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Interaction of an electron beam with whistler waves in magnetoplasmas

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Abstract

The present paper studies the whistler wave interaction with an electron beam propagating through magnetized plasma. A dispersion relation of whistler waves has been derived, and first-order perturbation theory has been employed to obtain the growth rate of whistlers in the presence of parallel as well as oblique electron beam. For whistler waves propagating parallel to the magnetic field, that is, parallel whistlers, only the cyclotron resonance appears with a parallel beam, while for whistler waves propagating at an angle to the magnetic field, that is, oblique whistlers interaction with parallel beam or parallel whistlers interaction with oblique beam, the Cerenkov and the cyclotron resonances both appear. The growth rate is found to increase with an increase in the transverse component of beam velocity and with an increase in the strength of magnetic field. The whistler wave frequency decreases with an increase in the beam velocity. The obliqueness of the whistler mode modifies its dispersion characteristics as well as growth rate of the instability. For purely parallel-propagating beams, it is essential for the growth of whistler mode that the wave number perpendicular to the magnetic field should not be zero. The results presented may be applied to explain the mechanisms of the whistler wave excitation in space plasma.

Keywords: Beam velocity; Dispersion; Electron beam; Growth rate; Whistler wave

1. INTRODUCTION

A whistler wave is a low-frequency electromagnetic wave generated for instance by lightnings. Whistlers travel along earth's magnetic field from Northern hemisphere to Southern hemisphere or vice versa. They are circularly polarized waves in the audio frequency range. Whistler modes excited by energetic electrons are often observed in the outer earth's radiation belt and in the auroral kilometric radiation zone. Low-frequency whistlers have low velocity and therefore suffer dispersion in ionosphere and magnetosphere. They can be driven in two ways, either by electron temperature anisotropy or by electron beams or heat fluxes (Cipolla *et al.*, 1977; Talukdar *et al.*, 1989; James *et al.*, 1995; Borcia *et al.*, 2000; Jalori *et al.*, 2004; Dorf *et al.*, 2010; Sharma *et al.*, 2010).

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The theory shows that beam–plasma interactions lead to the excitation of various kinds of waves depending on plasma and beam parameters (Gupta & Sharma, 2004; Prakash & Sharma, 2009; Gupta *et al.*, 2010; 2014; Prakash *et al.*, 2013; 2014). Experiments performed with charged particle beams, modulated or unmodulated, have exhibited Cerenkov and cyclotron emission of whistler waves (Baranets *et al.*, 2012).

Whistler wave excitation by electron beam injection has been studied extensively in many works during past decades (Volokitin *et al.*, 1995; Krafft & Volokitin, 1998; Starodubtsev *et al.*, 1999). In particular, a large number of papers have appeared regarding the theory of whistler wave excitation by a pulsed or modulated thin beam injected parallel to the magnetic field in an unbounded homogeneous magnetoplasma. The interest in such studies has been spurred by several space and laboratory experiments on generation of the whistler emissions due to the interaction of electron beams with plasma (Krafft *et al.*, 1994*a*; 1994*b*; Shoucri & Gagne, 1978). Krafft *et al.* (1994*a*) have studied whistler wave excitation in a magnetized laboratory plasma by a density-modulated electron beam for frequency modulation below the gyro frequency. In this case, the maximum emission of the whistler waves occurred when the phase velocity of the whistler wave was comparable with the beam velocity. Krafft *et al.* (1994*b*) have studied the emission of whistler waves by a density-modulated electron beam in a laboratory plasma and results have been compared with the excitation by loop antenna.

In this paper, we present a local theory of whistler wave excitation by an electron beam in magnetoplasmas. Different whistler excitation mechanisms have been studied with an electron beam injected parallel to the static magnetic field B_s and with an electron beam injected at an angle to the magnetic field. In Section 2, we study the plasma response to whistler wave perturbation and in Section 3, the beam response has been analyzed. Section 4 gives the dispersion relation and growth rate of excited whistlers for different propagation angles of beam and whistlers. In Section 5, we discuss our results.

