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CLASSIFICATION OF MULTIVARIATE LIFE DISTRIBUTIONS BASED ON PARTIAL ORDERING

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Barlow and Proschan presented some interesting connections between univariate classifications of life distributions and partial orderings where equivalent definitions for increasing failure rate (IFR), increasing failure rate average (IFRA), and new better than used (NBU) classes were given in terms of convex, star-shaped, and superadditive orderings. Some related results are given by Ross and Shaked and Shanthikumar. The introduction of a multivariate generalization of partial orderings is the object of the present article. Based on that concept of multivariate partial orderings, we also propose multivariate classifications of life distributions and present a study on more IFR-ness.

1. INTRODUCTION

Classification of life distributions is of immense importance for reliability analysis, especially during the early stages of product designing. Various reliability bounds obtained from different class properties provide conservative estimates of system reliability under minimum workable assumptions. Classification results also play a vital role in the selection and validation of stochastic models. As a result, one can find a large number of research works in this field. Standard univariate life distribution classes, extensively examined in the literature, are the following: increasing failure rate (IFR), increasing failure rate average (IFRA), decreasing mean residual life (DMRL), new better than used (NBU), new better than used in expectation (NBUE), harmonic new better than used in expectation (HNBUE), and their respective dual classes.

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Stochastic ordering is another approach for making a comparison among various probability distributions. Ross [11] has briefly covered the initial work and Shaked and Shanthikumar [17] have presented the different orderings of statistical implications in detail. Among these orderings, usual stochastic ordering, likelihood ordering, convex ordering, dispersive ordering, peakedness ordering, and transform orderings are worth mentioning. Barlow and Proschan [4], Ross and Schechner [12], and Kim and Proschan [9] have established relationships between classification of life distributions and stochastic ordering. Attempts have also been made to define stochastic ordering in terms of reliability measures. The hazard rate ordering of Ross [11], the mean residual life ordering of Boland, El-Neweihi, and Proschan [5], the cumulative hazard rate ordering of Alzaid [2], and the memory ordering of Ebrahimi and Zahedi [6] are some such concepts which have widened the art of stochastic ordering. Some results are also available on multivariate extensions of hazard rate and stochastic orderings (see Shaked and Shanthikumar [15,16]) and the general idea of multidimensional stochastic ordering (see Bacelli and Makowski [3]).

Barlow and Proschan [4] established a few equivalent relationships between the classifications of life distributions and various partial orderings. For two continuous distributions F and G, where G is strictly increasing in the support, an interval, and if $F(0) = G(0)$, then the following hold:

(i) $F <_c G$; that is, F is convex with respect to G if $G^{-1}F(x)$ is convex.
(ii) $F <_* G$; that is, F is star-shaped with respect to G if $G^{-1}F(x)$ is star-shaped [i.e., $G^{-1}F(x)/x$ is increasing in x for $x > 0$]
(iii) $F <_{\text{sa}} G$; that is, F is superadditive with respect to G if $G^{-1}(x)$ is superadditive [i.e., $G^{-1}F(x+y) \geq G^{-1}F(x) + G^{-1}F(y)\ \forall x \geq 0, y \geq 0$].

Barlow and Proschan [4, pp. 106–108] put forward the following equivalent definition of life distribution classes in terms of partial orderings:

For a choice of $G(x) = 1 - \exp(-\lambda x)$, the distribution function of an exponential distribution:

(a) F is IFR if and only if $F <_c G$
(b) F is IFRA if and only if $F <_* G$
(c) F is NBU if and only if $F <_{\text{sa}} G$.

These results, along with stochastic ordering and dispersion order, have been examined in detail in Shaked and Shanthikumar [17, Chap. 3] and also in Ahmed, Alzaid, Bartoszewicz, and Kochar [1]. To cover the other life distribution classes, Kochar [10] examined the extension of the DMRL and related partial orderings of the life distributions.

