Delusion proneness and 'jumping to conclusions': relative and absolute effects

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Background. That delusional and delusion-prone individuals 'jump to conclusions' is one of the most robust and important findings in the literature on delusions. However, although the notion of 'jumping to conclusions' (JTC) implies gathering insufficient evidence and reaching premature decisions, previous studies have not investigated whether the evidence gathering of delusion-prone individuals is, in fact, suboptimal. The standard JTC effect is a relative effect but using relative comparisons to substantiate absolute claims is problematic. In this study we investigated whether delusion-prone participants jump to conclusions in both a relative and an absolute sense.

Method. Healthy participants (n = 112) completed an incentivized probabilistic reasoning task in which correct decisions were rewarded and additional information could be requested for a small price. This combination of rewards and costs generated optimal decision points. Participants also completed measures of delusion proneness, intelligence and risk aversion.

Results. Replicating the standard relative finding, we found that delusion proneness significantly predicted task decisions, such that the more delusion prone the participants were, the earlier they decided. This finding was robust when accounting for the effects of risk aversion and intelligence. Importantly, high-delusion-prone participants also decided in advance of an objective rational optimum, gathering fewer data than would have maximized their expected pay-off. Surprisingly, we found that even low-delusion-prone participants jumped to conclusions in this absolute sense.

Conclusions. Our findings support and clarify the claim that delusion formation is associated with a tendency to 'jump to conclusions'. In short, most people jump to conclusions, but more delusion-prone individuals 'jump further'.

Received 20 March 2014; Revised 26 August 2014; Accepted 26 August 2014; First published online 30 September 2014

Key words: Decision making, delusion proneness, delusions, experimental economics, jumping to conclusions.

Introduction

In a now classic study, psychiatric patients suffering from delusions observed a researcher draw coloured beads from one of two hidden jars: a mostly pink jar (85 pink beads, 15 green beads) or a mostly green jar (85 green, 15 pink). The patients had to decide which jar the beads were being drawn from and to indicate when they were ready to make this decision. Relative to control participants, deluded individuals required fewer draws before making a decision (Huq *et al.* 1988). This effect, referred to as the 'jumping-toconclusions' (JTC) bias, has been replicated numerous times with both deluded and delusion-prone participants (Colbert & Peters, 2002; Moritz & Woodward, 2005). As a result, a tendency to gather insufficient evidence when forming beliefs and making decisions is thought to be a core cognitive component of delusion formation (Fine *et al.* 2007; Garety & Freeman, 2013).

Although the JTC effect is well replicated and robust to many modifications of the basic 'beads task' paradigm, there are some fundamental limitations with the way this task is typically administered that call the above interpretation into question. The key problem is that the term 'jumping to conclusions' implies gathering insufficient evidence and reaching decisions prematurely, yet the standard JTC effect is a relative effect. The notions of premature and late decisions are only meaningful if there is some optimal point at which a rational individual should decide, and for such a point to exist there needs to be both an (opportunity) cost of incorrect decisions and a cost associated with collecting extra information. Investigations using the beads task paradigm typically make no attempt to incentivize participants explicitly (cf. Woodward et al. 2009; Lincoln et al. 2010), and no previous study has incorporated both of these elements. Without

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Fig. 1. A still from the lakes and fish task. The fisherman displays a series of six fish from his catch: B-W-W-B-W-B.

these aspects there is no objective basis for the decision to stop collecting data. As a result, although deluded and delusion-prone participants may 'reach conclusions' on this task more quickly than control participants (gathering less evidence), the standard unincentivized paradigm cannot justify the suggestion that they 'jump to conclusions' (objectively suboptimal evidence gathering).

To illustrate this point using a different example from the clinical literature, consider the relationship between depression and rational belief. It is well known that, relative to healthy individuals, depressed individuals have negative thoughts and expectations about the future (Beck, 1976). It might be assumed that, whereas healthy individuals are essentially rational, depressed individuals have beliefs that are unwarrantedly pessimistic; perhaps they update their beliefs about future outcomes in a distorted fashion. However, recent research has clarified that healthy individuals have distorted beliefs about the future, being relatively disinclined to revise their beliefs in response to undesirable information (e.g. Sharot et al. 2011; Eil & Rao, 2012). Indeed, there is even evidence that people with mild depression show no bias when predicting future events (Sharot, 2011; Garrett et al. 2014). This research demonstrates the perils of using relative comparisons to substantiate claims about deviations from rationality (Shah, 2012). To show that a particular group of people is irrational, their beliefs and decisions must be compared to a rational standard, not simply to those of a different group.

