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ANTICIPATED FUTURE CONSUMPTION IN AN ENDOGENOUS GROWTH MODEL

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We devise an endogenous growth model in which agents' utility depends not only on current consumption but also on the pleasure of anticipated future consumption. We consider the case in which agents derive satisfaction from their own anticipatory feelings—inward-looking or internal anticipation—and the case in which agents derive utility from anticipation of other people's future consumption—outward-looking or external anticipation. We characterize the effects of introducing a forward-looking consumption reference on the dynamics of the economy. Whereas the inward-looking economy features transitional dynamics, the outward-looking economy does not. The distortions caused by the externality in the economy with external habits can be corrected by subsidizing income at a time-varying rate or by means of a tax on consumption at a decreasing rate. We contrast the equilibrium dynamics of our specification to the more standard specification of the habit formation consumption reference point. Numerical simulations supplement the theoretical analysis.

Keywords: Anticipated Consumption, Endogenous Growth, Optimal Policy

1. INTRODUCTION

This paper explores the effect that introducing a reference consumption stock into utility has on the dynamics of the economy, contrasting the case when the reference stock is forward-looking versus backward-looking. Thus, we leave the standard time separable utility specification, where an agent's welfare depends exclusively on his own current consumption. This framework has long been recognized as being simplistic and implausible (see, e.g., Samuelson (1952), and Koopmans (1960)). Its popularity is mainly because it simplifies the analysis of intertemporal choice. Recently, modern economic growth theory started using

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time nonseparable preferences, that is, when the individuals' utility depends not only on current consumption but also on a consumption reference level.

The two most obvious ways to include intertemporal nonseparability in utility can be described by the statements: "Humans are creatures of habits" and "I am looking forward to that (event)," that is, the consumption reference benchmark can be defined as: (i) "backward-looking" or (ii) "forward-looking." The "backward-looking" case considers that the consumption reference level—or habit stock—depends on past consumption levels. This idea was first introduced by Ryder and Heal (1973), but the notion that current utility depends on both current and past consumption can be traced back to Rae (1834). The "forward-looking" case can be traced back to Jevons (1871) and Marshall (1890) and considers that the consumption reference level is based on future consumption anticipations or "pleasures of anticipation."

Models of habit formation have been widely used with strong enthusiasm in various fields including, but not limited to, asset pricing puzzle (Abel (1990), Constantinides (1990), Galí (1994), Campbell and Cochrane (1999)), rational addiction Becker and Murphy (1988), Dockner and Feichtinger (1993)), monetary models (Fuhrer (2000)), the effect of government debt (Aloui (2013)), and economic growth (e.g., Carroll et al. (1997, 2000), Alvarez-Cuadrado et al. (2004), Turnovsky and Monteiro (2007), Bossi and Gomis-Porqueras (2009), Monteiro et al. (2013), Gómez (2015)). Habits in consumption is also a key ingredient of New Keynesian DSGE models (e.g., Smets and Wouters (2003), Christiano et al. (2005), Born and Pfeifer (2020), Li et al. (in press)).

The importance of anticipation as a source of pleasure and pain has long been recognized by psychologists. Countless studies find evidence that people derive happiness from waiting (anticipatory emotions) for an experience, such as going to a concert, on vacation, a dinner at a fancy restaurant (Kumar et al. (2014), Gilovich and Kumar (2015), Gilovich et al. (2015)), participating in lotteries (Kocher et al. (2014)), or that people's motivation to perform nontrivial activities or take risks depends on how they feel about the future, that is, optimistic, pessimistic, or anxious (Harris (2012)). The importance of anticipation as a source of pleasure and discomfort can be traced back to Jevons (1871) and Marshall (1890). Contrary to habit formation, however, whose origins are linked to empirical properties of the consumption function (e.g., Duesenberry (1949)), the origin of anticipation hypothesis is psychological and theoretical in nature, and thus its use has been slow to take off in economics. The first formal application of this concept to economics is Loewenstein (1987) who focused on the pleasure associated with the process of anticipating future consumption (or its pain in some circumstances) to the standard practice of time discounting of future consumption. He showed how the utility from anticipation may cause individuals to delay positive consumption experiences, thereby enabling them to prolong the pleasures of their anticipatory experience. Another area where anticipation can play an important role is the definition of a consumption reference benchmark, especially if agents are forward-looking. Loewenstein and Elster (1992) looked at the

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idea that anticipated experiences also affect current well-being via the contrast effect, by serving as a point of comparison against which current consumption is measured. More recently, the notion that the reference point should be based on expectations has been emphasized by Köszegi and Rabin (2006, 2007) in their seminal work on consumption and gain–loss utility.

Despite the evidence that anticipated future consumption plays an important role in the determination of current utility, the literature examining its macroeconomic consequences is almost nonexistent. Kuznitz et al. (2008) examine the implications of anticipated consumption on portfolio choice. Faria and McAdam (2013) discuss its consequences for equilibrium in a monetary economy and its implications for the effectiveness of monetary policy. Both papers find that the introduction of anticipated future consumption can have significant consequences. Monteiro and Turnovsky (2016) contrast the effects of anticipation versus habit formation for economic development and conclude that the way the reference benchmark is defined has serious implications for the dynamic adjustment of the economy. More recently, Faria and McAdam (2018) look at the effects of habit formation and anticipation in the context of the Green Golden Rule and show that agents are more environmentally friendly in the presence of habit formation than in the case of anticipation.

This paper analyzes the dynamics of an economy in which the individual's utility depends on a forward-looking reference consumption stock. Since our concern is with the nature of the reference benchmark and how this affects economy-wide dynamics, we keep the production side of the model as simple as possible using the AK endogenous growth framework of Rebelo (1991). In this way, we follow Carroll et al. (1997), who introduce a backward-looking consumption stock—the habits stock—in the utility function of an AK growth model. We also compare the implications of introducing forward-looking versus backward-looking consumption benchmarks.

Several key results arise from the combination of formal analysis and numerical calibration. From the formal analysis, we show that the introduction of forward-looking consumption reference point has important consequences for the transitional dynamics. First, the introduction of an external forward-looking consumption reference into the AK model does not add transitional dynamics to the model. This contrasts with the more traditional habit formation case derived in Carroll et al. (1997), where the introduction of a backward-looking reference benchmark adds transitional dynamics to the AK model. This happens because in the forward-looking case, anticipated consumption reference can jump on impact when the economy suffers a shock, whereas in the habit formation case the reference stock must adjust slowly over time. Second, the presence of an internal anticipated reference stock leads to a much richer dynamics than the internal habit (IH) reference case. In the forward-looking reference case, the transitional dynamics toward the balanced growth path (BGP) can be either monotonic or non-monotonic depending on the starting point of the economy, whereas in the backward-looking reference case the transitional dynamics is always monotonic,

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as shown by Carroll et al. (1997). Third, we characterize the optimal tax policy to correct for the distortionary effects caused by the externality in the model with external anticipation (EA) of future consumption. To this end, we compare the market equilibrium of the AK model with EA with the socially planned solution, which coincides with the market equilibrium of the AK model with internal anticipation (IA). Since the steady states for both economies are the same, the long-run optimal tax policy is no taxation. However, replication of the transition dynamics requires a time-varying policy. We show that the optimal growth path can be attained by subsidizing income or by taxing consumption at a decreasing rate.

On the numerical front, we show that in the presence of a forward-looking consumption reference, the intertemporal welfare changes and the immediate consumption response to a shock is always larger in the case when consumption reference benchmark is forward-looking, with the result being independent of how the way households perceive the reference benchmark, that is, internal or external. Intuitively in the case of habits, any decision made today cannot change habits but will affect it in the future, whereas in the anticipation case, the anticipated future consumption level is based on expectations that are not yet realized and thus any shock will immediately be incorporated in the anticipation reference stock, allowing it to change on impact. This has serious implications for how agents adjust consumption at the time of the shock, and for welfare, with the gains and loses being magnified in the expectation case.

The rest of the paper is organized as follows. Section 2 compares the backward-looking versus the forward-looking reference specifications. Section 3 analyzes the economy with IA, and Section 4, the economy with EA. Section 5 devises an optimal fiscal policy capable of decentralizing the first-best solution in the model with EA. Section 6 performs some numerical simulations. Finally, Section 7 concludes.

2. BACKWARD-LOOKING VERSUS FORWARD-LOOKING REFERENCE SPECIFICATION

In general, the idea that the agent's welfare depends not only on her own current consumption but also on a consumption reference level can be expressed as:

$$\int_0^{\infty} U(C,Z)\,\mathrm{e}^{-\beta t}dt, \quad \beta > 0,$$

where C denotes current consumption, Z denotes the consumption reference level, and β is the rate of time preference. As discussed in Dupor and Liu (2003), the effect of the consumption reference level, Z, can be separated into two categories: (i) the effect on the agent's utility and (ii) the effect on the marginal utility of current consumption. In the former, agents have been classified as jealous, $U_Z < 0$, or as altruistic, $U_Z > 0$. On the other hand, when looking at the effect of consumption reference on the marginal utility of consumption, agents are classified as "keeping up with the Joneses," $U_{CZ} > 0$, or "running away from the Joneses," $U_{CZ} < 0$.[1]

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One important issue, discussed in the previous section, is how should the reference level be measured. One approach, pioneered by Ryder and Heal (1973), is to consider that the reference level is "backward-looking," that is, to consider that the consumption reference level—or habit stock—depends on the time path of past consumption levels. Furthermore, they assume that agents suffer from jealousy.[2] Under the backward-looking approach, the intertemporal utility is specified to be of the form:

$$\int_0^\infty U(C,H)\,\mathrm{e}^{-\beta t}dt, \quad U_C > 0,\ U_H < 0,$$

where

$$H(t) = \theta \int_{-\infty}^{t} C(\tau)\,\mathrm{e}^{-\theta(\tau-t)}d\tau, \quad \theta > 0,$$

so that the consumption habit level at time t, denoted by $H(t)$, is an exponentially declining weighted average of past consumption.[3] Differentiating this expression with respect to time yields

$$\dot{H}(t) = \theta[C(t) - H(t)].$$

However, given that agents are forward-looking, particularly with respect to their consumption decisions, there is no reason why the reference level should be backward-looking, that is, based on past consumption levels, and not forward-looking, that is, based on future consumption anticipations.[4] In this case, the utility would be specified by:

$$\int_0^\infty U(C,A)\,\mathrm{e}^{-\beta t}dt,$$

where the reference level, Z, is now articulated in terms of future consumption anticipations, A, so that welfare at each instant depends on current consumption and anticipated future consumption. To preserve comparability with the more familiar backward-looking benchmark, and following Faria and McAdam (2013) and Monteiro and Turnovsky (2016), we specify the anticipation-based reference consumption benchmark at time t by:

$$A(t) = \rho \int_t^\infty C(s)\,\mathrm{e}^{-\rho(s-t)}ds, \quad \rho > 0,$$

the time derivative of which implies

$$\dot{A}(t) = \rho[A(t) - C(t)].$$

Despite of the symmetry with the backward-looking specification, used in part for comparison purposes, it is important to point some key differences between the two models. First, in the habit formation case, the reference level is based on past observed consumption levels, and it is clearly known and well defined for any value of θ. In contrast, in the forward-looking case, the reference anticipated future consumption level is based on expectations that are not yet realized

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and thus inherently uncertain. By assuming that the weights given to anticipated future consumption, ρ, are known and decline exponentially into the future, we are clearly abstracting from uncertainty involving anticipated future consumption. A second important difference is that in the habit case, present consumption is affected by past decisions that cannot be altered, whereas in the forward-looking case the anticipated future consumption can be affected by the current consumption decisions. To avoid the possibility of time inconsistency, we assume that when agents make a decision they stay committed to that initial decision. In other words, under the current deterministic environment, the expected and actual anticipated consumption stock are the same. Third, it makes no sense for an agent's anticipated future consumption index to be unattainable in the sense of being incompatible with the intertemporal budget constraint. In other words, if anticipated consumption was incompatible with the intertemporal budget constraint, there would come a point in time when the agent would not realize her expectations and thus would suffer an unexpected loss of consumption which would need to be taken into account. Hence, for the forward-looking anticipated future consumption point to be feasible, it must be consistent with the agent's intertemporal budget constraint from time t onward. The conditions resulting from this requirement will be derived analytically in the next section.

One final observation is that for purposes of comparison, as for the conventional backward-looking habits case, we consider that the forward-looking anticipated consumption reference point, $A(t)$, may be internally or externally generated. In the former case, the individual's well-being depends upon his own personal expected future consumption, whereas in the latter case it depends upon society's expected future consumption profile.

3. THE MODEL WITH IA

We study a closed economy populated by a constant population of identical infinitely lived agents. The utility derived by the agent depends both on her current consumption, C, and a reference consumption level or anticipation consumption stock, A. To obtain further insights into the role of anticipated future consumption and contrasting role of the two specifications of the reference consumption levels on the transitional dynamics, we adopt the specific constant elasticity utility function for preferences:[5]

$$U = \int_0^\infty \frac{[C(t)A(t)^\gamma]^{1-\epsilon} - 1}{1-\epsilon} \mathrm{e}^{-\beta t} dt, \quad \epsilon > 0, \quad \beta > 0, \quad \gamma > 0, \tag{1}$$

where β is the rate of time preference, $1/\epsilon$ is the elasticity of intertemporal substitution, and γ is the weight of anticipated consumption in utility.

