Robotica (2014) volume 32, pp. 907–934. © Cambridge University Press 2013
doi:10.1017/S0263574713001124

Modeling, stability and walking pattern generators of biped robots: a review

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(Accepted October 29, 2013. First published online: December 5, 2013)

SUMMARY

Biped robots have gained much attention for decades. A variety of researches have been conducted to make them able to assist or even substitute for humans in performing special tasks. In addition, studying biped robots is important in order to understand human locomotion and to develop and improve control strategies for prosthetic and orthotic limbs. This paper discusses the main challenges encountered in the design of biped robots, such as modeling, stability and their walking patterns. The subject is difficult to deal with because the biped mechanism intervenes with mechanics, control, electronics and artificial intelligence. In this paper, we collect and introduce a systematic discussion of modeling, walking pattern generators and stability for a biped robot.

KEYWORDS: Biped robot; Stability; Zero moment point; Orbital stability; Walking pattern generators.

1. Introduction

Industrial robots are often rigidly attached to the ground. In contrast, mobile robots are able to move in specific environments depending on the tasks they are required to perform. Much attention has been drawn to the design and control of mobile robots due to their significant characteristics such as their versatile mobility, sensing and reacting in their specific environments.[1] Mobile robots can be classified as legged robots, wheeled robots and tracks.

Legged robots offer significant advantages over wheeled robots and tracks in view of the workable environments, the energy consumption and the adaptability.[2] Although wheeled robots are simple and lightweight structures, they need regular terrains for motion and lack, e.g. the ability to climb stairs. Tracks help to overcome this drawback, but they consume a lot of energy due to the high friction between the chains and the ground. In contrast, legged robots are able to move in regular and irregular terrains with versatile mobility. With configuration changes, they can easily adapt to irregular environment. Due to the small contact areas of their feet, legged robots can be efficiently operated.[2]

The biped robot is designed to imitate human-like locomotion and perform certain tasks such as activities in danger environments, assistances to the elderly and entertainment.[3] The biped robot can consist of a trunk and legs with/without feet or even entire human-like mechanism depending on the desired application (see Section 3).

The biped robots offer advantages over multi-legged robots. The biped robots have considerably higher adaptability enabling them to easily surpass obstacles like narrow paths or stairs. This adaptability is particularly important when the robot is required to perform human-centered tasks. Due to the smaller ground contact area and fewer actuators used, their energy consumption can be lower than multi-legged robots. This is consistent with the conclusion that two-legged animals have higher efficiency and adaptability than multi-legged animals.[4] For a historical review of legged machines, we refer readers to Bekey[3] and Raibert.[5]

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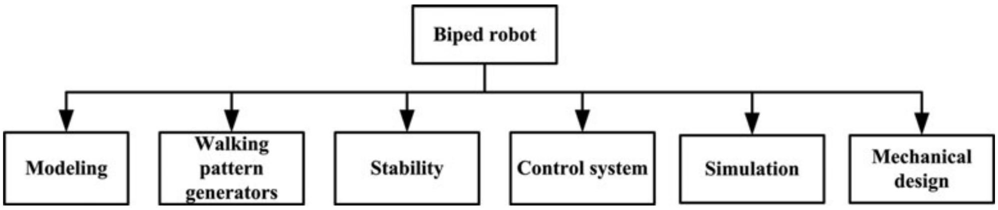

Fig. 1. Important design issues of a biped robot.

Some challenges encountered in the design of biped robots are follows:

- Biped robots have unstable structures due to the passive joint located at the unilateral foot–ground contact.[6–9]
- Due to the unilateral foot–ground contact and the varying configurations throughout the gait cycle, their mechanical description is highly nonlinear. During the single-support phase, the robot is under-actuated, while turning into an over-actuated system during the double-support phase.[10] Consequently, the dynamic description and control laws change during transition from one phase to another.[7] It is noted that the biped robot could be fully actuated during the swing phase but with some limitations (see Sections 2 and 3.2).
- Biped robots have many degrees of freedom (DOFs). A humanoid robot may have more than 30-DOF, making their mechanical behavior and control difficult.[11–13] To avoid this difficulty, most researchers have used simple models based on approximations and assumptions. A trade-off between simplicity and accuracy becomes necessary.
- Biped robots interact with different unknown environments. This requires robust algorithms for the generation of reference trajectories and control. These algorithms should be insensitive to the possible disturbances and noises. However, stabilization and online adaptive control schemes could solve this dilemma.

These challenges are associated with mechanics, control, electronics, artificial intelligence (AI) and human anatomy so that studying biped robots is interdisciplinary. Unified solutions are therefore difficult to develop. The possible issues to be encountered in the design of biped robots are illustrated in Fig. 1.

In this paper, we focus on the mechanical aspects of the biped robots. The gait cycle of biped locomotion is presented in Section 2. Section 3 discusses the modeling of biped mechanism, while Section 4 considers the stability of biped robots. Section 5 introduces classification of the methods used in the generation of walking patterns. The conclusions are given in Section 6.

2. Gait cycle

The complete gait cycle of human walking consists of two main successive phases: the double support phase (DSP) and the single support phase (SSP) with intermediate sub-phases.[14–16] The DSP arises when both feet contact the ground resulting in a closed chain mechanism, while the SSP starts when the rear foot is not supported by the ground with the front foot flatting on the ground. One should note that the percentage of DSP is about 20% of time during one stride of the gait cycle whereas SSP is about 80% of time.[8,16]

Due to the complexity of biped mechanisms, most researchers have simplified the gait cycle of biped walking to understand the kinematics, biomechanics and control schemes of bipeds. Studies have shown that there are three essential patterns used for the generation of periodic biped walking.[10] Fig. 2 illustrates the third pattern grading from simple to complex configurations.

Pattern 1[7]: It consists of successive DSP and SSP without sub-phases, as shown in Fig. 2(a). The swing feet are always level to the ground during leaving and striking the ground. The biped mechanism that adopts this pattern may undergo under-actuation during the SSP if one considers the ankle joints as passive joints. In contrast, if the ankle joints are active (powered), the system will be fully actuated with allowable ankle torque to keep the zero moment point (ZMP) within the contact area of the stance foot.[12] However, the legs constitute over-actuated system during the DSP. In this pattern, maximum ground support is provided during each step resulting in a slow or moderate biped

