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New Transformation and Analysis of a N-DOF LINAPOD with six struts for higher accuracy

Tim Boye* and Günter Pritschow

Institute for Control Engineering of Machine Tools, University of Stuttgart, 70174 Stuttgart (Germany)
E-mail: tim.boye@isw.uni-stuttgart.de

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SUMMARY

Parallel kinematic machines (PKMs) and, in particular linapods, are being increasingly used in the industrial workplace. The complex control required for various linapod kinematics, each having different numbers and types of Degrees-of-Freedom (DOF), require corresponding transformations to be generated. This paper introduces a general form of transformation that can be adapted to a wide range of linapods. The approach is illustrated by an example and the concept of a five-DOF linapod for the milling process is proposed. Furthermore, the advantages from the two types of three-DOF Linapods are discussed, and it is shown how the position accuracy can be increased.

KEYWORDS: LINAPODS; 5-DOF; Transformation and analysis; Struts.

I. INTRODUCTION

Since the early 1970s, researchers have investigated the characteristics and abilities of PKMs. Their main structural advantages, compared to serial kinematic machines, are increased stiffness and load carrying capability. These advantages are accompanied by better dynamics due to a lower moved mass. However, these advantages are faced with the limitation of a poor ratio between the installation space to workspace. This mainly results in collisions of the moving elements.

One group of PKMs have struts with a fixed length. Moving the mounting point of these struts effects the movement of the platform. These PKMs are known as linapods and are discussed in this paper. Linapods can be found with different DOF.

Three-DOF linapods, those with a Prismatic-Spherical-Spherical (PSS) configuration, have been described in several papers. These machines are pure translational linapods, realized by using pairs of parallel struts to avoid the tilting of the platform. The V100[1] is a three-DOF industrial turning machine and the URANE SX[1] (Renault Automation), the QUICKSTEP[2] (Krause & Mauser) and the Pegasus[3] are examples of milling machines used in industry. Another linapod, the Prismatic-Rotational-Universal (PRU) three-DOF Ecospeed, is introduced in reference[4], and has one translational DOF and two rotation DOFs.

A four-DOF linapod with six struts is introduced in reference [5], which is able to move in three translational directions and one rotational direction. Finally, there are several six-DOF linapods that are designed and realized in several laboratories. These are, for example, the Dynamil[6], the Paralix[7] and the Hexaglide.

For each linapod with a different number of DOF, a special transformations has to be calculated. This is especially true for those that are based on kinematic constraints, as these are controlled by complex calculations that have to be formulated for each linapod. In reference [8] a transformation for a three-DOF linapod is explored, which can only be used with this single linapod, having one translational and two rotational DOFs. A transformation will now be presented that is capable of controlling all of the above mentioned linapods. This transformation also provides the possibility to control a five-DOF linapod and will be more precisely explained.

II. THE BASIC ANALYSIS OF THE LINAPOD

Many of the mentioned types of linapod have in common the fact that they have six struts. The linapods with three legs, like the Ecospeed, can also be expressed as a special form of a six-legged linapod with two struts representing one leg. The main difference is the number of drives, which is equal to the number of DOFs.

To design and analyze linapod kinematics a general transformation, accompanied by a set of analysis tools, has to be available. These can be achieved by the following approach:

The inverse transformation is the operation that can be used to calculate from a known tool center point (TCP) the position of the corresponding drive positions. For linapods, it can be calculated by closing six vector chains which are declared as a drive loop and illustrated in Fig. 1. A single drive loop consists of a position vector of the moving platform, a position vector of a single platform joint, a position vector of the drive direction's mounting point, a drives direction vector and a known length of the strut

$$l_i^2 = (\vec{\mathbf{k}}_i + \vec{\mathbf{e}}_i \cdot q_i)^2 \quad l_i^2 = (\vec{\mathbf{k}}_i + \vec{\mathbf{e}}_i \cdot q_i)^2. \tag{1}$$

When equation (1) is solved, it leads to two solutions:

$$q_{1,2} = -\vec{\mathbf{k}}_i^T \vec{\mathbf{e}}_i \pm \sqrt{\left(\vec{\mathbf{k}}_i^T \vec{\mathbf{e}}_i\right)^2 - \vec{\mathbf{k}}_i^T \vec{\mathbf{k}}_i + l_i^2}. \tag{2}$$