Unfortunately, a direct generalization of partial ordering by considering both G and F to be multivariate cumulative distribution functions (c.d.f.'s) is not possible because $G^{-1}F(\mathbf{x})$, where $\mathbf{x}$ is vector of order p ($\geq$2), gives rise to one-to-many function. To resolve this problem, Roy [14] made an attempt to define multivariate

IFR (MIFR), multivariate IFRA (MIFRA), multivariate NBU (MNBU), and their dual classes by considering $G(x)$ as the univariate exponential c.d.f. and $F(x)$ as a multivariate c.d.f. Since $G(x)$ is a univariate c.d.f., this conceptual framework cannot be claimed to be a proper multivariate extension of the univariate concept of partial ordering.

The purpose of the present work is to suggest a multivariate concept of partial ordering and exploit the same for defining MIFR, MIFRA, MNBU, and their dual classes. In Section 2, we present the suggested definitions for partial ordering, introduce multivariate classes based on partial ordering, and interlink these multivariate classes with the system of multivariate classes developed by Roy [13] from an altogether different consideration. In Section 3, we examine the case of more IFRness in the multivariate setup and include a related example for the bivariate situation.

2. MULTIVARIATE PARTIAL ORDERINGS

Let us begin with a remark that the univariate concepts of convex, star-shaped, and superadditive orderings are based on convexity, star-shapedness, and superadditivity of the function $G^{-1}F(x)$ with respect to x. It may be noted that if we simultaneously replace G and F by $\bar{G}(=1-G)$ and $\bar{F}(=1-F)$, the functional form under study remains unchanged. Thus, one may modify the definitions (i)–(iii) by replacing $G^{-1}F(x)$ by $\bar{G}^{-1}\bar{F}(x)$ in the following way:

(i)′ $F <_c G$ if $\bar{G}^{-1}\bar{F}$ is convex.
(ii)′ $F <_* G$ if $\bar{G}^{-1}\bar{F}$ is star-shaped.
(iii)′ $F <_{\text{sa}} G$ if $\bar{G}^{-1}\bar{F}$ is superadditive, with $\bar{G}(0)=\bar{F}(0)=1$.

Similarly, we may redefine the life distribution classes as follows:

(a)′ F is IFR if and only if $\bar{G}^{-1}\bar{F}$ is convex,
(b)′ F is IFRA if and only if $\bar{G}^{-1}\bar{F}$ is star-shaped,
(c)′ F is NBU if and only if $\bar{G}^{-1}\bar{F}$ is superadditive,

where $\bar{G}(x)=\exp(-\lambda x)$, $\lambda > 0$.

In the multivariate setup, let us denote by $\mathbf{x}=(x_1, x_2, \ldots, x_p)$ a vector of dimension p. Let $F(\mathbf{x})$ and $G(\mathbf{x})$ be two p-variate c.d.f.'s and let $\bar{F}$ and $\bar{G}$ be the corresponding p-variate survival functions ($p \geq 1$) such that $\bar{F}(\mathbf{0})=\bar{G}(\mathbf{0})=1$. To propose the concept of multivariate partial orderings, let us draw analogy from the definition of the multivariate hazard rate given by Johnson and Kotz [8]. The basic idea of Johnson and Kotz was to consider p conditional distributions, which are of univariate type.

Now, writing

$$\bar{G}_i(\mathbf{x}) = \frac{\bar{G}(x_1,\ldots,x_{i-1},x_i,x_{i+1},\ldots,x_p)}{\bar{G}(x_1,\ldots,x_{i-1},0,x_{i+1},\ldots,x_p)} \quad \text{and} \quad \bar{F}_i(\mathbf{x}) = \frac{\bar{F}(x_1,\ldots,x_{i-1},x_i,x_{i+1},\ldots,x_p)}{\bar{F}(x_1,\ldots,x_{i-1},0,x_{i+1},\ldots,x_p)},$$