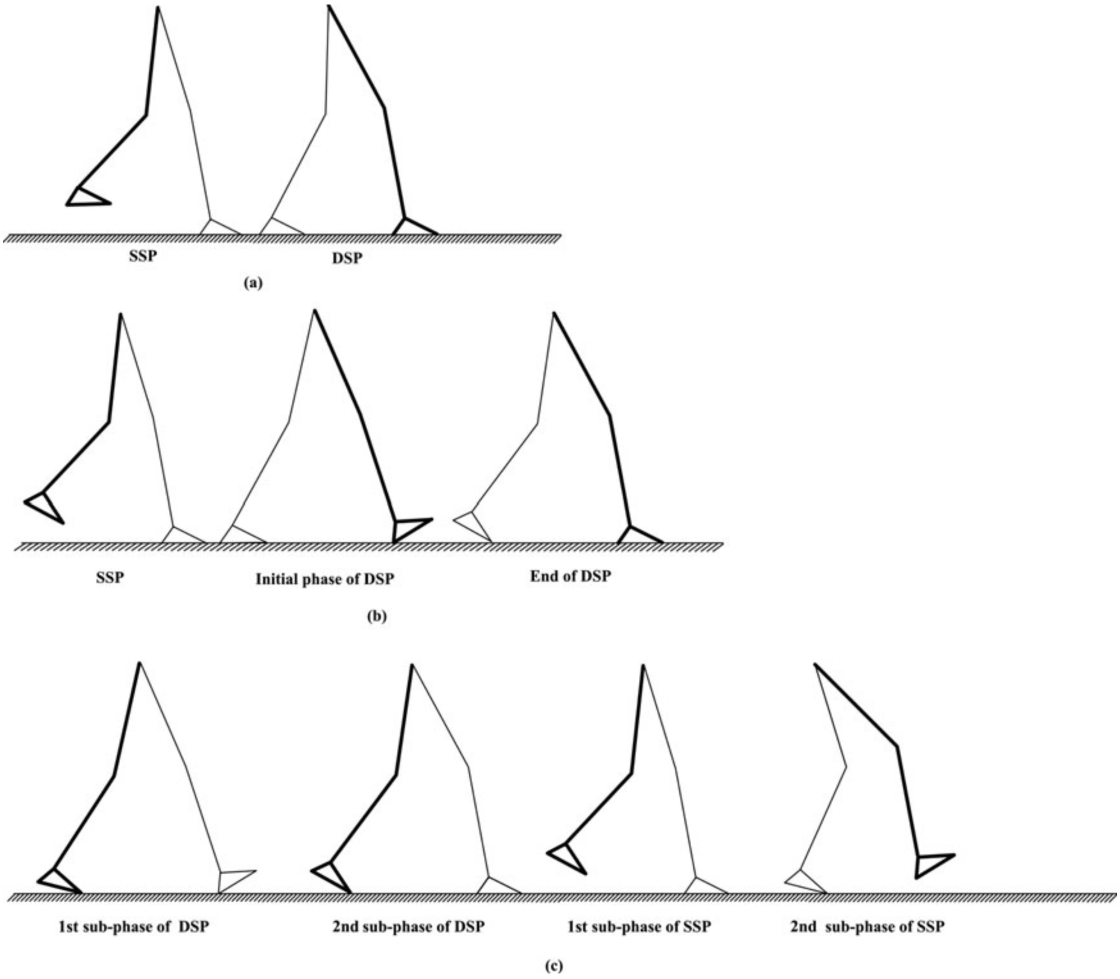

Fig. 2. Types of biped walking patterns.

walking.[14] Huang and co-workers[17,18] have stated that this type of pattern could result in unstable walking due to the sudden landing of whole sole on the ground at the beginning of the DSP. This drawback can be overcome by pattern 2.

Pattern 2[14,17,18]: This is analogous to the first pattern with an exception that the swing leg will leave and land the ground with specified angle as shown in Fig. 2(b). This results in a smooth transition of the striking foot from the heel to the whole sole at the beginning of the DSP. This pattern consists of two sub-phases of DSP and one phase of SSP.

Pattern 3[10]: This pattern is close to the human walking consisting of two sub-phases of DSP and SSP each. The first sub-phase of DSP starts when the rear foot begins to rotate about its front edge during a small rotation of the front foot about the heel. Its second sub-phase starts when the front foot is level to the ground while the rear foot continues to rotate about its front edge. The first sub-phase of the SSP arises in the moment at which the rear foot is lifted above the ground meanwhile the stance foot is level to the ground. Finally, the second sub-phase of the SSP occurs when the stance foot starts to rotate about its front edge. Fig. 2(c) shows the walking stages of this pattern.

Additional DOF is gained during the second sub-phase of the SSP. Consequently, the system is under-actuated during this sub-phase. This motivates the researchers to investigate the stability issue of biped mechanism without foot or with a point foot because the robot coinciding with this walking pattern is a system of under-actuation.[19]

Remark. It is possible to modify the mentioned patterns to generate the desired motion. For example, pattern 2 can be performed in one phase of DSP instead of two sub-phases such that the front and rear feet can rotate simultaneously at the same time. Another modification to pattern 3 can be seen in Sato *et al.*[20]

3. Modeling

Humans and animals have matchless mobility due to their versatile reconfigurations. Consequently, these creatures can walk easily and smoothly using their amazing control systems. The functionality of human and animal locomotion is still not entirely analyzed.[21] During motion, humans utilize only about 20-DOF, although the human musculoskeletal system provides more than 300-DOF.[8,22,23] This fact allows humans to use a large number of DOFs that may be locked or released in order to adapt their motion patterns to the environment and the desired tasks. Most researchers have attempted to understand human walking principle using different simplified models of biped robots. Possible choices are as follows:

- *2-link model*: It uses one leg consisting of thigh, knee and shin without foot.[5,24]
- *3-link model*: It consists of a trunk (torso) and two links for thigh and shin (shank) with locked knees and without feet. This sagittal plane model has 5-DOF.[25]
- *5-link model*: A sagittal plane model consisting of a trunk and two legs with knees, but without feet. This model is frequently used, since it is considered a reasonable trade-off between simplicity and leg anatomy.[10,26,12] The leg motion can be extended from planar to spherical if a joint with more than 1-DOF is chosen for the hip joint such as a universal joint or a ball-and-socket joint.
- *7-link model*: It is a 5-link model with additional feet.
- *9-link model*: It is analogous to the latter model adding two links connecting the ankle joints to the sole plates. This is the closest model that most researchers proposed.[10,17,18]

A humanoid robot may comprise a total of more than 30-DOF.[12,13]

3.1. Kinematics

After a suitable model is selected, equations are derived to mathematically describe the kinematic and dynamic behavior during walking. Direct kinematics can be written when the joint coordinates are known and the position and orientation of robot's links are formulated as a function of these coordinates.

Define the rotation matrix $R(t) \in SO(3)$ that transforms a 3×1 velocity expressed in a body frame to the same vector expressed in the inertial frame, then

$$\dot{R}(t) = (w\times)\, R(t) \tag{1}$$

with

$$(w\times) = \begin{bmatrix} 0 & -w_z & w_y \\ w_z & 0 & -w_x \\ -w_y & w_x & 0 \end{bmatrix} \tag{2}$$

where w_x, w_y and w_z denote the components of the instantaneous angular velocity of the frame.

The relationship between the vector of the joint velocity of the biped mechanism and the velocity of the end effector (swing foot in case of biped robot) can be described in Eq. (3),

$$\dot{\xi} = J_q \dot{q}, \tag{3}$$

where $\dot{\xi} \in \mathbb{R}^m$ represents the augmented linear/angular velocity vector of the end effector such that $1 \leq m \leq 6$, $J_q \in \mathbb{R}^{m\times n_q}$ is the Jacobian matrix that relates the vector of the joint velocity of biped mechanism to the body velocity of the end effector, n_q is the number of generalized coordinates of biped mechanism and $q \in \mathbb{R}^{n_q}$ represents the generalized angular displacement vector of biped mechanism.[27]

In contrast, the inverse kinematics is used to obtain the time history of joint coordinates as a function of position and orientation of the robot's links in Cartesian space. Therefore, the inverse kinematics is very useful in determining the actuator motion necessarily needed throughout the entire gait cycle of biped robots. The output of the inverse kinematic calculation provides the reference motion of actuators for control purposes.[28] Thus, the angular displacements of the biped mechanism can be found as follows:

$$\dot{q} = J_q^{\#}\dot{\xi}, \tag{4}$$

where

$$J_q^{\#} = J_q^T \left(J_q J_q^T\right)^{-1}. \tag{5}$$

Appropriate methods for the derivation of kinematic equations of biped robots can be found in the refs. [29–31].

Remark. In effect, the optimal control theory can be used alternatively for determining the optimal reference trajectory of joint displacements of biped mechanism. In addition, it can be used successfully to control the biped mechanism with free reference trajectory (see Section 5.1.4).

3.2. Dynamics

Two approaches are commonly used to obtain the differential equations of the motion describing system dynamics: Lagrange's equation and the Newton–Euler equation. The dynamic equations can govern the dynamic responses of the robot system to the input joint torques generated by actuators and other active forces.[31] Inverse dynamic calculations provide actuator torques as a function of the desired motion. In forward dynamics, the differential equation of the motion is solved for the system motion with active force/torque vector as an input. In control problem, we select appropriate control law to generate input controls (torques/forces) that track the desired reference trajectory of the robot mechanism.

In the following, the dynamic equations will be described assuming that the biped mechanism walking consists of three successive phases (SSP, contact phase (impact phase), and DSP).

During the SSP, the biped robot behaves as an open-chain mechanism, therefore the governed Lagrangian (or Newton–Euler[32]) dynamic equation can be written as

$$M\ddot{q} + C\dot{q} + g = A\tau, \tag{6}$$

where $M \in \mathbb{R}^{n_q \times n_q}$ is the mass robot matrix, q, $\dot{q}$ and $\ddot{q}\cdot \in \mathbb{R}^{n_q}$ are the absolute angular displacement, velocity and acceleration of the robot links, $C \in \mathbb{R}^{n_q \times n_q}$ represents the Coriolis and centripetal robot matrix, $g \in \mathbb{R}^{n_q}$ is the gravity vector, $A \in \mathbb{R}^{n_q \times n_\tau}$ is the mapping matrix derived by the principle of virtual work,[33,34] $\tau \in \mathbb{R}^{n_\tau}$ is the actuating torque vector and n_τ represents the number of actuators.

