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BANKING, LIQUIDITY EFFECTS, AND MONETARY POLICY

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We study liquidity effects and monetary policy in a model with fully flexible prices and explicit roles for money and financial intermediation. Banks hold some fractions of deposits and money injections as liquidity buffers. The higher the fraction kept as reserves, the less liquid the money is. Unexpected money injections raise output and lower nominal interest rates if and only if the newly injected money is more liquid than the initial money stocks. If banks hold no liquidity buffers, liquidity effects are eliminated. In an extended model with temporary shocks, we show that failure to withdraw state-contingent money injections does not make the stabilization policy neutral, though the economy may undergo higher short-run fluctuations than otherwise. Under this circumstance, the success of stabilization policy relies on unexpected money injections being more liquid than the initial money stock.

Keywords: Liquidity Effects, Money, Banking, Monetary Policy

1. INTRODUCTION

In a world where people are subject to trading shocks, banks can play an efficiency role by channeling funds from people with idle money to those who are cash constrained. This role, however, is limited by banks' holding reserves, due to regulations and liquidity management considerations.[1] For instance, the recent Basel III proposal requires that banks hold liquidity buffers sizable enough to enable them to withstand a severe short-term shock.[2] Banks' liquidity buffers may affect the monetary transmission mechanism and the conduct and effectiveness of monetary policy. To explore the potential effects, we use a general equilibrium framework with frictions, which give rise to the roles of money and financial intermediation [see, e.g., Lagos and Wright (2005), Berentsen et al. (2007)].

In this economy, banks take deposits from people with idle cash and lend to those who need liquidity to finance unanticipated consumption. The central bank injects

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money through financial intermediaries. Agents make decisions about money holdings before they learn the shocks of preferences and money injections. The preference shock determines whether an agent is a seller or a buyer. Sellers deposit idle cash, whereas buyers can borrow money from banks. Fiat money is used as the medium of exchange due to limitations on record keeping, enforcement, and commitment. Agents, however, are not subject to the standard cash-in-advance constraint because, before trading, they can borrow cash from banks to replenish their money holdings. The amount that agents can borrow is affected by banks' holdings of liquidity buffers. Banks hold (different) fractions of deposits and newly injected money as reserves. Money kept as bank reserves does not provide liquidity to lubricate economic activity, and the higher the fraction kept as reserves, the less liquid the money is.

An unexpected money injection results in two effects on the economy: It increases the nominal amount of loans (loanable-funds effect) and it also raises inflation expectations (Fisher effect). We find that unexpected money injections raise output and reduce nominal interest rates if and only if the fraction of money injections used to finance spending is larger than the fraction of the initial money stock used to finance spending, i.e., when the injected money is more liquid than the initial money stock. This condition implies that the loanable-funds effect outweighs the Fisher effect, increasing buyers' real balances. The lower nominal interest rates also stimulate borrowing, leaving higher total liquidity to support economic activity. Note that if the money injection is anticipated, it always raises nominal interest rates and generates no liquidity effect.

If the newly injected cash is as liquid as the initial money stock (e.g., banks hold no liquidity buffers), the liquidity effect is eliminated. This is so because agents' decisions on money holdings are in line with any money growth rate. Consequently, unexpected money injections do not distort anything, and agents make portfolio decisions *as if* they knew the amount of future money injections. Thus, in contrast to the previous literature, though agents make portfolio choices before the realization of monetary shocks, the informational friction in our model does not necessarily generate a liquidity effect.[3]

After establishing the monetary transmission mechanism in the basic model, we first extend it by incorporating temporary demand shocks to study optimal monetary policy. In the second extension, we motivate banks' holding reserves by resorting to the need for liquidity management due to random deposit withdrawals. Our analysis shows that in conducting stabilization policy, the central bank needs to weigh the payoff to managing inflation expectations against the effectiveness of implementing policy, measured by the magnitude of a liquidity effect. In an economy with high level of economic activity, it does little good to raise money injections (and, therefore, inflation expectations) to increase consumption, whereas its small liquidity effect requires higher money injections to achieve the goal. When aggregate demand is high, the central bank weighs less on the risk of raising inflation expectations, and thus it injects more money in an economy with higher economic activity than otherwise; the opposite occurs when aggregate demand is

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low. It is thus observed that the central bank reacts to shocks more aggressively in an economy with a relatively high level of economic activity.

Our extended model is similar to Berentsen and Waller (2011); however, in their model, lenders do not hold liquidity buffers. In Berentsen and Waller (2011), the critical element for effective stabilization policy is the central bank's price-level targeting policy, which will undo the current money injections at a future date. In this paper, failure to withdraw money injections does not make the stabilization policy neutral, though the economy undergoes higher fluctuations than otherwise. The management of inflation expectations is different under the two policy regimes. Under the price-level targeting policy studied by Berentsen and Waller (2011), the central bank controls long-run inflation expectations; in our model where the central bank lacks the ability to withdraw the state-contingent money injections at a specified future date, it must at least commit itself to a short-run expected inflation to implement the stabilization policy. Under this circumstance, the existence of a liquidity effect and the success of stabilization policy rely on unexpected money injections being more liquid than the initial money stock.

The rest of this paper is organized as follows. Section 2 describes the basic model. Section 3 derives equilibrium conditions. In Section 4, we study the liquidity effect and discuss further what distinguishes our paper from the literature. Section 5 illustrates some extensions of the basic model. We conclude in Section 6. All proofs and omitted derivations of equations are contained in the Appendix.

2. THE BASIC MODEL

The environment is based on Lagos and Wright (2005) and Berentsen et al. (2007). There is a $[0, 1]$ continuum of infinitely lived agents. Time is discrete and continues forever. Each period is divided into two subperiods, and in each subperiod trades occur in competitive markets. There are perishable and perfectly divisible goods, one produced in the first subperiod and the other (called the *general good*) in the second subperiod. The discount factor across periods is $\beta = \frac{1}{1+\rho} \in (0, 1)$, where ρ is the rate of time preference.

In the beginning of the first subperiod, an agent receives a preference shock that determines whether he consumes or produces. With probability θ an agent can consume but cannot produce; with probability $1 - \theta$ the agent can produce but cannot consume. We refer to consumers as buyers and producers as sellers. This is a simple way to capture the uncertainty of the opportunity to trade. Consumers get utility $u(q)$ from q consumption, where $u'(q) > 0$, $u''(q) < 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. Producers incur disutility $c(q)$ from producing q units of output, where $c'(q) > 0$ and $c''(q) \geq 0$. To motivate a role for fiat money, we assume that all goods trades are anonymous, and there is no public record of individuals' trading histories.

In the second subperiod, all agents can produce and consume the general good, getting utility $U(x)$ from x consumption, where $U'(x) > 0$, $U''(x) \leq 0$,

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$U'(0) = \infty$, and $U'(\infty) = 0$. Agents can produce one unit of the general good with one unit of labor, which generates one unit of disutility. This setup allows us to introduce an idiosyncratic preference shock while keeping the distribution of money holdings analytically tractable.

A government is the sole issuer of fiat money. The evolution of the money stock is $M_t = (1 + z_t)M_{t-1}$, where M_t denotes the per capita currency stock and z_t is the money growth rate, in period t. Assume $z_t = \mu + \varepsilon_t$, where μ is the long-run money growth rate and ε_t is a serially uncorrelated random variable with the density function f on $[\underline{\varepsilon}, \overline{\varepsilon}]$. The random variable, ε_t, generates a monetary shock, which becomes known at the beginning of period t. In the first subperiod, the central bank injects money, $\tau_t = z_t M_{t-1}$, or it can levy nominal taxes from banks' reserves to extract cash from the economy, which implies $\tau_t < 0$ and $z_t < 0$.

Competitive banks accept nominal deposits and make nominal loans. Sellers in the first subperiod can deposit their money holdings in banks at the nominal interest rate, i_d, and are entitled to withdraw funds in the second subperiod. Buyers can borrow money from banks at the nominal loan rate, i, and repay their loans in the second subperiod. We assume that loans and deposits are not rolled over, and so all financial contracts are one-period contracts.[4] Moreover, banks have zero net worth, and there are no operating costs.

Banks keep records on financial histories but not on trading histories in the goods market. The record-keeping technology is not available to individuals, so credit between private agents is not feasible. We assume full enforcement of debt repayment, and so default is not possible.[5] In equilibrium, the loan rate i clears the loan market. Assume that banks are owned by private agents. The central bank injects money through financial intermediaries, which extend funds to borrowers. This transfer scheme is merely an analytical device to mimic open-market operations. Because banks lend out money injected by the central bank, it is possible for competitive banks to obtain positive profits. A bank's profits are distributed to private agents as dividends, or are withdrawn from agents' bank accounts in the case of $z_t < 0$.

We consider an economy in which banks keep a buffer stock of reserves, due to liquidity risk management considerations and regulations. (See Section 4.2 for a more detailed discussion.[6]) Specifically, we assume that banks lend out a constant fraction, $\nu \in (0, 1]$, of deposits, and a fraction, $\chi_m \in (0, 1]$, of money that the central bank injects into banks. In the basic model, we treat ν and χ_m as parameters, and in Section 5, we consider random deposit withdrawals, in the spirit of Diamond and Dybvig (1983), to justify banks' holding reserves.

The timing of events is summarized as follows. At the beginning of the first subperiod of period t, each agent receives a preference shock, and money injections take place so that z_t is known to the public. Then, sellers make deposits, buyers take loans, and both trade in the goods market. In the second subperiod, agents settle financial claims, receive dividends from banks, and adjust money holdings. In Section 5, we extend the basic model to incorporate demand shocks to discuss stabilization policy, and introduce liquidity shocks to motivate banks' holding reserves, where we will describe the time sequence in more details.

3. EQUILIBRIUM

Let ϕ_t denote the value of money in terms of the goods produced in the second subperiod. We study symmetric stationary equilibria in which end-of-period real balances are time-invariant, i.e., $\phi_t M_t = \phi_{t-1} M_{t-1}$. Thus, $\frac{\phi_{t-1}}{\phi_t} = \frac{M_t}{M_{t-1}} = 1 + z_t$. As such, the money growth rate, z_t, also represents the inflation rate in the second subperiod of period t. In the following discussions, to simplify notations, we let variables corresponding to the next period be indexed by $+1$, and variables corresponding to the previous period be indexed by -1.

Let $V(m)$ denote the expected value of entering the first subperiod with m units of money. Let $W(m, b, d)$ denote the expected value of entering the second subperiod with m units of money, b debt, and d deposits, where loans and deposits are in the units of fiat money. We study a representative period t and work backward from the second to the first subperiod, using a similar approach to that in Berentsen et al. (2007), to characterize equilibria.

