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Stability and dynamics of a cosh-Gaussian laser beam in relativistic thermal quantum plasma

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Abstract

This paper presents an investigation on the self-focusing of a cosh-Gaussian laser beam in the thermal quantum plasma (TQP) by taking into account the effects of relativistic nonlinearity. An appropriate nonlinear Schrödinger equation has been solved analytically by applying the variational approach. The self-focusing and the self-phase modulation are examined under a variety of parameters. The self-trapping of a cosh-Gaussian laser beam is further studied at various values of the decentered parameter, b with different absorption levels, k_i'. Numerical analysis shows that these parameters play a vital role in propagation characteristics. The significant contribution of the quantum effects to enhance the self-focusing and minimize the longitudinal phase has been observed. Further, a comparison has been made with the classical relativistic (CR), the relativistic cold quantum (RCQ), and the thermal quantum (TQ) regimes. The self-focusing is found to occur earlier and is strongest for the case of TQP in the present analysis.

Introduction

Technological development in the field of laser physics has ushered a new era where highly intense lasers are available. The interaction of these intense laser radiations with plasma has been known to produce various nonlinear phenomena such as self-focusing, self-phase modulation (SPM), harmonic generations, etc. Theoretical and experimental study of these nonlinear phenomena is an active area of research due to its importance in potential applications such as plasma-based accelerators (Sarkisov *et al.*, 1999), inertial confinement fusion (ICF) (Tabak *et al.*, 1994; Regan *et al.*, 1999), ionospheric modification (Guzdar *et al.*, 1998; Gondarenko *et al.*, 2005), and other applications (Askar'yan, 1962; Parashar *et al.*, 1997; Honda *et al.*, 2000; Liu and Tripathi, 2001; Mulser and Bauer, 2004; Gupta and Suk, 2007; Hora, 2007; Winterberg, 2008; Uhm *et al.*, 2012). In order to practically realize these applications, it is important that a laser beam should propagate hundreds of Rayleigh lengths. The development of high-intensity laser beams makes the investigations of such nonlinear effects feasible. When high power intense laser beam/pulse propagates through a plasma medium, many instabilities and nonlinear phenomena such as the SPM, the filamentation instability, the group velocity dispersion (GVD), the finite pulse effects, and the relativistic and ponderomotive self-focusing effects, become important. Therefore, it is important to study analytically and numerically some of these effects. This leads us to understand physical insight of basic and fundamental processes.

Among the fundamental processes, the self-focusing and self-trapping are genuinely nonlinear phenomena which have become a fascinating field in the modern plasma physics research. For the first time, these two mechanisms were reported by Askar'yan (1962) and Chiao *et al.* (1964), respectively, and had been focus of attention for nearly five decades because of their relevance to a number of newly discovered processes. The self-focusing is a process in which a beam of light comes to focus as consequences of a nonlinear response of a material medium. In a nonlinear medium, a high power electromagnetic beam creates a refractive index profile across its cross-section corresponding to its own intensity profile. The refractive index of the medium increases with the beam intensity. As a result, the beam focuses of its own. In laser–plasma interaction, the generic process of the self-focusing of laser beams (Chiao *et al.*, 1964; Kelley, 1965; Sodha *et al.*, 1974, 1976; Milchberg *et al.*, 1995; Saini and Gill, 2006; Yu *et al.*, 2007; Gill *et al.*, 2010a, 2010b, 2010c, 2011; Kaur *et al.*, 2010, 2011; Mahajan *et al.*, 2010) has been focus of attention as it affects many other nonlinear processes. It plays crucial role in the beam propagation.

The self-focusing of laser beams in different plasma environments using a fundamental Gaussian beam has been confined mostly to the classical regimes. With the advent of ultra-high-intensity laser pulses, the study of laser–plasma interactions has undergone a paradigm shift. The relativistic nonlinearity is a fundamental nonlinear effect which occurs in highly intense field. This nonlinearity is particularly interesting when laser power is sufficiently large. The electric field associated with high power laser pulses leads to a quiver motion of electrons of the order of the speed of light in vacuum. The quiver motion of electrons leads to their expulsion from the region of high intensity. The expulsion due to ponderomotive force sets up a space charge field that retards electrons and eventually a quasi-steady state is reached. The effect of quiver motion of electrons modifies the refractive index. The transverse gradient of the nonlinear refractive index is responsible for the relativistic self-focusing (Zhou *et al*., 2011; Patil *et al*., 2012; Niknam *et al*., 2013; Bokaei and Niknam, 2014). The relativistic self-focusing is counterbalanced by the tendency of the beam to spread because of diffraction. In the absence of nonlinearities, the beam will spread substantially in a Rayleigh length, R_d ($\sim ka_0^2$), where k is the wavenumber and a_0 is the spot size of laser beam.

The relativistic self-focusing is not limited to low density and high temperature plasma. Last decades have witnessed an increasing interest in propagation of laser beams in dense plasmas where the quantum effects are important (Manfredi, 2005; Shukla *et al*., 2006; Shukla and Eliasson, 2010, 2011; Haas, 2011; Habibi and Ghamari, 2012a, 2012b, 2015a, 2015b; Patil and Takale, 2013; Patil *et al*., 2013a, 2013b). The combined effects of the relativistic variation of mass and quantum correction would significantly change the dielectric function and hence modify the nonlinear behavior of electromagnetic wave (EMW) propagation in the quantum plasmas. In principle, the classical plasma is identified by virtue of high temperature and low density, while the quantum plasma is characterized by high density and low temperature (Shukla and Eliasson, 2010, 2011). The parameter $\chi = T_F/T$ is often used to characterize the relevance of quantum effects. Here T_F is the Fermi temperature and T is the corresponding plasma temperature. For $\chi \geq 1$, the quantum effects are dominant and Fermi–Dirac distribution statistics is relevant for description of plasma. The quantum effects can also be measured by the thermal de-Broglie wavelength $\lambda_B = \hbar/((m_e k_B T)^{1/2})$ where $\hbar$ is the rationalized Planck's constant, k_B is the Boltzmann constant, and m_e is the electron mass. Here λ_B is the measure of spatial extension of the particle wave function. Thus, the quantum effects are relevant when λ_B of the electrons is equal or greater than the average inter-electron distance $n_e^{-1/3}$. Further in the classical regime, λ_B is small enough to ignore the overlapping of wave functions and quantum interferences. Eliasson and Shukla (2012) have presented a quantum relativistic model for nonlinear interaction between the large amplitude waves and quantum plasmas. The relativistic effects in such plasmas arise due to an increase in fermion number density in the case of degenerate plasma and influence the dynamics of high power EMWs. The plasma dielectric function for an unmagnetized and collisionless electron quantum plasma including the fermion gas pressure and Bohm potential has been derived by Ali and Shukla (2006). Na and Jung (2009) extended this model to study the ponderomotive magnetization in quantum plasma. The significant research work on different aspects of nonlinear wave propagation in quantum plasmas has been reported (Kremp *et al*., 1999; Andreev, 2000; Azechi and FIREX Project, 2006; Marklund and Shukla, 2006; Shukla *et al*., 2006; Shukla and Eliasson, 2007; Glenzer and Redmer, 2009). With the progress in technology of short-pulse and high-intensity radiation, creation and diagnosis of dense quantum plasmas with X-ray-free electron lasers (FELs) and modern high power lasers is possible (Peyrusse *et al*., 1995; Kodama *et al*., 2001; Neumayer *et al*., 2006; Vinko *et al*., 2012). Powerful X-ray sources (Landen *et al*., 2001; Lee *et al*., 2003; Faenov *et al*., 2015) replaced the optical lasers in penetrating dense or compressed matter and access the dense plasma physics regimes with electron density of the solid. Kodama *et al.* (2001) have reported that in laser-driven implosion of spherical polymer shells, an increase in density of 1000 times relative to the solid state (Azechi *et al*., 1991) has been achieved. These densities are large enough to enable controlled fusion provided the compressed fuel is heated to a temperature of about 10^8 K. The X-ray-FELs explore matter on the scale of a few angstroms and the quantum effects which play an important role in the degenerate electron gas and the warm dense matter (Glenzer *et al*., 2007; Glenzer and Redmer, 2009; Neumayer *et al*., 2010) have been measured experimentally. The quantum plasmas are encountered in many environments such as in quantum dots (Shpatakovskaya, 2006), astrophysical systems (Opher *et al*., 2001), and neutron stars (Chabrier *et al*., 2002). Further, it has been recognized that the quantum mechanical effects play an important role in the intense laser solid density plasma interaction experiments (Andreev, 2000; Marklund and Shukla, 2006; Mourou *et al*., 2006). It has been noticed that X-ray lasers and the FELs also undergo compression (Malkin and Fisch, 2007; Malkin *et al*., 2007). Propagation losses, deflection, and critical issues of relativistic ultra-intense laser–plasma interactions along with energy transport from interaction region to core plasma are the key issues of ICF-related research (Kodama *et al*., 2001; Lindl *et al*., 2004; Azechi and FIREX Project, 2006; Remington *et al*., 2006).