2. PLASMA RESPONSE

Consider a plasma immersed in a dc magnetic field $B_s \parallel \hat{z}$, with electron density n_{e0}, electron mass m_e, and electron charge, e. An electromagnetic whistler wave propagates through it, with electric field

$$\mathbf{E} = Ae^{-i(\omega t - \mathbf{k} \bullet \mathbf{r})},$$

where $\mathbf{k} = k_x\hat{x} + k_z\hat{z}$.

The magnetic field of the wave is $\mathbf{B} = \mathrm{c}\mathbf{k} \times (\mathbf{E}/\omega)$.

The equation of motion, governing the drift velocity of electron fluid is

$$m_e\left[\frac{\partial \mathbf{v_e}}{\partial t} + \mathbf{v_e} \cdot \nabla \mathbf{v_e}\right] = -e\mathbf{E} - \frac{e}{c}\mathbf{v}_e \times (\mathbf{B_S} + \mathbf{B}). \tag{1}$$

In equilibrium (i.e., in the absence of the wave), $\mathbf{v}_e = 0$. When wave is present, $\mathbf{E}$ and $\mathbf{B}$ of the wave are treated as small or perturbed quantities and the resultant electron drift $\mathbf{v_e}$ becomes the perturbed electron velocity $\mathbf{v}_{e1}$.

On linearizing Eq. (1), we obtain

$$\frac{\partial \mathbf{v}_{e1}}{\partial t} = -\frac{e\mathbf{E}}{m_e} - \mathbf{v}_{e1} \times \hat{z}\omega_{ce}, \tag{2}$$

where $\omega_{ce} = \mathrm{e}\mathbf{B_s}/m_e c$.

Writing x, y, z-components of Eq. (2), we obtain the perturbed electron velocities

$$v_{e1x} = -\frac{e\left(i\omega E_x + \omega_{ce}E_y\right)}{m_e\left(\omega^2 - \omega_{ce}^2\right)}, \tag{3}$$

$$v_{e1y} = \frac{e\left(\omega_{ce}E_x - i\omega E_y\right)}{m_e\left(\omega^2 - \omega_{ce}^2\right)}, \tag{4}$$

$$v_{e1z} = \frac{eE_z}{m_e i\omega}. \tag{5}$$

The perturbed electron current density

$$\mathbf{J}_{e1} = -n_{e0}e\mathbf{v}_{e1}. \tag{6}$$

Substituting Eqs. (3)–(5) into Eq. (6), we get

$$J_{e1x} = n_{e0}\frac{e^2}{m_e}\frac{\left(i\omega E_x + \omega_{ce}E_y\right)}{\left(\omega^2 - \omega_{ce}^2\right)}, \tag{7}$$

$$J_{e1y} = -n_{e0}\frac{e^2}{m_e}\frac{\left(\omega_{ce}E_x - i\omega E_y\right)}{\left(\omega^2 - \omega_{ce}^2\right)}, \tag{8}$$

$$J_{e1z} = -n_{e0}\frac{e^2 E_z}{m_e i\omega}. \tag{9}$$

3. BEAM RESPONSE

A uniform electron beam is propagating inside the plasma with density n_{b0}, mass m_e, and equilibrium beam velocity $\mathbf{v}_{b0} = v_{b0x}\hat{x} + v_{b0z}\hat{z}$. The quasineutrality condition at equilibrium is given by $en_{e0} + en_{b0} = en_{i0}$. The equilibrium is perturbed by an electromagnetic whistler wave. We consider two cases of beam propagation viz. parallel to the magnetic field, along the $\hat{z}$-direction and oblique propagation, in the x–z plane, in the presence of parallel whistlers or oblique whistlers in the plasma.

The response of parallel/oblique beam electrons to the parallel/oblique whistler wave perturbation is governed by the equation of motion, which on linearization yields the perturbed beam velocities as

$$\begin{aligned} v_{b1x} = {} & \frac{e}{im_e}\frac{\bar{\omega}^2}{\omega\left(\bar{\omega}^2 - \omega_{ce}^2\right)}E_x - \frac{e}{m_e}\frac{\bar{\omega}\omega_{ce}}{\omega\left(\bar{\omega}^2 - \omega_{ce}^2\right)}E_y \\ & - \frac{ie}{m_e}\frac{k_x v_{b0}\bar{\omega}}{\omega\left(\bar{\omega}^2 - \omega_{ce}^2\right)}E_z, \end{aligned} \tag{10}$$

