directions. The separation technique considers only the development year and calendar year, and the chain-ladder technique includes only the accident (underwriting) year and development year. In both cases, it is assumed that the other direction is either already dealt with somehow or that it does not matter that it is not included in the model. For the chain-ladder technique, this means that the data have already been adjusted for inflation, or that past inflation in the data will, somehow, be projected forward in a desirable way. Barnett & Zehnwirth (2000) criticise the chain-ladder technique for this reason (amongst others): that it ignores potentially significant effects that manifest themselves in calendar time. The same criticism could also be made of the separation technique, which does not include accident year in the model.

Like Barnett & Zehnwirth (2000), we believe that the way in which the chain-ladder technique deals with past inflation, and projects it forward into the estimates of outstanding claims, are important topics and we believe that it may happen that the chain-ladder technique is used in inappropriate situations. Our conclusion is that the chain-ladder technique should not be regarded as being somehow "correct", or the industry-standard method. Instead, it should be applied in its conventional form only when it is clear that it provides the appropriate statistical basis for the data.

The paper is set out as follows. In Section 2 we define the basic notation and models, and in Section 3 we describe the methodology which is used to investigate the calendar year effects implicit in the chain-ladder technique. The main section of the paper is Section 4, which considers a number of experiments in order to illustrate the way the chain-ladder technique deals with any calendar year effects in the data. In Section 5 we present some conclusions and suggestions for the way the chain-ladder technique should, and should not, be used.

2. The Chain-Ladder Technique and Calendar Year Effects

Without loss of generality, we assume that the data consists of a triangle of aggregated claim amounts and we do not specify whether the data refer to paid or incurred claims. The methods can be applied to other data shapes without changing the conclusions of this paper: using a triangle simply makes the notation more convenient. Also, this paper is concerned with the intrinsic properties of the chain-ladder technique and not with any particular aspects of the data, and so the issue of paid or incurred data is irrelevant.

We denote cumulative claims for accident year i and development year j by D_{ij}. The triangle of data is then $\{D_{ij} : i = 1, 2, \ldots, n; j = 1, 2, \ldots, n-i+1\}$ and the usual forecasts of future cumulative claims are $\{\hat{D}_{ij} : i = 2, 3, \ldots, n;$ $j = n-i+2, n-i+3, \ldots, n\}$. Note that this assumes that there are no tail factors. Since the purpose of the paper is to consider the properties of the forecasts, with particular emphasis on the way that any calendar year effects are implicitly included by the chain-ladder technique, it is sufficient for our purposes not to include any tail factors.

The incremental claims are denoted by $\{C_{ij} : i = 1, 2, \ldots, n; j = 1, 2, \ldots,$ $n-i+1\}$, where $C_{i1} = D_{i1}$ and $C_{ij} = D_{ij} - D_{i,j-1}, j \geq 2$.

The standard chain-ladder development factors, f_j, are usually calculated using the following formula:

$$f_j = \frac{\sum_{i=1}^{n-j+1} D_{ij}}{\sum_{i=1}^{n-j+1} D_{i,j-1}}.$$

Also, the estimate of the cumulative claims in the lower triangle is obtained by successive multiplication by the development factors:

$$\hat{D}_{ij} = D_{i,n-i+1} \prod_{l=n-i+2}^{j} f_l, \quad \text{for} \quad j = n-i+2, n-i+3, \ldots, n.$$

Finally, the estimates of outstanding claims, without tail factors, are then found by subtracting the cumulative payments made so far from the estimated ultimate claims, $\hat{D}_{in}$.

Any reserving method which includes just row effects and column effects, and which does not include calendar year effects, will project forward in a way that implicitly includes the calendar year effects in the other directions somehow. It is not easy to assess how this is done, but Section 3 puts forward a suggestion as to how to approach this issue. We concentrate on the chain-ladder technique, but there is no reason why this approach should not be used for other methods.