Remark. It is known that the biped robot during the SSP behaves as an open-chain mechanism with fully actuated- ($n_q = n = n_\tau$) or under-actuated-chain mechanism ($n > n_\tau$) depending on the desired walking patterns, where n represents the number of DOFs.

During the impact phase, the robot configuration remains unchanged with abrupt change in joint velocity. Thus, the resulting dynamic equation can be written as

$$M(q)(\dot{q}^+ - \dot{q}^-) = J^T \int_{t_o}^{t_o+\Delta t} \lambda dt, \tag{7}$$

$$J^T(\dot{q}^+ - \dot{q}^-) = 0, \tag{8}$$

where $\dot{q}^+$ and $\dot{q}^-$ refer to the angular velocity vector of the biped mechanism after and before impact respectively, $J \in \mathbb{R}^{n_\lambda \times n_q}$ represents the Jacobian matrix resulting from the constrained motion of the biped robot, $\lambda \in \mathbb{R}^{n_\lambda}$ is the constrained force/moment vector at the foot–ground interaction with the dimension of n_λ and t is the time. Equation (7) expresses the theorem of conservation of momentum, while Eq. (8) denotes the constraint of the motionless stance foot.

During DSP, the configuration of the biped mechanism changes, therefore the constrained Lagrangian dynamic equation during this phase is

$$M\ddot{q} + C\dot{q} + g = A\tau + J^T\lambda, \tag{9}$$

$$\phi(q) = 0, \tag{10}$$

where $\phi(.)$ represents the constraint equation with the same notations mentioned earlier.

The difficulties encountered in the analysis of the dynamics and control of the biped robots are the over-actuation during DSP and the under-actuation during SSP. The inverse dynamic problem of an over-actuated system is undetermined, and hence infinite combinations of torques can be applied to achieve the desired output trajectory. There are two schemes that are used commonly for the control of the over-actuated biped mechanism. The first one includes minimization of joint torques using the algebraic optimization,[12,35–37] whereas the second control method is represented by the kinematic Jacobian.[7,38] However, this strategy cannot guarantee the generation of continuous dynamic response (actuating torques/ground reaction forces) at transition instances (from SSP to DSP and *vice versa*).

The under-actuation during the swing phase poses particular difficulty for the control of biped robot. Four state-of-the-art control methods are used to solve this problem: the time scaling method,[12,39] hybrid zero dynamics,[40] the differentially flatness-based approach[25] and the port-Hamiltonian method.[41] Further elaboration of these methods is left for subsequent papers.

For biped robots, moderate path deviations are acceptable in the sense of stability. Therefore, elastic joints are preferred over stiff joints as they absorb shock and store energy.[7] Adding elastic joints means increasing the DOF of the system, and flexible joints further complicate the dynamics of the robots. In addition, it is found that the compact biped model is insufficient to describe some aspects of human walking; therefore, compliant legs are employed to improve the behavior of biped mechanism. The field of compliant legs has been widely investigated by Seyfarth and co-workers.[42–46]

The foot–ground contact poses another particular challenge to the dynamic behavior of biped robots. The consequences of these unilateral contacts with impulsive forces were well discussed in refs. [10, 47].

4. Stability

The biped mechanism is unstable during the SSP. One of the challenges in the design and control of biped robots is to keep its balance during walking in different kinds of environments. The reason for instability is the under-actuation due to the passive joint of the foot–ground contact. This means that the control of the feet is dependent on the control of the mechanism that is above the feet.[6,48] Common stability theories, such as analysis of eigenvalues, gain and phase margins and the Lyapunov stability can be applied to particular modes of biped robot gait. However, these approaches cannot guarantee the biped stability for all modes of motion.[49]

In general, there are two types of stability criteria on which the trajectories of a biped mechanism depend: static stability and dynamic stability. Static stability restricts the vertical projection of the center of mass of the biped to remain inside the support polygon. The support polygon is defined as the area represented by the stance foot during the SSP and the bounded area between the supported feet during the DSP.[10] This type of stability leads to slow gait and biped robot with large feet.[50] Thus, the position of the center of mass, p_{com}, can be calculated as

$$p_{com} = \frac{\sum_{i=1}^{n_l} m_i p_i}{\sum_{i=1}^{n_l} m_i}, \tag{11}$$

where n_l is the number of biped links, m_i is the mass of link (i) and p_i is the position of the center of mass of link (i). The ground projection of p_{com} can be easily found by determining its components.

Dynamic stability provides more freedom than static stability since the projected center of mass of the biped may leave the support polygon and thus allows for faster gait.[50] Following are the four specific techniques to analyze dynamic stability[49]:

(a) ZMP, (b) the periodicity-based gait, (c) theory of capture points[51,52] and (d) the foot placement estimator.[53] In this paper we discuss the first two strategies.

4.1. Zero-moment point (ZMP)

The notion of ZMP was proposed by Vukobratovic and Stepaneko,[54] exploiting the passive joint of the foot–ground contact. It is applied to the biped mechanism in designing walking patterns and control schemes. The ZMP is the point on the ground where the net moment vector of the inertial

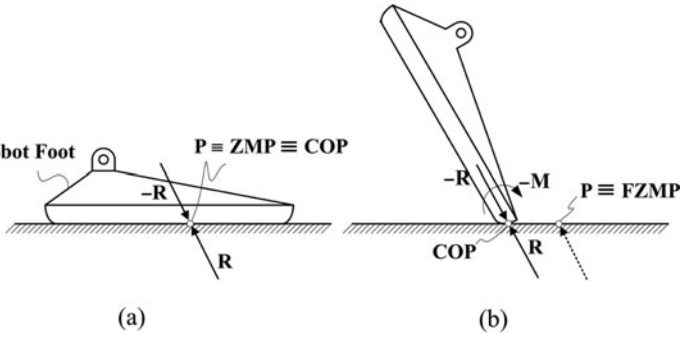

Fig. 3. Relationship between ZMP, FZMP and COP. (a) Dynamically stable, (b) dynamically unstable (this figure is cited from Vukobratovic and Borovac[6]).

and gravitational forces of the entire body has zero components in horizontal planes.[6,14,48] In brief, if ZMP is located inside the support polygon, then the system is stable and the center of pressure (COP) of the foot coincides with ZMP. If the ZMP is outside the support polygon, the system is unstable and the ZMP will be outside the stability margin comprising fictitious ZMP (FZMP)[6] as shown in Fig. 3. Point P in the mentioned figure represents the location of the zero components of the net moments affecting the foot at horizontal planes. It is clear that in the stable case, one can determine the position of ZMP by calculating the position of COP. This is conducted by using force sensors at the sole plate of the foot.[14] In effect, the theoretical calculation of ZMP can be performed by the following two formulations:

(a) Formulation 1 (more computational formulation)

By knowing the inertial and gravitational forces, the ZMP coordinate can be found as in Eqs. (12) and (13)[55]

$$x_{zmp} = \frac{\sum_{i=1}^{n} m_i (\ddot{z}_i + g) x_i - \sum_{i=1}^{n} m_i \ddot{x}_i z_i - \sum_{i=1}^{n} I_{iy} \ddot{q}_{iy}}{\sum_{i=1}^{n} (\ddot{z}_i + g)}, \tag{12}$$

$$y_{zmp} = \frac{\sum_{i=1}^{n} m_i (\ddot{z}_i + g) y_i - \sum_{i=1}^{n} m_i \ddot{y}_i z_i + \sum_{i=1}^{n} I_{ix} \ddot{q}_{ix}}{\sum_{i=1}^{n} (\ddot{z}_i + g)}, \tag{13}$$

where x and y axes are parallel to the horizontal plane for the biped motion, z-axis is pointing upwards, $(I_{ix}, I_{iy})^T$ is the inertial vector of link i, $\ddot{q}_{ix}$ and $\ddot{q}_{iy}$ are the angular acceleration of link (i) about x and y respectively, g is the gravitational acceleration, $(x_{zmp}, y_{zmp}, 0)$ is the coordinate of ZMP and (x_i, y_i, z_i) is the coordinate of the mass center of link (i). The above equations can be used to investigate the stability of biped mechanism during SSP and DSP. It has been noted that Eqs. (12) and (13) were described wrongly in most references as shown in Table I.

(b) Formulation 2 (less computational formulation)

Equations (10) and (11) are highly nonlinear; therefore, most researchers have used Eqs. (14) and (15) alternatively during the SSP using the static equations of fixed stance foot,[7]

$$x_{ZMP} = -\tau_y / F_z, \tag{14}$$

$$y_{ZMP} = \tau_x / F_z, \tag{15}$$

where τ_y and τ_x are the ankle joint torques about the referred axes and F_z is the normal component of the ground reaction force.