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for $i = 1, 2, \ldots, p$, we would like to view $\bar{G}_i(\mathbf{x})$ and $\bar{F}_i(\mathbf{x})$ as functions of x_i with x_j's, $j = 1, 2, \ldots, p\ (\neq i)$, as the given parameters. In that case, a solution of u_i, where

$$\bar{G}_i(x_1, \ldots, x_{i-1}, u_i, x_{i+1}, \ldots, x_p) = \bar{F}_i(x_1, \ldots, x_i, \ldots, x_p), \tag{2.1}$$

can be viewed as $\bar{G}_i^{-1}\bar{F}_i(\mathbf{x})$, where u_i is a function of x_i given $x_j, j = 1, 2, \ldots, p\ (\neq i)$. Thus, one can form a vector $\mathbf{u} = (u_1, \ldots, u_i, \ldots, u_p)$, where

$$\mathbf{u} = (\bar{G}_1^{-1}\bar{F}_1(\mathbf{x}), \ldots, \bar{G}_i^{-1}\bar{F}_i(\mathbf{x}), \ldots, \bar{G}_p^{-1}\bar{F}_p(\mathbf{x})), \tag{2.2}$$

which is the proposed vector representation of $\bar{G}^{-1}\bar{F}(\mathbf{x})$. Given this definition of $\bar{G}^{-1}\bar{F}(\mathbf{x})$, we propose the following multivariate definitions for convex, star-shaped, and superadditive orderings:

Definition 2.1: *$F <_{\text{mc}} G$; that is, F is multivariate convex with respect to G if for each $i = 1, 2, \ldots, p$, u_i is convex in x_i given x_j's of $\mathbf{x}$, for $j \neq i$.*

Definition 2.2: *$F <_{m*} G$; that is, F is multivariate star-shaped with respect to G if for each $i = 1, 2, \ldots, p$, u_i is star-shaped in x_i for given x_j's of $\mathbf{x}$ for $j \neq i$.*

Definition 2.3: *$F <_{\text{msa}} G$; that is, F is multivariate superadditive with respect to G if for each $i = 1, 2, \ldots, p$, u_i is superadditive in x_i for given x_j's of $\mathbf{x}$ for $j \neq i$.*

It may be noted that when p reduces to 1, the multivariate definitions reduce to corresponding univariate definitions.

It may further be observed from the following theorem that a multivariate ordering between F and G of p dimension implies similar ordering between corresponding marginal distribution of F and G of dimension q, where $1 \leq q \leq p$.

Theorem 2.1: *Let $\mathbf{X} = (X_1, X_2, \ldots, X_p)$ and $\mathbf{Y} = (Y_1, Y_2, \ldots, Y_q)$ be two p-dimensional random variables with distribution functions $F(\mathbf{x})$ and $G(\mathbf{x})$. Let $\mathbf{X}^{(1)} = (X_{i_1}, X_{i_2}, \ldots, X_{i_q})$ and $\mathbf{Y}^{(1)} = (Y_{i_1}, Y_{i_2}, \ldots, Y_{i_q})$ be any two similar subsets of $\mathbf{X}$ and $\mathbf{Y}$ with the corresponding distribution functions as $F^{(1)}$ and $G^{(1)}$ where $(i_1, i_2, \ldots, i_q) \subset (1, 2, \ldots, p)$, $1 \leq q \leq p$. If F is multivariate convex/star-shaped/superadditive with respect to G, then $F^{(1)}$ is multivariate convex/star-shaped/superadditive with respect to $G^{(1)}$.*

The proof follows from Definitions 2.1–2.3 and the appropriate choices of $\mathbf{x}$.

Theorem 2.2: *For three multivariate distribution functions F, G, and H of identical dimension the following results hold true:*

(i) If $F <_{\text{mc}} G$ and $G <_{\text{mc}} H$, then $F <_{\text{mc}} H$.
(ii) $F <_{\text{mc}} F$.

Proof:

(i) Writing the vector representation of $\bar{G}^{-1}\bar{F}(\mathbf{x})$ as $\mathbf{u}$ and that of $\bar{H}^{-1}\bar{G}(\mathbf{x})$ as $\mathbf{v}$ we note from Definition 2.1 that u_i is convex in x_i and v_i is convex in u_i where u_i and v_i satisfy the following conditions:

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$$\bar{F}_i(x_1,\ldots,x_{i-1},x_i,x_{i+1},\ldots,x_p) = \bar{G}_i(x_1,\ldots,x_{i-1},u_i,x_{i+1},\ldots,x_p) \quad \textbf{(2.3)}$$

$$\bar{G}_i(x_1,\ldots,x_{i-1},u_i,x_{i+1},\ldots,x_p) = \bar{H}_i(x_1,\ldots,x_{i-1},v_i,x_{i+1},\ldots,x_p). \quad \textbf{(2.4)}$$

Now, combining (2.3) with (2.4) we obtain

$$\bar{F}_i(x_1,\ldots,x_{i-1},x_i,x_{i+1},\ldots,x_p) = \bar{H}_i(x_1,\ldots,x_{i-1},v_i,x_{i+1},\ldots,x_p) \quad \textbf{(2.5)}$$

where v_i is a convex function of x_i.

