OPTIMAL MONETARY AND FISCAL POLICIES IN A SEARCH-THEORETIC MODEL OF MONEY AND UNEMPLOYMENT

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In this paper we study optimal policies in an environment where search frictions both in labor and goods markets give rise to unemployment and fiat money, as in Berentsen, Menzio, and Wright (*American Economic Review*, 2011). The underlying frictions that give rise to endogenous unemployment and fiat money also result in inefficient outcomes. Here we show that efficiency can be restored whenever lump sum monetary transfers are possible and a decentralized production subsidy financed by money printing and a vacancy subsidy financed by a dividend tax exist. This is the case even when the Hosios and Friedman rules do not hold.

Keywords: Matching, Money, Unemployment, Fiscal Polices

1. INTRODUCTION

The relationship between inflation and unemployment has been extensively analyzed over the past 50 years. One of the most robust monetary features of post-war U.S. data is their positive correlation.[1] Many economists view this as an important fact that has to be taken into account in conducting monetary policy. Understanding the origins of the correlations associated with the Phillips curve is a crucial first step to understand the implications of that relationship when designing policies.

Given that agents trading in frictionless Walrasian markets do not require fiat money and are always employed, environments with underlying frictions are necessary in studying the relationship between unemployment and inflation. The literature has departed from the standard Arrow–Debreu world and has considered different frictions. Within a fully flexible cash-in-advance framework, Cooley and Quadrini (1999) integrate a model of equilibrium unemployment. When price

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rigidities are considered, Walsh (2003), among others, studies the relationship between inflation and unemployment. When search-theoretic models of money are considered, Lehmann (2006), Kumar (2008) and Berentsen et al. (2011), among others, explore the relationship between inflation and unemployment.

Despite the increasing use of search-theoretic models of unemployment, little is known about the nature of optimal fiscal and monetary policies in this class of models. An exception is Cooley and Quadrini (2004), who study optimal monetary policy in a model that integrates the modern theory of unemployment with firms facing cash-in-advance constraints to purchase intermediate inputs. These authors show that when the economy is subject to productivity shocks, the optimal policy is procyclical, and with commitment, the optimal inflation rate is inversely related to the bargaining power of workers. Within the search-theoretic models of money, Lehmann (2006) shows that the optimal monetary growth rate decreases with the workers' bargaining power, the level of unemployment benefits, and the payroll tax rate. Finally, Berentsen et al. (2007) study optimal monetary policy in an economy with endogenous search decisions. They show that the same frictions that give fiat money a positive value generate an inefficient quantity of goods in each trade and an inefficient number of trades. The Friedman rule eliminates the first inefficiency and the Hosios rule the second. A monetary equilibrium attains the social optimum if and only if both rules are satisfied. When they cannot be satisfied simultaneously, optimal monetary policy achieves only the second best.

In this paper we follow the tradition of search-theoretic models of money and unemployment and study the design of optimal monetary and fiscal policies. The underlying environment is based on Berentsen et al. (2011). In each period, agents trade in three distinct submarkets where economic activity takes place. In two of these markets, trade takes place bilaterally and agents face search frictions. In the labor market, unmatched agents become unemployment, and in the specialized goods market, fiat money is essential in that it expands the set of feasible trades because of anonymity. In the last submarket, agents trade in a frictionless Arrow–Debreu type of market.

In monetary exchanges, agents pay a cost today (production) to receive a future benefit (money that can be used to purchase goods in future trades), thus facing the standard intertemporal distortion found in monetary models. The other distortions in the environment are a direct consequence of the properties of Nash's solution to the bargaining problem when agents trade goods for money and the matching frictions in the labor market.[2] Here we abstract from revenue-raising motives to focus on the welfare-enhancing properties of optimal policy. This paper tries to answer the following questions: Can fiscal and monetary policy restore efficiency of equilibria? Is the Friedman rule an optimal policy once unemployment exists and fiscal policy is available?

In this paper we propose different fiscal and monetary policies that can restore efficiency of monetary equilibrium even when the Hosios and Friedman rules do not hold. We find that production subsidies on the goods exchanged in the specialized goods market, paid in money, can be used to increase production in the

decentralized market, but they may be inflationary. If costless lump-sum monetary transfers are available, these can be used to extract the money introduced through the subsidy and thus inflation can easily be contained. Thus a simple combination of production subsidy and inflation rate can help to achieve efficiency in the specialized goods market. If the Hosios rule is satisfied, efficiency is achieved in both markets. However, if the Hosios rule is not satisfied, an efficient specialized goods production subsidy does not imply that efficiency in the labor market is achieved. We then propose a vacancy subsidy financed by a dividend tax that achieves efficiency even when the Hosios rule does not hold.

2. MODEL

Our model builds on Berentsen et al. (2011). Time is discrete and continues forever. There are two types of agents, firms and households, indexed by f and h. The set of households denoted by h belong to the $[0, 1]$ interval; the set of firms denoted by f is arbitrarily large, but not all are active at any point in time. Households work, consume, and enjoy utility, whereas firms maximize profits and pay dividends.

In each period, there are three distinct markets where economic activity takes place: a labor market in the spirit of Mortensen and Pissarides, a specialized goods market in the spirit of Kiyotaki and Wright, and a general market in the spirit of Arrow and Debreu. We refer these submarkets as LM, DM, and CM, respectively.

The timing of the environment is such that LM convenes first, then DM, and finally CM. In the first subperiod, agents enter the labor market with different employment status. Note that in the LM market, h and f can match bilaterally to create a job. To denote the outcome of that bilateral meeting we have an index, e, that represents the employment status: $e = 1$ if an agent is matched and $e = 0$ otherwise. Employed workers are matched with firms to produce output y, whereas unemployed workers do not produce. After the labor market takes place, both types of workers (employed and unemployed) who are matched enter into the next submarket, DM, where they purchase the goods that have been just produced, whereas the identity of agents is private information, ruling out credit. In the final subperiod, employed (unemployed) workers receive their wage payments (unemployment benefits), as well as dividend income from firms. Through exchanges in a Walrasian market, workers acquire consumption of a general good and rebalance their currency to carry into the next period.

We define value functions for LM, DM, and CM submarkets as $U_e^j(z)$, $V_e^j(z)$, $W_e^j(z)$ respectively, which depend on type $j \in \{h, f\}$, employment status $e \in \{0, 1\}$, and real money balances $z \in [0, \infty)$, where $z = m/p$ and p is the current price level. As in Berentsen et al. (2011), we adopt the following convention for measuring real balances. When an agent brings in m dollars to the CM, the agent then takes $\hat{z} = \hat{m}/p$ out of that market and into the next period. In the next CM market the price level is $\hat{p}$, so the real value of the money is $\hat{z}\hat{p}$,
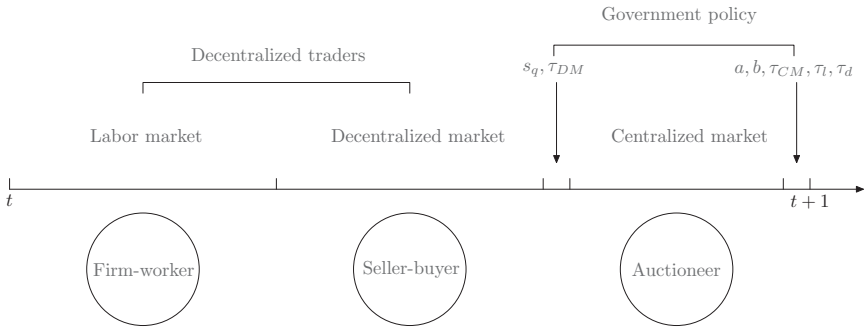
FIGURE 1. Timing.

where $\hat{\rho} = p/\hat{p}$ converts $\hat{z}$ into units of the numeraire general consumption good X in that market.

In addition to firms and households, there exists a government that conducts monetary and fiscal policies. In the CM, the government collects dividend taxes to finance vacancy subsidies and unemployment benefits. The government also conducts monetary policy by printing money at different times of the day. In particular, at the end of the DM, the government provides a monetary subsidy on production exchanged in DM. On the other hand, at the end of the CM, new money is injected through a lump-sum transfer (or tax). This operating procedure for monetary policy allows redistribution of resources between buyers and sellers. See Figure 1 for the exact timing.