Thus, the necessary associated constraint is

$$-l_f \leq x_{ZMP}(t) \leq 0, \quad \text{(for sagittal plane)} \tag{16}$$

where l_f is the length of the stance foot assuming the origin of the Cartesian coordinate system placed at the tip of the stance foot.

Table I. Wrong ZMP equations used in the literature.

References	The ZMP equation
[17]	$x_{zmp} = \frac{\sum_{i=1}^{n} m_i\,(\ddot{z}_i + g)\,x_i - \sum_{i=1}^{n} m_i \ddot{x}_i z_i - \sum_{i=1}^{n} I_{iy}\ddot{q}_{iy}}{\sum_{i=1}^{n}(\ddot{z}_i + g)}$
	$y_{zmp} = \frac{\sum_{i=1}^{n} m_i\,(\ddot{z}_i + g)\,y_i - \sum_{i=1}^{n} m_i \ddot{y}_i z_i + \sum_{i=1}^{n} I_{iy}\ddot{q}_{iy}}{\sum_{i=1}^{n}(\ddot{z}_i + g)}$
[18]	$x_{zmp} = \frac{\sum_{i=1}^{n} m_i\,(\ddot{z}_i + g)\,x_i - \sum_{i=1}^{n} m_i \ddot{x}_i z_i - \sum_{i=1}^{n} I_{iy}\ddot{q}_{iy}}{\sum_{i=1}^{n}(\ddot{z}_i + g)}$
	$y_{zmp} = \frac{\sum_{i=1}^{n} m_i\,(\ddot{z}_i + g)\,y_i - \sum_{i=1}^{n} m_i \ddot{y}_i z_i - \sum_{i=1}^{n} I_{ix}\ddot{q}_{ix}}{\sum_{i=1}^{n}(\ddot{z}_i + g)}$
[56]	$x_{zmp} = \frac{\sum_{i=1}^{n} m_i\,(\ddot{z}_i - g)\,x_i - \sum_{i=1}^{n} m_i \ddot{x}_i z_i + \sum_{i=1}^{n} I_{iy}\ddot{q}_{iy}}{\sum_{i=1}^{n}(\ddot{z}_i - g)}$
[57]	$x_{zmp} = \frac{\sum_{i=1}^{n} m_i\,(\ddot{z}_i - g)\,x_i - \sum_{i=1}^{n} m_i \ddot{x}_i z_i - \sum_{i=1}^{n} I_{iy}\ddot{q}_{iy}}{\sum_{i=1}^{n}(\ddot{z}_i - g)}$
	$y_{zmp} = \frac{\sum_{i=1}^{n} m_i\,(\ddot{z}_i + g)\,y_i - \sum_{i=1}^{n} m_i \ddot{y}_i z_i + \sum_{i=1}^{n} I_{ix}\ddot{q}_{ix}}{\sum_{i=1}^{n}(\ddot{z}_i + g)}$
	The authors have considered $g = -9.8m/s^2$.
[21]	$x_{zmp} = \frac{\sum_{i=1}^{n} m_i\,(\ddot{z}_i + g_z)\,x_i - \sum_{i=1}^{n} m_i(\ddot{x}_i + g_x)z_i}{\sum_{i=1}^{n}(\ddot{z}_i + g_z)}$
	$y_{zmp} = \frac{\sum_{i=1}^{n} m_i\,(\ddot{z}_i + g_z)\,y_i - \sum_{i=1}^{n} m_i(\ddot{y}_i + g_y)z_i}{\sum_{i=1}^{n}(\ddot{z}_i + g_z)}$
	where $g_z = -g$ for level ground

Since ZMP coincides with COP, the ZMP coordinate can be found during DSP according to Eq. (17),[58]

$$ZMP = COP = cop_f \frac{F_{zf}}{F_{zf} + F_{zr}} + cop_r \frac{F_{zr}}{F_{zf} + F_{zr}}, \tag{17}$$

where COP, cop_f and cop_r represent the center of pressure for the biped robot during DSP, the front foot COP and the rear foot COP respectively, F_{zf} and F_{zr} are normal components of ground reaction forces for front and rear feet respectively.

In general, the ZMP-based biped mechanisms are characterized by the following points.

- The stance foot of the biped should remain in full contact all the time.[59]
- All the joints of the biped mechanism are actuated and rigidly controlled to track predetermined trajectories that can simplify the control task during the SSP.
- Unnatural motion with high-energy consumption.
- Most conventional humanoid biped robots are always driven by electric motors via gears, and their walking patterns are generated based on the linear inverted pendulum mode (see Section 5.1.3 and Table II).

Table II. The most distinctive biped/humanoid robots built in different countries with their characteristics.

Year	Country	Structure	Gait speed (m/s)	Stability criterion	Walking pattern generators
1973	Japan/Waseda University	WABOT-1 is a hydraulically powered biped walking robot with two 5-DOF legs, a 1-DOF trunk and artificial hands.[163]	0.0043	Static	Model-based gait[a]
1980	Japan/Waseda University	Wl-9DR is a hydraulically powered biped walking robot with total 10 DOFs.[5,163]	0.045	Static/quasi-dynamic walking	Model-based gait[a]
1980–1984	Japan/University of Tokyo	A series of BIPER biped robots have been developed. BIPER-3 is a biped robot with four actuators at the hip and point feet. While BIPER-4 is equipped with eight actuators (4 actuators for each leg), BIPER-5 is similar to BIPER-3.[128]	-	Periodic	COG-based gait
1982–1985	USA/CMU Leg Laboratory	A series of running legged (one-legged, biped and quadruped) prototypes have been developed. The planar biped robot has 3 DOFs with tether boom for constraining the biped motion in 2.5-m radius circle.[5]	4.3	Periodic	Natural dynamics-based gait
1985	Japan/Waseda University	WL-10RD is a 12-DOF hydraulic-powered biped walking robot with microcomputers installed on the upper body.[63]	1.3 s/step	ZMP	Model-based gait[b]
1990	Canada/T. McGeer	A 2D passive (gravity-based powered on a shallow incline) walker with two straight legs (without knees) and semicircular feet.[78,164] In addition, the knee-jointed legs have been designed.[79]	0.46	Periodic	Natural dynamics-based gait
1996	Japan/Honda R&D Co.	A series of Honda humanoid robots have been developed. In 2000, the updated version called ASIMO was developed with a height of 1.2 (m), a weight of 43 (kg) and electrically actuated (DC servomotors with harmonic drive gears) 34 DOFs (3 DOFs for the head, 7 DOFs for each arm, 6 DOFs for each leg and 1 DOF for each hand). In 2011 the updated version of ASIMO was announced with 57 DOFs.[81,95,165–167]	Walking: 0.75, running: 2.5	ZMP	COG-based gait/model-based gait
1996	Japan/Waseda University	Wabian family has been created. The updated version is called Kobian with about 1.4 (m) tall, 62 (kg) in weight and 48 electrically actuated (DC motors/harmonic drive gears) DOFs (12-DOF legs, 3-DOF waist, 14-DOF arms, 8-DOF hands, 4-DOF neck and 7-DOF face).[168–173]	0.5	ZMP	Model-based gait[b]
1996–2000	USA/MIT	Spring Flamingo is a planar biped with 13.5 (kg) weight and 0.9 (m) height, and six electrically actuated (series elastic actuator) DOFs (3 DOFs for each leg).[160–162,174–177]	0.75–1.2	Capture point/ periodic	Natural dynamics-based gait
1997	France[c]	Rabbit is a planar five-link biped robot with 4 degrees of actuation (two actuators at the hip and two at the knees) and point feet.[100,178,179]	Walking: 0.75,[d] running: 1.25	Periodic	Optimization-based gait/model-based gait

Table II. Continued.