The second subperiod. In the second subperiod, an agent consumes x, produces h goods, redeems deposits, repays loans, receives dividends, F, and adjusts his money holdings. He solves the following problem:

$$W(m, b, d) = \max_{x,h,m_{+1}} U(x) - h + \beta V(m_{+1}) \tag{1}$$

$$\text{s.t. } x + \phi m_{+1} = h + \phi(m + F) + \phi(1 + i_d)d - \phi(1 + i)b.$$

If an agent has deposited d in the first subperiod, he receives $(1 + i_d)d$ units of money, and if he has borrowed b, he should repay $(1 + i)b$ units of money. Substituting h from the budget constraint into the objective function, we obtain

$$W(m, b, d) = \phi(m + F) + \phi(1 + i_d)d - \phi(1 + i)b$$
$$+ \max_{x,m_{+1}}[U(x) - x - \phi m_{+1} + \beta V(m_{+1})].$$

The first-order conditions are as follows:

$$U'(x) = 1, \tag{2}$$

$$\beta V_m(m_{+1}) \le \phi, \text{ “ = ” if } m_{+1} > 0, \tag{3}$$

where $V_m(m_{+1})$ is the marginal value of an additional unit of money taken into the first subperiod of $t + 1$. Equation (2) implies $x = x^*$ for all agents and for all t. The intertemporal equation (3) determines m_{+1}, independent of the initial holdings of m when entering the second subperiod. Therefore, the distribution of money holdings is degenerate at the beginning of a period. The envelope conditions are as follows:

$$W_m = \phi, \tag{4}$$

$$W_b = -\phi(1+i), \tag{5}$$

$$W_d = \phi(1+i_d). \tag{6}$$

The first subperiod. Let q_b and q_s denote the quantities consumed by a buyer and produced by a seller, respectively, and p denote the nominal price of the good, in period t. Because agents trade in the goods market after the monetary shock is realized, the price and quantities consumed and produced should depend on the money injection; for example, $p(z)$ denotes the price when the money injection is z. For notational simplicity, we suppress the dependence of p on z. As well, we suppress the dependence of z for other variables such as interest rates and quantity consumed and produced.

An agent may be a buyer with probability θ, spending pq_b units of money to get q_b consumption, or he may be a seller with probability $1-\theta$, receiving pq_s units of money from q_s production. Because buyers do not make deposits and sellers do not take out loans, in what follows we let b denote loans taken out by buyers and d denote deposits by sellers, and drop these arguments in $W(m, b, d)$ where relevant for notational simplicity. The expected utility of an agent from entering the first subperiod of period t with money holdings m is

$$V(m) = \int \{\theta[u(q_b) + W(m + b - pq_b, b)] \\ +(1-\theta)[-c(q_s) + W(m - d + pq_s, d)]\} f(z)dz. \tag{7}$$

Agents trade in a centralized market, so they take the price p as given. A seller solves

$$\max_{q_s, d} -c(q_s) + W(m - d + pq_s, d)$$

$$\text{s.t. } d \leq m.$$

Let λ_d denote the multiplier on the deposit constraint. The first-order conditions are as follows:

$$-c'(q_s) + pW_m = 0,$$

$$-W_m + W_d - \lambda_d = 0.$$

Using (4) and (6), the first-order conditions become

$$p = \frac{c'(q_s)}{\phi}, \tag{8}$$

$$\lambda_d = \phi i_d.$$

Equation (8) implies that a seller's production is such that the marginal cost of production, $\frac{c'(q_s)}{\phi}$, equals the marginal revenue, p. For $i_d > 0$, the deposit constraint binds, and sellers deposit all money balances, i.e., $d = M_{-1}$. Moreover, the production q_s is independent of the seller's initial portfolio brought to the first subperiod.

A buyer's problem is

$$\max_{q_b, b} u(q_b) + W(m + b - pq_b, b)$$

$$\text{s.t. } pq_b \leq m + b.$$

The buyer faces the cash constraint that his spending cannot exceed his money holdings, m, plus borrowing, b. He should have faced a constraint stating that his borrowing cannot exceed a certain credit limit. However, because banks can force borrowers to repay loans at no cost, the borrowing constraint does not bind, i.e., $b \leq \infty$, and hence, we ignore this constraint. Let λ be the multiplier on the buyer's cash constraint. Using (4), (5), and (8), the first-order conditions are as follows:

$$u'(q_b) = c'(q_s)(1 + \frac{\lambda}{\phi}), \quad \textbf{(9)}$$

$$\phi i = \lambda. \quad \textbf{(10)}$$

If $\lambda = 0$, (9) reduces to $u'(q_b) = c'(q_s)$, implying $i = 0$. If $\lambda > 0$, the cash constraint binds, and the buyer spends all of his money, i.e.,

$$q_b = \frac{m + b}{p}. \quad \textbf{(11)}$$

Combining (9) and (10), we obtain

$$\frac{u'(q_b)}{c'(q_s)} = 1 + i, \quad \textbf{(12)}$$

which implies that buyers borrow up to the point at which the marginal benefit of an additional unit of borrowed money, $\frac{u'(q_b)}{c'(q_s)}$, equals the marginal cost, $1 + i$.

To find an agent's optimal money holdings, we take the derivative of the value function in (7) with respect to m, and use (4) and (6) to get the marginal value of money:

$$V_m(m) = \int [\theta \frac{u'(q_b)}{p} + (1 - \theta)\phi(1 + i_d)] f(z) dz. \quad \textbf{(13)}$$

An agent receives $\frac{u'(q_b)}{p}$ from spending the marginal unit of money as a buyer, and if he is a seller, he deposits the idle cash in banks, which is valued $\phi(1 + i_d)$ in the second subperiod. Using the (3) lagged one period to eliminate $V_m(m)$ from

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(13), an agent's optimal money holdings at period $t-1$ satisfy

$$\beta \int [\theta \frac{u'(q_b)}{p} + (1-\theta)\phi(1+i_d)] f(z)dz \leq \phi_{-1}, \text{“ = ” if } m > 0. \tag{14}$$

Condition (14) states that the cost of acquiring an additional unit of money must be greater than the expected discounted benefit, with the equality holding if agents choose to hold money.

In a symmetric equilibrium, the market-clearing conditions for goods, money, and loan markets are, respectively,

$$(1-\theta)q_s = \theta q_b, \tag{15}$$

$$m = M_{-1}, \tag{16}$$

$$\theta b = \nu(1-\theta)d + \chi_m \tau. \tag{17}$$

In the loan-market-clearing condition, (17), the loanable funds per capita include ν fraction of deposits, $(1-\theta)d$, and χ_m fraction of money injection, $\chi_m \tau_t$, while the loan demand is θb.

Competitive banks accept nominal deposits and make nominal loans, and they take as given the loan rate and the deposit rate. In this model, banks are also the channel through which the central bank injects money into the economy. To ensure that a bank cannot become infinitely profitable by attracting an infinitely large amount of funds, we have the following zero marginal profit condition:

$$\nu i = i_d, \tag{18}$$

which is a result of the competition between numerous banks. (See Appendix A for the details on deriving solutions to the bank's problem.) Banks receive money injections, τ, a fraction χ_m of which is lent to buyers at the nominal loan rate i. In the second subperiod, banks receive repayments, $(1+i)\chi_m \tau$, which, together with the unloaned money injections, $(1-\chi_m)\tau$, are distributed as dividends, F, to agents who own the bank.[7] That is, the bank dividend payments are

$$F = (1 + i\chi_m)\tau.$$

DEFINITION 1. *A monetary equilibrium with credit is* (p, ϕ, i, q_b), *satisfying (8), (12), (14), and* $\phi = \frac{\theta q_b c'(\frac{\theta}{1-\theta} q_b)}{[\theta + (1-\theta)\nu + \chi_m z] M_{-1}}$.[8]

3.1. The First-Best Allocation

In a stationary equilibrium, the expected lifetime utility of the representative agent at the beginning of period t is

$$(1-\beta)W_s = \theta u(q_b) - (1-\theta)c(\frac{\theta}{1-\theta} q_b) + U(x) - x.$$

Imagine that the social planner is destined to maximize the representative agent's expected utility. The first-best allocation, (x^*, q_b^*, q_s^*), thus satisfies

$$U'(x^*) = 1,$$

$$u'(q_b^*) = c'(\frac{\theta}{1-\theta}q_b^*).$$

These are the quantities chosen by a social planner who could force agents to produce and consume.

4. THE LIQUIDITY EFFECT

In this section, we derive conditions for the existence of a liquidity effect—unexpected money injections raise output and lower nominal interest rates—and then discuss differences between the current model and the literature.

4.1. Existence of a Liquidity Effect

We first derive the total funds available per buyer to finance consumption in the first subperiod. The total funds include an agent's money holdings, m, and the money borrowed from a bank, b. From the loan-market-clearing condition, (17),

$$b = \frac{\nu(1-\theta)d + \chi_m \tau}{\theta}. \quad \textbf{(19)}$$

Substituting $d = m$, $\tau = zM_{-1}$, and the market-clearing condition for money, $m = M_{-1}$, into (19), we obtain the total funds available per buyer as

$$m + b = \frac{(\chi + \chi_m z)}{\theta}M_{-1}, \quad \textbf{(20)}$$

where

$$\chi = \theta + (1-\theta)\nu.$$

Note that χ and χ_m are the fractions of the initial money stock, M_{-1}, and the newly injected money, zM_{-1}, respectively, that can be used to finance spending in the first subperiod.

In the equilibrium where the cash constraint binds, $q_b = \frac{m+b}{p}$, from which we derive the relationship between q_b and the money injection, z:

$$q_b c'(\frac{\theta q_b}{1-\theta}) = \frac{(\chi + \chi_m z)\phi_{-1}M_{-1}}{\theta(1+z)}, \quad \textbf{(21)}$$

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by using (8), (20), and $1+z=\frac{\phi_{-1}}{\phi}$. Taking the derivative of (21) with respect to z, we obtain

$$\frac{\partial q_b}{\partial z}=\frac{\phi_{-1}M_{-1}(\chi_m-\chi)}{\theta(1+z)^2[c'(\frac{\theta q_b}{1-\theta})+\frac{\theta q_b}{1-\theta}c''(\frac{\theta q_b}{1-\theta})]}\begin{cases}>\\=\\<\end{cases}0 \text{ iff } \chi_m-\chi \begin{matrix}>\\=\\<\end{matrix} 0. \quad \textbf{(22)}$$

Observe from (22) that the existence of a liquidity effect ($\frac{\partial q_b}{\partial z}>0$) depends on the relative magnitudes of χ_m and χ. The intuitive reason is as follows. Money kept as banks' reserves does not provide liquidity to lubricate economic activity. For instance, when less of the bank's deposits are lent out, χ is smaller, which implies that the fraction of the initial money stock used to finance spending is lower. One can interpret χ and χ_m as the liquidity parameters for the initial money stock and newly injected money, respectively. Intuitively, (22) says that if the central bank injects money that is more liquid than the initial money stock ($\chi_m>\chi$), total liquidity rises to support economic activity, and, therefore, output rises.