2.1. Household's Problem

We now examine households' value functions in each of the subperiods, starting with the final one. A household that enters the CM with employment status e and real money holdings z chooses consumption and real money holdings for the next period, $\hat{z}$, to solve the following problem:

$$W_e^h(z) = \max_{X,\hat{z}}\{X + (1-e)l + \beta U_e^h(\hat{z})\},$$

$$\text{s.t.} \quad X = ew + (1-e)b + \triangle(1-\tau_d) + z - \hat{z} + \tau_z,$$

where X is the general CM consumption good, l represents leisure, $\triangle$ denotes dividend income, b is the unemployment benefit, τ_d represents the dividend income tax rate, and τ_z is the lump-sum real monetary transfers. We have chosen to consider a dividend tax because the dividend payment from firms to households acts just like a lump-sum transfer, thus not affecting decisions at the margin. This is the case because households already own the firm and households do not trade their ownership. Moreover, given that households have quasi-linear preferences, a tax on dividends is less distortionary than a sales tax, which affects the consumption

of general goods and money-holding decisions. Thus a dividend tax is more likely to help achieve efficiency than a sales tax.

Substituting X into the objective function, we obtain the following simplified problem:

$$W_e^h(z) = \max_{\hat{z}}\{ew + (1-e)(l+b) + \triangle\,(1-\tau_d) + z - \hat{z} + \tau_z + \beta U_e^h(\hat{z})\}. \quad (1)$$

As in Lagos and Wright (2005), with quasi-linear preferences the optimal choice of future real balances, $\hat{z}$, is independent of the current ones, z. It depends, however, on the workers' employment status e. To determine this link, let us now explore the problem of a representative household in the DM, which is given by

$$V_e^h(z) = \alpha_h v(q) + \alpha_h W_e^h[\rho(z-d)] + (1-\alpha_h)W_e^h(\rho z), \quad (2)$$

where $v(q)$ denotes the utility that a buyer obtains when he or she consumes q units of the specialized DM good, d represents the monetary payment from buyers to sellers to obtain the DM good, and α_h is the probability of trading. The probability of trade is given by a matching function $\alpha_h = \mathcal{M}(B,S)/B$, where B and S are the measures of buyers and sellers in the market. Assuming that $\mathcal{M}(B,S)$ satisfies the constant–returns to scale property, $\alpha_h = \mathcal{M}(Q,1)/Q$, where $Q = B/S$ is the queue length or market tightness. All households participate in the DM market, so $B = 1$; only firms with $e = 1$ participate, so $S = 1-u$, where u is unemployment out of the labor market. Thus, $\alpha_h = \mathcal{M}(1, 1-u)$.

Now, exploiting the linearity of $W_e^h(z)$, we can rewrite the DM value function as follows:

$$V_e^h(z) = \alpha_h[v(q) - \rho d] + W_e^h(0) + \rho z. \quad (3)$$

To close the household problem, we need to specify the choices of the representative household in LM, which are characterized by

$$U_1^h(z) = V_1^h(z) + \delta[V_0^h(z) - V_1^h(z)] \quad (4)$$

and

$$U_0^h(z) = V_0^h(z) + \lambda_h[V_1^h(z) - V_0^h(z)],$$

where δ denotes the exogenous rate at which matches are destroyed and λ_h is the endogenous rate at which they are created. The latter is determined by another standard matching function, $\lambda_h = \mathcal{N}(u,v)/u$, where u is unemployment and v is the number of vacancies posted by firms at the beginning at the LM market.

Implicit in this setup is that wages are determined when f and h meet in the LM, although they are paid in the next CM market. Moreover, wages can be renegotiated each period.

Now that all the value functions for the different submarkets have been specified, the problem of the representative household can be rewritten as follows:

$$W_e^h(z) = I_e + z + \max_{\hat{z}}\{-\hat{z} + \beta\alpha_h[v(q) - \rho d] + \beta\rho\hat{z}\} + \beta\mathbf{E}W_{\hat{e}}^h(0), \quad (5)$$

where $I_e = ew + (1-e)(l+b) + \triangle (1-\tau_d) + \tau_z$ and $\mathbf{E}W_{\hat{e}}^h(0)$ is the expectation with respect to the next period's employment status.

2.2. Firm's Problem

Firms carry no real balance out of the CM. The problem of a firm in the LM is given by

$$U_1^f = V_1^f + \delta(V_0^f - V_1^f) \tag{6}$$

and

$$U_0^f = V_0^f + \lambda_f(V_1^f - V_0^f),$$

where λ_f is the endogenous rate at which matches are created, which is given by $\lambda_f = \mathcal{N}(u,v)/v$. By constant returns, $\lambda_f = \mathcal{N}(u/v, 1)$, where v/u is labor market tightness.

In the first subperiod, firms with $e = 0$ have no production activities. If a firm enters the LM with a job match, it can produce y units of output in the LM, which can then be sold in the DM with a probability α_f. The problem of the firm in the DM is given by

$$V_1^f = \alpha_f W_1^f[y - c(q), \rho d(1+s_q)] + (1-\alpha_f)W_1^f(y,0), \tag{7}$$

where s_q is the monetary subsidy to production in DM and $c(q)$ is a transformation cost with 100% depreciation, as in Berentsen et al. (2011). Any leftover $y - c(q)$ is carried into the CM to be sold as a general good. Thus one could interpret X and q as the same physical good, which a firm can store across markets, bearing in mind that consumers generally value it differently in the two markets.

The CM value of a firm with inventory, X, real money balances, z, and wage commitment, w, is given by

$$W_1^f(X,z) = X + z - w + \beta U_1^f. \tag{8}$$

Thus the DM value of a firm is given by

$$V_1^f(z) = R - w + \beta[\delta V_0^f + (1-\delta)V_1^f], \tag{9}$$

where $R = y + \alpha_f[(1+s_q)\rho d - c(q)]$ is the expected revenue.

To model entry, any firm with $e = 0$ can pay k, in units of X, in the CM to enter the next labor market. Thus we have that

$$W_0^f = \max\{0, -k + a + \beta\lambda_f V_1^f + \beta(1-\lambda_f)V_0^f\}, \tag{10}$$

where a denotes the vacancy subsidy that the government gives to firms. The free entry condition then implies that $k - a = \beta\lambda_f V_1^f$. Now taking into account equation (9), we have that the entry condition becomes

$$k - a = \frac{\beta\lambda_f(R-w)}{1 - \beta(1-\delta)}.$$

The resulting profits over all firms are then given by

$$\widetilde{\Pi} = (1 - u)(R - w) - vk + va.$$

Because households own shares of all firms, profits define the dividend payment, which is given by $\widetilde{\Pi} = \triangle$.[3]

2.3. The Government

In this environment, the government's available monetary and fiscal policy tools are used to attenuate the frictions in the economic environment. The government pays unemployment benefits, b, and vacancy subsidies, a, which are financed by dividend taxes, τ_d, in the CM market. Thus the fiscal budget constraint is given by

$$va + ub = \tau_d \triangle,$$

which holds at every date.

Following Gomis-Porqueras and Peralta-Alva (2010), the monetary authority injects money at different times within a period to finance a production subsidy, s_q, in the DM as well as to affect the money supply at the end of CM. Thus the corresponding monetary budget constrains are given by

$$(1 - u)s_q \rho d = \tau_{\text{DM}} M$$

$$(1 + \tau_{\text{DM}})(1 + \tau_{\text{CM}}) = 1 + \pi,$$

where τ_{DM} (τ_{CM}) corresponds to the DM (CM) money growth rate, M is the money supply, and π equals the inflation rate in the steady state. The production monetary subsidy in DM is important, as a potential sales tax in DM would imply that the government has access to a record-keeping device rendering money inessential in this environment.[4] Moreover, because the government can inject/withdraw fiat money at different times, the production subsidy in DM together with the lump-sum monetary transfers in CM redistribute resources from buyers to sellers. The production subsidy is financed by money injection, τ_{DM}, and the size of the lump sum transfer, τ_z, is determined by the end-of-period money injection/withdrawal τ_{CM} for a given inflation rate.