Year	Country	Structure	Gait speed (m/s)	Stability criterion	Walking pattern generators
1998	USA[e]/Cornell University	Tinker toy walking model is a two-leg toy with no control system and gravity-based power.[80]	0.0277	Periodic	Natural dynamics-based gait
2001–2006	Japan[f]/Sony Co.	A series of SDR humanoid biped robots have been developed. The latest version is QRIO (SDR-XII) with small-scale size (height of 0.58 (m) and weight of 6.5 (kg)) and 38 electrically actuated (servo motor with gear unit) DOFs (4 DOFs for the head, 2 DOFs for the torso, 5 DOFs for each arm, 6 DOFs for each leg and 10 DOFs for the fingers).[180–186]	0.23	ZMP	COG-based gait/model-based gait
2001	USA/Cornell University	A 3D-passive dynamic walker with two legs, knees, curved feet, compliant heels and two swinging arms for stable gait.[155]	0.51	Periodic	Natural dynamics-based gait[g]
2002	Japan/AIST and KAWADA Industries Inc.	HRP family (HRP-2P, HRP-2, HRP-3P, HRP-3, HRP-4C and HRP-4) has been developed. HRP-4C is a humanoid biped robot with Japan female appearance, weight of 43 (kg), length of 1.58 (m) and electrically actuated (DC servomotors with harmonic drive gears) 42 DOFs (6 DOFs for each arm, 2 DOFs for each hand, 6 DOFs for each leg, 3 DOFs for the waist, 3 DOFs for the neck and 8 DOFs for the face). While the updated version HRP-4 is characterized by weight of 39 (kg), length of 1.51 (m) and 34 DOFs (2 DOFs for the neck, 7 DOFs for each arm, 2 DOFs for each hand, 2 DOFs for the waist and 6 DOFs for each leg).[187–197]	HRP-4C: 0.5, HRP-2: 0.694	ZMP	COG-based gait/model-based gait
2002	Japan/Waseda University	WL-15, WL-16, WL-16R and WL-16RII have been developed with electric-powered 6-DOF parallel mechanisms, two legs and a waist for multi-purpose use.[198,199]	0.2500	ZMP	Model-based gait[b]
2003	The Netherlands[h]/Delft University	Mike is a 2D passive dynamic walker with weight of 7 (kg), length of 0.75 (m) and McKibben muscles (pneumatic actuators) at the hip and knee joints for active control. It is similar to the close copy of McGeer by Garcia *et al.*[200,201]	0.4	Periodic	Natural dynamics-based gait
2003	Germany[i]/Technisce Universität München	Johnnie is a humanoid biped robot with length of 1.8 (m), weight of 40 (kg) and electrically actuated (DC brush motor with harmonic drive gears) 17 DOFs (2 × 3 DOFs for the hip, 2 × 1 DOFs for the knees and 2 × 2 DOFs for the ankles).[98,99]	0.6667	ZMP	COG-based gait/model-based gait

Table II. Continued.

Year	Country	Structure	Gait speed (m/s)	Stability criterion	Walking pattern generators
2004	USA/MIT	Toddler is a 3D passive dynamic walker with a single passive joint at the hip, two active joints at each ankle (pitch and roll) and large curved feet.[202]	0.2238	Periodic	Natural dynamics-based[202] gait/reinforcement learning control based on optimization strategy[203]
2004	The Netherlands/Delft University	Denise is an autonomous 3D biped robot with 8 (kg) weight, 1.5 (m) tall, five internal DOFs (1 × 2 DOFs for ankles which are equipped with spring and damper, 1 × 2 DOFs for passive knees and 1 DOF for pneumatically actuated (McKibben muscles) hip).[204,205]	0.4	Periodic	Natural dynamics-based gait
2004	Republic of Korea[j]/KAIST	A series of KHR humanoid biped robots have been developed. The updated version is HUBO with height of 1.25 (m), weight of 55 (kg) and 41 electrically actuated (servo motor + harmonic speed reducer + drive unit) DOFs (6 DOFs for each leg, 4 DOFs for each arm, 6 DOFs for the head, 7 DOFs for hand and 1 DOF for the trunk).[206–214]	0.3472	ZMP	COG-based gait/model-based gait
2004	China/Beijing Institute of Technology	BHR humanoid biped robot family has been developed. The updated version is called BHR-2 with 1.6 (m) length and 63 (kg) weight and 32 electrically actuated (DC motors) DOFs (2 × 3 DOFs for hips, 2 × 1 DOFs for the knees, 2 × 2 DOFs for the ankles, 2 × 3 DOFs for the shoulder 2 × 1DOFs for the elbows, 2 × 2 DOFs for the wrists, 2 × 3 DOFs for the hands and 2 DOFs for the head).[215,216]	0.1389	ZMP	COG-based gait/model-based gait
2005	USA/Cornell University	Cornell's biped is a 3D passive dynamic walker with 12.7 (kg) in weight, two arms, two 0.81-(m) long legs, a small upright torso with a leg-angle-bisecting mechanism and wide curved feet. There are 5 internal DOFs: one free hip joint, two periodically locked knees and two controlled ankle joints.[102,103]	0.44	Periodic	Natural dynamics-based gait
2006	Germany/Technische Universität München	LOLA is a humanoid biped robot with 1.8-(m) height and equipped with electrically actuated (permanent magnet synchronous motors PMSM) 22 DOFs (2 × 7 DOFs for the legs, 2 × 3 DOFs for the arms and 2 DOFs for torso).[217]	0.9278	ZMP	COG-based gait/model-based gait
2006	Japan/Toyota Central R&D Labs. Inc. and Toyota Motor Corp. Inc.	A humanoid biped robot has been developed with 1.30-(m) height, 50 (kg) weight and 15 DOFs (7 DOFs per leg and 1 DOF for the waist).[96,97]	Running: 1.9444	ZMP	COG-based gait/model-based gait

Table II. Continued..

Year	Country	Structure	Gait speed (m/s)	Stability criterion	Walking pattern generators
2009	Spain[k]/University Carlos III of Madrid	The RH-1 is a humanoid biped robot with 1.50-(m) height, 50 (kg) weight and 21 electrically actuated (servo motors) DOFs (6 DOFs for each leg, 4 DOFs for each arm and 1 DOF for the chest).[218–220]	0.194	ZMP	COG-based gait/model-based gait

[a] The walking patterns of the biped robot have been performed by constraining the COM of biped to be within the stability margin. The interpolation technique could be used for generation stable swing foot trajectory.

[b] To find appropriate mathematical relationship between the COG of the biped robot and the ZMP trajectory, Eqs. (10) and (11) were approximated using FFT. The interpolation technique was used for generating stable swing foot trajectory.

[c] There are other prototypes built in France, such as BIP2000,[221–228] SHERPA biped robot[229] and the amazing small-scale humanoid robot NAO.[230,231]

[d] RABBIT has been designed to be able to walk with average forward speed of at least 5 (km/h) and to run more than 12 (km/h).

[e] For more details on the prototypes built by Ruina and colleagues, we refer to the website Andy Ruina-Biorobotics and Locomotion Lab – Cornell University: http://ruina.tam.cornell.edu/research/index.php

[f] HR18[232,233], Morph[234] and PINO[235,236] are examples of small-scale humanoid robots manufactured in Japan.

[g] To make the passive dynamic robots walk on level ground, the authors have substituted the gravitational power with simple actuation at the hip and/or the ankles.[102]

[h] There are other prototypes built by Delft Lab (please see Delft Biorobotics Lab: http:// www. dbl.tudelft.nl/).

[i] There are other biped prototypes manufactured in Germany such as the small-scale humanoid robots TONI,[237] Kidsize[238,239] etc.

[j] AM12 is a humanoid biped robot manufactured in Republic of Korea based on ZMP criterion.[240]

[k] Other examples of Spanish humanoid biped robot are SILO2[241–243] and REEM-B.[244]

Several algorithms have been used to generate stable reference trajectory for the biped mechanism satisfying the ZMP constraint (see Section 5). This implies that the solution of Eqs. (12) and (13) to find a relationship between the COM of the biped and the ZMP trajectory requires a lot of computational effort. Therefore, these algorithms can be applied offline.[60,61] Alternatively, most researchers have used simple models to generate the desired biped walking patterns, such as the linear inverted pendulum for online implementation (see Section 5.1.3). Biped robot stability approaches based on ZMP still lack efficiency, robustness, easy handling and natural motion.[59,62] For further reading, we refer to refs. [18, 63–72].

4.2. *Periodicity-based gait*

For more than 40 years, researchers have tried to understand the locomotion of biped mechanism using the inverted pendulum model due to similarity between the two mechanical systems both in static and periodic stability.[26,73–77] This demands the investigation of the phase-plane portraits of generalized coordinates and velocities to see the periodic (cyclic/orbital) stability of the inverted pendulum.

In 1990, McGeer[78] designed a two-dimensional passive walker with two straight legs (without knees) and semicircular feet. In addition, the knee-jointed legs were designed to imitate the human-like motion on shallow slops without using any actuators.[79] The stability of this simplified prototype adopted the Poincare map to investigate periodic stability.