* Corresponding author. E-mail: tim.boye@isw.uni-stuttgart.de

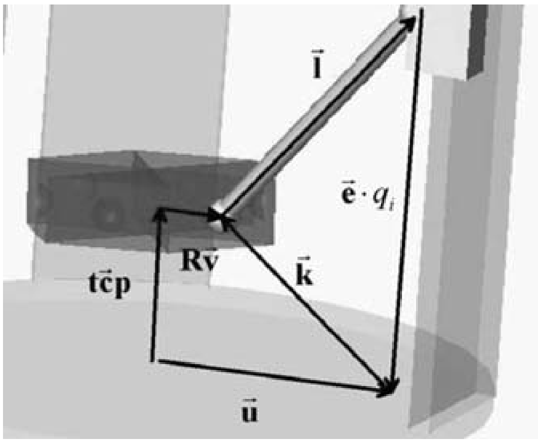

Fig. 1. Vector chain of a drive loop.

This equation is solvable in an analytical way, and for this it is also possible to differentiate it in an analytical way. The partial differentiation of the drive coordinates with respect to the generalized coordinates is called the inverse Jacobian matrix. It is defined as:

$$\mathbf{J}^{-1} = \frac{\partial q}{\partial x}. \tag{3}$$

And can be calculated by partial differentiation:

$$\begin{aligned}\mathbf{J^{-1}} &= -\frac{\partial \mathbf{k}_i^T}{\partial x_i}\vec{\mathbf{e}}_i \pm \frac{2\left(\vec{\mathbf{k}}_i^T\vec{\mathbf{e}}_i\right)\frac{\partial \vec{\mathbf{k}}_i^T}{\partial x_i}\vec{\mathbf{e}}_i - \frac{\partial \vec{\mathbf{k}}_i^T}{\partial x_i}\vec{\mathbf{k}}_i - \vec{\mathbf{k}}_i^T\frac{\partial \vec{\mathbf{k}}_i}{\partial x_i}}{2\sqrt{\left(\vec{\mathbf{k}}_i^T\vec{\mathbf{e}}_i\right)^2 - \vec{\mathbf{k}}_i^T\vec{\mathbf{k}}_i + l_i^2}} \\ &= \begin{bmatrix} \frac{\partial q_1}{\partial x_6} & \cdots & \frac{\partial q_1}{\partial x_6} \\ \vdots & \ddots & \vdots \\ \frac{\partial q_6}{\partial x_6} & \cdots & \frac{\partial q_6}{\partial x_6} \end{bmatrix}.\end{aligned} \tag{4}$$

The inverse Jacobian Matrix can be used either for the iterative forward transformation, or for the assessment of a kinematical design. For design decisions either the eigenvalues are calculated or the singularity values of the Jacobian matrix are calculated. These are defined as:

$$\sigma = eig(\mathbf{J}^{-1}\mathbf{J}^{-1T}). \tag{5}$$

The higher the quotient is of the smallest and highest singularity values, the better conditioned the linapod kinematic is. This is caused by the fact that the kinematic is not close to a singularity. This will happen when a singularity value is zero or infinite.

III. PROPOSAL OF A NEW TRANSFORMATION FOR THE DRIVE COMBINATION

In the following we present the transformation in which several drive pairs can be combined so that two struts are mounted on a single drive. Carrying on with this concept the normal six-DOF linapod can be reconfigured to combinations with less than six-DOF, all of which have the same transformation. Schematic examples are shown in Fig. 2. The figure

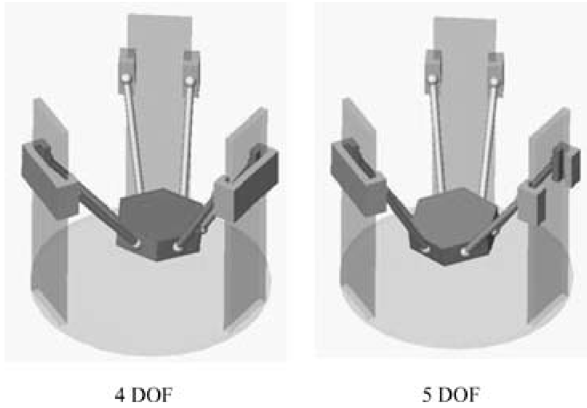

Fig. 2. Linapods with different DOF.

shows that the system's structure itself does not change dramatically. However, for each DOF less than six, the inverse transformation gains one more constraint. This results in the TCP having one less DOF. For each constraint one DOF of the TCP has to be fixed. The calculation of the TCP's-fixed DOF, is the key issue of the drive combination transformation.