The necessary condition for $\chi_m>\chi$ is that the fraction of deposits banks lend out, ν, is less than χ_m. Central banks usually impose reserve requirements against specified deposit liabilities, and so $\nu<1$, while there are no such requirements on reserves that banks acquire from open-market operations.[9] In our model, the injection of money is as a "helicopter drop" onto all banks, which is a simple way to mimic open-market operations. The interests lost from selling interest-bearing securities to the central bank must be offset by the interests earned from lending out reserves. Therefore, a bank may well lend out money injected, and $\chi_m=1$. In sum, banks do not lend out all deposits due to liquidity risk management and regulations, whereas there are often no such considerations for the money injected by the central bank, which implies $\chi_m>\chi$.[10]

To see more clearly the mechanism underlying the existence of a liquidity effect, note that an unexpected money injection results in two opposite effects: It increases the nominal amount of loans (loanable-funds effect) and it also raises inflation expectations and lowers the future value of money (Fisher effect). A liquidity effect exists if the loanable-funds effect outweighs the Fisher effect, and thus leads to higher real balances to finance spending. To illustrate this point, consider a case with a linear cost function, $c'(q_s)=1$. This implies $p=\frac{1}{\phi}$ from (8), and thus the inflation rate in the first subperiod, $\frac{p}{p_{-1}}$, rises one-for-one with the inflation rate in the second subperiod, $\frac{\phi_{-1}}{\phi}$. We rewrite (21) as

$$\theta q_b=\frac{\chi(1+\frac{\chi_m}{\chi}z)\phi_{-1}M_{-1}}{1+z}. \quad \textbf{(23)}$$

The right side of (23) is a buyer's real balances, adjusted by the second subperiod's inflation rate, $1+z$. From (23), if $\chi_m>\chi$, an increase in z causes a stronger loanable-funds effect, which dominates the Fisher effect, raising a buyer's real balances to support higher consumption.[11]

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When output rises in response to unexpected money injections, interest rates fall. From (12) and (15), $u'(q_b) = c'(\frac{\theta q_b}{1-\theta})(1+i)$. Thus, given $u'' < 0$ and $c'' \geq 0$, i falls as q_b rises. Note that χ and χ_m affect not only the existence, but also the magnitude of the liquidity effect. Equation (22) shows that the magnitude of the liquidity effect, measured by $\frac{\partial q_b}{\partial z}$, increases in χ_m and decreases in χ. The larger the difference between χ_m and χ, the stronger the liquidity effect.[12]

From (22), when $\chi = \chi_m$, the liquidity effect is eliminated. That is, an increase in the money growth rate does not affect output and interest rates, a prediction that is different from the previous literature with trade frictions. Though agents make portfolio choices before the realization of money injections, their decisions on money holdings are in line with any z, as long as they can borrow, and the newly injected cash is as liquid as the initial money stock. In this case, unexpected money injections do not distort anything. That is, agents make portfolio decisions *as if* they knew the amount of future money injections. Thus, in contrast to the previous literature, the information friction in our model does not necessarily generate a liquidity effect.

We emphasize that money injections must be unanticipated to generate the liquidity effect. By contrast, if the money growth rate is known when agents choose money holdings, an increase in the money growth rate reduces output. To see this, let γ denote the money growth rate in period t, which is known when agents choose money holdings in $t-1$. Removing the expectation operator from (14) and using the stationary condition, $\frac{\phi_{-1}}{\phi} = 1+\gamma$, we obtain the condition on optimal money holdings in $t-1$ as

$$\frac{1+\gamma}{\beta} = \theta \frac{u'(q_b)}{c'(\frac{\theta q_b}{1-\theta})} + (1-\theta)[(\frac{u'(q_b)}{c'(\frac{\theta q_b}{1-\theta})} - 1)\nu + 1]. \tag{24}$$

From (24), the period-t output, q_b, is pinned down by the anticipated money growth rate, γ, and an increase in γ reduces output, as is predicted in previous studies that also assume uncertainty in trading opportunities [e.g., Lagos and Wright (2005)].[13] On the contrary, the decision on money holdings described by (14) in period $t-1$ implies that, given the distribution of z, an agent chooses his money holdings to the point where the expected discounted benefit of holding an additional unit of money must equal its cost. Once there is an unanticipated increase in money injections in period t, the buyer's real balances increase if $\chi_m > \chi$, and consequently, output rises and nominal interest rates fall.

We summarize our main results as follows.

PROPOSITION 1. *If banks hold liquidity buffers, unexpected money injections raise output and lower nominal interest rates if and only if the fraction of injected money used to finance spending is larger than the fraction of the initial money stock used to finance spending* ($\chi_m > \chi$)*. If banks hold no liquidity buffers* ($\chi = \chi_m = 1$)*, the liquidity effect is eliminated.*

4.2. Discussion

We discuss distinctions between this paper and the literature. The mechanism underlying the liquidity effect in our paper is similar to that in Williamson's (2004) model with segmented markets. Unanticipated money injections in Williamson's model cause a redistribution of wealth between shoppers in the cash-goods market and the credit-goods market, because households cannot reallocate the newly issued fiat money between the two markets. If the issuance of private money is permitted, then after they learn the shock of money injections, households can issue private money and reallocate it to cash shoppers, which eliminates the liquidity effect. In our model, banks' lending of injected cash is similar to the private money issuance in Williamson's model, which effectively removes the cash-in-advance constraint. While Williamson (2004) assumes that fiat money and privately issued money are perfect substitutes, which is similar to the case with $\chi = \chi_m$ in our model, we go a step further to identify how different liquidity properties between the newly issued money and the initial money stock affect the existence and magnitude of the liquidity effect.[14]

In Berentsen and Waller (2011), stabilization policy works through a liquidity effect. In their model, lenders do not hold liquidity buffers, and the central bank injects money in response to temporary aggregate shocks. We use a two-subperiod setup to illustrate the mechanism of their model. Consider the example with a linear cost function, $c'(q_s) = 1$, and so $p = \frac{1}{\phi}$. Recall that $z = \mu + \varepsilon$, where μ is the long-run money growth rate and ε is the state-contingent money injection in period t. The price-level targeting policy implies that the central bank injects zM_{-1} in the first subperiod, while it withdraws εM_{-1} in the second subperiod so that $\frac{M}{M_{-1}} = \frac{\phi_{-1}}{\phi} = 1 + \mu$. Thus, $p = \frac{1+\mu}{\phi_{-1}} = (1 + \mu)p_{-1}$, i.e., the current price depends on the long-run money growth rate and last period's price. Using $p = \frac{1+\mu}{\phi_{-1}}$, we express the buyer's binding budget constraint, $pq_b = m + zM_{-1}$, as

$$q_b = \frac{1 + \mu + \varepsilon}{1 + \mu}\phi_{-1}M_{-1}. \tag{25}$$

The right side of (25) is the buyer's real balance in the first subperiod, which is raised by the state-contingent money injection, ε. The price-level targeting policy implies that state-contingent money injections do not cause inflation in the second subperiod when sellers will spend. Consequently, sellers are willing to produce more, while buyers' real balances increase to support higher consumption. It is observed from (25) that the promise of the central bank to undo state-contingent money injection is key to the existence of a liquidity effect and the effectiveness of stabilization policy, for, otherwise, $q_b = \phi_{-1}M_{-1}$.

A liquidity effect exists in our model as long as $\chi_m > \chi$, and failure to withdraw money injections does not eliminate the liquidity effect. The price-level targeting policy, however, would enlarge the liquidity effect more than otherwise. To see this, using $\frac{M}{M_{-1}} = \frac{\phi_{-1}}{\phi} = 1 + \mu$ to rewrite (23), we obtain

$$\theta q_b = \frac{\chi(1 + \frac{\chi_m}{\chi} z)}{1 + \mu} \phi_{-1} M_{-1}.$$

The second subperiod's inflation under the price-level targeting policy, $1 + \mu$, is lower than $1 + z$ in (23), and thus the buyer's real balances are raised more to support higher q_b.[15]

Though this paper highlights the bank lending channel by assuming that money is injected through the banking system, the main results may still hold in an economy without banks. To see this, consider a version of our model without banks, where money injections take the form of lump-sum transfers. Assume that there are exogenous restrictions on the use of cash: A buyer is allowed to spend a fraction χ of his initial money holdings, and a fraction χ_m of the injected cash. Obviously, the injected money is more liquid than the initial money stock if $\chi_m > \chi$. The buyer's budget constraint satisfies

$$pq_b = \chi m + \chi_m z M_{-1}, \tag{26}$$

where the right side of (26) is the total funds available to the buyer in the first subperiod. Thus, one can use the same approach as in Section 4.1 to derive the condition for the existence of a liquidity effect. In an environment without banks, however, there are no explicit nominal interest rates. The merit of considering the bank lending channel in this paper is that we do not need to impose restrictions on the use of cash, but, rather, we allow it to be determined by regulations, or banks' liquidity management facing random deposit withdrawals (see Section 5.2). One can also use our framework to study how regulations and monetary policy affect banks' operations, and the macroeconomic consequences of these effects.

Finally, compared to Berentsen et al. (2007), one new feature of our model is that banks lend out a constant fraction, ν, of deposits, and a fraction, χ_m, of injected money. So far, we have treated ν and χ_m as parameters, and justified banks' holding reserves by liquidity management considerations and regulations. We now give a more explicit interpretation of ν and χ_m by considering banks' optimal response to regulatory constraints. Suppose for now ν is solely determined by minimum reserve requirements, i.e., $(1 - \nu)$ is the required reserve ratio. Thus, required reserves per capita are $R_r = (1 - \nu)(1 - \theta)d$. After the central bank injects τ, the bank's total reserves before lending are $(1 - \theta)d + \tau$. Let L denote the loanable funds per capita:

$$L = \nu(1 - \theta)d + \chi_m \tau. \tag{27}$$

Subtracting loanable funds, (27), from the bank's total reserves, $(1 - \theta)d + \tau$, and using $d = m = M_{-1}$, $\tau = zM_{-1}$, and $(1 - \theta)(1 - \nu) = 1 - \chi$, we obtain the bank's reserve after lending as

$$R_1 = [(1-\chi) + (1-\chi_m)z]\, M_{-1}.$$

The excess reserves held by the bank are $ER = R_1 - R_r$, and

$$ER = (1-\chi_m)zM_{-1}.$$

Consider first the case where the central bank injects money, $z > 0$. Obviously, the optimal response for the bank is setting $\chi_m = 1$, because for otherwise, $ER > 0$, and it could have earned more profits by lending out excess reserves. When the central bank withdraws money, $z < 0$, the bank cannot meet the reserve requirements, unless it sets $\chi_m = 1$.[16] To make our model more flexible, we do not interpret ν as solely determined by regulations; rather, banks' holding reserves are also affected by liquidity management considerations.