The basis for the assessment of the treatment of the calendar year effects is the work of Kuang *et al.* (2008a and 2008b), who analyse a model with 3-way effects. We refer to this model as the age-period-cohort model. Kuang *et al.* (2008a,b) did not make any explicit distributional assumptions, but in this paper we follow the approach of Renshaw & Verrall (1998) and consider the over-dispersed Poisson distribution. In terms of the results in this paper, which look at just the point forecasts, the same results could be obtained using the Poisson distribution. However, we do not believe that the overall conclusions are affected by the choice of underlying model. The age-period-cohort over-dispersed Poisson model can be expressed in additive or multiplicative form. For the additive model, it is assumed that the incremental claims, C_{ij}, are independent over-dispersed Poisson random variables with $E[C_{ij}] = \exp(\mu + \alpha_i + \beta_j + \gamma_{i+j-1})$, where $\alpha_1 = \beta_1 = \gamma_1 = \gamma_2 = 0$. The over-dispersion parameter is denoted by ϕ. According to the theory developed in Kuang *et al.* (2008a) this parameterisation and set of constraints has identifiable unique optimal estimates for the parameters μ, α_i, β_j and γ_k. The multiplicative form is similar to this, except that it has a multiplicative structure for the mean. Thus, incremental claims, C_{ij}, are assumed to be independent over-dispersed Poisson random variables with $E[C_{ij}] = x_i y_j z_{i+j-1}$, where $\sum_{j=1}^{n} y_j = 1$ and $z_1 = z_2 = 1$. Again, the over-dispersion parameter is denoted by ϕ.

Note that

$$\alpha_i = \ln\left(\frac{x_i}{x_1}\right), \beta_j = \ln\left(\frac{y_j}{y_1}\right), \gamma_k = \ln(z_k), \mu = \ln(x_1 y_1) \quad \text{and} \quad y_1 = \frac{1}{\sum_{j=1}^{n} \exp(\beta_j)}.$$

Lemma

The additive form is equivalent to the multiplicative parameterisation.

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Proof

First assume that the multiplicative form holds. Then

$$E[C_{ij}] = x_i y_j z_{i+j-1} = \exp(\ln(x_i) + \ln(y_j) + \ln(z_{i+j-1}))$$
$$= \exp\left(\ln(x_1 y_1) + \ln\left(\frac{x_i}{x_1}\right) + \ln\left(\frac{y_j}{y_1}\right) + \ln(z_{i+j-1})\right)$$
$$= \exp(\mu + \alpha_i + \beta_j + \gamma_{i+j-1}).$$

Clearly, the conditions of the multiplicative model also imply that $\alpha_1 = \beta_1 = \gamma_1 = \gamma_2 = 0$.

Now assume that the additive form holds. The conditions $\gamma_1 = \gamma_2 = 0$ clearly imply that $z_1 = z_2 = 1$. Also, $\sum_{k=1}^{n} y_j = y_1\left(\sum_{j=1}^{n} \exp(\beta_j)\right) = 1$, which concludes the proof.

It follows from this Lemma that finding the parameters x_i, y_j, z_{i+j-1} for $1 \leq i, j \leq n$ in the multiplicative model is equivalent to finding the parameters $\mu, \alpha_i, \beta_j, \gamma_{i+j-1}$ in the additive model, provided each of the arguments of the natural logarithms are positive.

For the additive model, and the over-dispersed Poisson distribution, it is straightforward to obtain the optimal parameter values using an Excel spreadsheet. This involves maximising the log-likelihood function which takes the following form:

$$\frac{1}{\phi}\sum_{i=1}^{n}\sum_{j=1}^{n-i+1}\left(C_{ij}(\mu + \alpha_i + \beta_j + \gamma_{i+j-1}) - \exp(\mu + \alpha_i + \beta_j + \gamma_{i+j-1})\right).$$

Thus, it is straightforward to estimate the parameters for the additive model, and the theory of Kuang *et al.* (2008a,b) shows that these estimates are unique. If it is more convenient to consider the parameters of the multiplicative model, these can be estimated using the results from the additive model.

It is assumed that the dispersion parameter is estimated in the usual way using the sum of squares of the residuals (see England & Verrall, 2002, for more details). It is also assumed for simplicity that the dispersion parameter is constant over the triangle, although it would be straightforward to extend this to the case where the dispersion parameter depends on the development year, for example.