During the past few years, theoretical work on the quantum plasmas has staged an impressive comeback due to their tendency to behave as an active nonlinear medium. Ali and Shukla (2006) presented analytical and numerical studies of potential distributions around a moving test charge in quantum plasmas. They used quantum hydrodynamic (QHD) and Poisson equations to obtain the potential, which depends on the dielectric susceptibility of quantum plasmas. Ren *et al.* (2007) derived the dispersion relation of linear waves in uniform cold quantum plasma (CQP) using the QHD with the magnetic field of the Wigner–Poisson system. The quantum effects on the magnetization due to ponderomotive force are investigated by Jung and Murakami (2009). The results showed that the quantum effects cause the magnetization and cyclotron motion in quantum plasmas. The temperature effects on the nonstationary Karpman–Washimi ponderomotive magnetization are investigated in quantum Fermi plasmas by Na and Jung (2009). It has been observed that the frequency dependence on the ponderomotive magnetization diminishes with an increase in the Fermi temperature. Hefferon *et al.* (2010) studied the beam dynamics of a Gaussian laser pulse propagating through quantum plasma. They applied a quantum fluid model and Maxwell's equations for the beam dynamics and the quantum dielectric response (Jung and Murakami, 2009; Na and Jung, 2009) to derive a nonlinear Schrödinger equation for the electromagnetic (EM) field envelope. The results showed a longitudinal compression and stronger beam localization due to quantum effects. Habibi and Ghamari (2012a) studied the self-focusing of an electromagnetic Gaussian beam in an inhomogeneous CQP. A better self-focusing with higher oscillations is obtained because of the quantum effects and upward density profile. In addition to the ramp density profile, the role of relativistic nonlinearity on nonstationary self-focusing of intense laser beam in the CQP has been reported in another investigation (Habibi and Ghamari, 2012b). Stronger self-focusing at the rear part of the pulse is obtained. Further, Patil *et al.* (2013a, 2013b) have studied the self-focusing of a Gaussian laser beam in the relativistic cold quantum (RCQ) plasma. Additional self-focusing using the CQP has been observed in comparison with the classical relativistic (CR) case. Similar results were reported by the relativistic self-focusing of a Gaussian laser beam in collisional quantum plasma by Zare *et al.* (2015). Improved focusing of a cosh-Gaussian laser beam in the CQP using higher order paraxial ray approximation (PRA) is presented in a recent investigation by Habibi and Ghamari (2015a). Further, they studied the significant enhancement in the relativistic self-focusing of a cosh-Gaussian laser beam in dense plasmas using the ramp density profile (Habibi and Ghamari, 2015b). They used modified refractive index of an inhomogeneous CQP with quantum correction in the relativistic regime. In relatively recent studies, the stationary self-focusing of a Gaussian laser beam in the relativistic thermal quantum plasma (TQP) has been studied in detail (Patil and Takale, 2013). Patil and Takale (2014) further provided the evidence of strong self-focusing in the TQP as compared with the case of the CQP. The relativistic self-focusing of ultra-high-intensity X-ray laser beams using upward density profile in the warm quantum plasma is reported by Habibi and Ghamari (2014). It is observed that the quantum effects enhance the self-focusing of laser beams.

There are several approximate analytical approaches to describe the effects of self-focusing such as the PRA (Sodha *et al.*, 1974, 1976), the moment theory approach (Firth, 1977; Lam *et al.*, 1977), and the source-dependent expansion (SDE) method (Sprangle *et al.*, 2000). Each of these theories has limitations in describing completely the experimental/computer simulation results. Most of the investigations related to the self-focusing are based on Wentzel–Kramers–Brillouin (WKB) approximation and PRA given by Akhmanov *et al.* (1968) and developed by Sodha *et al.* (1976). This theory being local in character overemphasizes the field closest to the beam axis and lacks global pulse dynamics. Furthermore, it also predicts the unphysical phase relationship (Karlsson *et al.*, 1991). However, study of certain moments is done by the moment theory approach. The moment theory is not applicable for the beams of all types of irradiance and it also lacks the phase description. It has also been pointed out that the PRA is not applicable when high power laser beams are used (Subbarao *et al.*, 1998). Another global approach is the variational approach (Firth, 1977; Anderson and Bonnedal, 1979), though crude to describe the singularity formation and collapse dynamics, it is fairly general in nature to study the propagation of laser beam and also correctly predicts the phase.

Several research investigations on the self-focusing of laser beams have been confined to the cylindrically symmetric Gaussian beams symmetry (Akhmanov *et al.*, 1968; Esarey *et al.*, 1997; Sharma *et al.*, 2004), a very few have been reported on Hermite-sinusoidal-Gaussian (HSG) laser beams in the turbulent atmosphere (Baykal, 2004), Hermite–Gaussian beams (Takale *et al.*, 2009), and Hermite-cosh-Gaussian (HChG) laser beams (Belafhal and Ibnchaikh, 2000; Patil *et al.*, 2010). Apart from these, great interest has been evinced in the cosh-Gaussian beams. This is due to the fact that the propagation properties of the cosh-Gaussian laser beams have important technological issues as these beams possess high power in comparison with that of a circular Gaussian laser beam. A review of the literature highlights the fact that the propagation characteristics of the cosh-Gaussian laser beams in the TQP have not been studied to a significant extent. The objective of the present research work is to investigate the self-focusing of a cosh-Gaussian laser beam in the relativistic TQP by making use of the variational approach.

This paper has been structured as follows: in the section Basic formulation, a brief description of the effective plasma permittivity is given and the evolution equations governing the beam width parameter and the longitudinal phase are derived. In the section Self-trapping, authors have studied the self-trapped mode, and the section Numerical results and discussion is devoted to discussion. The stability characteristics of cosh-Gaussian laser beams are studied in the section Stability criterion of beam dynamics. Conclusions of the present analysis are presented in the last section.

Basic formulation

The present model is set up in an unmagnetized and collisionless TQP considering the relativistic nonlinearity. Assuming wave propagation in the $z-$ direction, the electric vector of laser beam E satisfies the following wave equation:

$$\frac{\partial^2 E}{\partial z^2} + \nabla_\perp^2 E + \frac{\omega^2}{c^2}\varepsilon E = 0 \qquad (1)$$

which may be derived directly from Maxwell's equations by neglecting the term $\nabla(\nabla \times E)$. The effective dielectric constant of homogeneous gaseous plasma can be expressed as:

$$\epsilon = \epsilon_0 + \Phi(EE^*) \qquad (2)$$

where $\epsilon_0 = 1 - (\omega_p^2/\omega^2)$ and Φ are the linear and nonlinear parts of the dielectric constant respectively. $\omega_p^2 = 4\pi n_0 e^2/m_0$ is the plasma frequency and e and m_0 are charge and rest mass of electron, respectively.