Since (2.5) is true for each i, we have, by Definition 2.1, $F <_{\text{mc}} H$.

(ii) As x_i is convex in itself $F <_{\text{mc}} F$ is an easy consequence of Definition 2.1.

THEOREM 2.3: *For three multivariate distribution functions F, G, and H of identical dimension, the following results hold true:*

(i) If $F <_{m^*} G$ and $G <_{m^*} H$, then $F <_{m^*} H$.
(ii) $F <_{m^*} F$.

PROOF:

(i) Let $F <_{m^*} G$ *and* $G <_{m^*} H$. Following the setup of the proof of Theorem 2.2, u_i/x_i is increasing in x_i and, hence, u_i is increasing in x_i. Further, v_i/u_i is increasing in u_i. Also, u_i itself is increasing in x_i. Thus, $\{(v_i/u_i)(u_i/x_i)\} = (v_i/x_i)$ is increasing in x_i. This being true for all $i = 1, 2, \ldots, p$, we have $F <_{m^*} H$.

(ii) Because $x_i/x_i = 1$, the proof is an easy consequence of the definition. Hence, follows the theorem. ■

THEOREM 2.4: *For three multivariate distribution functions F, G, and H of identical dimensions, the following results hold true:*

(i) if $F <_{\text{msa}} G$ and $G <_{\text{msa}} H$, then $F <_{\text{msa}} H$.
(ii) $F <_{\text{msa}} F$.

The proof is similar to those of Theorems 2.2 and 2.3.

THEOREM 2.5: *The following implicative relationships hold for multivariate partial orderings:*

Multivariate convex ordering ⇒ Multivariate star-shaped ordering
⇒ Multivariate superadditive ordering.

The proofs are easy consequences of the univariate implicative relationships of the p conditional survival functions of the type $\bar{F}_i(x_1,\ldots,x_i,\ldots,x_p)$, $i = 1,2,\ldots,p$, and the proposed multivariate definitions.

Once the multivariate concepts of convex, star-shaped, and superadditive orderings have been framed, one can easily order multivariate life distribution based on a standard choice of $G(\mathbf{x})$. For the purpose of the multivariate classification of life distribution, a probable choice of $G(\mathbf{x})$ is the c.d.f. of the multivariate exponential distribution due to Gumbel [7], where

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$$\bar{G}(\mathbf{x}) = \exp\left[-\sum_i \lambda_i x_i - \sum_{i>j}\sum \lambda_{ij} x_i x_j - \cdots - \lambda_{12,\ldots,p} x_1 x_2 \cdots x_p\right]. \tag{2.6}$$

Such a choice of $G(\mathbf{x})$ gives rise to equivalent definitions of life distribution classes for higher dimensions (see Roy [13]). Writing $R(\mathbf{x}) = -\log \bar{F}(\mathbf{x})$, the multivariate hazard function of $\bar{F}(\mathbf{x})$, we have, for $i = 1, \ldots, p$,

$$r_i(\mathbf{x}) = \frac{\partial}{\partial x_i} R(\mathbf{x}),$$

the ith multivariate failure rate, and

$$A_i(\mathbf{x}) = \frac{1}{x_i}\int_0^{x_i} r_i(x_1, \ldots, x_{i-1}, y, x_{i+1}, \ldots, x_p)\,dy,$$

the ith multivariate failure rate average.

