RUIN PROBABILITY FOR TRANSLATED COMBINATION OF EXPONENTIAL CLAIMS

BY BEDA CHAN

University of Toronto, Canada

ABSTRACT

An alternative expression for the coefficients in the ruin probability for the classical ruin model with translated combination of exponential claims is derived.

KEYWORDS

Probability of ruin; translated combination of exponentials.

In a compound Poisson claim process with claim amounts distributed as a mixture of exponentials

$$p(x) = \sum_{i=1}^{n} A_i \beta_i e^{-\beta_i x}$$

for $x > 0$ where all $A_i > 0$ and $\sum_{i=1}^{n} A_i = 1$, it is well known that the ruin probability is also a linear combination of exponentials

$$\psi(u) = \sum_{i=1}^{n} C_i e^{-r_i u}$$

where $\{r_1, \ldots, r_n\}$ are solutions to the adjustment coefficient equation

$$(1+\theta)\, p_1 = \frac{M_X(r)-1}{r}$$

and $\{C_1, \ldots, C_n\}$ are determined by the partial fractions of

$$\sum_{i=1}^{n} \frac{C_i r_i}{r_i - r} = \frac{\theta}{1+\theta} \cdot \frac{\dfrac{M_X(r)-1}{r}}{(1+\theta)\, p_1 - \dfrac{M_X(r)-1}{r}}\,.$$

See BOWERS et al. (1986), § 12.6 for details. This result was later extended by DUFRESNE and GERBER (1989) to the case when the claim distribution is a

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translated (density function moved by τ to the left) combination of exponentials. (Note that the A_i's need not be positive). They found that the coefficients C_i's are the solution to the system:

$$(1) \qquad \sum_{k=1}^{n} \frac{\beta_i}{\beta_i - r_k} C_k = 1, \qquad i = 1, \ldots, n,$$

and gave C_k explicitly. In this note we give an alternative expression for the solution for (1):

$$(2) \qquad C_k = \prod_{\substack{i \neq k \\ i=1}}^{n} \frac{r_i}{r_i - r_k} \prod_{i=1}^{n} \frac{\beta_i - r_k}{\beta_i}.$$

To verify (2), consider

$$\sum_{i=1}^{n} \frac{x}{x - r_i} C_i = 1 - \prod_{i=1}^{n} \frac{r_i(x - \beta_i)}{\beta_i(x - r_i)}$$

where the two sides are different expressions for the same rational function of (degree n/degree n) which has simple poles $\{r_1, \ldots, r_n\}$ and takes the value 1 at $x = \beta_1, \ldots, \beta_n$ and the value 0 at $x = 0$. Multiply by $x - r_k$ and let $x = r_k$ to obtain (2).

Two different expressions for C_k, (49) and (54) in DUFRESNE and GERBER (1989), arise naturally when a more detailed problem including the severity of ruin is studied. These two expressions can be obtained from summing (9) and (22) in DUFRESNE and GERBER (1988) over j respectively.

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BEDA CHAN
Department of Statistics, University of Toronto, Toronto, Canada M5S 1A1.