In the model with IA, the anticipated consumption stock is formed as an exponentially declining average of own future consumption:

$$A(t) = \rho \int_t^\infty \mathrm{e}^{-\rho(s-t)} C(s) ds, \quad \rho > 0. \tag{2}$$

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Differentiating (2) with respect to time, the rate of adjustment of the anticipation consumption stock is

$$\dot{A}(t) = \rho\,[A(t) - C(t)], \qquad \lim_{t\to\infty} A(t)e^{-\rho t} = 0. \tag{3}$$

Gross output per capita Y is determined by:[6]

$$Y = BK, \quad B > 0,$$

where K is the capital stock per capita. The single good of the economy can be either consumed or invested so, in the absence of depreciation, the agent's budget constraint is

$$\dot{K} = BK - C, \quad K(0) = K_0. \tag{4}$$

As discussed above, for the forward-looking anticipated future consumption stock, $A(t)$, to be feasible, it must be consistent with the agent's intertemporal budget constraint from time t on. From (4), the intertemporal resource's constraint at time t is

$$\int_t^\infty C(s)\,\mathrm{e}^{-B(s-t)}ds = K(t). \tag{5}$$

Anticipated future consumption must be consistent with this constraint for all t which, using (2), means that:

$$\frac{A(t)}{\rho} = \int_t^\infty C(s)\,\mathrm{e}^{-\rho(s-t)}ds \le \int_t^\infty C(s)\,\mathrm{e}^{-B(s-t)}ds = K(t).$$

This equation emphasizes how the expected lifetime resources of the agent constrain his rational anticipations of future consumption. For these consumption expectations to be consistently viable, we must have for all s and t that:

$$\mathrm{e}^{-\rho(s-t)} \le \mathrm{e}^{-B(s-t)}$$

which reduces to $\rho \ge B$, that is, the parameter determining the relative weight of each future consumption in the current consumption reference A_t must be larger than the marginal product of capital. If $\rho < B$, after some time, the present value of the agent's anticipated consumption exceeds his resources and is unrealizable. If $\rho = B$, the agent's anticipations are constrained by his current wealth. Thus, the utility function is of the form $U(C, K)$ and is equivalent to the utility function proposed by Kurz (1968) and pursued further in the "spirit of capitalism" literature discussed by Zou (1998) and others. In this regard, we may note that Kuznitz et al. (2008) assume $B = \rho$. In that case, the intertemporal budget constraint reduces to $A(0) = \rho K_0$, thereby imposing an initial condition on $A(0)$. Hence, we shall impose the restriction $\rho > B$, in which case the anticipations stock is unconstrained by the agent's intertemporal resources. Constraining $\rho > B$ allows us to focus on the interaction of anticipations for the relatively near future with current consumption, arguably the most relevant comparison. Therefore, we will assume

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henceforth that the following condition is fulfilled as:

$$\rho > B. \tag{6}$$

3.1. Equilibrium

The agent chooses the path of consumption to maximize the lifetime utility (1) subject to the budget constraint (4) and the constraint on the consumption anticipation stock accumulation (3), taking as given the initial condition on capital, $K(0) = K_0 > 0$.

Let J be the current value Hamiltonian of the agent's maximization problem:

$$J = \frac{(CA^{\gamma})^{1-\epsilon} - 1}{1-\epsilon} + \lambda(BK - C) + \mu\rho(A - C),$$

where λ and μ are the shadow values of capital and the anticipated consumption stock, respectively. The first-order conditions for an interior optimum are[7]

$$C^{-\epsilon}A^{\gamma(1-\epsilon)} = \lambda + \rho\mu, \tag{7}$$

$$B = \beta - \dot{\lambda}/\lambda, \tag{8}$$

$$\gamma C^{1-\epsilon}A^{\gamma(1-\epsilon)-1}/\mu + \rho = \beta - \dot{\mu}/\mu, \tag{9}$$

and the transversality condition is

$$\lim_{t\to\infty} e^{-\beta t}\lambda K = \lim_{t\to\infty} e^{-\beta t}\mu A = 0. \tag{10}$$

The optimality conditions (7)–(9) have the following interpretation. First, equation (7) equates the marginal utility of current consumption to its cost, comprised of the (usual) shadow value of the current capital forgone plus the shadow value of future anticipated consumption. Intuitively, an increase in the shadow value of anticipations induces the agent to reduce current consumption, thereby enabling him to prolong his enjoyment of anticipations. This term incorporates the delay of current consumption in response to anticipations emphasized by Loewenstein (1987), Kumar et al. (2014), Gilovich and Kumar (2015), Gilovich et al. (2015), and Chun et al. (2017). Equation (8) equates the rate of return on capital to the rate of return on consumption, whereas equation (9) is an arbitrage condition that links the rate of return of current consumption expressed in terms of units of anticipation, on the right-hand side, to the return on anticipations, given by the left-hand side. Furthermore, given that $A(0)$ is free, this implies that the shadow value of anticipated future consumption stock must be 0 at the initial value, that is, $\mu(0) = 0$ (see, e.g., Hestenes (1996), Leonard and Long (1992), or Bertsekas (2005)). Intuitively, agents set their initial expectations so that their shadow value is 0.

Let us define $c \equiv C/A$ as the ratio of consumption to the anticipated consumption stock, $k \equiv K/A$ as the ratio of capital to the anticipated consumption stock, and $g_C \equiv \dot{C}/C$ as the growth rate of consumption. The system that drives the dynamics of the economy in terms of the variables c, k and g_C, which are constant

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along a BGP, is (see Appendix A)

$$\dot{c} = c\left(\frac{\dot{C}}{C} - \frac{\dot{A}}{A}\right) = c[g_C - \rho(1-c)], \tag{11}$$

$$\dot{k} = k\left(\frac{\dot{K}}{K} - \frac{\dot{A}}{A}\right) = Bk - c - \rho(1-c)k, \tag{12}$$

$$\begin{aligned}\dot{g}_C = {} & \epsilon g_C^2 - 2\gamma(\epsilon-1)\rho c g_C + \gamma[1+\gamma(\epsilon-1)]\rho^2 c^2 \\ & + \frac{\gamma\rho}{\epsilon}\{(\epsilon-1)[\rho-\beta-\gamma(\epsilon-1)\rho] - \epsilon\rho + \epsilon[B-\beta-\gamma(\epsilon-1)\rho]\}\, c \\ & - [B+\rho-2\beta-2\gamma(\epsilon-1)\rho]g_C + \frac{1}{\epsilon}[\rho-\beta-\gamma(\epsilon-1)\rho] \\ & \times [B-\beta-\gamma(\epsilon-1)\rho].\end{aligned} \tag{13}$$

3.2. Balanced Growth Path

The BGP is obtained when $\dot{c} = \dot{k} = \dot{g}_C = 0$. Appendix B shows that we can state the following result:

PROPOSITION 1. *Let $\rho > B$. The IA economy has a unique interior saddle-path stable steady state with positive long-run growth:*

$$\bar{c} = 1 - \frac{B-\beta}{[\epsilon+\gamma(\epsilon-1)]\rho} = \frac{\rho-\bar{g}}{\rho}, \tag{14}$$

$$\bar{k} = \frac{\bar{c}}{B-\rho(1-\bar{c})} = \frac{\bar{c}}{B-\bar{g}}, \tag{15}$$

$$\bar{g} = \frac{B-\beta}{\epsilon+\gamma(\epsilon-1)}, \tag{16}$$

if and only if

$$B > \beta > (1+\gamma)(1-\epsilon)B. \tag{17}$$

3.3. Phase Diagram Analysis

The system given by (11) and (13) is accessible to phase diagram analysis in (g_C, c)–plane. From (11), the $\dot{c} = 0$–locus is given by the $c = 0$–axis and the line:

$$l_c(g_C) = (\rho - g_C)/\rho. \tag{18}$$

We have that $\partial\dot{c}/\partial c = g_C - \rho(1-c) + \rho c$. Evaluating this expression at the locus $c = 0$, we get that $\partial\dot{c}/\partial c = g_C - \rho$ and, therefore, the arrows point south (north) above (below) the locus for $g_C < \rho$, and the arrows point north (south) above (below) the locus for $g_C > \rho$. Evaluating $\partial\dot{c}/\partial c$ at the locus $c = l_c(g_C)$, we get that $\partial\dot{c}/\partial c = \rho - g_C$. Hence, the arrows point south (north) below (above) the locus $c = l_c(g_C)$ for $g_C < \rho$, and the arrows point north (south) below (above) the locus for $g_C > \rho$.[8]

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Appendix C shows that the $\dot{g}_C = 0$–locus is an ellipse with a positively sloped major axis. In light of this result, the $\dot{g}_C = 0$–locus crosses the $c = 0$–axis at two points, namely $\hat{g}_C = [B - \beta + \gamma(1-\epsilon)\rho]/\epsilon$ and $\breve{g}_C = [\rho - \beta + \gamma(1-\epsilon)\rho]/\epsilon > \hat{g}_C$. Furthermore, the $\dot{g}_C = 0$–locus will cut the $c = l_c(g_C)$ line at $\tilde{g}_C = [(1+\gamma)\rho - \beta]/[(1+\gamma)\epsilon]$ and $\bar{g}_C = (B-\beta)/[\epsilon + \gamma(\epsilon - 1)] < \tilde{g}_C$. We have that:

$$\frac{\partial \dot{g}_C}{\partial g_C}(\hat{g}_C, 0) = -(\rho - B) = -\frac{\partial \dot{g}_C}{\partial g_C}(\breve{g}_C, 0),$$

so the arrows point west inside the ellipse and point east outside the ellipse. With these data, the left panels of Figure 1 depict a phase diagram in the (g_C, c)–plane. Given the configuration of the two loci, the steady state $(\bar{g}_C, \bar{c})$ is saddle-path stable. The steady state $(\tilde{g}_C, \tilde{c})$, if it is feasible, is unstable, whereas the steady states with $c = 0$ do not satisfy the terminal condition in (3).

The two left panels of Figure 1 show the two possible cases that may arise. Let us denote by (g_C^M, c^M), the point in the ellipse in which the maximum value of g_C is attained. Figure 1(a) depicts the case in which (g_C^M, c^M) is to the right and below the point $(\tilde{g}_C, \tilde{c})$, that is, the case in which $c^M \leq \tilde{c}$. In this case, the stable saddle path starts from the point $(\tilde{g}_C, \tilde{c})$ and evolves toward the steady state $(\bar{g}_C, \bar{c})$ in a monotonic fashion, with c increasing and g_C decreasing steadily. Figure 1(b) depicts the case in which (g_C^M, c^M) is to the right and above the point $(\tilde{g}_C, \tilde{c})$, that is, the case in which $c^M > \tilde{c}$. In this case, the transition path is not monotonic. Now c decreases steadily toward its stationary value $\bar{c}$, but g_C first increases and then decreases as it evolves toward its steady-state value $\bar{g}_C$. This case happens when $c^M > \tilde{c}$. An alternative characterization can be given in terms of the slope of the ellipse at $(\tilde{g}_C, \tilde{c})$:

$$\left.\frac{dg_C}{dc}\right|_{\dot{g}_C=0}(\tilde{c}) = -\frac{\frac{\partial \dot{g}_C}{\partial c}(\tilde{g}_C(\tilde{c}), \tilde{c})}{\frac{\partial \dot{g}_C}{\partial g_C}(\tilde{g}_C(\tilde{c}), \tilde{c})}$$

$$= \frac{\gamma\rho(1-\gamma\rho\tilde{k})}{[1-(1+2\gamma)\rho\tilde{k}]} \begin{cases} > 0, \text{ if } \tilde{k} \leq \frac{1}{\rho(1+2\gamma)} \text{ or } \tilde{k} > \frac{1}{\gamma\rho}, \\ = 0, \text{ if } \tilde{k} = \frac{1}{\gamma\rho}, \\ < 0, \text{ if } \frac{1}{\rho(1+2\gamma)} < \tilde{k} < \frac{1}{\gamma\rho}. \end{cases}$$

If the slope is negative, the maximum value of g_C in the ellipse is to the right and below $(\tilde{g}_C, \tilde{c})$, and so, the transition path is monotonic as shown in Figure 1(a). If the slope is zero, the maximum value of g_C in the ellipse is attained at $(\tilde{g}_C, \tilde{c})$, and so, the transition path is also monotonic as shown in Figure 1(a). If the slope is positive, the maximum value of g_C in the ellipse is to the right and above $(\tilde{g}_C, \tilde{c})$, and so, the transition path is non-monotonic as shown in Figure 1(b).

The right panels of Figure 1 depict a phase diagram in the (k, c)–plane. Given that the economy is on its saddle path in the (g_C, c)–plane, c converges monotonically. Hence, the $\dot{c} = 0$–locus given by $c = \bar{c}$ is stable in (k, c)–plane, that is, the arrows point north (south) below (above) the locus, whereas the locus given by $c = \tilde{c}$ is unstable, that is, the arrows point south (north) below (above) the locus.