Starting by considering the dielectric constant in an unmagnetized and collisionless electron quantum plasmas including the Fermi gas pressure term as well as the Bohm potential effect caused by the collective interactions (Ali and Shukla, 2006; Na and Jung, 2009):

$$\epsilon = 1 - \frac{\omega_p^2}{\gamma\omega^2}\left(1 - \frac{\delta}{\gamma} - \beta\right)^{-1} \qquad (3)$$

where $\beta = k^2 v_{Fe}^2/\omega^2$, $v_{Fe} = \sqrt{(2k_B T_{Fe}/m_0)}$ is the Fermi speed, $\delta = 4\pi^4 h^2/m_0^2\omega^2\lambda^4$, $\gamma = \sqrt{1 + \alpha EE^*}$ is the relativistic factor with $\alpha = e^2/m_0^2\omega^2 c^2$, and λ is the wavelength of the laser used. It may be mentioned that simple expressions can be found only in the limiting cases: $T \gg T_{Fe}$ (corresponding to the classical regime) and $T \ll T_{Fe}$ (corresponding to the fully degenerate quantum plasma case). A smooth transition cannot be achieved in a straightforward manner (Manassah *et al.*, 1988) using dimensional arguments. However, the thermal speed becomes meaningless in very low temperature limit and should be replaced by the

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Fermi velocity. The nonlinear part of the dielectric constant can be written as:

$$\Phi(EE^*) = \frac{\omega_p^2}{\omega^2}\left[1 - \frac{1}{\gamma}\left(1 - \frac{\delta}{\gamma} - \beta\right)^{-1}\right] \quad (4)$$

The amplitude of the electric field of laser beam is given by:

$$E = \psi(r, z)\exp[\iota(\omega t - kz)]$$

where $\psi(r, z)$ is a complex function of its argument. Substituting the above expression for E into Eq. (1), one may neglect $\partial^2\psi/\partial z^2$ and obtain the envelop equation governing the propagation of the beam in nonlinear media as shown below:

$$\left[-2\iota k\frac{\partial}{\partial z} + \nabla_\perp^2 + \frac{\omega^2}{c^2}\left(\frac{\omega_p^2}{\omega^2}\left(1 - \frac{1}{\gamma}\left(1 - \frac{\delta}{\gamma} - \beta\right)^{-1}\right)\right)\right]\psi(r, z) = 0 \quad (5)$$

where $\nabla_\perp^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$.

Equation (5) is a nonlinear parabolic partial differential equation. The variational approach, which has rigorous basis, is employed here to investigate nonlinear wave propagation. The exact solution to Eq. (5) is not available and we therefore seek numerical or approximate analytical methods. Further, we choose the latter using a powerful variational method that has been used in several similar investigations (Anderson and Bonnedal, 1979; Gill et al., 2010a, 2010b, 2011; Kaur et al., 2010, 2011; Mahajan et al., 2010). Following the procedure of Anderson and Bonnedal (1979), we reformulate Eq. (5) into a variational problem corresponding to a Lagrangian L to make $(\delta L/\delta z) = 0$. Solving Eq. (5) is equivalent to making a certain function an extremum. Lagrangian L corresponding to Eq. (5) is given by:

$$\begin{aligned} L = \iota k\left(\psi\frac{\partial\psi^*}{\partial z} - \psi^*\frac{\partial\psi}{\partial z}\right) - \left[\left|\frac{\partial\psi}{\partial r}\right|\right]^2 \\ + \frac{\omega_p^2}{c^2}\left[-(\delta+\beta)\alpha|\psi|^2 + 0.5(\alpha|\psi|^2)^2\left(\delta - \frac{1}{2}\right)\right. \\ \left. + \frac{\beta}{4}(\alpha|\psi|^2)^2 - \frac{1}{12}\delta(\alpha|\psi|^2)^3\right] \end{aligned} \quad (6)$$

Thus, the solution to the variational problem

$$\delta\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} L\,dr\,dz = 0 \quad (7)$$

also solves the nonlinear Schrödinger Eq. (5). We use the cosh-Gaussian field distribution ansatz for the amplitude ψ as trial function:

$$\begin{aligned} \psi(r, z) = \frac{\psi_0(z)}{2}\exp\left(\frac{b^2}{4} - 2k_i z\right) \\ \times\left(\exp\left[-\left(\frac{r}{a(z)} + \frac{b}{2}\right)^2\right] + \exp\left[-\left(\frac{r}{a(z)} - \frac{b}{2}\right)^2\right]\right) \\ \times\exp(\iota q'(z)r^2 + \iota\phi(z)) \end{aligned} \quad (8)$$

where $a(z)$ is the beam width of the Gaussian amplitude distribution, ψ_0 is the amplitude at the central position, k_i is the absorption coefficient, b the decentered parameter, also termed the normalized modal parameter, $q'(z)$ is the spatial chirp, and $\phi(z)$ is the phase of laser beam. Using the expression for ψ as trial function into the Lagrangian L of Eq. (6) and after integrating L, we obtain:

$$< L > = \int_{-\infty}^{\infty} L\,dr \quad (9)$$

The reduced variational problem is obtained by solving the integration in Eq. (9) using some standard integrals to get:

$$\begin{aligned} < L > = \frac{\iota k a\sqrt{\pi}}{8\sqrt{2}}e^{-4k_i z}(b^2+8)\left(\psi_0\frac{\partial\psi_0^*}{\partial z} - \psi_0^*\frac{\partial\psi_0}{\partial z}\right) \\ + \frac{k|\psi_0|^2}{32\sqrt{2}}\frac{dq'}{dz}a^3\sqrt{\pi}e^{-4k_i z}(b^2+8) \\ + \frac{k|\psi_0|^2}{4\sqrt{2}}\frac{d\phi}{dz}a\sqrt{\pi}e^{-4k_i z}(b^2+8) \\ - \frac{|\psi_0|^2}{4\sqrt{2}}e^{-4k_i z}\sqrt{\pi}\left[\frac{2}{a} + \frac{b^4}{2a} + 2q'^2a^3 + \frac{q'^2b^2a^3}{4} - \frac{7b^2}{4a}\right] \\ - \frac{\omega_p^2}{c^2}\frac{|\alpha^2\psi_0|^2}{8\sqrt{2}}(\delta+\beta)e^{-4k_i z}\sqrt{\pi}(b^2+8) \\ + \frac{\omega_p^2}{64c^2}a(\alpha^2|\psi_0|^2)^2(\delta - 0.5)e^{-\frac{b^2}{2} - 8k_i z}\sqrt{\pi}(b^2+8) \\ + \frac{\omega_p^2}{64c^2}a(\alpha^2|\psi_0|^2)^2\beta e^{-\frac{b^2}{2} - 8k_i z}\sqrt{\pi}(b^2+8) \\ - \frac{\omega_p^2}{768c^2}a\delta(\alpha^2|\psi_0|^2)^3e^{-12k_i z}\sqrt{\pi}\left(\frac{10}{\sqrt{6}} + \frac{1.5b^2}{\sqrt{6}}\right) \\ - \frac{\omega_p^2}{768c^2}a\delta(\alpha^2|\psi_0|^2)^3e^{\frac{b^2}{2} - 12k_i z}\sqrt{\pi}\left(\frac{3}{\sqrt{6}} + 6 + \frac{3b^2}{4}\right) \end{aligned} \quad (10)$$

