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Speed/accuracy trade-offs in target-directed movements

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Abstract: This target article presents a critical survey of the scientific literature dealing with the speed/accuracy trade-offs in rapid-aimed movements. It highlights the numerous mathematical and theoretical interpretations that have been proposed in recent decades. Although the variety of points of view reflects the richness of the field and the high degree of interest that such basic phenomena attract in the understanding of human movements, it calls into question the ability of 'many models to explain the basic observations consistently reported in the field. This target article summarizes the kinematic theory of rapid human movements, proposed recently by R. Plamondon (1993b; 1993c; 1995a; 1995b), and analyzes its predictions in the context of speed/accuracy trade-offs. Data from human movement literature are reanalyzed and reinterpreted in the context of the new theory. It is shown that the various aspects of speed/accuracy trade-offs can be taken into account by considering the asymptotic behavior of a large number of coupled linear systems, from which a delta-lognormal law can be derived to describe the velocity profile of an end-effector driven by a neuromuscular synergy. This law not only describes velocity profiles almost perfectly, it also predicts the kinematic properties of simple rapid movements and provides a consistent framework for the analysis of different types of speed/accuracy trade-offs using a quadratic (or power) law that emerges from the model.

Keywords: central limit theorem; delta-lognormal law; Fitts' law; neuromuscular synergy; power law; quadratic law; speed/accuracy trade-offs; velocity profile

1. Introduction

Speed/accuracy trade-offs in target-directed movement have been studied for more than a century. Fullerton and Cattell (1892) referenced some experimental work by both German and French investigators on the psychophysics of movement. Woodworth (1899) is often credited with being the founder of research on the speed/accuracy trade-offs in movement. The main contribution of Woodworth's article is the proposition of a cohesive account of the accuracy of voluntary movements, which had never been done before. Woodworth's observations implied an intricate relationship between movement duration, amplitude, and velocity in the determination of movement accuracy. He suggested the separation of a rapid movement into two phases: (1) the initial adjustment phase and (2) the current control phase. Woodworth attributed the effect of movement speed on accuracy to the characteristics of current control and eliminated any effects on the accuracy of the initial adjustment.

Following Woodworth's work, many investigators pursued descriptions of speed/accuracy functions (Brown & Slater-Hammel 1949; Craik & Vince 1943/1963a; 1944/1963b; Garrett 1922; Philip 1936; Searle & Taylor 1948. See also Meyer et al., 1990, for a historical review). The next major contribution to research on speed/accuracy trade-offs was done by Fitts (1954), who was the first to propose a formal relationship linking movement time (MT) and the inverse of the relative spatial error:

$$MT = a + b \log_2 \left(\frac{2A}{W} \right) \tag{1}$$

where A represents the amplitude of the movement and W represents the target width; a and b are empirically determined constants. In this formulation, the logarithmic term is called the "index of difficulty (ID) of the movement,"

$$ID = \log_2 \left(\frac{2A}{W} \right) \tag{2}$$

and is linked to the maximum rate of information transmission of the human motor system.

In subsequent years, Fitts' tapping tasks or Fitts' paradigms were used in a systematic way for numerous studies. Equation (1) was found to be so general that it became known as Fitts' law (Crossman & Goodeve 1963/1983; Keele 1968). In spite of its generality, Fitts' relationship was found to be of limited use for spatially constrained tasks of low precision, and even inappropriate in timely constrained tasks (see Schmidt 1988 for a survey). Moreover, from a theoretical point of view, a few models have been developed to explain the origin of these phenomena (see Meyer et al. 1988 for a review), but no satisfying interpretation covering the various aspects of speed/accuracy trade-offs has been proposed.

This target article constitutes an attempt toward such an interpretation, one that explains the origin of the different speed/accuracy trade-offs by linking them to some fundamental properties of the kinematic behavior of a large

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number of coupled neuromuscular networks (Plamondon 1995a). The text is divided into four parts. First, a list of domains and contexts where Fitts' law has been studied is reported. In this short review, the previous theoretical models used to explain speed/accuracy trade-offs are presented. This part also summarizes the various experimental limitations that have been reported concerning Fitts' law as well as the different mathematical formulations that have been proposed to take these discrepancies into account. In the second part, the kinematic theory of rapid human movements developed by Plamondon (1993b; 1993c; 1995a; 1995b) is summarized and discussed to highlight the origin of speed/accuracy trade-offs in complex neuromuscular systems. This new theory is used in the context of spatially-constrained and temporally-constrained tasks to predict the different relationships between movement time and the inverse of the relative spatial error (Plamondon 1993c; 1993d; 1995b). In the third part, the resulting laws are tested using the data available from the numerous studies published in the field. The implications of Plamondon's kinematic theory for movement control and understanding are discussed in section 4.

2. Speed/accuracy trade-offs: A short survey

The majority of experimental procedures used in the study of speed/accuracy trade-offs fall into one of two categories: spatially constrained movements or temporally constrained movements. In movements with spatial constraints, subjects are asked to move as quickly as possible to a target placed at a distance (A). The target width (W) may or may not be explicitly given. In movements with temporal constraints, subjects are asked to move to a fixed target at a specified time (MT). The required timing precision (ΔMT) may or may not be explicitly given.

In this section we report on these two categories of movements. For spatially constrained movements we present Fitts' (1954) law as the case where both W and A are given and MT is measured, and Howarth et al.'s (1971) approach as the case where only A is specified and results on W and MT are reported. For temporally constrained movements, A and MT are specified. We present the work of Schmidt et al. (1979), who provide data on spatial variability, and the work of Newell et al. (1979), who provide data on temporal variability.

2.1. Spatially constrained movements where both A and W are given

The popularity of Fitts' law is mainly the result of the large number of experiments and studies that have been reported to support it. It has been roughly validated for a variety of movements, and some studies have adopted the model as a tool for investigating other issues. The experimental validation of Fitts' law has been confirmed, totally or partially, for a variety of movements, limbs and muscle groups, experimental conditions and manipulating devices, subjects, and a wide range of performance indices (see Table 1).

It should be noted that not all these studies support Fitts' law. Many of them report violations, new formulations, or new explanations of this law. Indeed, although Fitts' law has been generally accepted by many researchers as a good and practical working tool, many have worked on its experimental verification, and several modifications to his original equation (1) have been advanced, taking into account specific experimental conditions and thus providing better data fitting.

Several factors indicate limitations of Fitts' law as a potentially general description of the movement speed/accuracy relationship. An upward curvature of MT data, away from the regression line, has been observed for low indices-of-difficulty values, so the lawful relationship fails at very low IDs (Buck 1986; Crossman 1957; Crossman & Goodeve 1963/1983; Drury 1975; Klapp 1975; Langolf et al. 1976; Meyer et al. 1988; 1990; Wallace et al. 1978; Welford 1960). In Fitts' reciprocal tapping experiment, the constant a becomes negative when movement time is plotted against his index of difficulty, as it also does for zero information per response; in addition, the best regression line through the data is not straight but curved slightly upwards (Knight & Dagnall 1967).

Several discrepancies between Fitts' experimental data and his theoretical formulation have been deduced by many authors who changed the definition of the index of difficulty, as shown in Table 2. Welford (1968) suggested that subjects utilize only the half near the target area, and he modified the equation accordingly. Subsequently, Welford et al. (1969) advanced an equation relating movement time to amplitude and to target width, separately. While studying moving targets, Jagacinski et al. (1980b) proposed an alternate index of difficulty that explicitly incorporates a velocity factor.[1] Recently, Hoffmann (1991a) has proposed two models for capturing the speed of a moving target; these models provide an excellent fit to the experimental data of Jagacinski et al. The main improvement of the Hoffmann models over that of Jagacinski et al. is in the theoretical explanations. For situations where there is a transmission delay between control movements and the feedback of the system response to the operator, Hoffmann (1992) proposed a model incorporating the delay[2] in its formulation. Recently, MacKenzie (1989; 1992) showed that Fitts' choice of an equation that deviates slightly from the fundamental theorem 17 of Shannon is unfounded and proposed a corrected equation. Gan and Hoffmann (1988) found that when the index of difficulty is small, MT can be predicted by an equation depending only on A. Johnsgard (1994) proposed a modification to the equation of MacKenzie that includes the effect of the device gain (G).

Another problem stems from the relative contributions of W and A in the equation for MT. In contrast to the implicit assumption of Fitts' law, it has been suggested – following the example of Welford et al. (1969) – that amplitude and target width do not possess equal weight in the determination of movement time (Sheridan 1979). The disproportionate increase in movement time caused by reductions in target width compared to similar increases in target amplitude has also been noted directly from their data (Buck 1986; Jagacinski et al. 1980a; Jagacinski & Monk 1985; Keele 1973; Meyer et al. 1988; Welford et al. 1969), or indirectly from the analysis of the error-rate, which was found to increase as target width decreases independently of target amplitude (Card et al. 1978; Wade et al. 1978).

A few models have been proposed to give a theoretical explanation for speed/accuracy trade-offs, and particularly for Fitts' law. We review these briefly below.

2.1.1. Fitts' information theory hypothesis. Using the information theory (Miller 1953; Pierce 1961; Shannon 1948;

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Table 1. *Applications of Fitts' law*

Category	Study	Authors
Movements	a) Serial or continuous	a) Fitts (1954); Kvalseth (1975)
	b) Discrete	b) Carlton (1979; 1980); Fitts & Peterson (1964).
	c) Tapping	c) Fitts (1954); Fitts & Peterson (1964); Kantowitz & Elvers (1988); Megaw (1975);
	d) Object transferral	d) Fitts (1954); Raouf & Tsui (1978)
	e) Dart throwing	e) Kerr & Langolf (1977)
	f) Three-dimensional	f) MacKenzie et al. (1987)
	g) Rotary	g) Knight & Dagnall (1967)
	h) Pointing and dragging	h) Gillan et al. (1990)
Limbs and muscle groups	a) Wrist flexion and rotation	a) Crossman & Goodeve (1963/1983); Meyer et al. (1988); Wright & Meyer (1983)
	b) Foot movements	b) Drury (1975); Hoffmann (1991b)
	c) Head movements	c) Andres & Hartung (1989a; 1989b); Jagacinski & Monk (1985)
	d) Finger manipulation	d) Hoffmann & Sheikh (1991); Langolf et al. (1976)
	e) Arm extension	e) Kerr & Langolf (1977)
	f) Rapid elbow flexion	f) Corcos et al. (1988)
	g) Speech	g) Jafari & Kondraske (1988)
	h) Hand movements	h) Beggs & Howarth (1972); Howarth et al. (1971)
	i) It has been suggest that the law would hold for the mouth or any other organ for which a suitable motor task could be devised	i) Glencross & Barrett (1989); MacKenzie (1992)
Experimental conditions	a) Microscopic movements	a) Hancock et al. (1973); Langolf & Hancock (1975)
	b) Underwater movements	b) Kerr (1973; 1978)
	c) Aircraft flight	c) Hartzell et al. (1982)
	d) Controlled visual feedback	d) Carlton 1981; Crossman (1960); Glencross & Barrett (1989); Keele & Posner (1968); Meyer et al. (1988); Prablanc et al. (1979); Wallace & Newell (1983); Zelaznik et al. (1981)
	e) Inert gas narcosis	e) Fowler et al. (1982)
	f) Handedness studies	f) Flowers (1975)
	g) Moving targets	g) Hoffmann (1991a); Jagacinski et al. (1980a; 1980b)
	h) Tasks with transmission delay	h) Ferrell (1965); Hoffmann (1992); Sheridan & Ferrell (1963)
	i) Tasks with unusual body dynamics	i) Newman & Bussolari (1990)
Manipulation devices	a) Hand-held stylus	a) Fitts (1954); Kvalseth (1978)
	b) Rotary handle	b) Crossman & Goodeve (1963/1983)
	c) Joystick	c) Card et al. (1978); Epps (1986); Hartzell et al. (1982); Jagacinski et al. (1980a)
	d) Computer mouse	d) Boritz et al. (1991); Card et al. (1978); Epps (1986); Johnsgard (1994)
	e) Keyboard	e) Card et al. (1978); Drury & Hoffmann (1992)
	f) Food pedal	f) Drury (1975)
	g) Teleoperator	g) Draper et al. (1990); Drascic et al. (1989); Repperger & Remis (1990); Shinhar (1986)
	h) Head-controlled computer input device	h) Lin et al. (1992); Radwin et al. (1990); Spitz (1990)

(*continued*)

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Table 1. (*Continued*)

Category	Study	Authors
	i) Ballpoint pen or digitizer tablet	i) Spitz (1990); van Galen & Schomaker (1992)
	j) Trackball	j) Arnault & Greenstein (1987); Epps (1986)
	k) Touchpad	k) Epps (1986)
	l) Eyetracker	l) Ware & Mikaelian (1987)
	m) Telerobotic system	m) Cannon & Leifer (1990)
	n) Touch tablet	n) Arnault & Greenstein (1987)
	o) Virtual reality glove	o) Johnsgard (1994)
Subjects	a) Youths and children	a) Jones (1991); Salmoni (1983); Salmoni & McIlwain (1979); Sugden (1980); Wallace et al. (1978)
	b) The aged	b) Welford et al. (1969)
	c) Intellectually handicapped patients	c) Wade et al. (1978)
	d) Patients with Parkinson's disease	d) Flowers (1976)
	e) Patients with cerebral palsy	e) Bravo et al. (1990)
	f) Drugged subjects	f) Kvalseth (1977)
	g) Monkeys	g) Brooks (1979)
	h) Humans of different ages and sexes	h) Brogmus (1991)
Performance indices (*ID*)	a) Less than 1 bit/sec	a) Hartzell et al. (1982)
	b) More than 60 bits/sec	b) Kvalseth (1981)
	c) Most studies report *ID*s in the 3 to 12 bit/sec range	c) MacKenzie (1992)

Shannon & Weaver 1949), Fitts (1954) hypothesized that the task difficulty could be measured in bits using an information metric and that, in carrying out a movement task, information is transmitted through a noisy stochastic communication channel that models the behavior of the human motor system. Theorem 17 of Shannon (1948) expresses the effective information capacity C (in bits/sec) of a communications channel of bandwidth B (in Hz) as:

$$C = B \log_2 \left(\frac{S + N}{N} \right) \tag{3}$$

where N is the noise power and S is the signal power. Fitts claimed that, at the maximum rate of information transmission, the human motor system behaves in accordance with the logarithmic relation by identifying $1/MT$ with B, A with $S + N$, and $W/2$ with N to obtain the speed/accuracy trade-off relation proposed in equation (1).

One of the major hypotheses in Fitts' information theory interpretation is that the human motor system behaves like a stochastic communication channel. This theoretical framework has been criticized. Crossman and Goodeve (1963/1983) have pointed out "the empirical difficulty of establishing the existence of the postulated 'noise' or initial uncertainty" (p. 253). Kvalseth (1979) has claimed that "the Fitts' *ID* measure does indeed yield false estimates of the information capacity of the human motor system" (p. 292), and he proposed a power law as an alternative to Fitts' law, based on the superior data fit obtained by reanalysing previously published results. He did not support this power law theoretically. Recently, MacKenzie (1989) showed that Fitts' law is in fact derived from Goldman's equation 39 (Goldman 1953), which is an approximation of Shannon's theorem, instead of the original theorem 17 of Shannon (1948), given in equation (3):

$$C = B \log_2 \left(\frac{S}{N} \right) \tag{4}$$

The resulting variation of Fitts' law (see Table 2) has recently (Brogmus 1991; Welford 1990) been shown to be one of the best, although the origin of the constant a is still problematic and not predicted by the theory.

2.1.2. The deterministic iterative-corrections model of Crossman and Goodeve. An alternative to Fitts' information theory approach is known as the deterministic iterative-corrections model, which originated in the model of Crossman and Goodeve (1963/1983), and was subsequently developed by Keele (1968) and by Langolf et al. (1976). Under this model, movements intended to reach a target region quickly and accurately are executed through iterations of feedback-guided corrective submovements. A submovement is assumed to take a constant time t to cover a distance equal to $(1 - p)$ times the remaining distance. By applying these assumptions, the submovement sequence continues under either visual or kinesthetic feedback until the target region has been reached. So, if X_i denotes the distance remaining up to the target center after the i^{th} submovement, then:

$$X_o = A$$

$$X_n = \frac{W}{2}$$

$$X_i = pX_{i-1} \tag{5}$$

$$\text{and} \quad X_n = p \cdot X_{n-1} = \ldots = p^n \cdot A = \frac{W}{2}$$

$$n = \frac{1}{\log_2(p)} \log_2 \left(\frac{W}{2A} \right) \tag{6}$$

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Table 2. *Mathematical formulations of Fitts' law*

Authors	Equation	Remarks
Crossman (1956)	$MT = a + b \log_2 \left(\frac{A}{W}\right)$	Applying this expression to experimental data, Crossman found that the fit was better and that the constant a had a value of 0.05 sec, which was the time he found that the subject spent lingering on the target.
Welford (1968)	$MT = k \log_2 \left(\frac{A}{W} + 0.5\right)$	k is an experimentally determined constant.
Welford et al. (1969)	$MT = a + b_A \log_2 (A) + b_W \log_2 \left(\frac{1}{W}\right)$	
Jagacinski et al. (1980b)	$MT = c + dA + e(V + 1)\left(\frac{1}{W} - 1\right)$	V is the mean velocity of the target movement and c, d, e are fitting constants.*
Jagacinski et al. (1980b)	$MT = p + q \log_2 \left\{2\left[A + \frac{V}{W}(MT + T)\right]\right\}$	T is a constant, corresponding to the length of time the cursor had to be held over the target to capture it, and p, q or x, y, z are fitting parameters. It should be noted that this equation is transcendent and, as such, does not provide an analytical solution for MT.*
Jagacinski et al. (1980b)	$\mathrm{MT} = \mathrm{x} + \mathrm{y} \log_2 \left(\frac{2A}{W}\right) + z \log_2 \left[\frac{V}{W/T} + 1\right]$	T is a constant, corresponding to the length of time the cursor had to be held over the target to capture it, and p, q or x, y, z are fitting parameters.**
Hoffman (1991a)	$MT = \frac{1}{K} \ln \left[\frac{A + \frac{V}{K}}{\frac{W}{2} - \frac{V}{K}}\right]$ and $MT = a + b \log_2 \left(A + \frac{V}{K}\right) - c \log_2 \left(\frac{W}{2} - \frac{V}{K}\right)$	K, a, b, c are fitting parameters.*
Hoffman (1992)	$MT = -a + b(c + D) \log_2 \left(\frac{2A}{W}\right)$	D is the delay; a, b, and c are regression coefficients.***
MacKenzie (1989; 1992)	$MT = a + b \log_2 \left(\frac{A}{W} + 1\right)$	
Gan & Hoffmann (1988)	$MT = a + b\sqrt{A}$	
Johnsgard (1994)	$MT = a + b \log_2 \left(\frac{A/W}{G} + 1\right)$	
Kvalseth (1980)	$MT = a \left(\frac{A}{W}\right)^b$	

*If $V = 0$, we obtain an expression of MT for the case of static targets, which is different from Fitts' law.
**If $V = 0$, we obtain an expression of MT for two cases of static targets, similar to Fitts' law.
***If $D = 0$, we obtain an expression of MT for the no-delay case, similar to Fitts' formulation.

Since the movement time is $MT = nt$, it can be expressed as:

$$MT = \frac{-t}{\log_2(p)} \log_2 \left(\frac{2A}{W}\right) \quad (7)$$

Since the first move should take less than t (by a constant a), because the time it takes to decide how far to move initially occurs before a move begins (Keele 1968), then:

$$MT = (t - a) + (n - 1)t = nt - a \quad (8)$$

$$MT = -a + b \log_2 \left(\frac{2A}{W}\right) \quad \text{with} \quad \left(b = \frac{-t}{\log_2(p)}\right) \quad (9)$$

To verify these assumptions, the time it takes to process visual feedback was determined to be in the 135 msec to 290 msec range (Beggs & Howarth 1970; Carlton 1981; Crossman & Goodeve 1963/1983; Keele & Posner 1968; Zelaznik et al. 1983) and the proportional error constant p was estimated to be between 0.04 and 0.07 (Langolf et al. 1976; Meyer et al. 1988; Pew 1974; Schmidt 1988; Vince 1948), giving b in the 29.1 msec/bits to 75.6 msec/bits range. This estimate of the slope of the logarithmic trade-off relation was sufficiently impressive that the determinis-

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tic iterative-corrections model soon became accepted as the best available account of Fitts' law (Meyer et al. 1990). In addition, with this model Crossman (1956) suggested that the *ID* could be interpreted as the difference between two fundamental quantities – $\log_2(W)$ measuring the entropy of the endpoint distribution, and $\log_2(2A)$ measuring the entropy of a hypothetical initial distribution of motion amplitudes.

Despite its simplicity, the assumptions of the deterministic iterative-corrections model have been found to be suspect, and since the 1970s it has become increasingly clear that this model is seriously flawed. Langolf et al. (1976) and Jagacinski et al. (1980a) have found considerable variation in the duration of the initial submovement, which is contrary to the model's hypothesis of constant-duration submovements. Langolf et al. found that some movements exhibiting the logarithmic trade-off relationship have only one correction despite the model's prediction of several corrective submovements for large-movement-difficulty indices. Wallace and Newell (1983) observed that aimed hand movements produced in the absence of visual feedback obey Fitts' law, which gives additional evidence against the dichotomy of feedback-controlled movements obeying Fitts' law and feedforward-controlled movements obeying a linear speed/accuracy relation. More fundamentally, one handicap of this model is that it is completely deterministic. This implies that for a fixed target distance and width the sequence of submovements would always be the same. So, the model cannot explain why subjects sometimes miss a target and commit an error (Fitts & Peterson 1964; Meyer et al. 1988; Wallace & Newell 1983).

2.1.3. Connelly's control model. Connelly (1984) proposed an alternative interpretation of Fitts' law based on a control model linking the error rate of a movement to the error. To illustrate the mathematical development for a simple control law, he assumed that the error rate dX/dt is a linear function of the error X.

$$\frac{dX}{dt} = -KX \quad \text{with} \quad K > 0 \tag{10}$$

The solution to this equation is

$$X(t) = X(0)e^{-Kt} \tag{11}$$

It can be written as

$$\log_2\left(\frac{X(t)}{X(0)}\right) = -Kt\,\log_2(e) \tag{12}$$

to obtain

$$t = C\,\log_2\left(\frac{X(0)}{X(t)}\right) \tag{13}$$

where

$$C = \frac{1}{K\,\log_2(e)} \tag{14}$$

Finally, by identifying $X(0)$ with A and $X(MT)$ with $W/2$, the expression for MT is obtained:

$$MT = C\,\log_2\left(\frac{2A}{W}\right). \tag{15}$$

This is the same equation as Fitts' law, except for the constant a, which again is not predicted here. Connelly (1984) concluded his study by noting that a specific model had not yet been identified and that numerous models and control laws, both linear and nonlinear, could be formulated.

If we analyze the equations given above, it can be seen that the control law proposed in equation (10) is no more than a generalization of Crossman and Goodeve's (1963/1983) deterministic iterative-corrections model. In fact, Crossman and Goodeve's model is discrete, whereas Connelly's model is continuous. In addition, Crossman and Goodeve's model has a specific control law ($X_i = pX_{i-1}$), which is the discrete representation of $dX/dt = pX$, whereas Connelly's model does not have a specific control law.

2.1.4. The unifying noise/velocity relationship of Chan and Childress. Chan and Childress (1990) have proposed a relationship that relates the variance of the human-machine noise to the mean square velocity of the human-machine output. Furthermore, they showed that this noise-velocity relationship is verified for some human-machine models (McReur's [1980] crossover model, Elkind's human-machine model [Elkind & Forgie 1959], and Fitts' law), and that it can be considered as a more fundamental human-machine behavior property.

Noting R as the radius of the region from the target center where human-machine output is most likely to be at any time t, the derivation of their relationship from Fitts' law is obtained by writing:

$$R = Ae^{-(t-a)/c} \tag{16}$$

where $R = W/2$; $c = b/\ln(2.0)$ for $t = MT$.

In this context, the authors represented the random variable, corresponding to the position from the target center at any time t, by a zero-mean Gaussian random variable with a standard deviation σ_n related to R by

$$\sigma_n = k_1 R = k_1 Ae^{-(t-a)/c} \tag{17}$$

where k_1 is a proportionality constant.

From this,

$$\frac{\partial \sigma_n}{\partial t} = -\frac{k_1}{c}Ae^{-(t-a)/c} = -\frac{\sigma_n}{c} \tag{18}$$

since the position is a Gaussian random variable, the velocity is also a random variable with mean square v^2 and

$$v = \frac{\partial \sigma_n}{\partial t} = \frac{\sigma_n}{c} \tag{19}$$

finally to obtain a unifying noise-velocity relationship:

$$\sigma_n^2 = c^2 v^2 \tag{20}$$

Chan and Childress (1990) did not provide any theoretical support for this new relationship.

2.1.5. The stochastic optimized-submovement model of Meyer et al. This model (Meyer et al. 1988; 1990) represents the movement production process as an ideal compromise between the duration of primary and secondary submovements. The model assumes the existence of noise in the neuromotor system that may affect the primary submovement, causing it either to overshoot or undershoot the target. The stochastic optimized-submovement model assumes that the effect of the motor noise increases with the velocity of the submovements and that the relationship between primary submovement endpoint standard deviation S_1 and the average velocity V_1 of the primary submovement is:

$$S_1 = KV_1 = K\frac{D_1}{T_1} \tag{21}$$

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where K is a positive constant, D_1 is the mean distance travelled by the primary submovements, and T_1 is their mean duration. Another assumption of the model is that for the secondary submovement,

$$S_2 = K \frac{\Delta}{T_{2\Delta}} \tag{22}$$

where Δ is the distance travelled in a mean time $T_{2\Delta}$ by the secondary submovement. K is the same constant in equations (21) and (22). Finally, another key assumption is that the average velocities of the primary and secondary submovements are programmed to minimize the average total movement duration (MT).

Under these assumptions, the stochastic optimized-submovement model predicts that

$$MT = A + B \sqrt{\frac{D}{W}} \tag{23}$$

where A and B are non-negative constants. Many other interesting predictions are made by this model with respect to the mean primary submovement durations, the proportion of secondary submovements, and the values of error rates (see Meyer et al. [1988; 1990] for more details). Using computer simulations of this model, Meyer et al. have suggested that for multiple submovements, a quasipower function might be a better predictor of MT:

$$MT = A + B \left(\frac{D}{W}\right)^{1/n} \tag{24}$$

where n is the number of submovements.

As stated previously, a few studies of temporally constrained movements have suggested relationships that are other than linear between the standard deviation of movement endpoints and the average velocity (Hancock & Newell 1985; Schmidt et al. 1985). Also, the constant K in equations (21) and (22) may differ for primary and secondary submovements. In these cases, the polynomial speed/accuracy trade-off represented by equation (23) will not be verified, and neither will equation (24).

2.1.6. The VITE model of Bullock and Grossberg. The VITE model consists of a set of channels controlling the length of a particular muscle (Bullock & Grossberg 1988). Motor planning occurs in the form of a target position command (*TPC*) that specifies the length to which all trajectory-controlling muscles are intended to move, and a command signal *GO* that ties together all these channels and specifies the movement's overall speed. Mutual interactions exist between channels for antagonistic muscles. The present position command (*PPC*) subtracted from the *TPC* command specifies the difference vector (*DV*) that is integrated through time by the VITE circuit after being multiplied by the *GO* signal. Thus the *PPC* is gradually updated and generates an outflow movement command to reach the target. To generate a movement, a *TPC* different from the *PPC* generates a nonzero *DV* that is multiplied by the *GO* signal to generate an input to the *PPC*. By integrating this signal through time, the *PPC* will be updated until it equals TPC.

In its simplest form, the VITE circuit obeys these equations (Bullock & Grossberg 1988):

$$\frac{d}{dt} DV = \alpha(-DV + TPC - PPC) \tag{25}$$

$$\frac{d}{dt} PPC = GO[DV]^+ \tag{26}$$

where $[DV]^+ = max(DV,0)$; α is a given parameter of the model.

Bullock and Grossberg showed that

$$MT = \frac{2}{\alpha} \log \left(\frac{TPC(0) - PPC(0)}{E}\right) \tag{27}$$

where E is the amount of overshoot error in the VITE command. In this relation, $TPC(0) - PPC(0)$ represents the distance to be moved. It should be noted that this relationship was mathematically proven only in one specific case (Bullock & Grossberg 1988, Appendix I), and, using computer simulations, the authors suggest that it occurs with greater generality.

2.2. Movements with spatial constraints, where only A is given

Howarth et al. (1971) have proposed a theory of intermittent visual control that is not based on linear control assumptions. Their theory specifies that the hand should be as close as possible to the target at the time of initiation of the last corrective movement. In an attempt to determine the precise relationship between distance and time as the hand approaches the target, they found experimentally that

$$d = 814.9 \left(\frac{t}{MT}\right)^{1.4} \tag{28}$$

where d is the distance to the hand from the target in mm, t is the time remaining before the target is hit in seconds, and MT is the time for the total movement in seconds.

They found, from experimental analysis, that

$$E^2 = E_0^2 + (814.9)^2 \sigma_0^2 \left(\frac{t_u}{MT}\right)^{2.8} \tag{29}$$

where E^2 is the mean square error from the target center, E_0^2 is a square error due to tremor, σ_0^2 is the angular error of the final corrective movement, and t_u is the uncontrolled movement time evaluated at $t_u = 290$ msec by Beggs and Howarth (1970).

As presented above, equation (29) is empirically determined, and so it does not specify the nature of the control processes contained therein (see Langolf et al., 1976, for an extensive criticism of this theory). It should be noted that in their experimental procedure, Howarth et al. (1971) considered only one target distance, $A = 50$ cm. If equation (29) is written differently, we obtain

$$MT = t_u[(814.9)^2 \sigma_0^2]^{1/2.8} \left(\frac{1}{E^2 - E_0^2}\right)^{1/2.8} \tag{30}$$

which is similar to a power law (see Kvalseth, Table 2), where $\left(\frac{1}{E^2 - E_0^2}\right)$ corresponds to $\left(\frac{1}{W}\right)^b$; and, because A is constant (50 cm) here $t_u[(814.9)^2 \sigma_0^2]^{1/2.8}$ corresponds to $a(A)^b$. So, the empirical relationship of Howarth et al. can be seen as a manifestation of a power law.

2.3. Movements with temporal constraints, where MT is given: Focus on spatial variability

Another fundamental problem is the dependency of the speed/accuracy trade-off formulation on the subjects' temporal and spatial goals (Schmidt et al. 1979). It was claimed that a linear trade-off relationship is more appropriate for time-matching tasks or temporally constrained tasks, whereas

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a logarithmic trade-off relationship better explains time-minimization movement or spatially constrained tasks. The linear speed/accuracy trade-off can be characterized as

$$W_e = a + b\left(\frac{A}{MT}\right) \tag{31}$$

where W_e is the standard deviation of the endpoint coordinates.

Two main hypotheses have been considered for characterizing the conditions under which a linear rather than a logarithmic trade-off will occur for aimed movements (see Wright & Meyer, 1983, for a review).

The first hypothesis, named the movement-brevity hypothesis (Wright & Meyer 1983), was proposed by Schmidt et al. (1979) as a motor-output variability theory for ballistic movements. The proportional relationship between the within-subject variability in movement amplitude, called the effective target width (W_e), and the average velocity of the single aiming movement (A/MT) is derived in this model from two proportionality relationships – the first between the impulse and the average velocity:

$$impulse \propto velocity \tag{32}$$

and the second between the within-subject variability of the impulse and the size of the impulse:

$$\sigma_{impulse} \propto impulse. \tag{33}$$

Since W_e is proportional to the variability of the impulse (Schmidt et al. 1978; 1979):

$$W_e \propto impulse \tag{34}$$

W_e is also proportional to the average velocity of the movement (see Schmidt et al., 1979, for more details):

$$W_e \propto velocity \propto A/MT \tag{35}$$

The second hypothesis, named the temporal-precision hypothesis (Wright & Meyer 1983), was proposed by Meyer et al. (1982) as the symmetric impulse-variability model. This second approach provides a way to unify the linear and logarithmic trade-offs by attributing precisely timed movements to a single pair of opposing force pulses that minimizes temporal variability, and spatially precise movements to a preprogrammed series of overlapping force pulses that increases temporal variability. This approach includes a number of assumptions about the shapes of force pulses used to produce movements and about the stochastic variation of pulses across different movements (see Meyer et al., 1982, for more details). The linear speed/accuracy trade-offs have been observed by other investigators: stylus-tapping movements (Zelaznik et al. 1981; 1988); wrist rotations (Wright 1983; Wright & Meyer 1983); saccadic eye movements (Abrams et al. 1989). The scaling property of the force assumed by these models has been reported by Abrams et al. (1989), Armstrong (1970), Freund and Budingen (1978), Ghez (1979), Ghez and Vicario (1978), Gordon and Ghez (1987), and so on. However, some problems have been raised. Most of the detailed studies have reported nonproportional relationships between force and force variability in both isometric (Carlton & Newell 1985; Fullerton & Cattell 1892; Jenkins 1947; Newell & Carlton 1985; Noble & Bahrick 1956; Provins 1957) and anisometric tasks (Newell et al. 1982). Furthermore, this relationship was found to have an inverted U shape (Newell et al. 1984; Sherwood & Schmidt 1980; Sherwood et al. 1988).

A few investigators have questioned the model's assumptions about the scaling of force pulses (Schmidt et al. 1985; Zelaznik et al. 1986). Zelaznik et al. have reported that "the acceleration-time functions were not symmetrical and that symmetricality decreased as MT decreased" (p. 36), which caused difficulties for the Meyer et al. (1982) symmetric model (see Plamondon et al., 1993, for a review on the asymmetry of velocity profiles).

2.4. Temporally constrained movements, where MT is given: Focus on temporal variability

Newell et al. (1979) conducted three experiments in which subjects were asked to move, as soon as they were ready, through a target distance, and not to stop directly opposite the target, in a time as close as possible to a target time MT. Newell et al. examined MT_s of 100, 200, 500, 600, and 1,000 msec over distances of 0.75, 2.5, 5, and 15 cm. A controversial finding of their experiment, still without any convincing explanation, was that "the faster one moves, the more accurate the timing of the response" (Newell et al. 1979, p. 50; see Hancock & Newell, 1985, for a synthesis on the space-time approach to the speed/accuracy trade-off). They also found that the velocity effect was independent of the values of movement time, and that for very slow movements there was a reduction in timing accuracy and movement control broke down.

3. Plamondon's kinematic theory

As we can see, the speed/accuracy trade-offs in rapid human movements are far from being completely understood. None of the theoretical explanations proposed to date is able to take into account the major experimental observations in the field under a single scheme. In addition, the majority of the resulting mathematical equations presented in Table 2 produce good data fitting over a limited range. If a mathematical expression has to be considered as a law in a specific domain, it should be general enough to describe all the phenomena occurring in that domain and, ideally, should be supported both theoretically and experimentally.

Over the past six years, a research group led by R. Plamondon has been studying the theoretical and practical interests of modelling the impulse response of neuromuscular systems to describe speed/accuracy trade-offs (Plamondon 1990a; 1990b; 1991a; 1991b; 1992a; 1992b; 1993a; Plamondon et al. 1993). Recently, Plamondon (1993b; 1993c; 1995a; 1995b) has come up with a kinematic theory that seems to fulfill most of the previous requirements.

The major claim of this theory is that speed/accuracy trade-offs are inherent constraints that emerge directly from the delta-lognormal impulse response of the global neuromuscular system involved in a synergy. The intrinsic properties of this impulse response, combined with a simple perceptivo-motor condition that has to be met at some sensorimotor level, are sufficient to ensure the effective production of any rapid movements. In this context, the kinematic relationships that have been reported in the field can be taken into account under a single paradigm.

3.1. The delta-lognormal law

The production of a rapid movement involves the activation of a complex neuromuscular system made up of several

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components organized both hierarchically and in parallel (see Ghez, 1991, for an extensive survey). The primary motor cortex, the premotor cortex, and the supplementary motor area constitute the highest levels of control. These components contain somatotopic maps and receive information from the periphery via sensory relay nuclei. The next level of the hierarchy is the brain stem, which is made up of three neuronal systems (medial, lateral, and aminergic) that modulate motor neurons and interneurons in the spinal cord. This latter component constitutes the next level of the hierarchy. Its motor neurons interact, directly or indirectly, with proximal and distal muscles. In addition, two other components regulate motor functions: the cerebellum and the basal ganglia. At lower levels, various musculoskeletal networks are involved. Indeed, even the simplest movements require the coordination of several skeletal muscles acting in groups rather than individually. Such a group is generally referred to as a synergy (Bernstein 1967). The muscles that cause the desired action are called the agonist muscles; those causing the opposite effect are the antagonist muscles. Any of these muscles acts on bones via tendons by contracting parallel bundles of muscle fibers. For simple rapid movements it is generally accepted that sensory feedback is not continuously used to control the trajectory, but that advance information from sensory events is used as a feedforward control to adjust the properties of the neuromuscular system with respect to the task objective.

According to the kinematic theory of Plamondon (1993b; 1993c; 1995a; 1995b), a schematic view of the complete neuromuscular synergy involved in the production of a rapid-aimed movement can be represented as in Figure 1. This synergy is composed of two parallel systems, each made up of numerous components that represent the sets of neural and muscular networks involved in the generation of the agonist and antagonist neuromuscular activities resulting in a specific movement. Using such a model, it is possible to provide an analytical description of a synergy output, when a few basic hypotheses are made.

First, a representation space must be selected for this output. One of the most well-accepted invariants in rapid-aimed movements is the shape of the absolute velocity profile. Several authors (Abend et al. 1982; Atkeson & Hollerbach 1985; Beggs & Howarth 1972; Georgopoulos et al. 1981; Morasso 1981; Nagasaki 1989; Soechting & Laquaniti 1981; Uno et al. 1989, etc.) have shown that the velocity profiles of rapid-aimed movement have a global "asymmetric bell shape" that is invariant over a wide range of movement sizes and speeds. This invariance suggests that velocity might play a key role in movement control and that it is reasonable to assume that a synergy output can be described in the velocity domain.

Second, since the kinematic theory is concerned with the description of well-learned and well-practiced movements, we assume that for a specific task each subsystem works in a linear mode around some steady conditions. In this context, the agonist and antagonist systems described in Figure 1 can be considered globally as linear time-invariant systems producing a velocity output ($v_1(t)$ or $v_2(t)$) from an impulse command ($U_0(t - t_0)$) of amplitude D_1 or D_2, occurring at t_0. Although proprioceptive feedback as well as various forms of interaction and coupling exist in several places between these two systems, we assume that the global effect of all these mechanisms can be taken into account at the very end of the process by subtracting the two outputs. The resulting velocity of the end-effector of the synergy is thus represented by:

$$v(t) = v_1(t) - v_2(t) \tag{36}$$

$$= D_1 H_1(t - t_0) - D_2 H_2(t - t_0) \tag{37}$$

where subscripts 1 and 2 stand for the agonist and the antagonist systems, respectively, and $H(t - t_0)$ represents the impulse response of each system.

Equation (37) describes the output of the synergy as the difference between the impulse responses of the agonist and antagonist neuromuscular systems, weighted by the respective amplitude of their input activation commands. For each of these systems, their internal architecture, as depicted in Figure 1, is quite complex. Each component interacts serially with its nearest neighbor but also in a hierarchically parallel fashion with a large number of more distant components. If this model could be simplified, an analytical expression could be obtained for the global impulse response of both the agonist and antagonist systems using asymptotic predictions. On the one hand, if emphasis could be put only on the parallel interactions between the components of a system, its impulse response could be derived using the product of the integrals of the impulse response of each component, and under some specific conditions it could converge asymptotically toward different types of exponential functions: double exponential, power of exponential, and Weibull function (Galambos 1978; Leadbetter et al. 1983). For example, using a purely parallel model, Ulrich and Wing (1993) have proposed a specific sum of weighted exponential functions as an impulse response for a force-generating system. On the other hand, if emphasis could be put only on the sequential interactions between components, the impulse response of the system would be the convolution of the impulse responses of each component, and its mathematical description could be specified from the prediction of the central-limit theorem as applied to the convolution of a large number of positive functions (Papoulis 1987). So, if all the neural and muscular networks composing a system were to form an independent sequence of subsystems, the impulse response ($H(t - t_0)$) of the resulting linear system could be described by a Gaussian function (Plamondon 1991a; 1993b; 1995a).

Plamondon's kinematic theory stays in between these two extreme cases, using an argument based on the time delay introduced in a system by the different components that have to react to a specific command. In the purely parallel model, the total time delay characterizing a system, agonist or antagonist, would be limited by the distribution of the maximum time delays of the different components. In the purely sequential system, the total time delay would be defined by the sum of the individual time delays of each component.

With the mixed architecture depicted in Figure 1, the time delay associated with each component taken individually will affect the total time delay of the global system in a more complex way to reflect both the parallel and the sequential coupling between the components. One simple way to link the time delay of each component is to assume that the cumulative time delay (T_j) after j subprocessing steps is related to the cumulative delay time (T_{j-1}) of the previous $j - 1$ components by a law of proportional effect (Gibrat 1931), or, in other words, by a Weber law:

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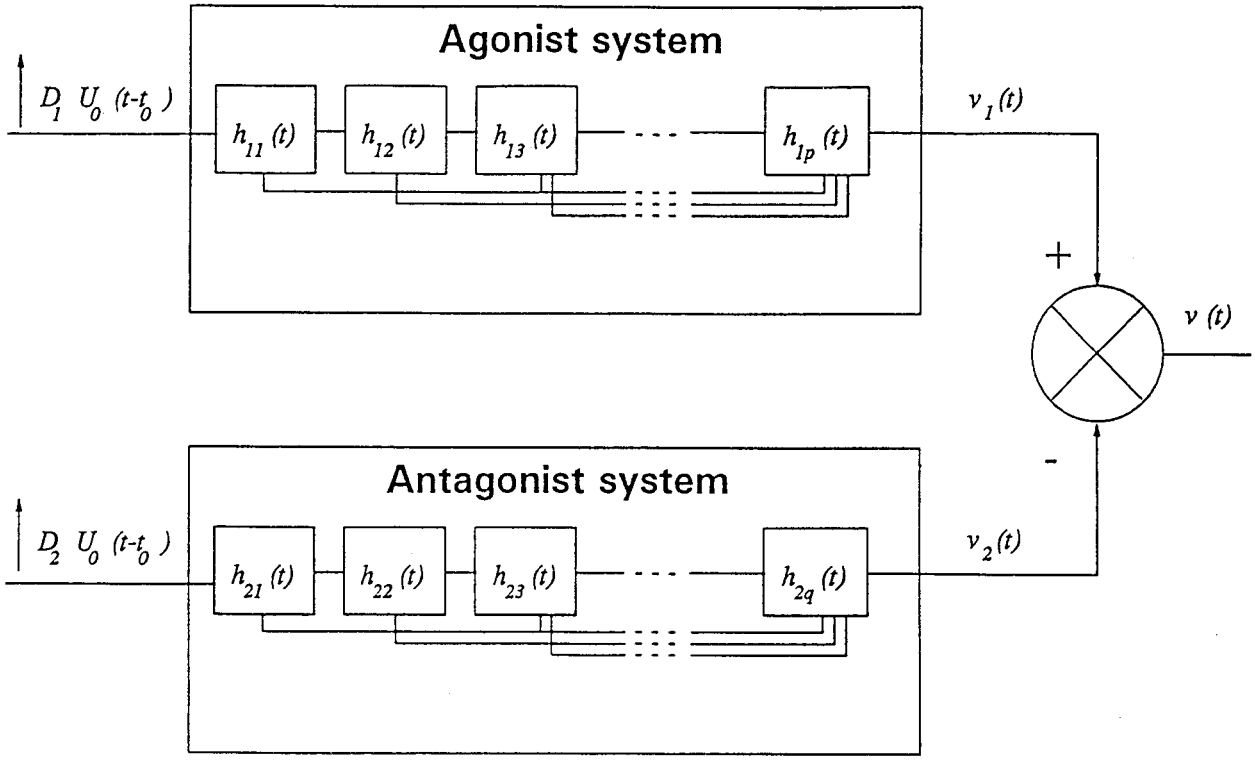

Figure 1. Schematic view of the neuromuscular synergy involved in the production of a rapid-aimed movement.

$$T_j = (1 + \varepsilon_j)T_{j-1} \tag{38}$$

where ε_j is a proportionality factor reflecting the coupling between step j and all the previous steps, each ε_j being independent of the others and independent of T_j.