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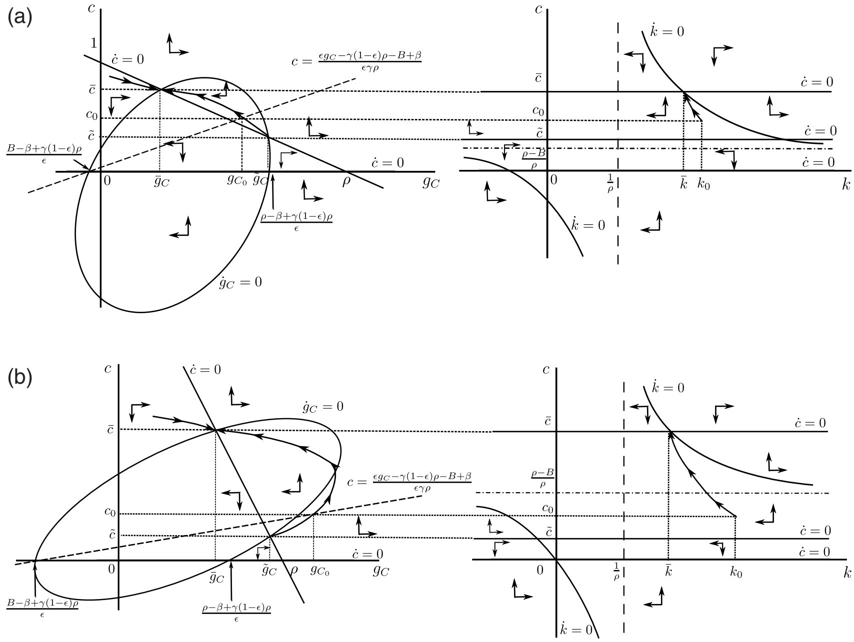

FIGURE 1. Phase diagram in the IA case. (a) Monotonic transition of $g_C = \dot{C}/C$ (b) Non-monotonic transition of $g_C = \dot{C}/C$

From equation (12), the $\dot{k}=0$–locus is given by $l_k(k)=(\rho-B)k/(\rho k-1)$. This locus has a vertical asymptote at $k=1/\rho$, and it is decreasing and concave to the left of $1/\rho$, decreasing and convex to the right of $1/\rho$, with $\lim_{k\to-\infty} l_k(k) = \lim_{k\to+\infty} l_k(k)=(\rho-B)/\rho>0$, $\lim_{k\to(1/\rho)^-} l_k(k)=-\infty$ and $\lim_{k\to(1/\rho)^+} l_k(k)=\infty$. The $\dot{k}=0$–locus is unstable in the relevant region ($k>0$ and $c>0$), that is, the arrows point east (west) to the right (left) of the locus, because $\partial\dot{k}/\partial k = B-\rho(1-c)$ which when evaluated at the $\dot{k}=0$–locus, using (14) and (15), reduces to $\partial\dot{k}/\partial k = B-\bar{g}=\bar{c}/\bar{k}>0$. Given the configuration of the two loci, there exists a unique and saddle-path stable steady state $(\bar{k},\bar{c})$.[9]

To determine the initial point (g_{C_0}, c_0) in the stable saddle path of the left panels of Figure 1, we use the condition that $\mu(0)=0$ because $A(0)$ is free. Substituting $\mu(0)=0$ into (A2), we have that the initial point (g_{C_0}, c_0) is located in the intersection of the stable manifold with the line $c=f(g_C)=[\epsilon g_C-\gamma\rho(1-\epsilon)-B+\beta]/(\epsilon\gamma\rho)$. This is the dashed line depicted in the left panels of Figure 1. This line is increasing, cuts the $c=0$–axis at $g_C=\hat{g}_C$, and cuts the locus $c=l_c(g_C)$ at $c^I=[\epsilon+\gamma(\epsilon-1)]\bar{c}/[(1+\gamma)\epsilon]<\bar{c}$ and $g_C^I=(B-\beta+\gamma\rho)/[(1+\gamma)\epsilon]=\bar{g}_C+\gamma\rho\bar{c}/[(1+\gamma)\epsilon]>\bar{g}_C$. Therefore, the initial values (g_{C_0}, c_0) satisfy that $c_0>c^I$ and $g_{C_0}>g_C^I$. Furthermore, the line $c=f(g_C)$ cuts the ellipse given by the $\dot{g}_C=0$–locus at the points $(\hat{g}_C, 0)$ and (c^J, g_C^J), where

$$c^J=\tilde{c}+\frac{2(\rho-B)}{(1+\gamma)\rho\epsilon}>\tilde{c},$$

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and

$$g_C^J = \tilde{g}_C + \frac{(\gamma-1)(\rho-A)}{(1+\gamma)\epsilon} \begin{cases} > \tilde{g}_C & \text{if } \gamma > 1, \\ = \tilde{g}_C & \text{if } \gamma = 1, \\ < \tilde{g}_C & \text{if } \gamma < 1. \end{cases}$$

Once determined in this way the initial value $c(0) = c_0$, this value determines the initial value $k(0) = k_0$ (and, as $K(0) = K_0$ is given, also determines $A(0)$) in the stable saddle path in the right panels of Figure 1. The former analysis shows that c increases and k decreases monotonically toward their respective long-run values. However, the growth rate of consumption g_C can exhibit two different behaviors: (i) decrease steadily toward its stationary value or (ii) first increase and then decrease steadily toward its stationary value.

4. THE MODEL WITH EA

In the model with external anticipated consumption, the reference stock is formed as an exponentially declining average of future consumption:

$$A(t) = \rho \int_t^{\infty} \mathrm{e}^{-\rho(s-t)} \bar{C}(s)ds, \quad \rho > 0, \tag{19}$$

where $\bar{C}$ denotes the economy-wide average level of consumption. Differentiating (19) with respect to time, the rate of adjustment of the anticipation consumption stock is

$$\dot{A}(t) = \rho\,[A(t) - \bar{C}(t)], \qquad \lim_{t\to\infty} A(t)e^{-\rho t} = 0. \tag{20}$$

4.1. Equilibrium

The agent chooses the path of consumption to maximize the lifetime utility (1) subject to her budget constraint (4), taking as given the path of economy-wide average consumption $\bar{C}$ and, therefore, the time path of anticipated consumption $A(t)$ given by (19), and the initial condition on capital $K(0) = K_0 > 0$.

Let J be the current value Hamiltonian of the agent's maximization problem:

$$J = \frac{(CA^{\gamma})^{1-\epsilon} - 1}{1-\epsilon} + \lambda(BK - C),$$

where λ is the shadow value of capital. The first-order conditions for an interior optimum are[10]

$$C^{-\epsilon} A^{(1-\epsilon)\gamma} = \lambda, \tag{21}$$

$$B = \beta - \dot{\lambda}/\lambda, \tag{22}$$

and the transversality condition is

$$\lim_{t\to\infty} e^{-\beta t} \lambda K = 0. \tag{23}$$

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Equation (21) equates the marginal utility of consumption to the shadow price of capital. Equation (22) equates the rate of return on capital to the rate of return on consumption.

Henceforth, we use that $\bar{C} = C$ in a symmetric equilibrium because all agents are identical. Let $c \equiv C/A$ and $k \equiv K/A$ be the ratios of consumption and capital to the anticipated consumption stock, and let $g_C \equiv \dot{C}/C$ be the growth rate of consumption. Equation (20) can be rewritten as:

$$\dot{A}/A = \rho(1-c). \tag{24}$$

Log-differentiating (21) with respect to time using (22), we get

$$-\epsilon\dot{C}/C + \gamma(1-\epsilon)\dot{A}/A = \beta - B. \tag{25}$$

The system that drives the dynamics of the economy in terms of the variables c and k, which are constant along a BGP is

$$\dot{c} = \frac{c}{\epsilon}\left\{B - \beta + [\epsilon + \gamma(\epsilon - 1)]\rho(c-1)\right\}, \tag{26}$$

$$\dot{k} = Bk - c - \rho(1-c)k. \tag{27}$$

Equation (26) results from (25) and (24) using that $\dot{c}/c = \dot{C}/C - \dot{A}/A$. Since $\dot{k}/k = \dot{K}/K - \dot{A}/A$, equation (27) results from (24) and (4). Equation (25) entails that the growth rate of consumption is

$$g_C = \frac{1}{\epsilon}[B - \beta + \gamma(1-\epsilon)\rho(1-c)]. \tag{28}$$

4.2. Balanced Growth Path

The BGP is obtained when $\dot{c} = \dot{k} = 0$. Appendix D shows that we can state the following result.

PROPOSITION 2. *Let $\rho > B$. The EA economy has a unique interior unstable steady state with positive long-run growth:*

$$\hat{c} = 1 - \frac{B - \beta}{[\epsilon + \gamma(\epsilon - 1)]\rho} = \frac{\rho - \hat{g}}{\rho}, \tag{29}$$

$$\hat{k} = \frac{\hat{c}}{B - \rho(1-\hat{c})} = \frac{\hat{c}}{B - \hat{g}}, \tag{30}$$

where the long-run growth rate of consumption, capital, and output per capita is

$$\hat{g} = \frac{B - \beta}{\epsilon + \gamma(\epsilon - 1)}, \tag{31}$$

if and only if

$$B > \beta > (1+\gamma)(1-\epsilon)B. \tag{32}$$

The steady state of the EA economy is unstable and, therefore, the economy jumps at the outset to its BGP. There are different dynamic implications when the

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reference stock is forward-looking versus the case when it is backward-looking. As Carroll et al. (1997) show, adding a backward-looking reference stock adds dynamics to the AK model, whereas adding a forward-looking reference stock does not add transitional dynamics to the model.

In summary, the evolution of the economy is described by:

$$K(t) = K_0 e^{\hat{g}t},$$
$$C(t) = C_0 e^{\hat{g}t} = (B - \hat{g})K_0 e^{\hat{g}t},$$
$$A(t) = A_0 e^{\hat{g}t} = \frac{1}{\hat{c}}(B - \hat{g})K_0 e^{\hat{g}t},$$

where we have used that $\hat{c}/\hat{k} = B - \hat{g}$, so that $C_0 = (B - \hat{g})K_0$ and $A_0 = C_0/\hat{c} = (B - \hat{g})K_0/\hat{c}$.

Comparison of equations (29), (30), and (31) with equations (14), (15), and (16) shows that the steady-state values of c, k, and g are the same whether the anticipation stock is formed in an external or internal form.[11] It should be noted, however, that introducing an external—or internal—forward-looking reference consumption stock into the standard AK model changes the BGP. As equation (31) shows, the inverse of the (effective) elasticity of substitution is $\epsilon + \sigma(\epsilon - 1)$ in the AK model with EA, whereas it is ϵ in the standard AK model. Thus, the long-run growth rate, the long-run ratios and, therefore, the trajectories of C and K change when we introduce anticipated future consumption.

4.3. Phase Diagram Analysis

Let us first note that we have assumed that the condition (6), $\rho > B$, is met. The system (26)–(27) is accessible to phase diagram analysis. From equation (26)—aside from the horizontal line $c = 0$— the $\dot{c} = 0$–locus is simply $l_c(k) = \hat{c}$. This locus is unstable in the relevant region ($c > 0$ and $k > 0$) because $\partial\dot{c}/\partial c = [\epsilon + \gamma(\epsilon - 1)]\rho c/\epsilon > 0$. From equation (27), and just like in the internal case, the $\dot{k} = 0$–locus is given by $l_k(k) = (B - \rho)k/(\rho k - 1)$. This locus has a vertical asymptote at $k = 1/\rho$, with $\lim_{k\to(1/\rho)^-} l_k(k) = -\infty$ and $\lim_{k\to(1/\rho)^+} l_k(k) = +\infty$, an horizontal asymptote at $c = \lim_{k\to\pm\infty} l_k(k) = (\rho - B)/\rho$, and it is decreasing and concave to the left of $1/\rho$, and decreasing and convex to the right of $1/\rho$. We have that $\partial\dot{k}/\partial k = (\rho - B)/(\rho k - 1)$ when evaluated at the $\dot{k} = 0$–locus and, therefore, the $\dot{k} = 0$–locus is unstable to the right of $1/\rho$ and stable to the left of $1/\rho$ in the relevant region ($c > 0$ and $k > 0$). Figure 2 depicts a phase diagram in the (k,c)-plane. Given the configuration of the two loci, there is an unstable steady state $(\hat{k}, \hat{c})$ and a stable steady state $(0, 0)$—which, however, does not satisfy the terminal condition in (20). Hence, the economy must jump to the BGP at the outset. The crucial point is that both C and A are jumpable variables and, therefore, so are $c = C/A$ and $k = K/A$. Hence, the stability result in this case means that given the initial value K_0, the agent set the initial value of her current

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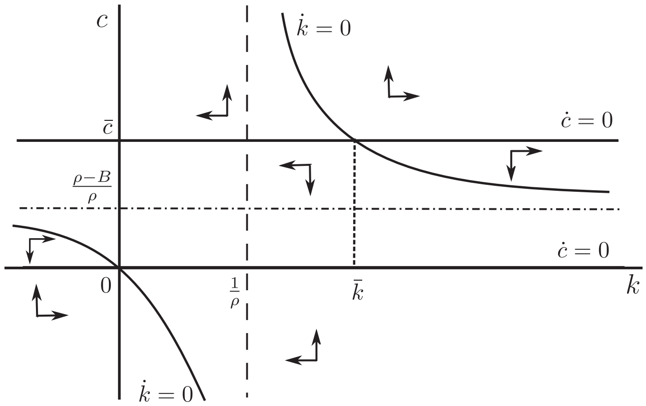

FIGURE 2. Phase diagram in the EA case.

consumption, C_0, and the initial value of the anticipated consumption stock, A_0, such that $c(0)=C(0)/A(0)=\hat{c}$ and $k(0)=K(0)/A(0)=\hat{k}$. Thus, this model does not exhibit transitional dynamics and instantaneously jumps to the BGP.

5. OPTIMAL FISCAL POLICY

In this section, we will analyze how the socially optimal solution path can be attained as a market equilibrium in the EA economy. A benevolent social planner would take into account that the average future consumption is equal to the agent's future consumption when solving the optimization problem. Thus, the first-best optimal solution of the EA economy coincides with the market equilibrium of the IA economy.

Since the steady states of the EA and the IA economies coincide, a long-run optimal policy would be no taxation (or subsidization). However, the phase diagram analysis clearly shows the different transitional adjustment that the EA and IA economies exhibit. Therefore, we have to determine an optimal fiscal policy that ensures that the EA economy evolves as the IA economy along the transition to the BGP. To this end, we introduce the government that can tax income and consumption at (time-varying) rates τ_Y and τ_C, respectively. The raised revenue is rebated as lump-sum transfers, S, to agents. Hence, the agent's budget constraint becomes

$$\dot{K}=(1-\tau_Y)BK-(1+\tau_C)C+S, \tag{33}$$

and the government budget constraint is

$$\tau_Y BK+\tau_C C=S. \tag{34}$$

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The agent in the EA economy maximizes utility (1) subject to the budget constraint (33). Now, the first-order conditions are

$$C^{-\epsilon}A^{(1-\epsilon)\gamma} = (1+\tau_C)\lambda_D, \tag{35}$$

$$(1-\tau_Y)B = \beta - \dot{\lambda}_D/\lambda_D, \tag{36}$$

where λ_D is the shadow value of capital in the (decentralized) EA economy with government.