The monetary production subsidy we consider can be mapped in real-life economies to policies that prescribe paying interest to some money holders, in our case firms. This type of policy prescription has a long tradition in monetary economics and has been advocated by Tolley (1957) and Friedman (1959), among others. Regarding the feasibility and implementability of this policy, Feinman (1993) notes that the Federal Reserve has explicitly supported legislation authorizing the payment of interest on reserves since the 1970s. Our paper then reexamines an established monetary policy prescription in the new search models of money and unemployment.

3. PRICING MECHANISMS

To characterize the terms of trade, we need to specify the trading protocol in each of the three markets. Following the literature, we consider price taking in the CM and Nash bilateral bargaining in the LM and DM submarkets. Although one can easily modify the model to allow other pricing options, we restrict our attention to the generalized Nash bargaining solution. The reason for this choice is that we can use the seller's bargaining power as a proxy for the degree of competition in the goods market, and examine how different market structures affect the implied inflation–unemployment relation.

Nash bargaining has the property that an action taken by an agent that expands the Pareto frontier can be punished for doing so, as first pointed out by Kalai and Smorodinsky (1975).[5] Thus, whenever Nash bargaining is the price-setting mechanism, policy instruments, monetary or fiscal, have a role in attenuating this "social inefficiency." The actual prescription for optimal policy in environments with a Nash price-setting mechanism crucially depends on how agents' choices affect the set of incentive-feasible outcomes.[6] This type of "social inefficiency" is especially important for monetary search models because money expands the set of feasible trades, as emphasized by Aruoba et al. (2007). The Friedman rule alone can not restore efficiency. This insight motivates our work in designing fiscal and monetary policies that can attenuate both the intertemporal and bargaining frictions in a search theoretic model of money.

3.1. Nash Bargaining in DM

Suppose that when a worker and firm meet in the DM, they bargain over the terms of trade (q, d), subject to the worker's cash constraint $d \leq z$, where d is the amount of real money balances the buyer gives to the seller in exchange for q. Let ϕ denote the seller's bargaining power. The solution is given by

$$\max_{q,d}[v(q) - \rho d]^{\phi}[\rho d(1 + s_q) - c(q)]^{1-\phi} \quad \text{s.t.} \quad d \leq z \quad \text{and} \quad c(q) \leq y.$$

We note that if $c(q) \leq y$ binds, efficiency can never be achieved regardless of any government intervention. This is a direct consequence of the underlying technology. Throughout the rest of the paper we do not consider this scenario.

The Nash bargaining solution is given a pair (q, d) that satisfies the following conditions:

$$d = z \quad \text{and} \quad q = g^{-1}(\rho z),$$

where $g(\cdot)$ is defined as follows:

$$g(q) = \frac{\phi c(q)v'(q) + (1 - \phi)v(q)c'(q)}{\phi(1 + s_q)v'(q) + (1 - \phi)c'(q)}.$$

Given this bargaining outcome, we can rewrite the choice of $\hat{z}$ by a household in CM as

$$\max_{\hat{z} \geq 0} \{-\hat{z} + \beta \alpha_h v[g^{-1}(\rho \hat{z})] + \beta (1 - \alpha_h) \rho \hat{z}\},$$

and the solution to this latter problem is given by

$$\frac{1}{\beta \rho} = \alpha_h \frac{v'(q)}{g'(q)} + (1 - \alpha_h),$$

which can be rewritten using the Fisher equation, $1/\beta = (1 + i)\rho$, as follows:

$$\frac{i}{\mathcal{M}(1, 1 - u)} = \frac{v'(q)}{g'(q)} - 1, \tag{11}$$

where i is the nominal interest rate. Equation (11) will be referred to as the LW curve. This curve represents the (q, u) pairs that are consistent with the household optimization problem. This curve determines the DM output, q, as in Lagos and Wright (2005), except that in their model α_h was fixed and now $\alpha_h = \mathcal{M}(1, 1-u)$. Moreover, in this environment, the production subsidy, s_q, also alters the incentives to produce in DM. The properties of the LW curve follow the well-known results that simple conditions guarantee that $\frac{v'(q)}{g'(q)}$ is monotone, so there is a unique q that solves the previous condition with $\frac{\partial q}{\partial u} < 0$.

PROPOSITION 1. *Let q^* solve $v'(q^*) = c'(q^*)$. For all $i > 0$ the LW curve slopes downward in (u, q) space, with $u = 0$ and $s_q = 0$ implying $q \in (0, q^*)$ and $u = 1$ implying $q = 0$. As $i \to 0$, $q \to q_0$ for all $u < 1$, where q_0 is independent of u.*

We refer the reader to the Appendix for the proofs of all our results.

An increase in u affects q because higher unemployment makes it less attractive to be a buyer by adversely affecting the probability and/or terms of trade. The different subsidies considered in this paper change the incentives to produce more and possibly increase welfare. The vacancy subsidy provides an incentive for firms to post more vacancies, which in turn affects how successful matches occur in the labor market. The fact that more successful matches are possible increases the total amount of resources in the decentralized market.

LEMMA 1. *The LW curve shifts upward, i.e., q is increasing for any given u, when the monetary subsidy to production s_q increases or the inflation rate π decreases.*

According to the Fisher equation $1/\beta = (1 + i)\rho$, the LW curve moves upward when the inflation rate $\pi = 1/\rho - 1$ decreases. This effect is intuitive: a high i increases the cost of holding money, and thus lowers the trading quantity, which is constrained by the money holding of buyers. When the subsidy to production s_q increases, a firm has a larger incentive to recruit in the labor market, and this helps to reduce unemployment.

3.2. Nash Bargaining in LM

In the labor market, when a firm with a vacancy meets with an unemployed worker they bargain over wage, w, with threat points given by their continuation values. Recall that the difference in value functions between different employment status for a household are given by

$$
\begin{aligned}
U_1^h - U_0^h &= (1 - \delta - \lambda_h)[V_1^h(z) - V_0^h(z)] \\
&= (1 - \delta - \lambda_h)[W_1^h(z) - W_0^h(z)] \\
&= (1 - \delta - \lambda_h)[w - b - l + \beta(1 - \delta - \lambda_h)\hat{S}^h],
\end{aligned}
$$

where $\hat{S}^h = \frac{w - (b + l)}{1 - \beta(1 - \delta - \lambda_h)}$ represents the steady state difference of the household's continuation values. From equation (6), we have that the difference in value functions between different employment status for the firm is given by

$$
\begin{aligned}
U_1^f - U_0^f &= (1 - \delta - \lambda_f)(V_1^f - V_0^f) \\
&= (1 - \delta - \lambda_f)[R - w + \beta(1 - \delta)\hat{S}^f],
\end{aligned}
$$

where $\hat{S}^f = \frac{R - w}{1 - \beta(1 - \delta)}$ is the steady state difference of the firm's continuation value and the firm's revenues, R, which are given by

$$
R = y + \alpha_f[(1 + s_q)\rho z - c(q)].
$$

The continuation values for the firm and the household having been specified, the Nash bargaining problem with firm's bargaining power η can be written as follows:

$$
\max_w \left[w - b - l + \beta(1 - \delta - \lambda_h)\hat{S}^h\right]^{1 - \eta} \left[R - w + \beta(1 - \delta)\hat{S}^f\right]^{\eta}.
$$

The solution to this problem is given by

$$
w = \frac{\eta[1 - \beta(1 - \delta)](b + l) + (1 - \eta)[1 - \beta(1 - \delta - \lambda_h)]R}{1 - \beta(1 - \delta) + (1 - \eta)\beta\lambda_h}.
$$

Knowing the household wage in the LM, w, as well as the firm's revenues, R, we can substitute these expressions and determine the corresponding free entry condition, which is given by

$$
k - a = \frac{\lambda_f \eta \left\{y - (b + l) + \alpha_f[(1 + s_q)\rho d - c(q)]\right\}}{(r + \delta) + (1 - \eta)\lambda_h},
$$

where r is the real interest rate.