3. Assessing the Inflationary Effects Implied by the Chain-Ladder Technique

Section 2 outlined the over-dispersed Poisson model for the age-period-cohort model described by Kuang *et al.* (2008a), in both additive and

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multiplicative form. The chain-ladder technique implicitly includes any calendar year effects somehow in the parameters it uses, and the aim of this section is to suggest how this might be examined in more detail. We suggest that a way to do this is to apply the chain-ladder technique to the data, and then to examine the calendar year effects using the age-period-cohort model. If this is done for artificially generated data, it should be possible to gain more understanding of the way the chain-ladder technique deals with any calendar year effects which have not already been removed from the data. Thus, the approach we use is to apply the chain-ladder technique and obtain the forecasts of future incremental claims — thereby obtaining a complete square consisting of past data and future forecasts. The age-period-cohort model is then applied, and the future inflation effects implied by the chain-ladder technique are estimated by maximising

$$\sum_{i=2}^{n} \sum_{j=n-i+2}^{n} \left(C_{ij}\left(\mu + \alpha_i + \beta_j + \gamma_{i+j-1}\right) - \exp\left(\mu + \alpha_i + \beta_j + \gamma_{i+j-1}\right)\right). \tag{3.1}$$

In the experiments described in Section 4, we generate the data using known values of the parameters. For this reason, we also examine the estimates of the implied future inflation when the row and column parameters are assumed fixed and known throughout. In other words, expression (3.1) is maximised over the calendar year effects, γ_k, only.

Using the identities derived in Section 2, it is possible to consider the calendar year effects either on the log scale (γ_k) or as multiplicative effects (z_k). It should be emphasised again that there are a number of drivers behind calendar year trends in addition to claims inflation. Thus, the parameters z_k should really be regarded as an index of calendar-year effects rather than simply inflation.

Using this approach, we have conducted a number of experiments to study the behaviour of the chain-ladder technique in different circumstances. These are described in Section 4, where the results are discussed in relation to the impact of the use of the chain-ladder technique.

4. Investigations

In order to investigate the way that the chain-ladder technique deals with any past inflation in the data, and the way it projects this forwards, a number of triangles of data were created. This was done by simply specifying the parameters x_i, y_j and z_k for $1 \le i, j, k \le n$. The parameters were chosen specifically in order to illustrate the way that the chain-ladder technique behaves under a number of different scenarios. Since the purpose of these experiments was to look at the behaviour of the chain-ladder technique in terms of its actual forecasts and not the variability of the forecasts, no random noise was added to the data.

Given the data sets created in this way, the chain-ladder technique was applied and then the calendar year parameters of the age-period-cohort model were estimated as described in Section 3. The focus here is on the way the chain-ladder technique projects forward the calendar year effects, and hence we look in detail at these.

For each of these experiments we consider a triangle with 10 rows and 10 columns. For simplicity, we set $x_i = 10$, for all i, and $y_j = 0.1$, for all j. These parameters are assumed known and are not re-estimated after the chain-ladder technique is applied: in this way, we focus attention on the calendar year effects.

Some definitions are needed at this stage. An I(1) (integrated of order 1) time series γ_k is one whose increments $d(\gamma_k) = \gamma_k - \gamma_{k-1}$ form a stationary series. An I(2) time series γ_k is one whose second increments $d^2(\gamma_k) = d(\gamma_k) - d(\gamma_{k-1})$ form a stationary series. An I(3) time series γ_k is one whose third increments $d^3(\gamma_k) = d^2(\gamma_k) - d^2(\gamma_{k-1})$ form a stationary series. And so on. It should be noted that Kuang *et al.* (2008b) always consider time series modulo linear trends. Hence an I(k) series in this paper would be referred to as an I(k-1) series in their publications.

4.1 *Experiment 1: Constant Calendar Year Trend in the Past*

The first experiment considers the situation which should be best suited to the chain-ladder technique. This occurs when it is assumed that the rate of claims inflation in the data is constant. Note that if the calendar year effect consists purely of claims inflation, then the rate of claims inflation can be measured by $\gamma_k - \gamma_{k-1}$. Thus, on the logarithmic scale, the rate of claims inflation is constant when the first differences of the calendar year parameters, γ_k, is constant. We denote this first difference by $d(\gamma_k)$, where $d(\gamma_k) = \gamma_k - \gamma_{k-1}$. For Experiment 1, we set $d(\gamma_k) = 0.182$ for $k = 2, 3, \ldots, 10$ (equivalent of an inflation rate of 20%). Given the commonly held views about the chain-ladder technique, we would expect the projected values of $d(\gamma_k)$ also to be 0.182. The data in the claims triangle were generated under these assumptions, the chain-ladder technique was applied, and then the age-period-cohort model was used to estimate the implied values of future claims inflation in the data projected by the chain-ladder technique. The results confirmed the initial assumption: the rate of claims inflation projected forward by the chain-ladder technique was indeed constant and equal to 0.182.