Regarding the IA economy, it will be useful to describe the dynamics of the economy in terms of c, k, and $q \equiv \mu/\lambda$, which is the relative shadow value of anticipated consumption. The system that drives the dynamics of the economy in terms of the variables c, k, and q is (see Appendix E)

$$\dot{c} = \frac{c}{\epsilon}\left\{B - \beta + [\epsilon + \gamma(\epsilon-1)]\rho(c-1) - \frac{\rho\dot{q}}{1+\rho q}\right\}, \tag{37}$$

$$\dot{k} = Bk - c - \rho(1-c)k, \tag{38}$$

$$\dot{q} = (B-\rho)q - (1+\rho q)\gamma c. \tag{39}$$

Comparing (35) and (7), we have that $(1+\rho q)\lambda = (1+\tau_C)\lambda_D$ and, therefore, $\rho\dot{q}/(1+\rho q) + \dot{\lambda}/\lambda = \dot{\tau}_C/(1+\tau_C) + \dot{\lambda}_D/\lambda_D$. Now, using (36) and (8), we get that:

$$\frac{\rho\dot{q}}{1+\rho q} = \frac{\dot{\tau}_C}{1+\tau_C} + \tau_Y B. \tag{40}$$

The optimal growth path can be attained in several ways. For example, it can be attained by means of an income subsidy at a rate:

$$\tau_Y = \frac{\rho\dot{q}}{B(1+\rho q)} = \frac{B - \beta + \gamma\rho(c-1)(\epsilon-1) - \epsilon g_C}{B} < 0,$$

where we have used equations (F1) and (F2) in Appendix F to substitute for q and $\dot{q}$, respectively. The negative sign follows because $\dot{q} < 0$ and $1+\rho q > 0$ along the transition path, as shown in Appendix F. In the steady state, the optimal income tax is zero, $\bar{\tau}_Y = 0$.

The optimal growth path can also be attained by taxing consumption at a rate satisfying that:

$$\frac{\dot{\tau}_C}{1+\tau_C} = \frac{\rho\dot{q}}{1+\rho q} = B - \beta + \gamma(c-1)\rho(\epsilon-1) - \epsilon g_C < 0,$$

using equations (F1) and (F2) in Appendix F, and the fact that $\dot{q} < 0$ and $1+\rho q > 0$ along the transition. Hence, the optimal consumption tax must be decreasing in time. Using that $q(0) = 0$, the solution to the former differential equation is simply:

$$\tau_C(t) = \tau_C(0) + [1+\tau_C(0)]\rho q(t) = \tau_C(0) + [1+\tau_C(0)] \times \frac{B - \beta + \gamma\rho(\epsilon-1)(c-1) - \epsilon g_C + \gamma\rho c}{\rho - \beta + \gamma\rho(\epsilon-1)(c-1) - \epsilon g_C + \gamma\rho c},$$

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TABLE 1. Benchmark parameters and steady-state values

Model	Anticipation	Habits
Production	$B = 0.08$	$B = 0.08$
Preferences	$\beta = 0.05$, $\epsilon = 1.33$, $\rho = 0.2$, $\gamma = 0.5$	$\beta = 0.05$, $\epsilon = 2$, $\rho = -0.2$, $\gamma = -0.5$
Steady state	$\bar{g} = 0.02$, $\overline{C/K} = 0.06$, $\overline{K/A} = 15$, $\overline{C/A} = 0.9$	$\bar{g} = 0.02$, $\overline{C/K} = 0.06$, $\overline{K/H} = 18.33$, $\overline{C/H} = 1.1$

where $\tau_C(0)$ can be set in an arbitrary manner. Equivalently, we have that:

$$\tau_C(t) - \tau_C(0) = [1 + \tau_C(0)]\rho q(t) = [1 + \tau_C(0)]\frac{B - \beta + \gamma\rho - \epsilon(g_C + \gamma g_A)}{\rho - \beta + \gamma\rho - \epsilon(g_C + \gamma g_A)}.$$

The steady-state value of the consumption tax is

$$\bar{\tau}_C = \tau_C(0) + [1 + \tau_C(0)]\rho\bar{q} = \tau_C(0) - [1 + \tau_C(0)]\frac{\gamma(\rho - \bar{g})}{(\rho - B) + \gamma(\rho - \bar{g})}.$$

Hence, $\tau_C(0)$ could be set so as to make its stationary value equal to 0, $\bar{\tau}_C = 0$, by choosing the initial tax rate on consumption as:

$$\tau_C(0) = \frac{\gamma(\rho - \bar{g})}{\rho - B} > 0.$$

6. NUMERICAL RESULTS

This section presents some numerical results to get an insight on what are the effects of several shocks on the long-run equilibrium and the transitional dynamics in the models with EA and IA and in the models with external habits (EH) and IH formation (see Carroll et al. (1997)).[12] To make the results comparable, we calibrate the models considered so that they yield the same long-run equilibrium values in the baseline for the "real" variables $\bar{g}$ and $\bar{c}/\bar{k} = \overline{C/K}$.

Table 1 summarizes the baseline parameterization in the models with habits and anticipated consumption, with the corresponding steady-state values. To set the parameter values, we follow Carroll et al. (1997) to calibrate the IH and EH models: the rate of time preference is $\beta = 0.05$, the instantaneous EIS is $1/\epsilon = 0.5$, the (negative) weight of habits in utility is $\gamma = -0.5$, and the speed of adjustment of the habits stock is $\theta = 0.2 = -\rho$. The productivity parameter B is then set so that the long-run growth rate is 2%, which yields a value of $B = 0.08$. The resulting steady-state ratio of consumption to capital is $\overline{C/K} = 0.06$. In the models with internal and external anticipated consumption, we choose the same parameter values for the rate of time preference, $\beta = 0.05$, the (positive) weight of anticipated consumption in utility, $\gamma = 0.5$, and the (positive) speed of adjustment of the anticipated consumption stock, $\rho = 0.2$. The values of B and ϵ are adjusted

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so that the long-run growth rate and the consumption–capital ratio are identical to those in the EH–IH models, that is, $\bar{g} = 0.02$ and $\overline{C/K} = 0.06$, which yields the parameter values $B = 0.08$ and $\epsilon = 1.33$. Following Fisher and Hof (2000), we shall refer to $1/[\gamma + \epsilon(1 - \gamma)]$ —which is positive given the assumptions made on the parameters' values—as the "effective" elasticity of intertemporal substitution (effective EIS); see equations (16) and (31). Thus, the "effective" EIS is kept equal to 1.5 in the models with habits and anticipated consumption.

To get a better understanding of the role of anticipated future consumption and the difference in the transitional dynamics of the forward- versus backward-looking specification of the reference consumption stock, we carry on some numerical simulations. Before we proceed, it is important to make a few observations. First, we consider two distinct shocks: (i) a 25% destruction of capital and (ii) a 20% increase in productivity. Second, to better understand the differences in reference consumption specification, we break each shock analysis into two subsections: (i) the transitional dynamics with externally generated reference points and (ii) the transitional dynamics with internally generated reference points. Finally, we contrast the expectations versus habits specification.

6.1. A 25% Destruction of Capital

Consider that the economy faces a temporary 25% destruction of capital, brought about by a natural disaster or war. The first thing to take into account is that this shock is stationary in the sense that, following the shock, the economy (big ratios) ultimately returns to its initial pre-shock equilibrium values. Table 2 summarizes the short-run and long-run effects of this destruction on key economic variables, whereas Figure 3 displays the transitional dynamics of several variables after this shock of an economy that was initially at its steady state.

6.1.1. Transitional dynamics with externally generated reference consumption point. Figure 3 and Table 2 show that the response to a 25% destruction of capital is significantly different for the EA and the EH formation cases. As shown in the previous section, the EA case model behaves like the standard AK model, that is, it has no transitional dynamics. In the absence of transitional dynamics, the economy remains in its steady state with the jump variables, C and A, both falling by 25% on impact (Table 2, panel A) and the transitional dynamics shown in Figure 3(e) and (g) and the constancy of the ratio $c = C/A$ shown in Figure 3(d). In addition, the growth rates of consumption and capital and the ratios C/K and S/Y remain unchanged on impact and during the transition (Table 2 panel B and Figure 3(a), (b) and (c)). Overall, the combined effect of the fall in C and A results in a welfare loss of about 35% with the transition shown in Figure 3(b).

Turning to the EH case, Figure 3 and Table 2 show that consumption drops less on impact by 20.72%. Intuitively, in the habit case, the household reduces consumption by less because she believes that habits will remain constant and thus utility will decrease by less. As shown in Figure 3(e), this allows the EH

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TABLE 2. Effect of a 25% destruction of capital

A. Quantities (%Δ)

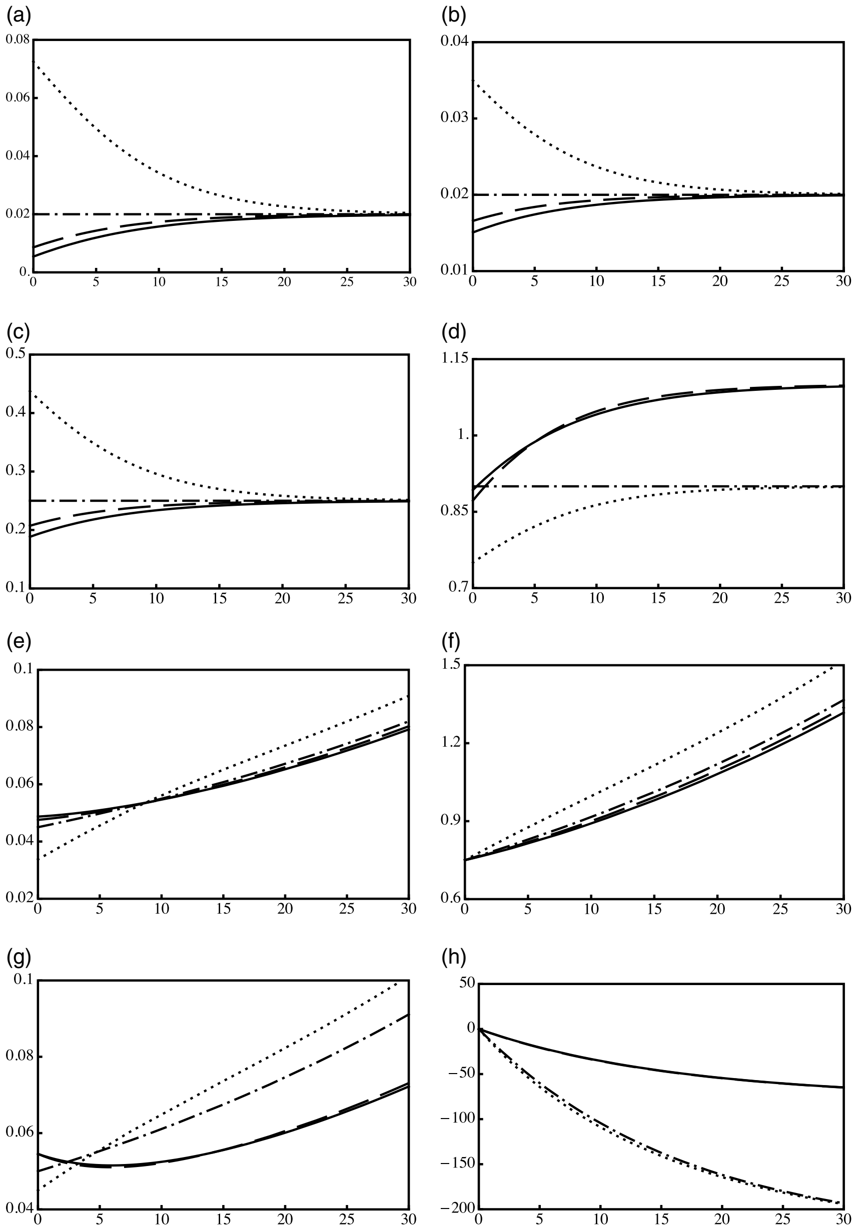
	Impact			After 25 years			Intertemporal		
	Capital	Cons.	Savings	Capital	Cons.	Savings	Capital	Cons.	Savings
Internal habits	−25	−18.83	−24.67	−27.64	−27.44	−0.82	−27.72	−27.72	0
External habits	−25	−20.72	−17.11	−26.60	−26.52	−0.31	−26.63	−26.63	0
Internal anticipation	−25	−57.81	75.00	−16.77	−37.87	1.39	−16.64	−34.48	0
External anticipation	−25	−25	0	−25	−25	0	−25	−25	0

B. Growth rates and ratios (percentage point change)

	Impact			After 25 years			Intertemporal		
	g_C	g_K	S/Y	g_C	g_K	S/Y	g_C	g_K	S/Y
Internal habits	−1.46	−0.49	− 6.17	−0.05	−0.016	−0.20	0	0	0
External habits	−1.14	−0.34	− 4.28	−0.02	−0.006	−0.08	0	0	0
Internal anticipation	5.25	1.50	18.75	0.11	0.028	0.35	0	0	0
External anticipation	0	0	0	0	0	0	0	0	0

C. Welfare evaluation (%Δ)

	Impact	After 25 years	Intertemporal
Internal habits	−18.83	−16.52	−16.19
External habits	−20.72	−16.71	−16.20
Internal anticipation	−53.79	−36.78	−34.20
External anticipation	−35.05	−35.05	−35.05



Notes: Internal habits (solid line), external habits (dashed line), internal anticipation (dotted line), external anticipation (dotdashed line).