This hypothesis can be interpreted to be assuming that both the agonist and antagonist systems depicted in Figure 1 can be represented by an equivalent sequential system made up of n coupled components, where the j^{th} is linked to the $j - 1$ previous ones. Then, by making an analogy with the predictions of the central-limit theorem (Papoulis 1987; 1965), it is predicted that under the hypothesis *described* by equation (38), the impulse response of a neuromuscular system will converge toward a lognormal curve (Plamondon 1991a; 1993b; 1993c; 1995a), provided that the individual impulse response $h_{ij}(t)$ of each component meets some very general conditions (real, normalized, non-negative conditions with a finite third moment and scaled dispersion). So, under these conditions, the complete velocity profile of the total synergy will be described by the weighted difference of two lognormals (Plamondon 1993b; 1993c; 1995a):

$$v(t) = D_1\Lambda(t;t_0,\mu_1,\sigma_1^2) - D_2\Lambda(t;t_0,\mu_2,\sigma_2^2) \tag{39}$$

where

$$\Lambda(t;t_0,\mu_i,\sigma_i^2) = H(t - t_0) = \frac{1}{\sigma_i\sqrt{2\pi}(t - t_0)} \exp - \left\{[\ln(t - t_0) - \mu_i]^2 \cdot \frac{1}{2\sigma_i^2}\right\} \tag{40}$$

As will be seen below, equation (39) is very general, and we refer to it as a delta-lognormal or $\Delta\Lambda$ law (Plamondon 1993b; 1995a). Similarly, we refer to μ and σ as the total logtime delay and logresponse time, respectively, since these parameters reflect the total time delay and response time of a lognormal impulse response on a logarithmic time scale.[3]

In other words, the synergetic execution of a rapid movement can be seen as resulting from the simultaneous activation (at $t = t_0$) of an agonist and an antagonist neuromuscular system, with commands of amplitude D_1 and D_2, respectively. Both systems react to their specific commands with an impulse response described asymptotically by a lognormal function, whose parameters μ_1, σ_1 and μ_2, σ_2 characterize the logtime delay and the logresponse time of the agonist and antagonist neuromuscular system involved in the synergy. From this point of view, t_0, D_1, and D_2 can be considered as command parameters and μ_1, μ_2, σ_1, σ_2 as synergetic or system parameters.

We have shown in previous studies (Alimi & Plamondon 1993a,b; 1994; Plamondon et al. 1993) that the $\Delta\Lambda$ law is actually the most powerful equation for reproducing complete velocity profiles of simple movements. Figure 2 shows a few examples of the optimum reconstruction of velocity profiles as obtained in these studies. Similar results were obtained for wrist flexions and extensions on a monkey subject as well as visual saccades, head rotations, and hand movements on a human subject (Plamondon 1995a; Plamondon et al. 1995a; 1995b).

In addition, Plamondon (1993b; 1993c; 1995a) has demonstrated that the $\Delta\Lambda$ law predicts the majority of phenomena consistently reported by many research groups studying these types of velocity profiles. First, the theory predicts that from a single pair of synchronous input commands D_1 and D_2 occurring at t_0, single, double, or triple peak velocity profiles can be generated. The main peak has an asymmet-

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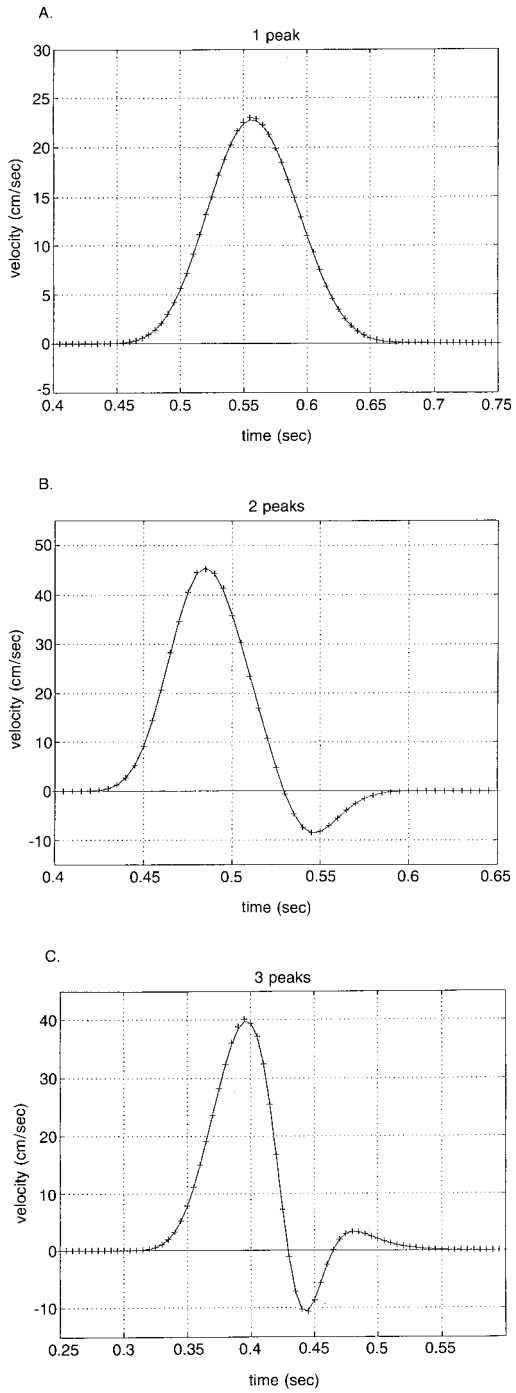

Figure 2. Typical results of an analysis-by-synthesis experiment for three typical velocity profiles. The solid lines represent the curvilinear velocity of a pentip movement as computed from digitizer data, whereas the crosses represent the best fitting $\Delta\Lambda$ law in each case. 2A: $D_1 = 2.33$, $D_2 = 3.36{\cdot}10^{-1}$, $\mu_1 = -1.28$, $\mu_2 = -1.10$, $\sigma_1^2 = 2.13{\cdot}10^{-2}$, $\sigma_2^2 = 9.10{\cdot}10^{-3}$, $t_0 = 2.88{\cdot}10^{-1}$
2B: $D_1 = 2.46$; $D_2 = 0.40$, $\mu_1 = -1.47$, $\mu_- = -1.25$; $\sigma_1^2 = 8.10{\cdot}10^{-3}$, $\sigma_2^2 = 2.5{\cdot}10^{-3}$, $t_0 = 0.25$
2C: $D_1 = 3.88$, $D_2 = 1.74$, $\mu_1 = -1.39$, $\mu_2 = -1.28$, $\sigma_1^2 = 1.96{\cdot}10^{-2}$, $\sigma_2^2 = 3.6{\cdot}10^{-3}$, $t_0 = 0.16$.

ric bell shape; its asymmetry depends upon the velocity and can be inverted at very high speeds. All these phenomena have been regularly and consistently reported (Abend et al. 1982; Atkeson & Hollerbach 1985; Beggs & Howarth 1972; Georgopoulos et al. 1981; Morasso 1981; Nagasaki 1989, Soechting & Laquaniti 1981; Uno et al. 1989, Zelaznik et al. 1986; and so on).

With regards to movements where subjects are asked to produce displacements of different amplitudes with the same duration, the $\Delta\Lambda$ law predicts that the maximum velocity of the dominant peak will increase almost proportionally to the distance covered, that the time to peak velocity will be constant, and that the different velocity profiles will be perfectly superimposable after displacement rescaling. All these facts have been reported in detail by Gielen et al. (1985).

As for movements where subjects are asked to cover a constant distance at different speeds, the $\Delta\Lambda$ law predicts that the maximum velocity of the dominant peak will increase for faster movements and that the time to peak velocity will decrease as the maximum velocity increases. These predictions are consistent with the results of Corcos et al. (1990), Gielen et al. (1985), Lestienne (1979), and Nagasaki (1989). Moreover, the velocity profiles of a specific family of curves will be approximately superimposable after appropriate amplitude and time rescaling, as reported by Corcos et al. (1990), Gielen et al. (1985), and Mustard and Lee (1987).

Finally, for movements where subjects are asked to move as quickly as possible to a target zone, computer simulation using the $\Delta\Lambda$ law predicts that the maximum velocity of the dominant peak will increase with movement time, and that the time to peak velocity will increase with an increase of maximum velocity. These predictions can be verified from the data reported by Brown and Cook (1981), Gielen et al. (1985), Goggin (1989), and Mustard and Lee (1987). It is also predicted that the maximum velocity increases with displacement, as observed by many researchers (Binet & Courtier 1893; Freeman 1914; Hoffman & Stick 1986; Jeannerod 1984; Milner 1986). For a specific distance interval, the latter relationship can be approximated by a straight line with a positive intercept; this is consistent with Wadman et al. (1979) and Milner (1986). Moreover, if the relationship is plotted on a log-log scale, it can also be approximated with a straight line for some specific distance intervals. Such an observation has been reported by Brown and Slater-Hammel (1949). Similar predictions are also made for the relationship between the mean velocity of the dominant peak as a function of the distance covered, in accordance with the linear approximation reported by Brook (1974), Brown and Slater-Hammel (1949), and Freund and Beudingen (1978).

3.2. The quadratic and power laws

One of the most striking predictions of Plamondon's kinematic theory is related to its predictions concerning the practical duration of a movement. Although a single lognormal curve reaches a null value after an infinite time (Plamondon 1991a), the subtraction of two lognormals may result in one or two zero crossings in the velocity profile (Plamondon 1993c; 1995b). The time occurrence (t_1) of these zero crossings can be calculated by canceling out the $\Delta\Lambda$ law. This automatically leads to a quadratic law that links the logarithm of the movement time[4] as defined here by $MT = t_1 - t_0$ and the logarithm of the ratio of the agonist-to-antagonist input commands:

$$a_1(\ln MT)^2 + a_2 \ln MT + a_3 = \ln \frac{D_1}{D_2} \qquad (41)$$

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where

$$a_1 = \frac{\sigma_2^2 - \sigma_1^2}{2\sigma_1^2\sigma_2^2} \quad (42)$$

$$a_2 = \frac{\mu_2\sigma_1^2 - \mu_1\sigma_2^2}{\sigma_1^2\sigma_2^2} \quad (43)$$

$$a_3 = \frac{\sigma_2^2\mu_1^2 - \sigma_1^2\mu_2^2 + 2\sigma_1^2\sigma_2^2 \ln\frac{\sigma_1}{\sigma_2}}{2\sigma_1^2\sigma_2^2} \quad (44)$$

So, depending on the general aspect of the velocity curve, zero, one, or two values will be observed for MT,[5] and these will be linked to the ratio D_1/D_2. In other words, the velocity profile will encompass one, two, or three peaks. These different cases can be analytically described under various parameter conditions (Plamondon 1993c; 1995b). For example, if $\sigma_1^2 = \sigma_2^2 = \sigma^2$ then equation (41) reduces to a linear equation with respect to $\ln MT$, and there is a single zero crossing in the velocity profile in this case, with a time occurrence defined by a power law:

$$MT = t_1 - t_0 = \left[\exp\left(\frac{\mu_1 + \mu_2}{2}\right)\right]\left(\frac{D_1}{D_2}\right)^{\frac{\sigma^2}{\mu_2 - \mu_1}} \quad (45)$$

$$= K\left(\frac{D_1}{D_2}\right)^{\alpha} \quad (46)$$

As previously said, the system parameters (μ_i, σ_i) reflect the global timing properties of the different neural and muscular networks recruited for the production of a specific movement of an end-effector. Since the limb inertial and viscoelastic properties are fairly constant in adults, it is conceivable that for a specific type of movement, the μ_i and σ_i will remain relatively constant or highly centered around their mean values from one movement to another. Therefore, for a set of synergetic movements of an end-effector characterized either by fixed values of μ_1, μ_2, σ_1, σ_2, or by the fact that μ_1, μ_2, σ_1, and σ_2 covary in such a way that a_i in equation (41) or K and α in equation (46) are constants, the logarithm of the movement time will be linked to the logarithm of the amplitude ratio of the input commands by a quadratic law in general, or by a linear equation (a power law in MT) in some specific cases (Plamondon 1993c; 1995b).

These latter conclusions highlight an efficient strategy that can be used by humans to control movement amplitude and movement time at the command level. Indeed, the movement amplitude (MA) – associated with the dominant pulse in the case of one velocity zero crossing or with the first two pulses when two zero crossings are observed in the velocity profile – is obtained by integrating the delta-lognormal law over movement time:[6]

$$MA = \int_{t_0}^{t_1} v(t)dt \simeq D_1 - D_2 \quad (47)$$

In addition, as can be seen from equations (41) and (45), the movement time can be controlled either at the input level, at the system level, or by both mechanisms together. The input control of the movement time is reflected by the ratio D_1/D_2 of the amplitudes of the agonist and antagonist input commands. In other words, for one set of experimental conditions, if it can be assumed that the system parameters μ_1, μ_2, σ_1, and σ_2 are fixed or covary in such a way that a_i in equation (41) or K and α in equation (46) are constant, then the quadratic law directly predicts the duration of the dominant peak of the velocity profile (as well as the duration of the second pulse when the profile has two zero crossings).

From this input level perspective, the theory explains how easy it can be to generate either one set of movements with different amplitudes and the same duration, or movements with the same amplitude but of different durations. In other words, if a subject is instructed to generate movements of different amplitudes within the same movement time, he should use a pair of input commands having a constant ratio (D_1/D_2 = constant) while changing $D_1 - D_2$ according to the required movement amplitude. If the goal is to produce movements with the same amplitude but of different duration, the subject must use a pair of input commands having a constant difference ($D_1 - D_2$ = constant) while modifying the ratio D_1/D_2 according to the required movement time.

3.3. Speed/accuracy trade-offs

In a Fitts' task, a subject has to move a pointer to a target positioned at a distance D from an origin and reach the target within an absolute spatial error of $\pm\Delta$D as quickly as possible. The first condition to be met by the subject can be expressed as putting a limit γ_1 on the maximum relative spatial error:

$$\left(\frac{\Delta D}{D}\right)_{\max} = \gamma_1 \quad (48)$$

where γ_1 is a constant for a specific task. The second condition is described by equation (41):

$$a_1(\ln MT_{\min})^2 + a_2\ln MT_{\min} + a_3 = \ln\left(\frac{D_1}{D_2}\right) \quad (49)$$

where it is assumed that the a_i are constants for an experiment where the same neuromuscular synergy is involved. The second constraint can thus be interpreted as putting a limit γ_2 on the maximum ratio of the antagonist to agonist input commands:

$$\left(\frac{D_2}{D_1}\right)_{\max} = \exp -[a_1(\ln MT_{\min})^2 + a_2\ln MT_{\min} + a_3] = \gamma_2 \quad (50)$$

To meet both conditions simultaneously, a subject has to map the spatial constraints with the command constraint. In this case, equations (48) and (50) are combined to obtain the general expression of the constraints of a Fitts task on the ratio of antagonist to agonist input commands:

$$\left(\frac{D_2}{D_1}\right)_{\max} = \frac{\gamma_2}{\gamma_1}\left(\frac{\Delta D}{D}\right)_{\max} \quad (51)$$

that is, to succeed in a Fitts task the ratio D_2/D_1 must be proportional to the relative spatial error required by the experimental protocol. In this context, substituting equation (51) with (49), the kinematic theory predicts that, in general, a quadratic law will be observed between the logarithm of the movement time and the logarithm of the inverse of the relative spatial error:

$$a_1(\ln MT_{\min})^2 + a_2\ln MT_{\min} + a_4 = \ln\left(\frac{D}{\Delta D}\right) \quad (52)$$

$$\text{where } a_4 = a_3 + \ln(\gamma_2/\gamma_1) \quad (53)$$

This quadratic law also reduces to a power law when $a_1 = 0$.

$$MT_{\min} = K_1\left(\frac{D}{\Delta D}\right)^{\alpha} \quad (54)$$

$$\text{where } K_1 = K\left(\frac{\gamma_1}{\gamma_2}\right)^{\alpha} \quad (55)$$

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In Figure 3, we have reproduced typical velocity profiles that are predicted by the kinematic theory (Plamondon 1993c; 1995b) as a function of different values of $D/\Delta D$. Figure 3A shows some profiles corresponding to the quadratic law (equation 52), whereas 3B depicts some examples of the power law (equation 54). These curves are similar to those reported by Corcos et al. (1988), Goggin (1989), and Soechting (1984). The maximum velocity decreases as $D/\Delta D$ increases. In addition, it is predicted that the asymmetry of the profile increases as the accuracy demand becomes greater, that is, as $D/\Delta D$ increases. This is consistent with the results of Corcos et al. (1988), Jeannerod (1984), MacKenzie et al. (1987), Marteniuk et al. (1987), Milner and Ijaz (1990), and Soechting (1984). Figures 3c and 3d also show that the profiles cannot be perfectly rescaled but their rising phase can be made almost similar after appropriate amplitude rescaling, as many others have reported.

The kinematic theory predicts that under experimental conditions, where the system parameters are held constant or covary in such a way that a_i in equation (52) or K and α in equation (54) are constants, an efficient strategy for producing, for example, a specific reaching movement would be to evaluate visually the distance D to be covered by the end-effector and the relative spatial error $\Delta D/D$ sufficient to accomplish the task. By coupling some sensory motor maps in such a way that $D = D_1 - D_2$ and $D_2/D_1 \propto \Delta D/D$, the target will be reached in a movement time predicted by equations (52) and (54) for each specific case.

Plamondon's (1991a; 1992a; 1993a; 1993b; 1993c; 1995a; 1995b) fundamental claim is that these relationships hold even when the D or ΔD information is not directly specified, in which case the subject performs some visual estimate and uses some kind of virtual or default values for the missing information. In this perspective, the same equation – (52) or (54) – can be used to analyze any single rapid movements made under different spatiotemporal conditions. In the Schmidt et al. (1979) protocol, for example, subjects have to reach a target at distance D within a certain movement time. The information about ΔD is not specified. If we assume that subjects are roughly estimating ΔD to succeed in this experiment, a strategy similar to the one described above can be used and the predictions of the kinematic theory (Plamondon 1993c; 1995b) can thus be found for these conditions, if equation (52) is rewritten as:

$$\Delta D = D(\exp - [a_1(\ln MT)^2 + a_2 \ln MT + a_4]) = \xi D \qquad (56)$$

Because MT is fixed here, this equation predicts a proportional relationship between ΔD and D for experiments requiring subjects to perform aiming tasks with the same neuromuscular synergy (μ_i, σ_i^2 constant) within a certain specified movement time (MT). In addition, in this context equation (56) predicts that MT will play the role of a scaling factor, decreasing the slope of the proportionality relationship for a longer movement time.

Similarly, for an experimental protocol such as the one used by Howarth et al. (1971), where subjects are asked to move from a home position to a constant distance (D) while measuring the error (ΔD) for different movement times (MT), the kinematic theory predicts a quadratic relationship between $\ln\Delta D$ and $\ln MT$ in the general case:

$$\ln\Delta D = \ln D - a_1(\ln MT)^2 - a_2 \ln MT - a_4 \qquad (57)$$

and a linear relationship between them for the specific case where $a_1 = 0$ (power law):

$$\ln\Delta D = \ln D - a_2 \ln MT - a_4 \qquad (58)$$

Since D, ΔD, and MT are fixed, it can be seen that to succeed in such a protocol the subjects will have to adapt their μ_i and σ_i to meet the constraint described by equation (57) or (58).

Finally, for experiments dealing with time accuracy, such as those reported by Newell et al. (1979), where subjects are required to reach a target of fixed width (which is equivalent to assuming ΔD = constant) under different distance and movement time conditions, if the subject uses a strategy governed by equation (52), the predictions of the kinematic theory (Plamondon 1993c, 1995b) concerning timing precision will be found under these conditions by estimating the movement time errors ΔMT. Differentiating equation (52) and assimilating the absolute errors to the differentials (Topping 1972), we obtain:

$$\Delta MT = \left(\frac{MT}{2a_1 \ln MT + a_2}\right)\frac{\Delta D}{D} = \frac{\Delta \mathrm{D}}{(2a_1 \ln MT + a_2)\overline{V}} \qquad (59)$$

where $\overline{V}$ = *mean velocity*

So, for experiments requiring the same absolute spatial accuracy ΔD at different movement times, the absolute timing error will be related to the movement time by a nonlinear relationship. Moreover, the distance (D) will act as a scaling factor under these conditions. For the specific case where $\sigma_1^2 = \sigma_2^2$ ($a_1 = 0$, power law), equation (59) reduces to a proportionality relationship:

$$\Delta MT = \frac{MT}{a_2}\frac{\Delta D}{D} = \frac{\Delta D}{a_2 \overline{V}} \qquad (60)$$

This summarizes the major list of predictions made by Plamondon (1993b; 1993c; 1995a; 1995b) concerning the kinematic theory. In the next section we report some validation results based on the data available in the numerous papers published on this topic.

4. Experimental results

Among the numerous studies dealing with speed/accuracy trade-offs, many provide tables of numerical data that can be used directly or indirectly to test the predictions of the kinematic theory concerning speed/accuracy trade-offs. We have grouped the tests under the four general headings previously used in section 2.

4.1. Movements with spatial constraints, where both D *and* ΔD *are given*

4.1.1. The data and the protocol. To test the validity of the relationships between movement time and the inverse of the relative spatial error, we have made an extensive survey of the studies dealing with Fitts' task. For any study where sufficient numerical data were available[7] we have run a regression analysis using three equations:

$$a_1(\ln MT)^2 + a_2 \ln MT = \ln\left(\frac{D}{\Delta D}\right) - a_4 \qquad (61)$$

$$\ln MT = b_1 + b_2 \ln \frac{D}{\Delta D} \qquad (62)$$

$$\text{and } MT = c_1 + c_2 \ln \frac{D}{\Delta D} \qquad (63)$$

The first two equations are the quadratic and power laws previously proposed by Plamondon (1993c; 1995b); the

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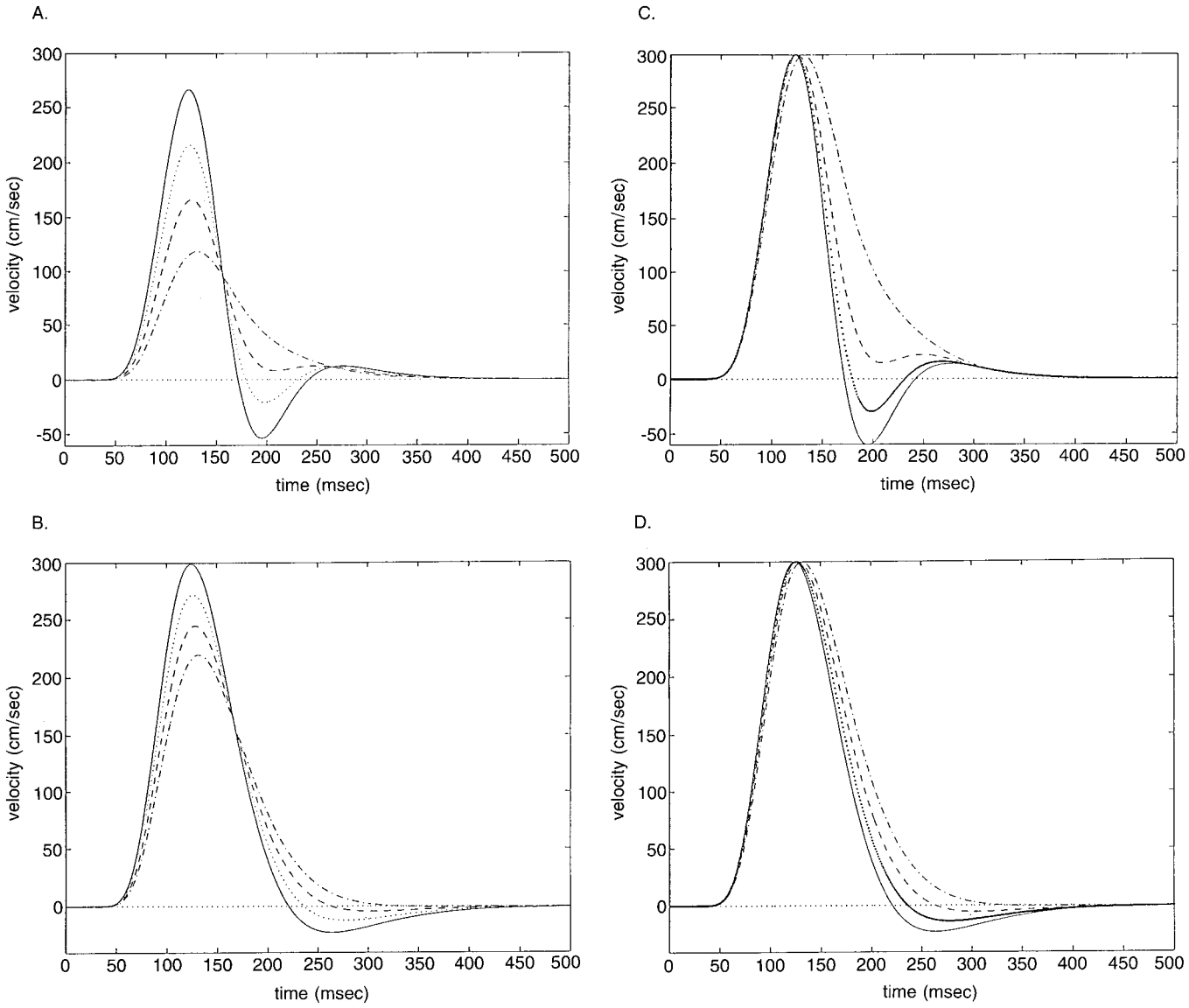

Figure 3. Properties of velocity profiles for movements with different accuracy demands. 3A: Typical velocity profiles corresponding to the quadratic law. ——— $\Delta D/D = 5.71 \cdot 10^{-1}$; .... $\Delta D/D = 5.00 \cdot 10^{-1}$; ---- $\Delta D/D = 4.00 \cdot 10^{-1}$; -·-· $\Delta D/D = 2.50 \cdot 10^{-1}$. 3B: Typical velocity profiles corresponding to the power law. ——— $\Delta D/D = 5.77 \cdot 10^{-1}$; .... $\Delta D/D = 5.05 \cdot 10^{-1}$; ---- $\Delta D/D = 3.85 \cdot 10^{-1}$; -·-· $\Delta D/D = 1.44 \cdot 10^{-1}$. (C) and (D) Approximate superimposition of the profiles of 3A and 3B, respectively, after amplitude rescaling.

third is the well-known equation generally referred to as Fitts' law (Fitts 1954), rewritten by us, using our own notation for the relative spatial error ($2A/W = D/\Delta D$) as well as using a natural logarithm instead of the base 2 logarithm. We found 11 studies, which provided 50 datasets. Their protocols are summarized in the Appendix and some of their basic features can be found in Table 3.

4.1.2. Results. Table 3 reports the results of our regression analysis over the 11 studies that were found with sufficient numerical data. For each study, the value of the fitting correlation coefficients for the quadratic law (R_Q^2), the power law (R_P^2), and Fitts' law (R_F^2) are listed under their specific headings.

As can be seen from these values, the quadratic law is the most powerful in predicting the data. It is either better than (84% of the cases) or equivalent to (14% of the cases) Fitts' law. The power law outperforms Fitts' law in about 78% of the cases. These results are in accordance with the facts that a power law can be considered as a sufficient approximationof the quadratic law in many cases (Plamondon 1993a; 1993b) and that a logarithmic law can be seen as the first-order approximation of a power law:

$$MT = K \left(\frac{D}{\Delta D} \right)^{\alpha} \tag{64}$$

$$= K \exp \left[\ln \left(\frac{D}{\Delta D} \right)^{\alpha} \right] \tag{65}$$

$$= K \left\{ 1 + \ln \left(\frac{D}{\Delta D} \right)^{\alpha} + \frac{1}{2!} \left[\ln \left(\frac{D}{\Delta D} \right)^{\alpha} \right]^2 + \frac{1}{3!} \left[\ln \left(\frac{D}{\Delta D} \right)^{\alpha} \right]^3 + \ldots \right\} \tag{66}$$

$$\simeq K + \alpha K \ln \frac{D}{\Delta D} \tag{67}$$

To illustrate better the differences in the prediction capacity of equations (61) and (62) over (63), we have

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Table 3. *Results of a regression analysis: The predictive power of a quadratic and a power law vs. Fitts' law*

Study	Features	Remarks	Quadratic law R_Q^2	Power law R_p^2	Fitts' law R_F^2
Andres & Hartung (1989a).	• 9 male subjects • A = 7.6, 15.2, 30.5 cm • W = 1.3, 2.5, 3.8 cm • 18 datasets	Session 1, subject 1	0.96	0.92	0.95
		Session 1, subject 2	0.95	0.91	0.83
		Session 1, subject 3	0.92	0.92	0.91
		Session 1, subject 4	0.86	0.86	0.84
		Session 1, subject 5	0.84	0.83	0.84
		Session 1, subject 6	0.89	0.89	0.86
		Session 1, subject 7	0.84	0.84	0.83
		Session 1, subject 8	0.81	0.81	0.80
		Session 1, subject 9	0.88	0.88	0.87
		Session 2, subject 1	0.89	0.89	0.87
		Session 2, subject 2	0.96	0.96	0.93
		Session 2, subject 3	0.88	0.88	0.87
		Session 2, subject 4	0.89	0.89	0.87
		Session 2, subject 5	0.89	0.87	0.81
		Session 2, subject 6	0.99	0.97	0.89
		Session 2, subject 7	0.78	0.78	0.77
		Session 2, subject 8	0.90	0.89	0.89
		Session 2, subject 9	0.83	0.83	0.81
Drury (1975)	• 10 male subjects • A = 150, 225, 300, 375, 525, 675 mm • W = 25, 50 mm • 1 dataset		0.98	0.95	0.92
Drury & Hoffmann (1992)	• 10 male subjects • A was kept constant at 160 mm • B = 2, 6, 10, 14, 18 mm • P = 0, 5, 10, 15 mm • $W = B + 0.6\,P$ in probe condition • $W = B + 10$ in finger condition • 2 datasets	Probe	0.96	0.96	0.96
		Finger	0.98	0.97	0.98
Fitts (1954)	• 16 male subjects participated in the first 3 experiments and 20 (10 men, 10 women) in the last one • Tapping task: (A = 2, 4, 8, 16 in; W = 0.25, 0.5, 1.0, 2.0 in) • Disk-transfer task: (A = 4, 8, 16, 32 in; W = 0.0625, 0.125, 0.25, 0.5 in) • Pin-transfer task: (A = 1, 2, 4, 8, 16 in; W = 0.03125, 0.0625, 0.125, 0.25 in) • 4 datasets	Tapping 1-oz stylus	0.99	0.99	0.96
		Tapping 1-lb stylus	0.99	0.99	0.96
		Disk-transfer task	0.87	0.87	0.84
		Pin-transfer task	0.91	0.91	0.89
Gan & Hoffmann (1988)	• 6 male and 6 female subjects • A = 4, 9, 16, or 25 cm • $ID = log\,(2A/W)$ of 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 6.0 bits • 4 datasets	A = 4 cm	0.98	0.97	0.93
		A = 9 cm	0.94	0.89	0.84
		A = 16 cm	0.92	0.84	0.79
		A = 25 cm	0.95	0.81	0.75
Hoffmann & Sheikh (1991)	• 5 males and 5 females • A = 100, 200, 400 mm • W = 2, 6, 10, 14, 18 mm • 2 datasets	Sharp probe	0.96	0.95	0.96
		Finger	0.97	0.97	0.97
Johnsgard (1994)	• 18 subjects • A = 2, 4, 8 in; W = 0.5, 1, 2 in • G = 1, 2, 3 • 6 datasets	Mouse Gain = 1	1.00	0.99	0.98
		Mouse Gain = 2	0.99	0.99	0.97
		Mouse Gain = 3	0.99	0.99	0.98
		Glove Gain = 1	1.00	0.99	0.99
		Glove Gain = 2	0.99	0.96	0.99
		Glove Gain = 3	1.00	1.00	0.99

(*continued*)

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Table 3. (*Continued*)

Study	Features	Remarks	Quadratic law R^2_Q	Power law R^2_P	Fitts' law R^2_F
Kerr & Langolf (1977)	• 8 male subjects • A = 8, 12, 16, 20 in • W = 0.25, 0.5, 1.0, 2.0 in • 9 datasets	Average (8 subjects)	0.94	0.94	0.93
		Subject #1	0.96	0.95	0.93
		Subject #2	0.84	0.84	0.81
		Subject #3	0.84	0.83	0.83
		Subject #4	0.43	0.43	0.42
		Subject #5	0.80	0.80	0.76
		Subject #6	0.90	0.88	0.83
		Subject #7	0.93	0.92	0.91
		Subject #8	0.81	0.79	0.81
MacKenzie et al. (1987)	• 6 subjects • W = 2.54, 1.27, 0.64, 0.32 cm • A = 7.62, 15.24, 30.48 cm • 1 dataset		0.98	0.97	0.96
Newman & Bussolari (1990)	• 19 subjects • Fitts *ID* in the 1.5-3.7 bits range • 2 datasets	Battery 2, no lag	0.99	0.98	0.99
		Battery 2, lag	1.00	0.99	0.97
Repperger & Remis (1990)	• 5 subjects • Fitts *ID* in the 6.35-11.73 bits range • 1 dataset		0.99	0.99	0.97

plotted on a log-log scale, in Figure 4, a few examples of the data reported by some authors of these studies, describing the predictions of the quadratic law, the power law, and Fitts' law. Even with a small (3%) difference in R^2_F, the predictions of Fitts' law can lead to large errors, particularly in curve extremities.

One might argue that part of the improvement gained by the quadratic law could be the result of its encompassing three parameters instead of two. However, this argument is not valid when we compare the performances of the power law and Fitts' law, where the number of parameters is the same in both cases. Figure 5 summarizes the results of this comparison. It shows that on a mean basis, the quadratic law gives a 2% increase in R^2_Q as compared with the R^2_P of the power law, which is itself 2% higher than the R^2_F of Fitts' law. A similar analysis using the equation proposed by MacKenzie (1989; 1992) gives results that are slightly better than Fitts' predictions (1% increase), but generally not as good as those of the power law or the quadratic law (1% and 3% worse, respectively).

In addition, it should be remembered that a larger number of parameters is not an automatic guarantee of a better fit (Alimi & Plamondon 1993a; Plamondon et al. 1993). For example, a regression analysis on the previous 50 datasets was executed using eight equations: quadratic law (3 parameters), power law (2 parameters), Welford's equation (1 parameter), Fitts' law (2 parameters), Welford et al.'s equation (3 parameters), Jagacinski et al.'s equation (3 parameters), MacKenzie's equation (2 parameters), and Meyer et al.'s equation (2 parameters). The results of this analysis (Alimi & Plamondon 1995) show that the quadratic law is generally the best among the three-parameters equations and that the three-parameters equations of Welford et al.'s and of Jagacinski et al.'s often perform less well than that of Meyer et al.'s or than the power law equation, both of which have only two parameters. Thus, a larger number of parameters with a less descriptive equation does not systematically improve the performances of a model in explaining real data.

When sufficient numerical data were available, we ran a regression analysis using the same three equations as above, but with the data grouped by amplitude values. The results of this regression analysis are reported in Table 4. A similar pattern is observed here that reflects the superiority of the quadratic law and the power law over Fitts' law.

In 4 out of 11 cases in Table 3, and in 2 out of 5 cases in Table 4, sufficient data were available, and a statistical analysis that was conducted shows that these differences are significant:

- In Andres and Hartung's (1989a) study (see Table 1), $R^2_Q > R^2_P$ ($T(17) = 2.6025$, $Prob > /T/ = 0.0186$), and $R^2_P > R^2_F$ ($T(17) = 2.3108$, $Prob > /T/ = 0.0337$).
- In Fitts' (1954) study (see Table 1), $R^2_P > R^2_F$ ($T(3) = 11.0000$, $Prob > /T/ = 0.0016$), but R^2_Q *and* R^2_P were not significantly different.
- In Gan and Hoffmann's (1988) study (see Table 1), $R^2_P > R^2_F$ ($T(3) = 12.2574$, $Prob > /T/ = 0.0012$), but R^2_Q and R^2_P were not significantly different.
- In Kerr and Langolf's (1977) study (see Table 1), R^2_Q R^2_P ($T(8) = 2.8000$, $Prob > /T/ = 0.0232$), and $R^2_P > R^2_F$ ($T(8) = 2.3570$, $Prob > /T/ = 0.0462$).
- In Fitts' (1954) study (see Table 2), $R^2_Q > R^2_P$ ($T(16) = 4.2885$, $Prob > /T/ = 0.0006$), and $R^2_P > R^2_F$ ($T(16) = 3.8454$, $Prob > /T/ = 0.0014$).
- In Kerr and Langolf's (1977) study (see Table 2), $R^2_Q > R^2_P$ ($T(35) = 2.5966$, $Prob$ $/T/ = 0.0137$), and $R^2_P > R^2_F$ ($T(35) = 4.2642$, $Prob > /T/ = 0.0001$).

Table 4 also highlights another phenomenon. As can be seen from the R^2_Q values, a better data fit is obtained when data are grouped under the same movement amplitudes than when they are grouped for all experimental conditions. In the context of the kinematic theory (Plamondon 1995a; 1995b), this suggests that, depending on the distance to be covered, a subject might use a slightly

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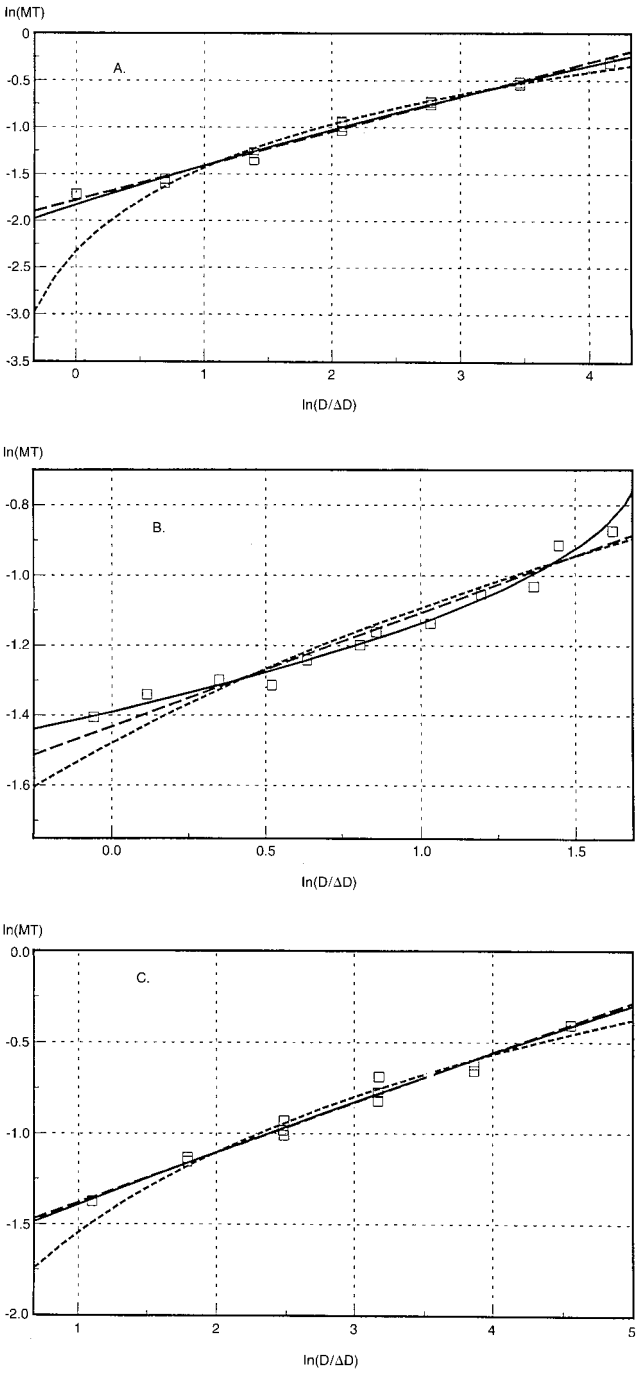

Figure 4. Fitting of data from typical experiments with a quadratic law (solid line), a power law (truncated line), and Fitts' law (dotted line). (A) Data from the Fitts (1954) experiment: Tapping 1 oz. stylus. (B) Data from the Drury (1975) experiment. (C) Data from the Mackenzie et al. (1987) experiment.

different neuromuscular synergy, and therefore slightly different μ_1, μ_2, σ_1, and σ_2 parameters from one distance to another. The data collected for each amplitude condition will obey a specific quadratic (or power) law and the grouped data will be a mixture of the slightly different laws due to slightly different parameters. From an external point of view, this could be misinterpreted as different relative contributions of ΔD and D in these conditions, as has been suggested by Sheridan (1979) and Welford et al. (1969).

4.2. Movements with spatial constraints where only D is given

The predictions of the kinematic theory (Plamondon 1993c; 1995b) are summarized in equations (57) and (58) for these conditions. Howarth et al. (1971) have reported such an experiment, where subjects were required to move from a home position to a constant distance target of varying size (ΔD) at different movement times (MT).[8] These authors could not predict their results from Fitts' law. Figure 6 presents their results, as plotted in a graph of lnΔD versus lnMT. The solid line shows the quadratic law predictions (equation 57), and the dotted line the power law approximation (equation 58). As can be seen, an almost perfect fit is obtained in the former case ($R_Q^2 = 99\%$) and a very good approximation is still reached in the second ($R_P^2 = 97\%$). In other words, the Howarth et al. experiment just reflects another aspect of the quadratic law.

4.3. Movements with temporal constraints where MT is given: Focus on spatial variability

The predictions of the kinematic theory in these conditions are summarized in equation (56) (Plamondon 1993c; 1995b). However, most of the papers dealing with this type of movement study the relationship between the movement distance and the standard deviation of the spatial error along or perpendicular to the overall direction of movement, not the absolute error (Schmidt et al. 1979; Wright 1983; Zelaznik et al. 1981; 1988). Two studies (Abrams et al. 1989; Wright & Meyer 1983) report both the standard deviation (S_D), also referred to as the variable error (VE), and the constant error (CE), from which the absolute error can be estimated under the assumption of a Gaussian distribution of errors (Schutz & Roy 1973):

$$E\{\Delta D\} = |CE|(2A_y) + 0.798\sqrt{VE} \cdot \left[\exp - \left(\frac{1}{2}\frac{CE^2}{VE}\right)\right] \quad (68)$$

where A_y represents the area between CE and the desired target in a normal distribution. Substituting equation (68) for (56), a strong correlation is predicted by the kinematic theory, between $E\{\Delta D\}$ and D, if the Gaussian distribution

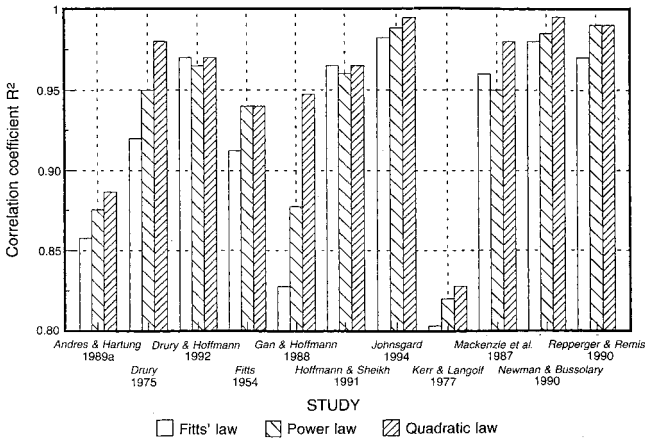

Figure 5. Summary of the comparison analysis between the quadratic, power and Fitts' laws for the 11 studies reported in the text.

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Table 4. *Results of a regression analysis: The predictive power of a quadratic and a power law vs. Fitts' law, by movements of similar amplitude*

Study	Remarks		Quadratic law R_Q^2	Power law R_P^2	Fitts' law R_F^2
Fitts (1954)	Tapping 1-oz stylus	A = 2	1.00	0.98	0.94
		A = 4	1.00	1.00	0.98
		A = 8	1.00	1.00	0.99
		A = 16	1.00	1.00	0.99
	Tapping 1-lb stylus	A = 2	1.00	0.97	0.93
		A = 4	1.00	0.99	0.96
		A = 8	1.00	1.00	0.98
		A = 16	1.00	1.00	1.00
	Disk-transfer task	A = 4	1.00	0.97	0.99
		A = 8	0.99	0.99	0.99
		A = 16	0.99	0.99	0.99
		A = 32	1.00	0.99	1.00
	Pin-transfer task	A = 1	1.00	0.98	0.94
		A = 2	1.00	0.98	0.95
		A = 4	1.00	0.98	0.96
		A = 8	1.00	0.98	0.95
		A = 16	1.00	0.98	0.95
Hoffmann & Sheikh (1991)	Sharp probe	A = 100	0.94	0.93	0.94
		A = 200	1.00	0.97	0.99
		A = 400	0.95	0.95	0.95
	Finger	A = 100	0.96	0.92	0.93
		A = 200	0.97	0.97	0.97
		A = 400	0.97	0.91	0.92
Kerr & Langolf (1977)	Average (8 subjects)	A = 8	1.00	0.98	0.94
		A = 12	1.00	1.00	0.98
		A = 16	1.00	0.94	0.90
		A = 20	1.00	1.00	0.98
	Subject #1	A = 8	1.00	1.00	0.96
		A = 12	0.99	0.98	0.92
		A = 16	0.97	0.97	0.96
		A = 20	1.00	0.99	0.96
	Subject #2	A = 8	0.96	0.95	0.93
		A = 12	1.00	1.00	1.00
		A = 16	0.84	0.80	0.82
		A = 20	0.99	0.97	0.99
	Subject #3	A = 8	0.96	0.94	0.90
		A = 12	0.99	0.96	0.90
		A = 16	0.69	0.55	0.63
		A = 20	0.99	0.98	0.97
	Subject #4	A = 8	0.86	0.86	0.84
		A = 12	0.92	0.23	0.21
		A = 16	0.98	0.95	0.95
		A = 20	0.99	0.69	0.70
	Subject #5	A = 8	0.83	0.83	0.81
		A = 12	1.00	1.00	0.99
		A = 16	0.90	0.83	0.79
		A = 20	0.98	0.97	0.85
	Subject #6	A = 8	0.97	0.96	0.94
		A = 12	0.94	0.90	0.86
		A = 16	0.89	0.76	0.72
		A = 20	0.95	0.90	0.86
	Subject #7	A = 8	0.97	0.91	0.85
		A = 12	1.00	1.00	0.96
		A = 16	1.00	0.97	0.91
		A = 20	1.00	1.00	0.98

(*continued*)

Table 4. (*Continued*)

Study	Remarks		Quadratic law R^2_Q	Power law R^2_P	Fitts' law R^2_F
	Subject #8	A = 8	1.00	0.99	0.99
		A = 12	0.88	0.79	0.74
		A = 16	0.97	0.94	0.97
		A = 20	0.95	0.95	0.93
Mackenzie et al.		A = 7.62	1.00	0.99	1.00
(1987)		A = 15.24	0.98	0.98	0.98
		A = 30.48	0.96	0.96	0.96

hypothesis (Schutz & Roy 1973) holds in these experiments. This prediction is confirmed by both studies (Abrams et al. 1989; Wright & Meyer 1983). The Wright and Meyer data are also consistent with the scaling effect of *MT*, that is, $E\{\Delta D\}$ decreases as *MT* increases. Figure 7 highlights these predictions by plotting the best linear regression between $E\{\Delta D\}$ and *D* over the different sets of data reported by Wright and Meyer.