Experiment 1 has shown that, as was expected, the chain-ladder technique projects a constant value of past "inflation" into the future. This result might lead one to think that the chain ladder implicitly uses an I(1) model for the calendar year parameters γ_k. However, it can be seen that this is not the case, and this is illustrated by Experiment 2.

4.2 *Experiment 2: Past Calendar Trend Not Constant*

This experiment is similar to Experiment 1, except that the change of the calendar year parameter is not constant. Table 4.1 shows the values of $d(\gamma_k)$, which have been chosen to follow an I(2) time series, where the differences of the $d(\gamma_k)$ values are constant.

The values of $d(\gamma_k)$ from Table 4.1, together with the implied values from the projected data when the chain-ladder technique is applied, are plotted in Figure 4.1. It can be seen that the chain-ladder method produces estimates where the future $d(\gamma_k)$ are not all equal. This shows that the chain-ladder method does not use an I(1) model for estimating future calendar year parameters. It also does not project the constant differences in the past $d(\gamma_k)$ into the future, i.e. the trend of rising "inflation" is not projected forward. What it appears to do is to try to project forward the pattern and use a weighted average of the past values, giving the more recent values the highest weight.

Table 4.1. Past values of $d(\gamma_k)$ for Experiment 2

$d(\gamma_2)$	0.182
$d(\gamma_3)$	0.278
$d(\gamma_4)$	0.373
$d(\gamma_5)$	0.468
$d(\gamma_6)$	0.564
$d(\gamma_7)$	0.659
$d(\gamma_8)$	0.754
$d(\gamma_9)$	0.849
$d(\gamma_{10})$	0.945

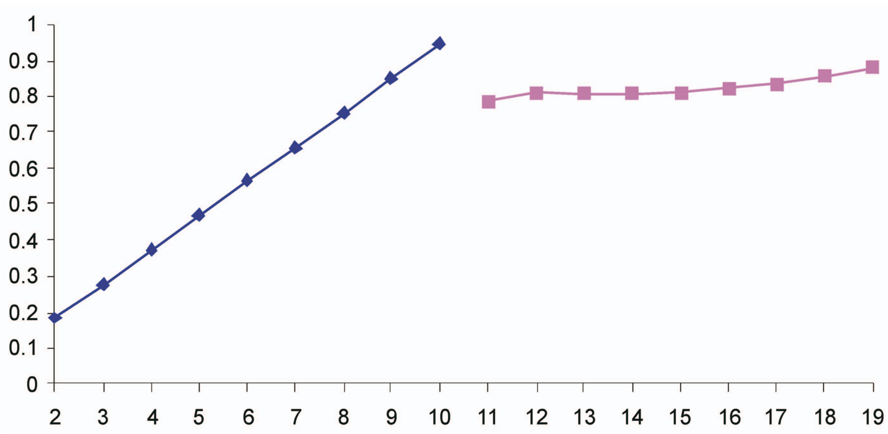

Figure 4.1. Past and future values of $d(\gamma_k)$, against k, for Experiment 2

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Examining this in a little more detail, it is useful to consider the way the values in the triangle are related to the projected values when the chain-ladder technique is applied. Since we have chosen to set $x_i = 10$, for all i, and $y_j = 0.1$, for all j, it can be seen from the multiplicative model in Section 2 that $C_{ij} = z_{i+j-1}$. When the chain-ladder technique is applied, there is a direct connection between z_{19} and z_{10}, since $z_{19} = z_{10}(f_{10} - 1) \prod_{j=2}^{9} f_j$. This expression is simple to prove as shown in Appendix 1.