5. EXTENSIONS

We have established the transmission mechanism of money injections, whereby banks hold exogenously imposed liquidity buffers. Unexpected money injections can raise output under certain conditions, but this scenario is not optimal. Indeed, the Friedman rule achieves the first-best allocation (see Appendix B). In a stochastic environment, what is the optimal monetary policy if the central bank is prohibited from implementing the Friedman rule due to, e.g., limited enforcement? We answer this question in the first extension by incorporating aggregate demand shocks into the basic model. In the second extension, we motivate banks' holding liquidity buffers by resorting to the need for liquidity management due to random deposit withdrawals. In so doing, we introduce shocks to the liquidity needs of depositors, in the spirit of Diamond and Dybvig (1983).[17]

5.1. Aggregate Demand Shocks and Stabilization Policy

We study how stabilization policy works in a stochastic environment where banks hold liquidity buffers. For the purpose of illustration, in this subsection we assume $c(q) = q$ and $u(q) = e^{\eta}(1-e^{-q})$, where η is a random variable with a probability density function $g(\eta)$ and support $[\underline{\eta}, \overline{\eta}]$. One can think of shocks to η as aggregate demand shocks. We consider the case $\chi_m = 1 > \chi$.

The central bank's objective is to maximize the expected utility of a representative agent. In so doing, it chooses the quantities consumed and produced, and the associated contingent money injection, $z(\eta)$, in each state such that the chosen quantities satisfy the compatibility constraint that agents hold money optimally, (14), and the buyer's cash constraint, $q_b(\eta) = \frac{[\chi + z(\eta)]\phi_{-1}M_{-1}}{\theta[1+z(\eta)]}$. We call it the state-contingent stabilization policy.[18] The buyer's binding cash constraint implies

$$z(\eta) = \frac{\theta q_b(\eta) - \chi\phi_{-1}M_{-1}}{\phi_{-1}M_{-1} - \theta q_b(\eta)}. \quad \mathbf{(28)}$$

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TABLE 1. The effects of stabilization policy

	Scheme 1	$(\chi = 0.95)$		Scheme 2	$(\chi = 0.9)$	
η	$\eta_L = 0.9554$	$\bar{\eta} = 0.9555$	$\eta_H = 0.9556$	$\eta_L = 0.9554$	$\bar{\eta} = 0.9555$	$\eta_H = 0.9556$
z	0.018	0.02	0.022	0.019	0.02	0.021
q_b	0.889604	0.889711	0.889817	0.885949	0.886056	0.886162
i	0.065796	0.065790	0.065783	0.069451	0.069444	0.069438

Substituting $z(\eta)$ from (28) into agents' optimality condition of holding money, $\beta \int_{\underline{\eta}}^{\overline{\eta}} \frac{1+\chi i(\eta)}{1+z(\eta)} g(\eta) d\eta = 1$, we obtain the central bank's problem:

$$\max_{x, q_b} \; W = U(x) - x + \int_{\underline{\eta}}^{\overline{\eta}} [\theta u(q_b) - (1-\theta) c(\frac{\theta}{1-\theta} q_b)] g(\eta) d\eta \qquad \textbf{(29)}$$

$$\text{s.t. } \beta \int_{\underline{\eta}}^{\overline{\eta}} \frac{(1+\chi i)(\phi_{-1} M_{-1} - \theta q_b)}{\phi_{-1} M_{-1}(1-\chi)} g(\eta) d\eta = 1. \qquad \textbf{(30)}$$

Because $1 + i = \frac{u'(q_b)}{c'(q_s)} = e^{\eta - q_b}$, we use the approximation, $\log(1+y) \simeq y$, to get $i(\eta) \simeq \eta - q_b(\eta)$. Let λ_A be the multiplier of the constraint (30). The first-order condition is $U'(x) = 1$, and

$$i(\eta) = \frac{\beta \lambda_A [\theta(1 - \chi \eta) + \chi \phi_{-1} M_{-1}]}{\theta[(1-\chi)\phi_{-1} M_{-1} - 2\beta \lambda_A \chi]}. \qquad \textbf{(31)}$$

(See Appendix C for the derivation.)

Table 1 reports the numerical results, from which we have the following observations.[19] First, in response to higher aggregate demand, the central bank chooses higher money injections to increase consumption.[20] Second, the economy in Scheme 1 features a larger χ, which results in a lower optimal money injection, $z(\eta_L)$, and higher $z(\eta_H)$, than in Scheme 2. That is, the central bank reacts to aggregate demand shocks more aggressively in an economy with a larger χ. When choosing the state-contingent money injection, the central bank weighs the payoff to managing inflation expectations, measured by the multiplier, λ_A, of the constraint on optimal money holding, (30), against the effectiveness of implementing policy, measured by the magnitude of a liquidity effect. A larger χ results in a smaller λ_A, and a smaller liquidity effect (because the difference between χ_m and χ becomes smaller, as we set $\chi_m = 1$). A small λ_A implies a low payoff to raising money injections (and, therefore, inflation expectations) to increase consumption, whereas a small liquidity effect implies that it requires a large money injection to achieve the goal. When aggregate demand is high, the second effect dominates; that is, the central bank weighs less on the risk of raising inflation expectations, and so $z(\eta_H)$ is higher in an economy with relatively high economic activity. The first effect dominates when aggregate demand is low, and we observe a lower $z(\eta_L)$ in an economy with a larger χ than otherwise.

Price-level targeting stabilization policy. Similar to Berentsen and Waller (2011), we now consider a policy whereby the central bank injects money in response to temporary shocks, and promises to withdraw the state-contingent money injection, ε_t, in the second subperiod. Under this price-level targeting stabilization policy, the money stock measured at the end of the second subperiod grows at the rate μ, i.e., $M = \mu M_{-1}$, where μ is fixed. Assume that the end-of-period real balances are state-independent, $\phi_{-1}M_{-1} = \phi M = \varsigma$. The central bank's problem becomes

$$\max_{x,q_b} \; W = U(x) - x + \int_{\underline{\eta}}^{\overline{\eta}} [\theta u(q_b) - (1-\theta)c(\frac{\theta}{1-\theta}q_b)]g(\eta)d\eta$$

$$\text{s.t. } \beta \int_{\underline{\eta}}^{\overline{\eta}} \frac{1+\chi i}{1+\mu} g(\eta)d\eta = 1. \tag{32}$$

We consider the same example where $c(q) = q$ and $u(q) = e^{\eta}(1 - e^{-q})$. Thus, one can solve for

$$i = \frac{1+\mu-\beta}{\beta\chi}. \tag{33}$$

Compared with the state-dependent interest rate shown in (31), (33) shows a perfect smoothing of interest rates. The perfect interest rate smoothing is due to the functional forms considered in this example. From the binding budget constraint, (21), we solve for

$$z(\eta) = \frac{\theta q_b(\eta)(1+\mu) - \chi\phi_{-1}M_{-1}}{\phi_{-1}M_{-1}}. \tag{34}$$

Using $i = \eta - q_b(\eta)$ and the fact that the interest rates are state-independent, q_b is higher when the economy is hit by a larger η. A higher q_b is supported by a higher money injection $z(\eta)$, as can be seen from (34). Under the price-targeting stabilization policy, the central bank always increases the state-contingent money injection in response to higher aggregate demand.

The price-level targeting policy results in smaller fluctuations in consumption than those in the state-contingent policy (see Appendix C). Moreover, the two policy regimes feature different management of expectations, which is reflected in the difference between the constraints in the planner's problems, (30) and (32). The central bank controls long-run inflation expectations via the price-level targeting policy, whereas when it lacks the ability to withdraw the state-contingent money injections, it must at least commit itself to a short-run expected inflation, as shown in the constraint, (30).

If the central bank cannot promise to withdraw state-contingent money injections, the liquidity effect does not exist when $\chi = \chi_m = 1$. Notice that if $\chi = \chi_m = 1$, the binding cash constraint becomes $q_b = \frac{\phi_{-1}M_{-1}}{\theta}$, and consumption is independent of state-contingent money injections. To summarize, when banks hold liquidity buffers and the central bank cannot commit itself to unraveling state-contingent money injections, the existence of a liquidity effect, and thus the success of stabilization policy, relies on unexpected money injections being more liquid than the initial money stock.

5.2. Random Deposit Withdrawals

So far, we have assumed that banks hold $1 - \nu$ fraction of deposits as reserves, where ν is an exogenous parameter. In this subsection, we consider random deposit withdrawals to motivate banks' holding reserves. The environment is the same as in the basic model, except that some depositors face a liquidity shock and withdraw deposits to finance consumption. Banks thus need to hold reserves to meet unanticipated withdrawals.

Liquidity shocks and deposit withdrawals. In the beginning of the first subperiod, an agent receives a preference shock. With probability θ_c an agent can consume but cannot produce (called the *buyer*), with probability θ_p the agent can produce but cannot consume (called the *seller*), and with probability θ_n he can neither consume nor produce (called the *nontrader*), where $\theta_c + \theta_p + \theta_n = 1$.

We consider the following timing sequence. Money injections take place after the realization of preference shocks. Then, sellers and nontraders make deposits, and banks make the decision of holding reserves and extending loans. Banks close after taking deposits and making loans. After banks close, nontraders receive a liquidity shock: With probability θ_{nc} a nontrader wants to consume (called the *late consumer*), and with probability $1 - \theta_{nc}$ he does not want to consume. Late consumers can withdraw deposits from automated teller machines to finance consumption.