FIGURE 3. A shock that reduces the stock of capital by 25%. (a) Evolution of $g_C = \dot{C}/C$ (b) Evolution of $g_K = \dot{K}/K$ (c) Evolution of S/Y (d) Evolution of $c = C/A$ (or C/H) (e) Evolution of $C(t)$ (f) Evolution of $K(t)$ (g) Evolution of $A(t)$ (or $H(t)$) (h) Evolution of Welfare.

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economy to experience about 10 years of higher consumption than in the EA case, after which the situation is reversed. This can be explained by the fact that, contrary to the EA case where the growth rate of consumption remains constant, in the EH economy it falls on impact by 1.14 percentage points and, as shown in panel B and Figure 3(a), growth is not fully recovered after 25 years. The same happens with the growth rate of capital, which drops on impact by 0.34 percentage points resulting from the initial fall in savings of 17.11%. After 25 years, these values have not completely recovered with savings loss of 0.31% and the growth rate of capital percentage point change of 0.006, with the transition shown in Figure 3(b) and (c). This results in capital in the EA case always being above capital in the EH case during the transition to the new equilibrium, as shown in Figure 3(f).

The smaller initial drop in consumption and unchanged habits results in an immediate welfare loss of 20.72% and an overall welfare loss of 16.2%, which is much lower than the immediate and permanent welfare loss of 35.05% observed in the EA case, with the evolution shown in Figure 3(b).

6.1.2. Transitional dynamics with internally generated reference consumption point. Figure 3(e) and Table 2 show that consumption drops less on impact when the consumption reference benchmark is backward-looking, that is in the presence of habits (IH case), than in the case of a forward-looking reference benchmark, that is in the presence of anticipated consumption (IA case). In other words, in the IH case, agents react to the destruction of capital by immediately reducing consumption by 18.83%, while in the IA case agents reduce consumption by 57.81% on impact. Intuitively in the IH case, the agent knows that a drop-in consumption will also drop habits in the future but not on impact (see Figure 3(g)) and thus reduces consumption by less on impact. In contrast, in the IA case, the agent is forced to not only adjust consumption but also keep in mind that his expected (anticipated) consumption must change too and thus it reduces consumption by more in anticipation of her loss in anticipated consumption, with the hope that it will allow for a faster recovery.

The larger drop in consumption in the IA case allows the agent to increase savings on impact by 75%, while in the IH case savings fall by 24.67%. The response of savings allows the growth rate of consumption and capital to increase on impact for the IA case by 5.25% and 1.5 percentage points respectively, while it falls on the IH case by 1.46% and 0.49 percentage points, respectively. Looking at the transition, Figure 3 shows a very different transition trajectory with the growth rates and savings converging to the steady state from opposite directions (Figure 3(a), (b), and (c)) and capital in the IA case always being above the corresponding one in the IH case (Figure 3(f)). The larger drop in consumption in the IA case results in lower consumption than that in the IH case for about 10 years, after which the faster growth and subsequent faster capital accumulation finally pays off and consumption in the IA case surpasses consumption in the IH case (Figure 3(e)).

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Looking at welfare, the dynamics described above leads to an immediate welfare loss of 18.83% in the IH case and 53.79% in the IA case. The faster consumption growth in the IA case allows for a substantial reduction in welfare loss to 36.78% after 25 years, whereas the loss in the IH case has only been reduced to 16.52%. Intertemporally, the big initial cut is reflected in a big welfare loss of 34.20% in the IA case, versus a loss of 16.19% in the IH case, as can be seen in Figure 3(b). This behavior can also be explained because after 25 years, the loss in consumption in the IH case has increased from 18.83% to about 27%, whereas in the IA case the consumption loss has improved from −57.81% to about 38% after 25 years.

6.2. A 20% Increase in Productivity

Contrary to the destruction of capital, this shock is nonstationary in the sense that following the shock, the economy (big ratios and growth rate) ultimately does not return to its initial pre-shock equilibrium values. Hence, after this shock, the long-run growth rate of the economy would increase to 3.07% in all the economies, with a higher long-run consumption–capital ratio of 0.0653. The information that describes the adjustment of the economy is summarized in Table 3 and Figure 4.

6.2.1. Transitional dynamics with externally generated reference consumption point. The first thing to notice is that except for the welfare impact, the behavior of these two models is very similar as shown not only in the table but also in the way that the transitional dynamics for both models almost overlap for most economic indicators. Intuitively, agents do not internalize the effect that their decisions have on the benchmark reference and thus respond very similarly. Looking at Table 3 and Figure 4(e), we see that consumption rises on impact by about 8% in both cases (8.09% in the EH and 8.89% in the EA case) with a compounded effect after 25 years of about 42% in both cases. This is made possible not only by the increase in productivity but also by the rise of about 115% in capital after 25 years as shown in Figure 4(f). Figure 4(g) and (d) reflect just that. In addition, savings drop by about 13% in the EH case and 14% in the EA case, with a 25-year compounded reduction of about 14.8% (see Figure 4(c)).

There are, however, differences in the way the reference benchmark (H or A) react to this increase in productivity, thus creating a different response in terms of welfare. Looking at Figure 4(g), we see that in the habits case, the reference benchmark, which is tied to the past, will slowly adjust over time as the increase in productivity is incorporated in the habits stock, whereas in the anticipation case the agent responds to the promise of higher productivity by reviewing is expectations up and thus increasing A on impact. Combining the consumption response with the reference benchmark, Figure 4(d) shows that the ratio of consumption–habit increases on impact, whereas ratio of the consumption–anticipation falls. The immediate rise in the anticipation and consumption in the EA case allows for an immediate consumption-welfare gain of 17.15% which translates to an

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TABLE 3. Effect of a shock that increases productivity by 20%

A. Quantities (%Δ)

	Impact			After 25 years			Intertemporal		
	Capital	Cons.	Savings	Capital	Cons.	Savings	Capital	Cons.	Savings
Internal habits	0	7.73	−12.88	116.25	42.78	−14.77	–	–	−14.81
External habits	0	8.09	−13.48	115.85	42.54	−14.80	–	–	−14.81
Internal anticipation	0	−18.83	30.56	144.42	86.56	−13.46	–	–	−14.81
External anticipation	0	8.09	−14.81	115.26	42.17	−14.82	–	–	−14.81

B. Growth rates and ratios (percentage point change)

	Impact			After 25 years			Intertemporal		
	g_C	g_K	S/Y	g_C	g_K	S/Y	g_C	g_K	S/Y
Internal habits	1.30	1.14	−4.83	1.07	1.07	−5.54	1.07	1.07	0
External habits	1.24	1.11	−5.05	1.07	1.07	−5.55	1.07	1.07	0
Internal anticipation	5.97	2.70	11.46	1.24	1.11	−5.04	1.07	1.07	0
External anticipation	1.07	1.07	−5.56	1.07	1.07	−5.56	1.07	1.07	0

C. Welfare evaluation (%Δ)

	Impact	After 25 years	Intertemporal
Internal habits	7.73	12.83	16.83
External habits	8.09	12.86	16.83
Internal anticipation	−16.44	32.62	53.60
External anticipation	17.15	36.08	51.25

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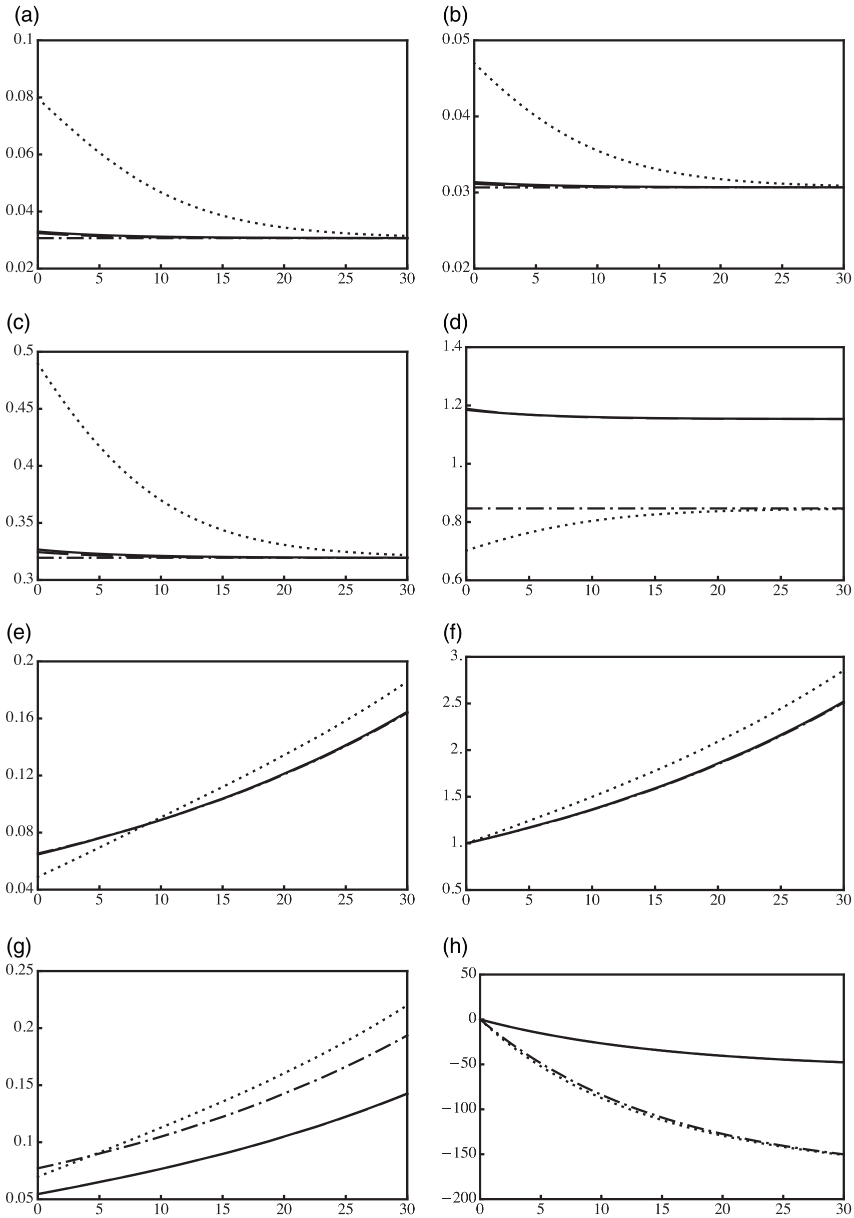

Notes: Internal habits (solid line), external habits (dashed line), internal anticipation (dotted line), external anticipation (dotdashed line).

FIGURE 4. A shock that increases productivity by 20%. (a) Evolution of $g_C = \dot{C}/C$ (b) Evolution of $g_K = \dot{K}/K$ (c) Evolution of S/Y (d) Evolution of $c = C/A$ (or C/H) (e) Evolution of $C(t)$ (f) Evolution of $K(t)$ (g) Evolution of $A(t)$ (or $H(t)$) (h) Evolution of Welfare.

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intertemporal gain of 51.25%, contrasting with the impact gain of 8.09% and 16.83%, respectively, in the EH case and the adjustment shown in Figure 4(h).

Finally, looking at Panel B of Table 3 and Figure 4(a), (b), and (c), we can see that the reaction of the growth rates and the ratios is very similar in both models on impact and exactly the same after 25 years.

6.2.2. Transitional dynamics with internally generated reference consumption point. In contrast to what happens in the external case, the response when the reference consumption is internally generated is very different for the anticipation and the habits cases. On impact, the response of consumption (or similarly savings) is exactly the opposite in both cases, hence making the adjustment that follows also different. In the IA case, consumption responds by decreasing by 18.33% on impact while savings rise by 30.56%, whereas in the IH case the responses are a rise of 7.73% and a fall of 12.88%, respectively. Although both scenarios consider the impact of agent's decisions on the reference consumption, in the IA case, agents incorporate the impact that a higher productivity will have on their expectations and thus reduce consumption on impact while setting a higher anticipation consumption and allowing the agent to save more and thus accumulate more capital in the future. In contrast, in the IH case, the agents' reference benchmark is backward-looking and thus agents must increase consumption on impact to allow habits to rise in the future and reflect the "incorporation" of the higher productivity into habits, thus reducing savings and resulting in a smaller rise in capital after 25 years.

Figure 4(c)–(g) show exactly the adjustment described in the previous paragraph. Figure 4(c) shows that savings jump up on impact for the IA case and then decrease toward the new lower long-run value, whereas in the IH case savings jump down and then proceed to adjust to the lower equilibrium value. This allows for a faster accumulation of capital in the IA case, as shown in Figure 4(f), whereas consumption (see Figure 4(e)) jumps down on impact but has recovered after approximately 10 years and surpassed the value of consumption in the IH case. The evolution of the reference consumption benchmark is shown in Figure 4(g), and it is clearly the case that the increase in productivity brings good expectations and thus the agents rises their anticipation, while habits must adjust over time and never really catch up to the anticipation benchmark. The reverse side if this story is shown in Figure 4(d) with the consumption–anticipation ratio jumping down on impact and converging to the new steady state from below, whereas the consumption–habit ratio jumps up and converges to the new long-run equilibrium from above. Panel B of Table 3 shows that the growth rates of capital and consumption jump on impact for both specifications with the IA case experiencing a substantial higher jump than in the IH case. But in both cases, the adjustment is done from below as shown in Figure 4(a) and (b). The magnitude of these jumps is a rise of 5.97 percentage point change in the growth rate of consumption and 2.7 percentage point change in the growth rate of capital for the IA case, and 1.3 and 1.14 for the IH case, respectively. Finally, the adjustments described above

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result in a consumption–equivalent welfare gain in both specifications, but larger in the IA case both on impact and intertemporal. The gains are of 17.15% on impact and 51.25% overall for the IA case and 7.73% and 16.83%, respectively, for the IH case, with the adjustment shown in Figure 4(h).