$$\begin{aligned} v_{b1y} = {} & \frac{e}{m_e}\frac{\bar{\omega}\omega_{ce}}{\omega\left(\bar{\omega}^2 - \omega_{ce}^2\right)}E_x - \frac{ie}{m_e}\frac{\bar{\omega}^2}{\omega\left(\bar{\omega}^2 - \omega_{ce}^2\right)}E_y \\ & + \frac{e}{m_e}\frac{k_x v_{b0}\omega_{ce}}{\omega\left(\bar{\omega}^2 - \omega_{ce}^2\right)}E_z, \end{aligned} \tag{11}$$

$$v_{b1z} = -\frac{ie}{m_e\bar{\omega}}E_z - \frac{ie}{m_e}\frac{k_z v_{b0x}}{\omega\bar{\omega}}E_x, \tag{12}$$

where

$$\bar{\omega} = \omega - \left(k_x v_{b0x} + k_z v_{b0z}\right).$$

Substituting the perturbed beam velocities given by Eqs. (10)–(12) in the equation of continuity, we obtain the perturbed beam density as

$$n_{b1} = \frac{i n_{b0} e k_x}{m_e \omega(\bar{\omega}^2 - \omega_{ce}^2)}\left[-\bar{\omega}E_x + i\omega_{ce}E_y - k_x v_{b0z}E_z\right] - \frac{i n_{b0} e k_z}{m_e \omega \bar{\omega}^2}[k_z v_{b0x}E_x + \omega E_z]. \tag{13}$$

The perturbed current density is given as

$$J_{b1} = -e n_{b0}\mathbf{v}_{b1} - e n_{b1} v_{b0x}\hat{x} - e n_{b1} v_{b0z}\hat{z}. \tag{14}$$

Writing the x, y, and z components of Eq. (14) and using Eqs. (10)–(13), we obtain

$$J_{b1x} = \frac{i e^2 n_{b0}}{m_e \omega}\left[\left(\frac{\bar{\omega}^2}{(\bar{\omega}^2 - \omega_{ce}^2)} + \frac{k_z^2 v_{b0x}^2}{\bar{\omega}^2}\right)E_x + \frac{\bar{\omega}\omega_{ce}}{i(\bar{\omega}^2 - \omega_{ce}^2)}E_y + \left\{\frac{k_x v_{b0z}\bar{\omega}}{(\bar{\omega}^2 - \omega_{ce}^2)} + \frac{k_z v_{b0x}\omega}{\bar{\omega}^2}\right\}E_z\right] \tag{15}$$

$$J_{b1y} = \frac{-e^2 n_{b0}}{m_e \omega(\bar{\omega}^2 - \omega_{ce}^2)}\left[\bar{\omega}\omega_{ce}E_x - i\bar{\omega}^2 E_y + k_x v_{b0z}\omega_{ce}E_z\right], \tag{16}$$

$$J_{b1z} = \frac{i e^2 n_{b0} k_x v_{b0z}}{m_e \omega(\bar{\omega}^2 - \omega_{ce}^2)}\left[\bar{\omega}E_x - i\omega_{ce}E_y + k_x v_{b0z}E_z\right] + \frac{i e^2 n_{b0}}{m_e \bar{\omega}^2}[k_z v_{b0x}E_x + \omega E_z]. \tag{17}$$

For parallel whistler wave perturbation to the oblique beam electrons, the perpendicular wave number $k_x = 0$, and the beam current densities from Eqs. (15)–(17) become

$$J_{b1x} = \frac{i e^2 n_{b0}}{m_e \omega}\left(\frac{\bar{\omega}^2}{(\bar{\omega}^2 - \omega_{ce}^2)} + \frac{k_z^2 v_{b0x}^2}{\bar{\omega}^2}\right)E_x + \frac{e^2 n_{b0}\omega_{ce}}{m_e \omega}\frac{\bar{\omega}}{(\bar{\omega}^2 - \omega_{ce}^2)}E_y + \frac{i e^2 n_{b0}}{m_e}\frac{k_z v_{b0x}}{\bar{\omega}^2}E_z, \tag{18}$$

$$J_{b1y} = \frac{-e^2 n_{b0}\omega_{ce}\bar{\omega}}{m_e \omega(\bar{\omega}^2 - \omega_{ce}^2)}E_x + \frac{i e^2 n_{b0}\bar{\omega}^2}{m_e \omega(\bar{\omega}^2 - \omega_{ce}^2)}E_y, \tag{19}$$

$$J_{b1z} = \frac{i e^2 n_{b0}}{m_e}\frac{\omega}{\bar{\omega}^2}E_z + \frac{i e^2 n_{b0}}{m_e}\frac{k_z v_{b0x}}{\bar{\omega}^2}E_x, \tag{20}$$