We differentiate $< L >$ with respect to the variables ψ_0 and ψ_0^*. Using the Euler–Lagrange's equations, we arrive at two equations. Further, we multiply these two equations with variables ψ_0 and ψ_0^*. On adding the resultant equations, we arrive at Eq. (11). The variation of $< L >$, i.e., $(\delta < L > /\delta S) = 0$ where S denotes $\partial\psi_0/\partial z$, $\partial\psi_0^*/\partial z$, a, $q\prime$, $dq\prime/dz$ etc., and following the procedure of Anderson and Bonnedal (1979) and Saini and Gill (2006),

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we arrive at the following equations:

$$\frac{\iota ka}{4\sqrt{2}}\left(\psi_0\frac{\partial\psi_0^*}{\partial z}-\psi_0^*\frac{\partial\psi_0}{\partial z}\right)=-\frac{k|\psi_0|^2}{16\sqrt{2}}\frac{dq'}{dz}a^3-\frac{k|\psi_0|^2}{2\sqrt{2}}\frac{d\phi}{dz}a$$
$$+\frac{|\psi_0|^2}{2\sqrt{2}}\frac{1}{(b^2+8)}\left(\frac{2}{a}+\frac{b^4}{2a}+2q'^2a^3+\frac{q'^2b^2a^3}{4}-\frac{7b^2}{4a}\right)$$
$$+\frac{\omega_p^2}{c^2}\frac{|\alpha^2\psi_0|^2}{4\sqrt{2}}(\delta+\beta)e^{-4k_iz}-\frac{\omega_p^2}{16c^2}a(\alpha^2|\psi_0|^2)^2(\delta-0.5)e^{-\frac{b^2}{2}-4k_iz}$$
$$-\frac{\omega_p^2}{16c^2}a(\alpha^2|\psi_0|^2)^2\beta e^{-\frac{b^2}{2}-4k_iz}$$
$$+\frac{\omega_p^2}{128c^2}a\delta(\alpha^2|\psi_0|^2)^3e^{-8k_iz}\left(\frac{10}{\sqrt{6}}+\frac{1.5b^2}{\sqrt{6}}\right)$$
$$+\frac{\omega_p^2}{128c^2}a\delta(\alpha^2|\psi_0|^2)^3e^{\frac{b^2}{2}-8k_iz}\left(\frac{3}{\sqrt{6}}+0.75b^2+6\right) \tag{11}$$

and

$$a^2|\psi_0|^2=a_0^2A_0^2=\text{constant} \tag{12}$$

($\delta\langle L\rangle/\delta a$) = 0 gives,

$$\frac{\iota k}{8\sqrt{2}}\left(\psi_0\frac{\partial\psi_0^*}{\partial z}-\psi_0^*\frac{\partial\psi_0}{\partial z}\right)=-\frac{3}{32}\frac{k|\psi_0|^2}{\sqrt{2}}\frac{dq'}{dz}a^2$$
$$-\frac{k|\psi_0|^2}{2\sqrt{2}}\frac{d\phi}{dz}+\frac{|\psi_0|^2}{4\sqrt{2}}\frac{1}{(b^2+8)}\left(\frac{-2}{a^2}-\frac{b^4}{2a^2}+6q'^2a^2+\frac{3q'^2b^2a^2}{4}-\frac{2b^2}{4a^2}\right)$$
$$+\frac{\omega_p^2}{c^2}\frac{|\alpha^2\psi_0|^2}{8\sqrt{2}}(\delta+\beta)-\frac{\omega_p^2}{64c^2}a(\alpha^2|\psi_0|^2)^2(\delta-0.5)e^{-\frac{b^2}{2}-4k_iz}$$
$$-\frac{\omega_p^2}{64c^2}a(\alpha^2|\psi_0|^2)^2\beta e^{-\frac{b^2}{2}-4k_iz}$$
$$+\frac{\omega_p^2}{768c^2}a\delta(\alpha^2|\psi_0|^2)^3e^{-8k_iz}\left(\frac{10}{\sqrt{6}}+1.5b^2\right)$$
$$+\frac{\omega_p^2}{768c^2}a\delta(\alpha^2|\psi_0|^2)^3e^{\frac{b^2}{2}-8k_iz}\left(\frac{3}{\sqrt{6}}+0.75b^2+6\right) \tag{13}$$

($\delta\langle L\rangle/\delta q'$) = 0 gives;

$$q'=-\frac{k}{4a}\frac{da}{dz} \tag{14}$$

$$\frac{dq'}{dz}=-\frac{k}{4a}\frac{d^2a}{dz^2}+\frac{k}{4a^2}\left(\frac{da}{dz}\right)^2 \tag{15}$$

Substituting Eq. (11) into Eq. (13) and using Eqs. (14) and (15), we arrive at the following equation for a:

$$\frac{d^2a}{dz^2}=\frac{32}{k^2a^3(b^2+8)}\left(2-\frac{7b^2}{4}+\frac{b^4}{2}\right)$$
$$+\frac{1}{6\sqrt{3}}\frac{\omega_p^2}{k^2c^2a}(\alpha^2|\psi_0|^2)^2\delta\frac{e^{-8k_iz}}{(b^2+8)}(1.5b^2+10)$$
$$+\frac{\sqrt{2}}{6}\frac{\omega_p^2}{k^2c^2a}(\alpha^2|\psi_0|^2)^2\delta\frac{e^{-8k_iz+\frac{b^2}{2}}}{(b^2+8)}\left(0.75b^2+6+\frac{3}{\sqrt{6}}\right)$$
$$-\sqrt{2}\frac{\omega_p^2}{k^2c^2a}(\alpha^2|\psi_0|^2)(\delta-0.5)e^{-4k_iz+\frac{b^2}{2}}$$
$$-\sqrt{2}\frac{\omega_p^2}{k^2c^2a}(\alpha^2|\psi_0|^2)\beta e^{-4k_iz-\frac{b^2}{2}} \tag{16}$$

Finally, the phase $\phi(z)$ of amplitude $\psi_0(z)$ is obtained by using $\psi_0(z)=|\psi_0|e^{\iota\phi}$ and also using Eq. (16):

$$\frac{d\phi}{dz}=\frac{2}{ka^2(b^2+8)}\left(1-\frac{7b^2}{8}+\frac{b^4}{4}\right)$$
$$+\left(\frac{1}{384\sqrt{3}}+\frac{1}{128}\right)\frac{\omega_p^2}{kc^2}(\alpha^2|\psi_0|^2)^2\frac{e^{-8k_iz}}{(b^2+8)}(1.5b^2+10)$$
$$+\frac{\omega_p^2}{4kc^2}(\delta+\beta)e^{-4k_iz}+\frac{\sqrt{2}}{96}\frac{\omega_p^2}{kc^2}(\alpha^2|\psi_0|^2)^2\delta\frac{e^{-8k_iz+\frac{b^2}{2}}}{(b^2+8)}$$
$$\times\left(0.75b^2+6+\frac{3}{\sqrt{6}}\right)$$
$$-\frac{5\sqrt{2}}{64}\frac{\omega_p^2}{kc^2}(\alpha^2|\psi_0|^2)(\delta-0.5)e^{-4k_iz+\frac{b^2}{2}}-\frac{5\sqrt{2}}{64}\frac{\omega_p^2}{kc^2}(\alpha^2|\psi_0|^2)$$
$$\times\beta e^{-4k_iz-\frac{b^2}{2}} \tag{17}$$