To simplify things, let us use the Beveridge curve, which describes the relationship between unemployment and the job vacancy rate. In the steady state we have that $(1 - u)\delta = \mathcal{N}(u, v)$, which implicitly defines $v = v(u)$, so we can rewrite

$\alpha_f = \frac{\mathcal{M}(1,1-u)}{1-u}$ $\lambda_f = \frac{\mathcal{N}[u,v(u)]}{v(u)}$, $\lambda_h = \frac{\mathcal{N}[u,v(u)]}{u}$. Recalling that $\rho d = g(q)$, the free entry condition can be written as

$$k - a = \frac{\frac{\mathcal{N}[u,v(u)]}{v(u)}\eta\{y - (b+l) + \frac{\mathcal{M}(1,1-u)}{1-u}[(1+s_q)g(q) - c(q)]\}}{(r+\delta) + (1-\eta)\frac{\mathcal{N}[u,v(u)]}{u}}, \qquad (12)$$

which we refer from now on as the MP curve. The MP curve, given by equation (12), relates the firm's optimal (u, q), which depends on different tax instruments, namely, the production and vacancy subsidies and the tax rate on labor income.

PROPOSITION 2. *The MP curve slopes downward in (u, q) space. It passes through $(1, \underline{q})$, where $\underline{q} > 0$, if $(k-a)(r+\delta) > \eta[y - (b+l)] + (1-s_q)g(q^*) - c(q^*)$, and it passes through $(\bar{u}, 0)$, where $\bar{u} > 0$, if $(k-a)(r+\delta) \leq \eta[y - (b+l)]$.*

The MP curve determines the unemployment rate u as in Mortensen and Pissarides (1994), except that the total surplus (the term in braces) includes not just $y - b - l$ but also the expected gain from trade in the decentralized market. Different from Berentsen et al. (2011), the labor market equilibrium can be driven by multiple fiscal policies. The following lemma summarizes the effects of fiscal policy instruments on MP curve.

LEMMA 2. *The MP curve moves upward when the unemployment benefit b increases. The MP curve moves downward when the monetary subsidy to production s_q or the vacancy subsidy a increases.*

Intuitively, there are three effects from an increase in u, all of which encourage entry: (i) it is easier for firms to hire; (ii) it is harder for households to get hired, which lowers w; (iii) it is easier for firms to compete in the DM market.

In contrast to the LW curve, a vacancy subsidy affects the firms' optimal (u, q), which translates into shifts in the MP curve.

4. MONETARY EQUILIBRIUM

We break the analysis of equilibrium into three parts. First, following Lagos and Wright (2005), we determine the value of DM production measured by q, taking unemployment u as given, the LW curve. We then determine u, taking q as given, as in Mortensen and Pissarides (1996), which yields the MP curve previously derived. It is convenient to depict these two relationships graphically in (u, q) space with the LW and MP curves; see Figure 2. Their intersection determines the equilibrium unemployment rate and the value of money (u, q), from which all of the other endogenous variables easily follow.

As in Berentsen et al. (2011), there may be multiple equilibria. In particular, the previous figure shows two cases, labeled 2 and 3, where the MP curve intersects the vertical axis at some q_1. In this case, there either exist multiple monetary steady states, as shown by curve 2, or no monetary steady states, as shown by curve 3. In either case there also exists a nonmonetary steady state at $u = 1$ and $q = 0$. In
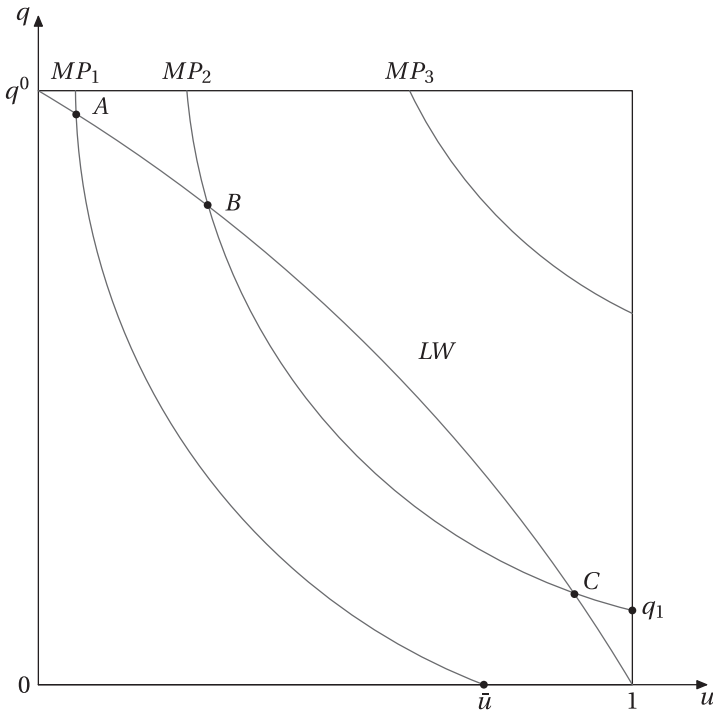
FIGURE 2. Monetary equilibrium.

these nonmonetary equilibria, the DM market shuts down, and this means the LW market also shuts down. In case 3, this is the only possible equilibrium; in case 2, however, there are also monetary equilibria with the DM and labor markets both open and $u < 1$. In case 1, recall that even if $q = 0$, there is still the standard LW equilibrium with $u < 1$.

PROPOSITION 3. *Steady state equilibrium exists. If $(k - a)(r + \delta) \geq \eta[y - (b + l)]$, there is a nonmonetary steady state at (1,0), and a monetary steady state may also exist. If $(k - a)(r + \delta) < \eta[y - (b + l)]$, there is a nonmonetary steady state at $(\hat{u}, 0)$, where $\hat{u} \in (0, 1)$ is the intersection between the MP curve and horizontal axis, and at least one monetary steady state. If the monetary steady state is unique, a rise in i or b decreases q and increases u in equilibrium, whereas a rise in a increases q and decreases u in equilibrium.*

Aruoba et al. (2007) emphasize that in the Lagos and Wright (2005) frameworks, Nash bargaining generates nonmonotonic surpluses for buyers, which lead to inefficient DM production. Moreover, from Mortensen and Pissarides (1994), we know that if the Hoisos condition is not satisfied, the economic environment will have inefficient outcomes. These two properties are inherited by our environment,

thus leaving a role for monetary and fiscal policies. To determine which precise policies, we first characterize the first best allocation in the next section.

5. EFFICIENT ALLOCATION

In this section, we explore the efficient allocation by establishing a social planner's problem. The social planner can choose the trade volume q in a pairwise matching in DM, which affects the intensive margin, and the vacancy number v, which affects the extensive margin. However, the social planner cannot affect the matching process in either the LM market or the DM market. Therefore, the efficiency in this model is constrained by matching frictions. Notice that, for the LM market in an arbitrary period t, u_{t-1} unemployed workers match with v_{t-1} vacancies match and generate u_t matches in the DM market through $u_t = u_{t-1} + (1 - u_{t-1})\delta + \mathcal{N}(u_{t-1}, v_{t-1})$. The social planner's problem is then as follows:

$$J(u) = \max_{q,v} \left\{ -\frac{vk}{\beta} + \hat{u}l + (1 - \hat{u})y + \mathcal{M}(1, 1 - \hat{u})[v(q) - c(q)] + \beta \hat{J}(\hat{u}) \right\}$$

$$\text{s.t.} \quad \hat{u} = u + (1 - u)\delta - \mathcal{N}(u, v).$$

Because v is chosen before a period starts, its value is discounted by $1/\beta$. Notice that the choice of q does not depend on the state variable u. The first-order condition with respect to q is simply $v'(q) = c'(q)$. The first-order condition with respect to v gives the expression for $\hat{J}'(\hat{u})$:

$$\hat{J}'(\hat{u}) = \frac{-\frac{k}{\beta} + \{y - l + \mathcal{M}_2(1, 1 - \hat{u})[v(q) - c(q)]\} \mathcal{N}_2(u, v)}{\beta \mathcal{N}_2(u, v)}.$$

Next, by applying the envelope theorem, we have

$$J'(u) = [1 - \delta - \mathcal{N}_1(u, v)] \left\{ l - y - \mathcal{M}_2(1, 1 - \hat{u})[v(q) - c(q)] + \beta \hat{J}'(\hat{u}) \right\}.$$

We only focus on the steady state, where $u = \hat{u}$. Substituting the expression of $\hat{J}'(\hat{u})$ into the last equation, we have the following social planner's Euler equation:

$$k = \frac{\mathcal{N}_2(u, v) \{y - l + \mathcal{M}_2(1, 1 - u)[v(q) - c(q)]\}}{\frac{1}{\beta} - 1 + \delta + \mathcal{N}_1(u, v)}.$$