For parallel beam propagation, $v_{b0x} = 0$ and the response of parallel beam electrons to the oblique whistler wave propagation yields current densities as

$$J'_{b1x} = \frac{i e^2 n_{b0}}{m_e \omega}\frac{\bar{\omega}^2}{(\bar{\omega}^2 - \omega_{ce}^2)}E_x + \frac{e^2 n_{b0}\omega_{ce}}{m_e \omega}\frac{\bar{\omega}}{(\bar{\omega}^2 - \omega_{ce}^2)}E_y + \frac{i e^2 n_{b0}}{m_e}\frac{k_x v_{b0z}\bar{\omega}}{\omega(\bar{\omega}^2 - \omega_{ce}^2)}E_z, \tag{21}$$

$$J'_{b1y} = \frac{-e^2 n_{b0}\omega_{ce}\bar{\omega}}{m_e \omega(\bar{\omega}^2 - \omega_{ce}^2)}E_x + \frac{i e^2 n_{b0}}{m_e \omega}\frac{\bar{\omega}^2}{(\bar{\omega}^2 - \omega_{ce}^2)}E_y - \frac{e^2 n_{b0}}{m_e \omega}\frac{k_x v_{b0z}\omega_{ce}}{(\bar{\omega}^2 - \omega_{ce}^2)}E_z, \tag{22}$$

$$J'_{b1z} = \frac{-e^2 n_{b0}}{m_e}\frac{\bar{\omega}}{i\omega}\frac{k_x v_{b0z}}{(\bar{\omega}^2 - \omega_{ce}^2)}E_x + \frac{e^2 n_{b0}}{m_e}\frac{\omega_{ce}}{\omega}\frac{k_x v_{b0z}}{(\bar{\omega}^2 - \omega_{ce}^2)}E_y + \frac{i e^2 n_{b0}}{m_e}\frac{k_x^2 v_{b0z}^2}{\omega(\bar{\omega}^2 - \omega_{ce}^2)}E_z + \frac{i e^2 n_{b0}}{m_e}\frac{\omega}{\bar{\omega}^2}E_z. \tag{23}$$

In case of parallel beam electrons perturbed by parallel whistlers, we substitute $k_x = 0$ and $v_{b0x} = 0$, therefore the beam current densities, Eqs. (15)–(17) can be written as

$$J''_{b1x} = \frac{i e^2 n_{b0}\bar{\omega}^2}{m_e \omega(\bar{\omega}^2 - \omega_{ce}^2)}E_x + \frac{e^2 n_{b0}\omega_{ce}}{m_e \omega}\frac{\bar{\omega}}{(\bar{\omega}^2 - \omega_{ce}^2)}E_y, \tag{24}$$

$$J''_{b1y} = \frac{-e^2 n_{b0}\omega_{ce}\bar{\omega}}{m_e \omega(\bar{\omega}^2 - \omega_{ce}^2)}E_x + \frac{i e^2 n_{b0}\bar{\omega}^2}{m_e \omega(\bar{\omega}^2 - \omega_{ce}^2)}E_y, \tag{25}$$

$$J''_{b1z} = \frac{-e^2 n_{b0}\omega}{i m_e \bar{\omega}^2}E_z. \tag{26}$$

4. DISPERSION RELATION AND GROWTH RATE

The mode structure of low-frequency electromagnetic whistler waves is governed by the wave equation given as

$$\nabla^2\mathbf{E} - \nabla(\nabla\cdot\mathbf{E}) + \left(\frac{\omega^2}{c^2}\right)\mathbf{E} = -\frac{4\pi i\omega}{c^2}\mathbf{J_1}. \tag{27}$$

We write the x, y, and z components of Eq. (27) and a nontrivial solution of these equations demands that the determinant of coefficients of E_x, E_y, and E_z must vanish; therefore, we get $\hat{\varepsilon}\mathbf{E} = 0$, where $\hat{\varepsilon}$ is the dielectric tensor, given as

$$\hat{\varepsilon} = \hat{\varepsilon}_p + \hat{\varepsilon}_b = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix},$$

where $\hat{\varepsilon}_p$ is the plasma dielectric tensor, and $\hat{\varepsilon}_b$ is the beam contribution to the dielectric tensor.