In the present study we incorporated rigorous procedures from experimental economics (e.g. monetary incentives, decision theoretic analysis, detailed written instructions and comprehension checks, absence of deception) to address these issues. We set both the cost associated with gathering information and the reward for correct decisions, so these were equal for all participants. To make the task more relatable and engaging we used the 'lakes and fish' adaptation of the beads task (Speechley *et al.* 2010). On each trial, participants were shown a fisherman who had caught a series of fish from one of two lakes (Fig. 1). After seeing a fish from his catch, participants had the opportunity to

see another fish or to decide which lake was the source of all fish in the series. Choosing the correct lake was rewarded but a small cost was incurred for each additional fish seen before making this decision. This combination of rewards and costs generated optimal decision points, that is points at which the expected payoff was maximal¹⁺. In line with the standard, relative JTC effect, we hypothesized that delusion proneness would significantly predict decisions in this task (controlling for intelligence and risk aversion), such that more delusion-prone participants would decide after seeing fewer fish. Importantly, we also hypothesized that delusion-prone individuals would make objectively premature decisions, that is decisions made in advance of the point at which the expected payoff would be maximal. Only by testing the latter hypothesis could we confirm whether delusion-prone individuals do in fact jump to conclusions.

Method

Participants

Participants (n = 112, 59 females, 53 males; mean age = 19.94 years, s.D. = 2.92) were students at Royal Holloway, University of London, recruited online (ORSEE; Greiner, 2004). Participants received £6 for participation and a decision-based bonus. The Psychology Department Ethics Committee of Royal Holloway, University of London approved the study.

Materials

Lakes and fish task

Before the start of the experiment, participants were issued written instructions providing information about the ratios of black to white fish in the two lakes (i.e. 25:75 or 75:25), the incentive structure (rewards and costs) and the means of responding. It was emphasized that, on a given fishing trip (corresponding to a trial), the fisherman visited one of the lakes only and was equally likely to visit either lake.

Table 1. The sequences of fish in the various trials (b = black fish; w = white fish); optimal decision points are shown in bold. Also shown are the number of participants who decided on the White lake (W) and on the Black lake (B), after each fish. Numbers in italics represent equivocal errors; numbers in bold italics represent inconsistent errors (see Jolley et al. 2014)

	Sequence A			Sequence B			Sequence C			Sequence D		Sequence E			
Draw	Fish colour	W (<i>n</i>)	B (<i>n</i>)	Fish colour	W (n)	B (<i>n</i>)	Fish colour	W (n)	В (n)	Fish colour	W (n)	B (<i>n</i>)	Fish colour	W (<i>n</i>)	В (n)
1	b	3	13	b	5	10	b	5	16	W	13	5	b	4	20
2	w	1	1	w	2	3	w	1	2	b	1	2	w	2	1
3	b	2	28	W	28	0	b	2	24	b	1	31	b	0	16
4	b	0	42	W	41	0	b	0	40	W	1	2	w	2	1
5	b	0	13	w	17	0	w	0	2	b	0	12	w	14	0
6	b	0	9	w	6	0	b	0	10	b	1	29	b	1	2
7	b	0	0	w	0	0	b	0	7	b	0	9	b	0	11
8	b	0	0	w	0	0	b	0	3	b	0	4	b	1	16
9	b	0	0	w	0	0	b	0	0	b	0	1	b	0	17
10	b	0	0	w	0	0	b	0	0	b	0	0	b	0	4
11	b	0	0	w	0	0	b	0	0	b	0	0	b	0	0
12	b	0	0	w	0	0	b	0	0	b	0	0	b	0	0
13	b	0	0	w	0	0				b	0	0	b	0	0
14	b	0	0	W	0	0				b	0	0	b	0	0
15	b	0	0							b	0	0	b	0	0
16										b	0	0			
17										b	0	0			

These instructions and the comprehension checks that followed were incorporated to minimize poor task comprehension, which has been shown to influence the JTC bias (Moritz & Woodward, 2005; Balzan *et al.* 2012*a,b*). Visual stimuli were based on those used by Speechley *et al.* (2010). Moreover, we aimed to reduce cognitive load through visual stimuli, which included explicit reminders of the ratios in the lakes, points to be earned and paid, and a string of the previously seen fish to avoid the confound of potentially impaired working memory (Dudley *et al.* 1997*a*; Moritz *et al.* 2007; Garety *et al.* 2013). We used the ratio of 25:75, rather than the 15:85 ratio used by Huq *et al.* (1988), to create optimal decision points later in the sequence.