After normalization using $\eta=cz/\omega a_0^2$, we arrive at the following equations for a_n and ϕ:

$$\frac{d^2a_n}{d\eta^2}=\frac{32}{a_n^3(b^2+8)}\left(2-\frac{7b^2}{4}+\frac{b^4}{2}\right)$$
$$+\frac{1}{6\sqrt{3}}\frac{\omega_p^2a_0^2}{c^2a_n}(\alpha^2|\psi_0|^2)^2\delta\frac{e^{-8k_i'\eta}}{(b^2+8)}(1.5b^2+10)$$
$$+\frac{\sqrt{2}}{6}\frac{\omega_p^2a_0^2}{c^2a_n}(\alpha^2|\psi_0|^2)^2\delta\frac{e^{-8k_i'\eta+\frac{b^2}{2}}}{(b^2+8)}\left(0.75b^2+6+\frac{3}{\sqrt{6}}\right)$$
$$-\sqrt{2}\frac{\omega_p^2a_0^2}{c^2a_n}(\alpha^2|\psi_0|^2)(\delta-0.5)e^{-4k_i'\eta+\frac{b^2}{2}}$$
$$-\sqrt{2}\frac{\omega_p^2a_0^2}{c^2a_n}(\alpha^2|\psi_0|^2)\beta e^{-4k_i'\eta-\frac{b^2}{2}} \tag{18}$$

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$$\frac{d\phi}{d\eta} = \frac{2}{a_n^2(b^2+8)}\left(1 - \frac{7b^2}{8} + \frac{b^4}{4}\right)$$
$$+ \left(\frac{1}{384\sqrt{3}} + \frac{1}{128}\right)\frac{\omega_p^2 a_0^2}{c^2}(\alpha^2|\psi_0|^2)^2 \frac{e^{-8k'_i\eta}}{(b^2+8)}(1.5b^2+10)$$
$$+ \frac{\omega_p^2 a_0^2}{4c^2}\delta e^{-4k'_i\eta} + \frac{\omega_p^2 a_0^2}{4c^2}\beta e^{-4k'_i\eta}$$
$$+ \frac{\sqrt{2}}{96}\frac{\omega_p^2 a_0^2}{c^2}(\alpha^2|\psi_0|^2)^2\delta\frac{e^{-8k'_i\eta+\frac{b^2}{2}}}{(b^2+8)}\left(0.75b^2 + 6 + \frac{3}{\sqrt{6}}\right)$$
$$- \frac{5\sqrt{2}}{64}\frac{\omega_p^2 a_0^2}{c^2}(\alpha^2|\psi_0|^2)(\delta - 0.5)e^{-4k'_i z+\frac{b^2}{2}}$$
$$- \frac{5\sqrt{2}}{64}\frac{\omega_p^2 a_0^2}{c^2}(\alpha^2|\psi_0|^2)\beta e^{-4k'_i\eta-\frac{b^2}{2}} \quad (19)$$

with $k'_i = k_i R_d$, k'_i is the normalized absorption coefficient.

Self-trapping

For an initially plane wave front, $a = a_0 = a_e$, $da/dz = 0$, and $a = 1$ at $z = 0$, the condition $d^2a/dz^2 = 0$ leads to propagation of a cosh-Gaussian laser beam without any change in its beam width. This is known as the uniform waveguide propagation. If we put $\frac{d^2a}{dz^2} = 0$ in Eq. (16), we obtain a relation between the dimensionless initial beam width parameter (ρ_0) and critical values of the intensity parameter Π (= $\alpha^2|\psi_0|^2$). The following general expression is obtained for determination of critical threshold for various values of b:

$$\frac{32}{k^2a^2(b^2+8)}\left(2 - \frac{7b^2}{4} + \frac{b^4}{2}\right)$$
$$= -\frac{1}{6\sqrt{3}}\frac{\omega_p^2}{k^2c^2}(\alpha^2|\psi_0|^2)^2\delta\frac{e^{-8k_iz}}{(b^2+8)}(1.5b^2+10)$$
$$- \frac{\sqrt{2}}{6}\frac{\omega_p^2}{k^2c^2}(\alpha^2|\psi_0|^2)^2\delta\frac{e^{-8k_iz+\frac{b^2}{2}}}{(b^2+8)}\left(0.75b^2+6+\frac{3}{\sqrt{6}}\right) \quad (20)$$
$$+ \sqrt{2}\frac{\omega_p^2}{k^2c^2}(\alpha^2|\psi_0|^2)(\delta-0.5)e^{-4k_iz+\frac{b^2}{2}}$$
$$+ \sqrt{2}\frac{\omega_p^2}{k^2c^2}(\alpha^2|\psi_0|^2)\beta e^{-4k_iz-\frac{b^2}{2}}$$

where $\rho_0 = a_e\omega/c$, is the initial dimensionless beam width parameter. The results are shown in the form of graphs. Such graphs are called the critical curves and these curves characterize the self-focusing region in the $\rho_0^2 - \Pi$ plane. The region below the critical curve corresponds to propagation of a cosh-Gaussian beam with self-focusing whereas the region above the critical curve corresponds to either oscillatory or steady state self-focusing of a cosh-Gaussian beam.

Numerical results and discussion

Equations (18) and (19) are nonlinearly coupled ordinary second order differential equations governing the normalized beam width parameter a_n and the phase ϕ as a function of distance of propagation η. These equations cannot be solved analytically. Therefore, we employ numerical computational techniques to study the dynamics of the beam. The first term on right hand side (R.H.S.) of Eq. (18) has its origin in the Laplacian ($\nabla_\perp^2$) appearing in the evolution Eq. (5) and it leads to the diffractional divergence of laser beam. The other terms in Eq. (18) arise due to the relativistic nonlinear effect when laser beam of high intensity is used. This effect arises due to the relativistic mass correction and it depends on various factors such as the intensity parameter ($\Pi = \alpha^2|\psi_0|^2$), relative plasma density, etc. The second, third, and the fourth terms have dependence on the cold quantum contribution δ as well as on the decentered parameter, b. However, the last term (fifth) is dominated by the TQP whose contribution comes from the term proportional to β which contains quantum effects *via* the Fermi temperature. A similar explanation can be given for the terms appearing in Eq. (19). The first term leads to the diffractional divergence of laser beam. The other terms in Eq. (19) appear because of the relativistic effects. The third term has dependence on the cold quantum contribution δ, the fifth and the sixth terms depend on δ as well as on the decentered parameter, b. The contribution of the fourth term comes from the term proportional to β which consists of the quantum effects. The last term is proportional to β, the decentered parameter, b and Π. The self-focusing/defocusing of laser beam is determined by the competing mechanisms on the R.H.S. of Eq. (18). The normalized beam width parameter, $a_n < 1$ corresponds to the self-focusing and $a_n > 1$ is the result of diffractional dominance over all the other terms leading to the defocusing of laser beam. If we set δ and β to zero, i.e., ignoring quantum effects, one obtains a differential equation for the normalized beam width parameter in the CR plasma. In Figure 1, the plot of normalized beam width parameter, a_n as a function of the dimensionless distance of propagation, η is displayed for $b = 0$ at different values of absorption levels k'_i considering the relativistic warm quantum plasma. In absence of the decentered parameter, a large value of k'_i is observed to weak the self-focusing effect. An analysis of evolution of the normalized beam width parameter as a function of distance of propagation, η is performed at $b = 0$, 1, 2 for three values of k'_i with other parameters chosen as follows: $a_0 = 0.002$ cm, $k = 0.53 \times 10^4$ cm^{-1}, $\omega_p = 2 \times 10^{-6} \times \omega$, $\omega = 1.778 \times 10^{20}$(rad/sec), $T_{Fe} = 10^9$ K, and $\lambda = 0.0106$ nm. Figure 2 plots a_n as a function of η for $b = 1$ with different values of absorption levels k'_i. The beam propagates oscillatory over several number of Rayleigh lengths. However, the situation has changed significantly in