According to the steady-state condition $\mathcal{N}(u, v) = (1 - u)\delta$, we can implicitly define $v = v(u)$. And given the fact that $\frac{1}{\beta} = 1 + r$, where r is the real interest rate, we know that the constrained efficient allocation (q^*, u^*) must satisfy the relationship

$$k = \frac{\mathcal{N}_2(u^*, v(u^*)) \{y - l + \mathcal{M}_2(1, 1 - u^*)[v(q^*) - c(q^*)]\}}{r + \delta + \mathcal{N}_1(u^*, v(u^*))}.$$

The next proposition proves the existence and uniqueness for the constrained efficiency under CES matching function specification.
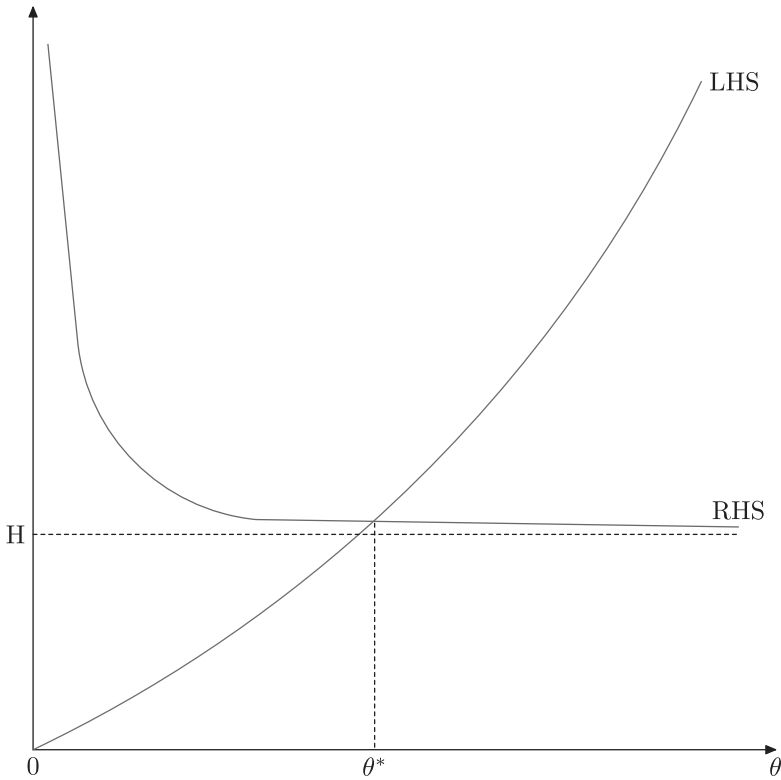
FIGURE 3. Uniqueness of efficient allocation.

PROPOSITION 4. *When the matching technologies in the LM and DM are given by $\mathcal{N}(u, v) = u^\sigma v^{1-\sigma}$ and $\mathcal{M}(B, S) = B^\gamma S^{1-\gamma}$, respectively, there exists a unique social optimal plan given by $\theta^* \equiv v^*/u^*$ and q^*.*

Figure 3 provides a graphical summary of Proposition 4.

Now that the first best is characterized, we can determine if fiscal and monetary instruments can implement such an allocation.

6. OPTIMAL MONETARY AND FISCAL POLICIES

The government's problem consists of choosing inflation, unemployment benefits, and tax and subsidy rates that maximize social welfare subject to the constraint that production and consumption in all markets are stationary monetary equilibria. Thus, we are contemplating an environment in which the government sets an inflationary and fiscal plan that will not change over time. In the next subsection we explore whether there exists a set of policies that can replicate the social planner's allocation (u^*, v^*, q^*).

6.1. Benchmark

As a baseline situation, let us consider an economy where the government does not provide unemployment insurance, b, nor the vacancy subsidy, a, so that there is no need to tax dividend income, $\tau_d = 0$. Let us further assume that the government sets $s_q = s_q^*$, the subsidy rate that can achieve q^* for α_h^*, which corresponds to the optimal matching where $u = u^*$. The next lemma ensures that it is possible to do so.

LEMMA 3. *Consider any given value of the buyer's bargaining power,* $0 < \phi \leq 1$, *and any given inflation rate,* $\pi \geq \beta - 1$. *Then there exists a value* s_q^* *that achieves the first-best allocation in the DM market.*

Lemma 3 indicates that, by using a proper production subsidy rate s_q^*, efficiency in a decentralized market can always be achieved for any unemployment level u even for inflation rates away from the Friedman rule. The subsidies on the production exchanged in DM encourages sellers to produce up to the efficient level. However, these subsidies cannot address the inefficiency in LM if the Hosios rule is not satisfied. Note that in a bargaining environment the Hosios rule depends solely on the underlying parameters of the model. If these do not satisfy the Hosios rule, inefficiency will be observed, allowing a role for active fiscal policies.

To correct the inflationary distortion, we follow Gomis-Porqueras and Peralta-Alva (2010). The monetary authority injects money at different times within a period to pay a production subsidy in the DM as well as to affect the money supply at the end of the CM. Thus the corresponding budgets constrains are

$$(1 - u^*)s_q \rho d^* = \tau_{\text{DM}} M^*,$$

$$(1 + \tau_{\text{DM}})(1 + \tau_{\text{CM}}) = 1 + \pi.$$

Observe that the availability of lump-sum monetary transfers at the end of the centralized market can neutralize any increase of the money supply from the payment of monetary subsidies in the centralized market (where we measure inflation). Thus we can deviate from the Friedman rule.

The resulting labor market equilibrium, MP curve, can be written as follows:

$$[(r + \delta)\theta^\sigma + (1 - \eta)\theta]k = \eta\{y - l + (1 + \delta\theta^{\sigma-1})^\gamma[(1 + s_q^*)g(q^*) - c(q^*)]\}.$$

Rearranging the social planner's solution and the equilibrium MP curve, respectively, we have the following relationships:

$$[(r + \delta)\theta^\sigma + \sigma\theta]\frac{k}{(1 - \sigma)} = y - l + (1 + \delta\theta^{\sigma-1})^\gamma(1 - \gamma)[v(q^*) - c(q^*)],$$

$$\textbf{(EF)}$$

$$[(r + \delta)\theta^\sigma + (1 - \eta)\theta]\frac{k}{\eta} = y - l + (1 + \delta\theta^{\sigma-1})^\gamma[(1 + s_q^*)g(q^*) - c(q^*)].$$

$$\textbf{(MP)}$$

In this search environment, there are efficiencies in both LM and DM markets. Although the government uses the subsidy s_q^* to correct the trade inefficiency in DM, the efficient unemployment rate θ^* is not guaranteed if there are no further active policies. We use θ^{MP} to denote the equilibrium market tightness when $s_q = s_q^*$ has been chosen. We summarize the relative value of θ^{MP} when compared to θ^* in the following proposition.

PROPOSITION 5. *When the trade efficiency is attained by setting $q^{\text{MP}} = q^*$, we have following properties regarding market tightness θ^{MP} in LM:*

- *when $\sigma = 1 - \eta$ (Hosios's rule) and $(1-\gamma)[\nu(q^*) - q^*] = (1+s_q^*)g(q^*) - c(q^*)$, $\theta^{\text{MP}} = \theta^*$;*
- *when $\sigma > 1 - \eta$ and $(1-\gamma)[\nu(q^*)-q^*] < (1+s_q^*)g(q^*) - c(q^*)$, $\theta^{\text{MP}} \geq \theta^*$;*
- *when $\sigma < 1 - \eta$ and $(1-\gamma)[\nu(q^*)-q^*] > (1+s_q^*)g(q^*) - c(q^*)$, $\theta^{\text{MP}} \leq \theta^*$.*

Proposition 5 is intuitive. The efficiency in LM depends on whether firms and workers are properly compensated in the matching and bargaining processes of both LM and DM. When Hosios's rule holds, the conventional inefficiency in LM is eliminated. However, on top of Hosios's rule, firms and workers need to be compensated by their contribution to DM matching so that the efficiency in LM can be implemented. This happens when $(1-\gamma)[\nu(q^*)-q^*] = (1+s_q^*)g(q^*) - c(q^*)$, where the LHS is a firm's contribution to generating surplus in DM and the RHS is the firm's real payoff in DM as an equilibrium outcome. When $\sigma \geq 1 - \eta$ and $(1-\gamma)[\nu(q^*)-q^*] < (1+s_q^*)g(q^*) - c(q^*)$, firms are favored in the bargaining of both LM and DM and have a larger incentive to post vacancies. Therefore, $\theta^{\text{MP}} > \theta^*$. The $\theta^{\text{MP}} < \theta^*$ result can be seen through similar reasoning. However, when bargaining in the LM is firm-biased (worker-biased) and bargaining in the DM is worker-biased (firm-biased), the value of market tightness is not clear, because these opposite effects may counterbalance each other.