The periodic stability exploits the under-actuation resulted from the passive joint of the foot–ground contact. It is noted that the Hamiltonian dynamical systems cannot have asymptotic stability whereas the conservative non-holonomic systems can have asymptotic steady stability in some variables.[80] Thus, the biped robots based on periodic stability can perform cyclic sequences of steps that are stable as a whole, but not locally at every instance of time.[59] The Poincare map is used to show the stability of this type of walking as described below.

The transition of the current biped state $\boldsymbol{v}_k$ to the successive state after one walking step $\boldsymbol{v}_{k+1}$ can be described by the stride function **S** as follows:

$$\boldsymbol{v}_{k+1} = \boldsymbol{S}(\boldsymbol{v}_k). \tag{18}$$

If the motion is perfectly cyclic, then the state $\boldsymbol{v}$ is a fixed point corresponding to the limit cycle,

$$\boldsymbol{v}_f = \boldsymbol{S}(\boldsymbol{v}_f). \tag{19}$$

Orbital stability is investigated by using a perturbation around the initial fixed point,

$$\boldsymbol{S}\left(\boldsymbol{v}_f + \Delta\boldsymbol{v}\right) \approx \boldsymbol{v}_f + \boldsymbol{P}\Delta\boldsymbol{v}, \tag{20}$$

where $\boldsymbol{P} = \frac{\partial \boldsymbol{S}}{\partial \boldsymbol{v}}$ denotes the linear return matrix which governs the orbital stability of the biped robot. If the eigenvalues of **P** are within the unit circle of the complex plane, the biped system is stable, otherwise it is unstable.[59]

In 1996, Honda R & D Co. announced the first self-contained humanoid biped robot called P2.[81] It was 1.82-m tall with 210-kg weight and depends on ZMP criterion to generate its desired stable trajectory. Consequently, a lot of energy can be consumed with unnatural motion due to bent knees. This motivates the community of the passive dynamic walking to re-examine and extend the work of McGeer;[82–93] see Table II for more details.

The periodicity-based biped robots can be characterized by the following points[94]:

- They can have human-like efficiency and actuation requirements.
- Their motions are mostly symmetrical and are extremely difficult to turn, go back, sit etc.
- Compliance plays an important role in regulating walking behavior.
- The velocity of the robot can be controlled by actuating hip and/or ankle joints.

Remark. Although it is well known that the periodic-based gait can be faster than the gait based on the ZMP criterion, ASIMO[95] and Toyota humanoid[96,97] (ZMP-based humanoids) can run with a speed of 9 and 7 km/h respectively, while Johnnie humanoid[98,99] can only walk with a speed of 3.34 km/h. In effect, literature can prove contrary among passive walking researchers related to the

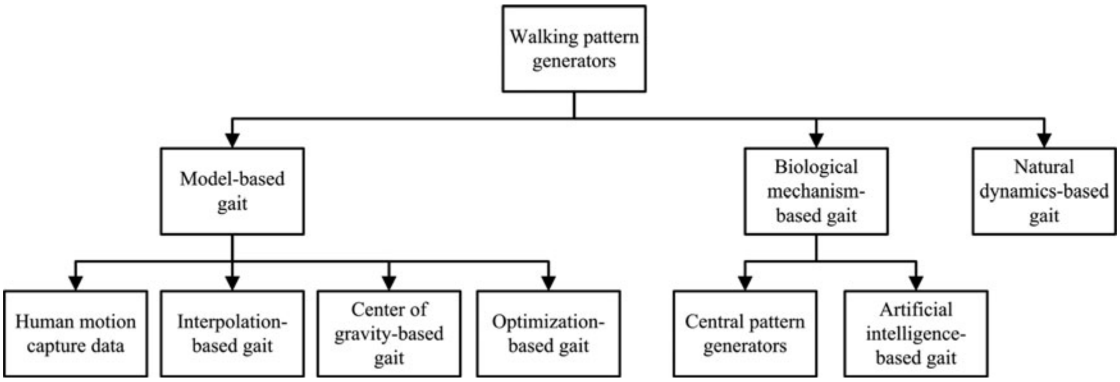

Fig. 4. Classification of the approaches used for generating walking patterns of biped robot.

speed of passive walking against ZMP-based gait. Chevallereau *et al.*[100] said: "... truly dynamic motions, such as balancing, running, or fast walking, are clearly excluded with this (ZMP) approach," whereas Hosoda *et al.*[94] said: "The velocity of the robot can be controlled by actuating hip and/or ankle joints, but the range of the velocity is relatively limited since the behavior of the robot is strongly governed by its passive dynamic."

Remark. The walking speeds of the human, ASIMO and Cornell's Biped are about 1.3 m/s (optimal speed for human[101]), 0.75 m/s and 0.44 m/s respectively. The energy efficiency of the biped locomotion is measured by the specific cost of transport, i.e. (energy used)/(weight) × (distance traveled). Thus, the energy efficiency of the human, ASIMO and Cornell's Biped are (0.2, 3.2 and 0.2) respectively.[102,103]

Methods for generating periodicity-based gait are as follows: central pattern generators (CPGs)-based gait (Section 5.2.1), using self-excited mechanism,[62] optimization techniques[104–106] or exploiting natural dynamics[49] (Section 5.3).

5. Walking pattern generators

One of the important issues of the biped locomotion is the generation of the desired paths that ensure stability while avoiding collision with obstacles.[56] Due to the similarity between the biped robot and the human locomotion, some important aspects should be considered in order to generate natural biped locomotion, which are as follows[107]:

- Learning (training), which needs a certain level of intelligence.
- A high level of adaptability to cope with uneven terrains and external disturbances.
- In specific circumstances, optimal motion to reduce energy consumption during walking.

It is hopeful to make the biped robot compete with the human system. Consequently, it needs at least an online adaptation system to deal with uncertain environments.[108,109] In contrast with this, there are many researchers who have attempted to design biped models using predefined reference trajectories which are insufficient for stability and adaptability purposes (Section 5.1.1 is an example of offline walking patterns). Online walking patterns require knowledge of the path changes online while guaranteeing stability and motion control of biped mechanism.[109] Examples of methods used to compensate ZMP online include preview control,[110] model predictive control,[109,111,112] and AI-based gait.[56,113] In general, the model-based methods can give useful explanations about the behavior of human walking. In contrast, AI can give robust results without explanation.[10,114] The first two methods are described in Section 5.1.4, while the latter method is discussed briefly in Section 5.2.2. Surely, there are other approaches used in the literature; however, the mentioned approaches serve as the typical online schemes so far.

There are numerous approaches of generating biped walking patterns. We have established an overview of walking pattern generators depending on refs. [7, 10, 49, 115]. These approaches can be classified as illustrated in Fig. 4.

5.1. Model-based gait

This method uses analytical schemes for the generation of reference trajectories of joints for biped mechanism. Then a robust control system is performed to track the desired trajectories.[7] The limitations of the model-based technique are characterized by the need of the full knowledge of the parameters of the dynamic biped model. These approaches are possibly accompanied with high burden of computations and large disturbances and noises resulted from measurement devices.

5.1.1. Human motion-capture data. One of the methods to develop walking pattern generators is to use the human motion-capture data that allows to extract recorded motion data from the examination of human gait, and provide this data for the target biped robot or the handicapped.[116–119] This method is implemented by attaching markers at the limbs and joints of a human subject to capture human movement. Once recorded, these data are processed numerically to get the coordinates of the limbs and joints from which the velocity and acceleration can be obtained. Hemami and Farnsworth[119] have applied this technique on a planar 5-link biped robot to analyze the posture and gait stability of their model. They have concluded that this method is useful in the designing of prosthetic devices for handicapped and in tuning the model to mimic human movement.