6.3. Habits Versus Anticipation

The way the reference consumption benchmark is formed, backward- versus forward-looking, results in very different economic adjustment. In a way the dynamics reflects exactly the idea that in the case of habits, the reference level is based on past observed consumption levels and thus is clearly known and well defined and must adjust slowly after a shock. Any decision made today cannot change the habits stock, H, but will affect it in the future. In contrast in the anticipation case, the anticipated future consumption level is based on expectations that are not yet realized and thus any shock will immediately be incorporated in the anticipated consumption stock, A, allowing it to change on impact. This has serious implications for how agents adjust consumption at the time of the shock, and for welfare, with the gains and loses being magnified in the expectation case.

Furthermore, the transitional dynamics is also very different for the two specifications. In the case of habits, Carroll et al. (1997) have shown that the introduction of habits leads to the presence of transitional dynamic adjustment, irrespective of whether habits are introduced in an internal or external way. In contrast, the introduction of expectation in an external way returns the AK model to its no transitional dynamics result in the EA model with the growth rate and ratios jumping to the new balance growth path at the outset. The introduction of a benchmark reference level of consumption leads to the presence of transitional dynamics in the IA model. However, there are also important differences with the IH model.

Carroll et al. (1997) show that the stable arm for the EH model always lies above (below) that for the IH model for values of $c = C/H$ lower (higher) than the steady state. In any case, the economy can approach the steady state from above if the capital–habits ratio $k = K/H$ is above its steady-state value, with a monotonically decreasing $g_C = \dot{C}/C$, k, and c along the transition, or from below if the capital–habits ratio is below its steady-state value, with an increasing g_C, k, and c. However, in the IA model, the economy approaches the steady state only in one way: the initial ratio of capital to the anticipated consumption stock, $k = K/A$, is always above its steady-state value, and the economy evolves with a monotonically decreasing $k = K/A$ and a monotonically increasing $c = C/A$. The transition path of the growth rate of consumption g_C can be monotonically decreasing or non-monotonic but, in any case, the growth rate of consumption along the transition remains above its steady-state value and approaches its stationary value from above. This is in sharp contrast with the transition dynamics of the growth rate of consumption in the model with IH, when it evolves monotonically along the transition path.

One final comparison can be made in terms of the speeds of convergence. Independently of the way the reference consumption benchmark is formed, the

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external case converges faster than the internal case.[13] In addition, the anticipation model also presents faster convergence speed than the habit formation case. Under our parameter specification, the models with habit formation have a convergence speed of 0.1401 with IH and 0.1650 with EH. In the models with anticipated consumption, the convergence speed is 0.1784 with IA and $+\infty$ with EA, as in this case there is no transitional dynamics. The convergence speed does not change after a stationary shock, as a destruction of capital. However, after a shock that increases productivity by 20%, the convergence speed in the IH model increases slightly to 0.14712 and in the EH model increases moderately to 0.1730, whereas in the IA model falls to 0.1597.

7. CONCLUSIONS

This paper contrasts the macrodynamic implications of two long held beliefs in economics. The first is that current consumption decisions are affected by our past decisions, that is, current consumption is affected by our habits. The second is that the anticipation of some future events will provide utility in advance of the occurrence of the event itself. With that goal in mind, the current paper introduces the idea of anticipated pleasure into an AK growth model by assuming that agent's current utility depends not only on current consumption but also on anticipated future consumption. We then proceed to contrast this with the case when agent's current utility depends not only on current consumption but also on habits and thus follow this literature by assuming two different specifications of the anticipated future consumption: the external and the internal index. To get a better understanding of the differences, we combine the theoretical analysis with a calibration exercise of a plausible economic growth model.

At the theoretical level, we show that the way the reference consumption benchmark is specified has serious implications for the dynamics of the AK growth model. We have three main sets of theoretical implications. First, in the presence of external anticipated future consumption, the model behaves like a standard AK growth model with no transitional dynamics, whereas the presence of IA leads to transitional dynamics if the economy starts at any point other than the steady state. This contrasts with the habit formation case, as shown in Carroll et al. (1997), where the introduction of habit formation in an AK model leads to transitional dynamics no matter how habits are specified. Second, the transitional dynamics of the growth rate of consumption in the economy with IA can be nonmonotonic. However, transitional dynamics are always monotonic in the model with habit formation. Third, we characterize an optimal tax policy that allows the external specification to replicate the entire optimal path of the internal specification. It requires that income be subsidized at a time-varying rate, or consumption be taxed at a decreasing rate.

Numerical simulations supplement the theoretical findings by quantifying the differences between models: habit versus anticipation and internal versus external reference consumption benchmark. One interesting finding is that when the

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economy suffers a sock, the impact response of consumption is always larger in the anticipation case and smaller in the habit formation case. Furthermore, the responses can be ranked as follows, IA, EA, EH, and IH from the larger in absolute value to the smallest. The difference reflects the fact that forward-looking index can jump on impact, whereas the backward-looking case is constrained to gradual adjustments.

Finally, it is important to remember that there is still a lot of work to explore in this field, especially in light of all the work done in psychology regarding anticipatory feelings on how they affect consumers' well-being or leisure labor decisions. This will be the subject of future research.

NOTES

1. An alternative interpretation, which is discussed in Houthakker and Taylor (1966) and Taylor and Houthakker (2010), is in terms of durability of the consumption good.
2. This assumption has been widely used in the literature.
3. The parameter θ characterizes the weight given to past consumption levels; the larger the θ, the faster the weight declines over time.
4. In fact, Z could even be a function of both past and expected future consumption. For simplicity, we treat them separately.
5. In light of empirical evidence that elasticity of substitution is less than 1, $1/\epsilon < 1$, this utility specification implies that anticipation, serving as a reference, will reduce utility $U_A < 0$ and the agents are classified as "keeping up with the Joneses," $U_{CA} > 0$.
6. The time argument is omitted whenever there is no risk of confusion.
7. We assume that $\epsilon + \gamma(\epsilon - 1) > 0$ (see Proposition 1 below), which ensures that the instantaneous utility function is concave in (C, A). Hence, the hamiltonian is concave in the states and the controls, and so, the first-order conditions, along with the initial and transversality conditions, characterize the interior optimal solution of the agent's problem.
8. The locus $c = l_c(g_C)$ would be stable if $c < 0$, which is not relevant from an economic viewpoint.
9. Note that this is confirmed by the local analysis performed above.
10. The first-order conditions are also sufficient because the utility function is concave in C and the budget constraint is linear.
11. This result occurs in the absence of endogenous leisure and is similar to what happens in the case when consumption reference is backward-looking (e.g., Carroll et al. (1997)).
12. In the model with internal habits (IH), the agent solves the problem:

$$\begin{aligned} \max \ & \int_0^\infty \frac{[C(t)H(t)^{-\gamma}]^{1-\epsilon} - 1}{1-\epsilon} \mathrm{e}^{-\beta t} dt \\ \text{s.t.:} \ & \dot{K} = BK - C, \\ & \dot{H} = \theta(C - H), \\ & K(0) = K_0, \ H(0) = H_0, \end{aligned}$$

where H is the habits stock. It can be readily noted that the solution to the IH model can be recovered from the solution of the IA model by replacing A, ρ, and γ with H, $-\rho$, and $-\gamma$, respectively. In the EH model, the rate of adjustment of the habits stock would be $\dot{H} = \rho\,(\bar{C} - H)$, where $\bar{C}$ is the economy-wide average consumption.

13. Gómez (2008) proved analytically this result for the model with habit formation, that is, the convergence speed is higher when habits are formed in an external way than when they are formed in an internal way.

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REFERENCES

Abel, A. B. (1990) Asset prices under habit formation and catching up with the Joneses. *American Economic Review* 80, 38–42.

Aloui, R. (2013) Deep habits and the macroeconomic effects of government debt. *Macroeconomic Dynamics* 17, 1659–1686.

Alvarez-Cuadrado, F., G. Monteiro and S. J. Turnovsky (2004) Habit formation, catching up with the Joneses, and economic growth. *Journal of Economic Growth* 9, 47–80.

Becker, G. S. and K. M. Murphy (1988) A theory of rational addiction. *Journal of Political Economy* 96, 675–700.

Bertsekas, D. P. (2005). *Dynamic Programming and Optimal Control. Volume I*. 3rd ed. Belmont, MA: Athena Scientific.

Born, B. and J. Pfeifer (2020) The New Keynesian wage Phillips curve: Calvo vs. Rotemberg. *Macroeconomic Dynamics* 24, 1017–1041.

Bossi, L. and P. Gomis-Porqueras (2009) Consequences of modeling habit persistence. *Macroeconomic Dynamics* 13, 349–365.

Campbell, J. Y. and J. H. Cochrane (1999) By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy* 107, 205–251.

Carroll, C. D., J. Overland and D. N. Weil (1997) Comparison utility in a growth model. *Journal of Economic Growth* 2, 339–367.

Carroll, C. D., J. Overland and D. N. Weil (2000) Saving and growth with habit formation. *American Economic Review* 90, 341–355.

Christiano, L. J., M. Eichenbaum and C. L. Evans (2005) Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy* 113, 1–45.

Chun, H. H., K. Diehl and D. J. MacInnis (2017) Savoring an upcoming experience affects ongoing and remembered consumption enjoyment. *Journal of Marketing* 81, 96–110.

Constantinides, G. M. (1990) Habit formation: A resolution of the equity premium puzzle. *Journal of Political Economy* 98, 519–543.

Dockner, E. J. and G. Feichtinger (1993) Cyclical consumption patterns and rational addiction. *American Economic Review* 83, 256–63.

Duesenberry, J. S. (1949). *Income, Saving and the Theory of Consumer Behaviour*. Cambridge, MA: Harvard University Press.

Dupor, B. and W. F. Liu (2003) Jealousy and equilibrium overconsumption. *American Economic Review* 93, 423–428.

Faria, J. R. and P. McAdam (2013) Anticipation of future consumption: A monetary perspective. *Journal of Money, Credit and Banking* 45, 423–447.

Faria, J. R. and P. McAdam (2018) The Green Golden rule: Habit and anticipation of future consumption. *Economics Letters* 172, 131–133.

Fisher, W. H. and F. X. Hof (2000) Relative consumption, economic growth, and taxation. *Journal of Economics* 72, 241–262.

Fuhrer, J. C. (2000) Habit formation in consumption and its implications for monetary-policy models. *American Economic Review* 90, 367–390.

Galí, J. (1994) Keeping up with the Joneses: Consumption externalities, portfolio choice, and asset prices. *Journal of Money, Credit, and Banking* 26, 1–8.

Gilovich, T. and A. Kumar (2015) We'll always have Paris: The hedonic payoff from experiential and material investments. In: J. M. Olson and M. P. Zanna (eds.), *Advances in Experimental Social Psychology*, vol. 51, Chapter 4, pp. 147–187. San Diego, USA: Academic Press.

Gilovich, T., A. Kumar and L. Jampol (2015) A wonderful life: Experiential consumption and the pursuit of happiness. *Journal of Consumer Psychology* 25, 152–165.

Gómez, M. A. (2008) Convergence speed in the Ak endogenous growth model with habit formation. *Economics Letters* 100, 16–21.

Gómez, M. A. (2015) Equilibrium dynamics in an endogenous growth model with habit formation and elastic labor supply. *Macroeconomic Dynamics* 19, 618–642.

https://doi.org/10.1017/S136510052000053X Published online by Cambridge University Press

Harris, C. R. (2012) Feelings of dread and intertemporal choice. *Journal of Behavioral Decision Making* 25, 13–28.

Hestenes, M. R. (1996). *Calculus of Variations and Optimal Control Theory*. New York: Wiley.

Houthakker, H. and L. D. Taylor (1966) *Consumer Demand in the United States, 1929–1970: Analyses and Projections*, 1st ed. Cambridge, MA: Harvard University Press.

Jevons, W. S. (1871) *The Theory of Political Economy*. London and New York: MacMillan & Co.

Kocher, M. G., M. Krawczyk and F. van Winden (2014) 'Let me dream on!' Anticipatory emotions and preference for timing in lotteries. *Journal of Economic Behavior and Organization* 98, 29–40.

Koopmans, T. C. (1960) Stationary ordinal utility and impatience. *Econometrica* 28, 287–309.

Köszegi, B. and M. Rabin (2006) A model of reference-dependent preferences. *Quarterly Journal of Economics* 121, 1133–1165.

Köszegi, B. and M. Rabin (2007) Reference-dependent risk attitudes. *American Economic Review* 97, 1047–1073.

Kumar, A., M. A. Killingsworth and T. Gilovich (2014) Waiting for Merlot: Anticipatory consumption of experiential and material purchases. *Psychological Science* 25, 1924–1931.

Kurz, M. (1968) Optimal economic growth and wealth effects. *International Economic Review* 9, 348–357.

Kuznitz, A., S. Kandel and V. Fos (2008) A portfolio choice model with utility from anticipation of future consumption and stock market mean reversion. *European Economic Review* 52, 1338–1352.

Leonard, D. and N. van Long (1992). *Optimal Control Theory and Static Optimization in Economics*. Cambridge, UK: Cambridge University Press.

Li, B., Q. Liu and P. Pei (in press) Investigating the role of money in the identification of monetary policy behavior: A Bayesian DGSE perspective. *Macroeconomic Dynamics*. https://doi.org/10.1017/S1365100519000762

Loewenstein, G. (1987) Anticipation and the valuation of delayed consumption. *Economic Journal* 97, 666–684.

Loewenstein, G. and J. Elster (eds.) (1992) *Choice Over Time*. New York: Rusell Sage Foundation.

Marshall, A. (1890) *Principles of Economics: An Introductory Volume*. London: MacMillan & Co.