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We consider the most general case of beam interaction with whistlers, where the electron beam or whistler waves are propagating at an arbitrary angle to the magnetic field.

Using Eqs. (15)–(17) in Eq. (27), we get the dielectric tensor elements as:

$$\varepsilon_{11} = -k_z^2c^2 + \omega^2 - \frac{\omega_{pe}^2\omega^2}{(\omega^2-\omega_{ce}^2)} - \frac{\omega_{pb}^2\bar{\omega}^2}{(\bar{\omega}^2-\omega_{ce}^2)} - \frac{\omega_{pb}^2k_z^2v_{b0x}^2}{\bar{\omega}^2},$$

$$\varepsilon_{12} = \frac{i\omega\omega_{ce}\omega_{pe}^2}{(\omega^2-\omega_{ce}^2)} + \frac{i\bar{\omega}\omega_{ce}\omega_{pb}^2}{(\bar{\omega}^2-\omega_{ce}^2)},$$

$$\varepsilon_{13} = k_xk_zc^2 - \frac{\omega_{pb}^2k_xv_{b0z}\bar{\omega}}{(\bar{\omega}^2-\omega_{ce}^2)} - \frac{\omega_{pb}^2\omega k_zv_{b0x}}{\bar{\omega}^2},$$

$$\varepsilon_{21} = -\frac{i\omega\omega_{ce}\omega_{pe}^2}{(\omega^2-\omega_{ce}^2)} - \frac{i\bar{\omega}\omega_{ce}\omega_{pb}^2}{(\bar{\omega}^2-\omega_{ce}^2)} = -\varepsilon_{12},$$

$$\varepsilon_{22} = -k^2c^2 + \omega^2 - \frac{\omega_{pe}^2\omega^2}{(\omega^2-\omega_{ce}^2)} - \frac{\omega_{pb}^2\bar{\omega}^2}{(\bar{\omega}^2-\omega_{ce}^2)},$$

$$\varepsilon_{23} = -\frac{ik_xv_{b0z}\omega_{pb}^2\omega_{ce}}{(\bar{\omega}^2-\omega_{ce}^2)},$$

$$\varepsilon_{31} = k_xk_zc^2 - \frac{\omega_{pb}^2k_xv_{b0z}\bar{\omega}}{(\bar{\omega}^2-\omega_{ce}^2)} - \frac{\omega_{pb}^2\omega k_zv_{b0x}}{\bar{\omega}^2} = \varepsilon_{13},$$

$$\varepsilon_{32} = \frac{ik_xv_{b0z}\omega_{pb}^2\omega_{ce}}{(\bar{\omega}^2-\omega_{ce}^2)} = -\varepsilon_{23},$$

$$\varepsilon_{33} = \omega^2 - \omega_{pe}^2 - \frac{\omega_{pb}^2\omega^2}{\bar{\omega}^2} - k_x^2c^2 - \frac{\omega_{pb}^2k_x^2v_{b0z}^2}{(\bar{\omega}^2-\omega_{ce}^2)},$$

where

$$\omega_{pb}^2 = \frac{4\pi n_{b0}e^2}{m_e}, \quad \omega_{pe}^2 = \frac{4\pi n_{e0}e^2}{m_e}.$$

The tensor elements proportional to k_x and k_x^2 arise due to the obliqueness of whistler waves and the tensor elements proportional to v_{b0x} and v_{b0x}^2 arise due to the obliqueness of electron beam, with respect to the external magnetic field. We examine here right-hand polarized electromagnetic whistler waves in the frequency range $\omega_{ci} = \omega = \omega_{ce}$. The ions contributions have been neglected due to the range of frequencies considered.