After participants were shown the first fish from the fisherman's catch, they had the choice between deciding on a lake or seeing another fish for a small price (2 points; 1 point = £0.05). They could request to see additional fish until they either decided on a lake or all fish in the catch were shown (maximum 17), after which they were forced to decide on a lake. Choosing the correct lake earned them a reward (100 points). This procedure was repeated for six series of fish (trials). The first trial consisted of a randomly drawn series of fish² and was used for payout (participants were informed that the series of fish on one trial would be drawn at random and that they would be paid based on their performance on this particular

trial; they did not, however, know which of the six trials was the random trial³). The next five trials were tailored to have varying optimal decision points. The optimal decision point was always reached at a difference of three fish of the majority colour over the minority colour (see the Appendix for the calculation of the optimal decision point). After this point, unbeknown to the participants, all subsequent fish in the trial were of the majority colour to strengthen the stopping rule and avoid introducing contradictory evidence once the optimal point was reached. The sequences of fish in each non-random trial, including the optimal decision points, are shown in Table 1.

Delusion proneness

We used the 21-item Peters et al. Delusions Inventory (PDI; Peters *et al.* 2004) to measure delusional ideation. Sample items include 'Do you ever feel as if there is a conspiracy against you?' and 'Do your thoughts ever feel alien to you in some way?' For each yes/no item endorsed, participants are required to indicate their levels of distress, preoccupation and conviction for that item on five-point scales. The yes/no endorsement summed scores range from 0 to 21; the separate dimensions summed scores range from 0 to 105; and the total summed scores range from 0 to 336.

Intelligence

We used the 12-item version of Raven's Advanced Progressive Matrices (12-APM) to measure intelligence (Arthur & Day, 1994). A time limit of 15 min was set for these 12 items. Intelligence was defined as the total number of correctly answered items (APM scores).

Risk aversion

We used Holt & Laury's (2002) measure to gauge how risk averse participants were. This measure involves 10 decisions between two gambles, for example a decision between option A 'a 4/10 chance of winning £2.00, a 6/ 10 chance of winning £1.60' and option B 'a 4/10 chance of winning £3.85, a 6/10 chance of winning £0.10' (Holt & Laury, 2002; based on p. 1645). With each successive decision, the probability of the high payoff outcome in each option increases by 10%, such that the expected value of the 'risky' option B increases. Risk aversion was defined as the total number of times option A was chosen, so that the higher the scores, the higher the risk aversion.

Procedure

Participants were tested in groups ranging from 20 to 26 people. All sessions were conducted on a local computer network using z-Tree software (Fischbacher, 2007) in the EconLab at Royal Holloway, University of London. After correctly answering a series of comprehension questions based on detailed instructions, participants started the task. As mentioned, the first trial was always a truly random sequence. The next five trials were presented in counterbalanced order across sessions. Incomplete counterbalancing was accomplished using a Latin square procedure to maximize the difference between the trial orders across the five sessions. The first decision (i.e. see another fish or decide on a lake) in the first trial had a time limit of 60 s, to allow participants to get acquainted with the task (note that this trial was not analysed). To discourage participants from making formal calculations (e.g. using mobile phones), each subsequent decision (i.e. see another fish or decide on a lake) in that trial and all decisions in subsequent trials had a time limit of 20 s. Once all participants had completed the task, a different decision task was presented (van der Leer et al., unpublished data). Then, participants completed the risk aversion measure, next the PDI, followed by the 12-APM, and finally some demographic questions.

Statistical analyses

There were two types of decisions in our study (as in all beads task studies). First, after each fish was

presented, participants had to decide whether or not to sample a further fish. Second, once they had decided not to sample any more fish (or when they had reached the end of the sequence), they had to decide which of the two lakes the fisherman was fishing from ('lake decision'). The former decisions were of primary interest, as the number of pieces of information (fish, in this case) that are requested (the number of 'draws to decision') comprise the main dependent measure in studies of JTC. To investigate relative effects of delusion proneness we regressed draws to decision on total summed PDI scores (McKay *et al.* 2006). We ran sequential regression analyses to check whether effects remained when intelligence and risk aversion were accounted for.