4.4. Movements with temporal constraints where MT is given: Focus on temporal variability

The predictions of the kinematic theory can be found under these conditions by equations (59) and (60). The study performed by Newell et al. (1979) provides a dataset that includes the absolute error ΔMT as a function of *MT* and for two distance values. Both a nonlinear regression based on equation (59) and a linear regression based on equation (60) can be used to describe these data, as can be seen from Figure 8. The nonlinear relationship provides a better fit, although equation (60) gives consistent predictions. The scaling effect of the movement amplitude is also clearly apparent in this graph. Similar predictions can be derived for $\Delta t_{v_{max}}$ as a function of $t_{v_{max}}$ in the same context. So, if the target is reached at zero speed or at maximum velocity, a proportionality relationship is predicted between the time to contact and its absolute error.

4.5. Other experimental protocols

The kinematic theory can also explain why consistent and simple patterns do not always emerge under some specific experimental conditions. For example, in the second experiment by Zelaznik et al. (1988), subjects are required to land in a spatial target zone and within a temporal bandwidth, in other words, *D*, ΔD, and ΔMT are fixed in this experiment. This protocol forces the subjects to change their synergetic parameters μ_1, μ_2, σ_1, and σ_2 from one condition to another, and the data collected reflect a behavior still obeying the quadratic law, but where nonconstant a_i are used from one task to another. In addition, since too many constraints are put on the system and since μ_1, μ_2, σ_1, and σ_2 certainly have upper and lower bounds, it is predicted that some task requirements will not be met by some subjects. This is clearly apparent in the data reported by Zelaznik et al., particularly for large and precise movements to be executed very fast.

Similar reasoning can be applied to explain the nonlinear speed/accuracy functions recently reported by Newell et al. (1993) for both spatial and temporal errors. The different tasks cannot be executed under a single set of μ_1, μ_2, σ_1, and σ_2, and a mixture of data from different quadratic laws are collected, depending on the experimental groupings. According to the kinematic theory, a complete analysis of these data would first require an analysis of each individual

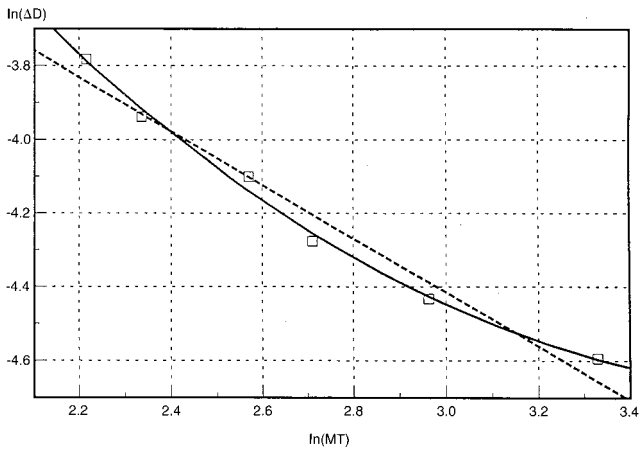

Figure 6. Fitting of the data from Howarth et al. (1971) with a quadratic law (solid line, $R^2_Q = 0.99$) and a power law (truncated line, $R^2_P = 0.97$).

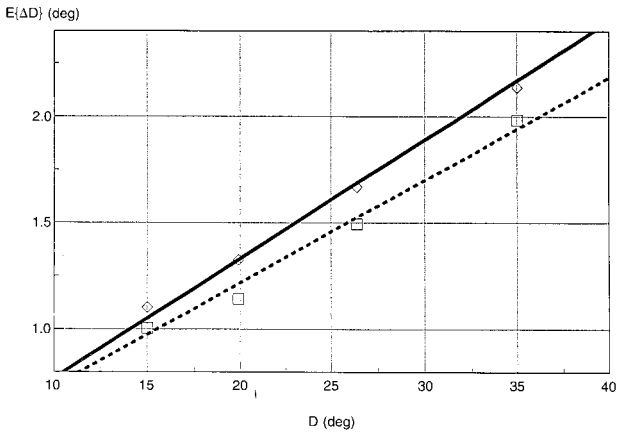

Figure 7. Best linear regression between $E\{\Delta D\}$ (equation 68) and *D* resulting from the proportionality between ΔD and *D* as predicted by equation (56) for movements of the same duration. Data from Wright and Meyer (1983, Tables 2 and 4). Solid line: *MT* = 239 msec, $R^2 = 0.97$. Dotted line: *MT* = 317 msec, $R^2 = 0.93$.

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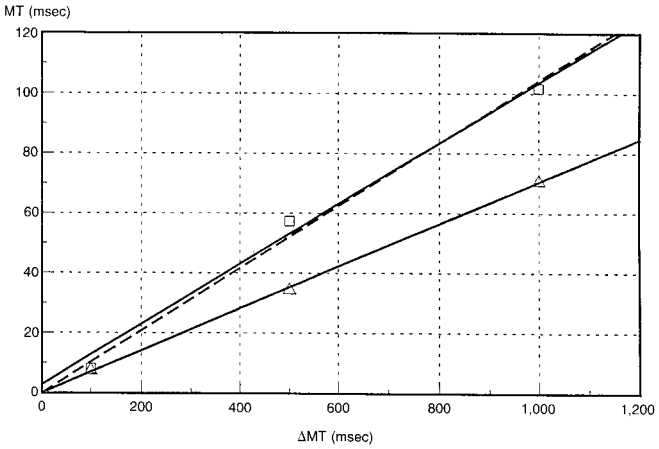

Figure 8. Best nonlinear regression (equation 59, solid line) and proportional regression (equation 60, truncated line) between ΔMT and MT. Data from Newell et al. (1979, experiment 1). □ D = 5 cm, R^2 = 0.99. Δ D = 15 cm, R^2 = 0.99.

velocity profile with a specific $\Delta\Lambda$ law, extracting the different parameters that allow an optimal reconstruction of that profile, followed by an analysis of the variations of these parameters under different experimental groupings. As long as the $\Delta\Lambda$ law parameters for each specific movement are not extracted and analyzed individually and statistically, these nonlinear observations will remain difficult to explain, although they probably reflect the effects of these specific experimental conditions on the synergetic parameters.

5. Discussion

As one can see from these results and predictions, the kinematic theory of Plamondon (1993b; 1993c; 1995a; 1995b) can take into account the various forms of the velocity profiles of an end-effector as well as its different properties as a function of various experimental conditions. Using a basic $\Delta\Lambda$ law (equation 41), all the resulting observations can be described and analytical predictions about movement time, time to maximum velocity, maximum velocity, and so on, can be performed. Using a quadratic law (or a power law approximation) that derives from the $\Delta\Lambda$ law, all the observations dealing with speed/accuracy trade-offs can be described under a single framework. Using a single model that schematizes the global asymptotic behavior of a neuromuscular synergy, a simple analytical description of very complex phenomena can be reached. This is consistent with the fact that for a specific set of tasks the same neuromuscular system is always being used, under different spatial or temporal constraints. One basic system – one basic description, but analyzed under the numerous points of view provided by the different experimental protocols that have been so cleverly designed over the last century.

Taking each of these various curve fittings individually, an improvement of a few percentage points would probably not justify the adoption of a new specific theory. It is interesting, however, that only one basic equation is sufficient to explain all the data and to provide a new paradigm for analyzing them from a new perspective. We summarize below a few examples of the new points of view provided by the theory, as reported and discussed in Plamondon (1993b; 1993c; 1995a; 1995b) and in Plamondon and Privitera (1995).

5.1. Origin of the speed/accuracy trade-offs

The kinematic theory of Plamondon (1993b; 1993c; 1995a; 1995b) provides a new insight into what might ultimately be responsible for speed/accuracy trade-offs – the asymptotic impulse response of a neuromuscular synergy, as described by the $\Delta\Lambda$ law (equation 39). This law predicts that even if there were absolutely no noise, no variability in the neuromuscular systems, a trade-off would have to be taken into account by a subject planning a rapid movement at the command level, because the distance to be covered is predicted by the difference between the agonist and antagonist input commands ($D = D_1 - D_2$), and the movement time is linked to the ratio of these two commands (see equation 41 or 45). Assuming that a coupling exists between some sensorimotor maps in such a way that the ratio of antagonist to agonist activities is made proportional to the desired relative spatial error $\Delta D/D$ of the planned movement, the different speed/accuracy trade-offs follow. These trade-offs result from the rapid-aimed movements being executed by an ensemble of networks that produce very specific velocity profiles.

Using proprioceptive feedback, for example, a subject can estimate the initial position of the end-effector to be moved. Using visual feedback, the distance to be covered as well as the required accuracy can be estimated. By coupling this information with $D_1 - D_2$ and D_1/D_2, respectively, an appropriate pair of input commands can be fed into a specific neuromuscular system. Depending on the pathways and units involved, the overall synergy will react with logtime delays μ_1 and μ_2 and logresponse times σ_1 and σ_2. Once the specific commands are fed into the agonist and antagonist systems, the subject "already knows" from previous learning that the target will be reached with a certain relative spatial error $\Delta D/D$ within a time MT, depending on the acuity and integrity of the sensory information available (visual, kinesthetic, or virtual). No visual feedback is needed during the movement, except as will be discussed below for the preparation of the next movement, if a complex sequence is being executed.

In this context, the variability of the different parameters describing a $\Delta\Lambda$ synergy is not the intrinsic cause of the speed/accuracy trade-offs. This variability constitutes a noise that is superimposed on the basic processes to produce motor output fluctuations around some mean values, each of which is a specific movement being described by the kinematic theory. In this context, the relationships that have been reported to depict the variability of space or time targets just emerge from the variability of the command and system parameters according to basic experimental constraints that are put on a subject who has to cope with its intrinsic neuromuscular limitations, as described in the context of the $\Delta\Lambda$ law (Plamondon 1993b; 1995a).

5.2. Control variables

The kinematic theory also provides an answer to the problem of control variables in rapid-aimed movement. Several variables have been proposed for controlling limb movements: force, velocity, length, stiffness, viscosity, and so on.

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(see Stein, 1982, for an extensive review). The kinematic theory shows that for target-directed movements, the input control variables D_1 and D_2 have to be coupled with distance information (D and ΔD), because the goal of the subject is to exploit the asymptotic shape and properties of the impulse response of its neuromuscular system. This approach seems in accordance with the results of Houk and Gibson (1987) and Gibson et al. (1985), who have shown that short high-frequency bursts transmitted from the red nucleus to the spinal cord in the rubrospinal track – as recorded from monkey subjects trained to perform a visual tracking task – code movement velocity in terms of the burst frequency of the firing cells. In addition, burst duration in these experiments correlates closely with movement duration, and the number of spikes in a burst correlates with movement amplitude. Based on these observations, it was suggested that red nucleus signals may serve to command velocity (Houk 1989).

Furthermore, Georgopoulos et al. (1984) have reported a strong similarity between the asymmetry of the velocity profile and the asymmetry of the vector cell population profile. This has been interpreted by Bullock and Grossberg (1988) as a manifestation of the properties of the neural networks that control movement.

A rapid-aimed movement may be seen as a motor task producing a certain spatial output within relatively stringent time limits. From a physics point of view, controlling velocity seems to be the simplest way to perform such a task. In this way, more complex movements, such as hand writing, can be segmented into strokes with relatively less activity at the beginning and at the end (Plamondon & Maarse 1989). These strokes can be interpreted in terms of spatial targets and a control strategy based on these virtual targets can be learned from the visual inspection of a trajectory (Plamondon & Privitera 1995).

In addition, the velocity vector is the sole dynamic information that is uniquely related to the end-effector trajectory: it is always tangent to the trajectory and can thus be recovered at least partially from visual observation of a real movement or of its trace image like, for example, in handwriting. Other representations, such as acceleration or force vectors, cannot. This is a major point to bear in mind when considering the importance of the visual system in the learning and control of more complex movements.

Some indirect evidence also support the importance of velocity in movement generation. For example, in a comparative simulation of 14 simplified handwriting models, it has been shown (Plamondon & Maarse 1989) that models controlling the velocity output yield the best reconstruction in an analysis-by-synthesis experiment. Similarly, in a study of comparative performance of position, velocity, and acceleration signals for automatic signature verification, it has been suggested that the velocity domain is one of the best representation spaces for a 2D signature verification system (Plamondon & Parizeau 1988).

5.3. Ballistic movements

The generation of a multiple peak velocity profile from a single pair of synchronous impulse commands is one of the most powerful characteristics of the $\Delta\Lambda$ law (Plamondon 1993b; 1995a). Most of the models published to date, if not all, ignore or neglect secondary peaks in velocity profiles or require specific commands for each peak (see Plamondon et al., 1993, for a comparative review). The $\Delta\Lambda$ law shows that secondary peaks emerge naturally from a single pair of synchronous impulse commands when the agonist and antagonist systems composing a synergy have different logresponse times, different logtime delays, or both.

In this context, the kinematic theory provides a practical definition of a simple ballistic movement: a rapid movement produced by a synergy made up of agonist and antagonist systems synchronously activated by a pair of impulse commands $D_1U_0(t - t_0)$ and $D_2U_0(t - t_0)$, each system is asymptotically characterized by a lognormal impulse response. As can be seen from the examples given in Figure 2, a simple movement does not necessarily result in a single peak velocity profile. Depending on the state of the neuromuscular system, as depicted globally by the values of μ_1 and μ_2, σ_1 and σ_2, a small velocity reversal or a double reversal may occur. These secondary peaks do not have to be associated with any closed-loop corrective submovements. They are not the result of new input commands for making trajectory corrections but are rather part of a single ballistic movement and result from the differences in the timing properties of the impulse responses of the agonist and antagonist systems to a single pair of synchronous inputs.

The occurrence of these vanishing oscillations in the system response to a single pair of synchronous commands seriously questions the concept of corrective movements or submovements that have been put forward in the past by several researchers to explain phenomena related to speed/accuracy trade-offs of simple movements. The kinematic theory provides a way to check if there are real closed-loop homing-in phenomena in more complex trajectories. Indeed, using equation (39), a complex experimental velocity profile can be reconstructed by concatenating and superimposing a few basic units (that might incorporate secondary peaks). The optimal solution to this signal reconstruction process would provide the number and the relative importance of the submovements needed to synthesize the complex velocity profile. In addition, if multiple solutions emerge in analyzing these data (Guerfali & Plamondon 1994), the $\Delta\Lambda$ law could serve as a basis for studying strategies and optimizing principles that might be involved in the production and control of complex movements (Guerfali & Plamondon 1995).[9]

5.4. Complex movements

In light of the kinematic theory, many previous results can be reanalyzed and new experiments planned to study the effect of the command and system parameters on movement kinematics. This will require the measurement of the complete velocity profiles and the extraction of movement parameters (Alimi & Plamondon 1994; Plamondon et al. 1995a; 1995b). With such an approach, a set of simple ballistic movements can be studied under different experimental conditions in terms of changes in the input commands and of changes in the global state of the system, instead of focusing on data-driven interpretations like single/multiple movements, accelerating/decelerating phases, closed- and open-loop portions, and so on. Since the theory does not require continuous feedback, it can also be useful in providing criteria to discriminate between open- and closed-loop conditions. If the velocity profile of a movement cannot be fitted by a $\Delta\Lambda$ law (as could happen, for example, for complex visual tracking tasks), this might

suggest that the corresponding movement, or part of it, has probably involved some kind of continuous feedback and that the $\Delta\Lambda$ law does not hold in these conditions.

It is interesting that the theory provides some cues on the necessity or possibility of using feedback information. Unless default or virtual values are used for D and ΔD, feedback is necessary prior to movement initiation to evaluate D and ΔD and to recruit the proper neuromuscular networks (as characterized by μ_i, σ_i). Once a movement is initiated, a specific $\Delta\Lambda$ velocity profile is generated and the end-effector will reach its target within MT within a relative spatial error $\Delta D/D$, and a relative timing error $\Delta MT/MT$. If some new sensory information is provided during that first movement, which requires a change in the planned trajectory to a new target, another movement with more or less similar $\Delta\Lambda$ characteristics (depending on the new neuromuscular networks recruited) would be initiated and a new velocity profile would start to become superimposed vectorially on the previous one. Using this methodology, more complex movements can be analyzed by vector superimposition of different ballistic units (Plamondon 1992b; 1992c; Guerfali & Plamondon 1994; 1995).

5.5. Sequence generation and control

Assuming that the built-in properties of the $\Delta\Lambda$ law are known and exploited by a subject, we have recently developed a neural network model that generates and learns rapid movement sequences – each described by a $\Delta\Lambda$ velocity profile (Plamondon & Privitera 1995). In other words, we have used the kinematic theory to describe a neuromuscular synergy that is coupled to a neural map representing the planning space and composed of leaky integrator elements. In light of the basic arguments previously introduced, the generation of a structured motor plan is then seen as the outcome of a predefined mental image of the movement, where only the principal targets and their corresponding time sequence is already represented before execution, with an indication related to the evaluation of the acceptable spatial error of the movement.

A movement sequence is instantiated by a recall of the movement sequencing plan from the long-term memory and the corresponding positioning of this plan on the surface of the planning space: all the virtual targets composing the sequence are settled on the surface of the leaky integrator grid and the map finally represents a sort of virtual imagery of the movement. By means of a simple competition mechanism, we can define a threshold process capable of detecting the descending part of the velocity of each motor stroke and consequently of instantiating the next ballistic stroke. This point of synchronization depends on the time requested for the movement: the higher the velocity of the movement the faster the next motor impulse has to be anticipated, finally risking losing the intended form of the movement. On the basis of this virtual imagery, which is kept for the entire course of the external movement sequencing, the command generator is able to activate the corresponding sequence of impulse commands D_1 and D_2 for the neuromuscular synergy. During movement learning, the same process is exploited: in this case the movement is executed by an external subject and the occurrence of a synchronization point is interpreted as the location of one of the virtual targets composing the new movement. The goal of the learning phase is to build a central representation of the observed movement by means of storing the sequence of corresponding virtual targets.

5.6. Motor-perception interaction

One key interpretation highlighted by the kinematic theory of Plamondon (1993b; 1993c; 1995a; 1995b) is the inherent relationship that must exist between the perceptual information and the motor commands. Indeed, to execute a target-directed movement, at least two basic cues must be perceived from the environment or from mental imagery previously acquired from learning: the distance (D) to be covered and the absolute error (ΔD) that is required in executing that movement. The knowledge of the absolute position of the target and the end-effector is not necessary at this stage, although it might be needed at another level to estimate the two previous pieces of information. Once this information is extracted from the optic flow, it has to be matched via different sensorimotor maps into two commands, D_1 and D_2, whose difference, $D_1 - D_2$, predicts the movement amplitude (D) and whose ratio, $D_1/D_2 \propto D/\Delta D$, predicts the movement time (MT) (Plamondon 1993b; 1993c; 1995a; 1995b). Depending on the status of the neuromuscular synergy recruited in the process, as described by the parameters μ_1, μ_2, σ_1, and σ_2, the resulting $\Delta\Lambda$ velocity profile will be more or less complex, in terms of its number of pulses (up to three for a single pair of commands). Proper coupling between the different stages involved in the continuous activation process will result in a faster or a slower movement, depending on the relative precision that is required by the task.

In other words, once a subject has learned how to control a specific neuromuscular synergy to execute a spatially constrained task with a specific end-effector, a specific $\Delta\Lambda$ velocity profile does emerge due to the coupling that is developed between the different components of the agonist and antagonist systems involved (equation 39). This allows the subject to "forget" about the synergy itself and concentrate on the goal of the action at a higher level, via a direct coupling between the global activation commands and the relevant perceptual information. In this context, the movement time does not have to be planned or programmed in advance as a specific goal, since it will automatically emerge from the selection of the ratio of two basic activation commands. Even when the maximum absolute error (ΔD) is not specified by the experimenter, the subject is probably able to extract from the environment or from past experience a significant or a default value for this cue (mental imagery). In any case, the spatial target will generally be reached within a movement time MT, as predicted by the quadratic law.

How is this mapping done? What types of maps are necessary, from a retinotopic to a motor representation? How are intermodality sensorimotor commands encoded as the target position and as spatial resolution? How does the learning affect the time delays of the different components? These are some of the questions that are raised by the new point of view provided by the kinematic theory. We hope to see some constructive suggestions made by the commentators, to help shed some light on these potential mechanisms.

6. Conclusion

This target article had two specific goals. First, we wanted to make clear that speed-accuracy phenomena, as observed in simple rapid-aimed movements, are still without a fully comprehensive interpretation. Second, we tried to show that the kinematic theory recently proposed by Plamondon (1993b; 1993c; 1995a; 1995b) provides a general framework in which such phenomena can be described and further studied.

To reach our first goal, a systematic survey of the scientific literature dealing with speed/accuracy trade-offs was presented to highlight the numerous mathematical and theoretical interpretations that had emerged over recent decades from the various studies that had been conducted on this topic. Although reflecting the richness of the studies in the field and the high degree of interest that such basic phenomena represent for the understanding of human movement, such a variety of points of view questions the validity of many of the models with respect to their capacity to explain all the basic observations consistently reported in the field.

In the second part of this article we have summarized Plamondon's kinematic theory of rapid human movement in which the basic properties of the proposed model emerge as a fundamental consequence of its architecture. By considering the asymptotic behavior of a large number of dependent linear systems, this theory provides a mathematical expression describing the velocity profile of an end-effector driven by the action of a synergy made up of an agonist and an antagonist system. The resulting equation that describes the velocity profile, referred to as a $\Delta\Lambda$ law (Plamondon 1993b; 1995a), can be used to depict the kinematic properties of simple ballistic movements and, more specifically, the various aspects of speed/accuracy trade-offs from a quadratic or a power law (Plamondon 1993c; 1995b), which emerges from it. So, within a single framework it is possible to take into account the basic observations consistently reported in the classical studies in the field.

The kinematic theory (Plamondon 1993b; 1993c; 1995a; 1995b) encompasses both similarities and dissimilarities with respect to the other models previously published. For example, unlike the minimum jerk (Hogan 1984) or the minimum torque (Uno et al. 1989) models, the theory does not require any minimization criterion to generate "bell-shaped" velocity profiles. The form invariance of the profile is evident in the asymptotic behavior of a large number of coupled neuromuscular networks. However, for the study of a more complex trajectory, where many $\Delta\Lambda$ equations will have to be superimposed to reproduce the complete velocity profile, it is expected that optimization principles will have to be incorporated into the theory to reduce the number of potential solutions. As in the impulse timing models (see Plamondon & Maarse, 1989, for a review), the kinematic theory succeeds in separating the command and the system parameters, except that the end of the activation does not have to specify directly a particular parameter at the command level (Plamondon 1993c; 1995b). Unlike the equilibrium point models (Feldman 1966; 1986; Polit & Bizzi 1979), where the focus is mainly on the modeling of the mechanical properties of the muscles, or the neural network models (Bullock & Grossberg 1988), where the emphasis is on the modeling of the architectural properties of the neural systems – the kinematic theory provides a global view based on the timing properties of both the neural and the muscular networks. The price for such generalization is that it is difficult, without further experiments, to provide a direct biological interpretation for the system parameters μ_1, μ_2, σ_1, and σ_2, and further study is needed in this context. Unlike the neural network models, the kinematic theory is based on the linear system theory. It provides an analytical solution for describing the trajectory of a well-learned rapid movement, but it does not provide too many cues on the learning process itself. However, using the basic knowledge that emerges from the theory, it is possible to construct higher-level neural networks that can learn to generate complex movements (Plamondon & Privitera 1995). Finally, the model can be considered to stand in between purely parallel (e.g., Ulrich & Wing 1993) and purely sequential models (e.g., Plamondon 1991a). The proportionality relationship (equation 38) that is assumed to exist between the cumulative time delays of the different stages of the equivalent sequential representation of the synergy is a way to take into account the different hierarchical couplings (both sequential and parallel) that exist in a synergy.

As will probably be reflected in the commentaries, although the kinematic theory is very powerful at a descriptive level, numerous questions remain unanswered in the study of well-learned tasks. To answer most of them, new experiments will have to be designed or old data will have to be reanalyzed. To progress in our understanding of rapid human movements and to take advantage of the new window provided by the kinematic theory summarized here, a large set of velocity profiles will have to be studied with an analysis-by-synthesis methodology based on an optimal parameter fitting of the $\Delta\Lambda$ law to these profiles (Alimi & Plamondon 1993a; 1993b; 1994; Plamondon et al. 1993; Plamondon et al. 1995a; 1995b). We hope that the analysis of the parameters that will be extracted from these studies will provide some answers to these unsolved problems.

APPENDIX

Eleven studies were selected to compare the performance of the quadratic law versus the power law and Fitts' law; these studies provided 50 datasets. Their protocols are summarized below.

Andres and Hartung (1989a). The study consisted of two experimental sessions. Nine male subjects were asked to move a chin stylus reciprocally between targets of various widths and separations. Using a Fitts' reciprocal tapping task, the subjects' head movement capability was predicted by recording the time intervals between movements of the chin stylus repetitively tapped between two target plates located along the horizontal axis of a reciprocal tapping apparatus. The experimental protocol consisted of nine A/W conditions (A = 7.6, 15.2, and 30.5 cm; W = 1.3, 2.5, and 3.8 cm), giving a Fitts' index of difficulty $ID = \log_2(2A/W)$ in the 2 to 5.58 bits range. The mean movement times were reported for the nine subjects and the two sessions for each A/W condition, providing a total of 18 datasets.

Drury (1975). Ten male subjects participated in experiment II of this study. Each subject started reciprocal tapping using his preferred foot. Six different amplitudes (A = 150, 225, 300, 375, 525, and 675 mm) were crossed with two pedal sizes (W = 25 and 50 mm), giving a Fitts' index of difficulty $ID = \log_2(2A/W)$ in the 0.918 to 3.335 bits range. The mean width of subjects' shoes (108.8 mm)

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was added to the target width as a reasonable adjustment, because any portion of a shoe touching the target was recorded as a hit. The mean movement times over subjects for experiment II were reported for each A/W condition, providing one dataset.

Drury and Hoffmann (1992). In this study, experiment I dealt with a standard Fitts' paradigm. Ten male subjects were asked to make discrete movements to hit simulated keyboard keys. The amplitude of movement A was kept constant at 160 mm (light key spacings) and five target boards were used onto which were attached target sets of width (B = 2, 6, 10, 14, and 18 mm). The subjects held four metal probes in their hands, flat across the palm so that the probes would produce a natural extension of the index finger. These probes had tip widths of (P = 0, 5, 10, and 15 mm). A further condition, in which the subjects used their index fingers as the probe, was also included (the mean pad width in this case was 11 mm).

To evaluate W in Fitts' index of difficulty $ID = \log_2(2A/W)$, the authors used $W = B + 0.6\,P$ in the probe condition (B was the target width and P the probe width) and $W = B + 10$ in the finger condition. The mean movement times over subjects for single targets were reported for each A/W condition for the probe and the finger conditions, providing two datasets.

Fitts (1954). In this original study four experiments were reported, consisting of a reciprocal tapping task with a stylus of 1 oz and 1 lb, a disk-transfer task, and a pin-transfer task. Sixteen male subjects participated in the first three experiments, and 20 subjects (10 male and 10 female subjects) in the last one. In the tapping task, four distances (A = 2, 4, 8, and 16 in) and four target widths (W = 0.25, 0.5, 1, and 2 in) were used, giving a Fitts' index of difficulty $ID = \log_2(2A/W)$ in the 1 to 7 bits range. In the disk-transfer task, four distances (A = 4, 8, 16, and 32 in) and four target widths (W = 0.0625, 0.125, 0.25, and 0.5 in) were used, giving a Fitts' ID in the 4 to 10 bits range. Finally, in the last pin-transfer task, five distances (A = 1, 2, 4, 8, and 16 in) and four target widths (W = 0.03125, 0.0625, 0.125, and 0.25 in) were used, giving a Fitts' ID in the 3 to 10 bits range. The mean movement times over subjects were reported for each A/W condition and for all the tasks, providing four datasets.

Gan and Hoffmann (1988). Six male and six female subjects were asked to make discrete tapping movements about the elbow in a left-to-right direction, and to make these as rapidly as possible. The apparatus consisted of boards having a starting plate and a target plate at a distance of (A = 4, 9, 16, or 25 cm) from the starting plate and each board had a constant Fitts' $ID = \log_2(2A/W)$ of 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, and 6 bits. The mean movement times over the subjects were reported for each ID and each A condition, providing four datasets.

Hoffmann and Sheikh (1991). Five male and five female subjects took part in this experiment. There were 15 experimental conditions (A = 100, 200, and 400 mm; W = 2, 6, 10, 14, and 18 mm). Movements were made by the subjects' either holding a sharp-pointed probe or using the finger as a probe. The probe was pointed at its tip. The mean finger pad size of the 10 subjects was P = 10.2 mm. Times for the discrete movements were measured from the moment the probe left the starting plate until the moment it contacted the target. The mean movement times over subjects were reported for each A/W condition for the sharp-probe condition and the finger condition, providing two datasets. For the finger condition, the value of the finger width P was added to the target width W to obtain the effective target width used in the calculus of the indices of difficulty.

Johnsgard (1994). In this study 18 subjects were asked to move a cursor from a starting position to a rectangular bar target drawn on the computer screen, using a mouse or a virtual reality glove. Three target amplitudes were chosen (A = 2, 4, and 8 in), fully crossed with three target widths (W = 0.5, 1, and 2 in). For each device, three values of gain were tested (G = 1, 2, and 3). The mean movement times over the subjects were reported for each ID and each G condition, providing three datasets for each device.

Kerr and Langolf (1977). Eight male subjects participated in the experiment. The task consisted of moving a stylus forward from a back contact plate to hit a target placed at specified distances in front of the subject. Four levels of movement distance were chosen (A = 8, 12, 16, and 20 in), and for each of these, four target widths were chosen (W = 0.25, 0.5, 1, and 2 in), giving a Fitts' ID in the 3 to 7.32 bits range. The mean movement times were reported for the eight subjects for each A/W condition. So nine datasets[10] were available (the eight subjects and the mean of all the subjects).

MacKenzie et al. (1987). Six subjects participated in the experiment with a discrete tapping task. There were 12 experimental conditions with the same IDs as in experiment 1 of Fitts and Peterson (1964). Four target diameters (W = 2.54, 1.27, 0.64, and 0.32 cm) were crossed with three amplitudes (A = 7.62, 15.24, and 30.48 cm), giving a Fitts' ID in the 2.58 to 7.58 bits range. The mean movement times over all subjects were reported for each A/W condition, providing one dataset.

Newman and Bussolari (1990). Nineteen subjects performed two batteries of tests, where unusual body dynamics were imposed on subjects to assess altered environment performance. The results of the first battery of tests are not used here, because only two points were given in the ID versus MT dataset. Eleven subjects (six females and five males) participated in the second battery of tests, where an altered environment was electronically created by introducing a first-order lag characteristic between the graphic input device and the computer. The dynamics of the arm motion-input device with a lag bear little resemblance to human arm motion-input device dynamics in microgravity. However, an astronaut performing tasks in the weightless state is in an analogous situation by being exposed to unusual body dynamics and responses. The average movement times over the subjects were reported for a Fitts' ID in the 1.5 to 3.7 bits range, for two conditions (with and without lag), providing two datasets.

Repperger and Remis (1990). Five subjects ran five trials each of the five different stylus diameters selected with the same task hole. Subjects were placed in an exoskeleton device to investigate the feasibility of using such devices in the performance of a Fitts' task. The mean movement times over all subjects were reported for a Fitts' ID in the 6.35 to 11.73 bits range, providing one dataset.

NOTES

1. It should be noted that if $V = 0$, we obtain an expression of MT for the case of static targets, which is still different from Fitts' formulation.

2. It should be noted that if D = 0, we obtain an expression of MT for the no-delay case, which is similar to Fitts' formulation.

3. One should not confuse between what we call the logtime delay (μ) and the logresponse time (σ), which refers to ($\log t$), a logarithmic time scale, with the time delay $\left(\exp\left(\frac{\mu + \sigma^2}{2}\right)\right)$ and the response time ($\{\exp[2\mu + \sigma^2]\exp[\sigma^2 - 1]\}^{1/2}$) of a lognormal on a conventional time scale (Plamondon 1995a).

4. We should use the term "activation time" here, since the definition of ($t_1 - t_0$) is more general than the operational definition of the movement time that is normally used in most studies. According to the kinematic theory, movement time refers to the whole activation time, beginning at the initiation of the input command t_0 and finishing at t_1, when the velocity reaches zero. Therefore, the time reference for computing movement time is not the reaction time (t_r), that is, the time where the end-effector begins to move, but t_0, the time of occurrence of the commands. Since, for a specific set of identical movements produced by the same synergy, $t_1 - t_r$ should be proportional to $t_1 - t_0$, we will not

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distinguish between activation time and movement time, to avoid confusion.

5. A supplementary zero velocity value is also predicted for $t_1 = \infty$. See Plamondon (1993b; 1995a) for a discussion about this asymptotic prediction.

6. Depending on the relative importance of the surface under the velocity curve from the longest time occurrence of the zero crossing to $t_f = \infty$, the approximation will be more or less realistic. For example, for many results reported in Figures 2 and 3a,b, the approximation defined by equation [47] would certainly be acceptable. In general, since the remainder will be a function of the system parameters (μ_1, μ_2, σ_1, σ_2), control from both input command and system processes might have to be taken into account in some extreme cases.

7. A few studies were eliminated for these reasons: Beggs and Howarth (1972), who reported only three values of $D/\Delta D$; Carlton (1980), who reported only two values of $\log_2 D/\Delta D$; Kerr (1973, p. 177), included the trials that missed the target in the calculation of *MT*; Kvalseth (1977, p. 43), for the same reason as Kerr (1973); Spitz (1990, p. 407), included in the measure of *MT* the time to stabilize the cursor on the target.

8. The ΔD values were computed from the effective target values (W_e), reported by the authors, using their method $\Delta D = 4.11 * W_e$ (Howarth et al. 1971).

9. We always assume here that the recording devices or the manipulanda used by the subjects have a negligible effect on the neuromuscular impulse response. If these mechanical devices strongly interfere with the end-effector under study, the $\Delta\Lambda$ velocity profile might be partially masked by the impulse response of the recording devices itself, making data analysis really more complex.

10. For subject 7, the value 000.3 (probably a typing error) in Table I was replaced by 600.3, which is consistent with the mean value reported (Kerr & Langolf 1977).

ACKNOWLEDGMENTS

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Dynamics of trajectory formation and speed/accuracy trade-offs

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Abstract: Though capable of reproducing experimentally observed velocity profiles, the model proposed by Plamondon & Alimi (P&A) does not provide a viable theoretical framework for the understanding of trajectory formation and speed/accuracy trade-offs. The issues of variation and stability can be better understood by considering movement as resulting from an underlying dynamic rather than from an impulse-response of the system.

Starting from the point of view that the asymmetric bell-shape of the velocity profile of aiming movements constitutes an essential invariant of the movement pattern, over the last few years Plamondon and collaborators have refined a model capable of reproducing this invariant. Assuming that the kinematic pattern observed is the result of the difference between the velocity outputs of agonist–antagonist neuromuscular systems, each responding in a lognormal fashion to a discrete activation impulse, Plamondon & Alimi (P&A) aim to provide a general theoretical framework in which trajectory formation and speed/accuracy trade-off phenomena can be understood.

Plamondon's model is indeed capable of generating adequate velocity profiles for simple unperturbed aiming movements directed toward stationary targets. To do this, seven parameters are proposed, three of which are supposed to reflect the command level and four the system level. Because of the metatheoretical position taken, with the command level dominating a somehow known system, all the neural and (bio)mechanical properties of the effector chain as well as those of the environment are regrouped under the same heading, rendering a meaningful biological interpretation of the so-called system parameters practically impossible. Thus, while the model can, under specific circumstances, reproduce the velocity profiles experimentally observed, it does not provide a coherent set of interrelated theoretical (explanatory) concepts and as such does not, at present, constitute a theory of trajectory formation.

P&A's claim that speed/accuracy trade-offs "are inherent constraints that emerge directly from the delta-lognormal impulse response of the global neuromuscular system" (sec. 3, para. 3) is not warranted, because the relationship between *MT* and relative spatial accuracy (equations 51 and 52) is not an emergent property of the delta-lognormal law of equation 39. Rather, this relation is externally imposed, based on the observation that in a Fitts' task a subject is confronted with both speed and accuracy constraints, followed by the equation of these two constraints in the language of the model. Obviously, we are not suggesting that the quadratic law relating $\ln(MT)$ to $\ln(D/\Delta D)$ does not give a good description of the speed/accuracy trade-off in Fitts' type tasks; we are simply suggesting that this relationship is empirically rather than theoretically founded. In fact, the most surprising feature of the model in this respect is that, while dealing with movement amplitude and movement time, it does not address spatial or temporal accuracy (i.e., variation) in a principled manner and therefore cannot constitute a theoretical framework for understanding the origin of the speed/accuracy trade-off.

Not only is the model completely deterministic, it is inherently unstable: a small perturbation already suffices to ensure that the target is never reached and that the movement never ends. This lack of stability results from the pure feedforward character of the model but may also be related to the absence of a positional anchoring, thus rendering its generalization to continuous movements tasks even more difficult.

These shortcomings can be overcome by considering the movement as resulting from an underlying dynamics rather than from an impulse-response of the effector system. The equilibrium point models, developed on the basis of the mass-spring analogy, constitute a first (but in our view insufficient) step in this direction. The full advantage is obtained by adopting a nonlinear dynamical systems perspective.

From such a perspective, the actual behaviour of the system is considered to reflect the presence of an attractor plus noise, where the attractor is the behavioural expression of the self-organisational properties of the environment-actor system under the pressure of the task constraints, and the noise represents microscaled random fluctuations. This attractor-plus-noise characterization has important consequences. The movement kinematics and the (attractive and hence stable) behavioural pattern are

both emergent properties of the operational régime of the underlying dynamical structure. Apart from its ability to deal with coordination phenomena (Haken et al. 1985), the power of the dynamical systems approach can be seen in its capability of reproducing the main aspects of trajectory formation for both discrete and continuous movements (e.g., Beek et al. 1995; Schöner 1994). Trajectory formation thus considered has direct consequences for speed/accuracy trade-offs, since variability is an intrinsic component of any such dynamical model.

Using a five-parameter nonlinear dynamical model, the movement kinematics in Fitts' tasks can be accurately reproduced (Mottet & Bootsma 1995). Moreover, the speed/accuracy constraints operating in task space can be meaningfully redefined in the model's parameter space: the movement time and the spatial accuracy requirements act, respectively, as (1) a constraint on the (nonlinearly varying) stiffness (the globally stiffer the system, the faster the motion) and (2) a constraint on the stability (the stabler the system, the lower its spatial dispersion). Because this stability is essentially due to a Rayleigh-type damping term, the increase in peak velocity with movement amplitude and the increase in asymmetry of the velocity profile with accuracy are emergent properties. Thus, contrary to Plamondon & Alimi's position, such an approach allows a theoretically founded perspective on trajectory formation and the reasons underlying the emergence of speed/accuracy trade-offs.

Speed/accuracy relations: The kinetic–kinematic link and predictions for rapid timing tasks

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Abstract: Recent accounts of the speed/accuracy relation for motor tasks have focused on the concept of motor output variability. We outline the advantages of this approach and the limitation of Plamondon's model in explaining movement error. We also examine and present complimentary data for rapid timing tasks. While these tasks do not meet the presented assumptions, the data still fit the model predictions.

Plamondon has formulated a neuromuscular synergy model for the control of rapid aimed movements that accounts for speed/accuracy relations under a number of task conditions. The strength of his work is that it predicts a number of speed/accuracy related findings from a single model. There are also some limitations to the model. Two points that we will address concern the importance of linking kinetic variability to movement accuracy and the predictions of the model related to rapid timing tasks.

Force variability and movement accuracy. The approach used by Plamondon & Alimi (P&A) focuses on space-time tasks but for the most part ignores the linking of accuracy in these tasks to accuracy in force production itself. This is in contrast to recent models and approaches that have linked accuracy to motor output (e.g., Schmidt et al. 1979). One of the prime advantages of this approach for studying motor functioning is the integration of kinetic and kinematic accuracy functions. The implicit and sometimes explicit assumption is that such a link is possible and will lead to a unified theory that can account for variability and, therefore, accuracy in a variety of tasks (Carlton & Newell 1993). Plamondon's model does not address the locus of the accuracy function and makes no predictions for the isometric force production case.

In order to explain the speed/accuracy relation in motor tasks, it is necessary to explain the source of the error. The concept of absolute spatial error is introduced by P&A in equation 48 by defining a variable (γ_1) equal to maximum relative spatial error. To perform a space-time task the performer must evaluate the distance, the relative spatial error ($\Delta D/D$), and couple "some sensory motor maps in such a way that $D = D_1 - D_2$ and $D_2/D_1 \alpha \Delta D/D$" (sect. 3.3, para. 4). What are these sensory motor maps? How are they developed? The important point is that there is no theoretical basis for this process – a process that is critical for moving from the general model to the specifics of the speed/accuracy predictions. One way to conceive of these spatial-temporal maps would be to think of them as large look-up tables. It appears that the maps would need to be specified for various combinations of synergy parameters because changes in the synergy parameters would result in a change in the relation between movement time and relative accuracy (equation 54). Even so, this would not explain the nature of the error.

Timing accuracy. P&A use experiments and data from Newell et al. (1979) to demonstrate that their model can account for time accuracy. Unfortunately, these experiments do not match the description provided or the assumptions that are used to produce the equations to predict time accuracy. Yet, the model accounts for the data quite well! In fact, with a few modest assumptions, the model predicts not only these timing data but also those from other rapid timing experiments (Newell et al. 1993a; 1993b).

Spatial errors in these experiments are not constant. In rapid aiming tasks the subject attempts to move through a set distance in a fixed movement time. Spatial errors are an outcome measure based upon arriving at the target too soon or too late. There were no measures of spatial error in Newell et al. (1979). Even though the model's assumptions do not match the characteristics of the data, it is surprising how well it predicts the empirical findings. Two experiments by Newell et al. (1993a; 1993b) demonstrate this. The experiments either held distance constant and varied movement time (MT) or held MT constant and varied distance. We would argue that the data that emerged from these experiments are consistent and simple patterns, contrary to P&A's suggestion (sect. 4.5, para. 1). In fact, the data from both these experiments fit equation 60 very well. Equation 60 can be rewritten to obtain:

$$\Delta MT/MT = 1/a_2\ \Delta D/D. \quad (1)$$

Assuming small constant errors, true for most of Newell et al.'s data, variable error can be substituted for absolute error. With this adjustment, the coefficient of variation of movement time [((sd of MT)/MT)] is proportional to the coefficient of variation of spatial error.

Figure 1 shows the relationship between spatial and temporal coefficients of variation for the three experiments from Newell et al. (1993a). For each set of data the fit was highly linear with a near zero intercept. This relation also holds for Newell et al. (1993b, Fig. 7).

Summary. We see the model as providing predictive and curve fitting power, but little explanation for the inaccuracy of movement. There are other limitations to the model. Two that we did not focus on are (1) the validity of its internal architecture and its testability, and (2) the difficulty of disproving the model because discrepant data can be accounted for by hypothesized changes in synergy patterns. Plamondon & Alimi are probably right in their conclusion: "Speed/accuracy phenomena . . . are still without a fully comprehensive interpretation" (sect. 6, para. 1).

Visual control of target-directed movements

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Abstract: Visual feedback regulation during movement is not fully captured in Plamondon's kinematic theory. However, numerous studies indicate that visual response-produced feedback is a powerful determinant of performance and kinematic characteristics of target-directed movement.