During walking, the human center of gravity oscillates vertically and horizontally. The vertical oscillation adopts a cycloid path with an amplitude of about 75 mm, while the horizontal oscillation is a sinusoidal path with an amplitude of about 30 mm.[10] Therefore, some researchers have exploited this feature to provide directly the hip and the feet of the robot model with sinusoidal or cycloid functions. Although this technique does not include human motion-capture data, it is integrated in this section for two reasons. First, the sinusoidal trajectories provided for the biped robot are inspired from the human gait. Second, the two described techniques have the same limitations that will be illustrated at the end of this section. Juang and Lin[120] used a cycloid profile developed by Kurematsu *et al.*[121] in generating the trajectories for the hip and the ankle joints during the swing leg, while Nicholls[50] used sinusoidal functions to generate paths for the feet and the trunk. However, there exist kinematic and dynamic inconsistencies, such as length, mass etc., between the human subject, whose motion is captured, and the proposed model of the biped robot. In fact, the human subject has a high level of training, adaptability and optimal motion, which are lost in biped robots.[7]

5.1.2. Interpolation-based gait. This method is widely used in generating reference trajectories for the end-effectors of industrial robotic arms. It describes the trajectory of the robot manipulator by a function such as a polynomial satisfying certain continuity conditions and guaranteeing smooth motion for the manipulator.[122] The interpolation can be developed in computer animation for describing the motion of a sequence of images by interpolating the intermediate key-frames. Brotman and Netravali proposed that this method can produce unnatural motion due to the absence of the dynamics of the problem.[123] Alternatively, the authors used the optimal control theory for this purpose.

We should notice that the interpolation of motion for biped robots is associated with either the optimization problem or stability criteria such as ZMP or both of them. Shih[124] realized the walking of 3D biped robot in different environments. The reference trajectories of the biped robot body and its feet have been generated by a third-order piecewise cubic polynomial. Huang *et al.* proposed a method to generate walking patterns for the hip and the feet of biped mechanism.[17,18] The authors generated the foot trajectory by using a third-order spline function considering the constraints of foot trajectories. By adjusting the parameters of the foot trajectory, different patterns of foot motion can be realized. The hip trajectory has been produced by a third-order spline function such that it can realize the desired ZMP trajectory. A similar work was implemented by Mu and Wu.[125]

5.1.3. Center of gravity (COG)-based gait. Due to the complex nonlinear dynamics of biped robot, it is difficult to obtain a closed form solution. Therefore, many researchers have focused their attention on simplifying the complex dynamics of biped robots in order to use simple algorithms for generating walking patterns and control. As aforementioned, the inverted pendulum model is used as an approximate model for biped mechanism due to the similarity between two mechanical systems in terms of static and periodic stability. However, this approximation can be connected with the periodic stability of the inverted pendulum. Kajita and Toni[126,127] proposed the linear inverted pendulum mode (LIPM) exploiting the previous related works[5,26,73–77,128]. The LIPM includes finding a simple mathematical relationship between the trajectory of COG of the biped robot and the ZMP to ensure stable biped motion. It is assumed that all masses of the biped model are concentrated in its COG and there is a reaction force between the ground and the foot without ankle torque.[61,129] Depending on the reference trajectories of COG and the feet, all the motions of the joints can be deduced. Thus, the relationship between COG and ZMP during the SSP can be computed as[7,130–132]

$$x_{ZMP} = x_{COG} - \frac{H}{g}\ddot{x}_{COG}, \quad \text{(for sagittal plane)} \tag{21}$$

where x_{COG} is the position of the COG, and H is the height of COG, which is assumed fixed. Consequently, the COG trajectory motion during SSP can be calculated solving differential Eq. (21),

$$x_{COG} = C_1 exp(wt) + C_2 exp(-wt), \tag{22}$$

where C_1 and C_2 are constants that can be obtained from the boundary conditions, and

$$w = \sqrt{g/_H}. \tag{23}$$

In a similar manner, the COG-based model can be used successfully for the modeling of biped mechanism during DSP. For details, we refer to refs. [130–132].

The LIPM concept could be accepted for quadrupeds due to their slender legs. However, one-third of the mass of human subjects is contained in the legs. Therefore, it is questionable to use LIPM in bipeds.[8] Therefore, the preview control described in the next section is used as a servo control technique for compensating the ZMP error produced by the COG-based model. Consequently, Park and Kim[61] suggested to use the gravity-compensated inverted pendulum mode (GCIPM) with two proposed masses to reduce the errors accompanied by LIPM. One mass denotes the dynamics of the swing leg and the other represents the stance foot and the trunk. Other modifications to LIPM have been proposed to avoid the deviation of the desired trajectories of ZMP. These modified approaches include the Two Masses Inverted Pendulum Mode (TMIPM),[133] the Multiple Masses Inverted Pendulum Mode (MMIPM)[134] and the Virtual Height Pendulum Mode (VHIPM).[135] For a comprehensive comparison of the mentioned methods, we refer to ref. [135].

5.1.4. Optimization-based gait. Studies have proved that human beings attempt naturally to minimize their energy consumption during walking according to the environment.[108] Consequently, many researchers have applied miscellaneous optimization schemes on biped robot to get feasible optimal design. One should mention that the optimization could be applied to the construction of the biped robot, its control system or its adaptation to the motion terrain etc.[2] For an elaborated discussion of optimization approaches used in robotics, we refer to refs. [2, 136, 137]. This section discusses briefly the optimal trajectory of the biped robot using different optimal control methods. In general, the optimization technique includes the selection of a suitable cost (objective) function that satisfies some definite constraints such as the step length, the foot clearance, the hip height, or the speed of walking. Then a set of parameters can be found to achieve the optimal path.[108]

In general, the optimal control problem can be classified as: dynamic programming, indirect methods and direct methods as shown in Fig. 5. Although the dynamic programming is less sensitive to the initial guess of design parameters, it suffers from the curse of dimensionality.[138] The indirect approach represented by Pontryagin's Maximum Principle (PMP) demands necessary conditions for optimality that can result in highly nonlinear Ordinary Differential Equations (ODEs) and the two-boundary value problem.[139,140] In addition, the indirect approach is extremely sensitive to the initial guess of the co-state equations. In effect, the indirect approach can be intricated for complex dynamic systems such as biped robot.[10,141] Despite this difficulty, Rostami and Bessonnet[142] and Bessonnet *et al.*[143] investigated the optimal motion of biped robot during SSP and during the complete gait cycle respectively using PMP, assuming the boundary conditions of the biped robot are known.

A more flexible method for the optimal control problem is the direct method, by transcribing the infinite dimension problem into the finite-dimensional nonlinear programming (static or parameter optimization). This can be implemented by the discretization of controls or states or both, depending on the selected discretization approach. The solution uses one of the nonlinear programming algorithms such as sequential quadratic programming (SQP), interior points, genetic algorithm

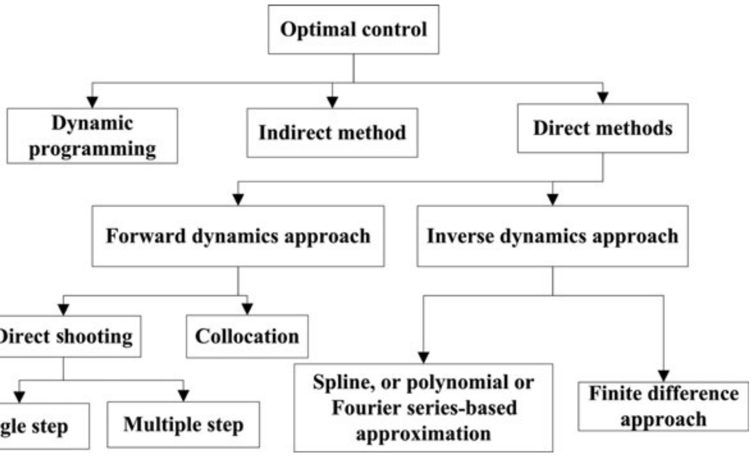

Fig. 5. Classification of optimal control methods.