Our paradigm, however, enables more than just relative comparisons. The incentive structure of our task (incorporating both rewards and costs) generates an optimal decision point for any given sequence of fish (i.e. an optimal number of draws to take before deciding which of the two lakes the fisherman was fishing from). To investigate objectively premature decisions (bona fide 'jumps to conclusions'), we compared participants' decision points to the optimal decision points across the five non-random trials, using one-sample ttests. We investigated whether objectively premature decisions were made by our sample as a whole and/ or by low- and high-delusion-prone subsets of the sample.

Ethical considerations

All procedures contributing to this work comply with the ethical standards of the relevant national and institutional committees on human experimentation and with the Helsinki Declaration of 1975, as revised in 2008.

Results

Data screening

There was no multicollinearity between predictor variables and predictors were linearly related to the outcome variable. Residuals were normally distributed, and the assumption of homoscedasticity was met. One PDI score was an outlier according to the Mahalanobis distance criterion, but it was a valid score (i.e. well within the range of possible scores) so we included it and applied a square-root transformation. After this transformation no more outliers were detected through Cook's and Mahalanobis' distances; the standardized residuals indicated that less than 5% of participants were outliers (Field, 2009).

Descriptive statistics

Descriptive statistics for risk aversion, PDI scores and APM scores are provided in Table 2. PDI scores were not significantly different between males (mean = 68.83, s.D. = 43.137) and females (mean = 80.66, s.D. = 37.691); $t_{110} = 1.816$, p = 0.072. Table 1 shows the numbers of lake decisions made at each point of each sequence along with the associated errors. As in Jolley *et al.* (2014), we defined two types of lake decision errors: deciding when the evidence collected to that point favoured neither lake ('equivocal errors') and deciding on a lake that was disfavoured by the evidence collected to that point ('inconsistent errors')⁴. Table 3 shows the mean draws to decision in each sequence.

Lakes and fish task

Relative effects

PDI scores were a significant predictor of draws to decision $F_{1,110}$ = 5.520, p = 0.021, $R_{adjusted}^2$ = 0.039; see Table 4, model 1). The higher a participant's PDI score, the earlier decisions were made.

A sequential regression was subsequently run to investigate whether adding PDI scores could predict additional variance not already accounted for by intelligence and risk aversion. Intelligence was a significant predictor of draws to decision in both models, but the addition of PDI scores significantly improved the model ($\Delta F_{1.108} = 6.128$, $\Delta p = 0.015$, $\Delta R^2 = 0.042$; $F_{3.108} =$ 12.076, p < 0.001, $R_{adjusted}^2 = 0.230$; see Table 4, model 2; and Fig. 2)⁵. Risk aversion was not a significant predictor of draws to decision in either model and did not correlate with draws to decision when considered alone $(r_{112}=0.032, p=0.741)$. We thus replicate the standard, relative, JTC finding in a fully incentivized paradigm: higher delusion proneness was associated with earlier decisions on the task, even accounting for risk aversion and intelligence⁶.

Absolute effects

Considered as a whole, participants in our sample gathered significantly fewer pieces of information than would have been optimal, that is they jumped to conclusions (mean deviation from optimal number of fish = -2.566, s.D. = 1.75, $t_{111} = 15.506$, p < 0.001)^{7.8}. To further investigate whether this effect was limited to participants high in delusion proneness, we divided participants into low-delusion-prone and high-delusion-prone groups based on a quartile split of their continuous PDI scores (as in Colbert & Peters, 2002). We then conducted separate one-sample *t* tests to investigate whether each group drew significantly fewer fish than would have been optimal. These tests

Table 2. Descriptive statistics for risk aversion, PDI scores and APM scores

Variable	Median	Mean	S.D.	Range
Risk aversion	6.0	5.41	1.732	0–9
PDI scores	68.5	75.06	40.611	10-253
APM scores	7.0	6.62	2.945	1–12

PDI, 21-Item Peters et al. Delusions Inventory; APM, 12-item version of Raven's Advanced Progressive Matrices; s.D., standard deviation.

revealed that both groups of participants jumped to conclusions [mean deviation from optimal for low-delusion-prone participants (PDI < 46) = -1.946, s.D. = 1.679, t_{25} = 5.912, p < 0.001; mean deviation for high-delusion-prone participants (PDI > 95) = -2.993, s.D. = 2.018, t_{28} = 7.987, p < 0.001]^{9.10}.

These deviations from optimal decision points could have had considerable financial consequences. For Bayesian individuals, deciding at the optimal point would have led to a gain in expected value of 10.6 points compared to deciding after seeing just one fish (averaged across the non-random trials). On average, however, our participants would have forgone more than 80% of this potential gain if these trials had been used for payout.