An immediate implication of our previous results is that, when we implement efficiency in DM by choosing s_q^*, efficiency in the labor market may not be achieved. This depends on the underlying parameters governing the matching processes in the labor and goods markets as well as the bargaining power of firms and workers. In the next subsection we explore whether active fiscal policies can help us achieve efficiency.

6.2. Active Fiscal Instruments and Efficiency

An important insight of the previous section was the importance of the DM production subsidy in helping achieve efficient production in the DM. Thus, after setting s_q^*, the government needs to choose a vacancy subsidy, a, unemployment insurance (or tax), b, and a labor income tax rate to move the MP curve so that efficiency can be achieved in the LM and DM markets. In other words, we need to specify policies such that $\theta^{\text{MP}} = \theta^*$ and $q = q^*$. In order to determine the right set of policies, we need to determine the efficient labor market gap, i.e., $\theta^{\text{MP}} - \theta^*$.

Recall that the MP curve was given by

$$\left[(r+\delta)\theta^{\sigma} + (1-\eta)\theta\right]\frac{k-a}{\eta} = y-(b+l)+(1+\delta\theta^{\sigma-1})^{\gamma}[(1+s_q^*)g(q^*)-c(q^*)].$$

From Lemma 2, we know that the MP curve shifts downward (i.e., u decreases for any fixed q) in (u, q) space when the vacancy subsidy, a, increases or unemployment benefits, b, decrease. Intuitively, if the government wants a larger vacancy–unemployment ratio $\theta = \frac{v}{u}$, the government can either raise firms' incentive to recruit, or lower unemployed workers' welfare, or do both.

For an initial set of fiscal and monetary policies, if the vacancy–unemployment ratio is higher than the efficient level, $\theta^{MP} > \theta^*$, government may want to tax firms or subsidize the unemployed. Thus in principle (u^*, q^*) is a possible allocation that belongs to the equilibrium MP curve.

Before we determine the appropriate policy instruments, we need to consider the different possibilities when $s_q = s_q^*$. First, if the MP curve is to the right of (u^*, q^*), the government can change the MP allocations by increasing a or decreasing b until the MP curve crosses (u^*, q^*). Then using the appropriate combination of policies we can construct a pair of LW and MP curves that crosses at (u^*, q^*). To be an efficient market equilibrium, the policy instruments τ_d, a, and b need to satisfy the government budget constraint, which is given by

$$v^*a + u^*b = \tau_d \, \triangle^*,$$

which holds at every date. The fact that τ_d has no effect on the LW and MP curves is going to be important, as this instrument can help the government balance its budget. After the government sets a and b to shift the MP curve so that the LW and MP curves cross at (u^*, q^*), the fiscal authority chooses τ_d to balance the budget.

However, if the MP curve is to the left of (u^*, q^*) when $s_q = s_q^*$, $\theta^{MP} < \theta^*$, it is still possible to achieve efficiency by setting $a < 0$, thus moving the MP curve to the right. But because we can at most set $a = -k$, efficiency in the DM market cannot be attained if the MP curve is too far away from (u^*, q^*). See Figure 4.

Define ζ as follows:

$$\zeta = \frac{(1-\gamma)[v(q^*) - q^*]}{(1+s_q^*)g(q^*, s_q^*) - c(q^*)}.$$

Generally, if $\zeta > 1$, the government can always achieve labor market efficiency by setting positive a and b as stated in the following proposition.

PROPOSITION 6. *When the Hosios rule holds, labor market efficiency can be achieved through appropriate fiscal policies $\{a^*, b^*, \tau_d^*\}$ that satisfy the following*
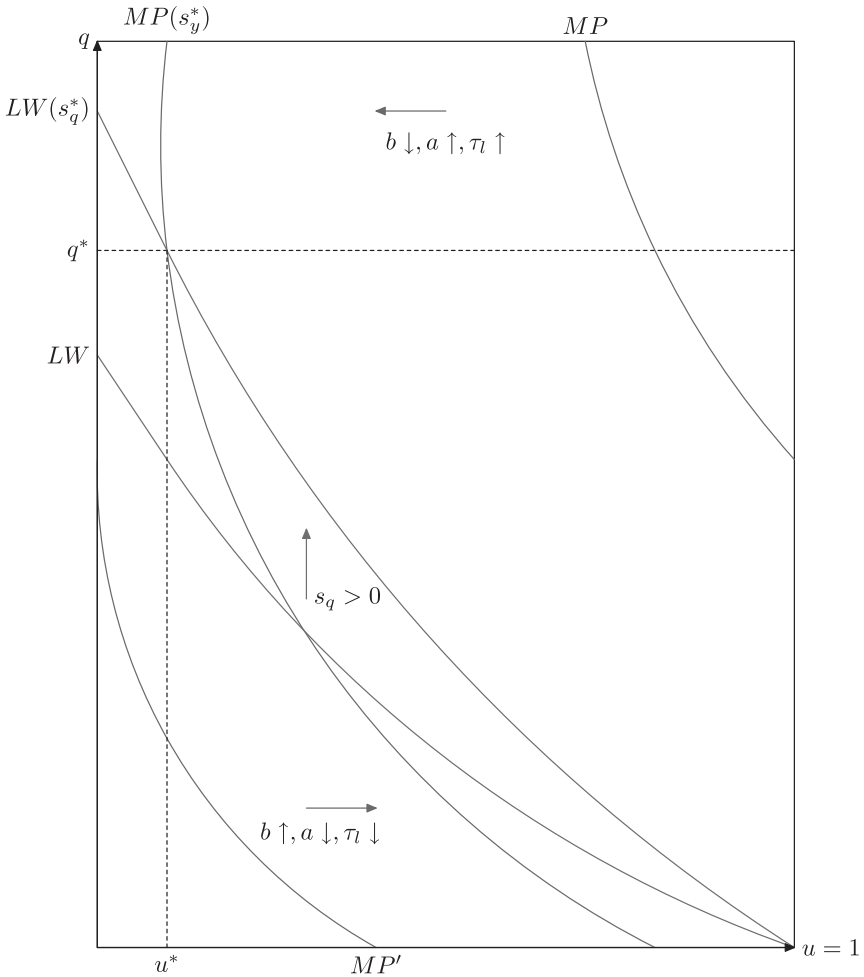
FIGURE 4. Existence of optimal policies.

conditions:

$$\frac{k}{k-a} = \frac{y}{y-(b+l)} = \zeta,$$

$$v^* a^* + u^* b^* = \tau_d^* \, \triangle^* \, .$$

- *When firms are favored in both the LM and DM, i.e.,*

$$\sigma \geq 1 - \eta \quad and \quad (1-\gamma)[v(q^*) - q^*] \leq (1 + s_q^*)g(q^*) - c(q^*),$$

to achieve labor market efficiency, the government needs to implement a small positive or even negative a and increase b.

TABLE 1. Welfare-maximizing policies

Policies	Values
s_q	0.356
τ_d	0.547
a	0.282

- *When workers are favored in both the LM and DM, i.e.,*

$$\sigma \leq 1 - \eta \quad and \quad (1 - \gamma)[v(q^*) - q^*] \geq (1 + s_q^*)g(q^*) - c(q^*),$$

to achieve labor market efficiency, government needs to increase a and decrease b.

6.3. Numerical Example

For a given parameterization, the government's problem can easily be solved with standard numerical methods. For ease of comparison with existing analyses, we follow closely the parameterization of Berentsen et al. (2011). These authors consider the preferences

$$v(q) = \frac{Aq^{(1-\alpha)}}{1 - \alpha},$$

and the values for the underlying parameters are $\beta = 0.992$, $l = 0.1$, $y = 3$, $A = 1.86$, $\delta = 0.05k = 8$, $\sigma = 0.28$, $\gamma = 0.28$, $\eta = 0.28$, $\phi = 0.275$, and $\alpha = 0.17$. Given this structure, we analyze the quantitative implications for optimal fiscal and monetary policies.