(GA) etc. Although being easy and robust, this method can only give suboptimal/approximate solution.[139,140,144,145] The formulation of the original optimal control problem can be described as follows:

Determine: $\boldsymbol{u}$.

$$\text{Minimize}: \quad J = c_0(\boldsymbol{x}, t) + \int_{t_0}^{t_f} L(\boldsymbol{x}(t), \boldsymbol{u}(t), t)\, dt. \tag{24}$$

Subject to:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(x(t), \boldsymbol{u}(t), t), \tag{25}$$

$$\begin{gathered} \boldsymbol{a}_1(\boldsymbol{x}(t_0), \boldsymbol{u}(t_0), t_0) \le \mathbf{0} \\ \boldsymbol{a}_2(\boldsymbol{x}(t_0), \boldsymbol{u}(t_0), t_0) = \mathbf{0}, \end{gathered} \tag{26}$$

$$\begin{gathered} \boldsymbol{b}_1(\boldsymbol{x}(t_f), \boldsymbol{u}(t_f), t_f) \le \mathbf{0} \\ \boldsymbol{b}_2(\boldsymbol{x}(t_f), \boldsymbol{u}(t_f), t_f) = \mathbf{0}, \end{gathered} \tag{27}$$

$$\begin{gathered} \boldsymbol{c}_1(\boldsymbol{x}(t), \boldsymbol{u}(t), t) \le \mathbf{0} \\ \boldsymbol{c}_2(\boldsymbol{x}(t), \boldsymbol{u}(t), t) = \mathbf{0}, \end{gathered} \tag{28}$$

$$\begin{gathered} \boldsymbol{u}_l \le \boldsymbol{u}(t) \le \boldsymbol{u}_u \\ \boldsymbol{x}_l \le \boldsymbol{x}(t) \le \boldsymbol{x}_u, \end{gathered} \tag{29}$$

where $\boldsymbol{u} \in \mathbb{R}^n$ is the input control vector, c_0 and L are scalar functions of the indicated arguments, J is the scalar performance index, $\boldsymbol{x} \in \mathbb{R}^n$ is the state vector, t, t_0 and t_f are the time, initial and final time respectively, $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ are the initial constraints, $\boldsymbol{b}_1$and $\boldsymbol{b}_2$ are the final constraints, $\boldsymbol{c}_1$ and $\boldsymbol{c}_2$ are the path constrains and Eq. (29) refers to the bound constraints of the input control and the states.

The formulation of the discretized optimal control problem can be described as a nonlinear programming as follows:

Determine: $\boldsymbol{Y}$, which may be control variables or states or both.

$$\text{Minimize}: \quad J = c_0(\boldsymbol{x}(t_N)) + \sum_{k=0}^{N-1} l_k(\boldsymbol{x}(t_k), \boldsymbol{u}(t_k), t_k)\, \Delta t, \tag{30}$$

where N is the number of time intervals and $\Delta t = (t_f - t_0)/N$. Equation (30) can be solved by any numerical integration approach such as the trapezoidal or composite Simpson's rule etc.

Subject to:

$$\boldsymbol{Z}(\boldsymbol{Y}) = \boldsymbol{0}, \tag{31}$$

$$\boldsymbol{C}_l \le \boldsymbol{C}(\boldsymbol{Y}) \le \boldsymbol{C}_u, \tag{32}$$

$$\boldsymbol{Y}_l \le \boldsymbol{Y} \le \boldsymbol{Y}_u. \tag{33}$$

Due to the complexity of the computation of these approaches, these may not be appropriate for on-line implementations. Moreover, the trajectories obtained may not be the true optimal path. This scenario may occur either because of the imperfect model, or the solution of the optimization problem may get stuck at the local minima.[56,108] Therefore, a group of researchers prefer to use intelligent methods for generating the desired paths. This will be discussed in the next section.

There are two interesting approaches associated with the optimal control problem that can give a feasible solution for online walking adaptation and rejection of disturbances. These two approaches are the preview control and the model predictive control. The objective of the preview control is to improve the performance of transient responses of the dynamic system when the future reference signals or the disturbances are available.[7] Kajita *et al.*[110] designed a new method for generating online walking patterns using the preview control approach of ZMP. The authors approximated the biped model with a cart-table model for a simple treatment with ZMP. Then they used the preview control to compensate the ZMP errors produced by the differences between the proposed model of the biped robot and the actual multi-body model. Alternatively, the model predictive control (MPC) is an optimal control-based approach that can deal with constrained dynamic systems for realizing online walking or any affecting disturbance.[109,111,112] It is a notable approach for walking adaptation. Using MPC, the biped mechanism can be controlled without using predetermined reference trajectories.

5.2. Biological mechanism-based gait

5.2.1. Central pattern generators (CPGs). Human and animal locomotion can generate stable rhythmic patterns of movements. They have versatile capabilities to change their speed or even their patterns of movements.[146] The central pattern generators in the spinal cord, which are neural networks (NNs), can produce these locomotion rhythms without any sensory signals.[146–148] Therefore, researchers have exploited this biological mechanism with feedback controllers to obtain stable natural gaits for legged robots. There are two commonly used models adopted by researchers for modeling CPGs, which are the Matsuoka neural oscillator[139] and the Van der Pol oscillator.[60,147,149–151] We will not discuss here the details of the mathematical models of CPGs; instead, we will mention some merits and disadvantages of this method. The CPGs-based gait does not need the full knowledge of dynamic biped model. In addition, a legged robot that adopts CPGs for generating walking patterns is governed by the periodicity-based gait (see Section 4.2). Furthermore, CPGs exhibit natural human-like motion with less energy consumption. However, there are some limitations included in the actuator capability,[149] and in the highly unstructured environments.[113] For further reading, we refer to Yang *et al.*[152] and Ijspeert.[153]

5.2.2. Artificial intelligence-based gait. Artificial intelligence approaches include neural networks, fuzzy logic (FL) and genetic algorithms (GA). In these approaches, the designer does not need a precise dynamic model to solve target problem. Although AI-based approaches can work in specific circumstances, they cannot create reliable walking patterns[114] because there might be no explanations of how onegets these interesting results.

Nonetheless, the difficulties encountered in the generation of biped walking patterns using model-based methods can justify the use of AI-based approaches by many researchers. These difficulties could include the resulting disturbances and noises of measurement devices and the possible computational complexity of gait planning. Vundavilli and Pratihar[56] made an interesting comparison study between analytical and AI methods for the gait generation of a planar 7-link biped robot. First, they solved the problem by using analytical methods, including inverse kinematics and static balance to generate the reference trajectories of the biped joints during walking in different environments. Then the same problem was solved by using NN- and FL-based approaches respectively. The databases of NNs and FL have been optimized using GA off-line. The authors have proposed that the AI-based approaches are accessible to be used online. For a detailed literature review of this interesting topic, we refer to Katic and Vukobratovic.[154]

Table III. Comparison of the methods of the walking pattern generators for biped walking.[152].

The approach	Advantages	Disadvantages
Model-based gait	1. It provides explanations about the behavior of human walking. 2. It can adopt ZMP and periodicity as stability criteria. Consequently, it can guarantee the dynamic stability of biped mechanism.	1. It needs full knowledge of the dynamic model of biped robot. 2. The online walking algorithms require large computations. 3. It rarely employs the natural dynamics of the biped system. 4. Information of the motion terrain should be known.
Biological mechanism-based gait	1. It does not need precise modeling of biped mechanism. 2. Online walking algorithms could be applied easily 3. It is robust to disturbances. 4. It can guarantee the stability of biped mechanism.	1. The limitations of CPG are widely investigated in Yang *et al.*[152] 2. Significant results are obtained without explanations.
Natural dynamics-based gait	1. It employs the natural dynamics of the biped system. 2. Less energy consumption could be produced. 3. It can produce natural motion.	1. No unified strategies could be adopted to achieve the desired results. 2. All strategies depend on the experiences of the designer.

5.3. Natural dynamics-based gait

This type of gait does not need predefined reference trajectories for biped mechanism and it is designed according to the natural dynamics of the biped. This strategy is used extensively to generate the natural human-like passive dynamic walkers.[155–157]

The intuitive control achieves the walking patterns' generation and the control of active biped robots using the physics of the system and the virtual elements such as springs or dampers.[158–162] Pieter van Zutven *et al.*[49] proposed a strategy citing intuitive control for designing periodicity-based gait. The authors used four controllers for this purpose. The first one called "controller symmetries" enforces the biped robot to imitate the passive dynamic walker compensating the actual gravity force of the biped system. The second controller prevents feet scuffing, while the third one adds some compliance for the ankle joint. The last controller regulates the size of walking steps.

The advantage of the intuitive control base gait is that it is easy to apply and is a powerful method so that the walking of the biped robot is characterized with natural appearance having less energy consumption. However, there are no unified strategies that a designer can follow to achieve desired results. Most of these strategies depend on designer's experience.[7,162]

6. Conclusions

In this paper, we attempted to collect and introduce a systematic discussion on the modeling, stability and walking patterns of biped robots. The subject is complicated to deal with, because it is interdisciplinary. An important remark one should notice is that the designer should decide in advance the stability criterion that he/she may use for generating walking patterns of biped mechanism. After that he/she should select the suitable approach described in Fig. 4 for planning the motion of biped locomotion. Finally, the controller structure should be selected. Table II illustrates with a sequential history the most distinctive biped/humanoid robots manufactured in different countries with their characteristics. Table III shows the advantages and disadvantages of the methods for the walking pattern generators of biped robots.

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