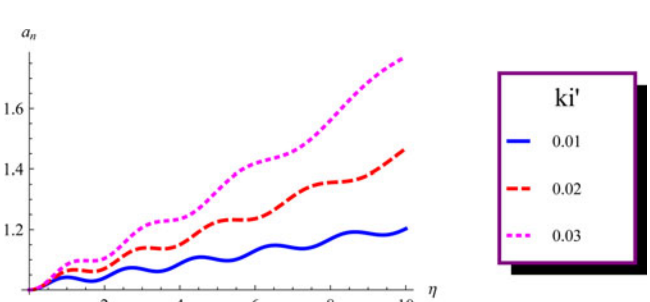

Fig. 1. Variation of normalized beam width parameter $a_n(\eta)$ as a function of dimensionless distance of propagation η considering relativistic nonlinearity for $b = 0$ with the following set of parameters for the various values of k'_i: $a_0 = 0.002$ cm, $\omega_p = 2 \times 10^{-6} \times \omega$, $\omega = 1.778 \times 10^{20}$(rad/sec), $k = 0.53 \times 10^4$ cm^{-1}, and intensity parameter, $\Pi = 0.1$. Solid curve corresponds to $k'_i = 0.01$, dashed to $k'_i = 0.02$, and dotted to $k'_i = 0.03$.

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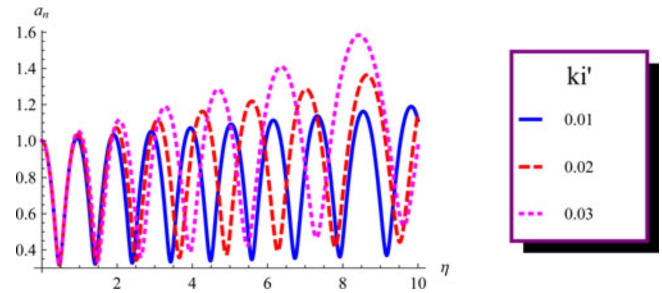

Fig. 2. Plot of $a_n(\eta)$ as a function of dimensionless distance of propagation η for $b = 1$ for the same set of parameters as in the caption of Figure 1.

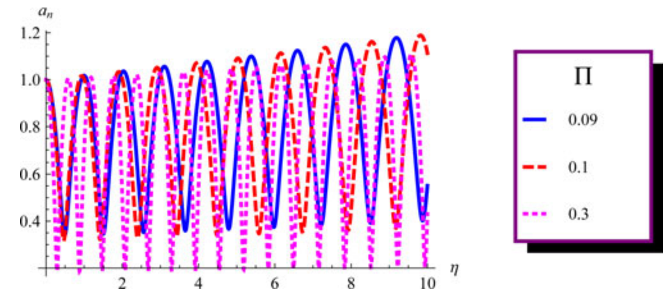

Fig. 4. Graph of $a_n(\eta)$ as a function of dimensionless distance of propagation η for $b = 1$ and $k_i' = 0.01$ at varying values of intensity parameter for the same set of parameters as in the caption of Figure 1. Solid curve corresponds to $\Pi = 0.09$, dashed to $\Pi = 0.1$, and dotted to $\Pi = 0.3$.

Figure 3, where we have plotted a_n as a function of η for $b = 2$ with varying k_i'. All curves are seen to exhibit sharp self-focusing effects.

The variation of the normalized beam width parameter, a_n as a function of the dimensionless distance of propagation, η with various intensity parameters ($\Pi = \alpha^2|\psi_0|^2$), keeping b and k_i' fixed is displayed in Figure 4. It is observed from Figure 4 that the self-focusing takes place at lower values of Π in comparison with the earlier investigations (Gill *et al.*, 2011). However, there is a substantial increase in the self-focusing with increase in Π. Figure 5 highlights the variation of the beam width parameter with η for the three cases of plasma. If we consider all the terms in Eq. (18), we obtain a plot between a_n and η for the case of relativistic TQP (dotted curve). If we put $\beta \to 0$, a curve for the RCQ plasma is obtained (dashed curve). Further if $\beta \to 0$ and $\delta \to 0$, i.e., ignoring the quantum effects, one obtains a curve for the CR plasma (solid curve). The results show much higher oscillations and better self-focusing in the TQP than the RCQ and the CR cases. Another aspect of this phenomenon observed here is that there is a decrease in the focusing length for the TQP in comparison with the earlier RCQ and the CR cases. The stronger pinching effect offered by the quantum effects enhances laser self-focusing. The largest self-focusing length is observed in the CR regime. Further, Eq. (19) describes the evolution of the longitudinal phase, $\phi(\eta, \alpha^2|\psi_0|^2)$ with the dimensionless distance of propagation (η) in Figure 6. The R.H.S. of Eq. (19) is a complicated function of b, k_i', the normalized beam width a_n and the intensity parameter, $\alpha^2|\psi_0|^2$. Though we can fix b, k_i', and $\alpha^2|\psi_0|^2$, the evolution of the normalized beam width parameter, a_n significantly affects the longitudinal phase with η. The longitudinal phase may be positive or negative depending upon the value of $\alpha^2|\psi_0|^2$ and k_i'. It is positive for the present model and does not show an oscillatory character as the R.H.S. of Eq. (19) for $b = 0$ depends only on $\alpha^2|\psi_0|^2$ and k_i' maintaining a nonlinear relationship. The regularized phase, ϕ_{reg}, is defined as:

$$\phi_{\text{reg}} = \phi(\eta) - \phi(\eta)|_{\alpha^2|\psi_0|^2=0}. \tag{21}$$

In the linear limit ($\alpha^2|\psi_0|^2 = 0$), the phase will have a more rapid growth during the propagation. But the presence of nonlinear forces counteracts the diffraction and tends to keep the pulse intensity higher in the plasma medium. However, the regularized phase always has a negative value. This is a consequence of frequency chirp as the time derivative of the phase will have same sign as in the case of a conventional plane wave. Thus, the red will always lead blue in the super continuum. This aspect of the phase change is well presented using the variational approach in the present investigation. Figure 7 displays the plot of the regularized phase versus η. This feature confirms the finding of Karlsson *et al.* (1992) and is contrary to the results of Manassah *et al.* (1988). The latter predicted that the regularized phase may be positive or negative during the beam propagation. This

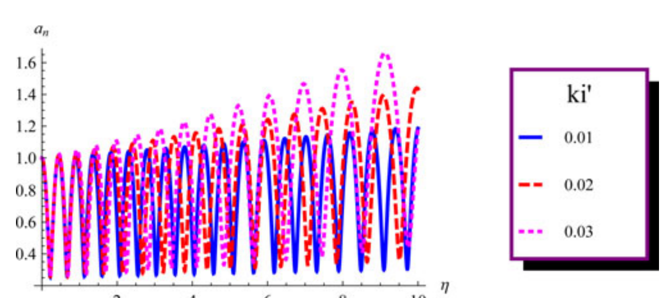

Fig. 3. Variation of normalized beam width $a_n(\eta)$ as a function of dimensionless distance of propagation η for $b = 2$ for the same set of parameters as in the caption of Figure 1.