The dispersion relation can be obtained by putting determinant

$$|\hat{\varepsilon}| = 0, \tag{28}$$

or

$$(\varepsilon_{11}\varepsilon_{22} + \varepsilon_{12}^2)\varepsilon_{33} + 2\varepsilon_{12}\varepsilon_{13}\varepsilon_{23} - \varepsilon_{13}^2\varepsilon_{22} = 0. \tag{29}$$

Considering the following cases:

(a) Parallel beam and parallel whistlers.

For parallel beam, $v_{b0x} = 0$ and for parallel whistlers, $k_x = 0$, therefore $\varepsilon_{13} = 0$, $\varepsilon_{23} = 0$, $\varepsilon_{31} = 0$, $\varepsilon_{32} = 0$, and $\varepsilon_{11} = \varepsilon_{22}$.

Substituting these values into Eq. (29), we get

$$\varepsilon_{11}^2 + \varepsilon_{12}^2 = 0. \tag{30}$$

It gives two distinct modes of wave propagation, one is left circularly polarized wave and another is right circularly polarized whistler wave. Considering only the latter wave mode, we get from Eq. (30)

$$(\omega - \omega_1)(\omega - \omega_1') = \frac{\omega_{pb}^2\bar{\omega}}{(\bar{\omega} - \omega_{ce})}, \tag{31}$$

where

$$\omega_1 = \frac{-\omega_{pe}^2}{2\omega_{ce}} + \frac{1}{2}\left[\frac{\omega_{pe}^4}{\omega_{ce}^2} + 4k_z^2c^2\right]^{1/2}, \tag{32}$$

which leads to the dispersion relation of parallel whistler waves for a dense plasma, and

$$\omega_1' = \frac{-\omega_{pe}^2}{2\omega_{ce}} - \frac{1}{2}\left[\frac{\omega_{pe}^4}{\omega_{ce}^2} + 4k_z^2c^2\right]^{1/2}.$$

It may be noted from Eq. (31) that Cerenkov interaction between parallel beam and parallel whistlers is not possible as there is only a cyclotron interaction term on right-hand side (RHS) of Eq. (31).

Assuming perturbed quantities $\omega = \omega_1 + \delta$ and $\bar{\omega} = \omega_{ce} + \delta$ in Eq. (31), we get

$$\delta = -\left[\frac{\omega_{pb}^2\omega_{ce}}{(\omega_1 - \omega_1')}\right]^{1/2}.$$

Therefore, Growth rate,

$$\gamma = \mathrm{Im}(\delta) = 0. \tag{33}$$

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The real part of the whistler wave frequency is $\omega_{1r} = \omega_1 + \mathrm{Real}(\delta)$,

$$\omega_{1r} = \frac{-\omega_{pe}^2}{2\omega_{ce}} + \frac{1}{2}\left[\frac{\omega_{pe}^4}{\omega_{ce}^2} + 4k_z^2c^2\right]^{1/2} - \left[\frac{\omega_{pb}^2(\omega - k_z v_{b0z})}{(\omega_1 - \omega_1')}\right]^{1/2}. \quad (34)$$

In resonance, the background electrons and beam electrons satisfy $\omega_1 = \omega_{ce} + k_z v_{b0z}$ or $v_{b0z} = (\omega_1 - \omega_{ce})/k_z$.

Since $\omega_1 = \omega_{ce}$; therefore the resonant electrons move in the direction opposite to the beam. From Eq. (34), we can say that as the beam velocity increases, the real frequency of the resonant whistler electrons shifts to lower frequencies, while remaining in the whistler range $\omega_{ci} = \omega = \omega_{ce}$. Equation (33) implies that the parallel whistlers do not show any growth in the presence of parallel electron beam.

(b) Oblique beam and parallel whistlers

For oblique beam, the beam velocity has a parallel component v_{b0z} as well as a perpendicular component v_{b0x}. Also for parallel whistlers $k_x = 0$; therefore $\varepsilon_{23} = -\varepsilon_{32} = 0$ and Eq. (29) becomes

$$\varepsilon_{11}\varepsilon_{22} + \varepsilon_{12}^2 = 0, \quad (35)$$

where we have retained only those terms which go as $\bar{\omega}^2$ or $(\bar{\omega}^2 - \omega_{ce}^2)$.