Following Roy [13], a distribution with survival function $\bar{F}(\mathbf{x})$ is MIFR (MDFR) if $r_i(\mathbf{x})$ is increasing (decreasing) in x_i for all $\mathbf{x}$ and each i, MIFRA (MDFRA) if $A_i(\mathbf{x})$ is increasing (decreasing) in x_i for all $\mathbf{x}$ and each i, and MNBU (MNWU) if

$$\begin{aligned}
&\bar{F}(x_1, \ldots, x_{i-1}, x_i + y_i, x_{i+1}, \ldots, x_p)\bar{F}(x_1, \ldots, x_{i-1}, 0, x_{i+1}, \ldots, x_p) \\
&\quad \le \bar{F}(x_1, \ldots, x_i, \ldots, x_p)\bar{F}(x_1, \ldots, x_{i-1}, y_i, x_{i+1}, \ldots, x_p), \\
&\qquad\qquad \forall y_i \ge 0 \text{ and for all } \mathbf{x} \text{ and each } i.
\end{aligned}$$

In terms of multivariate stochastic ordering, we have the following equivalent definitions as given in the next theorem.

Theorem 2.6: *For a choice of $G(x)$ given in (2.6), the following hold:*

(i) F is MIFR (MDFR) if and only if $F <_{\mathrm{mc}} G$ $(G <_{\mathrm{mc}} F)$.
(ii) F is MIFRA (MDFRA) if and only if $F <_{m^*} G$ $(G <_{m^*} F)$.
(iii) F is MNBU (MNWU) if and only if $F <_{\mathrm{msa}} G$ $(G <_{\mathrm{msa}} F)$.

Proof: By definition, $\mathbf{u}$ is the vector representation of $\bar{G}^{-1}\bar{F}(\mathbf{x})$, from which we have from (2.1),

$$\begin{aligned}
&\left(\lambda_i + \sum_{j(\neq i)} \lambda_{ij} x_j + \cdots + \lambda_{12,\ldots,p} \prod_{j(\neq i)} x_j\right) u_i \\
&\quad = -\log \bar{F}(\mathbf{x}) + \log \bar{F}(x_1, \ldots, x_{i-1}, 0, x_{i+1}, x_p),
\end{aligned} \tag{2.7}$$

$$\begin{aligned}
u_i &= \{R(x) - R(x_1, \ldots, x_{i-1}, 0, x_{i+1}, \ldots, x_p)\} \\
&\quad \times \left\{\lambda_i + \sum_{j(\neq i)} \lambda_{ij} x_j + \cdots + \lambda_{12,\ldots,p} \prod_{j(\neq i)} x_j\right\}^{-1}.
\end{aligned} \tag{2.8}$$

(i) $F <_{\mathrm{mc}} G \Leftrightarrow u_i$ is convex in x_i, given x_j's $\forall j(\neq i)$, $i = 1, 2, \ldots, p$
$\Leftrightarrow R(\mathbf{x})$ is convex in x_i, given x_j's $\forall j(\neq i)$, $i = 1, 2, \ldots, p$
$\Leftrightarrow r_i(\mathbf{x})$ is increasing in x_i, $i = 1, 2, \ldots, p$, for all $\mathbf{x}$
$\Leftrightarrow F$ has MIFR distribution.

(ii) $F <_{m^*} G \Leftrightarrow u_i$ is star shaped in x_i given x_j's $\forall j(\neq i)$, $i = 1,2,\ldots,p$
$\Leftrightarrow (1/x_i)\{R(x) - R(x_1,\ldots,x_{i-1},0,x_{i+1},\ldots,x_p)\}$ is increasing in x_i given x_j's, $\forall j(\neq i)$, $i = 1,2,\ldots,p$
$\Leftrightarrow (1/x_i)\int_0^{x_i} r_i(x_1,\ldots,x_{i-1},y,x_{i+1},\ldots,x_p)\,dy$ is increasing in x_i given x_j's $\forall j(\neq i)$, $i = 1,2,\ldots,p$
$\Leftrightarrow F$ has MIFRA distribution.