Discussion

That delusional and delusion-prone individuals 'jump to conclusions' on probabilistic reasoning tasks is perhaps the most important and influential claim in the entire literature on cognitive theories of delusions. Surprisingly, however, although the notion of JTC implies that delusion-prone individuals gather insufficient evidence and reach premature decisions, no previous study has investigated whether the evidence gathering of such individuals is in fact suboptimal. The standard JTC effect is a relative effect but using relative comparisons to substantiate absolute claims is problematic (Shah, 2012). Bankers may earn less money than film stars but this does not mean they have 'insufficient money'; likewise, the finding that delusion-prone individuals gather fewer data than controls does not mean they gather insufficient data; this is an empirical question, hitherto unaddressed.

In the present study we investigated evidence gathering using an incentivized task modelled after the classic beads task, the paradigm most commonly used to study the JTC bias. After observing the colour of a fish caught from one of two lakes, participants could choose to see further fish or to decide on one of the lakes as the source of the sequence of fish. We

	Sequence A	Sequence B	Sequence C	Sequence D	Sequence E
Optimal decision point	5	5	7	7	9
Mean (s.D.) draws to decision					
Full sample	3.54 (1.37)	3.52 (1.34)	3.64 (1.82)	4.25 (2.13)	5.21 (3.07)
Low-delusion-prone	3.96 (1.11)	4.04 (1.25)	4.27 (2.15)	4.88 (1.93)	6.12 (2.86)
High-delusion-prone	3.03 (1.61)	3.21 (1.63)	3.10 (2.04)	3.90 (2.43)	4.79 (3.30)

Table 3. Optimal decision points and mean (s.D.) draws to decision in each sequence, for both the full sample (n = 112) and the two outer quartiles reflecting low-delusion-prone (n = 26) and high-delusion-prone participants (n = 29)

s.D., Standard deviation.

Table 4. *B* values, standard errors (s.E.), β values and *p* values for each of the predictors in the steps of the regression models for draws to decision

Model	Step	Predictor	В	S.E.	β	р
1	1	PDI scores	-0.166	0.071	-0.219	0.021
2	1	APM scores	0.271	0.051	0.456	< 0.001
		Risk aversion	0.009	0.086	0.009	0.913
	2	APM scores	0.269	0.050	0.452	< 0.001
		Risk aversion	-0.029	0.086	-0.029	0.737
		PDI scores	-0.160	0.064	-0.210	0.015

PDI, 21-Item Peters et al. Delusions Inventory; APM,

12-item version of Raven's Advanced Progressive Matrices.

rewarded correct decisions but imposed costs for gathering extra information (seeing additional fish). This combination of rewards and costs generated an optimal number of 'draws to decision' for each sequence of fish, that is a number of draws that would maximize the expected payoff. We found that more delusionprone participants made earlier decisions, even when accounting for risk aversion and intelligence. This result replicates the standard relative JTC finding and indicates that this standard finding is not attributable to differences in intelligence or risk preferences; although, like Falcone et al. (2014), we did find that intelligence significantly affected JTC. Importantly, high-delusion-prone participants also decided in advance of an objective rational optimum, gathering fewer data than they should have to maximize their expected payoff. This reflects bona fide JTC. Surprisingly, we found that even low-delusion-prone participants jumped to conclusions in this objective, absolute sense, gathering significantly fewer pieces of information than would have been rationally optimal. By deciding too quickly, participants forwent more than 80% of the gain in expected value they could have obtained if they had decided at the optimal decision points.

Our results count against one alternate explanation of the standard relative JTC finding using the unincentivized beads task. The finding that deluded and delusion-prone participants take fewer draws to decision in unincentivized variants could simply be due to these participants being less motivated than controls to persevere with a seemingly worthless task and in more of a 'rush' to end the study (White & Mansell, 2009). Although our replication of the standard relative finding in a fully incentivized variant of the task casts doubt on this explanation, it is possible that exogenous incentives or disincentives (e.g. intrinsic motivation to perform well, fatigue) were nevertheless still confounded with delusion proneness in this study. In other words, the subjective cost of sampling further information could have been greater for delusion-prone participants, equal explicit incentives notwithstanding, and might have led them to decide earlier. In an important study, however, Moutoussis et al. (2011) analysed data from an unincentivized beads task using a costed-Bayesian model and found no support for this 'high-sampling-cost hypothesis'.