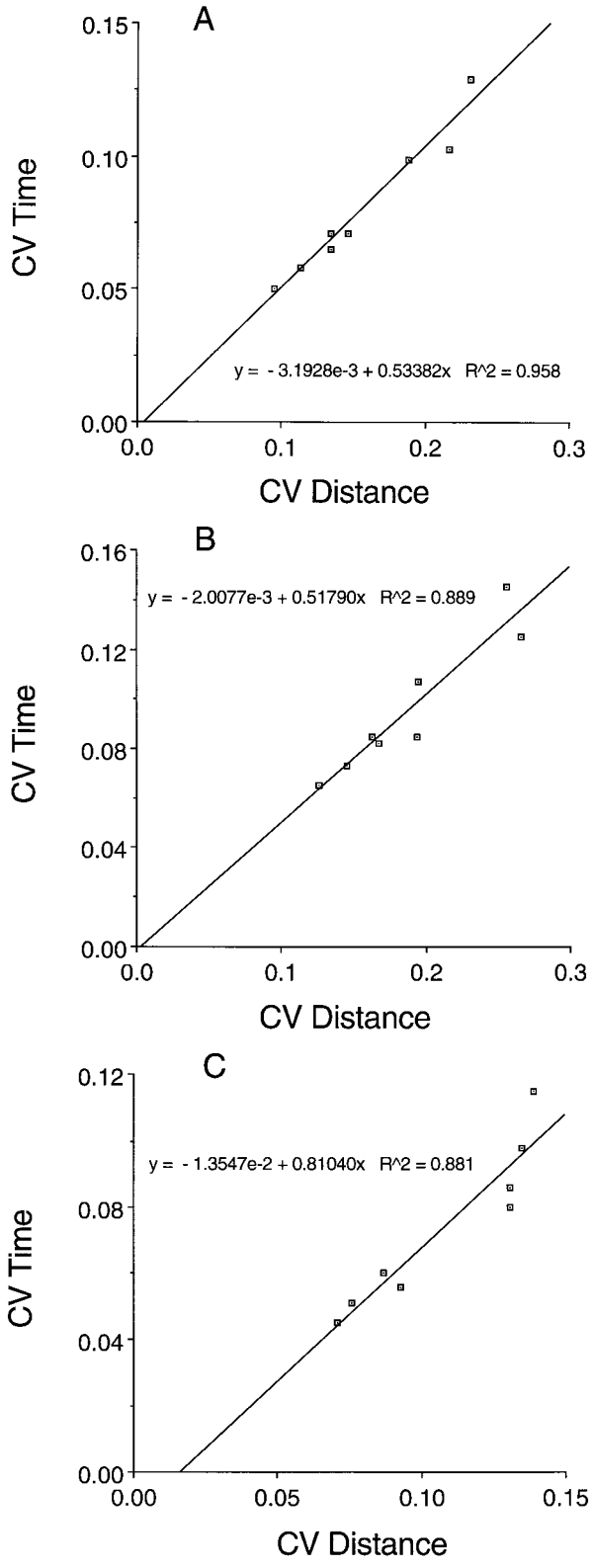

Figure 1 (Carlton & Liu). Relationship between relative time error and relative space error from Newell et al. (1993a). Panel A = Experiment 1 (30 degrees), Panel B = Experiment 2 (10 degrees), Panel C = Experiment 2 (50 degrees).

This influence remains despite extensive practice, with subjects learning to use the sensory information available during movement acquisition more efficiently.

Plamondon & Alimi (P&A) present a model that provides an account of speed/accuracy trade-offs and predicts kinematic characteristics of rapid goal-directed movements. Similar to recent theories of speed/accuracy trade-offs (Meyer et al. 1982; 1988; Schmidt et al. 1979), Plamondon's (1995a; 1995b) model is primarily open-loop in nature. That is, "sensory feedback is not used to control the trajectory," but rather, "advance information from sensory events is used as a feedforward control to adjust the properties of the neuromuscular system with respect to the task objective" (Plamondon 1995a, p. 296). An assumption of the model is "that linearity and feedforward control emerge through practice and learning" (Plamondon 1995a, p. 296). We take issue with these assumptions of Plamondon's kinematic theory of rapid human movements. Evidence from our labs and elsewhere indicates that when visual response-produced feedback is available, it is used to modify aiming trajectories and reduce error. Moreover, recent studies indicate that rather than becoming less dependent on visual feedback with practice, improved skill at aiming involves learning to use visual feedback more rapidly and efficiently.

Since the classic work of Woodworth (1899), it has been apparent that the elimination of vision during the execution of target-directed movements results in increased movement error and end-point variability. Thus the assumption of the kinematic theory that visual feedback is not needed during movement seems at odds with many studies showing accuracy advantages when visual feedback is available, even for very rapid movements (i.e., 125 msec; Carlton 1992). Studies that have, for example, manipulated visual conditions through the use of occlusion spectacles (Elliott et al. 1995b), removal of ambient lighting (Elliott & Allard 1985; Zelaznik et al. 1983), or extinguishing a graphic cursor from a monitor (Chua & Elliott 1993; Elliott et al. 1995a) have all demonstrated that the presence or absence of visual feedback is a potent influence on the outcome and kinematic characteristics of the movement.

Presumably, the performance advantages observed when visual feedback is available reflect rapid closed-loop corrective processes. This position is consistent with kinematic differences between aiming movements completed with full vision and those in which vision of the effector is eliminated upon movement initiation. Particularly when the instructional set emphasizes accuracy (Carson et al. 1993; Elliott et al. 1991) or when the error tolerance is small (Chua & Elliott 1993; Elliott et al. 1995b), subjects take more time to complete movements when full vision is available than when it is eliminated upon movement initiation. A greater proportion of the movement time is spent after peak velocity. Moreover, if one examines discontinuities in acceleration profiles after peak velocity, such as secondary zero-crossings, these occur more frequently when vision is available during that portion of the trajectory (Carson et al. 1993; Chua & Elliott 1993). Typical velocity and acceleration profiles for a visually closed-loop and a visually open-loop aiming movement by the same subject appear in Figure 1.

Trajectory modifications in the form of secondary velocity peaks and deviations in acceleration have been interpreted as reflective of feedback-based corrective processes (Chua & Elliott 1993). In contrast, P&A question the notion that such trajectory deviations are necessarily corrective in nature and show that secondary peaks in the velocity profile could simply emerge as a result of the neuromuscular dynamics. Thus, an interesting issue here might be how to distinguish between secondary peaks that reflect neuromuscular dynamics and those that reflect corrective attempts. Another issue pertaining to trajectory modifications is the information used to prepare such modifications. According to the kinematic theory, a trajectory modification could come in the form of a secondary movement generated and superimposed on the initial trajectory. Given that movement trajectories fail to exhibit

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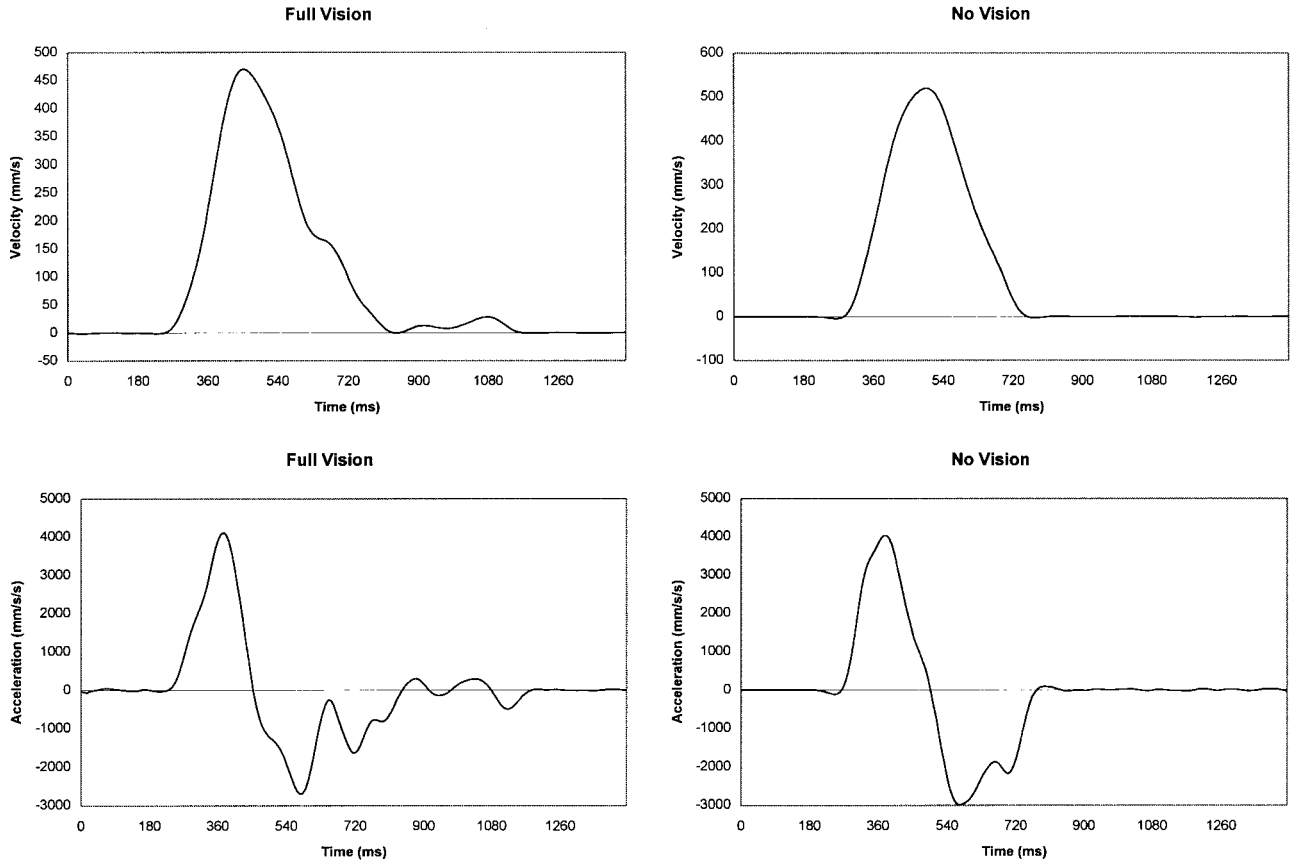

Figure 1 (Chua & Elliott). Velocity and acceleration profiles from a single subject from Experiment 1 of Chua & Elliott (1993). The left panels are profiles from a full-vision condition, and the right panels are from a no-vision condition. Movements were made with a mouse on a graphics tablet to a 1 cm target at a distance of 13 cm (index of difficulty = 4.7). Note the deviations in the full-vision profiles.

periods of "dead time," during which velocity is zero for an extended period of time, and the presence of processing delays, the sensory information required for the preparation of a secondary movement must be derived during the initial movement (Carson et al. 1993; Chua & Elliott 1993). Such a requirement would seem at odds with the kinematic theory's assumption that visual feedback is not needed during the movement.

While one might argue that performance and kinematic differences between visual conditions are transient and disappear after participants have had sufficient practice to develop the type of central representation(s) necessary for feedforward control (Plamondon 1995a; Schmidt & McCabe 1976), there is also evidence to suggest the contrary. Proteau et al. (1987) had subjects practice a simple discrete aiming movement either 200 or 2,000 times with full vision before transferring them to a situation in which vision was eliminated upon movement initiation. Contrary to the view that the learner progresses from a closed-loop to open-loop mode of control with practice, subjects with extensive practice under full vision showed greater performance decrements than those with limited practice. Further, in a study in which the amount of vision available during spatially constrained aiming was manipulated (vision eliminated either 400 or 600 msec into the movement), we found that, regardless of practice condition, subjects quickly adjust their movement trajectories to make maximal use of the visual feedback available (Elliott et al. 1995a). This suggests that if some sort of central representation develops with practice, we have argued that this phenomenon may be a result of subjects becoming more efficient in utilizing feedback or becoming more accurate in their movements in general, or that movement regulation is becoming more continuous in nature. A key point is that performance is nevertheless affected by the presence of visual feedback during the movement.

Although many of the studies we have cited were not intended to specifically examine speed/accuracy tradeoffs, we have proceeded with the assumption that the kinematic theory is meant to be a model of movement production in general. However, it is not entirely clear whether the model holds for feedback-regulated movements. It is our view that any comprehensive account of speed/accuracy relations in goal-directed movement must consider the important role played by vision both prior to and during movement execution. It is through the exclusion of the latter criterion that the kinematic theory fails to fully appreciate the visual regulation of movement.

Are speed/accuracy trade-offs caused by neuromotor noise, or not?

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Abstract: Notwithstanding its overwhelming descriptive power for existing data, it is not clear whether the kinematic theory of Plamondon & Alimi could generate new insights into biomechanical constraints and psychological processes underlying the way organisms trade off speed for accuracy. The kinematic model should elaborate on the role of neuromotor noise and on biomechanical strategies for reducing endpoint variability related to such noise.

The kinematic theory of Plamondon & Alimi (P & A) has a number of attractive features. It incorporates, for example, the fundamen-

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tal antagonistic nature of the motor apparatus, and it uses the idea that the motor system consists of many serially and linearly coupled elements. Both features seem to be inspired by a need to design a biologically realistic theory of aiming behavior. The ΔΛ model is able to fit the velocity profile of a wide variety of ballistic movements almost perfectly, and it is also capable of capturing the empirical results of many studies on spatially as well as temporally constrained aiming movements. In light of these attractive features and their descriptive power, what could possibly be wrong with the ΔΛ model? Perhaps a serious flaw is that the model does not generate *new* insights into motor control theory, in spite of the many assumptions and parameters that the model seems to need. This is not to say that the theory could not, in principle, do so. However, before that stage, some fundamental questions need to be resolved. We will concentrate on two of these issues, the first being that it is unclear how the parameters μ and σ are related to biomechanical control parameters such as stiffness and viscosity. The second issue is the apparent deterministic nature of the ΔΛ model, which does not seem to rely on any inherent noisiness of the human motor system, whereas the latter is such an apparent feature to explain the speed/accuracy trade-off phenomenon. Again, we think that the kinematic theory could indeed generate new insights on these issues, but presently it is silent about them. An answer to the first question is already given by P&A, who state that the kinematic theory gives only a global overview and that further experiments are needed "to provide a direct biological interpretation for the system parameters" (Conclusion). The second question will be discussed below.

Where variability vanishes. Some theories rely on the concept of motor variability, or *neuromotor noise*, to explain the speed/accuracy trade-off seen in rapid aiming movements (Fitts et al. 1954; Fitts & Peterson 1964; Meyer et al. 1988; Schmidt et al. 1979; Van Galen et al. 1995), whereas others explicitly do not (Crossman & Goodeve 1983). The kinematic theory, discussed in the target article, clearly belongs to this last category. The delta-lognormal model (ΔΛ) is, at first sight, completely deterministic and does not need any kind of variability to explain the speed/accuracy trade-off normally observed in spatial- or time-constrained movements. Can it be true that neuromotor noise indeed fails to play any significant role in the control and outcome of ballistic movements? And does the kinematic theory of P&A give us evidence that it does not?

What is neuromotor noise? The term *neuromotor noise* is used in the present context to indicate that the motor system is inherently noisy, like any other dynamic system, and that no part of the motor system, from the neurons in the brain to the motor units in the muscles, ever behaves in a totally predictable way. The reader should note that this interpretation is different from the static interpretation of noisiness of the human motor system according to which the *outcome* of a motor act is unsure. The actual endpoint of an aiming movement is the outcome of a noisy control signal and the noise-reducing properties of the biomechanical system (Van Galen & De Jong 1995). It is well known that humans are not capable of exerting a completely constant force; there is always some kind of uncontrolled variability. In our modeling of goal-directed movements, this phenomenon is equivalent to neuromotor noise. There is, of course, a large difference between such an isometric force and the normal force pulses seen in ballistic movements, but it is likely, and demonstrated in many experiments, that these brief force pulses in ballistic movements contain uncontrolled variability as well (Schmidt et al. 1979; Van Galen & Schomaker 1992).

Is there no noise in the delta-lognormal (ΔΛ) model? At first sight the ΔΛ model does not contain any noisy components. It nevertheless describes the speed/accuracy trade-off in a convincing manner, showing that there is a relation between the minimum movement time needed to reach a target and the maximum allowed relative spatial error (equation 52). This corroborates the major claim of the kinematic theory that the speed/accuracy trade-offs are inherent constraints that emerge directly from the delta-lognormal impulse response of the global neuromuscular system involved in a synergy. So, what kind of system does actually cause this impulse response?

According to Plamondon (1993), the neuromuscular system involved in a synergy consists of many interacting components. All these components react with a slightly different *random* time delay (ϵ_j) to the central impulse command (equation 38), resulting in a bell-shaped velocity profile that can be mathematically described with the lognormal function. We think that this random proportional time delay (ϵ_j) of the individual components does indeed imply that there is some kind of neuromotor noise present, even in the ΔΛ model. Van Galen & De Jong (1995) implemented a computational model, not so different from the ΔΛ model, which does explicitly simulate the effects of this type of neuromotor noise on endpoint variability. Moreover, the model also simulates the noise-reducing effects of stiffness and viscosity. In the present version of kinematic theory, the latter variables are implicitly controlled by the subject through the learned choice of an appropriate D_1 and D_2 (and also the parameters μ and σ). It is not made clear, however, how such an agonist–antagonist regime relates to noise-exaggerating and noise-reducing properties of the motor system. In our view, any new theory of motor control should refer to the optimization of these biomechanical constraints. P&A's theory seems to offer a global explanation of fast-aiming movements in which neuromotor noise is apparently not needed. At the same time, it is also important to note that there is indeed a random variable (ϵ_j) present in the definition of the lognormal function, which could be responsible for the global speed/accuracy trade-off.

In conclusion, an intriguing thing about the kinematic theory of Plamondon & Alimi is the emphasis on the slightly different response delays of the many components out of the motor system. This uncertainty in the timing of the components of the neuromotor systems is very similar to the way others have tried to model the role of neuromotor noise. It is, in the end, this *random* delay in the response to the command parameters that causes the bell-shaped velocity curve and also the speed/accuracy trade-off. The theory does not thereby demonstrate, however, that neuromotor noise does not contribute significantly to this trade-off.

Kinematic theory: From numerical fitting to data interpretation

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Abstract: Plamondon's kinematic theory is very powerful from a descriptive point of view. Unfortunately, the fact that it neglects some fundamental features of the motor system, such as nonlinear inertial torque interactions or joint redundancies, limits its explanatory power and biological validity. As a consequence, the data presented by Plamondon & Alimi should be analyzed and interpreted with caution. There appears to be a gap between the observations reported by the authors and some of the conclusions they draw.

As emphasized by Plamondon & Alimi (P&A), numerous models have been developed during the last decades in the literature dealing with speed/accuracy trade-offs in rapid aimed movements. The short, but rather complete survey provided by P&A shows that most of these models rely on engineering notions such as information theory and control theory. This is also the case with Plamondon's kinematic theory. However, unlike most authors, P&A incorporate memory and learning as putative factors to account for the speed/accuracy trade-offs in rapid aimed movements. When considered together with the ability of the kinematic theory to account for most of the results published in the literature, this feature makes P&A's model very heuristic and powerful. These positive aspects, however, conceal some crucial questions we would like to address in this commentary.

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Our first concern is related to the biological plausibility of the model. Plamondon's theory relies on a quadratic law, which, by adjusting four parameters of the synergies between the agonist and antagonist muscles, accounts for the various speed/accuracy trade-offs in the literature. However, the presence of nonlinear inertial torque interactions in multijoint movements (Hollerbach & Flash 1982) and the nonlinear relation linking arm moment to joint angles (Hogan 1985) make this model into an equivalent circuit rather than a biologically plausible model at the level of the final joint torque control. In fact, like the minimum jerk model (Flash & Hogan 1985), the kinematic theory, which neglects the redundancy problem and focuses on the global output of the motor system, tells us little about the organization of movement at the joint level. The regularities observed by P & A could only reflect higher levels of movement representation, that is, the fact that both the time-varying structure and the path followed by the hand have a direct neural representation for certain types of movements (Flash & Hogan 1985; Desmurget et al. 1997).

Another important point is related to what is usually called "additive theory." According to this theory, "experimental velocity profile[s] can be reconstructed by concatenating and superimposing a few basic units" (target article, sect. 5.3), and "movements can be analyzed by vector superposition of different ballistics units" (sect. 5.4). This assumption, already proposed to account for the eye and arm movement modifications in double step trials (Van Gisbergen et al. 1987; Flash & Henis 1991), and for the arm movement control in single step trials (Crossman & Goodeve 1983; Meyer et al. 1988; Milner 1992), has been challenged by several studies demonstrating the inability of the additive procedures to account for the movement reorganisation observed during double step trials. For instance, Massey et al. (1986) have observed that the trajectory modification observed when target location was modified during movement initiation involved first the application of antagonist forces to break the initial movement and second the implementation of a new movement joining the current position of the arm and the second target. A similar strategy has been reported in several experiments with prehension movement (Paulignan et al. 1991; Gentilucci et al. 1992).

Another major challenge to the additive theory was provided by Pélisson et al. (1986), who demonstrated the existence of smooth trajectory reorganisation when the target position was modified at the peak velocity of the ocular saccade (which corresponds roughly to hand movement onset for synergic eye-hand responses). Contrary to the predictions of the additive scheme, these authors showed: (1) that movement durations were not increased when the target location was modified, and (2) the acceleration and velocity profiles related to the perturbed movements did not exhibit secondary peaks. These results suggested the existence of an internal feedback loop continuously comparing the spatial representation of the target location with the internal representation of hand location, the latter being centrally and/or peripherally derived. Additional support for this view has recently been provided by Hoff and Arbib in a computational study (Hoff & Arbib 1993). The fact that the movement durations observed in Pélisson et al.'s study (1986; see also Prablanc & Martin 1992) were equivalent to the durations recorded in several studies quoted by P&A to illustrate the existence of invariant velocity profiles for "fast movements" (sect. 3.1) makes the foregoing remarks relevant to the present discussion.

Our last and probably most important point concerns the dissociation between feedback and feedforward processes. According to P&A, their theory "can also be useful in providing criteria to discriminate between open- and closed-loop conditions. If the velocity profile of a movement cannot be fitted by a $\Delta\Lambda$ law, . . . this might suggest that the corresponding movement . . . has probably involved some kind of continuous feedback and that the $\Delta\Lambda$ law does not hold in these conditions" (sect. 5.4). The implications of this assertion are two: (1) the velocity profiles that can be fitted by a $\Delta\Lambda$ law involve a purely feedforward control; (2) the velocity profiles observed for ballistic movements (purely feedforward control) should be significantly different from the velocity profiles observed for controlled movements (feedback control). There seems to us to be a long stretch from the application field of the kinematics theory to these conclusions. Indeed, as demonstrated by Prablanc and colleagues in a series of experiments on rapid aimed movements performed without vision of the limb (or more exactly with movements that can be considered rapid according to the examples provided by P&A, sect. 3.1), the final accuracy of the motion is significantly deteriorated when the target is turned off at hand movement onset (Prablanc et al. 1986). Moreover, when the position of the target is modified around movement onset unknown to the subject (see above), the error induced by the perturbation is almost fully corrected, without modification of the shape of the velocity profile (Goodale et al. 1986; Pélisson et al. 1986; Prablanc & Martin 1992). As shown by these results, the simple presence of the target is sufficient to trigger feedback processes, and the shape of the hand velocity profile is not always a relevant criterion for distinguishing between feedforward and feedback controls. These observations cast some doubt on the strong conclusions in P&A's discussion.

In summary, although P&A's kinematic theory is both very heuristic and powerful from a descriptive point of view, its explanatory power remains debatable when one considers the neural mechanisms involved in movement control and generation. In fact, like previous engineering models, the biological validity of Plamondon's theory seems to be limited, as it does not take into account some critical features of the motor system, such as nonlinear inertial torque interactions or joint redundancies. This limitation dictates some caution when extrapolating from numerical agreement to biological functions.

How do neuronal and muscle-mechanical properties contribute to the performance of the "delta lognormal" model?

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Abstract: Plamondon & Alimi's model will gain substantially in credibility when it is able to come up with predictions for new (rather than old) experimental results that discriminate between various models. Moreover, the present model is nothing more than a descriptive "black box," not an explanation for motor performance. A link to the contribution of various neuronal mechanisms involved in motor control and of muscle properties to the performance of the model is crucial.

Plamondon and Alimi (P&A) present a synthesis of their recently developed model and provide a comparison of its performance with that of other models available in the literature. Based on the results presented in their target article, the results of this comparison are favorable for their model. Although this may seem to provide strong support, theirs, like any model, should be treated with a healthy dose of scepticism. For a new model, it is not enough to provide a better fit to the available data in the literature. It should also suggest very specific hypotheses that allow falsification and can discriminate between various models. Hence a prediction of the results of new experiments should follow in order to show that the model can contribute to the understanding of "what might ultimately be responsible for speed/accuracy trade-offs."

P&A's model is based on the assumption that many subsystems are involved in the process of motor control and that the contribution of this large number of subsystems justifies the use of the central limit theorem, which states that the sum of a large number of randomly chosen distributions will converge to a Gaussian distribution. In this context it should be remarked that the central-limit theorem guarantees a Gaussian distribution, but the parameters of the Gaussian distribution (e.g., the mean and standard deviation) depend on the nature of the underlying probability distributions. This might be used to test the model and to relate its performance to various parts of the motor system.

It is of crucial importance for any model of motor control to at some point allow the identification of the contribution of the various (neuronal and muscle-mechanical) components of the motor system. This could be done in various ways. For example, various serial and parallel neural pathways are involved, as explained in the target article. It would accordingly be extremely important to study the parameters of P&A's model in conditions in which different neural pathways are active. A possible experiment might involve movements to visible targets and "remembered" targets. Movements to a remembered target require an internal cuing mechanism that is known (see Cunnington et al. 1996, for an overview) to be related to the involvement of the supplementary motor area (SMA). It would be interesting to see how involvement of the SMA would affect the parameters of the lognormal model.

It is well known that the relative contribution of muscles depends on the type of motor task. For example, the relative contribution of the elbow flexor muscles biceps, brachialis, and brachioradialis is different in isometric contractions and in movements (Gielen & van Ingen Schenau 1992; Theeuwen et al. 1994). As a consequence, one might expect that if the number of active muscles or motor units, or the type of muscle (mono- or biarticular) is relevant for the Gaussian distribution in the lognormal model, the parameters of the lognormal model might be different for isometric contractions and isotonic movements.

Along the same lines, one might expect to find different parameters for the lognormal model for periodic movements at various frequencies. This follows from experiments in cats (Smith et al. 1980) that demonstrated that in fast repetitive movements of the hind limb, the fast gastrocnemius muscle is active and that m. soleus is quiet, whereas in slow movements it is the other way around. Other evidence in favor of different contractile properties of muscles at different movement frequencies comes from recent experiments (Gielen et al. 1994; van Bolhuis et al. 1997), which have revealed differences in recruitment order in isometric contractions and repetitive movements. Since it is well known that each muscle has a large number of motor units, each with different contractile properties, a different distribution of active motor units will result in different contractile properties. Motor units with a low isometric recruitment threshold are not fatiguable and have twitches with a relatively small amplitude and long duration. Motor units with a high isometric recruitment threshold are fatiguable and have motor units with large twitches with short twitch contraction times. Since it has been demonstrated that the recruitment order of motor units is different in isometric contractions and in lengthening contractions (Nardone et al. 1989), one should expect differences in contractile properties of muscle in these conditions. This indicates that the properties of muscle are not the same but can vary in different conditions.

These observations indicate that some of the basic assumptions that underlie the lognormal model (e.g., the time-invariant properties of the system) may not be justified. Since the present version seems to give a better fit than previous models, this is a step forward. However, before it can give an ultimate explanation for human motor performance based on the neurophysiological and muscle mechanical properties, we have to go a long way, which may eventually lead to a different model.

Accuracy and variability of the movement in Fitts' and Schmidt's laws

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Abstract: In Schmidt's experiments, only properties of actually produced movements systems are measured; in Fitts' experiments, external task parameters are measured too (target size and distance). Thus, the laws contain variables of different natures and cannot be reduced to each other even formally. These difference especially reveals itself in modeling: a model of variability can be simpler if it deals with the performance variables only. On the other hand, modeling Fitts' law, one should take into account not only the human effector system but features of the experimental apparatus as well. These differences have not been sufficiently reflected in Plamondon & Alimi's target article.

The number of topics mentioned in Plamondon & Alimi's (P&A's) target article is very large, so we will make only three observations:

(1) Concerning prediction from "the kinematic theory of Plamondon" (see sect. 3), we would like to make a terminological observation about the use of the words "theory" and "prediction." Webster's Dictionary gives the word "hypothesis" as a synonym for the word "theory." If P&A's paper contains a hypothesis, it also should contain predictions based on it that can be rejected or affirmed. Indeed, there are plenty of predictions in the target article (the word "predict" is used about 70 times), and all of them are affirmed. As it can be understood from the article, P&A have predicted already published results, that is, confirmed "postdictions" rather than predictions (while traditionally, predictions are followed by experiments). This can confuse a reader, especially at the beginning of the article.

(2) The second observation is about the nature of the notions "accuracy" and "variability" (which, alas, also can be considered terminological). Unfortunately, the difference between these notions is repeatedly ignored. Accuracy can be estimated from one measurement; variability characterizes a set of measurements. In Fitts' law [$MT = a_1 + b_1 lg_2(2D/W)$, MT is movement time, D is the movement distance, W is target size, a_1 and b_1 are some constants], there is a dependence between movement accuracy and movement time. Schmidt's law ($SD = a_2 + b_2 D/MT$, SD is distance standard deviation, a_2 and b_2 are some constants) describes a dependence between movement variability and movement velocity. In Fitts' experiments, parameter W is the external task parameter (target size); in Schmidt's experiments, SD describes properties of the studied human movement system. Thus, the laws contain variables of different natures and cannot be reduced to each other even formally. This difference reveals itself especially in modeling: A model of variability is simpler; it deals with the actual moving system only, and SD comes out as a parameter of the set of measured movement distances. Vice versa, modeling Fitts' law, one should take into account not only the human moving system but part of the experimental apparatus (the target with its parameter, D and W), and its perception by the subject. These differences have not been sufficiently reflected in the target article, although it was expected from the title.

Just combining the known formulae, one can come to a new result. On the one hand, this result is a tautology; on the other hand, it could be interpreted in an important and unexpected way. However, the usefulness of the new formula needs to be demonstrated. For example, excluding distance from the pair of equations given above, we can obtain a new law connecting target size, distance standard deviation, and movement time:

$$aW/(SD - b) = 2^{-MT/c}\, MT,$$

where a, b, and c are constants. Analysis of this formula is still ahead.

(3) The third observation concerns the analytical expression modeling a measured dependence. A rich collection of such expressions fitting trajectories of reaching movements was represented by Plamondon et al. (1993). P&A prefer to work with their own expression, because it fits the experimental curves better than others. However, a comparison with other expressions cannot be made only by the number of parameters, as is attempted in the target article. A more adequate criterion should combine accuracy of fitting with the number of parameters, analogous, for example, to Akike's criterion used in linear regression. What is more important, the expressions (i.e., the models) have to be referred to the biomechanical and physiological ideas underlying the formulae. Unfortunately, even for their own model, P&A have not explained the idea of "the asymptotic behavior of a large number of coupled linear systems, from which a delta-lognormal law can be derived" so as to make it understandable for readers. However,

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if a reader does not understand the main issue of the theory, the matter of formalization used is lost and any discussion is moot.

In conclusion, P&A write that their kinematic theory "encompasses both similarities and dissimilarities with respect to the other models previously published. . . . unlike . . . (Hogan 1984) . . . (Uno et al. 1989), . . . (Feldman 1966; 1986; Polit & Bizzi 1979) . . . [and] (Bullock & Grossberg 1988), . . . kinematic theory provides a global view. . . . The price to pay for such generalization is that it is difficult, without further experiments, to provide a direct biological interpretation for the system parameters . . ." (sect. 6, para. 4).

Unfortunately, the price for "such generalization" is higher. If a view is too global and generality is too high there is the risk of saying nothing about everything. P&A's terminology is insufficiently accurate.

Neural models of reaching

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Abstract: Plamondon & Alimi (P&A) have unified much data on speed/accuracy trade-offs during reaching movements using a delta-lognormal form factor that describes "the asymptotic behavior of a large number of dependent linear systems," notably neuromuscular systems. Their approach raises questions about whether a large number of systems is needed, whether they are linear, and whether the results disclose the neural design principles that control reaching behaviors. The authors admit that "it is difficult . . . to provide a direct biological interpretation for the system parameters" (sect. 6, para. 4).

The VITE model (Bullock & Grossberg 1988) of neural trajectory formation implies Fitts' law, and various failures, as emergent properties of trajectory dynamics. VITE was derived to explain how motor *synergies* form, and how synergies contract *synchronously* at variable *speeds*. These three S's (synergy, synchrony, speed) of reaching behavior imply Fitts' law, as well as asymmetric velocity profiles and their invariances. They do so using a single, weakly nonlinear system rather than a large number of linear systems.

VITE multiplies a difference vector (*DV*), which codes the difference between desired target position and an outflow representation of present position – and a volitional GO signal. Are there other neural systems that use *DV*-style computations and that are cascaded together to provide multiple VITE-like contributions to Fitts' law, none of which involves neuromuscular computations?

The VITEWRITE model (Bullock, Grossberg & Mannes 1993) embeds VITE into a movement-planning circuit for generating handwriting movements. The script letters are an emergent property of circuit interactions that enable writing to preserve its form as volitional acts flexibly change its size or speed. The script letters have an invariant representation as a spatial pattern of synergy-controlling *DV*s that are stored in a working memory. As in VITE, GO volitional signals can alter their speed of execution. GRO volitional signals alter their size by multiplying the DV that is read out of working memory; this product is then input to the VITE circuit. Feedback from VITE to working memory releases the next working-memory *DV* only when the VITE *DV* is maximal or zero. Complex data about stroke coordination, such as the "two-thirds power law" of Lacquaniti et al. (1983), arise as emergent properties of these feedback interactions. Nowhere does the circuit need the virtual targets or minimization principles that the authors mention.

*DV*s also occur during visually guided control of motor-equivalent reaching to targets in space. The direct model (Bullock, Grossberg & Guenther 1993) shows how accurate reaches can be made with novel tools of variable lengths, clamped joints, distortions of visual input by a prism, and unexpected perturbations. The coordinate transformations from retinal, to head-centered, and finally to the body-centered coordinates that control reaches also use *DV* computations. Why are *DV*s so ubiquitous in the spatial planning and motor execution of reaches?

We propose that this is the correct computational format for autonomously learning the coordinate transformations and movement parameters that keep sensory-motor coordination accurate within a growing body (Grossberg et al. 1993; Guenther et al. 1994). P&A note that their approach "does not provide too many clues about the learning process itself."

P&A say that VITE does not describe "the mechanical properties of the muscles." This is because VITE concerns itself with outflow positional control. The FLETE model (Bullock & Grossberg 1991) links outflow VITE commands to spinal and cerebellar circuits that maintain positional accuracy of contracting muscles under variable tension. FLETE models identified spinal and motor components, such as Renshaw cells and gamma motoneurons, and simulated the multiple velocity peaks during ballistic movements (Bullock & Grossberg 1992) which P&A consider "one of the most powerful characteristics of" their model. VITE has since been extended to a model circuit for controlling reaching movements of variable speed and force in the presence of obstacles (Bullock et al. 1997). This model simulates the neurophysiological firing patterns of six identified cell types in cortical areas 4 and 5 during a wide variety of behavioral tasks.

P&A mention Weber law control of timed movements. A model of learning in the cerebellum describes how metabotropic glutamate receptors, acting at cerebellar Purkinje cell spines, may control adaptively timed learning that obeys a Weber law (Fiala et al. 1996).

In summary, whereas Plamondon & Alimi provide a stimulating account of how speed/accuracy data may arise from delta-lognormal processing, recent neural models of reaching behavior provide an alternative view of the design principles and nonlinear mechanisms whereby these data may arise as emergent properties.

Where in the world is the speed/accuracy trade-off?

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Abstract: Even though Plamondon's kinematic model fits the data well, we do not share the view that it explains movements other than ballistic ones. The model does not account for closed-loop control, which is the more common type of movement in everyday life, nor does it account for recent data indicating interference with ongoing processing.

Plamondon & Alimi (P&A) state two specific goals. The first is to demonstrate the absence of a cohesive account for aimed movements; the second is to advance Plamondon's kinematic theory as such an account. In general, P&A are successful with respect to these goals and are therefore to be congratulated. However, we have a number of questions, which principally concern real-world application of the findings, consistency with other data, and a potential weakness of the model itself.

It is our contention that P&A's work only relates to a very small and generally atypical segment of the full spectrum of movement capability. Only under highly constrained and artificial conditions, such as in the experimental laboratory or at sporting events, does any individual regularly engage in ballistic movements occurring at or near their maximum movement velocity. Very few daily skills require performance at the levels of velocity and accuracy typical in the cited research (although an obvious exception is keyboard-

ing). P&A are appropriately prudent about claims of improved curve fitting of the order of 1% to 2%. However, the form of behavior at hand represents perhaps 1% to 2% of the range of potential movement capabilities and the limit on generality is therefore indeed a critical one.

P&A refer to these movement as well-learned. Relatively, they are not. If any one individual research participant engaged in more than perhaps two hundred such movements it would be unusual. Compare this to the millions of repetitions at lower velocities for such actions as reach and grasp. Of course, P&A can protest that the latter movements are accounted for by their model, but there is a further objection to be raised: one hallmark of skill is energy minimization. Given this, high-level skills may well be composed of a single agonist burst, in which the constraints of the environment and/or the object/tool at hand could perform the antagonistic function. Where does this leave a model that depends crucially upon the interplay of signals from both agonist *and* antagonistic muscles to achieve the desired outcome? The absence of a D_2 signal in this situation only further delimits the task domain explained by the kinematic model.

Also, what is the nature of the outcome being accounted for? In certain conditions it is absolute error (AE), but AE is a derivative of the first two distributional moments and reflects neither in a pristine condition. Although P&A are right to focus on the relative accuracy of movement, they limit themselves to a combinatorial reflection that itself can be problematic (Newell & Hancock 1984; Schutz & Roy 1973). Besides, error in the sense of an obvious, gross mistake, rather than merely a close miss, also seems to be beyond the direct purview of the present approach.

P&A ignore various clear indications that slower aiming movements especially rely on closed-loop control in a limited capacity system. Many studies indicate the use of visual feedback and the existence of interference from concurrently performed tasks in aiming tasks. P&A do mention the possibility that deviations from the $\Delta\Lambda$ law might be explained by the use of continuous feedback in some aiming movements. However, they overemphasize the applicability of the Plamondon model in an attempt to show that it can do what all other models could not: explain the data. In line with this contention, P&A's ideas of sequence control in section 5.5 do fit current notions obtained in reaction time studies (e.g., Verwey 1996). However, those studies also show slowing effects of higher level processing on movement execution. Such data cannot be accounted for by Plamondon's kinematic theory, which basically rejects any form of on-line control.

A clear weakness of the model itself is that in section 4.5 P&A suddenly introduce the possibility that, besides D_1 and D_2, the *system* parameters μ_1, μ_2, σ_1, and σ_2 are programmable also. Apart from the biological implausibility of changing these parameters, adding the modifiability of these parameters as well would provide the model with sufficient degrees of freedom to explain virtually any movement outcome.

Finally, P&A are obviously enthusiastic proponents of the mathematical form of description. Indeed, they consider only explanatory constructs that use this notation. For example, they conclude, "None of the theoretical explanations proposed to date is able to take into account the major experimental observations in the field under a single scheme" (sect. 3, para. 1). This statement is true if only mathematical descriptions are considered. However, we would claim that the descriptive theoretical structure proposed by Hancock and Newell (1985) over a decade ago provides such an initial description and indicates the sequence of connections that must occur between intention, muscular activation, kinetic and kinematic descriptions, and related subsequent outcome. What was also avoided in the latter work was spurious theorizing about causation, relying upon underdeveloped constructs such as "noise" to redescribe the phenomena in different terms. It is unfortunate therefore, that P&A relapse into this "explanation" at a critical juncture of their work.

While we have focused upon some issues of concern, our comments should not be taken as overly critical. In general, the work is soundly conceived and presents new insights to the problem. It is in consequence a good step forward in understanding the intrinsic problems of movement control, and we applaud the effort.

Neuronal and muscular correlates consistent with Plamondon's theory: Velocity coding and temporal activation patterns

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Abstract: This commentary cites several findings of neuromuscular research that are consistent with aspects of Plamondon's kinematic theory. In addition, we point out certain biomechanical properties of the limb that influence the requirements for the production of accurate movement, and might thus compromise the global applicability of any law governing speed/accuracy trade-offs.

Plamondon & Alimi's (P&A's) idea that velocity is one of the main control parameters in the production of movements is consistent with several findings in neuromotor control research. In a study by Georgopoulos and colleagues (1992), the motor cortical population vector was found to be related to the change in force rather than to the force exerted by the subject, suggesting that cortical neurons are concerned with phasic commands. At the level of force output, Hollerbach and Flash (1982) have identified two distinct components of joint torque: a "gravity" torque with an invariant contribution to movements of different speeds, and a "drive" torque, scaling quadratically with linear changes in velocity. Accordingly, Flanders and Herrmann (1992) found two independent elements of muscle activation, whose weighted sum could account for the EMG signals observed during arm movements of different speed. While one "tonic" component had a constant weighting coefficient at all speeds, the other "phasic" component's coefficient scaled monotonically with velocity. The "drive" torque, or, concurrently, the "phasic" EMG component, could thus be interpreted as the result of a motor command concerned primarily with the control of velocity.

Similarly, certain temporal aspects of muscular activity patterns are consistent with P&A's notion that the delay time, with which a muscle is activated, depends on whether the muscle acts as an agonist or antagonist in the movement: Flanders and colleagues (1996) showed that the timing of the phasic EMG activity of human arm muscles during reaching gradually changed as a function of movement direction, from an early burst for the agonist direction to a later burst for the antagonist direction. This pattern of temporal shifting with direction persisted under dynamic isometric conditions, where it did not represent the mechanically optimal solution (Pellegrini & Flanders 1996). These robust features of muscle activation could thus be the result of a central neuromuscular control strategy like the one proposed in the target article.

Any theory attempting to explain the ubiquitous phenomenon of speed/accuracy trade-offs with a global law governing neurally generated movements will eventually have to be tested under broader conditions than those involved in the standard Fitts' task. More specifically, the law will have to deal with the nonuniform biomechanical requirements for multijoint movements in different directions and different parts of the workspace that arise from the pattern of the limb's elastic and inertial anisotropy (see Mussa-Ivaldi et al. 1985). In fact, Gordon and Ghez (1987) showed that for arm movements in different directions, subjects scaled their movement times in order to compensate for the differences in initial acceleration arising from the limb's inertial anisotropy. Soechting et al. (1995) found that for pointing movements, the arm's final posture depended on its initial posture in a way best

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predicted by the assumption that the work necessary to transport the arm from the starting position was minimized. It would thus be a rather profound finding if speed/accuracy trade-offs remained unaffected by these biomechanical factors and continued to obey one global law. It would therefore be interesting to examine whether endpoint errors also depended on initial arm position and whether speed/accuracy trade-offs can be described by the same law regardless of initial position.

What is Fitts' law about?

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Abstract: Movement time and accuracy, as defined by Plamondon & Alimi, do not conform to empirical definitions. When definitions are used that conform better to empirical ones, the original predictions of the kinematic theory are no longer valid – as is demonstrated by simulations. Thus the theoretically derived quadratic law and the successful empirical quadratic law seem to be independent of each other.

Fitts' law describes the relationships between movement time, movement amplitude, and accuracy under a variety of conditions. The generality requires an account that relies on fairly universal mechanisms, and the kinematic theory from which Plamondon & Alimi (P&A) derive their quadratic law seems to satisfy this requirement. Moreover, the quadratic law provides a more accurate fit to various experimental data than the power law, which has been derived from the model of Meyer et al. (1988) and the original logarithmic law of Fitts (1954). (As an aside, the fit of the quadratic law must be at least as good as that of the power law, because the latter is a special case of the former.) Given this state of affairs, it seems that the experimental data offer support for the kinematic theory. Nevertheless I shall argue that the successful empirical quadratic law is one thing, the theoretical quadratic law as based on the kinematic theory is another, and the two have little or nothing to do with each other. The reason is that the theoretical and the empirical quadratic laws are about different things, at least as far as movement time and accuracy are concerned.

What is movement time? P&A take $T_1 = t_1 - t_0$ as movement time, where t_0 is a parameter of the kinematic theory and t_1 is the time of the first zero crossing of the velocity profile. This definition of movement time differs from how movement time is defined in experiments, both at the start and at the end of a movement. At time t_0 there is not yet an overt movement which only begins at time t_r. Therefore, in footnote 4, P&A also refer to $t_1 - t_0$ as activation time and state that it should be proportional to $t_1 - t_r$, for which the start of the movement is defined in a more conventional manner. At time t_1, if it exists, the theoretical movement reaches its peak amplitude and reverses to approach the final amplitude $D = D_1 - D_2$. In equation (47) P&A state that the peak amplitude is proportional to $D_1 - D_2$.

Empirically, the end of a movement is typically not defined by a zero crossing of the velocity profile. When a stylus is moved from a home position to a target, contact with the target occurs at some point after peak velocity and before it is down to zero, with a larger velocity at impact for larger targets (cf. MacKenzie et al. 1987). In movements without a physical target to be hit, the end of the movement can be defined in terms of a tolerance range for acceleration (e.g., Heuer 1981) or velocity (e.g., Meyer et al. 1988); the end is defined as the time at which the signal enters the tolerance range and does not leave it for a defined duration.

To test the claim that $t_1 - t_0$ is proportional to $t_1 - t_r$ and to examine the eventual robustness of the theory's predictions against different definitions of movement time, I have run a number of simulations. The parameters were those given by Plamondon (1995a, Fig. 7), except that I have set $\sigma_2 = \sigma_1 = 0.3598$ to make the power law applicable, and that I have varied D_1 in somewhat irregular steps between 500 and 8 with $D_1 - D_2 = 7$. The results are shown in Figure 1.