The probability, θ_{nc}, is a random variable. For the purpose of illustration, we assume that θ_{nc} follows a discrete uniform distribution. In particular, θ_{nc} takes the value from the set $S_{nc} = \{\theta_{nc}^1, \theta_{nc}^2, \ldots, \theta_{nc}^k\}$, where $0 \leq \theta_{nc}^i < \theta_{nc}^j \leq 1$ if $i < j$, with an equal probability; that is, $\Pr(\theta_{nc} = \theta_{nc}^i) = \frac{1}{k}$, $i = 1, 2, \ldots, k$, where $k \geq 2$. We call shocks to θ_{nc} the liquidity shock, which arrives in the following way. Nature draws from the set, S_{nc}, to determine the realized value of θ_{nc}, denoted as θ_{nc}^j. Then, a nontrader receives the liquidity shock that he wishes to consume with probability θ_{nc}^j, or does not wish to consume with probability $1 - \theta_{nc}^j$. The liquidity shock is an aggregate shock, since it implies that, by the law of large number, there is a proportion, θ_{nc}^j, of nontraders who become late consumers. Given that Nature has drawn θ_{nc}^j, whether a nontrader wishes to be a consume is an idiosyncratic shock.

Because banks make the decision to hold reserves before the realization of liquidity shocks, and there are no alternatives to obtain reserves in the interim period, they will hold reserves to meet the demand for the largest amount of withdrawals from late consumers; that is, banks hold $\theta_n \theta_{nc}^k$ (per capita) fraction of deposits as reserves.[21] Banks do not pay interest on deposits withdrawn by late consumers, because deposits and withdrawals are made within the same subperiod. It is clear now why nontraders would make deposits: They can withdraw money if they wish to consume, and can earn interest payments otherwise.

To focus our attention on motivating banks' holding reserves by random withdrawals, in the current model we do not consider the case where nontraders make deposits as well as borrow from banks, due to, e.g., spatial frictions.[22] Moreover, we assume that banks do not take deposits after making loans. Therefore, it is not possible that nontraders borrow without making deposits, and deposit all money after they learn that they do not want to consume. Notice that restricting the opportunity for nontraders to borrow can distort allocations. In Appendix D, we relax these restrictions and derive the condition for nontraders to take loans.

Trade and bank operations in the first subperiod. The liquidity shock results in aggregate uncertainty, so we denote variables that depend on the state θ_{nc}^j with a superscript j, where $j = 1, 2, \ldots, k$. For example, p^j denotes the price in the first subperiod and q_l^j the consumption by a late consumer. Let d_n denote a nontrader's deposits.[23]

In the goods market, a seller's problem and a buyer's problem are similar to those discussed in Section 4. A nontrader maximizes his expected utility by choosing the deposits, d_n, and the quantity consumed if he becomes a late consumer, q_l^j:

$$\max_{q_l^j, d_n \geq 0} \frac{1}{k}\sum_{j=1}^{k}\theta_{nc}^j[u(q_l^j) + W(m - p^j q_l^j, 0)] + (1 - \theta_{nc}^j)W(m - d_n, d_n)$$

$$\text{s.t. } p^j q_l^j \leq m,$$

$$d_n \leq m.$$

With probability θ_{nc}^j a nontrader becomes a late consumer and uses money holdings left after deposits, $m - d_n$, plus deposits withdrawn, d_n, to finance consumption, $p^j q_l^j$, while with probability $1 - \theta_{nc}^j$ a nontrader does not want to consume, and he enters the second subperiod holding $m - d_n$ units of money and d_n deposits. The first-order condition implies that nontraders deposit all money holdings, and $d_n = m$. Moreover, $u'(q_l^j) = c'(q_s^j)$ when the cash constraint does not bind; otherwise, the late consumer spends all his money and

$$q_l^j = \frac{m}{p^j}. \tag{35}$$

The expected utility of an agent entering the first subperiod of period t with money holdings, m, is

$$\begin{aligned} V(m) = \frac{1}{k}\sum_{j=1}^{k}\int \theta_c[u(q_b^j) \\ &+ W(m + b - p^j q_b^j, b)] + \theta_n \left\{ \begin{array}{c} \theta_{nc}^j[u(q_l^j) + W(m - p^j q_l^j, 0)] \\ +(1 - \theta_{nc}^j)W(m - d_n, d_n) \end{array} \right\} \\ &+ \theta_p[-c(q_s^j) + W(m - d + p^j q_s^j, d)]f(z)dz. \end{aligned}$$

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The marginal value of money is

$$V_m(m) = \frac{1}{k}\sum_{j=1}^{k}\int \times \left\{\theta_c \frac{u'(q_b^j)}{p^j} + \theta_n[\theta_{nc}^j \frac{u'(q_l^j)}{p^j} + (1-\theta_{nc}^j)\phi(1+i_d)] + \theta_p\phi(1+i_d)\right\} f(z)dz. \quad \textbf{(36)}$$

The benefits of holding an additional unit of money to the first subperiod include expected gains from spending the money on goods as a buyer or as a late consumer and the deposits interest as a seller or as a nontrader who does not want to consume.[24] Using (3) lagged one period to eliminate $V_m(m)$ from (36), an agent's optimal money holdings satisfy

$$\beta\frac{1}{k}\sum_{j=1}^{k}\int \left\{\begin{array}{c}\theta_c \frac{u'(q_b^j)}{p^j} + \theta_n[\theta_{nc}^j \frac{u'(q_l^j)}{p^j} + (1-\theta_{nc}^j)\phi(1+i_d)] \\ +\theta_p\phi(1+i_d)\end{array}\right\} \times f(z)dz \leq \phi_{-1}, \text{“ = ” if } m > 0. \quad \textbf{(37)}$$

Sellers, buyers, and late consumers trade in the goods market in the first subperiod. In a symmetric equilibrium, the market-clearing conditions in state j for goods, loans, and money are

$$\theta_p q_s^j = \theta_c q_b^j + \theta_n \theta_{nc}^j q_l^j, \text{ for all } j, \quad \textbf{(38)}$$

$$\theta_c b = [\theta_p + \theta_n(1-\theta_{nc}^k)]d + \chi_m \tau, \quad \textbf{(39)}$$

and $m = M_{-1}$, respectively. In the loan-market-clearing condition (39), given that banks hold $\theta_n\theta_{nc}^k$ fraction of deposits as reserves, the per capita funds available for banks to lend out include $\theta_p + \theta_n(1-\theta_{nc}^k)$ fraction of deposits, and χ_m fraction of injected money, while per capita loans demanded is $\theta_c b$. Though we still let χ_m be a parameter, note that in this economy banks may well use up all injected money to extend loans.

The zero-marginal-profit condition, which ensures that a bank cannot become infinitely profitable by attracting an infinitely large amount of funds, is

$$\frac{\theta_p + \theta_n(1-\theta_{nc}^k)}{\theta_p + \theta_n(1-\overline{\theta}_{nc})} i = i_d,$$

where $\overline{\theta}_{nc} = \frac{1}{k}\sum_{j=1}^{k}\theta_{nc}^j$. In the basic model, the zero-marginal-profit condition is $\nu i = i_d$, where ν is a parameter. Here we have derived the fraction of deposits that banks lend out when facing random deposit withdrawals.

The liquidity effect under random deposit withdrawals. We use a similar approach to that discussed in Section 4 to derive conditions for the existence of a

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liquidity effect. (See Appendix D for the derivation.) We consider equilibria where the cash constraint binds for buyers and late consumers in all states. Let q_A^j denote the aggregate demand in state j. Then

$$q_A^j = \theta_c q_b^j + \theta_n \theta_{nc}^j q_l^j = \frac{(\chi_l + \chi_m z + \theta_n \theta_{nc}^j)\phi_{-1} M_{-1}}{c'(q_s^j)(1+z)}, \tag{40}$$

where

$$\chi_l = \theta_c + \theta_p + \theta_n(1-\theta_{nc}^k) = 1 - \theta_n \theta_{nc}^k.$$

Note that χ_l is the fraction of the initial money stock, M_{-1}, that can be used to finance spending. (We drop the superscript j below where there is no confusion.) Taking the derivative of q_A with respect to z, we obtain

$$\frac{\partial q_A}{\partial z} = \frac{\phi_{-1} M_{-1}(\chi_m - \chi_l - \theta_n \theta_{nc})}{(1+z)^2 \left[c'\left(\frac{q_A}{\theta_p}\right) + \frac{q_A}{\theta_p} c''\left(\frac{q_A}{\theta_p}\right)\right]} \begin{cases} > \\ = \\ < \end{cases} 0 \text{ iff } \chi_m \begin{matrix} > \\ = \\ < \end{matrix} \chi_l + \theta_n \theta_{nc}. \tag{41}$$

The existence of a liquidity effect ($\frac{\partial q_A}{\partial z} > 0$) depends on $\chi_m > \chi_l + \theta_n \theta_{nc}$. If $\theta_{nc} = \theta_{nc}^k$, we have $\chi_l + \theta_n \theta_{nc}^k = 1$, and even when banks lend out all money injected ($\chi_m = 1$), there is no liquidity effect. Because $\theta_{nc} = \theta_{nc}^k$ occurs with probability $\frac{1}{k}$, a liquidity effect is more likely to exist when k is larger.

When a liquidity effect exists, monetary policy has an (interim) redistribution effect between buyers and late consumers. Note that q_b is increased by z if $\chi_m > \chi_l$; that is, unanticipated money injections benefit buyers if the loanable-funds effect dominates the Fisher effect. However, q_l is decreased by z because the loanable-funds effect is absent for late consumers and only the Fisher effect applies. Consequently, money injections hurt late consumers. When a liquidity effect exists, unanticipated money injections redistribute consumption from late consumers to buyers. Moreover, for aggregate demand to increase by money injections, the increase in the buyer's consumption must outweigh the decrease in the late-consumer's consumption.

Using the stationary condition, $\phi_{-1} M_{-1} = \phi M$, and the total funds available per buyer to finance consumption in the first subperiod, $m + b = \frac{(\chi_l + \chi_m z)}{\theta_c} M_{-1}$, we have the following version of the equation of exchange:

$$p q_A = \frac{(\chi_l + \chi_m z + \theta_n \theta_{nc})}{(1+z)} M, \tag{42}$$

where the left side of (42) is the aggregate demand for goods, and on the right side, $\frac{(\chi_l + \chi_m z + \theta_n \theta_{nc})}{(1+z)}$ is the velocity of money. Compared to the velocity in the basic model, $\frac{\chi + \chi_m z}{(1+z)}$, in this environment the velocity is volatile due to the uncertainty from the consumption of late consumers. The larger the probability of being a late consumer, the higher the velocity. If there exists a liquidity effect ($\chi_m > \chi_l + \theta_n \theta_{nc}$), an unexpected money injection raises velocity and total liquidity, which leads to higher output and prices.