Monteiro, G., A. Cook and S. Dey (2013) Optimal tax policy under habit formation and capital utilization. *Journal of Macroeconomics* 37, 230–248.

Monteiro, G. and S. J. Turnovsky (2016) Anticipated consumption and its impact on capital accumulation and growth: 'Forward-looking' vs. 'backward-looking' consumption reference. *International Journal of Economic Theory* 12, 203–232.

Rae, J. (1834) *Statement of Some New Principles on the Subject of Political Economy: Exposing the Fallacies of the System of Free Trade, and Some Other Doctrines Maintained in the 'Wealth of Nations'*. Boston: Hilliard, Gray, and Co.

Rebelo, S. (1991) Long-run policy analysis and long-run growth. *Journal of Political Economy* 99, 500–521.

Ryder, H. E. and G. M. Heal (1973) Optimal growth with intertemporally dependent preferences. *Review of Economic Studies* 40, 1–31.

Samuelson, P. A. (1952) Probability, utility, and the independence axiom. *Econometrica* 20, 670–678.

Smets, F. and R. Wouters (2003) An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association* 1, 1123–1175.

Taylor, L. D. and H. Houthakker (2010) *Consumer Demand in the United States: Prices, Income, and Consumption Behavior*, 3rd ed. New York: Springer.

Turnovsky, S. J. and G. Monteiro (2007) Consumption externalities, production externalities, and efficient capital accumulation under time non-separable preferences. *European Economic Review* 51, 479–504.

Zou, H. (1998) The spirit of capitalism, social status, money, and accumulation. *Journal of Economics* 68, 219–233.

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APPENDIX A

A.1. DERIVATION OF $\dot{g}_C$ IN THE IA ECONOMY

From (7) and (9), we get

$$\dot{\mu} = (\beta - \rho)\,\mu - \gamma c(\lambda + \rho\mu). \tag{A1}$$

Differentiating (7) with respect to time, we get

$$C^{-\epsilon}A^{\gamma(1-\epsilon)}[-\epsilon g_C + \gamma(1-\epsilon)\rho(1-c)] = \dot{\lambda} + \rho\dot{\mu},$$

which, using (8) and (A1), can be expressed as:

$$(\lambda + \rho\mu)[-\epsilon g_C + \gamma\rho(1-\epsilon) + \epsilon\gamma\rho c] = (\beta - B)\lambda + \rho(\beta - \rho)\mu,$$

or, equivalently,

$$(\lambda + \rho\mu)[-\epsilon g_C + \gamma\rho(1-\epsilon) + \epsilon\gamma\rho c + B - \beta] = \rho(B - \rho)\mu. \tag{A2}$$

Differentiating the former expression with respect to time, we get

$$(\dot{\lambda} + \rho\dot{\mu})[-\epsilon g_C + \gamma\rho(1-\epsilon) + \epsilon\gamma\rho c + B - \beta] + (\lambda + \rho\mu)[-\epsilon\dot{g}_C + \epsilon\gamma\rho\dot{c}] = \rho(B-\rho)\dot{\mu}.$$

Using (8) and (9), we can get that:

$$(\lambda + \rho\mu)[-\epsilon g_C + \gamma(1-\epsilon)\rho(1-c)][-\epsilon g_C + \gamma\rho(1-\epsilon) + \epsilon\gamma\rho c + B - \beta]$$
$$+ (\lambda + \rho\mu)[-\epsilon\dot{g}_C + \epsilon\gamma\rho\dot{c} + \rho(B-\rho)\gamma c] = \rho(B-\rho)(\beta-\rho)\mu,$$

which, using (8) and (9), together with $\dot{c} = c[g_C - \rho(1-c)]$, can be expressed as:

$$\epsilon\dot{g}_C = [-\epsilon g_C + \gamma\rho(1-\epsilon)(1-c) + \rho - \beta][-\epsilon g_C - \epsilon\gamma\rho(1-c) + \gamma\rho + B - \beta]$$
$$+ \gamma\rho c[\epsilon g_C - \epsilon\rho(1-c) + B - \rho].$$

APPENDIX B

B.1. BALANCED GROWTH PATH OF THE IA ECONOMY

Starting with equation (11), imposing the BGP condition $\dot{c} = 0$ yields two solutions: (i) $\hat{c} = 0$ and (ii) $\bar{c} = (\rho - \bar{g}_C)/\rho$.

(i) Zero consumption, $\hat{c} = 0$. In this case, the condition $\dot{k} = 0$ yields $\hat{k} = 0$, whereas the solution to $\dot{g}_C = 0$ also yields two solutions, namely, $\hat{g}_C = [B - \beta + \gamma(1-\epsilon)\rho]/\epsilon$ and $\breve{g}_C = [\rho - \beta + \gamma(1-\epsilon)\rho]/\epsilon$. From expression (3), we see that both solutions yield $\hat{g}_A = \rho$ which violates the condition in (3) that $\lim_{t\to\infty} A(t)e^{-\rho t} = 0$, because $\hat{g}_A - \rho = 0$.

(ii) Nonzero consumption, $\bar{c} \neq 0$, and equal to $\bar{c} = (\rho - \bar{g}_C)/\rho$. In this case, the solution to $\dot{k} = 0$ is $\bar{k} = \bar{c}/[B - \bar{g}_C]$, and there are two solutions to the expression $\dot{g}_C = 0$, namely, $\tilde{g}_C = [(1+\gamma)\rho - \beta]/[(1+\gamma)\epsilon]$ and $\bar{g}_C = (B - \beta)/[\epsilon + \gamma(\epsilon - 1)]$. In both cases, we have that $\bar{g}_A = \bar{g}_K = \bar{g}_C$ in the long run. Let us now examine the properties of each steady state.

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(a) Let us first consider the steady state characterized by $\tilde{g} = \tilde{g}_C = \tilde{g}_A = [(1+\gamma)\rho - \beta]/[(\gamma+1)\epsilon]$, $\tilde{c} = (\rho - \tilde{g})/\rho$ and $\tilde{k} = \tilde{c}/(B - \tilde{g})$. If it is feasible, this steady state satisfies the transversality condition because

$$-\beta + \tilde{g}_K + \tilde{g}_\lambda = \tilde{g}_K - B = -\tilde{c}/\tilde{k}, \tag{B1}$$

$$-\beta + \tilde{g}_A + \tilde{g}_\mu = -\beta + \tilde{g}_A - [\epsilon + \gamma(\epsilon - 1)]\tilde{g}_A = -(1+\gamma)\rho\tilde{c}. \tag{B2}$$

To examine the stability of the steady state, we linearize the dynamic system (11)–(13) around the steady state $(\tilde{c}, \tilde{k}, \tilde{g}_C)$:

$$\begin{pmatrix} \dot{c} \\ \dot{k} \\ \dot{g}_C \end{pmatrix} = \begin{pmatrix} \rho\tilde{c} & 0 & \tilde{c} \\ (\rho - B)\tilde{k}/\tilde{c} & \tilde{c}/\tilde{k} & 0 \\ \gamma\rho\tilde{c}(1-\gamma\rho\tilde{k})/\tilde{k} & 0 & \tilde{c}[(1+2\gamma)\rho\tilde{k} - 1]/\tilde{k} \end{pmatrix} \begin{pmatrix} c - \tilde{c} \\ k - \tilde{k} \\ g_C - \tilde{g}_C \end{pmatrix}. \tag{B3}$$

Given the structure of the coefficient matrix, say M, its second diagonal element, $\tilde{c}/\tilde{k}$, is an unstable root. The other two roots are those of the submatrix obtained by deleting the second row and the second column of the matrix M:

$$M_{13} = \begin{pmatrix} m_{11} & m_{13} \\ m_{31} & m_{33} \end{pmatrix}. \tag{B4}$$

The trace and the determinant of M_{13} can be computed as:

$$\text{tr} = \rho - B + (1+2\gamma)\rho\tilde{c} > 0,$$

$$\det = (1+\gamma)(\rho - B + \gamma\rho\tilde{c})\rho\tilde{c} > 0,$$

where we have used that $\rho > B$. Given that both the trace and the determinant are positive, the steady state is unstable.

(b) Let us now consider the steady state characterized by $\bar{g} = \bar{g}_C = \bar{g}_A = (B - \beta)/[\epsilon + \gamma(\epsilon - 1)]$, $\bar{c} = (\rho - \bar{g})/\rho$ and $\bar{k} = \bar{c}/(B - \bar{g})$. If it is feasible, this steady state satisfies the transversality condition because

$$-\beta + \bar{g}_K + \bar{g}_\lambda = -\beta + \bar{g}_A + \bar{g}_\mu = \bar{g} - B = -\bar{c}/\bar{k}. \tag{B5}$$

To examine the stability of the steady state, we linearize the dynamic system (11)–(13) around the steady state $(\bar{c}, \bar{k}, \bar{g}_C)$:

$$\begin{pmatrix} \dot{c} \\ \dot{k} \\ \dot{g}_C \end{pmatrix} = \begin{pmatrix} \rho\bar{c} & 0 & \bar{c} \\ (\rho - B)\bar{k}/\bar{c} & \bar{c}/\bar{k} & 0 \\ m_{13} & 0 & B - \rho \end{pmatrix} \begin{pmatrix} c - \bar{c} \\ k - \bar{k} \\ g_C - \bar{g}_C \end{pmatrix} = M \begin{pmatrix} c - \bar{c} \\ k - \bar{k} \\ g_C - \bar{g}_C \end{pmatrix}, \tag{B6}$$

where

$$m_{13} = \gamma\rho\{(1-\epsilon)(B - \rho) + [\epsilon + \gamma(\epsilon - 1)]\rho\bar{c}\}/\epsilon$$

Given the structure of the coefficient matrix, say M, its second diagonal element, $\bar{c}/\bar{k}$, is an unstable root. The other two roots are those of the submatrix obtained by deleting the second row and the second column of the matrix M:

$$M_{13} = \begin{pmatrix} \rho\bar{c} & \bar{c} \\ m_{13} & B - \rho \end{pmatrix}. \tag{B7}$$

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The determinant of M_{13} can be computed as:

$$\det = -\frac{\bar{c}[\epsilon+\gamma(\epsilon-1)]\rho(\rho-B+\gamma\rho\bar{c})}{\epsilon} < 0.$$

Hence, the steady state is locally saddle-path stable.

APPENDIX C

C.1. THE $\dot{g}_C = 0$–LOCUS

Let us rewrite the $\dot{g}_C = 0$–locus defined by equation (13) as:

$$a_1 g_C^2 + 2a_2 c g_C + a_3 c^2 + a_4 g_C + a_5 c + a_6 = 0, \tag{C1}$$

where

$$a_1 = \epsilon > 0,$$
$$a_2 = -\gamma(\epsilon-1)\rho,$$
$$a_3 = \gamma[1+\gamma(\epsilon-1)]\rho^2 = \gamma\rho[\rho - a_2] > 0,$$
$$a_4 = -\frac{1}{2}[B-\beta-\gamma\rho(\epsilon-1)+\rho-\beta-\gamma\rho(\epsilon-1)],$$
$$a_5 = \frac{\gamma\rho}{2\epsilon}\{\epsilon[B-\beta-\gamma\rho(\epsilon-1)]+(\epsilon-1)[\rho-\beta-\gamma\rho(\epsilon-1)]-\epsilon\rho\},$$
$$a_6 = \frac{1}{\epsilon}[B-\beta-\gamma\rho(\epsilon-1)][\rho-\beta-\gamma\rho(\epsilon-1)].$$

The following determinant is positive:

$$D = \det\begin{pmatrix} a_1 & a_2 \\ a_2 & a_3 \end{pmatrix} = \gamma\rho^2[\epsilon+\gamma(\epsilon-1)] > 0,$$

which, given that we have already shown the existence of real solutions to the $\dot{g}_C = 0$ equation, entails that the $\dot{g}_C = 0$–locus is an ellipse.[14] The center of the ellipse is given by:

$$g_C^c = -\frac{1}{D}\det\begin{pmatrix} a_4 & a_2 \\ a_5 & a_3 \end{pmatrix} = \frac{1}{2}\left(\bar{g} + \frac{\rho-\beta+\rho\gamma(1-\epsilon)}{\epsilon}\right), \tag{C2}$$

$$c^c = -\frac{1}{D}\det\begin{pmatrix} a_1 & a_4 \\ a_2 & a_5 \end{pmatrix} = \frac{1}{2}\bar{c}. \tag{C3}$$

Denoting

$$\Delta = \det\begin{pmatrix} a_1 & a_2 & a_4 \\ a_2 & a_3 & a_5 \\ a_4 & a_5 & a_6 \end{pmatrix} = -\frac{\gamma\rho^2}{4\epsilon}[\epsilon+\gamma(\epsilon-1)]\left\{(\rho-B)^2+[\epsilon+\gamma(\epsilon-1)]\gamma(\rho-\bar{g})^2\right\} < 0, \tag{C4}$$

the $\dot{g}_C = 0$–locus can be rewritten as:

$$\epsilon\left[g_C - g_C^c + \frac{a_2}{\epsilon}(c-c^c)\right]^2 + \frac{D}{\epsilon}(c-c^c)^2 + \frac{\Delta}{D} = 0, \tag{C5}$$

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or, alternatively,

$$g_C = g_C^c - \frac{a_2}{\epsilon}(c - c^c) \pm \sqrt{-\frac{D}{\epsilon^2}(c - c^c)^2 - \frac{\Delta}{\epsilon D}}. \quad \textbf{(C6)}$$

Hence, for any value of c, there will be two values of g_C except at the left and right extremes of the ellipse.[15] In this case, there will be only one value of g_C. To determine the location of the left and right extremes, we should equal the square root to zero, so the previous expression returns only one value of g_C. This happens when c takes the following values:

$$c = c^c \pm \frac{1}{D}\sqrt{-\epsilon\Delta}.$$

The eigenvalues of the matrix D—which are positive because D has a positive trace and determinant—are given by:

$$\lambda_1 = \frac{1}{2}\left[(\epsilon + a_3) + \sqrt{(\epsilon - a_3)^2 + a_2^2}\right],$$

$$\lambda_2 = \frac{1}{2}\left[(\epsilon + a_3) - \sqrt{(\epsilon - a_3)^2 + a_2^2}\right].$$

An eigenvector associated with the greatest eigenvalue λ_1 is $(u, 1)$ where

$$u = \frac{\epsilon - a_3 + \sqrt{(\epsilon - a_3)^2 + 4a_2^2}}{2a_2} > 0.$$

Hence, the slope of the eigenvector $(u, 1)$ is positive, and so, the major axis of the ellipse is positively sloped. An eigenvector associated with the smallest eigenvalue λ_2 is $(v, 1)$ where

$$v = \frac{\epsilon - a_3 - \sqrt{(\epsilon - a_3)^2 + 4a_2^2}}{2a_2} < 0.$$

Hence, the slope of the eigenvector $(v, 1)$ is negative, and so, the minor axis of the ellipse is negatively sloped.