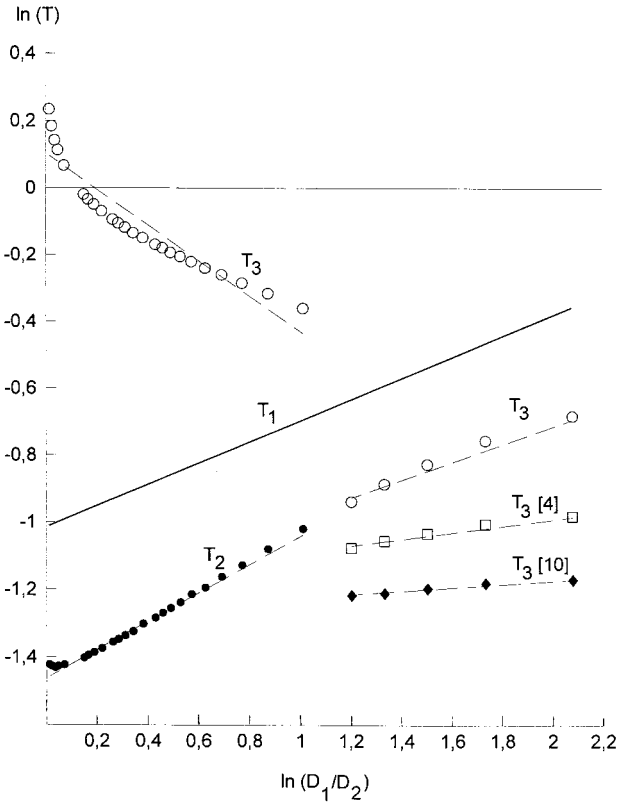

Figure 1 (Heuer). Movement times defined in different ways as a function of $\ln(D_1/D_2)$. $T_1 = t_1 - t_0$, $T_2 = t_1 - t_r$, $T_3 = t_e - t_r$; t_r is the start of the movement, t_e is the end of the movement as defined by a tolerance range for velocity.

For $T_1 = t_1 - t_0$, only the fitted straight line is shown, because the data points do not deviate from it (as, of course, they should not). For $T_2 = t_1 - t_r$, with t_r being defined as the time at which velocity exceeds a threshold of 0.3, the data points show some deviation from a straight line, but the fit is still reasonably good. (The line does not extend beyond $\ln(D_1/D_2) = 1.1$ because for higher ratios the negative velocity did not exceed a tolerance range of -0.3 and is thus negligible.) Across the 24 simulations there was a linear relation between T_2 and T_1 ($r^2 = .99$), but with almost 100 msec the intercept was nonnegligible ($T_1 = 0.0995 + 1.1428\ T_2$). The good fit of the power function for T_2 results from the fact that there is again a linear relation between the logarithms of T_1 and T_2, $\ln T_1 = 0.119 + 0.775 \ln T_2$, $r^2 = 0.99$. Although T_2 exhibits the same type of relation to D_1/D_2 as T_1 for constant $D_1 - D_2$, as shown in Figure 1, it does not remain constant – as T_1 does – for a constant ratio D_1/D_2 when the difference $D_1 - D_2$ is varied. This is illustrated in Table 1.

Figure 1 also shows the relation of $T_3 = t_e - t_r$ to the ratio D_1/D_2. The time of the end of the movement t_e is defined as the time at which velocity enters a tolerance range of $\pm$ 0.3 and stays within this range for at least 20 msec; T_3 [4] and T_3 [10] have equivalent criteria for the end of a movement, but with tolerance ranges of $\pm$ 4 and $\pm$ 10. For T_3 it is apparent that there is an optimal setting of the ratio D_1/D_2 such that overshoots are so small that the tolerance range is not left again. Only for movements without a noticable overshoot or no zero crossing of the velocity profile does T_3 increase as the ratio D_1/D_2 increases. This increase can again be fitted by a power function with a reasonable accuracy, but it tends to become progressively less steep as the width of the tolerance range becomes larger. Table 1 shows that T_3, again in contrast to T_1, is not constant for a constant ratio D_1/D_2 as the difference $D_1 - D_2$ is varied; for the particular parameter values

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Table 1 (*Heuer*). *Different measures of movement time (T_1, T_2, T_3) as a function of amplitude $D = D_1 - D_2$ for $In(D_1/D_2) = 0.773$. Also given is the overshoot (ΔD) at the time of the zero crossing of the velocity profile.*

D	T_1	T_2	T_3	ΔD	$D/\Delta D$
3.5	.472	.322	.669	0.3658	9.57
7.0	.472	.327	.752	0.7316	9.57
14.0	.472	.332	.832	1.4631	9.57
28.0	.472	.336	.912	2.9263	9.57
56.0	.472	.340	.991	5.8526	9.57

examined, the variation across different amplitudes is even stronger than for T_2.

The simple simulations show that the kinematic theory no longer predicts a power law (under parametrizations in which the quadratic law simplifies to the power law) when the definition of movement time is made more consistent with common empirical definitions. In particular, movement time turned out to be no longer invariant against variations of the difference $D_1 - D_2$ for constant ratios D_1/D_2, while power functions still gave a reasonable fit to its variations across different ratios D_1/D_2 for a constant difference $D_1 - D_2$.

What is accuracy? Empirically, Fitts' law refers to target width (with percentage of hits being roughly constant) or to a measure of variability of movement end points, sometimes referred to as effective target width. The important point is that, empirically, accuracy is measured in terms of variability; this can be done directly or indirectly, as when a target width is given that constrains the variability of movement end points in such a way that only a few movements miss the target. Variability, however, is unknown to the P&A model, and this is a feature that P&A's theory shares with other accounts of Fitts' law, such as the deterministic iterative-corrections model of Crossman and Goodeve (1963/1983) or the vite model of Bullock and Grossberg (1988).

I have tried to find a particular theoretical reason for the constraints specified in P&A's equations (48) and (50) (see also Plamondon 1995b, equations (16) and (18)). If I got it right, the reason could be seen in the overshoot ΔD that is present at the zero crossing of the velocity profile. The overshoot increases as the ratio D_1/D_2 decreases (or as D_1 and D_2 both increase with a constant difference $D = D_1 - D_2$). Thus the ratio $\Delta D/D$, which corresponds to the ratio of target width to target amplitude W_T/A_T (with A_T measured to the nearer edge of the target), imposes a lower bound on D_1/D_2. For a constant ratio D_1/D_2 the relative overshoot $\Delta D/D$ seems to be constant, as is illustrated in Table 1. Replacing $\ln(D_1/D_2)$ by $\ln(D/\Delta D)$ requires a linear relation between them for the quadratic law to hold also for $D/\Delta D$. In my 10 simulations with $\ln(D_1/D_2)$ in the range between 0.431 (large overshoot) and 1.012 (small overshoot), I found a linear relation with $r^2 = .99$. Thus, with the overshoot as a measure of accuracy, one could relate the theoretical term D_1/D_2 to the task constraint $A_T/W_T = D/\Delta D$.

As a measure of accuracy, the overshoot does not correspond to empirical measures of accuracy, and with a given target width, movements do not end systematically at the far edge of the target. Thus, as with movement time, there is also a discrepancy between the theoretical and the empirical definitions of accuracy. It seems that it is impossible in principle to account for a fundamentally stochastic phenomenon like the speed/accuracy tradeoff in deterministic terms (cf. Heuer 1991). A possible alternative could be to assume that D_1 and D_2 are random variables with a systematic relation between variability and expected value, and that A_T constrains the expected amplitude, $E(D) = E(D_1) - E(D_2) = A_T$, while W_T constrains the variability of endpoints, $k\sigma_D = W_T$. Such a stochastic version of the kinematic theory could also take into account that Fitts' law deals with mean movement times, not with durations of individual movements.

In conclusion, the theoretical definitions of movement time and accuracy that Plamondon & Alimi use in deriving the quadratic law do not correspond to the empirical definitions of movement time and accuracy. It could be an interesting exercise to go one step back and analyse velocity profiles of movements under different A_T/W_T conditions in terms of the kinematic model. To the extent that the velocity profiles can be fitted reasonably well, the model – with the addition of the proper definitions of start and end of a movement – should be able to predict the speed/accuracy tradeoff. Through such a procedure one might get a better idea about parametric variations across conditions that might be more complicated than variations of $D_1 - D_2$ and D_1/D_2, and one could also obtain some idea about the sources of variability within each condition.

Individual data and clear assumptions about movement

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Abstract: It is important to study movement data from *individual* subjects rather than by averaging data across subjects or trials, because averaged data may follow different laws than those followed by the individual data. This fact can be shown mathematically. In addition, clear assumptions and a thorough understanding of their consequences are a necessary component of any realistic model.

Importance of individual data. If a coin were flipped many times and the resulting data were averaged rather than listed individually, a scientist might conclude that a flipped coin typically lands on edge, balancing halfway between "heads" and "tails." Similarly, if movement data from a number of individuals are averaged, then the average may follow a law different from that followed by the individual neuromuscular systems. A strength of Plamondon & Alimi's (P&A's) analysis is the focus on individual neuromuscular systems, with individual variation of parameters allowed within the scope of a single model.

To illustrate the importance of individual data: if a delta-lognormal law is followed exactly by every rapid-aimed movement of every individual, then in general an *average* velocity profile *will not* follow the delta-lognormal law (Fig. 1A). Here, a delta-lognormal curve is P&A's

$$D_1\Lambda(t;\, t_0,\, \mu_1,\, \sigma_1^2) - D_2\Lambda(t;\, t_0,\, \mu_2,\, \sigma_2^2)$$

where

$$\Lambda(t;\, t_0,\, \mu,\, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}(t - t_0)} \exp\left(-\frac{(\ln(t - t_0) - \mu)^2}{2\sigma^2}\right).$$

Data on movement times of individuals are also important. If movements by subjects always follow, for example, the exact "power law" $\ln MT = b_1 + b_2 \ln(D/\Delta D)$ (where MT = movement time, D = distance, ΔD = absolute spatial error, and b_1 and b_2 are constants that may differ between individuals), then in general the *average* movement time over two or more subjects *will not* follow an exact power law (Fig. 1B). P&A use data on individuals – for example, their Table 3, Kerr and Langolf (1977) – in testing models of movement time (although P&A's Fig. 4 apparently uses averaged data).

Future models that aim to explain the physiology of individuals – as opposed to the emergent properties of averages – must also be tested on data from a variety of *individuals.*

Importance of clear assumptions. Do individuals exhibit more contraction of both agonist and antagonist muscles during movements of less accuracy? This is predicted by P&A's assumption

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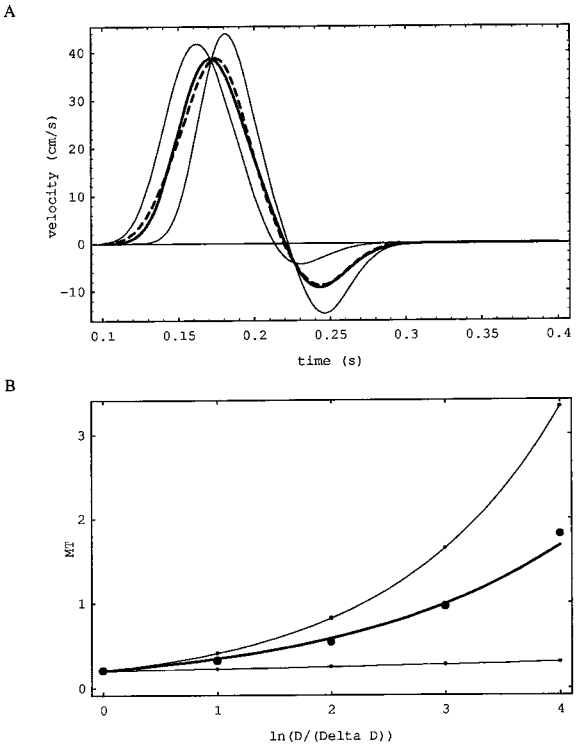

Figure 1 (Holly). (A) The average of two or more delta-lognormal curves is not, in general, a delta-lognormal curve. The average (dashed line) of two different delta-lognormal curves (thin lines) is shown along with an attempted fit (solid thick line) by a delta-lognormal curve. Parameters used for the two original curves are $t_0 = 0$ and: (1) $D_1 = 2.4$cm, $D_2 = 0.34$cm, $\mu_1 = -1.8$, $\mu_2 = -1.5$, $\sigma_1 = 0.14$, $\sigma_2 = 0.09$; (2) $D_1 = 2.0$cm, $D_2 = 0.65$cm, $\mu_1 = -1.7$, $\mu_2 = -1.4$, $\sigma_1 = 0.10$, $\sigma_2 = 0.07$. Parameters for the attempted fit are $t_0 = 0$, $D_1 = 2.17$cm, $D_2 = 0.50$cm, $\mu_1 = -1.75$, $\mu_2 = -1.42$, $\sigma_1 = 0.13$, and $\sigma_2 = 0.08$. Although this particular attempted fit may not be the best, these curves show the type of mis-fit that occurs when any reasonable attempt is made. (B) The average of two or more sets of exact power-law data cannot, in general, be fit by an exact power law. Shown is an attempted fit by a power law (thick line) to averaged data (large dots) from two simulated sets of exact power-law data (small dots), which are shown with their exact power-law fits (thin lines). Movement time (MT), which is being measured or specified in the studies under investigation, is given by the ordinate, while $\ln(D/\Delta D)$, which is linearly related to the index of difficulty, is given by the abscissa. The values of b_2 used for the simulated exact power-law data are 0.1 and 0.7, while the attempted fit uses $b_2 = 0.54$. All use $b_1 = -1.6$. In cases where the two sets of exact power-law data are more alike than those shown here, the fit will be better, but still will not be exact.

arising in section 3.3 that D_2/D_1 is proportional to $\Delta D/D$ (where D_1 = amplitude of agonist command and D_2 = amplitude of antagonist command). In particular, because D is proportional to $D_1 - D_2$ by the delta-lognormal law, P&A's assumption is equivalent to

$$\Delta D = k_1 D - k_2 \frac{D^2}{D_1}$$

where k_1 and k_2 are positive constants. This means that for a fixed target distance D, ΔD increases as D_1 increases (Fig. 2). In other words, accuracy is low when the agonist command is high; the antagonist command is also high in this case in order to maintain target distance D.

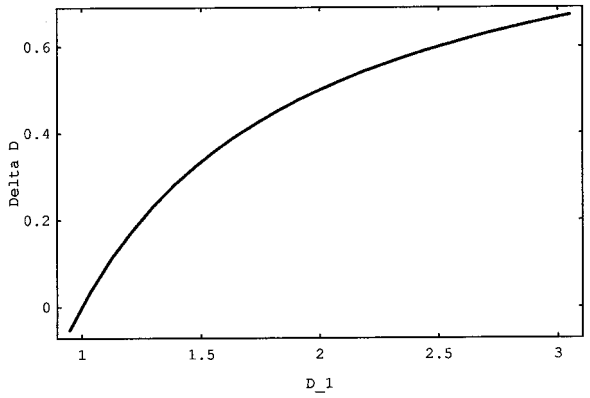

Figure 2 (Holly). P&A's equations predict that rapid movements of greater accuracy are achieved by weaker muscle contractions, and that rapid movements of less accuracy are achieved by stronger muscle contractions, when the target distance D is fixed. As the amplitude of the agonist command D_1 (the abscissa) and the antagonist command increase, ΔD (the ordinate) increases. Although $k_1 = k_2 = 1$ are used here, the trend is the same for all positive values of k_1 and k_2. See text for details.

The origin and justification for the D_2/D_1-proportional-to-$\Delta D/D$ assumption are not entirely clear. P&A's section 3.3 might seem to imply that this assumption follows from the delta-lognormal law, but in fact, proportionality in general of D_2/D_1 and $\Delta D/D$ does not follow mathematically from the proportionality of the single $(D_2/D_1)_{max} = \gamma_2$ and $(\Delta D/D)_{max} = \gamma_1$, at least not without a further (perhaps unstated) assumption. In other words, the delta-lognormal law and the proportionality of D_2/D_1 and $\Delta D/D$ are mathematically independent.

The quadratic and power laws for movement time are subsequently derived from the above proportionality assumption, an assumption that is not physiologically well justified by P&A. However, it is worth noting that, unless an additional assumption is added as discussed above, questions about the validity of D_2/D_1's proportionality to $\Delta D/D$ do not impinge upon the possible validity of the delta-lognormal law for velocity profiles.

At the same time, there are a number of alternative assumptions that would also lead to quadratic and power laws for movement time from a delta-lognormal law for velocity. For example, D_2/D_1 may be proportional to $\sqrt{\Delta D/D}$ or $(\Delta D/D)^2$ instead of $\Delta D/D$. In fact, any power of $\Delta D/D$ would lead to the quadratic and power laws in P&A's equations (61) and (62).

Which assumption, if any, best fits the physiology? Once again, a model – and its pursuit of clear assumptions – can help sharpen our investigation of the physiology.

ACKNOWLEDGMENT

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Flawed kinematic models cannot provide insight into the nature of motor variability

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Abstract: Plamondon & Alimi's derivation of the kinematic model is mathematically flawed. By simply naming a particular parameter combination "variability," the model fails to explicate the sources of variability. As a result, the model cannot distinguish between various sources of error, such as those resulting from task demands and those resulting from movement execution.

Natural variability is an important property of human voluntary movement. Variability of targeted movements has become a subject of study that has provided insights into the organization of human movement in general. In particular, speed/accuracy trade-offs are a primary tool for the analysis of motor variability.

Consider three major aspects of movement generation: (1) perception and comprehension of the task; (2) planning a movement (e.g., internal simulation); and (3) movement execution. Fitts' paradigm implies that a subject slows down when a distant and/or small target is presented. This trade-off depends on factors relevant to the levels of task comprehension and movement planning, so that the trade-off may be affected by changes in instruction and behavioral information. In fact, a proper instruction resulted in peak velocities of movements to a very small target that were nearly twice as high as characteristic velocities observed using the same setup, same distances, and four times larger targets combined with a typical Fitts'-type instruction (cf. Corcos et al. 1988; Latash 1993). Such a modulation with instruction can be modelled, for example, within the equilibrium-point hypothesis as a shift in strategy used to perform the task (Latash & Gottlieb 1990). Behavioral feedback can also be used to modify speed/accuracy trade-offs. In particular, the relation between movement time, distance, and target width depends upon the availability of visual feedback after movement initiation, the adequacy of perceptual information (Flanagan & Rao 1995), the effects of the short-term motor memory (Imanaka et al. 1996; Miall et al. 1995), and so forth.

Because Plamondon's model does not address any of these factors, the account of Fitts' law remains restricted to the level of movement execution. (Below, we shall argue that, even at this level, Plamondon & Alimi's [P&A's] account is flawed). The level of movement execution can contribute to other types of trade-offs, for example to the scaling of the standard deviation of final position with movement distance and time (Schmidt et al. 1979; Sherwood & Schmidt 1980). Thus, P&A's account fails to distinguish between these distinct forms of trade-offs and analyzes them erroneously as a single phenomenon.

Goodman/Gutman were the first to make a clear distinction between two types of trade-offs commonly described in the literature (Gutman & Gottlieb 1992; Gutman et al. 1993; Latash & Gutman 1993). Goodman's kinematic model (ignored in the target article), generates kinematic patterns that fit the data at least as well as Plamondon's model. Goodman's model is much superior in the lack of obvious flaws (see below) and in the clear display of its limitations related to the fact that it reflects hypothetical processes related to only some of the steps involved in the process of movement generation. These limitations are inherent to Plamondon's model but are not explicitly acknowledged or analyzed.

Specific critique of Plamondon's model. Plamondon's model essentially consists of an assumption about the functional form of Fitts' law, which is then fitted to data sets. This functional form is unrelated to the hypothesized functional form of the kinematic law suggested for unidimensional end-effector movement. The discussion purporting to derive Fitts' law from the kinematic law is incorrect and misleading. By definition, the coefficient γ_1 (equation 48 in the target article) covaries with the required relative precision, so the required precision cancels out of the model equations. (In equation 51, the expression $(1/\gamma_1)^*(\Delta D/D)_{max}$ equals 1 and can be dropped.) Thus, assuming proportionality between the model parameter ratio D_1/D_2 and required relative precision $\Delta D/D$ is an independent, additional assumption. Because this parameter ratio (D_1/D_2) has no other meaning in the model and in the subsequent fitting than to express the difference of logarithms of movement time, the power law relationship between movement time and required error is essentially freely assumed, unconstrained by the kinematic model.

The derivation of the kinematic law ("delta-lognormal") from a system-theoretic model is flawed. When the transformation of the time variable in the convolution of the individual transfer functions is attempted, a Gaussian distribution can be derived for the sum of the random variables associated with each transfer function. This sum is a sum of exponentials. The logarithm cannot be interchanged with the sum, and thus the derived distribution cannot be transformed to the claimed form. (In fact, if the proposed trick were to work, any functional form could be derived from the central limit theorem in contradiction with the theorem itself.) There is no simple logical connection between this kinematic law and system-theory. The discussion linking Fitts' law to ensembles of agonist and antagonist subsystems is misleading and the connection between the postulated formula for the Fitts' law and the level of movement execution remains unclear.

The derivation of the mean delay from the assumption of proportional scaling of the delay is correct. The adequate conclusion from this derivation is, however, that the serial subsystem model with this rule imposed on its components leads to a Gaussian distribution with the mean given by the logarithmic formula. The resulting distribution is still purely Gaussian in terms of the time variable, although its mean scales as desired.

Thus, the model presented by P&A is aimed to explain speed/accuracy trade-offs even though it lacks a source of variability! A particular parameter combination in a functional form used to fit trajectories is called variability by assumption. This is one reason why the model cannot distinguish between various sources of error such as those resulting from task demands and those resulting from movement execution. Apart from the mathematical errors in the derivation of the kinematic law, the sources of variability in a system-theoretical account have not been adequately addressed. What are the effects of variability in the timing of the motor command, in the shape of the motor command, its amplitude, and so on? P&A do not even try to address these questions.

Segregation of agonist and antagonist systems minimizes the benefits of polarity

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Abstract: A purely kinematic theory of movement runs the risk of having no explanatory power because it neglects the internal generative structures of the central nervous system. Distributed interaction between the agonist and antagonist systems would better simulate physiological mechanisms of oscillation, lateral inhibition, and synchronization, all of which have important roles in motor control.

Plamondon & Alimi (P&A) are taking great strides toward making the study of motor control an exact science. This in itself is to be applauded, and several features of their theory are appealing. First, the polarized design around opposing agonist and antagonist systems is physiological. Second, the simultaneous activation of agonist and antagonist systems makes a lot of functional sense. Physiologists often take "reciprocal innervation" too far and imagine the systems as operating in strict alternation. In fact that is an extreme case, whereas most EMG records show weak antagonist activity during agonist activity. The full gamut of muscle synergies requires simultaneous activity in both systems to varying degrees. Third, a sequence of limb velocity reversals is shown to potentially ensue from a single pair of impulse commands. This important property is too often overlooked by investigators searching for separate causes to link with every blip in a waveform. Despite these positive features, however, I have major reservations with the overall design of the model.

The goal of the extensive mathematical treatment is to simulate the motion of limbs. To do this with an inadequate understanding of the generative dynamic structure underlying body motion is much like the Ptolemaic modeling of celestial orbits prior to the Copernican/Newtonian revolution. Whether or not we presently have the requisite knowledge is moot, but the use of pure speed/accuracy data as a basis for motor theory is insufficient. The

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resulting simulation lacks variables that can be directly compared to neuronal and muscle activity patterns – or even joint torques – for rigorous testing. The model would be much more powerful if it simulated dynamic physiological elements that are known to generate motor programs.

Transfer functions have never been popular with biologists, who generally find their linearity too restricting and misleading as descriptions of system transformations. Current models of "central pattern generators" favor nonlinear dynamics, especially cellular and network oscillators (Arshavsky et al. 1993; Grillner et al. 1991). When you look into the nervous system, oscillating elements are ubiquitous (e.g., Steriade et al. 1993). It would make the greatest functional sense, therefore, to build a model using nested circuits of oscillators. In effect, the biology says oscillation is the primary goal, whereas the traditional engineering approach has viewed oscillation as an unwanted by-product.

If you stick with a purely linear model, then it makes no difference whether the agonist/antagonist subtraction is lumped at the end or distributed throughout. For simplicity, P&A have opted to keep the systems separate until the final output. Because this segregated approach, as the authors admit, is negated by CNS organization, I suspect that there must be important functional reasons for incorporating interaction at every level. The most obvious reason would be efficiency. By interacting from the start, you do not waste energy in the form of excessive signals to motor units that simultaneously oppose each other. Since the kinematic theory is concerned only with the velocity resulting from subtracted impulse responses at the effector, the absolute magnitude of opposing muscle forces is not considered. Unfortunately this is a recipe for chronic muscle pain and injury. A second benefit of distributed interaction occurs in the form of lateral inhibition to sharpen contours of active zones at every level of pattern generation. In the motor cortex, clusters of neurons related to joint motion in opposing directions are almost routinely juxtaposed (e.g., Kwan et al. 1978). This is no accident: it optimizes interaction either inhibitory or facilitatory, which can improve spatial definition or temporally synchronize populations. In addition, oscillators in the motor system are based on a polarized structure, ultimately representing opposing limb motions. Distributed interaction between opposing poles permits the embodiment of oscillators at every level of the control structure.

In short, the concept of polarized networks opposing one another needs to be broken down into subunits, with the larger constructs mimicking the polarized design of the smaller elements. Such a "self-similar" design is nonlinear, and more stable and efficient than any linear simplification. Agonist–antagonist polarity is an engine that drives the motor system. For maximal effect that engine must be allowed to express itself at every stage.

Furthermore, some brain structures are relatively insensitive to movement direction. The most obvious is the cerebellum (e.g., MacKay 1988; Thach et al. 1992), which may provide temporal precision of impulses to sets of motor elements, leaving it up to other centers to relegate extension or flexion routing. That is, many cerebellar neurons appear to straddle both the agonist and antagonist systems. The kinematic theory should contain provision for metasystemic elements that provide a common function to both systems.

I am inherently suspicious of mathematical descriptions of outputs, however accurate they may seem, if they do not contain an identifiable internal model of the biological system or include variables that can be directly compared to physiological measures such as EMG recordings. When Plamondon's kinematic theory is developed into a dynamic theory that accurately predicts, say, the EMG pattern or joint torques in parallel with angular velocity, then it will be a very useful tool.

Separating *A* and *W* effects: Pointing to targets on computer displays

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Abstract: We address two main issues: the distinction between time-constrained and spatially constrained tasks, and the separable A and W effects on movement time (*MT*) in spatially-constrained tasks. We consider *MT* and 3-D kinematic data from human adults pointing to targets in human-computer interaction. These are better fit by Welford's (1968) two-part model, than Fitts' (1954; Fitts & Peterson 1964) ID model. We identify theoretical and practical implications.

Plamondon & Alimi (P&A) make explicit for readers the two distinct paradigms for the study of speed/accuracy trade-offs: time-constrained and space-constrained tasks. They note that "Fitts relationship was found to be . . . even inappropriate for timely constrained tasks" (sect. 1, para. 3), referring readers to Schmidt (1988) for a survey. Fitts developed his formulation for repetitive (1954) or discrete (Fitts & Peterson 1964) spatially constrained tasks. Our commentary is restricted to the spatially constrained class of target-aiming movements, that is, where subjects try to make aiming movements as quickly and accurately as possible, and movement time (*MT*) is measured, not manipulated by the experimenter.

We have investigated how the constraints of the environment in human-computer interaction (HCI) affect planning and control of discrete aiming movements with the hand. In contrast to interacting directly with physical targets in the work space, in HCI pointing is indirect; hand movements are mediated by an input device (e.g., mouse or trackball), and motion and position are represented abstractly on a graphics screen. These constraints introduce several issues, e.g., the relative locations of hand space and display space, the relative scale of the spaces, and the mapping (often nonlinear) between hand movement and its displayed representation.

In a series of HCI experiments, Graham (1996a) examined pointing on a computer display. The displayed pointer (a small red arrow, 9 × 35 mm) was driven graphically in real time, with less than 25 msec lag, by optotrak 3D position data from markers placed on the index finger. Subjects pointed to displayed circular white targets (varying in width, direction, and amplitude) from a constant starting position (5 mm in diameter). Kinematic analyses of 3-D hand and 2-D cursor motion across different gain conditions revealed that the control was in hand space, not display space. That is, *MT* and kinematic profiles were best predicted by amplitude and target width in hand space, not display space (see Graham & MacKenzie 1995).

In one experiment, pointing directly at physical targets was compared to a virtual condition, where a mirror superimposed the 2-D display image on the workspace. In the virtual condition, the subject saw the target and, in place of the hand, a red arrow that accurately tracked the planar position of the index finger. Figure 1 compares *MT* for physical versus virtual pointing. A marked asymmetry in the magnitude of amplitude (*A*) and width (*W*) effects causes a spread of points within a single ID. A two-part model, modified from Welford (1968):

$$MT = \mathrm{a} + \mathrm{b1}\ \log_2 A - \mathrm{b2}\ \log_2 W \qquad (1)$$

was used to capture this asymmetry. Multiple regression using Equation 1 on *MT* for pointing to physical and virtual targets revealed:

$$\text{Virtual: } MT \text{ (msec)} = 22 + 123\ \log_2 A - 79\ \log_2 W \quad (R^2 = .99) \qquad (2)$$

$$\text{Physical: } MT \text{ (msec)} = -24 + 98\ \log_2 A - 32\ \log_2 W \quad (R^2 = .96) \qquad (3)$$

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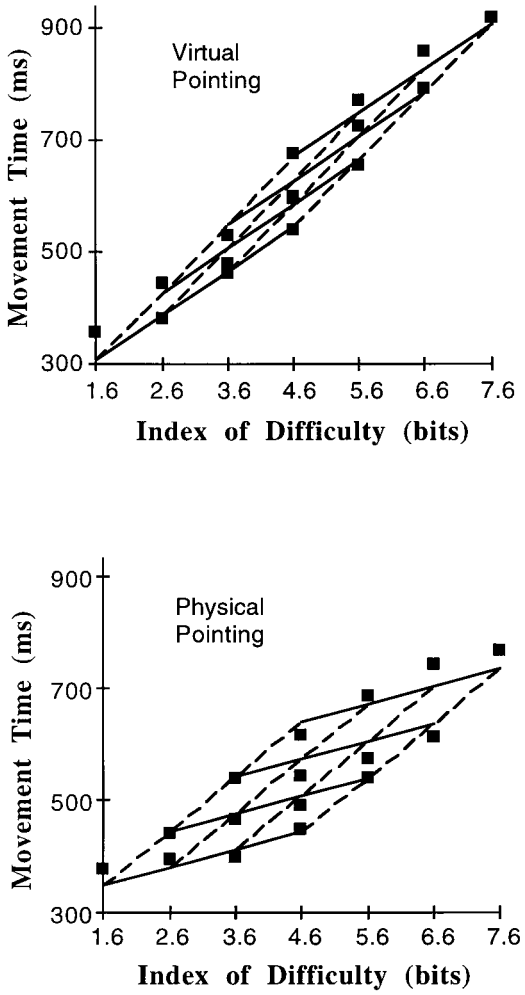

Figure 1 (MacKenzie & Graham). Actual mean movement time (black squares) and *MT* modelled from equations 2 and 3 (lines) for pointing to virtual and physical targets. Dashed lines show the effect of changing amplitude (*A*), solid lines show the effect on *MT* of changing target width (*W*). Note the greater sensitivity of *MT* to *A* than *W* for both physical and virtual pointing.

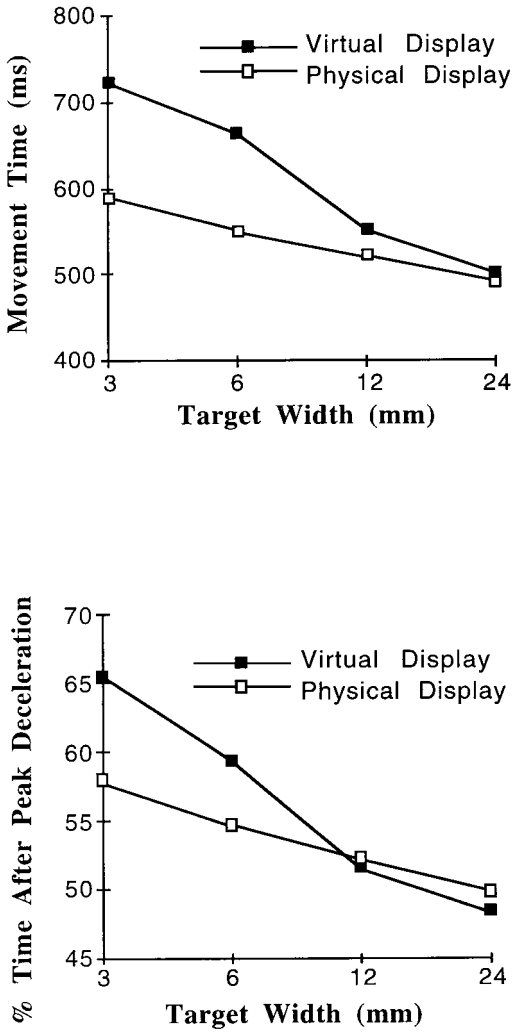

Figure 2 (MacKenzie & Graham). *MT* and percent of time after peak deceleration as a function of target width for pointing to physical and virtual targets.

The kinematics in these experiments replicate and extend our earlier 3-D analysis of discrete aiming movements in a Fitts' task (MacKenzie et al. 1987). In brief: (1) amplitude primarily affects the initial phase of movement (peak acceleration, peak velocity, time to peak velocity), and (2) target width almost exclusively affects the proportion of time in the deceleration phase. Figure 2 shows that the longer *MT* for pointing to virtual than physical targets is attributed primarily to a lengthening of the deceleration phase of the movements.

In a number of studies we have found that the separable effects of *A* and *W* on movement microstructure, (1) and (2) above, are consistent, but do not exactly cancel (as suggested in MacKenzie et al. 1987). Instead, they combine to produce the asymmetric *A* and *W* effects on *MT*, and a spread of data points for each ID as illustrated in Figure 1. It is not clear how P&A's quadratic and power law simulations can be arranged (e.g., their Fig. 3) to produce velocity profiles with the above properties. For example, they predict that "maximum velocity decreases as $D/\Delta D$ increases" (sect. 3.3, after equation 55). There are two ways to increase $D/\Delta D$: increasing D or decreasing ΔD. Using their terms, our interpretation of the data is that maximum velocity increases with increasing D, but is unaffected by decreasing ΔD. P&A state that as $D/\Delta D$ increases, the entire movement slows down, with increasing asymmetry in the velocity profiles. We suggest that the velocity profiles are "skewed" as a function of ΔD only, based on our data.

In conclusion, we wish to make several points. Following Welford's (1968) insight, and the results of other investigators (e.g., Buck 1986; Fowler et al. 1982; MacKenzie et al. 1987; Meyer et al. 1988; Sheridan 1979), we wish to urge other researchers to analyze *MT* and kinematics considering the separate effects of *A* and *W*. This may help to elucidate the underlying motor control processes. The two-part model has implications for design in HCI, where the scale of *A* and *W* for the hand can be altered by choice of control-display gain. For example, Figure 3 illustrates a parameter space for sensitivity of *MT* to changes in *A* and *W*, for four HCI pointing studies (Graham 1996a), showing *MT* more sensitive to *A* than *B*. These data, which all fall below the Fitts' law line, predict that *MT* for discrete, space-constrained aiming movements will be optimized by reducing the scale of the movement environment (both *A* and *W*) for the hand.

ACKNOWLEDGMENTS

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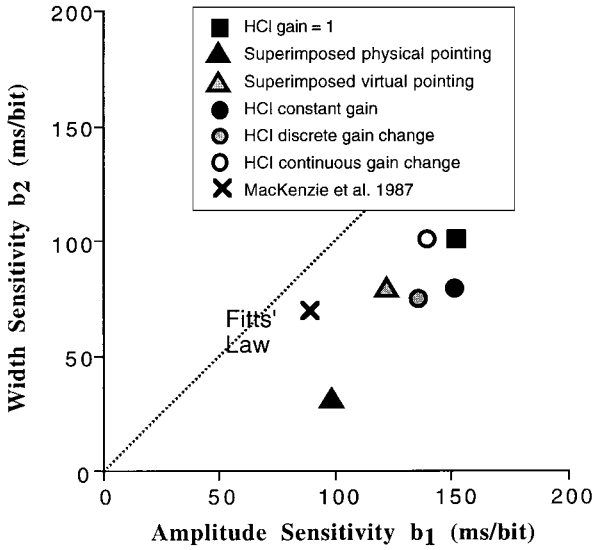

Figure 3 (MacKenzie & Graham). Parameter space showing sensitivity of *MT* to changes in *A* and *W*, from equation 1. Triangles show the results of physical and virtual pointing. For a detailed explanation and interpretation, see Graham (1996a) and Graham and MacKenzie (1996).

to E. D. Graham. Experimental data are from the doctoral dissertation of Evan D. Graham (Graham 1996a). Related published work can be found in Graham (1996b) and Graham & MacKenzie (1995; 1996).

Kinematic models cannot provide insight into motor control

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Abstract: In Plamondon & Alimi's target article, a bell-shaped velocity profile typically observed in fast movements is used as a basis for the "kinematic theory" of motor control. In our opinion, kinematics is a necessary but insufficient ground for a theory of motor control. Relationships between different kinematic characteristics are an emergent property of the system dynamics controlled by the brain in a specific way. In particular, bell-shaped velocity profiles with or without additional waves are a trivial consequence of shifts in the equilibrium state of the system as suggested, for example, in the λ-model of motor control.

Criteria for the selection of a model. In Plamondon & Alimi's (P&A's) target article, a mathematical model of fast movements is proposed, based on the observation of a bell-shaped velocity profile in many motor tasks. Experimental velocity patterns are approximated by a combination of log-normal functions (equation 39; Figs. 2 and 3). This approximation, combined with the assumption that the end-point error is proportional to the movement distance, are the essence of the Plamondon's kinematic theory. The quality of fit of velocity/accuracy trade-off is claimed to be better than in other kinematic models (P&A, sect. 4.1.2 and Table 3). In general, even a perfect fit of experimental curves is not sufficient to accept or reject a model. Plamondon's model introduces a lot of parameters in order to achieve a satisfactory fit. These parameters are not easily interpreted and cannot be reliably identified experimentally. A model, even with a large number of parameters, would be useful if it could predict nontrivial effects. It is doubtful that this is the case for Plamondon's model. Moreover, P&A often claim that the model "predicts" something, whereas actually they just offer an explanation of some kinematic features of prior experimental data. This is very confusing. The term "predictions" refers to new effects, previously unknown, and, in this sense, nothing has been predicted by the kinematic theory. The biomechanical "neuromuscular synergy" (Fig. 1) designed by P&A for the interpretation of the model's parameters can hardly be considered plausible from the physiological point of view.

Bell-shaped velocity profile or kinematics versus dynamics. The major failure of the kinematic model is that it ignores dynamical properties of fast movements. Any movement is produced by muscular forces. The nervous system can change muscular forces by specifying control parameters. Let us consider, from the dynamical point of view, the cornerstone of the kinematic theory, the bell-shaped velocity profile. In the beginning of movement, muscle forces act to accelerate movement and then to decelerate it. This is a dynamic basis of point-to-point movements, leading to a bell-shaped velocity profile. Indeed, the acceleration and deceleration phase may not be precisely balanced, leading to an asymmetry in the velocity profile and additional terminal oscillations. Plamondon's model was designed to describe all these oscillations ad hoc rather than to explain their origin. The statement that "most models, if not all, ignore or neglect secondary peaks in velocity" (sect. 5.3, para. 1) does not seem to be justified. For example, secondary peaks have been repeatedly reported and simulated in almost all studies related to the λ-model (e.g., Flanagan et al. 1993; Feldman & Levin 1995). Terminal oscillations can be suppressed by changing specific parameters of the neuromuscular system. In the λ-model, for example, the coactivation (C) command and a damping parameter (μ) are responsible for the control of terminal oscillations as demonstrated by simulation (Flanagan et al. 1993). In contrast to the kinematic theory, designed to explain neither dynamical phenomena nor electromyographic patterns, the λ-model was able to provide an integrated view of movement production. P&A's opinion that the λ-model takes into account "just [the] mechanical properties of muscles" (sect. 6, para. 4) is incorrect. In contrast to the Plamondon model, the λ-model was not designed ad hoc to describe any particular kinematics or EMG patterns. These patterns emerge as a result of the dynamic interaction between the control signals, afferent feedback, and mechanical components of the system including the environment. In Figure 1, the model of Flanagan et al. (1993) was used to simulate

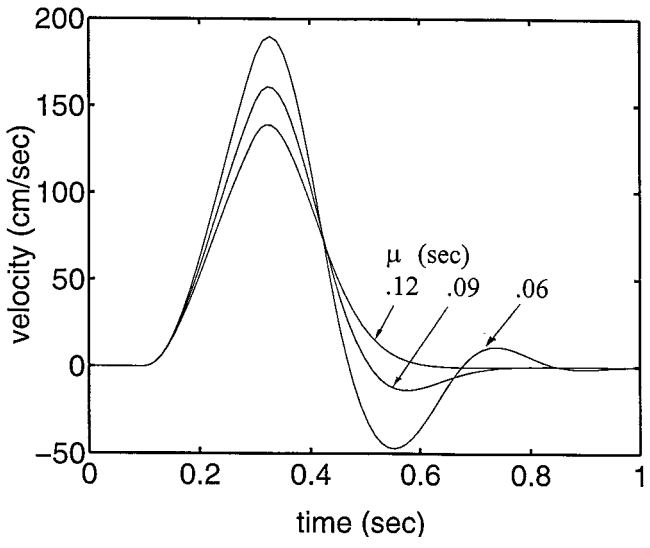

Figure 1 (Mitnitski). Velocity profiles derived for two-joint pointing movement according to the λ-model. A damping factor (μ) was the only parameter changing during the simulation. Note that the number of waves in the velocity profile as well as the peak velocity may be efficiently controlled by this parameter.

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a velocity profile for a two-joints reaching movement. Only one parameter (μ) a damping factor, was varied. The number of peaks in the velocity profiles may vary depending on μ. In addition, the magnitude of the main peak also varies, even though the control signals underlying the movement remain the same. This illustrates that some properties of movements that look very complicated in kinematic theories like Plamondon's may be a simple consequence of the dynamic properties of the system.

Movement dynamics in speed/accuracy trade-off

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Abstract: Fitts' law and the $\Delta\Lambda$ model are "weak" theories of motor control because they are limited to the kinematic aspects of movement and do not capture its essential dynamic nature. The internal source of "noise" that determines the speed/accuracy trade-off can be associated with the partial compensation of movement-generated "parasitic" forces.

Although model fitting and data fitting are frequently confused, they are far apart from the epistemological and pragmatical points of view. Quoting the classical system identification approach (Eykhoff 1974), a model can be defined as "a representation of the essential aspects of an existing system (or a system to be constructed) which presents knowledge of that system in a usable form (p. 1)." This notion of "essential aspects" for a complex system such as neuromuscular motor control is certainly multifaceted and entails qualitative as well as quantitative elements, where the latter may be related to the precise fitting of experimental data and the former include what we may call the "space of behavioral situations." Indeed, even if we limit ourselves to motor actions in everyday life, there is no doubt that the variability of loading conditions, task requirements, and so forth is much larger than for the rapid-aimed movements that form the main topic of Plamondon & Alimi's (P&A's) target article.

In a sense, limiting the study of motor control to the kinematic observation of the highly constrained act of constant-load, overpracticed reaching movement is like looking at a complex picture through a very small hole. In general, the narrower the observation angle for a complex system, the more precise can be the fitting of the observed data but the more unrelated will be the global system structure (the "essential aspects") from the specific fitting model. This does not mean that studying rapid-aimed movements is pointless, but merely that precision fitting of the speed profile per se is too narrow an observation window for inferring anything about the internal structure of the motor control system. The same argument also applies, in our opinion, to the speed/accuracy trade-off, which is indeed compatible with (and has been attributed to) a large and disparate number of explanatory theories, with a related set of fitting models.

In general, we feel that the main point is that Fitts' law is limited to some kinematic aspects of (a small class) of movements and, similarly, the $\Delta\Lambda$ model is a kinematic theory of motor control. Kinematic theories have certainly been motivated by the experimental observation of kinematic invariants in reaching movements (Morasso 1981) and apparently "explained" by equilibrium-point control models (see Bizzi et al., 1992 and Feldman & Levin, 1995, for extensive reviews). The underlying rationale was that the viscous-elastic properties of the muscles are strong enough to "absorb" the dynamic complexity of movement control, thus leaving the brain with a purely kinematic control problem. However, this elegant theory has recently been falsified (or at least its degree of plausibility has been greatly diminished) by the empirical observation that the physiological range of muscle stiffness is too low to compensate for the dynamic forces occurring during everyday movements (Bennett et al. 1992; Katayama & Kowato 1992; Tsuji et al. 1995) unless a complicated process of virtual trajectory planning is involved, which ends up as complex as solving the dynamic problem. In fact, if we consider the notion of "usable form" in the above-mentioned definition of "model," this is an example of a well-fitting (kinematic) model whose control parameters are in practice unusable for dominating the real dynamic phenomena – and the same comment applies to the D, μ, and σ parameters of the $\Delta\Lambda$ model as well.