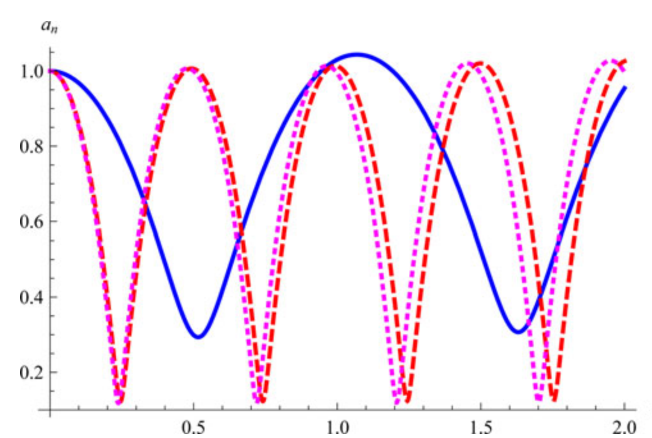

Fig. 5. Variation of $a_n(\eta)$ as a function of dimensionless distance of propagation η for $b = 1$, $k_i' = 0.01$, and $\Pi = 0.1$ for different plasma regimes using same set of parameters as in the caption of Figure 1. Solid curve corresponds to CR region, dashed to RCQ, and dotted to TQP.

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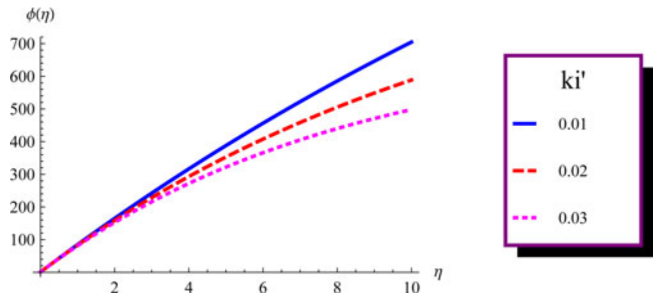

Fig. 6. Plot of longitudinal phase $\phi(\eta)$ versus dimensionless distance of propagation η for $b = 0$ for the same set of parameters as mentioned in the caption of Figure 1. Solid curve corresponds to $k_i' = 0.01$, dashed to $k_i' = 0.02$, and dotted to $k_i' = 0.03$.

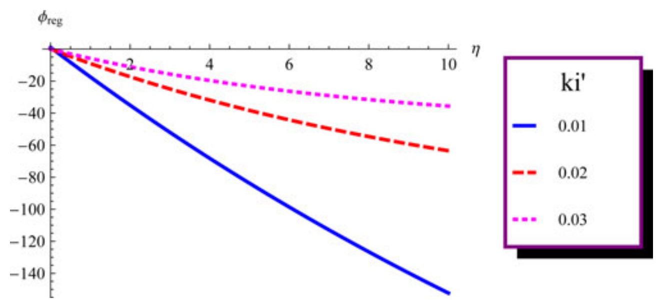

Fig. 7. Plot of regularized phase ϕ_{reg} versus dimensionless distance of propagation η for $b = 1$ for the same set of parameters as mentioned in the caption of Figure 1. Solid curve corresponds to $k_i' = 0.01$, dashed to $k_i' = 0.02$, and dotted to $k_i' = 0.03$.

inconsistent result stems from the fact that the PRA used by Manasaah *et al.*, does not correctly predict the ϕ_{reg}.

To further elucidate the results for delineating the underlying physics for the propagation of a cosh-Gaussian laser beam in the TQP, we numerically analyze the dimensionless initial beam width parameter (ρ_0) as a function of critical values of beam power Π ($= \alpha^2|\psi_0|^2$) for different values of b when the relativistic nonlinearity is considered. The results are depicted in the form of graphs. The critical curves for a cosh-Gaussian laser beam characterize the self-focusing region in the $\rho_0^2 - \Pi$ space. In Figure 8, ρ_0^2 versus Π is plotted for two values of b, i.e., $b = 0$ and $b = 1$. The solid curve corresponds to $b = 0$ and the dashed curve to $b = 1$. The solid curve depicts that the dependence of ρ_0^2 for $b = 0$ is much weaker on the high intensity, a result consistent with earlier calculations based on the variational approach (Anderson, 1978). The initial beam width is much higher than the earlier investigations. Higher the value of b, the faster is the initial decrease in ρ_0^2 with Π.

Lastly, Figure 9 depicts the variation of a_n with η in the relativistic TQP for three different values of the Fermi temperature. It is observed that the self-focusing length decreases with an increase in the Fermi electron temperature. A propagation to several Rayleigh lengths is observed. Also, it occurs at much lower value of the intensity parameter in comparison with the earlier investigations. This is apparently a consequence of the variational approach where the contribution of the whole wave front is considered in the averaging process. On the other hand, the PRA takes into account only those rays which are very close to the beam-axis.

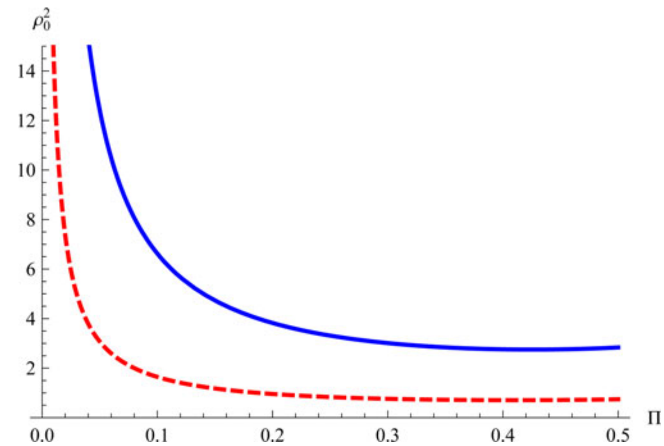

Fig. 8. Dependence of dimensionless initial beam width (ρ_0) as a function of Π with $b = 0$ and $b = 1$. Solid curve corresponds to $b = 0$ and dashed curve to $b = 1$.

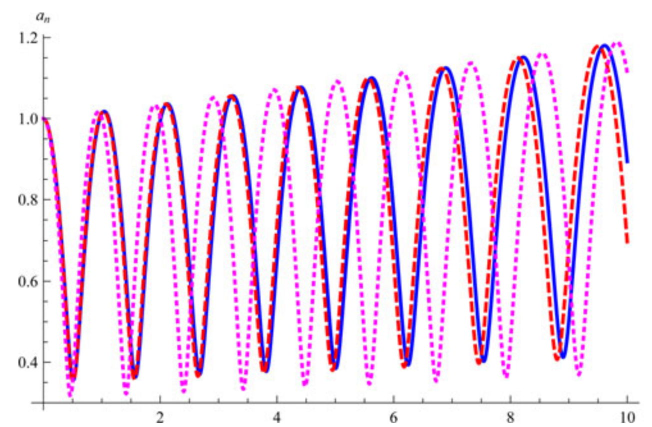

Fig. 9. Comparison of $a_n(\eta)$ with dimensionless distance of propagation η at different values of electron temperature (T_{Fe}). Other parameters are same as in the caption of Figure 1. Solid curve corresponds to $T_{\text{Fe}} = 10^7$ K, dashed to $T_{\text{Fe}} = 10^8$ K, and dotted to $T_{\text{Fe}} = 10^9$ K.