For this particular parameterization, we find that there exists a unique monetary equilibrium. Nevertheless, there are multiple fiscal and monetary policies that yield the efficient allocation. The optimal fiscal and monetary policies are reported in Table 1 where $\pi = \beta - 1$, $b = 0$ with optimal unemployment, vacancy, DM production 0.076, 0.037, and 12.863, respectively.

With this particular parameterization, we obtain a unique monetary equilibrium that exhibits an endogenous long-run Phillips curve. The government is able to implement a vacancy subsidy that shifts the MP curve enough so that q^* and u^* belong to both the MP and LW curves.

7. CONCLUSION

The objective of this paper is to provide a better understanding of the interactions between monetary and fiscal policies in an economic environment with microeconomic foundations for fiat money and unemployment. The underlying framework is tractable where search-and-bargaining frictions are considered to create a role for money and to deliver unemployment as equilibrium outcomes.

One of our main results is to show that, even when terms of trade in the decentralized market are given by Nash's bargaining solution, some of the inefficiencies in the Berentsen et al. (2011) framework can be restored with appropriate fiscal policies. In particular, when lump-sum monetary transfers are possible, a production subsidy financed by money printing can increase output in the decentralized market and a vacancy subsidy financed by a dividend tax even when the Hosios rule does not hold.

We also showed there exist multiple subsidies and (sometimes strictly positive) inflation rates that yield the efficient allocation. The Friedman rule is one of the possible policy options that yields efficiency. Finally, the Friedman rule is always an optimal policy regardless of the bargaining power of the buyer.

Finally, under any of the operating procedures for monetary and fiscal policy considered in this paper, with or without lump-sum taxes, large welfare gains are achieved by having fiscal and monetary policies in place. Thus, ignoring active fiscal policies can be quite costly.

The findings of this paper confirm the conjecture of Kocherlakota (2005) and Wright (2005) that fiscal and monetary policies have important interactions in frameworks with micro foundations for the existence of fiat money; thus, they should always be jointly considered in the design of optimal government 111 policy.

NOTES

1. See Ireland (1999), Ribba (2003), and Berentsen et al. (2011) for more on this issue.
2. As shown in Hosios (1990), if the worker's share of the matching surplus is too small, there will be an excessive creation of vacancies due to the high profitability of a match for the firm.
3. Recall that we have normalized $B = 1$.
4. Recall that agents in DM are anonymous, so that credit as a medium of exchange is not feasible.
5. This property of Nash bargaining disappears when the axiom of independence of irrelevant alternatives is replaced with the axiom of monotonicity, as first proposed by Kalai and Smorodinsky (1975).
6. For real economies we refer to Chae (2002), who investigates tax incidence in a two-person bargaining model where one party is taxed and derives some general results. The condition under which the nontaxed party shares the burden of taxes (or benefit of subsidies) is characterized. Sufficient conditions for the tax burden to fall entirely on the taxed party are given. Also, Chae (2002) provides a sufficient condition for the nontaxed party actually to benefit from taxation.

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APPENDIX

PROOF OF PROPOSITION 1

The LW curve determines q for a given u exactly as in Lagos and Wright (2005), except for the production subsidy q_s in $g'(q)$. We implicitly impose mild assumptions to guarantee that $v'(q)/g'(q)$ is globally decreasing in q. Examples of such conditions can be (i) v' is log-concave or (ii) $\phi \approx 1$. Thus, when u increases, the LHS of equation (11) increases; q will decrease, as $v'(q)/g'(q)$ is a decreasing function of q; and therefore the LW curve is downward-sloping.

If $u = 0$ and $s_q = 0$, $q < q^*$ as in Lagos and Wright (2005). If $u = 1$, it implies that the LHS of (11) goes to ∞ and therefore $q \to 0$ as $v'(0) = \infty$. When workers own full bargaining power, i.e., $\phi = 1$, and the Friedman rule $i \to 0$ is implemented, we have $v'(q)/c'(q) \to 1$, i.e., $q \to q^*$. Furthermore, when $i \to 0$, the RHS of (11) goes to 0 so that q is independent of u. ∎

PROOF OF LEMMA 1

It is straightforward to see that when u is fixed, q decreases as i increases, given that $v'(a)/g'(q)$ is decreasing in q. From the representation of $g(q, s_q)$ and by applying the implicit function theorem, we have that

$$\frac{\partial q}{\partial s_q} = -\frac{\phi z v'(q)}{\phi\{-v' + v''[z(1 + s_q) - q]\} - (1 - \phi)v'}.$$

Thus, given $v' \geq 0$, $v'' < 0$ and $z(1 + s_q) - q \geq 0$, we have $\partial q / \partial s_q \geq 0$. Therefore, a higher monetary subsidy to production shifts the LW curve upward. ∎

PROOF OF PROPOSITION 2

First, notice that $\frac{\mathcal{N}(u,v)}{v}$ and $\frac{\mathcal{M}(1,1-u)}{1-u}$ are increasing in u, whereas $\mathcal{N}(u, v)/u$ is decreasing in u. Then, when other parameters are unchanged, a higher u will lead to a lower q when $q \leq q^*$, given that $g'(q) > 0$. The MP curve is thus downward-sloping in (u, q) space. Second, multiplying both sides of equation (12) by $(r + \delta) + (1 - \eta)\frac{\mathcal{N}(u,v)}{u}$, we have

$$(k - a) \left\{ (r + \delta) + (1 - \eta)\frac{\mathcal{N}[u,v(u)]}{u} \right\}$$
$$= \frac{\mathcal{N}[u,v(u)]}{v(u)} \eta \{ y - (b + l) + \frac{\mathcal{M}(1,1-u)}{1-u}[(1 + s_q)g(q) - c(q)] \}.$$

The LHS reaches its minimum when $\mathcal{N}(u, v)/u \to 0$. The RHS reaches its maximum when $\mathcal{N}(u, v)/v \to 1$ and $\frac{\mathcal{M}(1,1-u)}{1-u} \to 1$. Thus, if

$$(k - a)(r + \delta) \geq \eta \left\{ y - (b + l) + [(1 + s_q)g(q^*) - c(q^*)] \right\},$$

the market will shut down. Finally, suppose $(k - a)(r + \delta) \geq \eta(y - [b + l])$; then q must be positive at $u = 1$, otherwise the equation cannot hold. ∎

PROOF OF LEMMA 2

The effect of policy instruments a and b on u and q are immediate from equation (12). Also notice that

$$\frac{d(1 + s_q)g(q)}{ds_q} = g(q) + (1 + s_q)\frac{dg(q)}{dq}\frac{dq}{ds_q} \geq 0.$$

This implies that when u and other parameters are fixed, q decreases as s_q increases. ∎

PROOF OF PROPOSITION 3

We only focus on steady state equilibrium. When $(k - a)(r + \delta) \geq \eta(y - (b + l))$, the MP curve crosses $(1, \underline{q})$ by Lemma 2. It is clear from Figure 2 that the MP curve may (e.g., MP$_2$) or may not (e.g., MP$_3$) intersect the LW curve. However, because the LW curve crosses $(1, 0)$, there always exists a nonmonetary steady state where there is no production and trade at all. When $(k - a)(r + \delta) < \eta[y - (b + l)]$, the MP curve enters (u, q) space from $(\bar{u}, 0)$(e.g., MP$_1$). Therefore, there exists a nonmonetary steady state $(\bar{u}, 0)$ and at least

one monetary steady state (e.g., point A in Figure 2). There may exist multiple monetary steady state (e.g., point B and C in Figure 2).