The dynamic equations of a kinematic articulated chain of a robot manipulator or a human limb

$$I(\vec{q})\ddot{\vec{q}} + C(\vec{q}, \dot{\vec{q}}) + G(\vec{q}) + \vec{\tau}_{load}(\vec{q}, \dot{\vec{q}}, \ddot{\vec{q}}) = J_M^T(\vec{q})\vec{f}_m(\vec{l}, \dot{\vec{l}}, \vec{u}) + J_s^T(\vec{q})\vec{f}_{ext}^{\,1} \quad (1)$$

where $\vec{q} = \vec{q}(t)$ is the desired law of motion in a given time window (with the corresponding muscle pattern $l(t)$), clearly show that the motor command waveform $\vec{u} = \vec{u}(t)$ is supposed to take into account nonlinearities related to accelerations and (squared) velocities of the different joints, which scale quite differently with movement duration, yielding patterns that strongly depend on load and speed. This is, in our view, the main source of "motor noise," and the (approximate) mechanism of compensation is likely to operate "in parallel" with the corticospinal pathways, not "in series," as is implied by the $\Delta\Lambda$ model. In a forthcoming target article in this journal by V. Braitenberg, D. Heck, and F. Sultan, it is argued that compensating for such "parasitic" dynamic forces, that is, internal disturbances generated by our own motion, is one of the main functions of the cerebellar circuitry. Without attempting here to articulate a model of how such compensation can actually take place, we wish to suggest that current knowledge about cerebral/cerebellar circuitry is compatible with a parallel "sculpting" action of the corticospinal patterns, which can at the same time explain the relative kinematic invariances under different loading conditions, and the speed/accuracy trade-offs attributed to pure kinematic effects.

In any case, the accurate fitting of kinematic curves in a variety of experimental conditions by the $\Delta\Lambda$ model is a reference point for any approach to understanding rapid movements in quantitative terms.

NOTE

1. I is the inertia matrix, C is the vector of Coriolis forces, G is the vector of gravity forces, τ_{load} is the vector of load forces, f_m is the vector of muscle forces, f_{ext} is the vector of external forces acting on the end-effector, J_m is the Jacobian matrix of the muscles, and J_s is the Jacobian matrix of the end-effector. While gravity forces do not depend upon movement time, inertial forces scale inversely, and Coriolis forces scale inversely to the square.

Predicting relationships between speed and accuracy of targetting movements is important

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Abstract: While explaining a large proportion of any variance, accounts of the speed and accuracy of targetting movements use techniques (e.g., log transforms) that typically reduce variability before "explaining" the data. Therefore the predictive power of such accounts are important. We consider whether Plamondon's model can account for kinematics of targetting movements of clinical populations.

Fitts' law describes but does not explain relationships between speed and accuracy of movement. It has unfortunately been such a good description that it tends to embarrass any further attempts to explain any additional few percent or so of variance. Indeed Fitts' law to some extent condones the behaviour of cognitive psycholo-

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gists who relegate motor coordination to the province of an irrelevant peripheral device whose behaviour has been adequately described. We therefore applaud Plamondon & Alimi's (P&A's) attempt to dissect relationships between speed and accuracy of movement in terms of agonist and antagonist systems, because we suspect some of the continued success of Fitts' law is spurious.

Fitts' law is replete with statistical methods for reducing variation in a relationship. It employs log transforms, which in other contexts are used to control severe variability in data. In addition Fitts' law tends to destroy any individual differences, or any intraindividual differences, since it describes the average movement times of a group of individuals' average performance; focusing upon means reduces variability by the square root of the number of observations. In the light of these observations, we are not surprised that it can account for a sizeable proportion of any "remaining" variance. Unfortunately this means that we are less than impressed with other models (including P&A's) using similar methods to describe relationships between movement speed and accuracy. Nevertheless, P&A's model has interesting properties, it has a potentially biological (rather than an information-theoretic) basis, and it can explain intraindividual differences, which might give it considerable predictive power. In particular, we are interested in the shape of the kinematic functions the model can produce.

P&A's model can cope with asymmetries and "reversed" asymmetries in velocity functions. This is of interest because most models predict prolongations of the decelerative phase of movement, whereas we have observed that prolongations of the accelerative phase of movement can occur in patients with movement disorders (Martin et al. 1994). In ongoing work, we have also observed prolongations of the accelerative phase of movement in situations where there is some uncertainty about target location.

P&A's model can also explain increases in the number of submovements during aiming movements without necessarily invoking a specific command for each submovement. This is certainly important; however, we doubt that this is realistic if it is always done by varying muscular parameters of the model. There are certain movement disorders, in particular Parkinson's disease, that cause patients to employ multiple cycles of acceleration and deceleration. However, this is not simply a physical property of the slower movements of such patients. We have noticed differences in the numbers of submovements (and cycles of acceleration and deceleration) in patients with Parkinson's disease when they are trained to move at the same speed as healthy age-matched controls (Phillips et al. 1994). We have also noticed differences in the number of submovements in older adults when matching their movement durations to those of younger adults (Morgan et al. 1994). Even if P&A's model could explain submovements in disorders such as Parkinson's disease, we very much doubt it could provide a sensible account of the action tremor associated with cerebellar dysfunction. From clinical impressions, the 4–5 Hz oscillations seen in cerebellar dysfunction are clearly corrective impulses (albeit corrective impulses that overshoot or undershoot their target), which can be reduced by removing visual feedback. Nevertheless, being able to explain submovements without specific commands could be valuable when considering some populations, for example, patients with Alzheimer's disease. Patients with Alzheimer's disease have inefficient movements, but we doubt whether these additional submovements necessarily reflect a greater utilisation of visually based corrections (Slavin et al. 1995).

The legacy of Fitts' law has been a propensity to congratulate ourselves on how much we have explained, with a tendency to overlook what we have yet to learn. Plamondon & Alimi potentially have provided a valuable step forward with a more biological model that explains the speed and accuracy of targetting movements. There is a lot more, however, that remains to be addressed, since their model is still unidimensional, and as such ignores any directional biomechanical constraints and neglects any differential roles of proximal-distal muscle groups in coordinating targetting movements.

Speed/accuracy trade-offs in rapid simultaneous and sequential actions: Evidence for carryover effects

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Abstract: The idea of a neuromuscular synergy involving independent amplitude commands to the agonist and antagonist musculature is quite an appealing part of Plamondon's theory. One question that might be raised relates to the relative independence of the two "D" commands. Evidence is presented that suggests that the two commands might be related in sequential or simultaneous rapid aiming movements.

I found the idea of a neuromuscular synergy involving amplitude commands to the agonist and antagonist musculature quite an appealing part of the theory presented by Plamondon & Alimi (P&A). It is also intriguing that by changing the difference between the two amplitude commands (i.e., $D_1 - D_2$) one can produce different movement amplitudes. I suppose one assumption that must be made is that the amplitude commands must be independent of one another, so that a wide range of amplitudes can be generated. However, there may be circumstances in which the amplitude commands are correlated or interact. For example, in some recent experiments, I asked subjects to make rapid positioning movements over different distances (20° and 60°) with the different limbs in movements separated by 250 msec. When the short (20°) movement preceded the long (60°) movement in the sequence, the short target was overshot and the long target was undershot relative to when the movements were performed independently. The same trend was shown when the long movement led the short movement in the sequence. In rapid sequential movements, it appears that movement accuracy is dependent not only the goal distance and the goal movement time, but also on the presence and absence of adjoining movements. One might conclude that the amplitude specifications to the agonists and antagonists in each limb may not be independent, but related, and result in the assimilation of movement amplitude.

The same point can be made involving simultaneous positioning movements. When subjects move different distances, for example, the shorter distance limb overshoots when paired with a longer distance limb showing spatial assimilations (Sherwood 1991). In addition, the shorter distance limb slows down when paired with a longer distance limb, demonstrating a temporal assimilation effect (Marteniuk et al. 1984). However, the assimilations effects are also limited to certain combinations of distances and times. For example, I combined 30°, 40°, 50°, and 60° movements in the left hand with a standard 60° reversal movement in the right hand. The left limb overshot and the right limb undershot in the 30°–60° dual condition relative to single movements. No other spatial assimilations were found, suggesting that spatial assimilations were limited to bimanual conditions where interlimb differences in end location were greater than 20°. However, there was no change in relative timing across limbs and conditions, suggesting that the same temporal structure was utilized in both single and bimanual movements. Also the assimilations noted here are dependent on movement speed, with greater assimilations noted with greater movement speeds. In simultaneous movements, as in sequential movements, spatial and temporal accuracy depends on kinematic factors (i.e., distance, time) and on the characteristics of the movement pair. In summary, the specification of movement amplitude via the D_1 and D_2 commands does not seem to be independent across limbs in sequential or simultaneous actions. Can the kinematic theory of speed and accuracy account for these data?

What's different in speed/accuracy trade-offs in young and elderly subjects

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Abstract: We question whether Plamondon & Alimi's model is useful in accounting for the nonsymmetrical and multiple-peaked velocity profiles observed in young and elderly subjects for ballistic aiming tasks. For these subjects, both data and observation suggest that a central representation initiates the movement in an appropriate direction but that multiple adjustments are made, both early and late, to achieve spatial accuracy.

Developing models to explain the speed/accuracy trade-off in normal adult subjects is important, but it is the young and the elderly who often have the greatest need for speed/accuracy trade-off as they control rapid aiming movements. Yet neither Plamondon & Alimi's (P&A's) model, nor any of the previous models (e.g., Crossman & Goodeve 1983; Fitts 1954; Meyer et al. 1988) even mention, much less account for, the dramatic influence of lifespan development on the speed/accuracy trade-off. As an example, look at Figure 1, which provides two sample velocity curves (with an acceleration curve beneath each) from a rapid, linear aiming hand/arm movement (20 cm movement to a 6 cm target, ID = 2.58) for a typical 6-year-old child (top) and a 74-year-old senior adult (bottom).[1] Note that the velocity curves are very different from the typical samples provided by Plamondon & Alimi (1997, p. 11). The velocity curves have multiple peaks (more than three), with multiple crossings of the 0 acceleration line. These changes in acceleration represent relatively uneven progress toward the target; this is further verified by measures of normalized jerk (third derivative of movement displacement normalized for movement duration), which are much greater in the young and the elderly than in college-age adults.

P&A argue that $\Delta\Lambda$ law results in a velocity curve with multiple peaks generated by single pair of synchronous impulse commands to the agonist/antagonist muscles (likely the biceps/triceps and anterior/posterior deltoid for this movement). It is difficult to accept that ballistic aiming movements like these (and their velocity profiles) that we observe in young and elderly subjects are controlled by a central representation with no need for visual feedback. In fact, our evaluation of the behavioral evidence related to these kinematic characteristics is that for the young and old, the movement goal (hitting the target with a rapid linear movement) is basically achieved through corrective actions on the part of the subject. We postulate that central parameters of the movement simply "get it started" in the right direction followed by multiple corrections. In fact, peak velocity is often not achieved before the first corrective action is initiated.

If we use Meyer et al.'s (1988) criterion (0 crossing of the acceleration profile) for separating primary and secondary submovements to evaluate velocity profiles like these, we find that typically about 15% to 30% of the total movement distance is governed by the primary submovement phase (closer to 15% in the 6-year-olds and closer to 30% in the 74-year-olds) with the rest associated with the secondary phase. Using a wrist rotation task, Pratt et al. (1994) reported that their elderly subjects covered about 50% of the movement distance with the primary phase. Although this is somewhat higher than our 30%, their subjects were not as old (about 66 versus about 74 years of age) as ours and their task was different. Regardless, this is a much smaller percent of the movement length in the primary phase than is typically reported for adult subjects (e.g., Meyer et al. 1988, 75%–90%). We found more than 80% of the movement occurred in the primary phase for our 24 year-old subjects, and Pratt et al. (1994) reported about 70% for their 20-year-old adults. In addition, normalized jerk is much higher in our young and elderly subjects, suggesting a less smooth movement with more corrections (e.g., see Yan 1996[2] for original data).

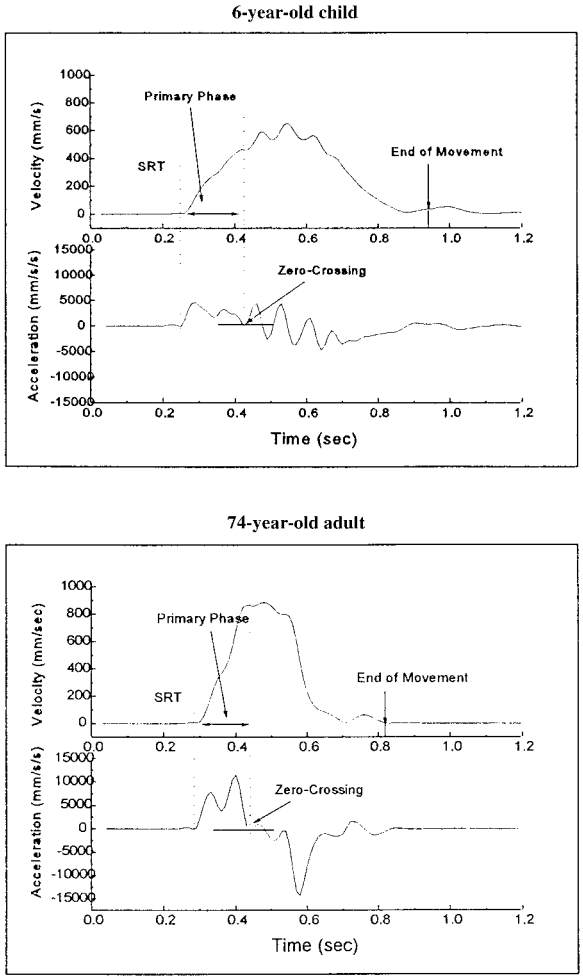

Figure 1 (Stelmach & Thomas). Typical velocity and acceleration profiles for a 6-year-old child (top) and a 74-year-old adult (bottom) performing a single ballistic arm linear movement to a target.

Our question from a developmental perspective is whether or not P&A's proposed model can accommodate the changes that occur over the lifespan. The major difficulty is that the model suggests that "bumps" in the velocity curve (reflected by 0 crossings in the acceleration curve) do not represent adjustments in motor control, but rather are conceived as part of a central movement map (or a motor program) without visual adjustments near the target. Simply observing the ballistic aiming movements of young children and senior adults suggests that the movements are not a single action that is completely programmed. If this is true, then the two-part (primary and secondary submovements) Optimized Submovement Model (Meyer et al. 1988), which allows for "on-line" adjustments after an initial ballistic phase, seems more adaptable and likely to fit our lifespan data better even though it may not fit the data quite as well for adult (generally college-age) subjects.

NOTES

1. The mean movement duration for the 6-year-old subjects was 510 msec (*SD* = 140). For 74-year-old subjects, the mean movement duration was 530 msec (*SD* = 123). Adult 24-year-old subjects had a mean movement duration of 280 msec (*SD* = 70).

2. A paper (Yan, Thomas, Stelmach & Thomas) based on data from this dissertation is in review.

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Modeling movement variability in space and time

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Abstract: Plamondon & Alimi propose a universal account of trajectory formation and speed/accuracy trade-off in rapid movements but fail, because: (1) the kinematic model ignores the more fundamental dynamics of movement generation, and (2) it does not capture the essential space-time constraints of movement accuracy. Hence, the modeling lacks a biologically and behaviorally principled foundation and is driven by pragmatic function fitting.

The attempt to formulate a theory that "explains" the large body of data that has been accumulated for rapid discrete movements and their variability is a valuable and simultaneously challenging undertaking. Plamondon's proposed model is valuable in that its formulation has progressed from accounting for the formation of continuous movement trajectories to capturing the outcome of such goal-directed trajectories and its variability. In this sense Plamondon's theory is an interesting counterpoint to much work in the speed/accuracy literature that limited its theoretical rationale to errors alone, irrespective of the underlying generative system. While on the metatheoretical level we acknowledge such an aspiration, we want to point out some major deficiencies in the model's assumptions, especially at the stage of modeling variability data, which has given the account the flavor of function fitting.

One of the conceptually plausible features is the model's strategic reduction of movement control to two generalized antagonistic impulse functions whose parametrization produces the various shapes of velocity profiles reported in the literature. Another interesting feature is the grounding of the generation of the macroscopic velocity profiles in an abstracted microstructure. This abstraction, however, assumes a linear time-invariant system in order to allow probability assumptions that obtain the lognormal velocity functions. Given the growing recognition of the importance of nonlinearities in complex systems such as the movement apparatus, linearization is a mathematical convenience that may ignore the essence of complex system behavior. Limit cycle behavior, for instance, which has proven a valid account for rhythmic coordination, is outside the model's scope. In the same spirit, it is questionable whether a model that focuses on the level of kinematics and ignores dynamics can be more than a superficial account of movement speed and accuracy.

Still, in this first stage of the model's development, the kinematic characteristics of continuous movement trajectories arise from structural properties of the model in both space and time. However, when Plamondon & Alimi (P&A) attempt also to "explain" the data on the speed/accuracy trade-off, the model ignores the fundamental fact that accuracy is likewise defined in both the space and time domain. Categorizing the various experimental protocols according to either space or time demands in order to model their results, P&A run the danger of requiring a separate theory for every task. P&A's formulation of constraints is precisely fitted to Fitts' original experimental protocol, which was replicated by many subsequent studies and which pursued a remarkably one-dimensional approach to capturing variability in goal-directed movements: accuracy is measured only in terms of spatial errors, which are then only correlated with the temporal duration of the movement. In strict analogy, γ_1 reflects the spatial constraints captured by the task criterion variables D and ΔD, and γ_2 captures the maximal movement time in terms of the command variables D_1 and D_2.

We contend that only a space-time approach to movement accuracy is a principled one that pays tribute to the axiomatic truth that actions take place in space and time. Spatial errors are always measured in regard to time and temporal errors in relation to space. For example, it seems ironic to say that a baseball batter missed the ball by 100 msec but *only* by 1 cm. Only both measures together tell the success or failure of the action. Also, temporal accuracy can be traded off against spatial accuracy. In many laboratory tasks (including Fitts' tapping protocol), however, the method of determining movement error is somewhat arbitrary and often follows either the task's measurement traditions or the experimenter's whims. However, when spatial and temporal errors are measured in the same frame of reference, the error profiles as a function of movement velocity are complementary (Hancock & Newell 1985; Newell 1980; Newell et al. 1993b).

P&A pass off the extensive work in timing error by Newell and colleagues as controversial, but the timing error functions have been replicated many times and, significantly for P&A, they are tightly linked to the functions for spatial error. We argue, therefore, that, analogous to the command signals that define the temporal and spatial extent, movement accuracy is constrained in both space and time. A complete set of constraints (within the approach offered by P&A) should also comprise the temporal accuracy specified by the task. $\Delta MT/MT$, as well as a constraint for movement amplitude on the command level formulated in terms of $D_1 - D_2$.

Instead, in their attempt to account for temporal variability, P&A propose an additional derivation (equation 59) with a different theoretical rationale, which is based on their initial equation for spatial constraints. Not surprisingly, the ability to fit the data breaks down when experimental protocols are addressed that have combined temporal and spatial constraints on accuracy – movements that are all too reminiscent of any real world task! P&A's alternative suggestion that under such complex conditions the internal parameters μ_1, μ_2, σ_1, and σ_2 need to be changed is nothing more than hand waving. Which system parameters should change? What task manipulations do the parameter modifications correspond to? Without seeing a systematic variation of parameter values across different task levels, P&A's model gives the flavor of arbitrary function fitting to meet the local task demands that are often equally arbitrary.

Do we need an encompassing speed/accuracy trade-off theory?

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Abstract: Even if we recognize that the delta-lognormal model provides an excellent fit to a large variety of data, the question remains as to what we actually learn from such a model, which could be seen as merely another multiparameter account? Do we welcome such an encompassing account, or do we expect to learn more from the limitations that become apparent when applying dedicated models addressing specific classes of movements?

If a formal theory, like the delta-lognormal model, appears to be successful in explaining a variety of phenomena, its likely fate is to be subjected to criticisms related either to what it does not account for or what the reviewer would have preferred instead. An example of a criticism of the former type would be the fact that the delta-lognormal law does not seem to explain why in aiming movements directional errors are generally smaller than amplitude errors (see Rosenbaum et al. 1995, pp. 46–47). However, the extensive set of data fits presented by Plamondon & Alimi (P&A) seems to leave particularly the latter option open for commentary. Doubtless the delta-lognormal law has great appeal for those who prefer a single, concise account for the speed/accuracy trade-off relationships. But those who are more keen to understand in detail what the actual mechanisms, processes, and limitations of the motor system are, and how they are under voluntary control may be dissatisfied by the mere information that these relationships can be modeled to involve sequential and parallel subsystems and agonist–antagonist synergies.

The trouble is that our insight into the origin and timing constraints of rapid targeted movements has not really increased

very much since 1954, or since 1899, for that matter. Do we really want better and better multiparameter model descriptions for a larger and larger variety of experimental data? Or do we need much more real and detailed understanding of the phenomena themselves, that is, of the nature and meaning of the speed/accuracy trade-off relationships, and (if it exists) of the single universal mechanism, or (which is more likely) of the different specific mechanisms involved in such trade-offs under different physical and experimental conditions? It is of interest to note here that the target article has the plural form "trade-offs" in its title. We would have liked a serious consideration of the types of approaches and models that are needed to understand the phenomena related to the timing of rapid, aimed movements. The principal question that emerges is whether or not we gain the desired insight by trying to cover all sorts of tasks performed by the diverse effector systems, often manipulating different kinds of devices as well. To the extent that such a model is more encompassing, it is necessarily of a more general and global nature. An in-depth approach that develops models addressing particular classes of movements, as distinguished from other classes, will be more fruitful in promoting our understanding.

On the face of it, these classes of movement are not hard to define. One of the distinctions that may lead to insight into the phenomena, but which are not addressed by P&A, pertains to relatively novel versus highly overlearned movements. Clearly, most laboratory tasks of the Fitts type are rather artificial, and the subjects' practice is usually restricted to the duration of the experimental sessions; this in contrast to the highly practiced "aiming" movements of professional musicians and craftsmen. Another such distinction is concerned with complexity, for example, the number of segments of the effector system involved in any particular aiming movement. Eye movements are much simpler in this respect than, for instance, graphic movements that involve an extremely complex, multijoint effector system holding a writing stylus with the additional constraints implied in its grip. A third distinction relates to single, discrete versus cyclically repeated movements. It is most likely that different use is made of proprioceptive as well as visual feedback in those types of tasks. In particular, the opportunity for exploiting the visco-elasticity and inertial dynamics of the effector is much larger in the case of cyclic or repeated movements than in discrete movements.

Distinctions such as the ones that these few examples illustrate, will no doubt remind us, in both modelling and parameter fitting, of the boundary conditions and limitations of any specific model, and will therefore highlight the unexplained aspects of human motor control in any particular domain. Only this will contribute to our understanding and it thus seems to be a valuable counterpart, if not a necessary complement, to the – admittedly admirable – approach taken by Plamondon & Alimi. The brilliance of their data fits unfortunately blinds us when looking for such darker, weak spots in the theory.

Distance versus position information in the control of aiming movements

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Abstract: Information about positions, from which differences in position are computed (as proposed in the vector-integration-to-endpoint model), provides a more plausible perceptual basis for the control of goal-directed arm movements than information about distance (as proposed in the kinematic model).

Plamondon & Alimi (P&A) present a mathematical theory for speed/accuracy trade-offs in target-directed movements. Their "kinematic" model involves a delta-lognormal law, which is demonstrated to describe the velocity profiles of fast target-directed movements equally well as or better than Fitts' law. P&A's model requires that three input control parameters (t_0, D_1, and D_2) and four system parameters (two logtime delays μ_1 and μ_2, and two logresponse times σ_1 and σ_2) are set prior to the initiation of the movement. Apart from the close fit with the data, the model has two theoretically attractive features. The model is realistic and parsimonious in that no cost-optimization principles are being applied, whereas movement times are not planned a priori but are emergent properties. Furthermore, the model assumes that velocity is the primary variable that is being controlled by the nervous system, which fits in well with currently evolving understanding of the motor control system as a first-order system (see, e.g., Beek et al. 1996).

P&A present their kinematic model, based on linear system theory, as a "global view based on the timing properties of both the neural and the muscular networks" (sect. 6). They continue, "The price for such generalization is that it is difficult, without further experiments, to provide a direct biological interpretation for the system parameters μ_1, μ_2, σ_1, and σ_2, and further study is needed in this context." This remark reflects the authors' wish to interpret or link their mathematical model to biological processes and substrates involved in the control of movement. Similar sentiments have been expressed by proponents of the application of nonlinear system theory to the study of human movement (Beek et al. 1995).

This should not be read to imply, however, that purely mathematical models without such an interpretation are useless, although they admittedly run the risk of becoming completely data driven if the curve fitting (that is part of any modelling approach) does not lead to insight into the control and organization of movement. Following this line of reasoning, we wonder whether the model proposed by P&A provides more insight into motor control than alternative models. In this context, we focus on the fact that the input commands to the system are based on (either perceived or remembered) distance information rather than on a difference between position vectors as in the vector-integration-to-endpoint (VITE) model proposed by Bullock and Grossberg (1988). We argue that this constitutes an apparent drawback in the light of the well-known reaching experiments of Bizzi et al. (1984) with deafferented monkeys.

In these experiments, the reaching behavior of monkeys that had undergone dorsal rhizotomy (eliminating prioprioceptive feedback) was investigated. In one of the conditions, the monkey's forearm, which the monkey could not see, was passively driven from the initial starting position to the target position (in the absence of the target light) and held there for a variable amount of time (1 to 3 sec). After the target light was activated and the arm released, the monkeys first moved their arms toward the initial starting position before reversing direction at a point intermediate between the initial and the target position and moving back toward the target position. The sooner the arm was released, the further it travelled in the direction of the initial starting position. Inasmuch as these results constituted a blow to the original formulation of the mass-spring model (i.e., with a single, instantaneously fixed equilibrium point), they also seem incompatible with the model proposed by P&A. After all, if the monkeys would set D_1 and D_2 on the basis of a remembered distance, they were always to move in the same direction after the appearance of the stimulus and cover (about) the same distance. Within the perspective of the VITE model proposed by Bullock and Grossberg (1988), however, this finding can be explained because, according to this model, the present position vector (corresponding to the initial starting position) is being integrated so that the difference with the target position vector becomes zero. Moreover, the position vector encoding for the control of arm movements proposed in the VITE model is supported by neurophysiological evidence on primate cells from several regions of the cerebral cortex (Georgopoulus et al. 1982; 1986; Kettner et al. 1988). To our knowledge, similar neurophysiological support for distance as a controlling variable in the control of arm movements is not available. Given that the VITE model also accounts for the behavioral data explained by the

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kinematic model, it occurs to us that, in comparison to the model proposed by Plamondon & Alimi, the VITE model has the edge in explaining both behavioral and physiological data.

The delta-lambda model: "Yes" for simple movement trajectories; "no" for speed/accuracy tradeoffs

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Abstract: Although it provides a useful description of elementary movement trajectories, we argue that the delta-lognormal model is deficient as an account of speed/accuracy tradeoffs in aimed movements. It fails in this regard because (1) it is deterministic, (2) its formulation ignores critical task elements, and (3) it fails to account for the corrective role of submovements.

We welcome the present discourse on speed/accuracy tradeoffs (SATs) in aimed movements and the delta-lognormal ($\Delta\Lambda$) model. This model describes well the shapes of elementary movement trajectories (Plamondon et al. 1993), especially what we call "time-matching" movements (cf. Meyer et al. 1988; 1990; Wright & Meyer 1983). Furthermore, Plamondon & Alimi (P&A) have made a valiant attempt at applying the $\Delta\Lambda$ model to the SATs in a variety of movement tasks. This is praiseworthy because SATs in rapid aimed movements are a major phenomenon that unified theories of motor control should explain. Unfortunately, the target article deals with these SATs in a seriously flawed and incomplete way.

Stochastic nature of speed/accuracy tradeoffs. The authors note that the study of SATs arose from and is part of psychophysics (sect. 1, para. 1). Behavioral variability, a central and inescapable aspect of the relationship between mental states and their physical manifestations, is one key topic of psychophysics (Woodworth 1899). We are therefore surprised and disappointed that the target article proclaims the $\Delta\Lambda$ model to be virtuous because it disregards the inherent variability of aimed movements (sect. 5.1). This proclamation overlooks the basic obligation of any serious motor-control theory to incorporate and account for effects of neuromotor "noise." Because trial-to-trial variability is typically large in movements like those considered here and because this variability is central to the dependent measures used, it is difficult to see how any model that ignores this variability could be correct.

Of course, we are not the only ones to reject deterministic models as explanations of SATs (e.g., Fitts 1954; Schmidt et al. 1979; Woodworth 1899). Ironically, P&A acknowledge this requirement initially before subsequently neglecting to honor it in their $\Delta\Lambda$ model. Summarizing Crossman and Goodeve's (1963/1983) theory, they state (sect. 2.1.2, para. 4): "More fundamentally, one handicap of this model is that it is completely deterministic. This implies that for a fixed target distance and width the sequence of submovements would always be the same." Unfortunately, exactly the same criticism, which is a very serious one, also applies to the $\Delta\Lambda$ model.

Application of the $\Delta\Lambda$ model to the Fitts' task. The $\Delta\Lambda$ model provides a flawed account for results in the Fitts' Task, in which subjects are instructed to move as quickly as possible from a starting position to a target region whose distance is D and whose width is $\pm\Delta D$. According to the $\Delta\Lambda$ model, velocity profiles for this situation, which we call the "time-minimization task," have one or two zero-crossings before returning to zero at $t = \infty$. The derivation for equation 52 ensures that the spatial position of the effector will be within $\pm\Delta D$ of the target at the first or second zero crossing. Although this formulation yields predictions for movement time (MT) that fit the data well, it ignores several important aspects of the task situation. First, the zero-crossing criterion is at odds with the procedures to define a movement's endpoint used in the experiments being modeled. Second, the $\Delta\Lambda$ model ignores a psychologically crucial aspect of the task situation: the subjects' requirement to hit the target is probabilistic. Subjects are instructed to produce distributions of movement endpoints with a nonzero probability (e.g., .05) of missing the target region. A resulting frequent complication is that a subject may generate movement–endpoint distributions that are much wider or narrower than the nominal target region for different targets. The $\Delta\Lambda$ model ignores this phenomenon.

Application of the $\Delta\Lambda$ model to the time-matching task. The $\Delta\Lambda$ model has more serious problems accounting for SATs in what we call the "time-matching task" (and the target article calls "movements with temporal constraints where MT is given: focus on spatial variability"). This task requires subjects to produce movements that travel a distance D in a prespecified total time MT; typically a linear SAT is observed between average velocity D/MT and movement-endpoint variability σ_D (Abrams et al. 1989; Schmidt et al. 1979; Wright 1983; Wright & Meyer 1983; Zelaznik et al. 1981; 1988). The target article suggests that subjects estimate D perceptually and then that they solve equation 56 to obtain a value of ΔD consistent with MT. The formulation of equation 56 predicts that ΔD will be proportional to D for fixed values of MT, but, contrary, to the experimental findings, equation 56 does not predict a linear relation when MT varies. Moreover, the derivation of equation 56 makes clear that ΔD indexes the maximum allowable absolute distance error rather than the variable error, σ_D, commonly studied in experiments with the time-matching task. (Contrary to what the target article implies in sect. 4.3, ΔD is also not absolute error, as it is typically defined in the motor control literature.)

Existence of corrective submovements. Yet another problem for the $\Delta\Lambda$ model is its failure to characterize aptly the function of submovements. P&A claim that subjects make few, if any, truly corrective submovements (sects. 5.1 and 5.3); rather, according to them, apparent submovements are artifactual. This claim is patently false. Although not every reversal in the velocity profile indicates the start of a corrective submovement and although some reversals may stem from passive oscillations or agonist/antagonist mismatches of the sort embodied in the $\Delta\Lambda$ model, there is strong evidence that (1) truly corrective submovements do occur frequently, and (2) visual feedback is used to guide those corrections.

Two kinds of evidence support the existence of corrective submovements. First, for sufficiently difficult task conditions, there are often more than two zero-crossings in the velocity profiles of Fitts' task movements. For example, Meyer et al. (1990) found three or more corrective submovements in almost one-half of the trials than during one representative experiment. Second, there are more apparent submovements during movements in a Fitts' task than during movements in a time-matching task (Carlton 1994). The $\Delta\Lambda$ model does not predict this between-task difference. This difference is predicted by the stochastic optimized-submovement model (SOS) of Meyer et al. (1988; 1990).

That the extra corrective submovements observed in the time-minimization task involve true corrections based on visual feedback has been amply demonstrated as well (e.g., Keele & Posner 1968; Meyer et al. 1988; Zelaznik et al. 1987). When visual feedback is eliminated selectively during the course of time-minimization movements to difficult targets, movement accuracy drops substantially. The $\Delta\Lambda$ model does not explain why; indeed it cannot, whereas the SOS model can.

For these and other reasons, if the $\Delta\Lambda$ model is to succeed as a general, unified theory of motor control it will require substantial further development that, we feel, may ultimately bring it closer to the SOS model.

Can one explanation serve two laws?

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Abstract: Several issues are raised concerning the notion that a single strategy explains Fitts' law and the linear speed/accuracy trade-off. Two additional concerns are discussed: (1) distance is programmed, (2) the fact that movements produced without the aid of vision obey Fitts' law does not mean that sighted movements must be explained without regard to vision.

Plamondon & Alimi (P&A) argue that the kinematic theory of rapid movement can be used to explain Fitts' law as well as the linear speed/accuracy trade-off. This is a very large undertaking, and at this time perhaps premature. In this commentary, we present behavioral evidence from which we infer that these two speed/accuracy relations do not derive from a common kinematic strategy.

The crucial question concerns whether a unified trajectory control strategy underlies performance on a Fitts' task (time minimization) and on a linear speed/accuracy trade-off type task (temporally constrained). P&A are not the first to argue for a common explanation (Hancock & Newell 1985); however, until recently, experimental tests of this proposition have been lacking. Carlton (1994) reported the results of a very simple experiment designed to determine whether the linear speed/accuracy trade-off relationship (Schmidt et al. 1979) and Fitts' law (Fitts 1954) can be derived from a common strategy. Subjects performed a 400 msec, 12 cm, temporally constrained aiming movement. Carlton measured the within-subject standard deviation in movement distance, that is, the effective target width, W_e, and then constructed a target width twice the W_e. It seems to us that Plamondon's kinematic model would view these two situations as identical. Thus, when subjects performed the spatially constrained, time minimization Fitts' law task, with a distance of 12 cm and a target width equal to twice their W_e derived from the temporally constrained task, the movement time should be approximately their average movement time in the temporally constrained situation. It was not. There was a 90-msec saving in movement time for the time minimization Fitts' law task. Moreover, the movements in the latter task were composed of two submovements, whereas those in the temporal-accuracy task were characterized by a single submovement. These results appear to be counter to the suggestion of P&A that these two speed/accuracy relationships are explained by the velocity profile strategy that they outline.

We believe that the Carlton experiment serves as a crucial test of the notion that subjects adopt different goals (i.e., strategies) in time minimization tasks, compared to temporally constrained tasks (Meyer et al. 1988; Wright & Meyer 1983). Zelaznik et al. (1988, Experiment 1) showed nicely how the manipulation of the temporal demands of an aimed hand movement causes changes in the nature of the speed/accuracy relation. When temporal demands were relaxed, aimed hand movements showed a speed/accuracy relation more like Fitts' law. P&A appear to be more concerned about the second experiment of Zelaznik et al. (1988), which showed no interaction between the target width and temporal precision on the nature of the speed/accuracy relation. Yet, the explanation for this finding is simple. The target widths were 1, 2, and 3 cm, clearly too large to produce strategic changes in the submovement optimization principles.

There are other aspects of their work with which we take issue. First, P&A rely heavily on the fact that Wallace and Newell (1983) found that unsighted movements obeyed Fitts' law. However, that result can be explained by Meyer et al.'s (1989) optimized submovement theory. Even without vision, subjects will choose a value for movement duration that minimizes the requirement for additional submovements. Just because unsighted movements obey Fitts' law, it does not follow that sighted movements do not utilize vision and, of course, corrective submovements. Second, an important implication of Plamondon's theory concerns distance programming. P&A discuss the need to know the initial condition (starting point) and the distance to be moved, in order to plan the velocity profile trajectory. Despite the recent challenge to the equilibrium point hypothesis (Gomi & Kawato 1996), the evidence is clear that location programming is more accurate than distance (see Rosenbaum 1991; Schmidt 1988), and that information about starting position is not necessary for short-term motor memory (Schmidt 1988).

Finally, many models in other domains of human performance presume that individuals have several routes to successful performance. In stimulus–response (S-R) compatibility research, for example, a distinction is often made between a direct, or automatic, response-selection route, via which a stimulus activates its corresponding response, and an indirect (or S-R) translation route, through which an assigned response to a stimulus is retrieved or generated by means of a rule (e.g., Kornblum et al. 1990). Compatibly mapped responses to stimuli are faster for "pure" trial blocks in which all S-R pairs have compatible mappings than for "mixed" blocks in which the mappings for some pairs are incompatible. This and other findings have been interpreted as suggesting that a strategy of responding on the basis of direct response activation is adopted for pure compatible blocks but not for mixed-mapping blocks (van Duren & Sanders 1988; Stoffels 1996). We propose that on the motor side of information processing, individuals can likewise utilize different strategies to generate movement trajectories. These strategies are based upon the task constraints, that is, temporal and spatial accuracy demands. The nature of the task will then determine how the subject achieves the goal and the type of speed/accuracy trade-off that will be observed.

Author's Response

The kinematic theory: A new window to study and analyze simple and complex human movements

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Abstract: To cover as much as possible the various questions raised by the commentators, I have divided my Response into three major sections. In section R1, I reply to the major comments and remarks dealing with the basic hypothesis upon which the kinematic theory is built (Plamondon 1993b; 1993c; 1995a; 1995b). I focus on linearity, determinism, kinematics, and the biological significance of the model parameters. I conclude this section by showing how, from a practical point of view, the delta-lognormal law can be used to group similar data prior to proceeding with further statistical analysis. In section R2, I address the main comments and remarks dealing with the speed/accuracy tradeoffs. First, I focus on the origin of speed/accuracy tradeoffs and the effect of visual feedback. Then I clarify some terminology problems and mathematical misinterpretations prior to providing new support for the theory using the experiments and data referred to by some commentators. I devote section R3 to a generalization of the kinematic theory and to some potential applications. I first provide new explanations of some classical experiments; then I show how more complex movements can be

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analyzed using the delta-lognormal law. I focus on perturbed and oscillatory movements as well as on cursive script and signature analysis. I conclude this section and my Response by showing some possible applications to the study of movement learning in children as well as to the study of aging phenomena in movement control. I complete my Response by recalling some technical problems that still need further analysis. Throughout the text, I try to convince the reader that the kinematic theory provides a new window on old and new data in the field.

R1. Introduction

The target article had two goals: the first was to make clear that speed/accuracy phenomena, as observed in simple rapid-aimed movements still lack a fully comprehensive explanation. The second goal was to show that the kinematic theory proposed and developed by Plamondon (1993b; 1993c; 1995a; 1995b) provides a general framework for describing and analyzing such phenomena. Although some commentators have directly or indirectly acknowledged that the target article was partly successful with respect to these two goals (**van Wieringen & Beek; Carlton & Liu; Desmurget et al.; Hancock & Verwey; De Jong & Van Galen**), many interesting questions and judicious criticisms have been raised within and beyond the specific topics covered in the target article. Reactions beyond the specific scope of the paper were expected. They reflect the fact that the kinematic theory is not limited to the simple study of speed/accuracy tradeoffs but can be helpful in many other areas, as will be seen below.

From a practical point of view, to address as fully as possible the major points raised by the commentators, I have grouped my various replies under three major headings. I will first deal with comments and objections regarding the basic hypothesis behind the kinematic theory. Then I will discuss the commentaries dealing with speed/accuracy tradeoffs, focussing mostly on fundamental questions, trying to clarify some points that seem to have been misunderstood. Finally, I will analyze a few experiments reported by some commentators, for which the kinematic theory can provide new insights. I will also take the opportunity, in this third section, to show new applications of the kinematic theory in the more general context of complex movement analysis. Throughout the Response, I will show that the theory is already in a "*usable form*" (**Morasso & Sanguineti**), for many applications.

R2. The kinematic theory

Several aspects of the kinematic theory were considered powerful and even promising by many commentators: the prediction of the different forms of velocity profiles (**Phillips et al.**), the realism and parsimony of the model (**van Wieringen & Beek**), the use of velocity as a primary variable (**van Wieringen & Beek; Sternad & Newell**), the agonist/antagonist representation (**Sherwood; MacKay**), the simultaneity of the two input commands (**MacKay**), the high descriptive power of the model (**Wright & Meyer; Thomassen & Meulenbroek; Desmurget et al.**), the interest of the model for the study of individual data (**Holly**), and the incorporation of memory and learning to account for the speed/accuracy tradeoffs (**Desmurget et al.**), to name a few. There are, however, at least five challenges to the basis of the theory. In the first section I will focus mainly on replying to these objections.

R2.1. Linearity versus nonlinearity. A few commentators (**Sternad & Newell; MacKay; Bootsma & Mottet; Desmurget et al.**) have pointed out that the kinematic theory (Plamondon 1993b; 1993c; 1995a; 1995b) has a major drawback because it is based on a linear system approach, whereas muscular and neural networks are better modeled using nonlinear dynamics. I certainly agree that nonlinear dynamics provides useful ways to describe and analyze individual networks, although many authors using this approach fail to report any *quantitative* comparisons of their nonlinear models with real data (Bullock & Grossberg 1991; Mottet & Bootsma 1995, etc.). The fundamental hypothesis behind the kinematic theory is that when a large number of neuromuscular networks work in a synergetic fashion to produce a natural, well-learned rapid movement, the overall system behaves as if each subsystem were working in a linear mode around some specific operating conditions. Because a large number of these coupled systems are involved in the production of a single movement, the central limit theorem can be used to predict the mathematical form of the asymptotic impulse response of such a neuromuscular system: a lognormal function. The organization of a synergy into a combination of agonist and antagonist systems then leads to the prediction that the velocity profile of an end-effector will be described by a delta-lognormal law (Plamondon 1993b; 1995a).

Let me make an analogy. Semiconductors or, more particularly, transistors are nonlinear devices. When they are polarized around some operating points, they behave like linear devices for small variations around that point. With proper interconnections, there are various ways to build a global system that will act as a linear amplifier using nonlinear devices. This is what a large part of the design of audio amplifier systems is all about. In this perspective, the kinematic theory suggests that throughout evolution and learning our neuromuscular system has found ways to interconnect each subsystem to each other in a way that mimics, at some global level of representation, a linear system.

This phenomenon can be illustrated by looking at some simulations that we have run at Laboratoire Scribens (Warmoes 1995) using different nonlinear subsystems. For example, we have computed the impulse response of a sequence of N subsystems, each one being described by a simple nonlinear shunting equation of the form:

$$\frac{dx_j}{dt} = -A_j x_j + B_j x_{j-1} - C_j x_{j-1} x_j \qquad \text{(R.2.1)}$$

where x_j is the output of the j^{th} subsystem, x_{j-1} is the output of the $(j-1)^{\text{th}}$ subsystem and input of the j^{th} subsystem, A_j is the passive decay factor, B_j is the positive feed-back factor, and C_j is the negative feed-back factor.

Figure R1 shows a typical evolution of the form of the impulse response of a sequence of these simple coupled nonlinear systems. As the number of systems in the sequence increases, the impulse response tends more and more toward a lognormal, even if each of these individual subsystems is nonlinear. In practice, after interconnecting only a few stages, the lognormal behavior is already apparent. The concept of a large number of systems (called into question by **Grossberg**) does not seem to be too restrictive. Another explanation for such a convergence might be based on some generalized form of the central limit theorem. In

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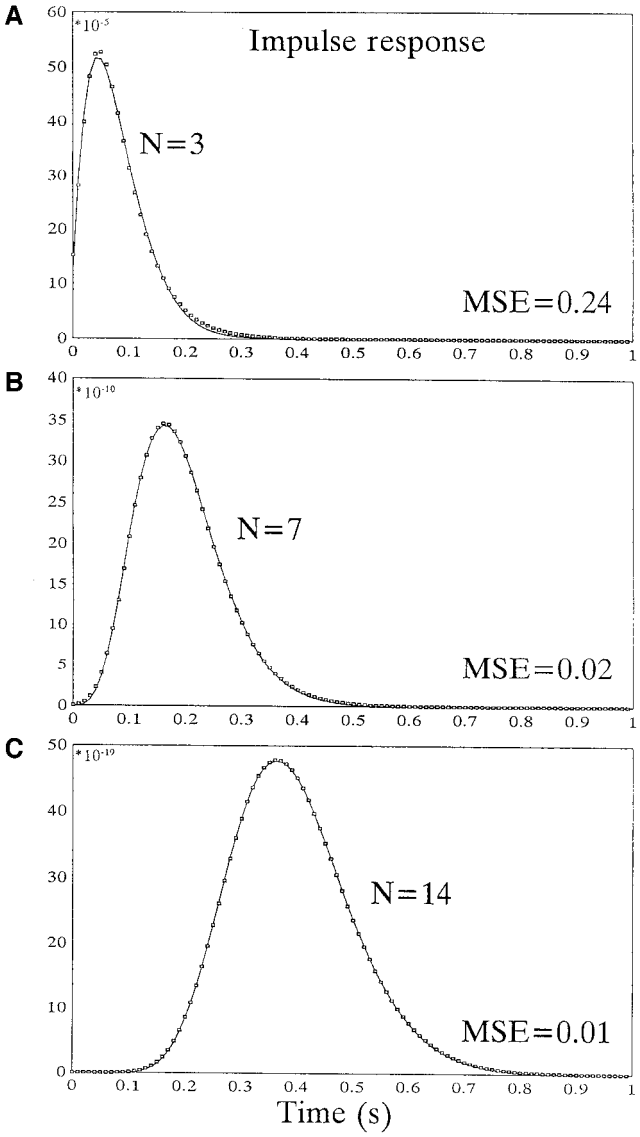

Figure R1. Evolution of the impulse response (*IR*) of a set of coupled nonlinear systems (equation R.2.1) as a function of the number (*N*) of subsystems in the set. The dotted lines represent the best fitting lognormal. The mean square error (*MSE*) obtained when fitting the *IR* with a lognormal decreases as *N* increases, as predicted by the kinematic theory.

any case, it seems that there are conditions where a lognormal function is a very good approximation to the global output of a sequence of coupled linear or nonlinear subsystems.