For a linapod with less than six-DOFs, a constraint applies that at least two drives have to be connected together. Thus, the distance between the drives (i,j) has to remain constant. With this constraint fulfilled, the two drives can be treated as a single drive. The constraint can be declared as:

$$q_i - q_j = const = \Delta q_{ij}. \tag{6}$$

In order to satisfy this constraint, one DOF of the TCP is no longer controllable. Instead a translational or rotational position of the DOF is dictated. One way to describe the TCP position in space is to declare its Euler-coordinates (x,y,z,α,β,γ). By combining two struts on one drive, the k-th DOF is directly related to the constraint (6). It can be calculated by the above mentioned (3) Jacobian matrix:

$$x_k = x_k + (q_i - q_j)\left(\frac{\partial x_k}{\partial q_i} - \frac{\partial x_k}{\partial q_j}\right). \tag{7}$$

The differentiation $\frac{\partial x_k}{\partial q_i}$ of the chosen DOF with respect to the drive coordinates must be calculated by inverting the terms from the analytically calculated inverse jacobian matrix $\mathbf{J}_{k,i}^{-1}$ and $\mathbf{J}_{k,j}^{-1}$.

Considering that more than one strut pair should be attached to a drive, (6) and (7) have to be expanded to tensors:

$$\mathbf{x_k} = \mathbf{x_k} + \Delta q_{i,j}\frac{\partial(\boldsymbol{\Delta} x_k)}{\partial(\boldsymbol{\Delta} q_{i,j})} \Rightarrow \mathbf{x_k} + \Delta\mathbf{q_{i,j}}\,{}^k\mathbf{D}^{-1} \text{ for } k = 1..3. \tag{8}$$

The deviation Jacobian matrix $\mathbf{D}$ is calculated by subtracting the Jacobian i-th row from the j-th row in the k-th column. This forms a square $k \times k$ matrix:

$${}^k\mathbf{D} = \begin{bmatrix} \frac{\partial q_{i,1}}{\partial x_1} - \frac{\partial q_{j,1}}{\partial x_1} & \cdots & \frac{\partial q_{i,1}}{\partial x_1} - \frac{\partial q_{j,1}}{\partial x_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial q_{i,k}}{\partial x_k} - \frac{\partial q_{j,k}}{\partial x_k} & \cdots & \frac{\partial q_{i,k}}{\partial x_k} - \frac{\partial q_{j,k}}{\partial x_k} \end{bmatrix}^{k\times k} \quad \text{for i, j} = 1\ldots k. \tag{9}$$