Stability criterion of beam dynamics

Variational method is used in several branches of physics and mathematics, can also be applied to study the stability characteristics of the evolution of cosh-Gaussian laser beam in TQP when relativistic effects are taken into account. The methods of nonlinear dynamics applied to dissipative solitons (Skarka *et al.*, 1997, 1999; Skarka and Aleksic, 2006) can also be used to study stability properties in the present investigation. The Euler–Lagrange equations are the starting point to establish stability criterion. The dependent variables are disturbed about their equilibrium values and method of Lyapunov's exponents (Lakshman and Rajasekar, 2003; Skarka and Aleksic, 2006) is used. Thus, for stability characteristics of the system, the following Jacobi determinant is constructed from derivatives with respect to amplitude, width and curvature in terms of S, F, and G where

$$S = \frac{d\psi_0}{dz} = \frac{-4q'|\psi_0|}{k} \tag{22}$$

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$$F = \frac{da}{dz} = \frac{-4aq'}{k} \quad (23)$$

$$G = \frac{dq'}{dz} = \frac{4q'^2}{k} - \frac{8}{ka^4(b^2+8)}\left(2 - \frac{7b^2}{4} + \frac{b^4}{2}\right) - \frac{1}{24\sqrt{3}}\frac{\omega_p^2}{kc^2a^2}(\alpha^2|\psi_0|^2)^2\delta\frac{e^{-8k_iz}}{(b^2+8)}(1.5b^2+10) - \frac{\sqrt{2}}{24}\frac{\omega_p^2}{kc^2a^2}(\alpha^2|\psi_0|^2)^2\delta\frac{e^{-8k_iz+\frac{b^2}{2}}}{(b^2+8)}\left(0.75b^2+6+\frac{3}{\sqrt{6}}\right) + \frac{\sqrt{2}}{4}\frac{\omega_p^2}{kc^2a^2}(\alpha^2|\psi_0|^2)e^{-4k_iz+\frac{b^2}{2}}(\delta-0.5) + \frac{\sqrt{2}}{4}\frac{\omega_p^2\beta}{kc^2a^2}(\alpha^2|\psi_0|^2)e^{-4k_iz-\frac{b^2}{2}} \quad (24)$$

$$\det|J-\lambda I| = \begin{vmatrix} \frac{\partial S}{\partial \psi_0}-\lambda & \frac{\partial S}{\partial a} & \frac{\partial S}{\partial q'} \\ \frac{\partial F}{\partial \psi_0} & \frac{\partial F}{\partial a}-\lambda & \frac{\partial F}{\partial q'} \\ \frac{\partial G}{\partial \psi_0} & \frac{\partial G}{\partial a} & \frac{\partial G}{\partial q'}-\lambda \end{vmatrix} = 0 \quad (25)$$

This leads to the following characteristic equation cubic in λ:

$$\lambda^3 + \alpha_1\lambda^2 + \alpha_2\lambda + \alpha_3 = 0 \quad (26)$$

where

$$\alpha_1 = -\frac{4q'}{k} \quad (27)$$

$$\alpha_2 = \frac{-32q'^2}{k^2} + \frac{128}{k^2a^4(b^2+8)}\left(2-\frac{7b^2}{4}+\frac{b^4}{2}\right) + \frac{1}{\sqrt{3}}\frac{\omega_p^2}{k^2c^2a^6}(\alpha^2A_0^2|\psi_{00}|^2)^2\delta\frac{e^{-8k_iz}}{(b^2+8)}(1.5b^2+10) + \sqrt{2}\frac{\omega_p^2}{k^2c^2a^6}(\alpha^2A_0^2|\psi_{00}|^2)^2\delta\frac{e^{-8k_iz+\frac{b^2}{2}}}{(b^2+8)}\left(0.75b^2+6+\frac{3}{\sqrt{6}}\right) - 4\sqrt{2}\frac{\omega_p^2}{k^2c^2a^4}(\alpha^2A_0^2|\psi_{00}|^2)e^{-4k_iz+\frac{b^2}{2}}(\delta-0.5) - 4\sqrt{2}\frac{\omega_p^2\beta}{k^2c^2a^4}(\alpha^2A_0^2|\psi_{00}|^2)e^{-4k_iz-\frac{b^2}{2}} \quad (28)$$

$$\alpha_3 = \frac{-128q'^3}{k^3} + \frac{512q'}{k^3a^4(b^2+8)}\left(2-\frac{7b^2}{4}+\frac{b^4}{2}\right) + \frac{4}{\sqrt{3}}\frac{\omega_p^2q'}{k^3c^2a^6}(\alpha^2A_0^2|\psi_{00}|^2)^2\delta\frac{e^{-8k_iz}}{(b^2+8)}(1.5b^2+10) + 4\sqrt{2}\frac{\omega_p^2q'}{k^3c^2a^6}(\alpha^2A_0^2|\psi_{00}|^2)^2\delta\frac{e^{-8k_iz+\frac{b^2}{2}}}{(b^2+8)}\left(0.75b^2+6+\frac{3}{\sqrt{6}}\right) - 16\sqrt{2}q'\frac{\omega_p^2}{k^3c^2a^4}(\alpha^2A_0^2|\psi_{00}|^2)e^{-8k_iz+\frac{b^2}{2}}(\delta-0.5) - 16\sqrt{2}q'\frac{\omega_p^2\beta}{k^2c^2a^4}(\alpha^2A_0^2|\psi_{00}|^2)e^{-4k_iz-\frac{b^2}{2}} \quad (29)$$

In order to have Lyapunov's stability, Hurwitz conditions must be fulfilled, i.e., $\alpha_1\alpha_2 - \alpha_3$ must be positive. According to the Routh–Hurwitz criterion, a necessary and sufficient condition for the stationary solutions to be stable is:

$$\alpha_1\alpha_2 - \alpha_3 > 0 \quad (30)$$

Equation (26) has a pair of purely imaginary roots at a critical point (Lugiato and Narducci, 1985):

$$\lambda = \pm\iota\nu, \quad \nu > 0 \quad (31)$$

Substituting Eq. (31) into Eq. (26), we get:

$$\nu^2 - \alpha_2 = 0 \quad (32)$$

and

$$\alpha_1\nu^2 - \alpha_3 = 0 \quad (33)$$

The critical condition of the Hopf-bifurcation is:

$$f = \alpha_1\alpha_2 - \alpha_3 = 0 \quad (34)$$

$f > 0$ is a necessary condition for the stationary solution to be stable, $f < 0$ is a necessary condition for the Hopf-bifurcation to emerge. It is observed that the condition $f \geq 0$ is satisfied for chosen set of parameters in the present analysis and therefore Hopf-bifurcation, resulting from the unstable fixed point does not come into play and it leads to overall stability of the beam dynamics (Wang, 1990).

Conclusions

In the present research work, authors have studied the evolution of a cosh-Gaussian laser beam in the TQP by taking into account the relativistic nonlinearity. An equation describing the envelope of the EM field is set up on prior knowledge based on the quantum fluid model, Maxwell's equations, and the earlier results of the quantum dielectric response and solved by using the variational approach. Approximate trial function for describing a cosh-Gaussian beam is used and equations for the beam width parameter and phase are derived and solved numerically for suitable set of parameters. It is observed that the combined quantum effects and the relativistic nonlinearity lead to a stronger self-focusing. The results are supported on the basis of comparison between the TQP, the CQP, and the CR regimes. The decentered parameter plays a crucial role in the beam dynamics. It is observed that the large value of k_i', the normalized absorption coefficient, weakens the self-focusing effect in the absence of b. However, an oscillatory self-focusing takes place for higher value of the decentered parameter. The aspect of the longitudinal phase and the regularized phase is well displayed in the form of graphs using the variational approach in the present investigation. Some features describing the critical curves for $b = 0$, 1 are also obtained for the chosen set of parameters. A better evolution of laser beam to several Rayleigh lengths is obtained using the variational approach as compared with the PRA. Because integration over the whole wave front is considered in the variational approach whereas in the PRA, only those rays are considered which are very close to the beam axis. Further, a cosh-Gaussian beam can handle high power in comparison with a circular Gaussian beam, the present investigation may shed some light on key aspects such as propagation, losses, diffraction, and critical issues of the relativistic ultra-intense laser–plasma interaction physics relevant to the ICF as well as the intense laser-solid density plasma experiments where the quantum effects are important. Lastly, stability properties of beam dynamics are studied and it is found that beam is marginally stable in the absence of any dissipation.

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