R2.2. Deterministic versus stochastic approach. One of the goals of the kinematic theory is to allow the study of individual movements (considered by **Holly** as one of the greatest points of interest in the theory). To do so, a deterministic approach is used to model the properties of a neuromuscular synergy in the production of a single rapid movement. This leads to a $\Delta\Lambda$ law that describes almost perfectly the velocity profile of the end-effector controlled by that synergy. In the context of the theory, this means that each specific rapid aimed movement can be described in terms of a set of parameters that represent the input command (D_1, D_2, t_0) to the neuromuscular synergy and the timing properties (μ_1, μ_2, σ_1, σ_2) of the agonist and antagonist systems involved in that specific synergetic action.

The fact that the kinematic theory associates with each individual gesture a set of specific parameters in no way implies that a stochastic approach cannot be useful when it comes to analyzing the statistical properties of a set of individual movements. I must accordingly disagree with the commentators who reject the theory on the basis of its deterministic nature (**Wright & Meyer; Bootsma & Mottet; De Jong & Van Galen**). The theory in fact provides an original approach to statistical modeling, based on the statistical analysis of the model parameters. In other words, the different parameters of the model take on a single specific value (I will come back to this point later on) for a single movement but in practice these parameters can be considered as *tunable random variables* that have specific distributions for specific experimental conditions. (I agree that this point was not developed clearly in the target article although most of my arguments in sections 4 and 5 reflected that this tunability was implied.) The stochastic properties of the neuromuscular systems involved in a specific task can thus be analyzed with the $\Delta\Lambda$ law rewritten as:

$$v(t) = \tilde{D}_1, \Lambda(t; \tilde{t}_0, \tilde{\mu}_1, \tilde{\sigma}_1^2) - \tilde{D}_2 \Lambda (t; \tilde{t}_0, \tilde{\mu}_2, \tilde{\sigma}_2^2) \quad \text{(R.2.2)}$$

where each $\sim$ parameter is a tunable random variable with its own statistical properties. Doing so, one should be able to analyze both the individual movements (equation 39 of the target article) and the statistical properties (equation R2.2) of their associated parameters, considered as random variables, using a full database of such movements. This might be an interesting way to avoid the masking effect (**Holly**) that occurs when more conventional statistical analysis is done. With this new approach, the variability of a group of movements can be easily simulated or studied. For example, I have performed such an analysis (Plamondon 1997) and all the observations previously reported on the variability of movement variables can be predicted on the basis of the same single fundamental equation: the $\Delta\Lambda$ law. For example, computer simulations predict that the standard deviation of the maximum velocity will increase in proportion to the average peak velocity. Conditions of departure from perfect proportionality can also be partly explained in terms of the number of trials per experiment as well as the physical limits associated with each parameter of the $\Delta\Lambda$ law. In this context, the basic hypothesis on which the stochastic optimized submovement model relies (Meyer et al. 1988) also seems to be another direct consequence of the $\Delta\Lambda$ law.

Another way to incorporate movement variability in the theory, proposed by **De Jong & Van Galen,** is via the study of the statistical properties of the random variable ϵ_j (equation 38). This is certainly an interesting suggestion that can be investigated using, for example, different subsystems in detailed models like the one used in Figure R1 (see equation R2.1). The relationship between the statistical properties of ϵ_j and the variability of the system parameters μ_i, σ_i^2 could then be explored in more detail and compared with real data.

R2.3. Kinematics versus dynamics. Apart from what I wrote in section 5.2, a few commentators have also provided new support to the idea that velocity might be the primary variable in movement control (**Hermann & Soechting; van Wieringen & Beek; Sternad & Newell**).

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Some commentators, however, criticize the kinematic approach (**MacKay**), some even considering kinematics a "trivial" consequence (**Mitnitski**) of dynamics.

Indeed, many models of human movement are inspired directly or indirectly by what is known about the basic description of a single muscle as a force generator. Although a single muscle can be described as a force generator, using many kinds of dynamic spring models, this does not lead automatically to the generalization that synergetic action of a large number of muscles is equivalent to the action of a single force generator. This might be an interesting working hypothesis but many alternative assumptions are possible. The one I use in the kinematic theory is that the description of a complete neuromuscular system involving numerous nerves and muscles can be better understood if the whole system is depicted as a velocity generator. (This is also consistent with the recent study of Wolpert et al. [1995] that suggests that trajectories are planned in kinematic coordinates.)

From the perspective of physics, there are two reasons for using velocity as a control variable. Velocity encompasses two types of basic information that can be helpful to the central nervous system (CNS) for movement control and generation:

$$\text{momentum} \qquad = \vec{p} = m\vec{v} \tag{R.2.3}$$

$$\text{the kinetic energy} = E = \frac{1}{2}\, m|\vec{v}|^2 \tag{R.2.4}$$

By controlling the velocity vector a subject obtains all information about the momentum of a gesture; this might be of practical importance for any type of movement involving, for example, a collision with an external object. Moreover, by specifically controlling the magnitude of that velocity vector, a subject gets control over the effective kinetic energy produced by that gesture. This is consistent with the findings of Soechting et al. (1995) on work minimization.

From this perspective, the kinematic theory provides a new way to look at human movements in terms of momentum and energy, two very fundamental variables in physics. Which is more important, force or energy? This is a sterile question as far as physics is concerned. Both variables provide a description of the movement and one can be theoretically recovered from the other. I did not refer to forces in the target article, not because the theory does not make any predictions about forces (**Carlton & Liu**), but because these are not necessary to understand speed/accuracy tradeoffs, the main themes of the target article. I have recently shown how isometric and isotonic force patterns can be described using the $\Delta\Lambda$ law and its first time derivative, respectively (Plamondon 1997). Here again, starting with the same basic law, numerous observations repeatedly reported in the field have been accounted for.

What is more important in this process is the question of what is gained by analyzing a movement with the delta-lognormal law. The target article shows a few examples of these gains; many of these cannot be as easily described when one analyzes movements with a nonlinear dynamic approach, particularly when it comes to making quantitative comparisons. From a practical point of view, the delta-lognormal law can be seen as a global constraint that any dynamic description of an "articulated chain of a robot manipulator or a human limb" (**Morasso & Sanguineti**) or a redundant multijoint arm motor system (**Desmurget et al.**) must respect to produce realistic outputs. For example, the law could be helpful in selecting the proper form of the `go` signals for the `vite` model (**Grossberg**).

R2.4. Biological significance of the model parameters. In its simplest form, the delta-lognormal law relies on a set of seven parameters to describe the magnitude of the velocity of an end-effector as a function of time in terms of the weighted difference in the impulse response of the agonist and antagonist neuromuscular systems involved in rapid movement. Some commentators acknowledge that the model has a potentially biological basis, particularly with regard to its power to explain intraindividual differences (**Phillips et al.; MacKay**) and its approach polarized around an opposing agonist and antagonist system (**MacKay; Sherwood**), described by two generalized impulse responses (**Sternad & Newell**) and based on a "Weber law control of *timed* movements" (**Grossberg**).

Many comments deal with the biological meaning of the individual parameters and what kind of insight they provide on the control and organization of the motor system (**Thomassen & Meulenbroek; van Wieringen & Beek; Goodman; MacKay; Bootsma & Mottet; De Jong & Van Galen**). This is certainly a major issue, probably the most difficult one, particularly if one tries to interpret the model parameters in the context of the classical approaches to date. A better way to explore these questions is to analyze movement parameters in the context of the representation space supported by the model, a representation related mostly to space, time, and energy.

According to the kinematic theory, a time t_0 can be conceived of as the moment when a given voluntary order is given. Because any movement generally involves the activity of cell populations, t_0 can be seen as a way to represent the time of volition, the time when the activity of agonist and antagonist cell populations initiating a movement is large enough to be interpreted as the instant of the voluntary command for that specific movement. The global activity of the agonist and antagonist cell populations at this specific time can then be interpreted as the amplitude D_1 and D_2 of the agonist and antagonist commands, respectively. Which variable describing the global activity of a cell population directly correlates with D_1 or D_2 is still an open question. The fact that $D_1 - D_2$ defines the distance that will be covered by the end-effector is consistent with the recent study of Nougier et al. (1996) that suggests that human movements are programmed in terms of amplitudes rather than in terms of a final position. [See also Libet: "Unconscious Cerebral Initiative and the Role of Conscious Will in Voluntary Action" *BBS* 8(4) 1985.]

Parameter interpretation in the kinematic theory would certainly be helpful in new research projects because it allows for the recovery of t_0, D_1, and D_2 by an analysis-by-synthesis of any individual simple movements. From a practical point of view, the parameter t_0 could be helpful, for example, in synchronizing rasters of action potentials in any kind of histogram analysis that could link the synchronized activities of different cell populations with a specific task on the basis that the amplitudes D_1 and D_2 of these cell activities are also linked to external variables such as movement amplitude (equation 47) and movement time (equations 41 or 46). A similar approach could also be used

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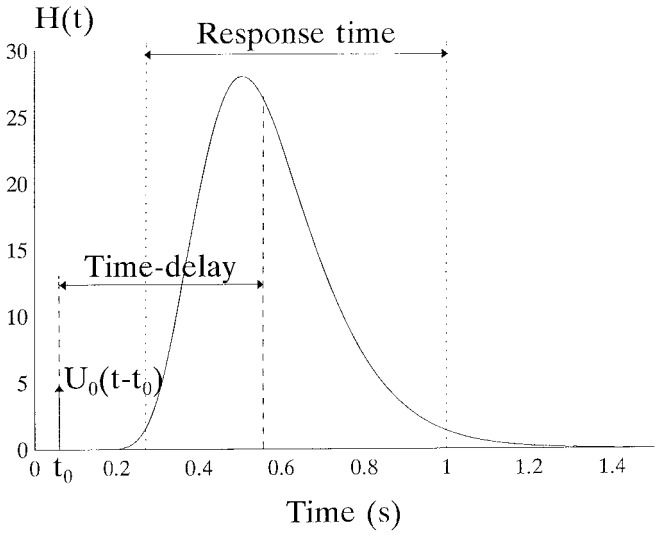

Figure R2. The concepts of time delay and response time of an impulse response.

to analyze EMG data to provide a deeper insight into the observation that "*the 'phasic' EMG* (electromyographic) *component could thus be interpreted as a result of a motor command concerned primarily with the control of velocity*" (**Hermann & Soechting**).

The temporal parameters μ and σ represent the global time delay and response time of a neuromuscular network on a logarithmic time scale. Figure R2 clarifies these two concepts. It depicts the lognormal impulse response (equation 40) of a neuromuscular system (agonist or antagonist) with a representation of the time delay and the response time.

When such a function is analyzed from a system engineering point of view, one is interested in characterizing the "time interval over which that function differs significantly from zero" (Brown 1963, p. 142) generally called the response time and also the delay "after the application of the input $[U_0(t - t_0)]$ at which the output attains a certain value" (Brown 1963, p. 143). In practice, for comparative purposes, if the impulse response is non-negative then the median of the impulse response is taken as a good representation of the time delay. The response time is often evaluated from the estimated width of $H(t - t_0)$ using the standard deviation of the impulse response as a reference, since 2.5 σ gives a good estimate of the spread of $H(t - t_0)$ on the time axis.

In this general context, for a lognormal impulse response starting at $t_0 = 0$:

$$\mu = \ln(\textit{median}) \qquad \text{(R.2.5)}$$

$$\sigma^2 = \ln\left(\frac{\textit{median}}{\textit{mode}}\right) \qquad \text{(R.2.6)}$$

can be used to depict the time delay and the response time on a logarithmic time scale (Plamondon 1993b; 1995a).

These parameters are probably linked, for example, to the response latency of the various motoneurons as well as to the viscosity or the stiffness of the individual muscles of a network. The kinematic theory emphasizes the idea that, at a global level of representation, the CNS controls a movement not in terms of the individual biomechanical properties of each individual neuromuscular component but in terms of a more general and familiar concept: *time*. This is consistent with the observation of Darling and Stephenson (1993) that individual joints or limb segments are not controlled in the same manner as is the terminal pointing segment. If this is true, there must be some links between the two representation spaces, biological versus time, but new data analysis will be required to make such a link.

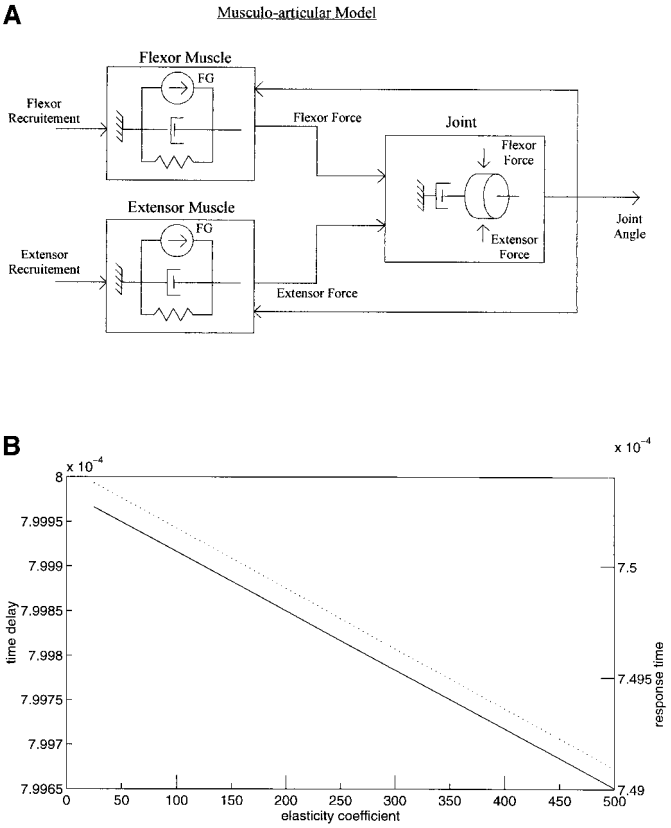

Figure R3. R3A: A simple muscle-joint model. R3B: Effect of elasticity coefficient on the time delay and the response time of the system depicted in (A) for a constant displacement.

Figure R3 highlights briefly some possible correlations that could be studied more extensively on real systems. It reports the results of a simulation using a simplified system (Fig. R3A) describing a joint controlled with a pair of agonist-antagonist muscles modelled according to Hill (1938). Using the impulse function $U_0(t)$ as input to such a system, one gets the time course of its impulse response. The latter curve can be characterized by its time delay and response time, as defined in Figure R2.

Figure R3B shows the relationship between the time delay (solid line, left vertical axis), the response time (dotted line, right vertical axis), and the coefficient of elasticity for a constant displacement. As one can see, for such a simple system, an increase in the elasticity factors results in a decrease in the time delay and the response time. Accordingly one can suspect that the kinematic theory will not be at odds with other biomechanical theories and that specific experiments could be designed to check for relationships between the neuromuscular temporal parameters μ_i and σ_i and any other specific physical variable describing the dynamic properties of the neuromuscular system.

From the specific point of view of the kinematic theory, what is more interesting is studying μ_i and σ_i by analyzing

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different sets of individual movements to provide a new way to think about movement. Indeed, once a given order (D_1, D_2) is given at time t_0, the CNS controls its propagation through the different neuromuscular subsystems of the agonist and antagonist pathways. This distributed control can be interpreted in terms of controlling the relative logtime delay and the relative logresponse time of both agonist and antagonist systems. The different forms of velocity profile predicted by the delta-lognormal law (see, for example, Fig. 2) occur only because within certain limits, a subject can change or tune the logtime delay and the logresponse time of the agonist system relative to the antagonist system. (I fail to understand why such a tuning of parameters would be biologically implausible [**Hancock & Verwey.** We will see other indirect evidence of such a capability later on. As will be seen, it has nothing to do with "hand waving" (**Sternad & Newell**)]. According to the kinematic theory, the CNS can thus control how to expend the kinetic energy for a specific task by controlling the global timing properties of the agonist and antagonist neuromuscular systems. Because the theory allows for the simultaneous study of both the agonist and antagonist systems, the results can be analyzed in conjunction with any other current recording techniques to check for relationship with any other model variables.

Moreover, the kinematic theory might provide a clue in our effort to link the parameters μ and σ to the biological properties of the individual subsystems of a pathway. The entire demonstration that the asymptotic impulse response of a neuromuscular network tends toward a lognormal function relies on equation 38, which describes the relationship that must exist between the cumulative time delays of the different subsystems involved in the production of a simple movement. In a multiple electrode recording protocol at both the brain and the muscular level, the theory predicts that for any individual movement described by a $\Delta\Lambda$ law, the cumulative time delay as computed from the recorded data at the frontiers of different subsystems will be linked by a proportionality relationship. If such a relationship is not observed, although a $\Delta\Lambda$ velocity profile is observed at the end-effector level, this might indicate that the set of electrodes is not properly positioned. (In the long term, with a little speculation, one can dream that the kinematic theory will even provide some clues for electrode positioning. . . .)

R2.5. Data fitting versus data selection. One point that seems to be acknowledged by numerous commentators is the power of the $\Delta\Lambda$ law to fit individual velocity profiles. However, as pointed out by **Morasso & Sanguineti,** one should not confuse model fitting and data fitting and I agree. Similarly, the remark by **Mitnitski** – "a perfect fit of experimental curves is not sufficient to accept or reject a model" – makes sense. But we should not forget that a perfect fit of experimental data is at least *a necessary condition* in accepting or rejecting a model. Working with a nice conceptual model that does not fit well with the real data can easily lead to speculation. The new suggestions in the foregoing paragraphs for finding a biological meaning for some of the $\Delta\Lambda$ parameters would be completely useless if the $\Delta\Lambda$ law did not have such a remarkable descriptive power. What would be the interest of exploring, for example, the biological meaning of t_0, D_1, D_2, etc., if the parameters extracted from various experimental curves could not allow for an almost perfect reproduction of the measured data? The "human like" example provided by **Mitnitski** is hence unfortunately quite trivial. There are numerous models that can show similar behavior. Simulations like this can be done with the $\Delta\Lambda$ law by modifying a single parameter too (Plamondon 1993b; 1995a).

The first challenge for a model is to reproduce real data. As long as such a conclusive comparison is not done using a large database, a model must be considered merely speculative. The delta-lognormal law has been found to be the most powerful model among 26 such models to describe velocity profiles (Plamondon et al. 1993; Alimi & Plamondon 1993a; 1994). Any model of rapid human movements should at least reach the quality of description obtained with the $\Delta\Lambda$ law. In these previous studies, the Guttman et al. (1992) model has been tested and compared with the other models. Contrary to what is claimed by **Latash & Schöner** in their comments, the Guttman et al. (1992) model did not perform very well. With our 1,052 handwritten stroke databases, it was ranked tenth among 26, resulting in a mean square error of 1.12 cm^2/s^2 compared with 0.09 cm^2/s^2 for the delta-lognormal law (Alimi 1996).

The second challenge for a model is to have meaningful parameters that can be helpful in understanding the process the model describes. I agree that the kinematic theory is far from having reached the descriptive power of the vite model (**Grossberg, van Wieringen & Beek**). It still needs further development and new experiments will have to be conducted to take up the challenge. The fact that a $\Delta\Lambda$ law is very powerful in data fitting will be an essential benchmark for further progress on this topic. Indeed, the interest of having a model that can fit the data almost perfectly is that the fitting process itself can be used to clean up a database prior to going into further analysis. Human movements encompass a high degree of variability and in a specific experiment where individual movements cannot be analyzed with some existing models, statistical tools are normally used to compare, among other things, the mean behavior of a specific variable as well as its variance with respect to the mean. In doing so, a large part of the collected data is considered, often without regard to the success of the human subject in executing the required task.

As indicated in section 5.3, the $\Delta\Lambda$ law provides a clear mathematical definition of a simple ballistic movement. It can be used to check whether all the data collected in a specific experiment on rapid movements are indeed ballistic. Figure R4 shows the result of applying this method to a database of wrist flexions and extensions from a monkey subject (Riehle & Requin 1989). The thin lines represent the experimental data and the thick lines the best fit obtained using the $\Delta\Lambda$ law. As one can see, for a large majority of these 28 curves, an excellent fit is obtained. In fact, for the whole database of 1,817 movements, an almost perfect fit was obtained in 85% of the cases. Looking at those cases where an almost perfect fit is not reached, one can see that very often these movements do not correspond to what the model describes as a simple ballistic movement. (See, for example, curves 3 and 4 in row 5.) In other words, these trials do not correspond to the types of movement that are under study; they often correspond to more complex movements. The quality of the fit with a $\Delta\Lambda$ law can be used to remove these untypical results from the database prior to any statistical analysis to make certain that a "purified" database is used. This is an example of what can

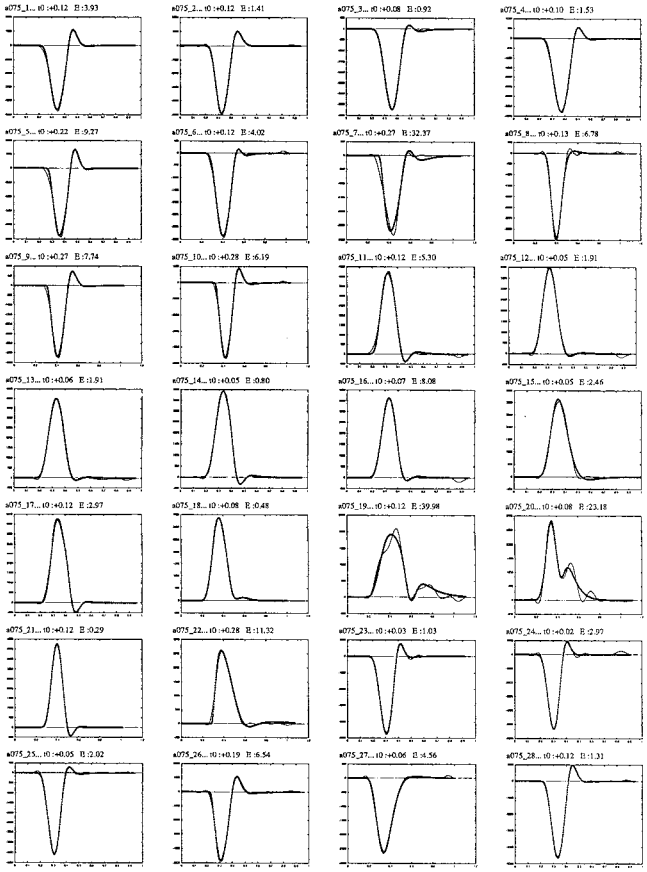

Figure R4. Typical examples of wrist flexion and extension movements of a monkey subject. Light line-original data; solid line-reconstructed data using the delta-lognormal law. At least two of these curves (Nos. 3 and 4, row No. 5) are badly reconstructed using single velocity profiles.

be done with the kinematic theory in terms of database cleaning. (I do not mean that these excluded data are of no interest but that they constitute more complex movements. They can be analyzed separately with the same theory using a more complex strategy likewise based on the $\Delta\Lambda$ law, as will be seen below.)

Without going further on this specific point, let me provide an example of an analysis that could be done next. The individual value of t_0 as extracted from each curve could be used to rank-order or synchronize rasters of action potentials, to generate time histograms of the corresponding cellular activity in the cortex (Plamondon et al. 1995b). Moreover, correlation among the different parameters extracted could be checked and analyses like the ones proposed by **MacKay** could be performed.

R3. Speed/accuracy tradeoffs

The kinematic theory provides a new approach to the analysis and explanation of the origin of speed/accuracy tradeoffs, based on the intrinsic properties of the delta-lognormal equation describing the velocity profile of a rapid aimed movement (Plamondon 1993c; 1995b). This integrated view seems to be at odds with the numerous explanations (see sec. 2 of the target article) that have so far been proposed to explain these phenomena. Numerous comments and criticisms deal with the potential explanation provided by the kinematic theory and its underlying hypothesis. I have grouped my answer on these topics under four headings.

R3.1. Origin of speed/accuracy tradeoffs. According to the kinematic theory, the origin of the speed/accuracy tradeoffs resides in the intrinsic properties of the lognormal impulse responses of the agonist and antagonist neuromuscular systems, as driven by a pair of synchronous commands. For any movement, a subject can control both the amplitudes (D_1, D_2) of the input commands and their time of occurrence t_0, as well as the logtime delays (μ_1, μ_2) and logresponse times (σ_1, σ_2) of the neuromuscular systems that react to these commands. How this control is performed is still an open question. What is the role of the various motor cortical areas, the cerebellum, the basal ganglia, the different motor neurons, and muscle fibres? We do not know yet. The theory assumes that there are two possible levels of movement control: a more localized command level (D_1, D_2, t_0) and a distributed level through the parameters μ_i, σ_i^2 that can be tuned to some specific range of values. A subject when intending to execute a specific simple movement or when asked to do so, must solve one fundamental problem: to link the external task goals with an internal command representation and to execute that command by selecting the proper set of neuromuscular networks.

The $\Delta\Lambda$ law provides some cues to control at the command level, a movement that is to be executed by a specific neuromuscular network. Indeed, for a specific set of μ_i, σ_i^2, it is possible to predict the distance that will be covered (equation 47) and to evaluate movement time *MT* by estimating the times of the velocity zero crossings (equation 41 or 45). The kinematic theory is, to my knowledge, the only analytical theory that links under a single basic equation both the kinematic properties of movement trajectories and the numerous data dealing with movement and task variables. For example, most of the attempts to explain speed/accuracy tradeoffs (see sec. 2) fail to make any prediction about the form and the invariance of the velocity profile.

For a specific Fitts' task, the goal is to cover a distance D with a specific absolute error ΔD within a minimum movement time. In mathematical terms, these requirements are met if the subject neuromuscular system is able to "solve" equation 47 to evaluate the distance and to "solve" simultaneously equations 48 and 50. In other words, it is the task requirements that force the dependence between these independent equations (see **Holly** and **Latash & Schöner**). The latter condition leads automatically to the quadratic law (equation 52) or its simplified version, the power law (equation 54). Therefore, according to the kinematic theory, any successful rapid aimed movement in a Fitts' task has this intrinsic property. Because all movements produced under a specific set of experimental conditions are slightly different, the general behavior can be described by equation R2.2. Equations 52 and 54 could also be rewritten using the statistical distribution of the model parameters to highlight how the noise in the neuromotor systems will mask, as pointed out by **Holly,** the very fundamental phenomenon observed for each individual movement.

Not pertinent here are the commentaries (**Wright &**

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Meyer; Bootsma & Mottet; Heuer; Hancock & Verwey) suggesting that variability is not integrated into the theory as a major cause of speed/accuracy tradeoffs. The kinematic explanation of these tradeoffs does not rely on variability. The phenomenon is a characteristic of any individual movement that can be described by a $\Delta\Lambda$ law (equations 41 or 45). Variability has to be taken into account to understand why the basic quadratic (or power) law is not always fully apparent in mean data (see correlation coefficients $\neq 1$ in Tables 3 and 4) of some specific tasks, because of the noise masking effect.

I am not suggesting that variability studies are unimportant. On the contrary, they are one of the key ways to study the neuromuscular system. Moreover, the kinematic theory can provide another perspective on these studies through the analysis of the statistical properties of the $\Delta\Lambda$ law parameters under different experimental conditions after performing an analysis-by-synthesis of each individual trial (Plamondon 1997).

Moreover, the theory shows that many kinds of speed/accuracy tradeoffs can be explained using the same scheme, based on a single fundamental equation: the delta-lognormal law. This goes against **Thomassen & Meulenbroek's** suggestion to develop models addressing particular classes of movements. It is unnecessary (and unjustified) to do so in kinematic theory. A single global and tunable neuromuscular system is involved in a specific type of aimed movement and a single model with different modes of operation is an efficient and parsimonious way to analyze and understand these movements in different tasks.

R3.2. Visual feedback. The kinematic theory provides a full description of the velocity profile of rapid human movements without directly referring to continuous visual feedback. This is consistent with numerous studies that have reported that removing vision does not drastically affect movement accuracy (see Carlton 1992 for a review). It does not necessarily mean, however, that vision is completely absent from the model. It only means that vision is not fully used in a continuous external feedback loop. It is indeed possible that for the same type of simple movements, the presence or the absence of vision will affect the value of model parameters as well as their variability. This must be studied by an analysis-by-synthesis experiment.

For example, the kinematic theory provides a method for determining how many individual $\Delta\Lambda$ velocity profiles are hidden in a signal. This can be used to propose another interpretation of the **Chua & Elliott** data concerning the fact that "rather than becoming less dependent on visual feedback with practice, improved skill at aiming involves learning to use visual feedback more rapidly and more efficiently." If one attempts to reconstruct the two velocity profiles provided by these commentators, a very specific effect of the use of vision is made apparent in this typical experiment based on the following observation: it is easy to reconstruct the velocity profile of the movement with no vision using a single $\Delta\Lambda$ law, it is not possible to get such good results on the velocity curve of a movement with full vision. At least two (and maybe three) individual submovements are needed, each having a velocity profile described by its own $\Delta\Lambda$ law, to get acceptable results (Fig. R5).

Full vision in this specific example seems to help the subject using a strategy based on a series of commands, the

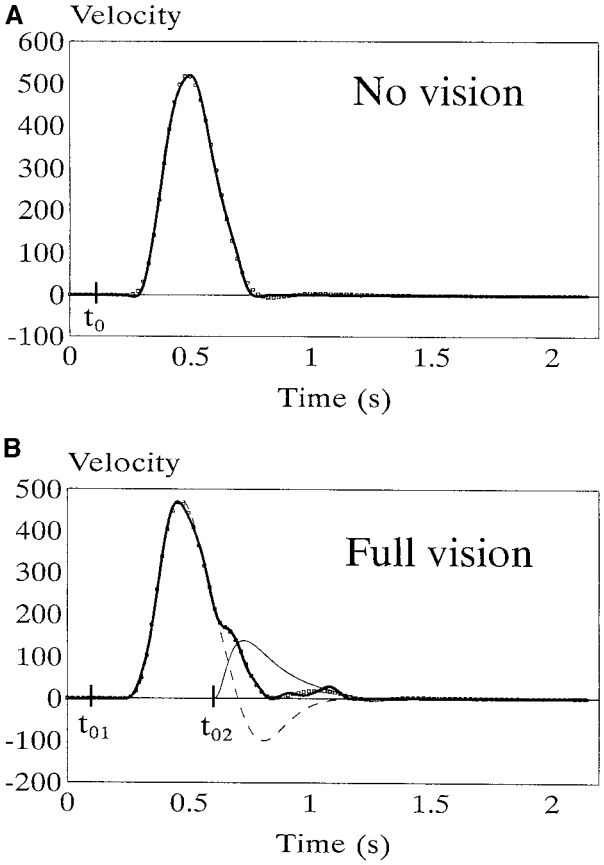

Figure R5. Reconstruction of **Chua & Elliott** data. R5A. No visual feedback condition: the velocity profile can be easily reconstructed from a single delta-lognormal equation. R5B. Visual feedback condition: at least two delta-lognormal velocity profiles are needed to reconstruct this more complex movement. The dotted line and the thin line show the two individual submovements.

first activated at $t_{01} = 115$ msec and the second at $t_{02} = 583$ msec. Analyzing the two classes of movements similar to those depicted in Figure R5 as if they were all simple movements is questionable. Here again, the kinematic theory would allow a detailed study of these two sets of velocity profiles, comparing statistically the changes in the parameters of each individual movement (or submovement) used to reconstruct the complex velocity profiles. Instead of mixing many types of velocity profiles, the theory would allow for a better separation of the two classes of movements (single versus complex) and would provide new ways to analyze the effects of vision in such an experiment. Knowing that two submovements are hidden in the signal of Figure R5B, we understand easily why "subjects take more time to complete movements when full vision is available" (**Chua & Elliott**). As one can see, the kinematic theory is not at odds with the concept of corrective submovements (**Wright & Meyer**); it provides a systematic method for extracting and analyzing these individual components and perhaps a new method for deepening our understanding of the effect of vision in rapid movement control.

I agree with **Hancock & Verwey** that "slower aiming movements especially rely on closed-loop control." I suspect that the kinematic theory alone will fail in analyzing, for example, tracking movements, where continuous visual feedback is probably used to track the on-line difference

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vector (**Grossberg**) and where very different velocity profiles are generally observed (**Desmurget et al.**). Here again the quality of the data fit might be one criterion among others for distinguishing between closed-loop and open-loop conditions, as suggested in section 5.4.

R3.3. Terminology. A couple of commentators criticize the use of the word *prediction* in the target article (**Mitnitski; Goodman**), suggesting that it be reserved for new results rather than for predicting old data. I partly disagree because I believe that when one is trying to develop a general theory in a domain, one should first check whether the predictions of one's model are consistent with what is already known, particularly if the domain is very rich in consistent data and observations. I agree, however, that once this first test is passed, new experimental results should be predicted, as suggested by **Gielen.** This will be the topic of forthcoming papers. For the moment, if I stick to the rigorous interpretation of the verb "predict," the only predictions in this target article should have been:

1. The velocity profile of an end-effector executing a simple rapid movement will follow a delta-lognormal law because the impulse response of a neuromuscular system made up of a large number of coupled subsystems tends toward a lognormal function.

2. All the kinematic observations reported in the field of simple rapid movements can be described using the delta-lognormal law and some corollary equations.

I used the word predict in the more general sense in which it "commonly implies inferences from facts or accepted laws of nature" (See "foretell" in *Webster's Ninth New Collegiate Dictionary*). Hence I started directly with the delta-lognormal law, an unknown equation, and studied the predictions that would emerge from that new equation in the different types of experimental conditions used so far to study rapid human movements. Unless one is a very powerful calculator, I think that none of my *predictions* can be derived from mere visual inspection of equation 39.

The terminological problem is probably still more profound than this. It is probably grounded in differences at not only the scientific level but probably also the cultural one. Whenever one attempts to take a global view of several phenomena previously studied independently, whenever one tries to bridge the gap between different disciplines, semantic problems do occur, as they have in some of the comments presented above. For example, some commentators refer incorrectly to "*transfer functions*" (**Latash & Schöner**), which are used to study linear systems in the frequency domain, although this paper deals only with time domain analysis using impulse responses. These same commentators also refer to statistical terms such as "*distribution*" when they talk about the delta-lognormal law, although the whole demonstration of section 3.1 is based on the central limit theorem as applied to a large number of coupled linear systems. It is a common mistake to associate the central limit theorem with statistical analysis although "the central-limit theorem can be stated independently of any probabilistic considerations. It is merely a property of convolutions involving a large number of positive functions" (Papoulis 1965, p. 267).

Another confusion that seems to arise from a terminological problem introduced by **Wright & Meyer, Goodman,** and **Heuer** concerning the notion of accuracy and variability, the first term being estimated from one measurement and the second being a characteristic of a set of measurements. This is a key point and I will focus on the intuitive approach that guided me when I developed the kinematic theory. One problem I faced was that when a subject is asked to perform one simple gesture he has both to measure visually the distance to be covered and to evaluate the acceptable error for success in the task. These estimates must be performed through a single "visual measurement," not through continuous iterated computation of statistical estimates. I accordingly sought a variable that had the same general meaning for single and multiple measurements. This suggested using the absolute error, which has both a unitary and a statistically consistent meaning. According to the physics of measurement, the absolute error is the unsigned difference between the measured value (x_m) and the ideal value (x_i) required by the task:

$$\Delta x = |x_m - x_i| \tag{R3.1}$$

It also has a statistical meaning for a set of measurements:

$$\Delta x = \frac{\sum_{m=1}^{n} |x_m - x_i|}{n} \tag{R3.2}$$

and the latter equation reduces to R3.1 when $n = 1$. A similar approach with, for example, the standard deviation is not straightforward because the mean ($\bar{x}$) is also required to compute it. In other words, standard deviation cannot be intuitively extracted from a single visual measurement. It might be a good tool for data analysis, but it seems useless for planning single movements. Therefore, using the concept of absolute error (to be more precise I should probably have used the term "mean absolute error" for the second case, equation R3.2), it was possible to link the subject's individual task, described by the delta-lognormal law, to the statistical values reported in the field.

The ΔD in Fitts' law is a task parameter specifying the largest permissible absolute error for any individual trial to be considered successful; it accordingly refers to equation R3.1. In practice, for any experiment where the required distance D is specified, ΔD can be computed as:

$$\Delta D = |MA - D| \tag{R3.3}$$

where MA is the amplitude of the executed movement. Alimi (1996), in his dissertation, has reported that this definition of ΔD provides higher correlation coefficients for the quadratic law than the required target width ΔD generally used in a Fitts' task. He did not however, follow **Heuer's** suggestion to use the overshoot for estimating ΔD. In another thesis (Guerfali 1996) on more complex movement in handwriting this idea was successfully used as a criterion to be minimized in selecting a proper set of initial parameter values for nonlinear regression. In most experiments reporting statistical results, it is not the absolute error as defined in R3.2 that is used but the standard deviation is reported. As shown by Schultz and Roy (1973) and contrary to **Goodman**'s comment, it is possible to estimate the absolute error from the variable and constant errors. In this context, I have used equation 68 to study Wright and Meyer's (1983) data (see Fig. 7), as these commentators fortunately provided both the variable and constant errors. Some commentators have gone further to hypothesize that if the constant errors were small in some experiments the kinematic theory could also explain other experimental results (Newell et al. 1993a; 1993b) on vari-

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able errors because in such cases the "*variable error can be substituted for absolute error*" (**Carlton & Liu**). I have not gone this far in my analysis because I had decided from the start to work only with directly published data in the target article. The proper analysis of the Newell et al. (1993a; 1993b) experiments would be to work with absolute errors and individual velocity profiles using not only temporal and spatial errors but also the neuromuscular parameters to evaluate the predictions of equation 60 (or preferably equation 59) which is the formal space-time relationship (**Sternad & Newell**) predicted from the kinematic theory.

This point highlights a confusion that might emerge if one only uses statistical values to study a phenomenon. In daily life, as stated by a few commentators (**Hermann & Soechting; Hancock & Verwey; Morasso & Sanguineti**) we do not keep repeating simple pointing as in Fitts' task. Many of our gestures can be assimilated to pointing tasks, but we do not repeat these continuously. We execute one movement and then do something else. If some statistical relationship does emerge when a simple task is repeated, this relationship must be grounded in some more fundamental phenomenon that applies to any single trial. I have tried to show in this target article that the typical velocity profile that emerges from the typical impulse response of the neuromuscular system might be a way to avoid this confusion. Most of the other models published to date require proceeding in the opposite direction and make only statistical predictions regarding any specific individual movement as too noisy to reflect anything about the system under study.

This bring me to another interesting point made by **Heuer,** as well as by **Wright & Meyer:** the theory does not conform to empirical definitions. I partly agree except that my reaction is not to reject the theory on these grounds but to propose that we modify our way of measuring some variables. For example, the kinematic theory defines movement time (*MT*) (perhaps I should have used activation time as suggested in footnote 5, to avoid confusion) as $t_1 - t_0$ and experimenters generally measure *MT* using $t_f - t_b$, where t_b and t_f are two time values defined by a threshold that must be exceeded to consider that a movement has started and finished. What is interesting here is the relationship between $t_1 - t_0$ and $t_f - t_b$. In the target article I assumed that these were proportional for a specific class of movements so that the theory dealing with $t_1 - t_0$ should apply directly to the known measured data, using $t_f - t_b$. The simulations of **Heuer** show that a direct proportionality is not necessarily a good assumption depending, as it does, on the definition of the threshold used to compute $t_f - t_b$ but "nevertheless the power function provided a good fit."

If one goes further on this avenue and tries to find a relationship that links $t_1 - t_0$ and $t_f - t_b$, an interesting observation can be made. For example, Figure R6 shows that for the four sets of distances used in the Fitts' experiment with the 1-ounce stylus, using ΔD values that correspond to the index of difficulties used by Fitts, a quasilinear relationship can be plotted between $t_f - t_b$ and $ln(t_1 - t_0)$ for each class of distances. This means that for velocity profiles described by a delta-lognormal law, $t_1 - t_0$ can be roughly approximated by:

$$t_1 - t_0 \cong u_1 exp[u_2(t_f - t_b)] \qquad \text{(R3.4)}$$

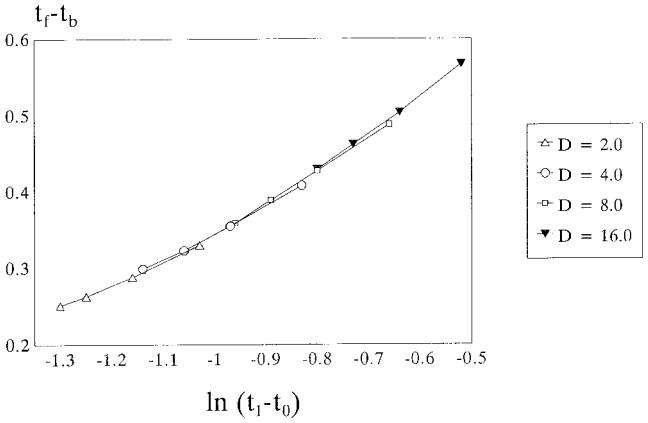

Figure R6. Plot of $t_f - t_b$ versus $lnt_1 - t_0$ for the four sets of distances used in Fitts' experiment with the 1-ounce stylus (see **Heuer**).

Since the kinematic theory predicts, in the simplest case, a power relationship between $t_1 - t_0$ and $D/\Delta D$, it is immediately concluded from equation R.3.4 that, at most, a logarithmic relationship will be observed between $t_f - t_b$ and $D/\Delta D$:

$$t_f - t_b \cong \frac{\alpha \ln \frac{D}{\Delta D} + \ln K - lnu_1}{u_2} \qquad \text{(R3.5)}$$

$$t_f - t_b \cong a + b \ln \frac{D}{\Delta D} \qquad \text{(R3.6)}$$

which corresponds to Fitts' law. So, not only does the empirical definition $t_f - t_b$ partly mask the fundamental power (or quadratic) relationship between movement time and the relative spatial error predicted by the kinematic theory, but it also constrains researchers to observe an apparent logarithmic relationship in this case where the parameters *a* and *b* (equation R3.6) or u_1 and u_2 (equation R3.5) are meaningless. Analyzing the movement in terms of a power law (equation 54) instead allows an interpretation of the parameters *K* and α (equation 54) in terms of the neuromuscular parameters μ_1 μ_2 σ_1 σ_2 that have a physical meaning, as depicted in Figure R2.

Similarly, some commentators mention that the experimental conditions "do not match the description provided or the assumptions that are used to produce the equations to predict time accuracy" (**Carlton & Liu**). Referring to experiment 1 of Newell et al. (1979, p. 51), the subject has "to move through the target distance and not to stop directly opposite the target in a line as close as possible to the target MT." So in this experiment *D* is fixed, as is *MT*. Moreover, ΔD is defined indirectly by "the gap in the metal unit which contained pairs of photoelectric cells." In this context, equations 59 or 60 apply directly as confirmed by Figure 8. The fact that the subjects do not stop directly on the target again means that the thresholds for defining *MT* differ from those described by the kinematic theory (which is based on $t_1 - t_0$). Hence there are some links between any of these thresholds, since any velocity profile is defined by a $\Delta\Lambda$ law.

R3.4. Mathematical clarifications and new support. A few commentators (**Latash & Schöner; Sternad & Newell**)

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have reported that some of the mathematical derivations reported in the paper were not correct. I will try to clarify these points, although I refer these commentators to the two original papers (Plamondon 1993b; 1993c; or 1995a; 1995b) where the theory was first developed for more detail.

The first concern of **Latash & Schöner** is with the derivation of the delta-lognormal law. First of all, my approach does not rely on the statistical version of the central limit theorem but on its interpretation in the context of convolutions of a large number of positive functions. These commentators accordingly miss the mark. However, an analogy can be made by analyzing a random variable that is the outcome of a discrete random process. Kapteyn and Van Uven (1916) and Gibrat (1931) have clearly shown that a lognormal distribution will be observed, provided that the change in the variable is a random proportion of its momentary value (cf. equation 38). There are also other ways to get a lognormal distribution and I refer the reader to classical monographs on the subject (e.g., Aitchison & Brown 1966 or Crow & Shimizu 1988).

Similarly, **Sternad & Newell** seem to misunderstand the derivation of equation 59. My approach is based on a method for deriving absolute error predictions once an equation describing a process is known. Taking the partial derivative of that equation with respect to any variable, one gets the predicted absolute error that will be observed experimentally as a function of a change in that variable. [The reader is referred to classical books like Topping (1972) for more details on this method.] This method could be generalized to study the effect of any parameter of the model by performing a partial differentiation of equations 39, 52, or 54 with respect to this specific parameter.

