it is conceded that plane curves yield plenty of good examples, and virtually all those in Stichtenoth's book come into this category.

I enjoyed reading this book and believe it is successful at doing its job. As a footnote, let me record that a preprint by Niederreiter, Xing and Lam, 'A new construction of algebraic-geometry codes', has recently come to hand. In it Goppa's construction is powerfully extended using higher degree places. This might be an incentive for the reader of Pretzel's text, or ammunition for a future edition.

S. D. COHEN

CVETKOVIĆ, D., ROWLINSON, P. AND SIMIĆ, S. *Eigenspaces of graphs* (Encyclopedia of Mathematics and its Applications, Vol. 66, Cambridge University Press, Cambridge, 1997), xiii + 258 pp., 0 521 57352 1, £45 (US$69.95).

It is over forty years since Collatz and Sinogowitz's seminal paper [1] on the eigenvalues of a graph. Since then, there have been over a thousand papers on the subject, as well as three important books by Cvetković, Doob and Sachs [2], Biggs [3], and Cvetković, Doob, Gutman and Torgašev [4]. These books reviewed the progress in the subject over the intervening years; in particular, the third edition of [2] describes recent developments up to 1995.

The book under review extends the subject further by concentrating on the eigenspaces of a graph. If G is a graph with vertex-set $\{v_1, \ldots, v_n\}$, then its adjacency matrix is the $n \times n$ matrix $\mathbf{A} = (a_{ij})$, where a_{ij} is the number of edges joining the vertices v_i and v_j. The *spectrum* of G is the set of eigenvalues of $\mathbf{A}$, and does not depend on the particular labelling of the vertices; the *index* of G is the largest eigenvalue of $\mathbf{A}$. Although the eigenvalues of certain families of graphs specify the graphs completely, this is far from being true in general; for example, Schwenk [5] has proved that almost no tree is determined by its eigenvalues. Such considerations lead to a search for sets of graph invariants that specify a graph uniquely, and in this book the authors obtain such a set of invariants – the distinct eigenvalues and a certain type of basis for $\mathbb{R}^n$.

There are nine chapters. The first two provide a masterly summary of earlier spectral results, designed to bring the reader up to speed on the results needed for later chapters; in particular, there is an introductory discussion of graphs that are characterized by their spectrum. In Chapter 3 the authors introduce several eigenvector techniques and use them to investigate the indices of various families of graphs. Chapters 4–6 inject a more geometrical flavour by discussing eigenspace invariants such as the *angles* of a graph. Although the eigenvalues and angles do not specify the graph completely, except in small cases and for particular families of graphs, they prove to be a most useful tool in the general discussion, and contribute to our understanding of strongly regular graphs, the graph reconstruction conjecture, and graph perturbations (changes in the spectrum caused by adding or deleting individual vertices and edges).

Chapters 7 and 8 form the core of the book. Chapter 7 continues the geometrical approach through the notion of a *star partition* of vertices, an important concept that enables one to construct natural bases (*star bases*) for the eigenspaces of a graph. In Chapter 8 a unique canonical star basis is obtained for each graph, and it is this basis, together with the distinct eigenvalues, that forms the complete set of invariants for the graph. The authors also present efficient algorithms for finding star bases for a given graph. The final chapter is a survey of some interesting results that are related to graph eigenspaces, but which do not fit readily into earlier chapters.

This book is highly recommended for anyone interested in learning about current trends in spectral graph theory, especially those developments of a more geometrical nature.

R. J. WILSON

REFERENCES

1. L. Collatz and U. Sinogowitz, Spektren endlicher Graphen, *Abh. Math. Sem. Univ. Hamburg* **21** (1957), 63–77.

2. D. Cvetković, M. Doob and H. Sachs, *Spectra of graphs – theory and application* (Academic Press, 1980, 1982; Johann Ambrosius Barth Verlag, 1995).

3. N. L. Biggs, *Algebraic graph theory* (Cambridge University Press, 1974, 1993).

4. D. Cvetković, M. Doob, I. Gutman and A. Torgašev, *Recent results in the theory of graph spectra* (North-Holland, 1988).

5. A. J. Schwenk, Almost all trees are cospectral, in *New directions in the theory of graphs* (ed. F. Harary) (Academic Press, 1973), 275–307.

McKean, H. and Moll, V. *Elliptic curves: function theory, geometry, arithmetic* (Cambridge University Press, Cambridge, 1997), xiii+280pp., 0 521 58228 8 (hardback), £40 (US$59.95).

One of the main trends in mathematics during the last few decades has been the development of links between what were originally regarded as separate disciplines. A major catalyst in this process has been the theory of elliptic curves, that is, Riemann surfaces of genus 1. These objects have so many different facets that they have served, since the early 19th century, as a common meeting-ground for mathematicians from a wide range of backgrounds. In recent years general interest in elliptic curves has been greatly enhanced by Andrew Wiles's work on the Taniyama-Shimura Conjecture, which relates elliptic curves to modular forms, with its spectacular corollary of Fermat's Last Theorem. This book's subtitle, *Function theory, geometry, arithmetic*, gives an indication of the authors' broad approach, tracing the development of elliptic curves through the 19th and early 20th centuries. In addition they briefly outline links with several other topics such as Galois theory, partitions and even applied mathematics (solitons and the KdV equation for instance).

Chapter 1 is a brisk introduction to Riemann surfaces and projective curves, while Chapter 2 covers the classical theory of elliptic integrals and elliptic functions, due to Abel, Gauss, Jacobi and Legendre, and shows how they lead to complex tori $\mathbb{C}/L$, where L is a lattice, or equivalently to elliptic curves such as $y^2 = (1 - x^2)(1 - k^2x^2)$. Chapter 3 explains the development of theta functions, from Jacobi to Ramanujan, with interesting digressions into quadratic reciprocity, sums of squares, and partitions. The modular group $\Gamma_1 = PSL(2, \mathbb{Z})$, with its associated modular forms and congruence subgroups, is the main theme of Chapter 4; if this can be seen as a modern introduction to the work of Fricke and Klein, then Chapter 5 plays the same role for Klein's book on the icosahedron, with an algebraic and geometric discussion of the quintic equation, and Hermite's solution using $\sqrt[4]{k}$. Chapter 6 is about algebraic number theory, with special attention to imaginary quadratic number fields, where the connection is that ideal classes in the ring of integers correspond to equivalence classes of elliptic curves. The book closes with a rather brief chapter on the arithmetic of elliptic curves, leading up to a proof of the Mordell-Weil Theorem that the group of rational points on a nonsingular cubic curve has finite rank.

Throughout this book the style of writing is brisk, clear and informal with routine verifications generally left to the reader. There are plenty of references to alternative sources, such as the excellent book by Silverman and Tate **[1]**, for readers wanting a more detailed treatment. This approach allows an impressive range of topics to be covered and some fascinating connections to be explored. Although this is not a history book, there are numerous historical references and the authors often take care to explain not just what the 19th-century masters did but also how they did it; this approach can be of great benefit to those lacking the time, the library facilities or the linguistic skills to read the original papers. The bibliography alone, with more than 300 references, makes this book a valuable resource.