There appears to be no reason why the predicted calendar year effect should behave in this way: why would it be expected that the predicted "inflation" for 2018 should have a special relationship with that in 2009, for example? This illustrates an intrinsic difficulty with the chain-ladder technique, and it should be noted that this effect will also be manifest when the technique is applied to real data. We have used a simple triangle in order to remove all the other effects and reveal what is happening. Similar effects can be found by moving up the rows of the array. For example, it can be shown that z_{18} is directly connected with z_9 and z_{10}.

4.3 *Experiment 3*

In this experiment, we consider a similar situation to Experiment 1, except that one of the $d(\gamma_k)$ values is not equal to the others. In other words, $d(\gamma_k) = 0.182$ for all k, except $k = m$, and $d(\gamma_m) = 0.693$. This experiment is repeated (9 times) for $m = 2, 3, \ldots, 10$.

It can be seen from Figure 4.2 that the more recently the inflationary shock is applied, the more it affects the average future inflation. In other words, it appears that more weight is given by chain ladder to more recent calendar year effects.

Figure 4.2 also illustrates the relationship between the predicted effects and the past effects as described above. For example, in the top right hand plot, the results are shown when the shock is applied in the latest observed calendar year and the effect this has on the latest predicted calendar year can be clearly seen.

Another interesting feature of the results when the shock is applied in the most recently observed calendar year is that there is an "inflationary dip" observed in the year immediately following this. This appears to show that if there is a large change in the calendar year effect for the latest calendar year, the 1-year forecasts implied by the chain-ladder method should be treated with caution. This is of particular importance in the context of Solvency 2 if capital requirements are to be measured on a 1-year Value-at-Risk (VaR) basis.

For example, the stochastic chain-ladder method might be used in a practical context to calculate 1-year 99.5% VaR figures for the reserves to ultimate. Suppose the data triangle has n rows and n columns. The 1-year 99.5% VaR might be estimated by modelling a 1-in-200 year event occurring