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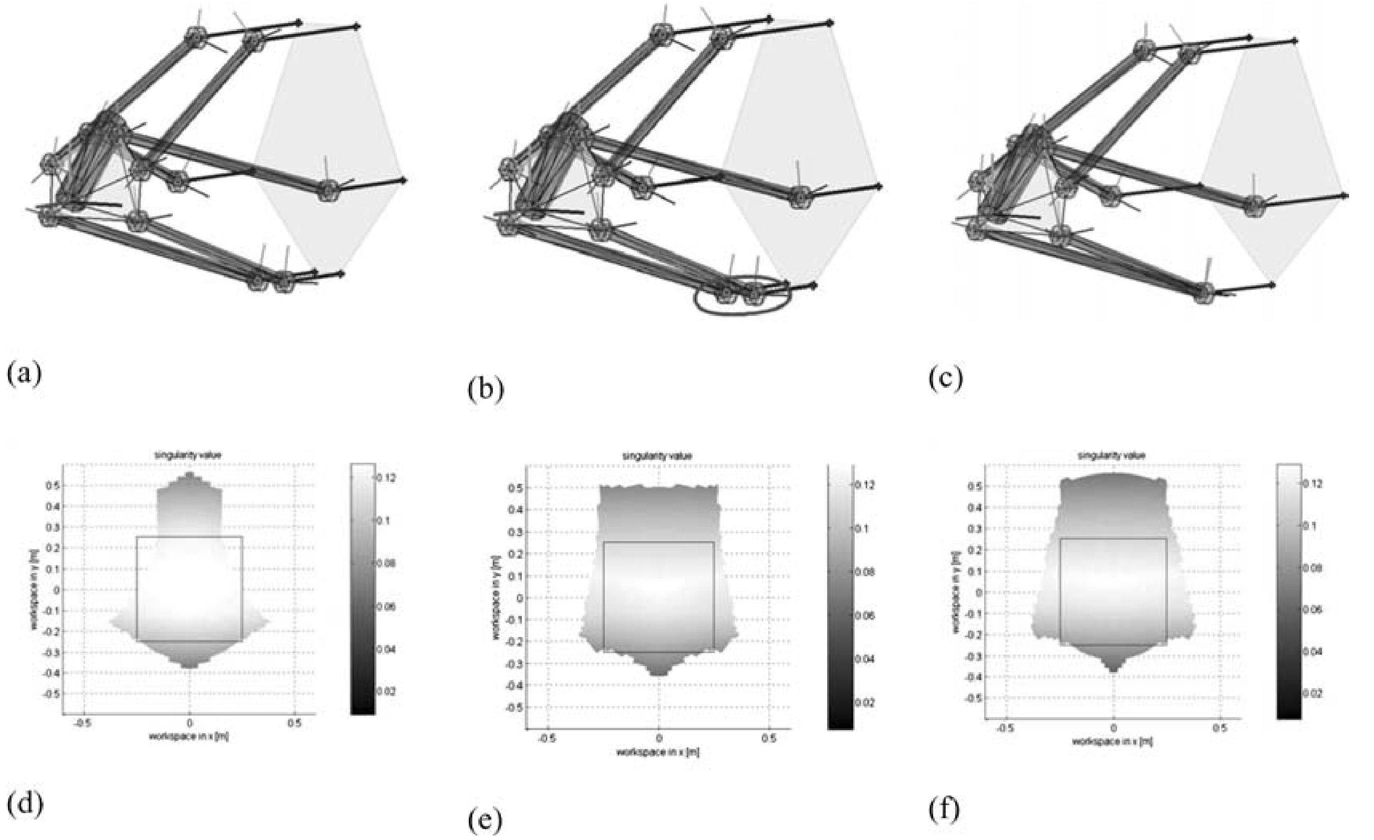

Fig. 3. Six-DOF Linapod versus two five-DOF Linapods.

The inverse transformation (3) can be expanded iteratively by this process. By using this convention, a single DOF can be freely chosen and fixed. These fixed DOFs of the TCP are generally not zero. To reduce the number of iterations per cycle, which is of relevance to control systems, the current position vector can be used as the start vector for the following iteration. This method help to ensure real-time performance.

The calculation of the singularity values is no longer possible using the inverse Jacobian matrix as described in (4), because all coordinates of the TCP position are now dependent on one another. However, the reduced Jacobian, which is defined as:

$$\mathbf{J}_r = \frac{\partial x}{\partial q_r} \tag{10}$$

can be calculated. The reduced Jacobian is, in general terms, a $6 \times n$ matrix. The term ∂q_r is the number of reduced drives. The matrix itself can be calculated by using the inverse of (4) of the non-reduced system and then adding the i-th and j-th columns of the combined drive joints:

$$\mathbf{J}_r = \lfloor(\mathbf{J}_{i,n} + \mathbf{J}_{j,n})\mathbf{J}_{k \neq ij,n}\rfloor \text{for } \; k = 1..6 \cap n = 1..6. \tag{11}$$

The reduced Jacobian matrix is not square anymore. This is caused by the reduced number of drives. It is still possible to calculate the singularity values of this matrix as in (5), because the quotient of the smallest and largest singularity values of an inverse Jacobian matrix is equal to that of the original matrix. The quotient provides the magnitude of the velocity ratio. For each combined drive pair the number of singularity values is reduced by one. This also means that the number of possible singularities is reduced. For thats reason it is worth decreasing the number of DOFs at the TCP that are required for the desired manufacturing process.