On simplifying Eq. (35) in the absence of beam, we get the dispersion relation of whistler waves as

$$\omega_2^2 = \frac{k_z^4 c^4 \omega_{ce}^2}{\omega_{pe}^2\left(\omega_{pe}^2 + 2k_z^2c^2\right)}.$$

In the presence of beam, Eq. (35) gives

$$k_z^4c^4 - 2k_z^2c^2\frac{\omega_{pe}^2\omega^2}{\omega_{ce}^2} - \frac{\omega_{pe}^4\omega^2}{\omega_{ce}^2} = \frac{-2\omega_{pb}^2\bar{\omega}^2}{(\bar{\omega}^2 - \omega_{ce}^2)}\left(k_z^2c^2 + \omega_{pe}^2\frac{\omega}{\bar{\omega}}\right) + \left(\frac{\omega_{pe}^2\omega^2}{\omega_{ce}^2} - k_z^2c^2\right)k_z^2v_{b0x}^2\frac{\omega_{pb}^2}{\bar{\omega}^2},$$

or

$$\omega^2 - \omega_2^2 = \frac{2\omega_{pb}^2\bar{\omega}^2\omega_{ce}^2}{(\bar{\omega}^2 - \omega_{ce}^2)}\frac{\left(k_z^2c^2 + \omega_{pe}^2\frac{\omega}{\bar{\omega}}\right)}{\omega_{pe}^2\left(\omega_{pe}^2 + 2k_z^2c^2\right)} + \left(\frac{\omega_{pe}^2\omega^2}{\omega_{ce}^2} - k_z^2c^2\right)\frac{k_z^2v_{b0x}^2\omega_{ce}^2\omega_{pb}^2}{\bar{\omega}^2}. \quad (36)$$

In Cerenkov interaction $\left(\omega - k_z v_{b0z}\right)^2 \approx 0$; therefore neglecting the first term on RHS in Eq. (36) and assuming perturbed quantities $\omega = \omega_2 + \delta$ and $\omega = k_z v_{b0z} + \delta$, where δ is the small frequency mismatch, the growth rate is obtained as

$$\gamma = \mathrm{Im}(\delta) = \frac{\sqrt{3}}{2}\left[\frac{(\omega_{pe}^2 + k_z^2c^2)}{2(\omega_{pe}^2 + 2k_z^2c^2)}\frac{v_{b0x}^2}{c^2}\omega_2\omega_{pb}^2\right]^{1/2}. \quad (37)$$

In cyclotron interaction $\left(\omega - k_z v_{b0z}\right)^2 \approx \omega_{ce}^2$; therefore neglecting the second term on RHS in Eq. (36), we get slow cyclotron interaction from $\omega - k_z v_{b0z} \approx -\omega_{ce}$ and fast cyclotron interaction from $\omega - k_z v_{b0z} \approx \omega_{ce}$.

Considering slow cyclotron interaction and assuming perturbed quantities $\omega = \omega_2 + \delta$ and $\omega = k_z v_{b0z} - \omega_{ce} + \delta$, the growth rate is obtained as

$$\gamma = \left[\frac{\omega_{pb}^2\omega_2\omega_{ce}}{2k_z^2c^2}\right]^{1/2}. \quad (38)$$

Now, considering fast cyclotron interaction and assuming perturbed quantities $\omega = \omega_2 + \delta$ and $\omega = k_z v_{b0} + \omega_{ce} + \delta$, the growth rate is obtained as

$$\gamma = 0. \quad (39)$$

That is, in case of fast cyclotron interaction, there is no growing mode, as was the case in parallel beam interaction with parallel whistlers, as $\omega = \omega_{ce}$. However, an oblique beam results in growth of whistler mode via slow cyclotron interaction [Eq. (38)] and via Cerenkov interaction [Eq. (37)]. From Eq. (37), we can say that as the transverse component of beam velocity increases, the growth rate increases. Also the growth rate increases with an increase in the strength of magnetic field in Cerenkov as well as cyclotron interactions.