Although our findings align with the notion that low-delusion-prone participants are more 'conservative' than high-delusion-prone participants (Dudley *et al.* 1997*b*), the fact that even low-delusion-prone participants jumped to conclusions in our incentivized draws-to-decision paradigm is surprising given previous evidence of generally 'conservative' belief revision in healthy participants (e.g. Phillips & Edwards, 1966). However, van der Leer & McKay (2014) have recently shown that incentivizing a probability estimate variant of the task, in which participants were shown a fish from the fisherman's catch and then supplied probability estimates for each lake, led low-delusion-prone participants to provide

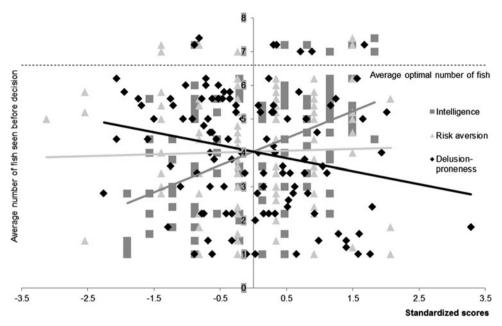


Fig. 2. Average number of fish seen before making a decision plotted against standardized scores of intelligence (12-item version of Raven's Advanced Progressive Matrices, 12-APM), risk aversion and delusion proneness (square-root transformed scores on the 21-Item Peters et al. Delusions Inventory, PDI). The vast majority of participants decided before the optimal number of fish (indicated by the dotted line).

probability estimates not significantly different from Bayesian probabilities, whereas high-delusion-prone participants still deviated from this objectively rational norm despite the included incentives. These findings resonate with a 'liberal acceptance account' (Moritz et al. 2006, 2007; White & Mansell, 2009; Averbeck et al. 2011), implying that participants, particularly those prone to delusions, might behave less conservatively than their estimates of relevant probabilities suggest they should. Future investigations with clinical populations may shed further light on this possibility. Given that delusional patients gather even less evidence than delusion-prone participants (Warman et al. 2007), we might expect deviations from optimal decisions to be even more pronounced for delusional patients than for our delusion-prone participants.

Conclusions

That delusional ideation is associated with JTC on probabilistic reasoning tasks is almost a received view in psychiatry (Huq *et al.* 1988; Colbert & Peters, 2002; Fine *et al.* 2007; Garety & Freeman, 2013). We adopted stringent experimental economics methods to minimize effects of potential confounds and, crucially, to generate rationally optimal decision points. Our procedure provides a generalized method of instantiating and computing optimal decision points, given any permutation of ratios, costs and rewards (see the Appendix). We found that nearly all of our participants jumped to conclusions, deciding before it was optimal to do so (from a risk-neutral perspective). Nevertheless, this tendency was greater the more delusion prone the participants were. No previous beads task investigation has included both rewards for correct decisions and costs for gathering information; the most that could justifiably be claimed from draws-to-decision studies using standard unincentivized paradigms is that delusional and delusionprone individuals 'reach conclusions more quickly' than controls. Our findings, however, support and clarify the claim that delusional ideation is associated with a tendency to 'jump to conclusions'.

Appendix

Calculation of optimal decision points¹¹

First we introduce some notation:

- w: the event that the next fish caught is white
- *b* : the event that the next fish caught is black
- *W* : the event that the true lake is 'White', the lake with predominately white fish
- *B* : the event that the true lake is 'Black', the lake with predominately black fish
- n_w : the number of white fish caught so far
- n_b : the number of black fish caught so far
- $\Delta = n_w n_b$
- Δ' = the value of Δ after catching one more fish

- *l* : the event that the next fish is of the currently 'leading' fish colour (l = w if $n_w > n_b$ and l = b if $n_w < n_b$)
- *L* : the event that the true lake is the currently 'leading' lake $(L = W \text{ if } n_w > n_b \text{ and } L = B \text{ if } n_w < n_b)$
- $p = \Pr(w | W) = \Pr(b | B) > 0.5$
- $\rho = p/(1-p) > 1$
- π : the probability of making a correct guess if guessing now
- *c* : the cost of seeing one more fish
- R : the reward for a correct guess