IV. EXAMPLE OF FIVE-DOF MANIPULATOR

Based on the principle that the milling process is rotationally symmetric, the kinematic structure moves the cutter in three translational directions (x, y, z), and two rotational directions (α,β). Using the introduced transformation, it is possible to use a five-axes linapod for this process. This configuration is evidently well suited to combine two legs on one drive. In Fig. 3 three different configurations can be seen. The first configuration (a) is based on the classic linapod. All its legs are mounted on separate drives and enables the platform to be move freely throughout the entire workspace. The second combination (b) is similarly configured as in (a), with the only difference being that the lower two legs are both mounted on one drive. In the third combination (c), the combined legs are mounted on the drive at the same point. This means that both legs must now be considered as a single part. The two combined legs form now a PUR configuration but does not influence the general transformation. In Fig. 3(d, e, f), below each configuration, the x-y plane of the usable workspace can be seen, and the quotient of the singularity value is plotted. A value of zero could have been caused by a singularity or by the workspace being bounded by a collision of the moving elements. For each point in the x-y plane the spindle was tilted

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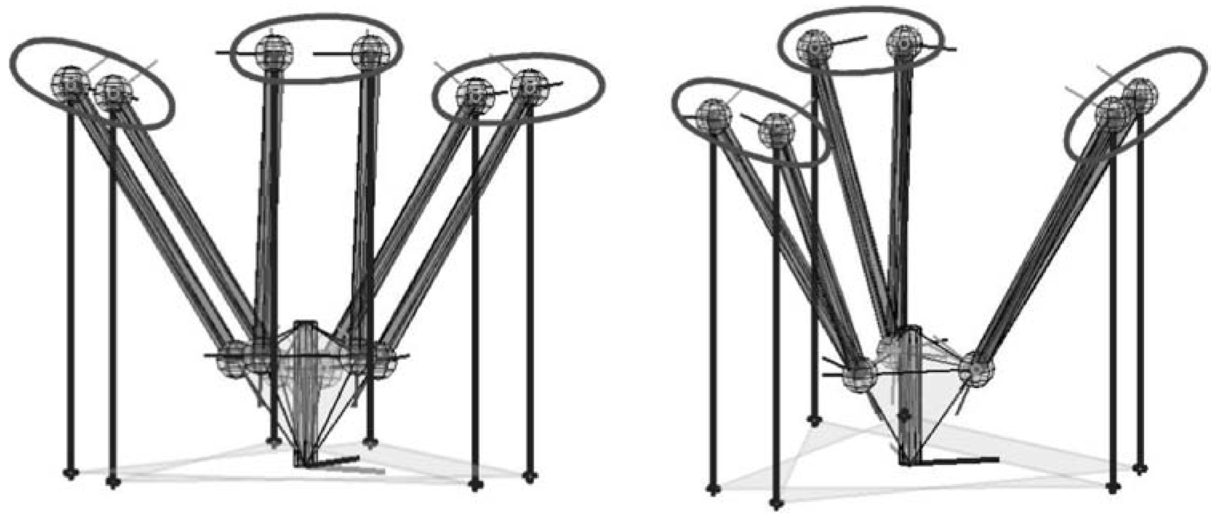

Fig. 4. Two types of three DOF Linapods (a) three translational DOF (b) one translational two rotational DOF.