(iii) $F <_{\text{msa}} G \Leftrightarrow u_i$ is superadditive in x_i given x_j's $\forall j(\neq i)$, $i = 1,2,\ldots,p$

$$\Leftrightarrow R(x_1,\ldots,x_{i+1},x_i + y_i,x_{i+1},\ldots,x_p) + R(x_1,\ldots,x_{i+1},0,x_{i+1},\ldots,x_p) \geq R(x_1,\ldots,x_i,\ldots,x_p) + R(x_1,\ldots,x_{i-1},y_i,x_{i+1},\ldots,x_p), \quad \forall x_i, y_i \geq 0,\ i = 1,2,\ldots,p$$

$$\Leftrightarrow \bar{F}(x_1,\ldots,x_{i-1},x_i+y_i,x_{i+1},\ldots,x_p)\bar{F}(x_1,\ldots,x_{i-1},0,x_{i+1},\ldots,x_p) \leq \bar{F}(x_1,\ldots,x_i,\ldots,x_p)\bar{F}(x_1,\ldots,x_{i-1},y_i,x_{i+1},\ldots,x_p), \quad \forall x_i, y_i \geq 0,\ i = 1,2,\ldots,p$$

$\Leftrightarrow F$ has MNBU distribution.

The proof for dual classes is similar. ■

Remark 2.1: The result of Theorem 2.6 remains the same for the case

$$\bar{G}(x) = \exp\left[-\sum_i \lambda_i x_i\right], \tag{2.9}$$

a special case of (2.6). Sometimes, it works as a natural choice satisfying multivariate lack of memory property in the strongest sense.

3. MULTIVARIATE IFR-NESS

In the univariate case, Barlow and Proschan [4, p. 105] stated as a result for the Weibull distribution that an increase in the value of the shape parameter (>1) increases the IFR-ness of the distribution. Given two multivariate distributions $F(\mathbf{x})$ and $H(\mathbf{x})$, one may similarly like to know which one is more MIFR. For addressing this problem, we recall that a distribution F is more IFR than H if $F <_c H$. We propose a similar definition for more MIFR-ness to keep analogy with the univariate concept.

Definition 3.1: *$F(\mathbf{x})$ is more MIFR than $H(\mathbf{x})$ if we have $F <_{\text{mc}} H$.*

Example 3.1: Let us consider two bivariate Weibull distributions given by survival functions

$$\bar{F}(x_1,x_2) = \exp\lfloor -a_1 x_1^{\alpha_1} - a_2 x_2^{\alpha_2} - a_3 x_1^{\alpha_1} x_2^{\alpha_2} \rfloor, \qquad 0 \leq a_3 \leq a_1 a_2, \tag{3.1}$$

$$\bar{H}(x_1,x_2) = \exp[-b_1 x_1^{\beta_1} - b_2 x_2^{\beta_2} - b_3 x_1^{\beta_1} x_2^{\beta_2}], \qquad 0 \leq b_3 \leq b_1 b_2. \tag{3.2}$$

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Now, if F is to be more BIFR than H, we need to have $F <_{\text{mc}} H$. *Writing* $\mathbf{w} = (w_1, w_2)$ as the vector representation $H^{-1}F(x_1, x_2)$, we have

$$b_1 w_1^{\beta_1} + b_3 w_1^{\beta_1} x_2^{\beta_2} = a_1 x_1^{\alpha_1} + a_3 x_1^{\alpha_1} x_2^{\alpha_2}$$

or

$$w_1 = x_1^{\alpha_1/\beta_1} \left[\frac{a_1 + a_3 x_2^{\alpha_2}}{b_1 + b_3 x_2^{\beta_2}} \right]^{1/\beta_1}. \tag{3.3}$$

Similarly,

$$w_2 = x_2^{\alpha_2/\beta_2} \left[\frac{a_2 + a_3 x_1^{\alpha_2}}{b_2 + b_3 x_1^{\beta_2}} \right]^{1/\beta_2}. \tag{3.4}$$

To ensure that $F <_{\text{mc}} H$, we need to show that w_1 is convex in x_1 given x_2, and w_2 is convex in x_2 given x_1. This necessarily implies that $\alpha_1 > \beta_1$ from (3.3) and that $\alpha_2 > \beta_2$ from (3.4). Thus, for $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, F is more BIFR than H.

This result is also in conformity with the corresponding univariate result that higher the value of the shape parameter, the more IFR is the corresponding Weibull distribution.

Remark 3.1: More MIFRA-ness and more MNBU-ness can be similarly defined in terms of multivariate star-shaped and multivariate superadditive orderings.

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