Suppose that *n* fish have been caught so far, of which n_w are white and n_b are black. We are interested in the probability that the true lake is White, conditional on having caught n_w white fish: $\Pr(W|(n_w, n_b))$ [note that $\Pr(B|(n_w, n_b)) = 1 - \Pr(W|(n_w, n_b))$]. We find this using Bayes' rule:

$$\Pr(W|(n_w, n_b)) = \frac{\Pr((n_w, n_b)|W)\Pr(W)}{\Pr((n_w, n_b)|W)\Pr(W) + \Pr((n_w, n_b)|B)\Pr(B)}$$
(1)

Because we have assumed a diffuse prior [i.e. participants are informed that both lakes are *a priori* equally likely, Pr(W) = Pr(B) = 0.5], the formula simplifies to:

$$\Pr(W|(n_w, n_b)) = \frac{\Pr((n_w, n_b)|W)}{\Pr((n_w, n_b)|W) + (\Pr(n_w, n_b)|B)}$$
(2)

The conditional probabilities that n_w of the *n* fish are white given the type of lake are:

$$\Pr((n_w, n_b)|W) = p^{n_w} (1-p)^{n_b} \binom{n_w}{n_w + n_b}$$
(3)

$$\Pr((n_w, n_b)|B) = (1-p)^{n_w}(p)^{n_b} \binom{n_w}{n_w + n_b}$$
(4)

Inserting these expressions into the formula for $Pr(W|(n_w, n_b))$ and dividing through by $p^{n_w}(1-p)^{n_b}$, we finally obtain:

$$\Pr(W|(n_w, n_b)) = \frac{1}{1 + \rho^{n_b - n_w}}, \quad \text{where} \quad \rho = \frac{p}{1 - p} > 1$$
(5)

Note that $\Pr(B|(n_w, n_b)) = 1 - \Pr(W|(n_w, n_b)) = 1/(1 + \rho^{n_w - n_b})$. Because p > 0.5, $\rho > 1$, so that $\Pr(W|(n_w, n_b)) > \Pr(B|(n_w, n_b))$ if and only if $n_w > n_b$. Therefore, if the decision maker decides to make a guess, they should always guess the lake corresponding to the most fish caught so far, and the probability of a correct guess is:

$$\int \Pr(W|(n_w, n_b)) = \frac{1}{1 + \rho_1^{n_b - n_w}} \quad \text{if } n_w > n_b$$

$$\pi = \begin{cases} \Pr(B|(n_w, n_b)) = \frac{1}{1 + \rho^{n_w - n_b}} & \text{if } n_b > n_w \\ \Pr(W|(n_w, n_b)) = \Pr(B|(n_w, n_b)) = \frac{1}{2} & \text{if } n_w = n_b \end{cases}$$
(6)

Note that this simplifies to the following extremely simple rule, where the probability of a correct guess depends only on the absolute value of the difference of the numbers of white and black fish caught so far:

$$\pi(\Delta) = \frac{1}{1 + \rho^{-\Delta}}, \quad \text{where} \quad \Delta = n_w - n_b \tag{7}$$

It follows that the only relevant state variable for our problem is Δ , and we can write the value function as $V(\Delta)$. Here $V(\Delta)$ is the expected value to the decision maker of having observed Δ more fish of one colour than of the other. As in any stopping-time problem, this value is the maximum of (*a*) the expected value of guessing immediately and (*b*) the expected option value of seeing one more fish.

The only missing element remaining in the formulation of this problem is the state transition matrix. This, however, is easy to find. Clearly, if one more fish is caught, Δ will change to either Δ +1 or Δ -1. Furthermore, if Δ =0, the change will be to Δ' = Δ +1 with probability one. If Δ >1, the probability of it changing to Δ +1 is simply the probability of getting one more fish of the currently leading colour:

$$Pr(\Delta' = \Delta + 1) = Pr(l|L) + Pr(l|L)$$
$$= p\pi(\Delta) + (1 - p)(1 - \pi(\Delta))$$
(8)

To summarize:

$$Pr(\Delta' = \Delta + 1) = \begin{cases} 1 & \text{if } \Delta = 0 \\ p\pi(\Delta) + (1 - p)(1 - \pi(\Delta)) & \text{otherwise} \end{cases}$$
(9)

Now, we are ready to formulate the Bellman equation for the optimal stopping time problem. The expected value of guessing now is $\pi(\Delta) \cdot R$. The expected value of drawing one more fish is $Pr(\Delta' = \Delta + 1)V(\Delta + 1) +$ $(1-Pr(\Delta' = \Delta + 1)V(\Delta - 1) - c)$. The value function is therefore defined recursively by:

$$V(\Delta) = \max\{\pi(\Delta)R; \Pr(\Delta' = \Delta + 1)V(\Delta + 1) + (1 - \Pr(\Delta' = \Delta + 1))V(\Delta - 1) - c\}$$
(10)

This equation is easy to solve by value function iteration. It can also be proven analytically that the stopping rule will always take the form 'Stop if and only if $\Delta > \Delta_0$ ' for some Δ_0 .