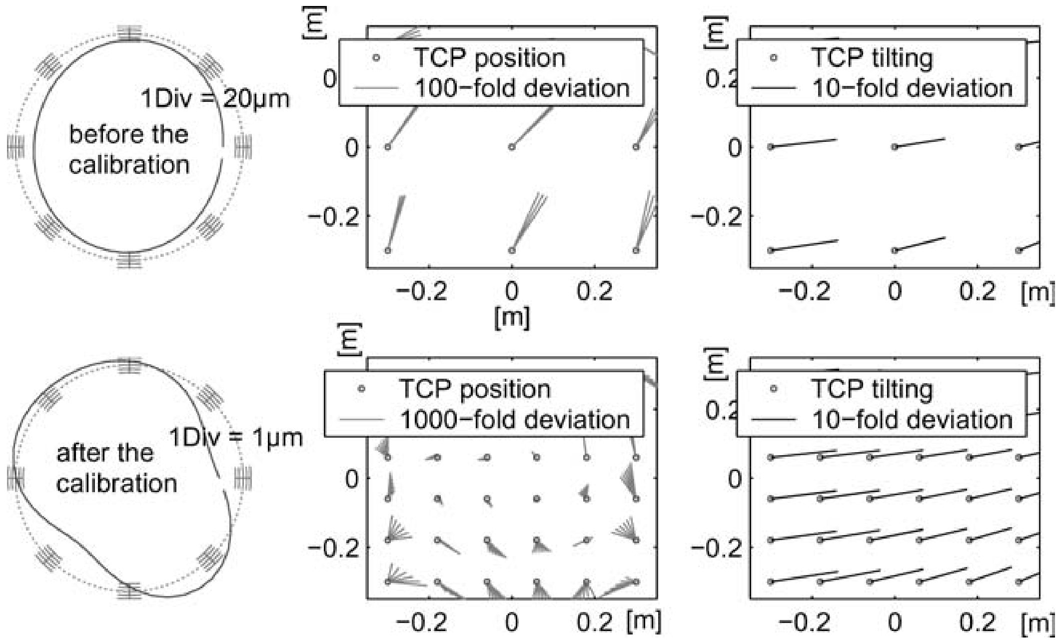

Fig. 5. Calibration of a tripod with a standard transformation.

from 0 to 90° about the x-axis, and the smallest quotient was plotted. The result of this particular investigation shows that not only the workspace enlarges by combining two legs on one drive, but also the area of well conditioned singularity values is enlarged. It follows that the correction angle around the z-axis of the spindle does not have a negative effect on the behavior of the linapod. It has to be mentioned that a six-axis machine can be controlled in a manner to achieve the same results as configuration (b), but the sixth drive is not necessary for these results. By combining the two legs into one, as in configuration (c), and mounting it on one drive, we can achieve an even better result. This result can be seen in Fig. 3(f). The area of well conditioned singularity values is again enlarged compared to the combination (b) with the best quotient of the singularity values rising to 0.27. This combination cannot be realized with a six-axis machine, because both joints must be mounted at a single point.

V. EXAMPLE FOR THREE-DOF MANIPULATOR

The three-DOF configurations is much more common in an industrial environment. Two types of three-DOF manipulators were described in the introduction Figure 4. In terms of control, they both have completely different transformations. We will now describe some of their characteristics. The Tripod (Figure 5a) achieves its pure translational movement by mounting three pair of struts on three drives, with the kinematical constraints, that all of these pairs must be parallel to each other. As a result, the tolerances have to be very tight and, furthermore, during the installation one of the joints has to be able to be adjusted in order to align the struts. Last but not least, it will not be possible to completely fulfil all the constrains. Using the standard transformation for the Tripod, it is assumed, that the platform will never tilt or yaw. The designer is not aware of how different tolerances will effect the swivelling of the TCP.