I agree that by "combining the known formulae one can come to a new result" as proposed by (**Goodman**). In his thesis, Alimi (1996) has shown how different relationships between task parameters could be derived from the delta-lognormal law. He focuses particularly on relationships between:

[*ln*(*time to maximum velocity*)]^2 versus *ln*(*time to maximum velocity*)

[*ln*(*movement time*)]^2 versus [*ln*(*movement time*)]

[*ln*(*time to maximum velocity*)] versus [*ln*(*movement time*)]

[(*maximum velocity* ÷ *movement amplitude*)] versus [*movement time*]

[*maximum velocity* × *movement time*] versus [*movement amplitude*]

[*maximum velocity*] versus [*movement amplitude*]

[*ln*(*maximum velocity*)] versus *ln*[*movement amplitude*]

[*maximum velocity*] versus [*movement amplitude*]

[*ln*(*maximum velocity*)] versus [*ln*(*movement amplitude*)]

In all these cases, Alimi (1996) demonstrates that the predictions of the kinematic theory for the four classes of movement described in section 2 of the target article are verified. This study was done using digitizer data, four subjects and more than 6,000 pen strokes. I agree with **Goodman** that the usefulness of these new formulae needs to be demonstrated but the fact that the kinematic theory leads to new analytical expressions might be an advantage over some other models – for example, over the vite and flete models (**Grossberg**) that are not analytic and where many parameters are hidden in the form of the input go signals.

Some commentators (**Holly; Latash & Schöner**) do not seem to understand the link between D_2/D_1 and $\Delta D/D$. The rationale behind my approach has been explained in section R3.1. I will try to focus only on the misunderstanding: Contrary to the interpretation of (**Latash & Schöner**), γ_1 in equation 48 is not a coefficient; it is a numerical value, for example, 0.1, describing the maximum relative spatial error associated with a specific Fitts' task. Similarly, γ_2 is a numerical value depending on the minimum movement time that a subject is able to achieve in executing the task with his neuromuscular network, as described by μ_i, σ_i. If, for example, $\gamma_2 = 0.3$, we are led by equation 51 to the conclusion that this subject should use a ratio of agonist to antagonist commands three times the required relative spatial error to succeed with the task. **Holly**'s suggestion to use the square root of $\Delta D/D$ or its squared value in the development would also lead to quadratic and power laws, but this choice does not make sense physically because the task requirement is specified in terms of the relative spatial error $\Delta D/D$. Unless it is clearly shown that a square root, a squared function or any power function of $\Delta D/D$ describes the output of the human perception system, such a generalization is not justified.

Wright & Meyer report that a linear relationship has been observed when *MT* varies in experiments where *D* is fixed and the standard deviation of *D* is measured, contrary to the predictions of equation 56. This observation does not invalidate equation 56 but again suggests that a more detailed analysis of individual data will be needed to check for the effect of *MT* because many parametric conditions could lead to an approximately linear relation over the range of measured values. For an example, let us have a closer look at the **Wright & Meyer** data. Equation 56 does in fact predict an *MT* effect more complex than linear in this case. For a given *D*, this equation can be rewritten as:

$$ln\Delta D = -a_1(ln\ MT)^2 - a_2 ln\ MT + a_4 + ln\ D \quad \text{(R.3.7)}$$

So, the kinematic theory predicts a quadratic relationship between *ln* ΔD and *ln MT* for a fixed *D*. To test this prediction, I use the data provided by Wright and Meyer 1983; Tables 2 and 4). Here again the ΔD values were not directly available because these authors reported only the constant and the variable errors. Using equation 68, I estimate $E\{\Delta D\}$ for the two conditions where a sufficient amount of data was available, that is, for $D = 19.9°$ and $D = 26.4°$.

Figure R7 shows a plot of $ln(E\{\Delta D\})$ as a function of $ln(MT)$ for these two conditions. The solid lines show the best quadratic fit using equation R3.7. In both cases, the quadratic prediction leads to correlation coefficients $R^2 \geq 0.993$. A similar analysis using a linear regression between *MT* and the variable error lead to a correlation coefficient of $R^2 = 0.991$ and 0.850 for $D = 26.4$ and 19.9, respectively. It seems that a linear relationship does not necessarily provide the best representation of the *MT* effect in this case, as compared with the predictions of equation R3.7 or 56. These new results provide further support for the kinematic theory and again calls into question the use of the variable error in these studies.

R4. Generalization and applications

Apart from what has already been said in section R3.4, a few commentators (**MacKenzie & Graham; Zelaznik &**

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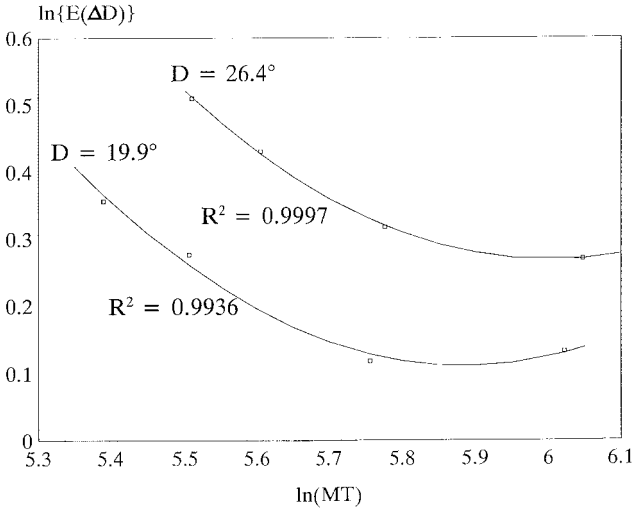

Figure R7. Best quadratic relationship between $ln(E\{\Delta D\})$ and $lnMT$ as predicted by equation R3.7 using equation (68) to estimate $E\{\Delta D\}$. Data from Wright and Meyer (1983; Tables 2 and 4).

Proctor; Stelmach & Thomas) have brought up other phenomena that can be given a specific interpretation in the context of the kinematic theory. I will focus on some of these examples in the first part of this last section. I will limit my analysis to single upper limb movements and will not discuss the interesting comments on bimanual tasks (**Sherwood**) and on experimental protocols using HCT that introduce external nonlinearities in the input data (**MacKenzie & Graham**). There is too much work to be done to describe these phenomena from the actual status of the theory without falling into the trap of pure speculation. Similarly, I will not discuss in detail the Bizzi et al. (1984) experiment, as summarized by **van Wieringen & Beek.** These commentators seem to overlook an important difference in the experimental protocol for normal and deafferented monkeys: "before each experiment with a deafferented monkey, the monkey was allowed to practice the pointing task for a few minutes with the opaque cover removed" (Bizzi et al., 1984, p. 2739). In this context, the fast motor relearning hypothesis cannot be neglected and a more detailed analysis of the individual data would be necessary to understand the experiment. If rapid motor relearning is taken into account, Bizzi et al.'s experimental results can easily be simulated using the superimposition of two individual movements described by the $\Delta\Lambda$ law. I also remind these commentators that Nougier et al. (1996) used a similar experimental protocol with deafferented human subjects to reject the equilibrum point hypothesis.

Some commentators (**Hermann & Soechting; Hancock & Verwey; Morasso & Sanguineti**) also raise a few questions concerning the limited scope of the kinematic theory if it is only valid for simple pointing movements; in everyday life, movements are generally more complex. One of the basic hypotheses underlying the kinematic theory is that many complex movements are made up of a temporal sequence of simple movements, each of these being described by a $\Delta\Lambda$ law (Plamondon 1995c; Plamondon & Guerfali 1996a; 1996b). The kinematic theory hence covers a broader field than the speed/accuracy tradeoffs on which this target article focuses. It can be expanded to more complex movements and has been applied as a key solution to a few engineering problems. I will provide some examples later in this Response to answer the most frequent comments on this point. Each example constitutes a research subject in itself and raises more questions than it answers. Most of my examples will come from handwriting data as collected from a digitizer. Here again I wish to emphasize the fact that, to my knowledge, none of the other models proposed so far to study speed/accuracy tradeoffs has been systematically challenged by an analysis-by-synthesis of real complex individual data. Finally, I will conclude my Response by recalling some of the remaining technical problems associated with the exploitation of the kinematic theory.

R4.1. New explanations to some classical experiments. **MacKenzie & Graham**'s point about separating the effects of D and ΔD in Fitts' tasks can be analyzed and interpreted in a slightly different way using the kinematic theory. This will highlight another interpretation of the command parameters with respect to the task parameters. Indeed, according to equations 47 and 51 (with $\gamma_1/\gamma_2 = 1$ for simplicity), one can easily demonstrate the relationship that must exist between the command parameters D_1 and D_2 and the task parameters D and ΔD to succeed in a Fitts' experiment:

$$D_1 = \frac{D}{(1 - \Delta D/D)} \qquad \text{(R4.1)}$$

$$D_2 = \frac{\Delta D}{(1 - \Delta D/D)} \qquad \text{(R4.2)}$$

The latter equations provide a link between the definition of the task in the external world and its internal representation in terms of input commands (see **Holly**). In other words, these equations predict that the input agonist (D_1) and antagonist (D_2) commands have to be coupled and linked, respectively, to the movement amplitude D and the required target width (ΔD), with the inverse of the relative spatial accuracy $\left[\left(\frac{1}{(1 - \Delta D/D)}\right)\right]$ playing the role of a scaling factor. For example, if a task requires a relative spatial accuracy of $\left(1 - \frac{\Delta D}{D}\right) \geq 90\%$ then the subject should select $D_1 \leq 1.1\ D$ and $D_2 \leq 1.1\ \Delta D$ to succeed.

So, in an experiment where D is changed while ΔD is kept constant, both the agonist D_1 and the antagonist D_2 commands have to be changed. The same is true for an experiment where D is fixed and ΔD varies. Figure R8A shows the delta-lognormal velocity profiles for the first situation, while Figure R8B depicts the second case. As can be seen from Figure R8A and equations R4.1 and R4.2, if, for example, the movement amplitude is multiplied by four from 7.5 cm (curve A) to 30 cm (curve B), with $\Delta D = 4.8$ cm, D_1 must be increased while D_2 must be reduced. This means that agonist activity will increase whereas antagonist activity will decrease. In contrast, for $D = 15$ cm (see Fig. R8B), when ΔD changes from 1.2 cm (curve A) to 4.8 cm (curve B), D_1 increases and D_2 likewise increases. So, both the agonist and antagonist systems will increase their activity provided that the neuromuscular parameters are kept constant or vary slightly around their means.

As can be seen from Figure R8, increasing D (Fig. R8A) results in an increase of movement time, of peak velocity, and of the first acceleration peak (as indicated by the slope

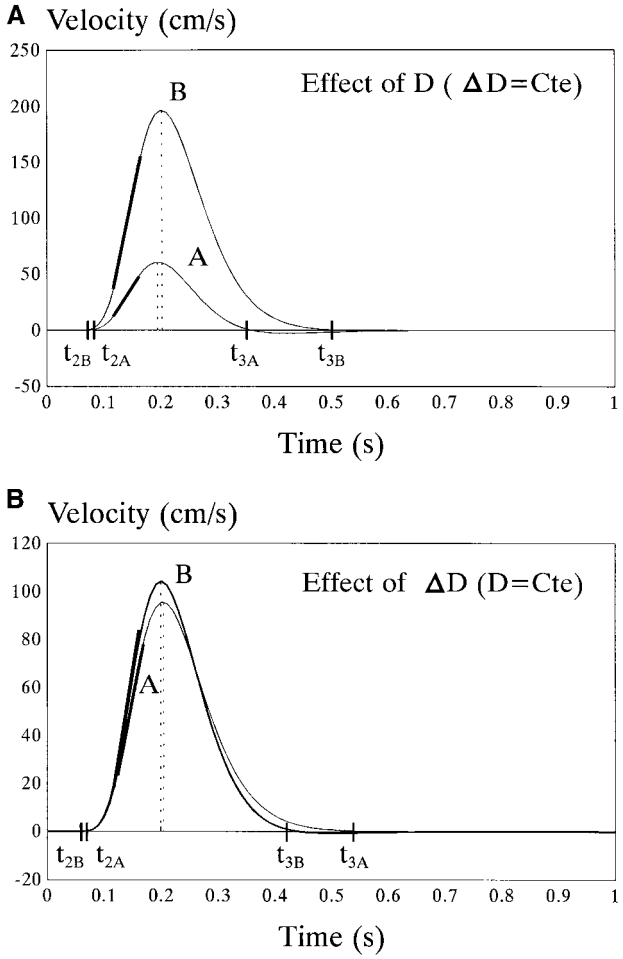

Figure R8. R8A: Effects of D on a velocity profile described by a $\Delta\Lambda$ law (see **MacKenzie & Graham**). R8B: Effects of ΔD on a similar velocity profile (see **MacKenzie & Graham**).

of the velocity curve at the first inflexion point). The time to peak velocity as well as the time from peak velocity both increase, the latter being more influenced. Increasing ΔD (Fig. R8B) results in a decrease of movement time, a slight increase in peak velocity and of the first acceleration peak. The time to peak velocity slightly decreases in these conditions and the time from peak speed decreases to a greater extent. These results are perfectly consistent with the data reported in Table 1 of MacKenzie et al. (1987, p. 635).

In the previous examples, we assumed that the neuromuscular parameters were not affected by D or ΔD. What happens if they are? Figure R9 shows a simulated example using the power law (equation 54). The circle #1 on the solid line represents a specific MT value for a given $D/\Delta D$ and a given set of $\mu_i\sigma_i$. If we wish, for example, to increase $D/\Delta D$ by a factor of 2, two extreme cases can be tested: doubling D or diminishing ΔD by a factor of two. If the μ_i and σ_i are affected by these changes, then two other power laws will be involved. For example, when D is doubled, MT will be predicted by, say, circle #2 on the corresponding new power law. Similarly, reducing ΔD by a factor 2 might lead to another power law at circle #3. Since μ_i and σ_i are not measured here, to make full MT predictions the changes observed in MT for both conditions leading to a doubling of the initial value of $D/\Delta D$ will be interpreted as an asymmetric effect of D versus ΔD. This might be an

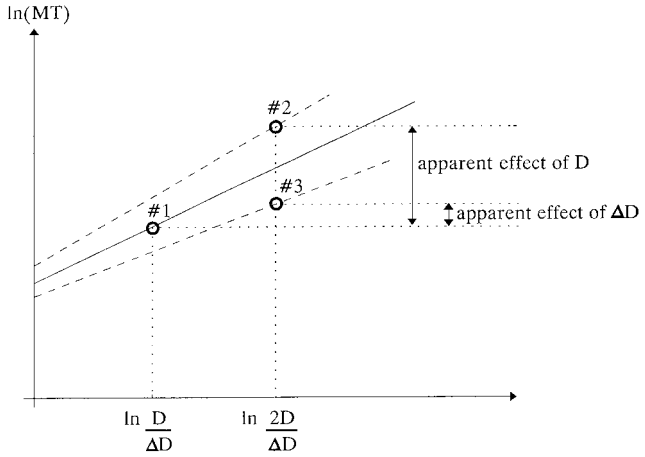

Figure R9. A possible interpretation of the apparent asymmetric effect of a change of D versus ΔD (see **MacKenzie & Graham**).

explanation for the phenomenon reported by **MacKenzie & Graham.** A complete data analysis will be needed to check this explanation of the asymmetric effect of a change of D versus ΔD.

In a similar fashion, the "crucial test" of Carlton's (1994) experiment can be interpreted in a quite different way from that of **Zelaznik & Proctor,** to show that the kinematic theory is indeed perfectly consistent with the reported data. To do so, we must take into account the fact that the μ_i and σ_i are tunable, within certain limits, according to task requirements. We then come to the conclusion that when the $\Delta\Lambda$ law is valid, any experiment can be interpreted with the same approach and the parameter tuning strategy used by the subject can be studied and modelled.

According to equation 54, for any task where the spatial error is specified, there is a power relationship (in the simplest case) between MT and $D/\Delta D$. So for a time accuracy condition in Carlton's experiment, MT (400 msec) as well as D (12 cm) are fixed, and ΔD is required to be minimal:

$$400\ ms = K_a \left(\frac{12}{\Delta D_{min}} \right)^{\alpha_a} \qquad \text{(R4.3)}$$

where K_a and α_a depend on μ_{1a}, μ_{2a}, and σ_a. So, to execute this task, the subject has to tune μ_{1a}, μ_{2a}, and σ_a to get the minimum absolute error permitted within the specified movement time because all the other parameters are fixed.

In the corresponding time minimization task, the same power relationship is valid:

$$MT_{min} = K_m \left(\frac{12}{\Delta D_{min}} \right)^{\alpha_m} \qquad \text{(R4.4)}$$

where K_m and α_m now depend on μ_{1m}, μ_{2m}, and σ_m. Here again, the subject has no choice. Since D and ΔD_{min} are fixed, the only way to succeed in this experiment is to tune μ_1, μ_2, and σ to some new values, μ_{1m}, μ_{2m}, and σ_m.

Substituting in equation R4.4, the values found for ΔD_{min} in the previous equation R4.3, the kinematic theory predicts the following relationship between MT_{min} and the 400 ms required for MT in the time accuracy task:

$$MT_{min} = \frac{K_m}{K_a} (400)^{\alpha_m/\alpha_a} \qquad \text{(R4.5)}$$

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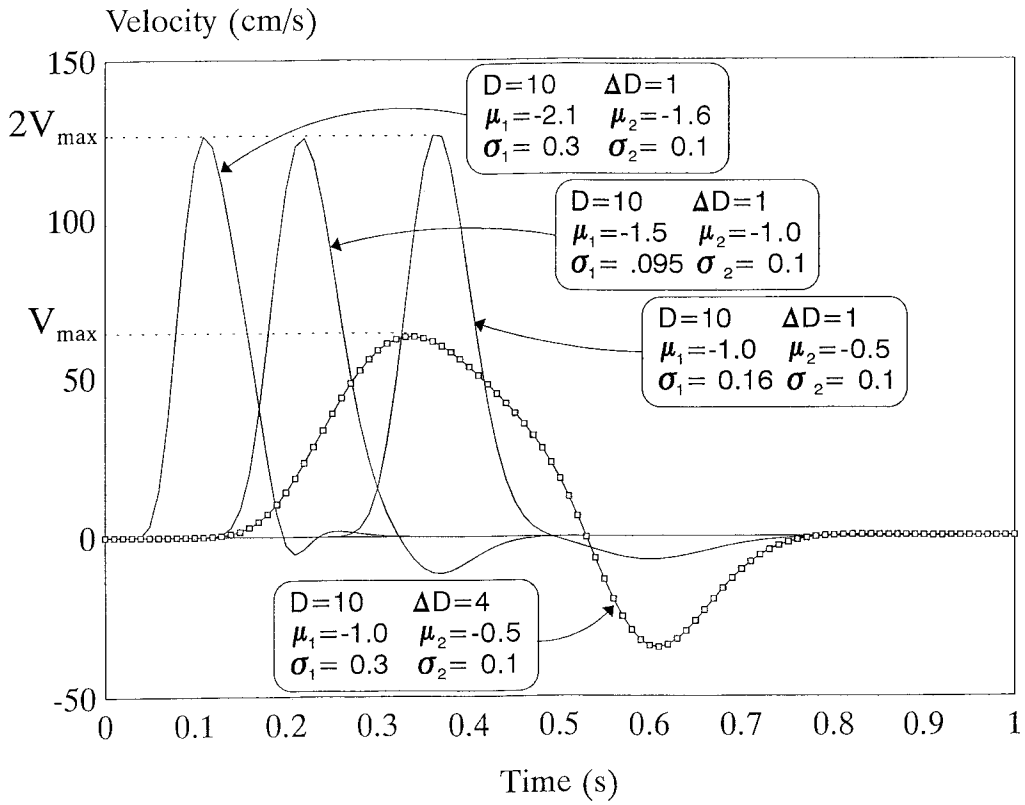

Figure R10. A simulation of the effect of a change of the neuromuscular parameters with instructions. Three simple scenarios leading to a doubling of the velocity peak, with a relative spatial error reduced by four (see **Latash & Schöner**).

So MT_{min} will differ from 400 msec, contrary to the interpretation of **Zelaznik & Proctor.** Moreover, one can easily see that:

$$\mu_{1m} < \mu_{1a}$$
$$\mu_{2m} < \mu_{2a} \qquad \text{(R4.6)}$$
$$\textit{and } \sigma_m < \sigma_a$$

to obey equation (R4.4). This will result in a tendency for the velocity curves to encompass two (or three) submovements in the time minimization task and a single submovement in the temporal accuracy task. In this context, the kinematic theory fully accounts for the data reported by Carlton (1994). A single basic equation, the delta-lognormal law, is thus the basis of the different control strategies that explain both time minimization and temporally constrained tasks. The mistake made by Zelaznik & Proctor in their interpretation of Carlton's experiment arises from their unrealistic assumption that the μ_i, σ_i parameters were fixed, once and for all, as if humans were simple robots.

Similarly, the phenomenon reported by **Latash & Schöner** concerning the effect on the peak velocity of a change in instructions and behavioral information provides further evidence that the neuromuscular parameters μ_i, σ_i are tunable. Figure R10 shows how this modulation with instructions can be modeled using the $\Delta\Lambda$ law. The dotted curve represents a typical simulation of a velocity profile for a Fitts' task with a given relative spatial error $\Delta D/D$. The other three curves show some possible ways to generate a velocity profile for a task requiring a spatial error reduced by four ($\Delta D/4D$), but covering the same distance D (same surface under the curve). By tuning either the μ_i alone, the σ_i alone or a combination of these, several curves with peak velocity twice as high as the one obtained in the former task (dotted curve) can be simulated.

What does this phenomenon suggest? It probably reflects the fact that for a typical task there are preferential values or a distribution of values for the neuromuscular parameters, according to the natural pathways that react to the commands. It is thus easy to succeed in a certain range of tasks by merely modifying the command parameters D_1 and D_2. If the task parameters exceed a certain limit then preferential values of the neuromuscular parameters must be tuned to another range of values. This tuning from one set of values to another can also be done voluntarily, even if the new set of values does not reflect the set of parameters that would be used by the subject under normal conditions.

What would be of interest in these experiments is again to perform an analysis-by-synthesis of each individual velocity profile and then to study the statistical properties of each group of parameters. This might provide new ways to analyze the various subjects' strategies under normal and forced conditions.

R4.2. Perturbed and oscillatory movements. The kinematic theory provides a powerful tool for analyzing more complex movements under the hypothesis that these movements result from the vector addition of the individual velocity profile composing them:

$$\vec{v}(t) = \sum_{i=n}^{n} \vec{v}_i(t - t_{0i}) \qquad \text{(R4.7)}$$

where each $|\vec{v}_i(t - t_{0i})|$ obeys a $\Delta\Lambda$ law.

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As previously mentioned, the theory does not include continuous feedback as it is used, for example, in tracking experiments, but it provides some clues for analyzing and understanding the role of discontinuous feedback as it occurs, for example, in perturbed movements. According to the kinematic theory (Plamondon 1995c; Plamondon & Guerfali 1996a; 1996b), once a given pair of commands D_1 and D_2 has been input to the agonist and antagonist neuromuscular systems, a subject is able to anticipate the distance that will be covered as well as the time it will take to execute that movement and reach the target (T_1). If in the course of action the trajectory for reaching a new target (T_2) has to be modified for one reason or another, the subject starts another movement as if he had already reached the first target T_1. The resulting velocity profile is thus the vector sum of the two single velocity profiles, each one being described by its specific $\Delta\Lambda$ law (Plamondon 1995; Plamondon & Guerfali 1996b). The speed/accuracy trade-offs are hence not only simple properties of single movements but constitute the basis upon which more complex movements are planned and controlled, providing a link between the acceptable relative spatial error tolerated for a trajectory and its motor planning (Plamondon & Privitera 1995).

I disagree with **Desmurget et al.** that the kinematic theory is to be rejected because "several studies demonstrate the inability of the additive procedures to account for the movement reorganisation observed during double step trials." In many of these studies, the way data were processed casts serious doubt on their conclusions, at least in the context of what I have written in section 5.3 about the superimposition of movements. To provide just one example, Massey et al. (1986) check for the possibility of reconstructing a target shift movement (from a point A to a point C via point B) by adding algebraically the velocity profiles of two unperturbed movements from point A to B and from point B to C, where A is the starting point and B is 8 cm above and C, 8 cm below A. To do so, these authors add algebraically (because the two movements are in the opposite direction) to a control velocity profile from A to B, a second velocity profile corresponding to a movement from B to C as measured when enough time was given between stimuli to consider that the second movement was not influenced by the first.

First of all, under these conditions, the kinematic theory predicts that the second peak of the complex velocity profile of the target shift movement can and probably will exceed the peak of the second velocity profile of the unperturbed movement since the complete velocity profile of the first unperturbed movement generally encompasses a sign reversal (see Fig. 2 or 3). This secondary inverted peak will add to the peak of the second constituent movement, making it larger. As reported by Massey et al. (1986, p. 249) "peak second velocities in the target shift trial often exceed those of the movements made all the way from the first to the second target." This observation, which was used by Massey et al. (1986) to reject the superimposition principle, in fact provides further support for the kinematic theory and the vectorial superimposition principle. Another drawback of Massey et al.'s (1986) approach is the way they take into account the time interval between the two signals before adding them. This operation was done empirically using a threshold on the change of the measured force. This approach is very sensitive to the selected threshold value and any small error in the estimated time interval can result in large errors in the algebraic addition, as shown clearly in Figure 6 of Massey et al. With the kinematic theory, each velocity profile is described by a delta-lognormal equation where the parameter t_{0i} defines the time of occurrence of the i^{th} velocity profile. Any addition of velocity profiles can thus be done by optimizing the reconstruction of the complex velocity profile of an individual target shift trial with two delta-lognormal laws, using the difference of t_{0i} to take into account the time interval between successive velocity profiles. When such a procedure is used, not only simple perturbed movements but also very complex movements such as handwriting or signing can be reproduced and described using the vector superimposition of velocity profiles (Plamondon 1995c; Plamondon & Guerfali 1996a; 1996b) (see also Figs. R12, R13, R14, and R15).

The interest of having an analytical description of individual velocity profiles is a key solution to the study of complex movements. The use of averaged values has a masking effect, as already pointed out by **Holly,** and nonoptimized signal synchronization simply destroys the information. It would be interesting to reanalyze the data from studies of perturbed movements with the analysis-by-synthesis methods that are currently in use in my laboratory (Guerfali & Plamondon 1997). For example, it is clear that there is more than one velocity profile in the data reported in Figure 4 of Pelisson et al. (1986). Although this figure refers to the averaged horizontal components of the velocity and acceleration instead of the whole velocity vector, the small "glitch" in the acceleration curve is typical of a movement composed of two submovements, the second smaller than the first (Plamondon 1995c; Plamondon & Guerfali 1996b). Moreover, as the delta-lognormal law also describes eye velocity profiles (Plamondon 1995a), both eye and hand movements could be synchronized and analyzed using the kinematic theory.

Figure R11 depicts the general scheme that can be used to analyze real perturbed trajectories. A first movement aiming at a target T_1 is modified in the course of action to reach T_2. Both the image of the trajectory and the components of the velocity vector can be reconstructed by vectorial addition using the kinematic theory. This makes it possible, for example, to recover t_{01} and t_{02} and to adjust these *timing* parameters in relation to the time of occurrence of the requisite visual stimuli for a change of trajectory.

Figure R12 shows a similar analysis-by-synthesis on an oscillatory pen tip movement (Guerfali 1996). The global curvilinear velocity output is reconstructed using a set of individual movements, each one described by a $\Delta\Lambda$ law. One interesting observation that emerges from this analysis is that the timing of each individual movement is very stable here with $(t_{0i+1} - t_{0i}) \cong 100$ msec ($SD = 12.2$ msec). As can be seen in Figure R12, the kinematic theory allows a very good and continuous reconstruction of the velocity profile.

Many types of oscillatory patterns can be reconstructed using the superposition of individual delta-lognormal velocity profiles. The kinematic theory could help in comparative studies of cyclic versus discrete movements to analyze the suggestions put forward by **Thomassen & Meulenbroek.** For example, the data of **Sherwood** on sequential movements can be analyzed for a possible correlation between movement parameters. Patterns similar to those studied by

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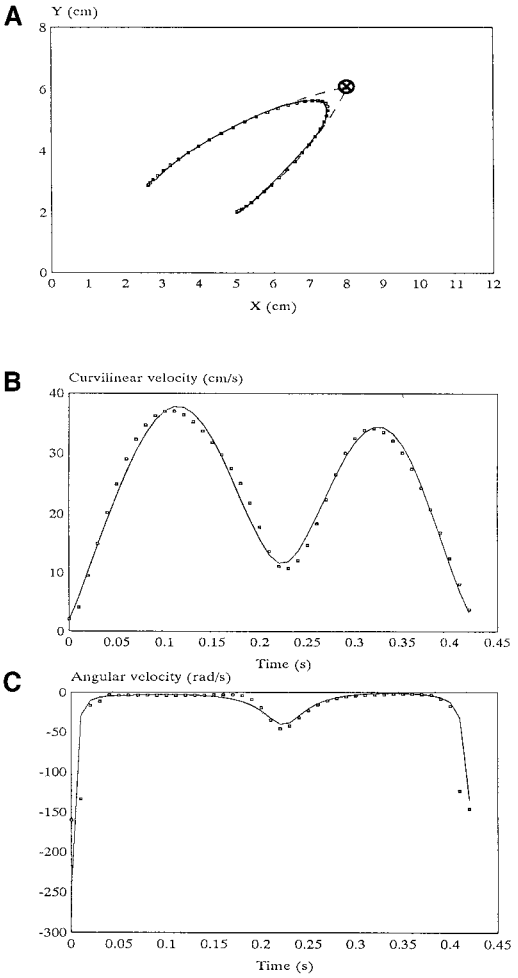

Figure R11. Result of an anlysis-by-synthesis of a perturbed movement. Solid lines–original digitizer data; dotted lines–reconstructed data using the kinematic theory. R11A: Trajectory. R11B: Curvilinear velocity. R11C: Angular velocity.

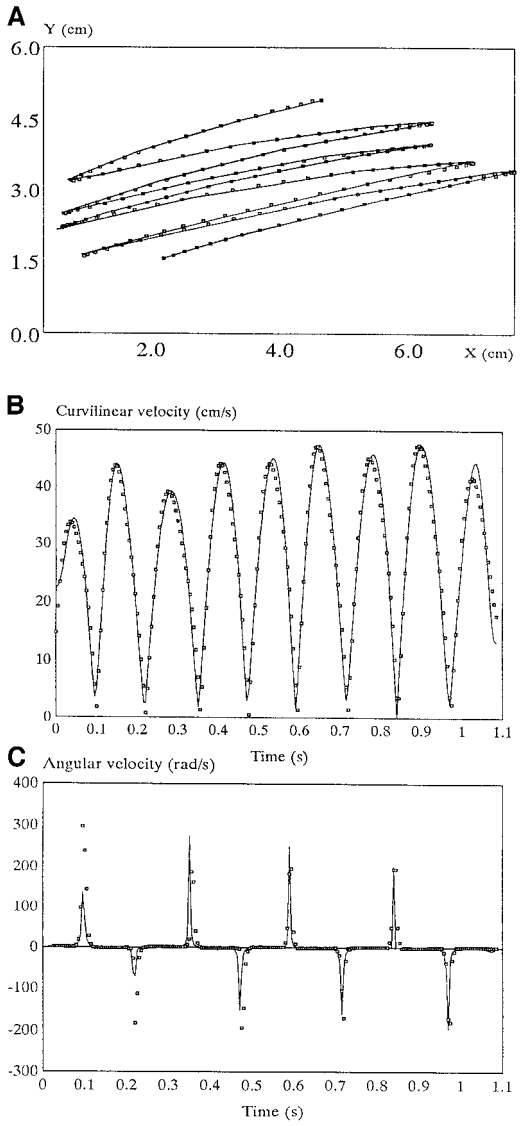

Figure R12. Result of an analysis-by-synthesis of an oscillatory movement. Solid lines–original digitizer data; dotted lines–reconstructed data using the kinematic theory. R12A: Trajectory. R12B: Curvilinear velocity. R12C: Angular velocity.

Mottet et al. (1995) can also be analyzed in the linear context of the kinematic theory. As the relative spatial error increases, peak velocity increases. Velocity profiles become more symmetric around their first peak and secondary peaks emerge (see Fig. 3). With proper tuning of the logtime delays and logresponse times, the superposition of velocity profiles corresponding to movements of larger spatial error will result in a more symmetric oscillatory pattern than the superposition of movements with smaller relative spatial error. Hence a plot of acceleration versus position will automatically look more similar to a straight line in the former condition. In the Mottet et al. (1995) interpretation, the dynamic system will look more linear at larger spatial error values.

R4.3. Cursive script segmentation and signature representation. The reconstruction of complex movements using temporal superposition of a set of simple basic movements is at odds with many models in which the basic unit of movement is an oscillatory one and a single movement is seen as a specific case of an interrupted oscillation. Using an oscillatory movement instead of a single movement as a basic unit often leads to discontinuities in the reconstructed velocity signal (Stettiner & Chazan 1994; Singer & Tishby 1994) that are not apparent in the real signals.

I have recently extended the kinematic theory to study bidimensional movements, providing analytical equations for both the curvilinear and the angular velocity (Plamondon 1995c). Figure R13A shows a handwritten cursive word with its curvilinear (R13B) and angular velocity (R13C) profiles (Guerfali 1996), while Figure R14 presents a similar set of data using a European hand-written signature (Leclerc 1996). In both cases, the whole bidimensional trajectory (both spatial and kinematic information) can be reconstructed using a certain number of circular strokes, by superimposing them in time, using vector addition.

The handwriting and signing action plans can be seen as a sequence of virtual targets distributed over a neural map representing external space. Contrary to **Bootsma & Mottet**'s comments, Plamondon & Privitera (1995) have shown that using this representation it is possible to develop a stable neural net model interfaced to a neuromuscular synergy described by a delta-lognormal law, one that can generate the proper sequence of input commands to pro-

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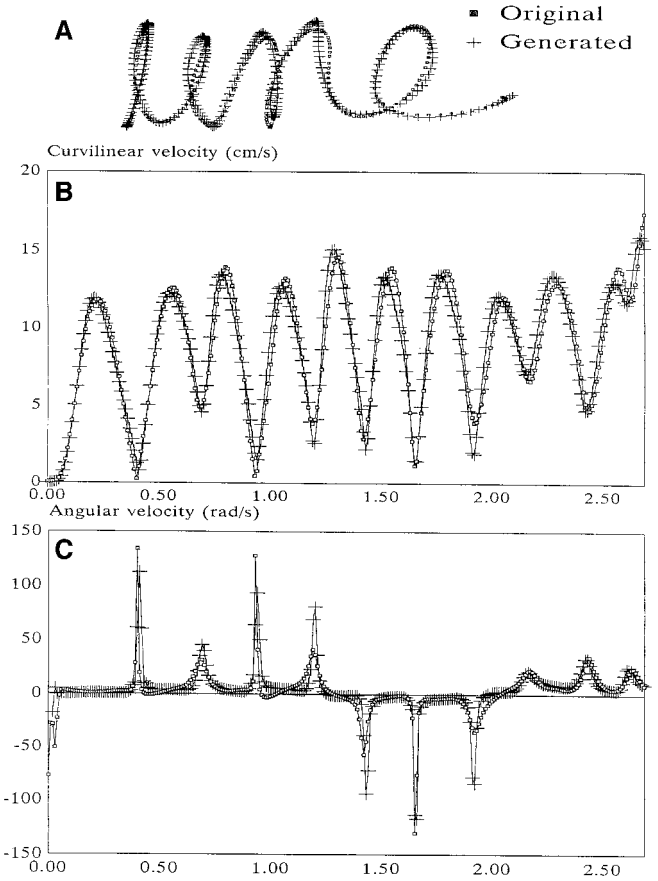

Figure R13. Result of an analysis-by-synthesis of a cursive handwriting movement. Solid lines–original digitizer data: crosses lines reconstructed data using the kinematic theory. R13A: Cursive word trajectory. R13B: Curvilinear velocity. R13C: Angular velocity. (Reprinted with permission from Guerfali 1996.)

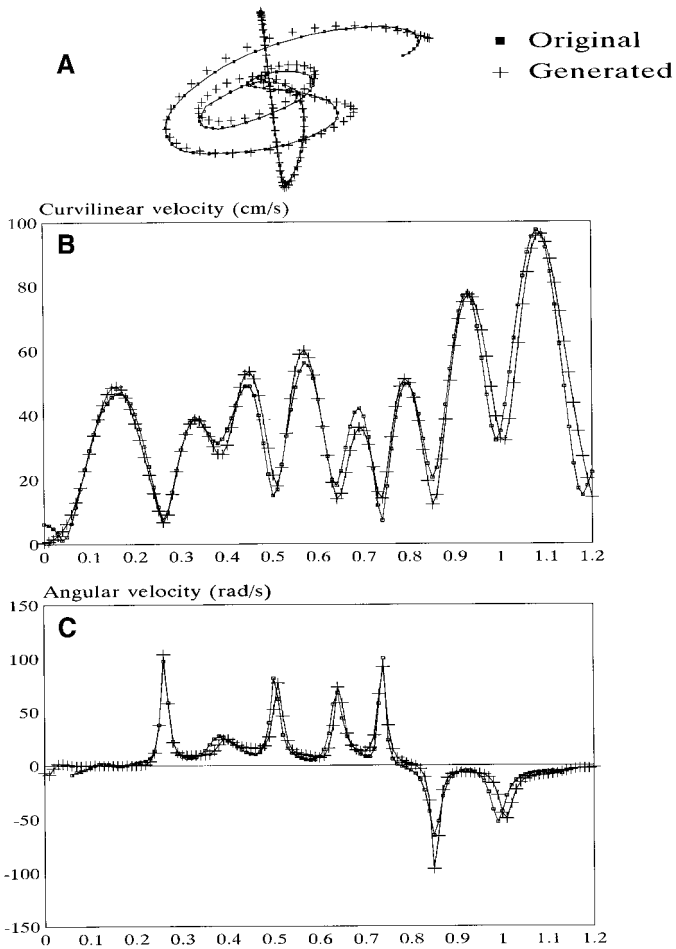

Figure R14. Result of an analysis-by-synthesis of a European signature. Solid lines–original digitizer data; crosses–reconstructed data using the kinematic theory. R14A: Signature trajectory. R14B: Curvilinear velocity. R14C: Angular velocity. (Reprinted with permission from Leclerc 1996.)

duce a specific instance of a trajectory. This approach also provides some cues about learning mechanisms (**Grossberg**) that extract a set of virtual targets from a given trajectory to construct an action plan.

Although many other models have been proposed over the last 10 years to describe handwriting generation (see Plamondon & Maarse 1989, for a review of the models published prior to 1989; Bullock et al. 1993; Morasso et al. 1994; Stettiner & Chazan 1994; Singer & Tishby 1994), none has been tested at such a level of details on real data. For example, the vitewrite model (Bullock et al., 1993) has been used mainly to simulate "human-like" letters and no direct quantitative comparison with real digitizer data has so far been reported.

As can be seen, in Figures R11C, R12C, R13C, and R14C, the angular velocity is also easily described by the kinematic theory (Plamondon 1995c; Plamondon & Guerfali 1996a; 1996b) (**MacKay**). It simply emerges from the vectorial summation process. Plamondon and Guerfali (1996b) have shown that the kinematic theory can easily take into account several well-known phenomena in the field of handwriting: the spatial scaling (Freeman 1914), the isochrony principle (Binet & Courtier 1893), the two-thirds power law (Laquaniti et al. 1983), the effector independence (Merton 1972), and so forth. They also show how handwriting characteristics such as dimension, slant, baseline, and shape are affected and controlled using an action plan made up of virtual targets fed into a neuromuscular synergy that is governed by a delta-lognormal law (Plamondon & Privitera 1996).

These typical analysis-by-synthesis studies are very helpful in segmenting a word or a signature into basic strokes (Plamondon & Guerfali 1996a). Although these strokes are hidden in the signal, they can be theoretically recovered using the kinematic theory and the $\Delta\Lambda$ law. In doing so, one gets information about the time of each stroke as well as about the different commands used by the writer and the system parameters that describe the status of its neuromuscular networks when the subject executes these strokes. The new representation provided by the theory could be used to develop robust handwriting recognition or signatures verification systems.

We are currently working on several related applications: the generation of oriental characters (Plamondon & Guerfali 1997), the recovery of odometric information from handwritten words (Plamondon & Privitera 1997), the segmentation of cursive script (Li et al. 1996), the generation of letter models (Guerfali & Plamondon 1995b), the recognition of cursive script using a syntactical approach (Parizeau & Plamondon 1995), the off-line segmentation of cursive script (Privitera & Plamondon 1995), the design of an on-line signature verification system (Plamondon 1994),

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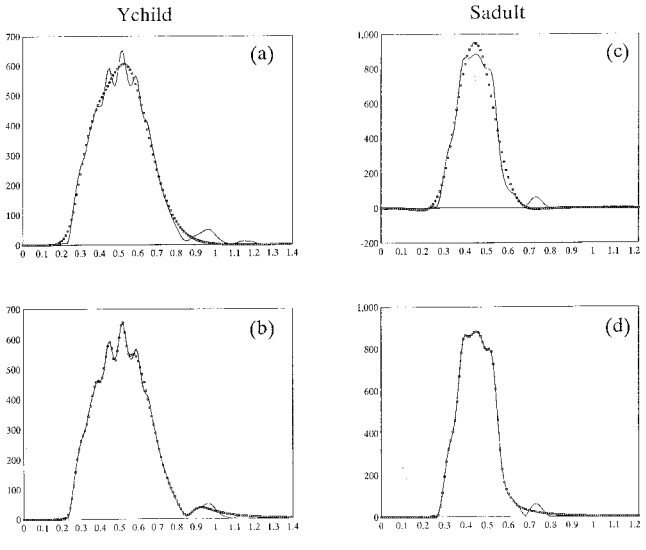

Figure R15. Reconstruction of **Stelmach & Thomas** data. R15A,B: Attempt to reconstruct velocity profile of a child and an aged subject with a single $\Delta\Lambda$ law. R15C,D: Successful reconstructions using four and three strokes, respectively.

the design of an interactive tool to help children learning handwriting (Carrières & Plamondon 1994), the development of a complexity measure to model the difficulty of forging signatures (Brault & Plamondon 1993a), the segmentation of handwritten signatures (Brault & Plamondon 1993b), and so forth.

R4.4. Still a long way to go. I hope to have convinced some readers with this Response that the kinematic theory (Plamondon 1993b; 1993c; 1995a; 1995b) offers a new window on human movement and a complement to the numerous methods used so far. If the theory produced a perfect fit for only a single specific set of experiments, the negative arguments put forward by some commentators about curve fitting would make sense. The fact that the theory explains, from a single basic equation, a vast number of experimental results consistently observed over the last century, certainly suggests that there is something there and that we should explore its possibilities in more detail. The field covered by the theory is not as narrow as some commentators (**Morasso & Sanguineti; Hancock & Verwey**) have suggested. I agree that it is still young and needs further development in terms of underlying processes in order to give its parameter a deeper meaning that leads to new insights into the control and organization of movement, which is certainly the ultimate goal of this work. If we do so we will find that the theory has a range of application and some limits, as is the case with any theory. For example, Figures R15A and R15B show the best result when one tries to fit the velocity profiles provided by **Stelmach & Thomas** with a single delta-lognormal equation. As one can see, the fit is not acceptable, compared with what I presented in the other figures of my Response. This might suggest that the theory does not apply to children or older subjects. However, if one tries to fit the same patterns with more than one velocity profile treating these data as the result of a complex movement instead of a simple one, it is evident in Figures R15C and R15D that quite good results can be obtained even in this case. The child velocity profile can be reconstructed using four individual $\Delta\Lambda$ curves while the older adult data require three single movements to be superposed. So, what seems to be a limit of the theory at first glance might rather reflect different control processes that are age dependent but still exploit the basic delta-lognormal law. A similar approach could be used to study "prolongations of the accelerative phase of movement that occurs in patients with movement disorders" (**Phillips et al.**). In the latter cases, the parameters of each individual constituent curve can be studied and analyzed to better understand the learning strategies used by children as well as the aging problem that senior subjects are faced with and how they *achieve corrective actions* (**Stelmach & Thomas**).

Specifying this range of validity of the theory will probably force us to include new concepts and to link the theory to some concepts that are already used in other models. There are conditions where a detailed analysis will call for other control principles. For example, as noted by **van Wieringen & Beek,** the kinematic theory does not seem to require cost-optimizing principles to describe a single movement, but it is clear that such a principle will be needed to explain human strategies in the generation of complex movements. The theory does not avoid the problem of reverse kinematics. When it comes to extracting model parameters, we are often faced with a set of almost equivalent solutions in terms of mean square errors but different ones in terms of parameter values. Optimization principles will certainly be needed to better analyze, compare, and select among these potential solutions. It emerges from the kinematic theory, however, that as is often done in the study of physical systems, it is probably preferable to track this problem as action ($\Delta E \cdot \Delta t$) minimization, rather than as minimizing jerk (Flash & Hogan 1985), snap (Edelman & Flash 1987), or torque (Uno et al. 1989).

The development of robust methods for automatic parameter extraction from a delta-lognormal law requires solving many problems to ensure the convergence of the algorithms. We have used several successful approaches so far but we still need to explore new techniques. We have principally compared nonlinear regression methods based on the Marquart (1963) approach (Plamondon et al. 1993; Guerfali & Plamondon 1994; Guerfali 1996; Leclerc 1996) as well as more general methods based on genetic algorithms (Ménier et al. 1997). The former approaches require some heuristics to define initial conditions (an initial approximation of the solution) for the search process and the solution obtained depends on these conditions. As long as a study focusses on changes of parameter values relative to changes in experimental conditions, this approach is probably sufficient, provided that the same heuristic is used to determine the initial values. If one is interested in a more global view of possible solutions, genetic algorithms are of interest because they are designed to explore a continuous space of parameters and to optimize an error function given an a priori condition or heuristic approximation of the solution. The latter method was used to produce Figure R4 while Figures R1, R5, R11, R12, R13, R14, and R15 were obtained using nonlinear regressions.

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