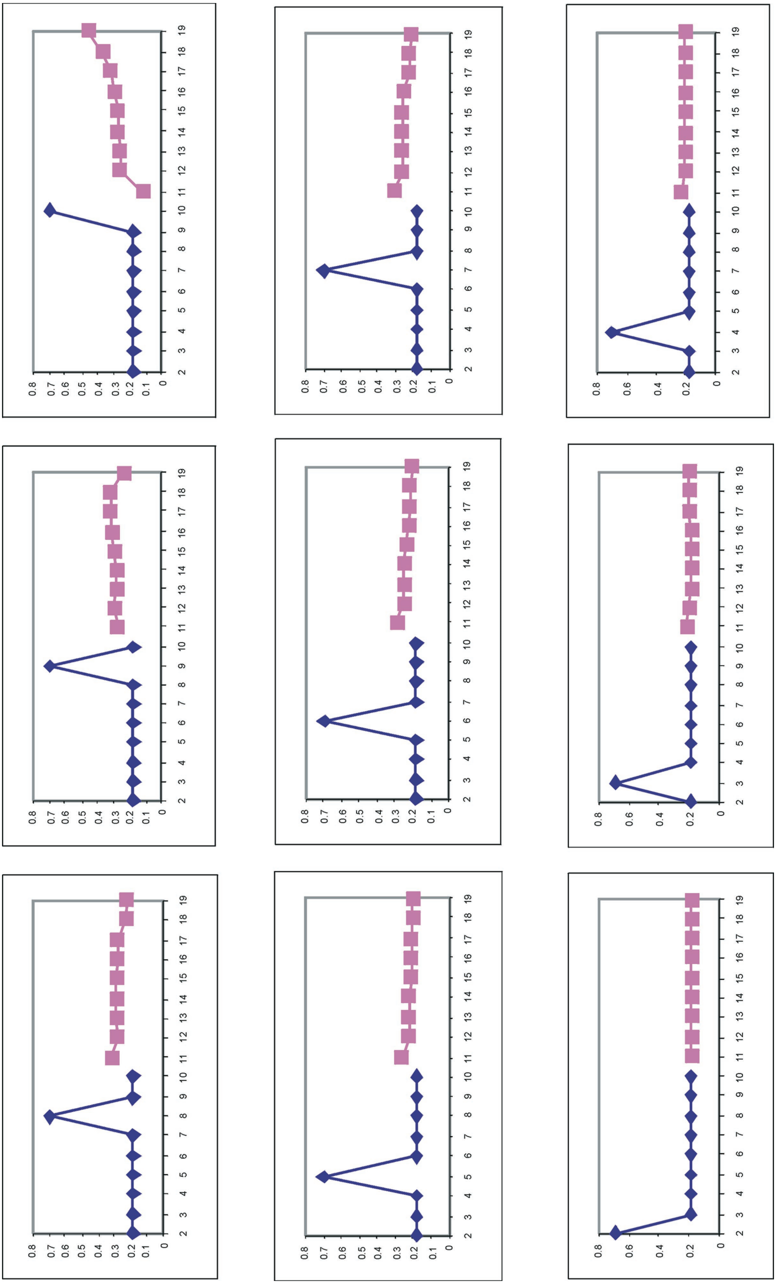

Figure 4.2. Past and future values of $d(\gamma_k)$, against k, for Experiment 3, with the shock at $m = 2$ (bottom left) to $m = 10$ (top right)

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in the next calendar year by bootstrapping the chain ladder and taking the 99.5% worst case in the $(n+1)th$ diagonal. A projection to ultimate may then be based on the original triangle augmented by the extra $(n+1)th$ diagonal whose entries correspond to the 1-in-200 year event. If the 1-in-200 year event stems from a simulation (or group of simulations 'averaged') in which the *nth* diagonal has a large inflationary increase on the $(n-1)th$ calendar year, it would be similar to the type of situation in the top right plot of Figure 4.2 with the danger that the 1-year-ahead forecasts are lower than would be expected. In other words, under these circumstances, it appears that the chain-ladder technique projects forward a lower than expected calendar year effect for the next calendar year, and it may result in a VaR estimate which is underestimated.

A further observation is that it does not appear that the chain-ladder technique implies the use of an I(2) model for future calendar year parameters as the projected $d(\gamma_k)$ values do not follow a straight line. This can be illustrated most clearly by considering the second differences of the values of γ_k: if an I(2) model is used, the projected values should be 0. Figure 4.3 shows the second differences of the values of γ_k for the case when there is a shock in the latest calendar year. Clearly, the projected values do not conform to an I(2) model.

Similarly, Figure 4.4, which shows third differences, shows that an I(3) model is not used by the chain-ladder technique to project forward the calendar year parameters.

In all these experiments, the conclusion is that, in a practical context, the chain-ladder technique does not project forward the calendar year effect in a sensible way except in very specific circumstances. The conclusion from this is that it is important to use a model which includes calendar year effects unless there is strong evidence that there is no calendar year effect, or that there is a constant trend.

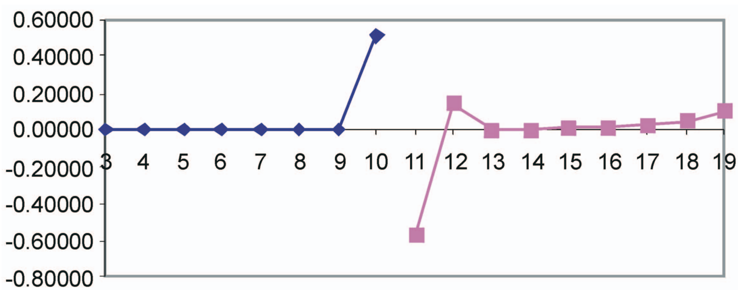

Figure 4.3. Past and future values of $d^2(\gamma_k)$, against k, for Experiment 3, when $m = 10$

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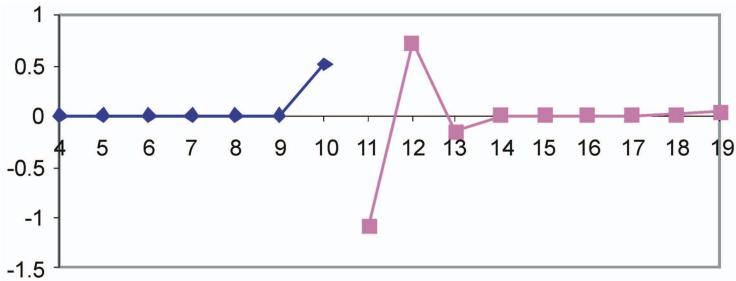

Figure 4.4. Past and future values of $d^3(\gamma_k)$, against k, for Experiment 3, when $m = 10$

5. Conclusions

In a practical context, the considerations in this paper should be taken into account when applying the chain-ladder technique. If it is reasonable to assume that all calendar year effects have been removed from the data, then it should give reasonable forecasts. This is not particularly surprising, since the chain-ladder technique does not include any parameters for the calendar years. Where there are any calendar year effects left in the data, it is questionable whether the chain-ladder technique will project these forward in a sensible way.