Suppose a monetary steady state exists. Because both the LW and MP curves are downward-sloping, any shift of one curve creates a new monetary equilibrium (if one exists), with u and q changing in opposite directions. By Lemma 1, a higher i makes the LW curve move downward and thus decreases q and increases u. By Lemma 2, a rise in a or a decrease in b makes the MP curve shift upward and thus leads to a steady state with higher q and lower u.   ∎

PROOF OF PROPOSITION 4

By choosing Cobb–Douglas matching technologies and using the Beveridge curve $u\mathcal{N}(1, \theta) = \mathcal{N}(u, v) = (1 - u)\delta$, we have $\mathcal{N}_1(u, v) = \sigma\theta^{1-\sigma}$, $\mathcal{N}_2(u, v) = (1 - \sigma)\theta^{-\sigma}$, and $\mathcal{M}_2(1, 1 - u) = (1 - \gamma)(1 + \delta\theta^{\sigma-1})^{\gamma}$. The social planner's optimal choice can be written as

$$[(r + \delta)\theta^{\sigma} + \sigma\theta]k = (1 - \sigma)\left\{y - l + (1 - \gamma)(1 + \delta\theta^{\sigma-1})^{\gamma}[v(q^*) - c(q^*)]\right\}.$$

The left-hand side (LHS) tends to 0 as $\theta \to 0$, whereas it tends to ∞ as $\theta \to \infty$. And

$$\frac{\partial \text{LHS}}{\partial \theta} = [(r + \delta)\theta^{\sigma-1} + 1]\sigma k > 0.$$

The right-hand side (RHS) tends to ∞ as $\theta \to 0$, whereas it tends to $H = (1 - \sigma)\left[y - l + (1 - \gamma)[v(q^*) - c(q^*)]\right]$ as $\theta \to \infty$. And

$$\frac{\partial \text{RHS}}{\partial \theta} = -(1 - \sigma)^2(1 - \gamma)\gamma\sigma[v(q^*) - c(q^*)](1 + \delta\theta^{\sigma-1})^{\gamma-1} < 0.$$

Thus, there exists a unique θ^* that satisfies social optimality. Given q^* is also unique, there exist an unique social optimal plan given by θ^* and q^*.   ∎

PROOF OF LEMMA 3

Take the L–W curve $\frac{i}{\mathcal{M}(1,1-u)} = \frac{v'(q)}{g_q(q, s_q)} - 1$ as given and assume $c(q) = q$, where

$$g_q(q, s) = \frac{\phi(1 + s_q)v'(q)^2 + (1 - \phi)v'(q) - (1 - \phi)\phi v'(q)[(1 + s_q)v(q) - q]}{[\phi(1 + s_q)v'(q) + 1 - \phi]^2}.$$

The optimal policy requires the government to pick s_q given q such that $g_q(q, s) = 1$, which is the true marginal cost. Such a policy eliminates any bargaining inefficiency that arises. Under such a subsidy policy we have

$$\frac{i}{\mathcal{M}(1, 1 - u)} = v'(q) - 1,$$

which means that q only depends on i. As a result setting $i = 0$ (i.e., $\pi = \beta - 1$) eliminates the time cost of holding money yields, q^*. Given this policy, we can now solve for the

optimal subsidy. We have

$$1 = \frac{\phi(1+s_q)v'(q)^2 + (1-\phi)v'(q) - (1-\phi)\phi v''(q)[(1+s_q)v(q)-q]}{[\phi(1+s_q)v'(q)+1-\phi]^2},$$

which is a quadratic equation in s_q. Imposing $v'(q^*) = 1$, we can solve for a positive s_q^*. Also notice that, given that $i = 0$ is chosen, s_q^* does not depend on u. ∎

PROOF OF PROPOSITION 5

Comparing the social planner's allocation (EF) and the market equilibrium allocation (MP), we can consider three different cases.

Case 1: $\sigma = 1 - \eta$. The matching elasticity with respect to u, $u\mathcal{N}_2(u, v)/\mathcal{N}(u, v)$, is σ under our Cobb–Douglas specification. Hosios (1990) states that the labor market efficiency is going to be achieved whenever the bargaining power reflects the party's contribution to matching, i.e., $\sigma = 1 - \eta$. Thus this scenario satisfies the so-called Hosios rule. In this case, the LHS of equation (EF) and the LHS of equation (MP) are equal for the same market tightness θ. Thus, if the RHSs of equations (EF) and (MP) satisfy

$$(1-\gamma)[v(q^*) - c(q^*)] = (1+s_q^*)g(q^*) - c(q^*)$$

or equivalently

$$\frac{v(q^*)}{c(q^*)} = \frac{(1-\gamma)(1+\phi s_q^*) - (1-\phi)}{(1-\gamma)(1+\phi s_q^*) - (1-\phi)(1+s_q^*)}$$

$$= 1 + \frac{(1-\phi)s_q^*}{(1-\gamma)(1+\phi s_q^*) - (1-\phi)(1+s_q^*)},$$

we must have that the equilibrium labor market tightness θ^{MP} equals the efficient tightness θ^*. This in turn implies that the optimal policy scheme (s_q^*, π^*) implements the efficient outcome in both markets.

Now suppose that $(1-\gamma)[v(q^*) - q^*] > (1+s_q^*)g(q^*) - q^*$. Then, for the same market tightness θ, the RHS of equation (EF) is larger than the RHS of equation (MP). As a result, we have that $\theta^{\mathrm{MP}} < \theta^*$ when $q^{\mathrm{MP}} = q^*$. Similarly, if $(1-\gamma)[v(q^*) - q^*] < (1+s_q^*)g(q^*) - q^*$, we have $\theta^{\mathrm{MP}} > \theta^*$, given that $q^{\mathrm{MP}} = q^*$.

Case 2: $\sigma > 1 - \eta$. The Hosios rule is no longer holding because of the excessive bargaining power of firms. The LHS of equation (EF) is larger than the LHS of equation (MP) for the same θ. If $(1-\gamma)[v(q^*) - q^*] \leq (1+s_q^*)g(q^*) - q^*$, we must have that $\theta^{\mathrm{MP}} \geq \theta^*$ when $q^{\mathrm{MP}} = q^*$. If $(1-\gamma)[v(q^*) - q^*] > (1+s_q^*)g(q^*) - q^*$, the relative value of θ^{MP} compared to θ^* is ambiguous.

Case 3: $\sigma < 1 - \eta$. The Hosios rule is not satisfied because of the excessive bargaining power of workers. The LHS of equation (EF) is smaller than the LHS of equation (MP) for the same θ. If $(1-\gamma)[v(q^*) - q^*] \geq (1+s_q^*)g(q^*) - q^*$, we must have $\theta^{\mathrm{MP}} \leq \theta^*$ when $q^{\mathrm{MP}} = q^*$. If $(1-\gamma)[v(q^*) - q^*] < (1+s_q^*)g(q^*) - q^*$, the relative value of θ^{MP} when compared to θ^* is ambiguous. ∎

PROOF OF PROPOSITION 6

First, suppose that Hosios's rule holds. From equations (EF) and (12), it is easy to see that θ^* can be achieved in the labor market if and only if $\frac{k}{k-a}$, $\frac{y}{y-(b+l)}$, and $\frac{(1-\gamma)[v(q^*)-c(q^*)]}{(1+s_q^*)g(q^*)-c(q^*)}$ are of the same value. The first two fractions can be adjusted by choosing policy instruments a and b. Whether the efficiency can be resorted depends on the value of the last fraction.

Suppose that $\frac{(1-\gamma)[v(q^*)-c(q^*)]}{(1+s_q^*)g(q^*)-c(q^*)} = \zeta > 1$. All we need is to choose a and b such that

$$\frac{k}{k-a} = \frac{y}{y-(b+l)} = \zeta.$$

This can be easily done by choosing nonnegative policy instruments. Suppose that $\zeta < 1$. Now we need to choose negative a. But because the entry tax a cannot be less than $-k$ and also is restricted by budget constraint, the feasible set of policies is restricted. Thus, only when ζ is not too small can, efficiency be achieved.

When Hosios's rule does not hold, choice of policy becomes complicated, as it depends on the detailed parameters of the matching and bargaining process. However, the direction that leads to efficiency is clear. When

$$\sigma > 1 - \eta \quad \text{and} \quad (1-\gamma)[v(q^*) - q^*] \leq (1 + s_q^*)g(q^*) - c(q^*),$$

the MP curve determined by (MP) is lower than (u^*, q^*) by Proposition 5. Thus, large b and small (or even negative) a should be chosen. A similar argument can be formed for the opposite case.

Of course, all there policy combinations should satisfy the government's budget constraint $v^*a^* + u^*b^* = \tau_d^* \triangle^* + \alpha_f^{**}w$. ∎