Acknowledgements

This research received no specific grant from any funding agency, commercial or not-for-profit sectors. We thank Sir Robin M. Murray and three anonymous reviewers for valuable feedback on a draft of this manuscript. We also thank K. Levine and R. J. Banis for their generous advice, and V. Bourne and R. Ross for helpful discussions. Finally, we are grateful to M. Naef for the opportunity to use the experimental economics laboratory (EconLab) at Royal Holloway, University of London, and to T. S. Woodward for providing the images that formed the basis of our stimuli.

Declaration of Interest

None.

Notes

- + The notes appear after the main text.
- ¹ Strictly speaking these decision points are only 'optimal' from a risk-neutral perspective. Risk-seeking or risk-averse individuals may make decisions that fail to maximize their expected outcome but that nevertheless maximize their expected utility (and thus are rational) given their risk preferences.
- ² To clarify: one of the two lakes was selected at random (each lake being equiprobable) and a sequence of fish was drawn at random from that lake, in accordance with the ratios of black and white fish in that lake (i.e. if the mostly black lake was selected, each fish drawn was black with probability 0.75).
- ³ By including this truly random sequence we avoided deceiving our participants and thus conformed to a key methodological principle of experimental economics (Hertwig & Ortmann, 2001).
- ⁴ In evaluating these errors, what matters is the evidence collected up to the point of the lake decision, not the evidence available in the full sequence. For example, in sequence E a participant who sampled five fish (B-W-B-W-W) and then decided on the mostly black lake made an inconsistent error, even though every subsequent fish in that sequence was black.
- ⁵ Although we analysed transformed data, we checked whether the same results were obtained when excluding the data point that comprised an outlier for untransformed data. Without this data point, PDI scores still significantly predicted draws to decision both as the sole predictor ($F_{1,109} = 4.150$, p = 0.044, $R_{adjusted}^2 = 0.028$) and when accounting for risk aversion and intelligence ($\Delta F_{1,107} = 5.413$, $\Delta p = 0.022$, $\Delta R^2 = 0.038$; $F_{3,107} = 11.262$, p < 0.001, $R_{adjusted}^2 = 0.219$).
- ⁶ Although there were no significant differences between males and females for PDI scores, for robustness an additional sequential regression was conducted. Adding PDI scores as a fourth predictor significantly improved a previous model with gender, risk aversion and intelligence as the three predictors ($\Delta F_{1,107}$ =4.024, Δp =0.047, ΔR^2 = 0.027; $F_{4,107}$ =11.077, p<0.001, $R^2_{adjusted}$ =0.266).
- ⁷ Negative and positive deviations indicate early and late decisions respectively.
- ⁸ This result was also found when excluding the outlier on untransformed data: mean deviation = -2.546, s.D. = 1.75, $t_{110} = 15.360$, p < 0.001.
- ⁹ The high-delusion-prone participants decided significantly earlier (and thus deviated significantly further from optimal decisions) than the low-delusion-prone participants [$F_{1,53}$ = 4.317, p = 0.043 (two-tailed), η_p^2 = 0.075]. This was also the

case when risk aversion and intelligence were accounted for in an ANCOVA [$F_{1,51}$ = 5.725, p = 0.020 (two-tailed), η_p^2 = 0.101].

- ¹⁰ As the median PDI score in our sample was higher than in Colbert & Peters' (2002) sample, the validity of our low-delusion-prone group could be queried. For this reason we repeated these analyses using groups based on Colbert & Peters' quartile values. One-sample *t* tests again showed that both groups of participants jumped to conclusions [mean deviation for low-delusion-prone participants (PDI < 34.5) = -2.0615, s.D. = 1.357, $t_{12} = 5.476$, p < 0.001; mean deviation for high-delusion-prone participants (PDI > 75) = -2.804, s.D. = 1.839, $t_{49} = 10.781$, p < 0.001].
- ¹¹ Our model is essentially a special case of the costed-Bayesian model analysed by Moutoussis *et al.* (2011) where we assume agents to act as in classical decision theory and classical game theory, rather than as in quantal response equilibrium models (McKelvey & Palfrey, 1995).

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