While it is true that the chain-ladder technique deals appropriately with a constant trend in the calendar year, this does not mean that it always projects forward using a constant trend for future calendar year effects. This is contrary to what might have been assumed: there appears to be a popular belief that the chain-ladder technique somehow takes an average value of past inflation and uses this as the constant trend for projecting forward. This paper has shown that this may not be true.

Furthermore, the projections to future calendar years implicit within the chain-ladder technique appear to be disrupted by correlations which result purely from the arithmetic of the procedure.

Our conclusion is that the calendar year effects (including any claims inflation) should be examined in detail in order to ensure that the projections have realistic properties. This is disappointing for the chain-ladder technique, whose simplicity and ease of application is very appealing. We would suggest that it can be used to give a quick forecast, as a "back-of-the-envelope" calculation, but that any serious reserving exercise should also look at the data in much greater detail. This should include consideration of calendar year effects, and would probably also entail looking at the numbers of claims and the claims amounts separately. The reason for looking at claim numbers and claim amounts separately is that the calendar year effects may well have different manifestations for each of these.

References

Barnett, G. & Zehnwirth, B. (2000). Best estimates for reserves. *Proceedings of the Casualty Actuarial Society*, **LXXVII**, 245-321.

England, P.D. & Verrall, R.J. (2002). Stochastic claims reserving in general insurance (with discussion). *British Actuarial Journal*, **8**, 443-544.

Jones, A.R., Copeman, P.J., Gibson, E.R., Line, N.J.S., Lowe, J.A., Martin, P., Matthews, P.N. & Powell, D.S. (2006). A change agenda for reserving. Report of the general insurance reserving issues taskforce (GRIT). *British Actuarial Journal*, **12**(III), 435-619.

Kuang, D., Nielsen, B. & Nielsen, J.P. (2008a). Identification of the age-period-cohort model and the extended chain-ladder model. *Biometrika*, **95**, 979-986.

Kuang, D., Nielsen, B. & Nielsen, J.P. (2008b). Forecasting with the age-period-cohort model and the extended chain-ladder model. *Biometrika*, **95**, 987-991.

Mack, T. (1993). Distribution-free calculation of the standard error of chain-ladder reserve estimates. *ASTIN Bulletin*, **23**, 213-225.

Renshaw, A.E. & Verrall, R.J. (1998). A stochastic model underlying the chain ladder technique. *British Actuarial Journal*, **4**(IV), 903-923.

Taylor, G.C. (1977). Separation of inflation and other effects from the distribution of non-life insurance claim delays. *ASTIN Bulletin*, **9**, 217-230.

Taylor, G.C. (2000). *Loss reserving: an Actuarial perspective*. Kluwer.

Verbeek, H.G. (1972). An approach to the analysis of claims experience in motor liability excess-of-loss reinsurance. *ASTIN Bulletin*, **6**, 195-203.

Wuthrich, M.V. & Merz, M. (2008). *Stochastic claims reserving methods in insurance*. Wiley.

APPENDIX 1

$$\begin{aligned}
z_{19} &= C_{10,10} = D_{10,10} - D_{10,9} \\
&= \left(C_{10,1} \times \prod_{j=2}^{10} f_j\right) - \left(C_{10,1} \times \prod_{j=2}^{9} f_j\right) \\
&= C_{10,1} \times \left(\prod_{j=2}^{10} f_j - \prod_{j=2}^{9} f_j\right) \\
&= C_{10,1} \times (f_{10} - 1)\prod_{j=2}^{9} f_j \\
&= z_{10} \times (f_{10} - 1)\prod_{j=2}^{9} f_j.
\end{aligned}$$