6. CONCLUSION

This paper features flexible prices and frictions, which give rise to the roles of money and financial intermediation. A liquidity effect exists if and only if the fraction of money injections used to finance spending is larger than that of the initial money stock. If banks hold no liquidity buffers, the liquidity effect is eliminated. We extend the basic model by incorporating aggregate demand shocks to study optimal stabilization policy, and the need for liquidity management due to random deposit withdrawals to motivate banks' holding reserves. In contrast to Berentsen and Waller (2011), failure to unravel state-contingent money injections in our model does not make the stabilization policy neutral. The price-level targeting policy, however, results in smaller fluctuations in consumption than those in the state-contingent policy. Finally, in an economy where banks hold liquidity buffers, when the central bank cannot commit itself to a price-level path, the existence of a liquidity effect and the success of stabilization policy rely on unexpected money injections being more liquid than the initial money stock.

NOTES

1. Requiring that banks hold sufficiently high liquid assets is a cost on financial intermediation. However, as Freedman and Click (2006) and Ratnovski (2009) have argued, developing countries have to rely on the quantitative liquidity regulation because of less available information on banks' net worth.

2. The short-term liquidity buffer (mostly comprising cash, central bank reserves, and domestic sovereign bonds), known as the liquidity coverage ratio, will require a bank to have enough highly liquid assets on the balance sheet to cover its net cash outflows over a 30-day period following a shock, such as a three-notch downgrade to its public credit rating. The liquidity coverage ratio was introduced in 2015, but the minimum requirement began at 60% and will reach 100% on 1 January 2019. See *Basel III: International Framework for Liquidity Risk Measurement, Standards and Monitoring,* December 2010.

3. For instance, Lucas (1990), Christiano (1991), and Fuerst (1992) attribute the source of the liquidity effect to a type of information friction that arises because agents are not able to adjust their portfolios at the time when money injections take place.

4. With the assumption on the linear utility costs of production in the second subperiod, agents do not gain by spreading the repayment of loans or redemption of deposits across periods.

5. But see, e.g., Berentsen et al. (2007) and Li and Li (2013), for considering the possibility of default to study the effects of inflation on credit arrangements, output, and asset prices. Chiu and Meh (2011) consider perfect enforcement with a finite fixed cost in the model of Berentsen et al. (2007), to study the welfare effect of inflation and banking.

6. A rationale for liquidity risk management is proposed by, for instance, Kashyap et al. (2002): Banks provide customers with liquidity on demand to satisfy their unexpected needs. Liquidity is provided by offering demand deposits and loan commitments, which give a borrower the option to take the loans on demand over a certain specified period of time. Both of these products require explicit liquidity risk management.

7. We follow Fuerst's (1994) setup in which the monetary authority injects money as a "helicopter drop" onto all banks, and, therefore, in equilibrium, banks may have positive profits [see also Christiano

(1991)]. As argued by Fuerst (1994), if open-market operations were modeled, the gains of the loanable reserves would be exactly offset by the loss of the interest-bearing securities.

8. From (8), (11), and (17), $pq_b = m + b = m + \frac{\nu(1-\theta)d+\chi_m\tau}{\theta} = \frac{\theta m+\nu(1-\theta)d+\chi_m\tau}{\theta}$. Substituting (8), $d = m$, $\tau = zM_{-1}$, and $m = M_{-1}$ into the aforementioned expression, we obtain $\phi = \frac{\theta q_b c'(\frac{\theta}{1-\theta}q_b)}{[\theta+(1-\theta)\nu+\chi_m z]M_{-1}}$.

9. The Fed imposes reserve requirements against specified deposit liabilities. For instance, net transaction accounts in excess of the low-reserve tranche are currently reservable at 10%. For details, see http://www.federalreserve.gov/monetarypolicy/reservereq.htm.

10. Though we can infer that in usual time $\chi_m > \chi$, it is not easy, if not impossible, to find the magnitudes of ν (and, therefore, χ) and χ_m from data. The recent financial crisis, however, may present a case in which $\chi_m < \chi$. Effective October 1, 2008, the Federal Reserve Banks pay interest on required reserve balances and on excess reserve balances. This would increase banks' incentives to hold reserves, and reduce χ_m. (In this model, however, the central bank does not pay interest on banks' reserves.) As mentioned by Cochrane (2014), during the last few years, the Fed has bought about \$3 trillion of assets, and created about \$3 trillion of bank reserves, in which required reserves are only about \$80 billion. Banks only held about \$50 billion of reserves before the crisis. Thus, he concluded that almost all of the \$3 trillion were excess reserves, held voluntarily by banks. One can interpret this case as χ_m being extremely small, and we may well have $\chi_m < \chi$.

11. This is so even if the first subperiod's inflation rises more than the second subperiod's inflation, as in the case with convex cost functions. The increase in the price, p, is in line with the seller's incentive to produce more.

12. From (23), we derive a version of the equation of exchange by using $\phi_{-1}M_{-1} = \phi M$:

$$p\theta q_b = \frac{\chi(1+\frac{\chi_m}{\chi}z)}{(1+z)}M.$$

The left side of the above equation is the aggregate demand for goods, and in the right side, $\frac{\chi(1+\frac{\chi_m}{\chi}z)}{(1+z)}$ is the velocity of money. If $\chi_m > \chi$, a money injection raises the velocity, leading to higher output and prices.

13. Using (12) we rewrite (24) as

$$\frac{1+\gamma}{\beta} = 1 + \chi i,$$

which is the Fisher equation, but in our model, the nominal interest rate is also influenced by the liquidity parameter, χ. An increase in γ increases the nominal interest rate, which is the inflation expectation effect as predicted by the Fisher question, and there is no liquidity effect.

14. Previous literature using limited participation models to study liquidity effects includes, for example, Grossman and Weiss (1983), Rotemberg (1984), and Williamson (2006). They identify the distributional effect of money injections as the underlying mechanism, but often the models are not analytically tractable, except Williamson (2006).

15. In our model, the central bank's promise to undo money injections implies that the value of money in the next second subperiod, ϕ_t, is known to the public when they choose money holdings in period $t-1$, though they do not know the money injection, z_t. The optimal condition of holding money (14) thus becomes

$$\beta\int[\theta\frac{u'(q_b)}{p} + (1-\theta)(1+i_d)]f(z)dz = 1+\mu.$$

16. In the case of $z < 0$, the central bank levies nominal taxes from banks' reserves to extract money. Banks reduce loans by χ_m fraction of the withdrawn money, while lending ν fraction of deposits.

17. See Bencivenga and Camera (2011) for another setup in which banks hold liquidity buffers. Unlike the current model where banks make loans, their paper assumes that banks invest in capital formation, and depositors make heterogeneous withdrawals. Banks, therefore, always hold some positive amount of reserves to satisfy the heterogeneous liquidity needs of buyers.

18. We do not consider that the central bank uses reserve requirements as a tool in response to shocks, but rather we take reserve requirements as given and focus on the optimal money injection. Also see Lagos (2011) for optimal monetary policies in stabilizing shocks to aggregate liquidity.

19. In Appendix C and numerical examples, we assume that η follows a Bernoulli distribution such that η takes a value from the set, $\{\eta_L, \bar{\eta}, \eta_H\}$, with an equal probability, $\frac{1}{3}$, where $\bar{\eta} = (\eta_H + \eta_L)/2$. Because λ_A is a complicated function of parameters, it is not feasible to show analytically how λ_A is affected by some parameters. From numerical examples, we find that, given other parameters, λ_A increases if χ is lower, or $\bar{\eta}$ is higher.

The parameter values in Table 1 are $M_{-1} = 100$, $\beta = .96$, and $\theta = .5$. Suppose that in period $t-1$, $\eta = (\eta_H + \eta_L)/2$, and the central bank chooses the long-run money growth rate, $\mu = .02$. In Scheme 1, we set $\nu = .9$ (so $\chi = .95$) and thus $\lambda_A = .0014$. In Scheme 2, we set $\nu = .8$ (so $\chi = .9$) and thus $\lambda_A = .0031$.

20. In Appendix C we prove that if χ is smaller than a threshold ($\chi < \frac{1}{\bar{\eta}}$), the central bank chooses a higher money injection in response to higher aggregate demand. If χ is larger than the threshold, the condition on whether the policy is leaning against the wind depends on λ_A, which is a complicated function of parameters. We, therefore, tried a lot of numerical examples and found that if equilibria exist, the central bank injects more money in response to higher aggregate demand. In the model, we consider an economy where $\chi < \frac{1}{\bar{\eta}}$. Therefore, the state-contingent policy is not leaning against the wind.

21. In reality, banks facing unexpected deposit withdrawals can resort to the interbank loan market, the central bank's discount window, or calling back loans. We do not consider those possibilities in this paper, and so banks need to meet the unexpected withdrawals with their own reserves. Even given those possible ways of obtaining reserves, weighing the cost of the alternative resorts against that of holding reserves, banks may still keep some excess reserves.

22. One type of such spatial frictions can work as follows. Suppose there is a location shock that is perfectly correlated to the preference shock. The shock locates sellers and nontraders to banks' deposits service units while locating buyers in units of lending money. Banks' deposits and lending service units are spatially separated, and agents are time constrained to visit more than one service unit.

23. As in the basic model, the price and quantities consumed and produced should also depend on the money injection, z, but we suppress the dependence of those variables on z. Because deposits and loans are made before the liquidity shock is realized, d_n and b do not depend on the state, θ_{nc}^j, neither do interest rates.

24. Because of the stationary condition, $\phi_{t-1}M_{t-1} = \phi_t M_t$, the value of money in the second subperiod of t, ϕ_t, is independent of θ_{nc}^j.

25. If $\chi > \frac{1}{\bar{\eta}}$, then $\mu_2 < \mu_3$, it is possible to find a $\mu \in (\mu_2, \mu_3)$ under which $\Omega_n < 0$. If in this case we can find $\Omega_d < 0$, and so the interest rate and real balance are positive, we cannot preclude the possibility that the central bank chooses a policy that is leaning against the wind. Since Ω_d depends on λ_A, which is a complicated function of parameters, we cannot obtain explicit conditions for $\Omega_d > 0$. From a lot of numerical examples, we found that if $\chi > \frac{1}{\bar{\eta}}$ and $\mu \in (\mu_2, \mu_3)$, equilibria do not exist, but when $\mu < \mu_2$, the central bank injects more money in response to higher aggregate demand.