APPENDIX D

D.1. BALANCED GROWTH PATH OF THE EA ECONOMY

Looking at the balanced growth path (or steady state), we can see that just like the internal case, there are two possible steady states. The first is characterized by $\tilde{c} = 0$, $\tilde{k} = 0$, $\widetilde{C/K} = B - \tilde{g}_K$, in which using (28), the long-run growth rate of consumption and capital per capita is $\tilde{g}_K = \tilde{g}_C = [B - \beta + \gamma\rho(1 - \epsilon)]/\epsilon$, and the growth rate of the anticipation stock is $\tilde{g}_A = \rho$. However, this steady state does not satisfy the terminal condition in (20), $\lim_{t\to\infty} A(t)e^{-\rho t} = 0$ because $\tilde{g}_A - \rho = 0$. The second is characterized by the case in which consumption, capital, and anticipation consumption stock grow at constant (possible distinct and/or zero) rates.

Henceforth, we assume that condition (32) is fulfilled. In particular, this entails that the steady state satisfies the terminal condition in (20), $\lim_{t\to\infty} A(t)e^{-\rho t} = 0$, because $\hat{g} - \rho < 0$. To investigate the (local) stability of the steady state, we linearize the dynamic system

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(26)–(27) around the steady state (29)–(30). The linearized system is

$$\begin{pmatrix} \dot{c} \\ \dot{k} \end{pmatrix} = \begin{pmatrix} [\epsilon + \gamma(\epsilon - 1)]\rho\hat{c}/\epsilon & 0 \\ -1 + \rho\hat{k} & \hat{c}/\hat{k} \end{pmatrix} \times \begin{pmatrix} c - \hat{c} \\ k - \hat{k} \end{pmatrix} = M \times \begin{pmatrix} c - \hat{c} \\ k - \hat{k} \end{pmatrix}. \quad \textbf{(D1)}$$

Given that the coefficient matrix is triangular, its eigenvalues are its diagonal elements which are both positive. As there are as much unstable roots as jumpable variables, c and k, the steady state is unstable and, therefore, the economy jumps at the outset to its balanced growth path.

APPENDIX E

E.1. DYNAMICS OF THE IA ECONOMY WITH THE VARIABLES *k*, *c* and *q*

In this Appendix, we derive the dynamics of the IA economy in terms of the variables $k = K/A$, $c = C/A$ and $q = \mu/\lambda$. From equation (7), we get

$$\lambda = C^{-\epsilon} A^{\gamma(1-\epsilon)}/(1 + \rho q), \quad \textbf{(E1)}$$

$$\mu = q C^{-\epsilon} A^{\gamma(1-\epsilon)}/(1 + \rho q). \quad \textbf{(E2)}$$

Log-differentiating (E1) with respect to time, using (22), we get

$$-\epsilon\dot{C}/C + \gamma(1-\epsilon)\dot{A}/A = \dot{\lambda}/\lambda + \frac{\rho\dot{q}}{1+\rho q} = \beta - B + \frac{\rho\dot{q}}{1+\rho q},$$

which, rearranging terms and using that $\dot{A}/A = \rho(1-c)$, can be rewritten as:

$$\dot{C} = \frac{C}{\epsilon}\left[B - \beta + \gamma(\epsilon - 1)\rho(c-1) - \frac{\rho\dot{q}}{1+\rho q}\right].$$

Using that $\dot{c}/c = \dot{C}/C - \dot{A}/A$, we get (37). Since $\dot{k}/k = \dot{K}/K - \dot{A}/A$, equation (38) results from (24) and (4). Since $\dot{q}/q = \dot{\mu}/\mu - \dot{\lambda}/\lambda$, (39) results from (8) and (9), taking into account (E1) and (E2).

The system (37)–(39) is accessible to phase diagram analysis. From (37), aside from the horizontal line $c = 0$, the $\dot{c} = 0$–locus is given by:

$$l_c(q) = -\frac{1}{(1+\gamma)\epsilon\rho}\left[\frac{B-\rho}{1+\rho q} - (1+\gamma)(\epsilon - 1)\rho - \beta\right]. \quad \textbf{(E3)}$$

This locus has a vertical asymptote at $q = -1/\rho$, with $\lim_{q\to(-1/\rho)^-} l_c(q) = -\infty$, and has a horizontal asymptote at $\lim_{q\to-\infty} l_c(q) = \lim_{q\to+\infty} l_c(q) = \tilde{c}$. The $\dot{c} = 0$–locus is strictly decreasing because $l_c'(q) = (B-\rho)/[\epsilon(1-\gamma)(1+\rho q)^2] < 0$. Furthermore, $l_c''(q) = 2(\rho - B)\rho/[(1+\gamma)\epsilon(1+\rho q)^3]$, so that the $\dot{c} = 0$–locus is concave (convex) to the left (right) of $-1/\rho$. We have that $\partial\dot{c}/\partial c = \rho(1+\gamma)c > 0$ when evaluated at the $\dot{c} = 0$–locus. Hence, the $\dot{c} = 0$–locus is unstable.

From (39), the $\dot{q} = 0$–locus is given by:

$$l_q(q) = -\frac{q(\rho - B)}{\gamma(1+\rho q)}. \quad \textbf{(E4)}$$

The $\dot{q} = 0$–locus has a vertical asymptote at $q = -1/\rho$, with $\lim_{q\to(-1/\rho)^-} l_c(q) = +\infty$ and $\lim_{q\to(-1/\rho)^+} l_c(q) = -\infty$, and has a horizontal asymptote at $\lim_{q\to-\infty} l_q(q) =$

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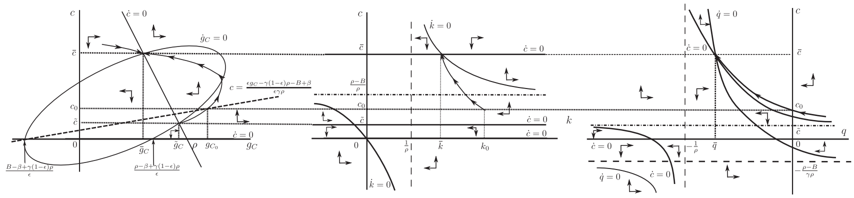

FIGURE E.1. Phase diagram in the IA case with k, c, and q.

$\lim_{q\to-\infty} l_q(q) = -(\rho - B)/(\gamma\rho)$. The $\dot{q} = 0$–locus is decreasing because $l_q'(q) = -(\rho - B)/[\gamma(1+\rho q)^2] < 0$. Furthermore, $l_q''(q) = -2(B-\rho)\rho/[\gamma(1+\rho q)^3]$, so that the $\dot{q} = 0$–locus is concave (convex) to the left (right) of $-1/\rho$. We have that $\partial\dot{q}/\partial q = (B-\rho)/(1+\rho q)$ when evaluated at the $\dot{q} = 0$–locus. Hence, the $\dot{q} = 0$–locus is unstable (stable) to the left (right) of $-1/\rho$. The horizontal asymptote of the $\dot{q} = 0$–locus is above the horizontal asymptote of the $\dot{c} = 0$–locus, that is,

$$\lim_{q\to\pm\infty} l_c(q) = -\frac{\rho - B}{\gamma\rho} < \frac{(1+\gamma)(\epsilon-1)\rho+\beta}{(1+\gamma)\epsilon\rho} = \tilde{c} = \lim_{q\to\pm\infty} l_q(q).$$

if condition (17) is met and $\rho > B$. This results from

$$\frac{\rho - B}{\gamma\rho} + \frac{(1+\gamma)(\epsilon-1)\rho+\beta}{(1+\gamma)\epsilon\rho} > \frac{\rho - B}{\gamma\rho} + \frac{(\epsilon-1)(\rho-\beta)}{\epsilon\rho} = \frac{[\epsilon+\gamma(\epsilon-1)](\rho-B)}{\gamma\epsilon\rho} > 0,$$

where the first inequality follows from $\beta > (1+\gamma)(1-\epsilon)B$ in (17), and the third one follows from $B > (1-\epsilon)(1+\gamma)B$, which entails that $\epsilon + \gamma(\epsilon - 1) > 0$. Furthermore, we have that

$$\lim_{q\to(-1/\rho)^+} [l_c(q) - l_q(q)] = -\infty, \tag{E5}$$

$$\lim_{q\to+\infty} \left[l_c(q) - l_q(q)\right] = \frac{(\epsilon-1)(1+\gamma)\rho+\beta}{(1+\gamma)\epsilon\rho} + \frac{\rho-B}{\gamma\rho} > 0, \tag{E6}$$

$$l_c'(q) - l_q'(q) = \frac{[\epsilon+\gamma(\epsilon-1)](\rho-B)}{\gamma(1+\gamma)\epsilon(1+\rho q)^2} > 0, \tag{E7}$$

so that there is a unique solution $(\bar{q}, \bar{c})$ to $\dot{q} = \dot{c} = 0$ in the interval $(-1/\rho, +\infty)$. Given that

$$\lim_{q\to-\infty} \left[l_c(q) - l_q(q)\right] = \frac{(1+\gamma)(\epsilon-1)\rho+\beta}{(1+\gamma)\epsilon\rho} + \frac{\rho-B}{\gamma\rho} > 0,$$

$$l_c'(q) - l_q'(q) = \frac{[\epsilon+\gamma(\epsilon-1)](\rho-B)}{\gamma(1+\gamma)\epsilon(1+\rho q)^2} > 0,$$

the $\dot{q} = 0$–locus and the $\dot{c} = 0$–locus do not cross to the left of $-1/\rho$. Hence, there exists a unique feasible stationary solution.[16]

The right panel of Figure E.1 depicts a phase diagram in (q, c)–plane.[17] Given the configuration of the two loci, there is a unique and saddle-path stable steady state $(\bar{q}, \bar{c})$, with $\bar{q} < 0$. The central panel of Figure E.1 depicts a phase diagram in (k, c)–plane (see the right panel of Figure 1). Given the configuration of the two loci, there exists a unique and saddle-path stable steady state $(\bar{k}, \bar{c})$.[18]

To determine the initial point in the stable manifold, note that $A(0)$ is free, so that $\mu(0) = 0$; that is, $q(0) = 0$. Hence, the economy starts at the point in which the stable saddle path

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in the right panel of Figure E.1 crosses the vertical $q = 0$ axis, say at the point $(0, c_0)$. Once determined in this way the initial value $c(0) = c_0$, this value determines the initial value $k(0) = k_0$ in the stable saddle path in the central panel of Figure E.1. The former analysis shows that c and k both increase monotonically toward their respective long-run values, whereas the relative shadow price q decreases steadily toward its stationary value.

APPENDIX F

F.1. EXPRESSING q AND $\dot{q}$ AS FUNCTIONS OF c AND g_C

To express q as a function of c and g_C, let us divide both sides of (A2) by λ, to get after simplification that

$$q = -\frac{1}{\rho}\left\{\frac{B + \gamma\rho[\epsilon(c-1)+1] - (\beta + \epsilon g_C)}{\rho + \gamma\rho[\epsilon(c-1)+1] - (\beta + \epsilon g_C)}\right\}. \quad \textbf{(F1)}$$

Differentiating this expression with respect to time, we get that

$$\dot{q} = -\frac{\epsilon(\rho - B)\,(\gamma\rho\dot{c} - \dot{g}_C)}{\rho\{\rho + \gamma\rho[\epsilon(c-1)+1] - (\beta + \epsilon g_C)\}^2},$$

which, using (11) and (13), yields

$$\dot{q} = -\frac{B-\rho}{\rho}\left\{\frac{B + \gamma\rho(\epsilon - 1)(c-1) - (\beta + \epsilon g_C)}{\rho + \gamma\rho[\epsilon(c-1)+1] - (\beta + \epsilon g_C)}\right\}. \quad \textbf{(F2)}$$

The phase diagram analysis in Appendix E shows that q is strictly decreasing—that is, $\dot{q} < 0$—as it converges toward its stationary value:

$$\bar{q} = -\frac{\gamma(\rho - \bar{g})}{\rho[(\rho - B) + \gamma(\rho - \bar{g})]}.$$

Given that $q(0) = 0$, this also entails that q is negative along the transition path. Furthermore, let us note that

$$1 + \rho q = \frac{\rho - B}{\rho + \gamma\rho[\epsilon(c-1)+1] - (\beta + \epsilon g_C)}.$$

Evaluating this expression at the steady state yields

$$1 + \rho\bar{q} = 1 + \frac{\gamma(\rho - \bar{g})}{B - \rho - \gamma(\rho - \bar{g})} = \frac{\rho - B}{\rho - B + \gamma(\rho - \bar{g})} > 0.$$

As q is strictly decreasing along the transition path and $\rho > 0$, this entails that $1 + \rho q > 0$ along the transition.

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