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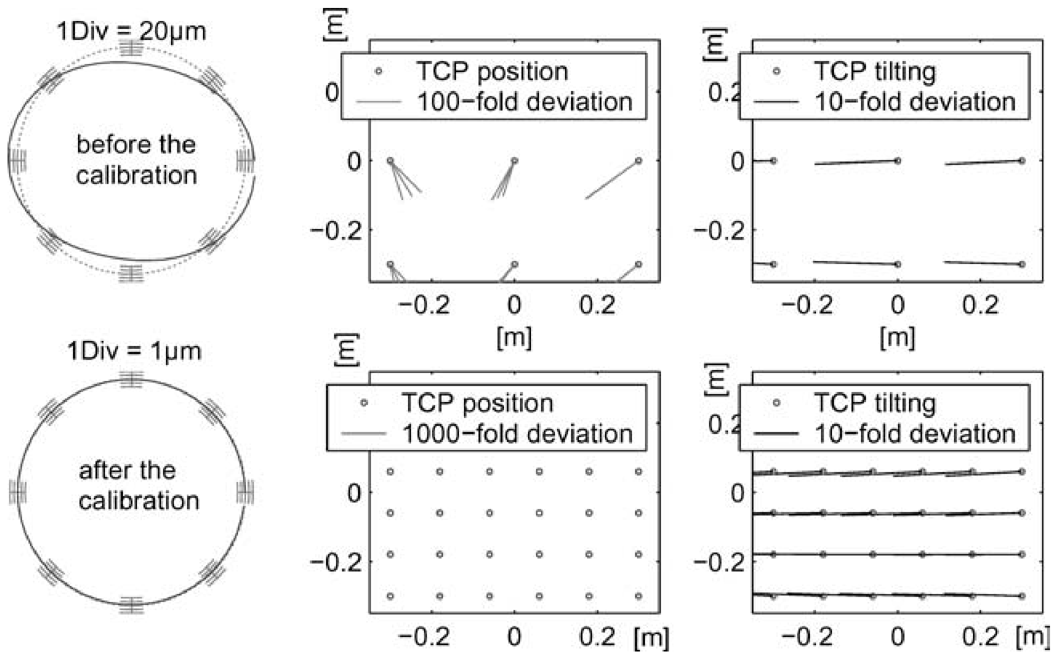

Fig. 6. Simulated calibration of a tripod with the new transformation.

Using the introduced transformation, a direct yaw and tilting angle for the chosen tolerance of kinematical errors will be obtained according to the TCP position. This can be used to find useful values for the tolerances, as we can directly interpret their influence on the TCP. On the other hand, using the transformation in the control system, it will be possible to determine the yaw and tilt angles of the platform. Of course, the transformation cannot prevent the swivelling of the platform, but it will reflect the system better than the standard transformation, which is based on pure translational movement.

A common way to identify the kinematical parameters is done by calibration as introduced in reference [3]. The calibration is based on a measurement of the TCP and of the drive positions in special positions. The error of the calculated TCP position and the one of the real system, must be compensated by the calibration algorithm. This means that the used transformation is able to reflect the real system. In Figures 5 and 6 the results of calibrations with the standard transformation and the new transformation can be compared. The circular test shows that the standard transformation is able to reduce the error, but it is not able to compensate it completely, whereas the new transformation could compensate the position errors completely (Figure 6). In the very left plots of Figures 5 and 6 the tilting errors can be seen, which are not possible to compensate. These errors can only be reduced by better manufactured tolerances.

The transformation is also very useful for the 3 DOF kinematics which can be seen in Figure(b). It is used, for example, as an additional processing head in five-axis milling machines for pocket milling. With this configuration, it is possible to move the platform in the z-Direction and tilt it about the A and B axis. However, tilting and yawing of the platform is always combined with a translational movements in x- and y-directions. These have to be compensated for by the serial kinematics of the base. For this purpose it is not necessary nor possible to prevent the translational movement, but it is evidently important to know the exact translational movement. To calculate this motion of the Linapod platform, the dimentions of the structure have to be precisely known. The current way is to use a simplified transformation, which is built on symmetric conditions of the kinematic as introduced in reference [8]. Therefore, small deviations of the symmetry cannot be accounted for by the transformation. This can only be done by the presented transformation which calculates the constraints.

For the mentioned example, the variation of the identified parameters will reflect those of the real system. With these parameters the actual position of the swivelling head can be calculated more exactly.

VI. CONCLUSION

In this paper a method was introduced which makes it possible to calculate a transformation for all linapod structures based on six legs. The number of DOFs of the entire kinematics can be reduced from six to five, four or three DOFs simply by mounting pairs of struts on a single drive. It is shown that special five DOF configurations have no disadvantages compared to the equivalently configured six axes linapods. Moreover the drive combination positively affects the workspace and singularity behavior.

For special combinations, like the mentioned five DOF parallel kinematics, the combining of struts to a single drive not only reduces the number of drives needed, but also positively effects the singularity conditions of the workspace. For three DOF kinematics the introduced transformation is able to reflect the real kinematics much better than a simplified symmetrical model. Manufacturing and assembling errors can be accounted for much better than in the actual standard model.

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