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APPENDIX A: SOLVING THE BANK'S PROBLEM

Banks are perfectly competitive with free entry they take as given the loan rate and the deposit rate. There is no strategic interaction among banks or between banks and agents, and no bargaining over the terms of the loan contract. A profit-maximizing bank turns out to solve the following problem per borrower:

$$\max_b (i - \frac{i_d}{\nu})b$$
$$\text{s.t.} \quad u(q_b) + W(m, b, d) \geq \Gamma,$$

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where Γ is the reservation value of the borrower, which is the surplus from obtaining loans at another bank. The first-order condition to the bank's problem is

$$i - \frac{i_d}{\nu} + \lambda_\Gamma[u'(q_b)\frac{dq_b}{db} + W_b] = 0, \tag{A.1}$$

where λ_Γ is the Lagrangian multiplier on the borrower's participation constraint. For $i - \frac{i_d}{\nu} > 0$, the bank would like to make the largest loan possible to borrowers. Therefore, this implies a zero-marginal-profit condition:

$$\nu i = i_d,$$

and the bank would choose a loan amount such that $\lambda_\Gamma > 0$.

From (8) and the buyer's budget constraint, $\frac{dq_b}{db} = \frac{\phi}{c'(q_s)}$. Using $W_b = -\phi(1+i)$, from (A.1) we have

$$\frac{u'(q_b)}{c'(q_s)} = 1 + i.$$

APPENDIX B: FINANCIAL INTERMEDIATION AND THE FRIEDMAN RULE

We now show that the Friedman rule achieves the first-best allocation in our model. Our discussion on the Friedman rule is similar to that in Berentsen et al. (2005), who consider the real effect of monetary injections without the banking system. Consider first the benchmark case where banks lend out all deposits and money injections, and so the loan rate equals the deposit rate. In a monetary economy, from (8) and (12), (14) becomes

$$\beta E_{-1}[\phi(1+i_d)] = \phi_{-1}. \tag{B.1}$$

We define the expected real return on money as $\frac{1}{1+\hat{\gamma}}$; that is, $E_{-1}\frac{\phi}{\phi_{-1}} \equiv \frac{1}{1+\hat{\gamma}}$. To implement the Friedman rule in this economy, the central bank must set the expected return on money equal to the real interest rate, i.e., $\frac{1}{1+\hat{\gamma}} = \frac{1}{\beta}$, which implies $i_d = 0$ by (B.1). Hence, $u'(q_b) = c'(q_s)$ by (12), and the Friedman rule achieves the first-best allocation. The Friedman rule ensures that agents can perfectly insure themselves against monetary shocks because holding currency has zero costs.

The Friedman rule, however, may require a positive money growth rate in the environment with unexpected money injections. Consider an example in which the money growth rate, z, is a random variable such that

$$z = \begin{cases} z_h = \mu(1+\epsilon) \text{ with probability } \frac{1}{2}, \\ z_\ell = \mu(1-\epsilon) \text{ with probability } \frac{1}{2}, \end{cases}$$

where $\mu, \epsilon > 0$. The stationary condition implies that $1 + z_h = \frac{\phi_{-1}}{\phi_h}$ and $1 + z_\ell = \frac{\phi_{-1}}{\phi_\ell}$. The average inflation is $\mu = Ez > 0$, and the average return on money is

$$\frac{1}{1+\hat{\gamma}} \equiv E_{-1}\frac{\phi}{\phi_{-1}} = \frac{1}{(1+\mu)(1-\epsilon^2)}.$$

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One can see that $\mu > \hat{\gamma}$ and if $\epsilon > \sqrt{1-\beta}$, the money growth rate in the Friedman rule is positive.

In a monetary economy where banks hold liquidity buffers, the Friedman rule also achieves the efficient allocation. From (8) and $\nu i = i_d$, equation (14) becomes

$$\beta E_{-1}\phi[\theta(1+i) + (1-\theta)(1+i_d)] = \phi_{-1}. \tag{B.2}$$

From (B.2) we see that, under the policy that sets $1+\hat{\gamma} = \beta$, $u'(q_b) = c'(q_s)$ as i_d and i approach to 0. The Friedman rule achieves the efficient allocation.

APPENDIX C: DERIVING OPTIMAL MONEY INJECTIONS UNDER DEMAND SHOCKS

The central bank's problem is

$$\max_{x,q_b}\ W = U(x) - x + \int_{\underline{\eta}}^{\overline{\eta}} [\theta u(q_b) - (1-\theta)c(\frac{\theta}{1-\theta}q_b)]g(\eta)d\eta$$

$$\text{s.t. } \beta \int_{\underline{\eta}}^{\overline{\eta}} \frac{(1+\chi i)(\phi_{-1}M_{-1} - \theta q_b)}{\phi_{-1}M_{-1}(1-\chi)} g(\eta)d\eta = 1.$$

Because $1+i = \frac{u'(q_b)}{c'(q_s)} = e^{\eta - q_b}$, we use the approximation, $\log(1+y) \simeq y$, to get $i(\eta) \simeq \eta - q_b(\eta)$. Let λ_A be the multiplier of the constraint (30). The first-order conditions are $U'(x) = 1$ and

$$\int_{\underline{\eta}}^{\overline{\eta}} [\theta e^{\eta - q_b} - \theta]g(\eta)d\eta$$

$$-\frac{\beta\lambda_A}{\phi_{-1}M_{-1}(1-\chi)} \int_{\underline{\eta}}^{\overline{\eta}} \{\chi(\phi_{-1}M_{-1} - \theta q_b) + [1+\chi(\eta - q_b)\theta\}g(\eta)d\eta = 0,$$

where λ_A is the multiplier of the constraint (30). Simplifying the above equation and using the approximation $i \approx \eta - q_b$, we obtain $i(\eta)$ defined in (31). Assuming a particular distribution of η, and substituting $i(\eta)$ from (31) into the constraint (30), one could obtain λ_A as a function of parameters such as η, β, θ, $\phi_{-1}M_{-1}$, and χ.

Assume that η follows a Bernoulli distribution such that η takes a value from the set $\{\eta_L, \bar{\eta}, \eta_H\}$ with an equal probability, $\frac{1}{3}$, where $\bar{\eta} = (\eta_H + \eta_L)/2$. This implies that $i = i_H = i(\eta_H)$ with probability $\frac{1}{3}$, $i = i_{\bar{\eta}} = i(\bar{\eta})$ with probability $\frac{1}{3}$, and $i = i_L = i(\eta_L)$ with probability $\frac{1}{3}$. Rewrite $i(\eta)$ as

$$i(\eta) = \frac{1+\chi(\frac{\phi_{-1}M_{-1}}{\theta} - \eta)}{\frac{(1-\chi)\phi_{-1}M_{-1}}{\beta\lambda_A} - 2\chi}. \tag{C.1}$$

We can solve for the real balance, $\phi_{-1}M_{-1}$, from the long-run equilibrium, in which $\eta = \bar{\eta}$ and $z = \mu$. Therefore, in the long-run equilibrium, $1+\chi i_{\bar{\eta}} = \frac{(1+\mu)}{\beta}$, $q_b = \frac{\phi_{-1}M_{-1}(\chi+\mu)}{\theta(1+\mu)}$,

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and $i_{\bar{\eta}} = \bar{\eta} - q_b$. We obtain

$$\phi_{-1}M_{-1} = \frac{\theta(1+\mu)}{\chi+\mu}(\bar{\eta} - \frac{1+\mu-\beta}{\beta\chi}), \tag{C.2}$$

$$i_{\bar{\eta}} = \frac{1+\mu-\beta}{\beta\chi}. \tag{C.3}$$

Note that $i_{\bar{\eta}} \geq 0$ if and only if $\mu \geq \beta - 1$. Denote $\mu_0 = \beta - 1$. Let Ω_n and Ω_d denote the numerator and denominator, respectively, of $i(\bar{\eta})$ in (C.1), i.e.,

$$\Omega_n = 1 + \chi(\frac{\phi_{-1}M_{-1}}{\theta} - \bar{\eta}),$$

$$\Omega_d = \frac{(1-\chi)\phi_{-1}M_{-1}}{\beta\lambda_A} - 2\chi.$$

Using (C.2) we obtain

$$\Omega_n = \frac{\chi(1-\chi)}{\chi+\mu}\bar{\eta} + 1 - \chi\frac{1+\mu}{\chi+\mu}\frac{1+\mu-\beta}{\beta\chi}.$$

The numerator of $i(\bar{\eta})$, $\Omega_n > 0$, if and only if $\mu_1 < \mu < \mu_2$, where $\mu_1 = -1 + \beta - \sqrt{\beta[\beta - (1-\chi)(1-\bar{\eta}\chi)]}$ and $\mu_2 = -1 + \beta + \sqrt{\beta[\beta - (1-\chi)(1-\bar{\eta}\chi)]}$. The real balance, $\phi_{-1}M_{-1}$, shown in (C.2), is strictly positive if and only if $\mu < \mu_3$, where $\mu_3 = -1 + \beta + \beta\bar{\eta}\chi$. Note that $\mu_1 < \mu_0$, and $\mu_3 < \mu_2$ if and only if $\chi < \frac{1}{\bar{\eta}}$. Therefore, when $\chi < \frac{1}{\bar{\eta}}$, the condition $\mu \in [\mu_0, \mu_3]$ implies that $\Omega_n > 0$, and that to get $i(\bar{\eta}) \geq 0$ we need the denominator of $i(\bar{\eta})$ from (C.1) to be positive.

We now show that under certain conditions the central bank would not choose a policy that is leaning against the wind [i.e., a policy that leads to $z(\eta_H) < z(\eta_L)$, and so $i_H > i_L$]. Differentiating $i(\eta)$ from (C.1) with respect to η, one obtains $\frac{\partial i}{\partial \eta} = \frac{-1}{[(1-\chi)\phi_{-1}M_{-1}/\beta\lambda_A - 2\chi]}$. Then, $\frac{\partial i}{\partial \eta} > 0$ if and only if $\frac{(1-\chi)\phi_{-1}M_{-1}}{\beta\lambda_A} - 2\chi < 0$, i.e., the denominator of $i(\bar{\eta})$, $\Omega_d < 0$. In order for $i(\bar{\eta}) \geq 0$, we need $\Omega_n \leq 0$, which implies that either $\mu \leq \mu_1$ or $\mu \geq \mu_2$. Because $\mu_1 < \mu_0$, we thus need $\mu \geq \mu_2$ to get $\Omega_n \leq 0$. But, if $\chi < \frac{1}{\bar{\eta}}$, then under the condition $\mu \geq \mu_2$, the real balance is negative, and the equilibrium does not exist. Therefore, if $\chi < \frac{1}{\bar{\eta}}$, then the central bank injects more money in response to higher aggregate demand. In the model, we consider an economy where $\chi < \frac{1}{\bar{\eta}}$.[25]

Comparing fluctuations of consumption under the state-contingent stabilization policy with those under the price-level targeting stabilization policy. Here we use the variance of consumption to measure fluctuations. Let q_{wb}^j and q_{ob}^j (i_w^j and i_o^j) denote the buyer's consumption (interest rates) under the policy with and without withdrawal of the state-contingent money injection, respectively, where $j = H, L$ represents that the state is $\eta = \eta_H$ or $\eta = \eta_L$. Let $\bar{q}_{wb}$ and $\bar{q}_{ob}$ (σ_w^2 and σ_o^2) denote the average (variance) of consumption under the policy with and without withdrawal of the state-contingent money injection, respectively, when $\eta = \bar{\eta} = \frac{\eta_H + \eta_L}{2}$.