(c) Parallel beam and oblique whistlers

For parallel beam, $v_{b0x} = 0$ and for oblique whistlers, the wave number **k** has a parallel component k_z as well as a perpendicular wave number k_x. Therefore, from Eq. (29), we get the dispersion relation of oblique whistlers in the absence of beam as

$$\omega_3 = \frac{kk_zc^2\omega_{ce}}{\left(\omega_{pe}^4 + 2k^2c^2\omega_{pe}^2 + k_x^2k^2c^4\right)^{1/2}}. \quad (40)$$

If $k_x = 0$ that is, for parallel whistlers, then Eq. (40) gives $\omega_3 = \left[\left(k_z^2c^2\omega_{ce}\right)/\omega_{pe}^2\right]$, which is the standard parallel whistler wave dispersion relation (Krall & Trivelpiece, 1973).

In the presence of beam, there can be Cerenkov or cyclotron interaction.

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Following the same method as used in case (b), we get the growth rate in Cerenkov interaction as

$$\gamma = \frac{\sqrt{3}}{2}\left[\frac{\omega_{pb}^2\omega_3^3}{2\omega_{pe}^2}\left\{1 - \frac{\omega_{pe}^2\left[\left(k^2+k_z^2\right)c^2+\omega_{pe}^2\right]}{\left(\omega_{pe}^4+2k^2c^2\omega_{pe}^2+k_x^2k^2c^4\right)}\right\}\right]^{1/3}. \quad (41)$$

Similarly, the growth rate in slow cyclotron interaction is obtained as

$$\gamma = \left[\frac{\omega_{pb}^2\omega_3}{4\omega_{pe}^2\omega_{ce}k^2k_z^2c^4}\left\{\omega_{pe}^2\omega_{ce}^2\left(k^2+2k_z^2\right)c^2 - 2\omega_{pe}^2\omega_3\omega_{ce}\left(\omega_{pe}^2+k_x^2c^2\right)+k_x^2k^2c^4\omega_{ce}^2 + 2k_x^2c^2k_zv_{b0z}\left(\omega_3\omega_{pe}^2-k^2c^2\omega_{ce}\right)\right\}\right]^{1/2}. \quad (42)$$

It may be noted from Eq. (41) that if $k_x = 0$, $\gamma = 0$, that is, for parallel whistler wave interaction with parallel beam, the growth rate turns out to be zero, which is the same result as obtained in case (a). The obliqueness of whistler waves modifies its dispersion characteristics as well as the growth rate in beam-wave interaction.

5. DISCUSSION

In this paper, we show the possibility to excite whistler waves by an electron beam interacting with a magnetized plasma. The excitation by a parallel beam and an oblique beam has been studied analytically for parallel as well as oblique whistlers. The dispersion relation of the whistler modes has been derived for parallel beam–parallel whistlers interaction, oblique beam–parallel whistlers interaction and parallel beam–oblique whistlers interaction.

In the case of parallel beam interaction with magnetized plasma, whistlers excite through Doppler resonance and propagate opposite to the beam direction. Cerenkov interaction between parallel beam and parallel whistlers does not take place. On the contrary, for parallel beam interaction with oblique whistlers, Cerenkov as well as cyclotron interactions are observed, which excite the whistlers. It should also be mentioned here that if the beam has a finite perpendicular velocity, that is, with oblique beam, even parallel whistlers with $k_x = 0$ can be excited. However, for purely parallel propagating beams, $k_x \neq 0$ is essential for the growth of wave. The main physical process occurring during beam–wave interaction is that the beam electrons bunch along the magnetic field, which are continuously accelerated or decelerated while keeping resonance with the emitted wave. The bunches are the main cause which supports the wave emission, whereas the non-resonant beam electrons practically do not exchange energy with the wave. All the loss of the resonant beam particles' energy is transformed into emitted wave energy and the wave grows. The growth rate is sensitive to beam velocity in the case of Cerenkov interaction but is quite insensitive in cyclotron interaction. The results of beam-excited whistlers presented here may be applied to explain the mechanisms of whistler wave excitation in space plasmas, either by artificial beams injected from spacecraft in the ionosphere or the magnetosphere like the recent satellite measurements in the Earth's radiation belt (Sauer & Sydora, 2010), which describe the discovery of large amplitude whistlers aboard the satellite STEREO-B .

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