Under the price-level targeting stabilization policy, from (33), $i_w = \frac{1+\mu-\beta}{\beta\chi}$. Then, $\bar{q}_{wb} = \bar{\eta} - i_w$. Under the state-contingent stabilization policy, whereby the central bank does not

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withdraw the state-contingent money injection, $\bar{q}_{ob} = \bar{\eta} - \frac{i_o^H + i_o^L}{2}$. Let $\omega = \frac{i_o^H - i_o^L}{2}$. Then,

$$\begin{aligned}\sigma_o^2 - \sigma_w^2 &= [(q_{ob}^H - \bar{q}_{ob})^2 + (q_{ob}^L - \bar{q}_{ob})^2] - [(q_{wb}^H - \bar{q}_{wb})^2 + (q_{wb}^L - \bar{q}_{wb})^2] \\ &= [(\eta_H - \bar{\eta} - \omega)^2 + (\eta_L - \bar{\eta} + \omega)^2] - [(\eta_H - \bar{\eta})^2 + (\eta_L - \bar{\eta})^2] \\ &= 2\omega(\omega - \eta_H + \eta_L).\end{aligned}$$

Under the state-contingent policy, $i_H < i_L$. Therefore, $\omega < 0$, and $\sigma_o^2 > \sigma_w^2$; the price-level targeting policy results in smaller fluctuations than those under the state-contingent policy.

APPENDIX D: RANDOM DEPOSIT WITHDRAWALS

Solving the bank's problem. The representative bank's expected profit is

$$\theta_c i b - \frac{1}{k}\sum_{j=1}^{k}\left[\theta_p + \theta_n(1-\theta_{nc}^j)\right] d i_d,$$

where b and d satisfy the market-clearing condition for loans, (39). Substituting d from (39) into the bank's expected profit, we obtain that a bank solves the following problem per borrower:

$$\max_b (i - \frac{i_d}{\tilde{\nu}})b$$

$$\text{s.t. } \frac{1}{k}\sum_{j=1}^{k}\left[u(q_b^j) + W(m,b,d)\right] \geq \Gamma_c,$$

where Γ_c is the reservation value of the borrower, which is the surplus from obtaining loans at another bank, and $\tilde{\nu} = \frac{\theta_p + \theta_n(1-\theta_{nc}^k)}{\frac{1}{k}\sum_{j=1}^{k}[\theta_p + \theta_n(1-\theta_{nc}^j)]}$. The first-order condition to the bank's problem is

$$i - \frac{i_d}{\tilde{\nu}} + \lambda_{\Gamma_c}\frac{1}{k}\sum_{j=1}^{k}[u'(q_b^j)\frac{dq_b^j}{db} + W_b] = 0, \quad \textbf{(D.1)}$$

where λ_{Γ_c} is the Lagrangian multiplier on the borrower's participation constraint. For $i - \frac{i_d}{\tilde{\nu}} > 0$, the bank would like to make the largest loan possible to borrowers. Therefore, this implies a zero-marginal-profit condition,

$$\tilde{\nu} i = i_d.$$

and the bank would choose a loan amount such that $\lambda_{\Gamma_c} > 0$.

From (8) and the buyer's budget constraint, $\frac{dq_b^j}{db} = \frac{\phi}{c'(q_s^j)}$. Using $W_b = -\phi(1+i)$, from (D.1) we have

$$\frac{1}{k}\sum_{j=1}^{k}\frac{u'(q_b^j)}{c'(q_s^j)} = 1 + i.$$

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We now consider the buyer's problem:

$$\max_{q_b^j, b \geq 0} \frac{1}{k} \sum_{j=1}^{k} [u(q_b^j) + W(m + b - p^j q_b^j, b)]$$

$$\text{s.t. } p^j q_b^j \leq m + b.$$

The first-order condition with respect to b is $\frac{1}{k} \sum_{j=1}^{k} \frac{u'(q_b^j)}{c'(q_s^j)} = 1 + i$. A buyer borrows to the point at which the expected benefit of financing consumption equals the borrowing cost.

Deriving the condition for a liquidity effect. We use an approach similar to that discussed in Section 4 to derive conditions for the existence of a liquidity effect. From the loan-market-clearing condition (39), we have

$$b = \frac{[\theta_p + \theta_n(1 - \theta_{nc}^k)]d + \chi_m \tau}{\theta_c}. \tag{D.2}$$

Substituting $d = m$, $\tau = zM_{-1}$, and $m = M_{-1}$ into (D.2), we obtain the total funds available per buyer to finance consumption in the first subperiod:

$$m + b = \frac{(\chi_l + \chi_m z)}{\theta_c} M_{-1}, \tag{D.3}$$

where

$$\chi_l = \theta_c + \theta_p + \theta_n(1 - \theta_{nc}^k) = 1 - \theta_n \theta_{nc}^k.$$

We consider an equilibrium where the cash constraint binds in all states, $q_b^j = \frac{m+b}{p^j}$, from which we derive

$$q_b^j c'(q_s^j) = \frac{(\chi_l + \chi_m z)\phi_{-1} M_{-1}}{\theta_c(1 + z)}, \tag{D.4}$$

by using (8), (D.3), and $1 + z = \frac{\phi_{-1}}{\phi}$. A similar condition for the late consumer is

$$q_l^j c'(q_s^j) = \frac{\phi_{-1} M_{-1}}{(1 + z)}. \tag{D.5}$$

The aggregate demand q_A^j is thus

$$q_A^j = \theta_c q_b^j + \theta_n \theta_{nc}^j q_l^j = \frac{(\chi_l + \chi_m z + \theta_n \theta_{nc}^j)\phi_{-1} M_{-1}}{c'(q_s^j)(1 + z)}.$$

From (D.5), q_l^j is decreased by the money injection, z, whereas from (D.4), q_b^j is increased by z if $\chi_m > \chi_l$. Taking the derivative of q_A^j with respect to z, we obtain condition (41).

Relaxing the restrictions so that nontraders may borrow. We first consider an environment with no spatial frictions, where nontraders can make deposits and take loans. Let b_n

and d_n denote the loan amount and deposits made by a nontrader, respectively, before the liquidity shock is realized. A nontrader's problem is

$$\max_{q_l^j, b_n, d_n \geq 0} \frac{1}{k}\sum_{j=1}^{k}\theta_{nc}^j[u(q_l^j) + W(m + b_n - p^j q_l^j, b_n, 0)] + (1-\theta_{nc}^j)W(m + b_n - d_n, b_n, d_n) \quad \textbf{(D.6)}$$

$$\text{s.t. } p^j q_l^j \leq d_n + b_n,$$

$$d_n \leq m.$$

The first term in (D.6) shows that with probability θ_{nc}^j a nontrader becomes a late consumer and uses money holdings left after deposits, $m - d_n$, plus deposits withdrawn, d_n, and loan, b_n, to finance consumption, $p^j q_l^j$. The second term in (D.6) implies that with probability $1 - \theta_{nc}^j$ a nontrader does not want to consume, and he enters the second subperiod holding $m - d_n + b_n$ units of money, b_n debt, and d_n deposits. The first-order condition with respect to b_n is

$$\frac{1}{k}\sum_{j=1}^{k}\theta_{nc}^j\frac{u'(q_l^j)}{c'(q_s^j)} \leq 1 + i, \text{ “ = ” if } b_n > 0. \quad \textbf{(D.7)}$$

Condition (D.7) implies that when the expected net benefit of borrowing to finance consumption offsets the loan interests, a nontrader has incentives borrow. Given the loan rate, condition (D.7) is less likely to hold at equality if θ_{nc}^j is smaller. That is, when the future opportunity for consumption is slim, nontraders are less likely to incur the borrowing cost to take out loans.

Next, consider an environment where banks take deposits after the liquidity shock is realized. Suppose a nontrader borrows b_n before the liquidity shock is realized. We consider the situation where once a nontrader learns that he does not want to consume, he deposits the money initially held and the borrowed money. (This is innocuous because if we assume there is a cost of withdrawing deposits, $\epsilon > 0$, nontraders would not make deposits before the liquidity shock is realized.) Let d_n^j denote deposits made by a nontrader after the liquidity shock is realized, when the state is θ_{nc}^j. A nontrader solves the following problem:

$$\max_{q_n^j, b_n, d_n^j \geq 0} \frac{1}{k}\sum_{j=1}^{k}\theta_{nc}^j[u(q_l^j) + W(m + b_n - p^j q_l^j, b_n, 0)] + (1-\theta_{nc}^j)W(m + b_n - d_n, b_n, d_n^j)$$

$$\text{s.t. } p^j q_l^j \leq m + b_n,$$

$$d_n^j \leq m + b_n.$$

With probability θ_{nc}^j a nontrader is a late consumer, and he spends his money holding and borrowed money, $m + b_n$, to finance consumption, and with probability $1 - \theta_{nc}^j$ he does not want to consume, and deposits d_n^j in the bank. Note that $d_n^j = m + b_n$, for all j, because making deposits would earn interests. The first-order condition is

$$\frac{1}{k}\sum_{j=1}^{k}\left[\theta_{nc}^j\frac{u'(q_l^j)}{c'(q_s^j)} + (1-\theta_{nc}^j)(1+i_d)\right] \leq (1+i), \text{ “ = ” if } b_n > 0. \quad \textbf{(D.8)}$$

When (D.8) holds with equality, a nontrader borrows to the point at which the borrowing cost equals the expected benefit of financing consumption plus the expected benefit of depositing money if he does not want to consume. Because here banks are ready to accept deposits after the liquidity shock is realized, nontraders do not need to make deposits before the realization of liquidity shocks, and so there is no withdrawal uncertainty facing banks. This is different from our motivation to justify banks' holding reserves to insure against random deposit withdrawals.